Is the Doubly Special Relativity Theory Necessary?

Absolute Maximum Speed of Light

Implies a Planck Length and a Planck Time

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Abstract

Giovanni Amelino-Camelia (2002) has proposed a theory whose hope (should it be confirmed by experiments) is to supersede Einstein's 1905 Special Theory *of* Relativity (STR). This theory is known as the *Doubly Special Relativity* (DSR) and it proposes a new observer-independent scale-length. At this scale, it is agreed that a particle that has reached this scale-length, has entered the Quantum Gravity regime. According to the STR, observers will – *in principle*; not agree on whether or not a particle has reached this length hence they will not agree as to when does a particle enter the Quantum Gravity regime. This presents the STR with a "paradox". Amongst others, the DSR is fashioned to solve this "puzzle/paradox". We argue/show here, that the STR already implies such a scale-length – it is the complete embodiment of the STR, thus we are left to excogitate; "Is the Doubly Special Relativity theory necessary?".

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"*Doubt everything or believe everything: these are two equally convenient strategies. With either, we dispense with the need for reflection.*"

– **Jules Henri Poincaré** (1854 – 1912)

1 Introduction

In 2002, Giovanni Amelino-Camelia of the University of Rome in Italy proposed a revision of Einstein's sacrosanct Special Theory *of* Relativity (Amelino-Camelia 2002*a*, *b*) by adding to it, a universal absolute minimum length (ℓ_p). The proposal by Amelino-Camelia is popularly known

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as the Doubly Special Relativity (DSR) theory. So, to the already well established absolute universal constant – the sacrosanct speed of light $c = 2.99792458 \times 10^8 ms^{-1}$, Giovanni Amelino-Camelia added a second, thus his theory contains not one, but two absolute universal constants $(c,\ \ell_p)$. Because the theory has two universal absolute constants, Giovanni Amelino-Camelia dubbed it *"Doubly Special"* hence the name Doubly Special Relativity.

Without the understanding that we shall provide in the next section, Giovanni Amelino-Camelia's theory has just reasons for its existence – it is "well" founded. Giovanni Amelino-Camelia choose the scale $\ell_p \simeq 10^{-35} m$ to coincide with the Planck scale and the reason will be made clear. The Planck scale can be thought of as the minimum possible separation between any two points in space. *Viz* Quantum Gravity (QG), most if not all [researchers in this field] will agree that there is a special scale – the Planck length ℓ_p, at which quantum gravitational effects will become so strong that a fully-fledged theory of QG must be used to describe the physics. That is to say, when objects – say a star, were to shrink down to the Planck length, it is expected that at this length, a fully-fledged theory of QG must takeover in order to describe the physics thereof. The Planck scale – as is argued in the next paragraph, possess a "puzzle/paradox" for the STR – this is only a puzzle/paradox if one is without the understanding provided here.

According to the STR, different observers (depending on their state of motion) will measure different lengths, thus they will (may) not agree on whether or not a particle has reached its Planck length since this is dependent on their relative state of motion. The just said, is the puzzle/paradox that the STR faces as an object approaches the Planck scale. If they agreed on the Planck scale, then their motions must be similar (that is to say, the relative velocity between them must be zero). If their motions are dissimilar (that is to say, the relative velocity between them is non-zero) and they agreed on the Planck scale, it would mean the Laws *of* Physics must be different for different observers – this goes against the very foundations of the STR. To solve this, Giovanni Amelino-Camelia proposed his DSR theory which has been welcome by a significant number of researchers (see *e.g.* Kowalski-Glikman 2003; Magueijo & Smolin 2002*a*, *b*: amongst many others).

We have known of Giovanni Amelino-Camelia's theory and actually thought of it as a brilliant solution to this apparent paradox until (while conducting research which is unrelated to the DSR) we figured out that the STR (*via* the absolute universal constant c) already implies the existence of such a minimum length! In the next section, we shall advance the thesis leading to this rather surprising result that the STR implies an absolute minimum length. This thesis is so simple and trivial one would easily not (even) think of it. This perhaps explains why such a thesis is not found in the literature.

2 Light-Speed Implies Lower Space and Time Limits

In this part of the reading, we establish lower space and time limits on spacetime. To achieve this, we use the simple and well accepted Law *of* Nature that the speed of light, c, is an upper absolute speed limit for all material bodies and energy in the Universe. Considering the case of motion in one dimension say along the $x-axis$, if a particle happens to be at a point x_1 at time t_1 and at a later time $t_2 > t_1$ this particle is located at x_2, we know that the speed V of this particle is given:

$$V = \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{x_2 - x_1}{t_2 - t_1}\right|. \tag{1}$$

It is clear from the above that if there exists no limits on the intervals $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$, that particle's speed will range from zero to infinity. That is, for any finite duration $\Delta t > 0$ for which $\Delta x = x_2 - x_1 = 0$, we will have $V = 0$ and for any finite separation $\Delta x > 0$ for which

$\Delta t = t_2 - t_1 = 0$ we will have $V = \infty$, hence: $0 \leq V \leq \infty$. So far, so good, no problem – lets proceed!

The fact that there is a maximum absolute speed c implies that $t_2 - t_1 \neq 0$ thus there must exist a minimum time interval. If we set this minimum time interval to be (say) t_p, such that for all $t_2 > t_1$, $\Delta t > t_p$ where t_p is smallest possible interval of time that can occur between any two events; then, for any space interval $\Delta x = x_2 - x_1$ separating these two events, there will exist a maximum speed for that particular space interval, let us write this as $V_{max}(x_2, x_1)$, and this will be given:

$$V_{max}(x_2, x_1) = \frac{|x_2 - x_1|}{t_p}. \tag{2}$$

Now, if there exists a minimum distance that any two points can ever come closest; that is, the points x_2 and x_1 can be brought closer together up until a certain minimum, call this minimum, call this minimum ℓ_p, then, we can talk of an absolute maximum speed, $V_{amax}(x_2, x_1)$, between the two-points. This absolute maximum speed, call it c, is, unlike $V_{max}(x_2, x_1)$, independent of the coordinates hence thus for any object moving in such a spacetime:

$$V < c = \frac{\ell_p}{t_p}. \tag{3}$$

Any abject that travels at this speed c is basically traveling the minimum possible distance in the least possible time duration or it travels an integral multiple ($n\ell_p : n = 1, 2, 3...$) of this distance in an integral multiple time of the least time ($nt_p : n = 1, 2, 3...$). From the above thesis, what this means is that spacetime must have space and time limits if it is to have a universal and absolute maximum speed; *i.e.*, for any two points x_2 and x_1 and any two-points on the time-axis t_2 and t_1 the following must hold: $x_2 - x_1 \geq \ell_p$ and $t_2 - t_1 \geq t_p$. This simple reasoning, is all that there is to it. The STR implies a minimum possible time and a minimum possible length!

Lets reason differently. The fact that there exists a maximum speed c implies there can be no object that can move from any two points in zero time interval – this is a bare and natural fact emerging from the constraint of a maximum absolute speed. The just said directly points to the undeniable fact that there must always be a finite duration in the time interval when a material object is moved from point x_1 to x_2, otherwise there won't any such phenomena as an absolute maximum constant speed. If there must be a finite duration, then, there must exist the least possible time interval because this time interval can not be infinitely small. By infinitely small we mean this. For example, the first greatest number after zero is $0.\dot{0}1$ where the $\dot{0}$ represents an infinite number of zeros; the number $0.\dot{0}1$ is infinitely small. The least possible time interval can not take this value or any infinitely small number. The number of zeros between the comma and the first significant figure must be finite – *i.e.*, $0.[\text{finite number of zeros}]ABC...$ where $A \in [1, 2, 3, 4, 5, 6, 7, 8, 9]$ is the first significant figure and $B, C, ... \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]$.

Now, the existence of a least time interval coupled with the existence of an absolute maximum speed implies there must exist a least possible space interval given by ct_p. This is a mathematical fact! From this, clearly, the very existence of an absolute maximum speed means there must exist an absolute least time interval and an absolute least space interval. The question naturally arises, "Is the DSR theory really necessary?" The DSR theory exists and firmly stands on the premises that the STR does not have within it an absolute least time and or distance. We shall leave this to the reader to decide for themselves.

Before leaving this section, we would like to conclude this section by noting and highlighting something interesting?! It is clear from the above that when time moves from one moment to the next, it must do so in intervals. These intervals ought to be the smallest possible and must not be infinitely small. This suggests, that time must itself be quantized in intervals of t_p. That is, the time evolution of an event from $t = 0$ must follow the sequence, $t = 0t_p \mapsto 1t_p \longmapsto 2t_p \mapsto 3t_p \mapsto$

$4t_p \mapsto etc$. This points directly to the idea that spacetime must be quantized in small spacetime volumes of $ct_p\ell_p^3$ and that spacetime should consist of Planck nodes, *i.e.*, the grid of spacetime must be such that all the points on this grid are separated by the distance $\ell_p = ct_p$.

3 Discussion and Conclusion

In our modest opinion – we believe and hold that, in the light of the presentation made here; the DSR theory is not necessary as it is founded on soils that are wholly part and parcel of the provincial soils of the STR. Simple said, the DSR has not gone any further than the STR if its bedrock is the existence of a minimum length because this is implied by the STR. We simply have lived without being aware of this simple, bare, basic and natural fact directly emergent from the implied existence of the upper sacrosanct speed limit c.

The implied existence of t_p and ℓ_p further implies that spacetime is quantized! If our thesis is correct, as we believe it is, then, it gives researchers seeking a theory of Quantum Gravity, ***solid reasons*** to do so because their reasons are founded on a founded theory – the STR. While this is good news, there is some uncomfortable news as-well.

Given that $V < c$, it means the length of a ponderable material object (*i.e.*, an object of none-zero rest-mass), l, will never actually reach the minimum length but only approach it asymptotically, *i.e.*: $l \longmapsto \ell_p$. Thus if QG is only attained at this length ℓ_p, then for any material particle, this regime is unattainable hence QG must be unattainable. Surely, QG must exist, thus the belief that it (QG), is only attained at this critical scale must be put to question. Perhaps, since spacetime must be quantized as argued above – maybe, a QG theory is one in which the spacetime continuum is quantized.

In closing, allow me to say that, given the simplicity of the arguments presented herein and the magnitude of the implications drawn from them, and more so that the literature appears to be devoid of this kind of argument/thesis whose implications is so rich; it may well be that we are all wrong – *we have misunderstood the facts at hand*. Our strong convictions emerging as a result of going through this many times is that we are on the correct path – we have not errored. To allow for that minute and small chance that we may be all-wrong in our thinking, we leave our reader to be the judge and if we have errored, let it be taken or known as just a slip not of the tongue but of the mind, let it pass quietly without due notice with the simple remark that "It is but just another of those common human errors that occur on our journey to fathom the inner and outer workings of *Nature*".

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