On Representing a Particle by a Standing Luminal Wave-II -Incorporating Spin

V.A.Induchoodan Menon,
9, Readers' Row Houses,
Gujarat University,
Ahmedabad-380009, Gujarat, India.
e-mail:induchoodanmenon@yahoo.co.in

Abstract

The author extends his approach which treats the elementary particle as a standing luminal half wave to the 3-dimensional situation incorporating spin. It is shown that the function representing the circularly polarized standing electromagnetic half wave is a solution to the Dirac equation and the two positive energy solutions turn out to be formed by the forward and the reverse luminal half waves belonging to two different spin states of the standing wave. In the process, the author presents the physical picture behind the spinor representation of the particle. The standing wave structure formed by the circularly polarized luminal wave offers a simple explanation for the "zitterbewegung" undergone by the electron. Besides, this structure of the half spin particle offers a simple but elegant explanation for the Pauli's exclusion principle.

PACS number: 03.00, 03.30, 03.65-w

1 Introduction

We know that the plane wave which represents a particle could be formed by the confinement of a luminal wave and the standing wave formed by such a confinement acquires rest mass [1]. The spatial component of the standing wave when given a translational velocity was seen to get converted into the amplitude wave which gets compacted into the internal coordinates while the time dependent component was seen to get converted into the plane wave that represents the particle in the external coordinates. In this paper, we shall take up the case where the standing wave is formed by the confinement of a circularly polarized luminal (CPL) wave. The obvious choice for the CPL wave is the circularly polarized electromagnetic wave. The idea that the electromagnetic wave forms the most basic structure of a particle is quite appealing in the light of the fact that the end products of the particle anti-particle collision are only high energy photons. However, such an approach attributing electromagnetic basis to the structure of particles may be alright so far as we confine ourselves to the electrons. But in the case of the particles like quarks which are subject to strong interactions, it may be quite inappropriate to attribute the sub-structure entirely to the electromagnetic waves. This is an issue which calls for a deeper study. For the moment we shall assume that the CPL wave stands for the electromagnetic wave and the particle we are dealing with is the electron.

Once we assume that the CPL wave stands for the electromagnetic wave, then, we may represent the standing wave formed by it having translational velocity v along the x-axis by [1].

$$\phi^R \quad = \quad 2\left\{\xi_y \cos\left[\text{þ}c\left(t'-x'/v\right)/\hbar\right]+\xi_z \sin\left[\text{þ}c\left(t'-x'/v\right)/\hbar\right]\right\}\ e^{-i\hbar^{-1}(Et-\text{þ}x)}. \qquad (1)$$

Note that ϕ^R represents a standing wave which is rotating in the clockwise direction and therefore we may treat it as the right handed wave with positive helicity. The corresponding left handed wave with negative helicity will be given by

$$\phi^L \quad = \quad 2\left\{\xi_y \cos\left[\text{þ}c\left(t'-x'/v\right)/\hbar\right]-\xi_z \sin\left[\text{þ}c\left(t'-x'/v\right)/\hbar\right]\right\}\ e^{-i\hbar^{-1}(Et-\text{þ}x)} \quad . \qquad (1A)$$

Here the two terms within the bracket represent the amplitude wave which we know can be shifted to the internal coordinates. Taking $\xi_y = \xi \cos \varphi$ and $\xi_z = \xi \sin \varphi$, we have

$$\begin{aligned} \phi^R &= 2\,\xi \cos\left[\mathrm{þ}c\left(t' - x'/v\right)/\hbar - \varphi\right] e^{-i\hbar^{-1}(Et - \mathrm{þ}x)} \\ &= 2\,\xi \cos\left[\mathrm{E}\left(x' - vt'\right)/\hbar\, c + \varphi\right] e^{-i\hbar^{-1}(Et - \mathrm{þ}x)} \,. \end{aligned} \quad (2)$$

Note that since φ is chosen arbitrarily, we may drop it conveniently in which case the helicity state gets suppressed and the standing wave could be expressed in the exponential form as

$$\phi = \xi\left[e^{iE(x'-vt')/\hbar c} + e^{-iE(x'-vt')/\hbar c}\right] e^{-i\hbar^{-1}(Et - \mathrm{þ}x)} \,. \quad (2A)$$

We should keep in mind that (2A) is nothing but (1) from which the helicity aspects are suppressed. The first and second terms in (2A) represent respectively the forward and the reverse waves constituting the standing wave with unspecified spin. As already discussed in the earlier paper [1] the phase function of the amplitude wave could be switched over from the internal coordinates to the external coordinates whenever we want. This is because the wave under study is taken to be localized in the external coordinate at the point $\widehat{x} = v\widehat{t}$ [1]. Therefore, the phase of the amplitude wave $\left[E\left(x' - vt'\right)/\hbar c\right] = \left[E\left(\widehat{x} + x' - v\widehat{t} - vt'\right)/\hbar c\right] = \left[E\left(x - vt\right)/\hbar c\right]$ where $x = \left(\widehat{x} + x'\right)$ and $t = \left(\widehat{t} + t'\right)$ are the external coordinates.

Till now we have been dealing with the one-dimensional case with the standing wave confined along one direction by a pair of perfect mirrors facing each other. If we have to deal with particles existing in the 3-dimensional space, then we will have to consider the confinement of the CPL wave within a hollow sphere with perfectly reflecting inner surface. Let us now take the case of a CPL wave with unspecified helicity travelling in the general direction of **r** within the hollow sphere. In that case the standing wave formed by the CPL wave can be expressed as a linear combination of two plane polarized standing waves with their planes of polarization perpendicular to each other given by

$$\phi = \boldsymbol{\xi}\left(e^{-i\hbar^{-1}(E't - \mathbf{p}'.\mathbf{r})} + e^{-i\hbar^{-1}(E''t + \mathbf{p}''.\mathbf{r})}\right) + i\boldsymbol{\xi}_\perp\left(e^{-i\hbar^{-1}(E't - \mathbf{p}'.\mathbf{r})} - e^{-i\hbar^{-1}(E''t + \mathbf{p}''.\mathbf{r})}\right). \quad (3)$$

Here E′ and **p′** represent the energy and momentum of the forward wave and E″ and **p″** those of the reverse wave. Further, the vectors $\boldsymbol{\xi}$ and $\boldsymbol{\xi}_\perp$ denote the amplitude of the waves which are orthogonal to the direction of propagation while being perpendicular to each other. Let us now take the case of the standing wave in a general direction in the rest frame of the hollow sphere. We know that to form a standing wave, the values of $p^o{}_x$, $p^o{}_y$ and $p^o{}_z$ will take the values

$$\lambda^o_x = h/p^o_x = 2d/l\,; \quad \lambda^o_y = h/p^o_y = 2d/m \quad and \quad \lambda^o_z = h/p^o_z = 2d/n\,. \quad (4)$$

where l, m and n are integers, while d denotes the diameter of the sphere. We shall confine ourselves to the standing wave having the lowest energy which is is the basic harmonic. It will be shown later that the basic harmonic represents particles with half spin. We know that for the basic harmonic, the three possibilities are given by l = 1, m = n = 0, or m = 1, l = n = 0, or n = 1, l = m = 0.

Note that in the case of the basic harmonic state, there can be only one half-wave. It can be along either one of the three axes. Of these orthogonal momentum states, only the one which forms a sharp peak in the amplitude will be able to represent the particle and we know that only the standing wave which is in the direction of the translational velocity satisfies this condition [1]. We shall not consider higher harmonics now. We shall show later by a separate paper why only the basic harmonic structure may be allowed for particles having rest mass. Needless to say, particles like π mesons which have rest mass and unit spin may be treated as composite particles constituted by various quarks. Therefore, assuming the translational velocity of the standing wave to be along the x-axis, (3) may be expressed as

$$\phi \quad = \quad \xi_y \left(e^{-i\hbar^{-1}(E't-\text{þ}'x)} + e^{-i\hbar^{-1}(E''t+\text{þ}''x)} \right) \pm i\xi_z \left(e^{-i\hbar^{-1}(E't-\text{þ}'x)} - e^{-i\hbar^{-1}(E''t+\text{þ}''x)} \right). \tag{5}$$

We may identify this function with either (1) or (1A).

Note that the CPL wave referred here stands for the electromagnetic wave which is circularly polarized. For the sake of convenience and clarity we may call a single circularly polarized electromagnetic wave by the name "photino". This means that the standing wave is formed by the confinement of a single photino. We may call such a standing wave by the name staphon which is the short form for standing photino. Interestingly, when a staphon is formed, its standing photino structure is compacted into the inner coordinates while in the external coordinate it is observed as the plane wave [1].

It is interesting to note that there have been attempts to represent a particle in terms of standing radial waves or Turing waves [2]. The electric field at the core of a particle is configured as a radially periodic stationary wave pattern which is termed a Turing wave. In fact, it is proposed that the linear wave propagation mode of the photons undergoes a change and takes up a new mode where the propagation is radial. While this sort of structure brings out the spatial symmetry of the particle, it calls for a radical re-appraisal of the Maxwell's equations and the concepts of the quantum field theory, which appears to be quite unwarranted. In the present approach the standing CPL wave structure resolves the problem of the spatial symmetry of the particle by its quantum mechanical behavior. These standing waves could be assumed to exist in a virtual state in all possible directions since their amplitude in the rest frame of reference is always zero [1]. When a translational motion is given, the standing wave in the direction of motion acquires non-zero amplitude whereby the spherical symmetry is destroyed. In this manner working with the well understood structure of the standing CPL wave, we are able to resolve the problem of the spatial symmetry of the particle. We shall now see how this standing CPL wave structure enables us to explain the spin of the particle.

2 Accounting for the Relativistic Transformations in the Electric and the Magnetic Fields

For the sake of simplicity, in the approach followed till now we had assumed the electric vector to be a constant. Now we shall take into consideration the relativistic transformation of the electric and the magnetic fields of the electromagnetic wave. We know that the relativistic transformation equations for the electromagnetic field is given by [3]

$$\boldsymbol{\xi}' = \gamma(\boldsymbol{\xi} - \boldsymbol{\beta} \times \mathbf{B}); \quad \mathbf{B}' = \gamma(\mathbf{B} + \boldsymbol{\beta} \times \boldsymbol{\xi}) \quad . \tag{6}$$

where $\boldsymbol{\xi}$ and $\mathbf{B}$ stand for the electric and magnetic fields in a given frame of reference while $\boldsymbol{\xi}'$ and $\mathbf{B}'$ represent the values when viewed from second frame of reference which has a uniform velocity $-\mathbf{v}$ with regard to the first one. In the above equations, we have taken $\boldsymbol{\beta} = \mathbf{v}/c$ and $\gamma = 1/\sqrt{(1-\boldsymbol{\beta}^2)}$. Taking into account the behavior of the forward and the reverse waves under relativistic transformation, the function representing the circularly polarized standing wave along the x-axis with positive helicity can be expressed as (see annexure 1)

$$\phi^R \quad = \quad 2\left\{ \boldsymbol{\xi}_{yo} \cos\left[E(x'-vt')/\hbar c - \theta\right] - \boldsymbol{\xi}_{zo} \sin\left[E(x'-vt')/\hbar c - \theta\right] \right\} e^{-i\hbar^{-1}(Et-\text{þ}x)} \quad .$$

Now taking $\boldsymbol{\xi}_{yo} = \xi_o \cos\varphi$ and $\boldsymbol{\xi}_{zo} = \xi_o \sin\varphi$, where ξ_o is the magnitude of the electric field of the CPL wave in the rest frame of the standing wave, we have

$$\phi^R \quad = \quad 2\,\xi_o \cos\left[E\,(x'-vt')/\hbar c - \theta + \varphi\right] e^{-i\hbar^{-1}(Et-\text{þ}x)} \,. \tag{7}$$

Since the angle φ can be chosen arbitrarily, we may drop it conveniently to obtain

$$\phi^{R} \quad = \quad 2\,\xi_{o}\,\cos\,\left[E\left(x'-vt'\right)/\hbar c-\theta\,\right]e^{-i\hbar^{-1}(Et-\text{þx})}\quad . \tag{7A}$$

Expressing the cosine term as a linear combination of two exponential terms, we have

$$\phi^{R} \quad = \quad \xi_{o}\left[\,e^{iE\left(x'-vt'\right)/\hbar c\,-\,i\theta}+e^{-iE\left(x'-vt'\right)/\hbar c\,+\,i\theta}\right]e^{-i\hbar^{-1}\left(Et-\text{þx}\right)}\quad . \tag{7B}$$

It can be seen that the first term in (7B) represents the forward wave and may be expressed as

$$\phi^{R_1} \quad = \quad \xi_{o}\;\; e^{iE\left(x'-vt'\right)/\hbar c\,-\,i\theta}\;\; e^{-i\hbar^{-1}\left(Et-\text{þx}\right)}\quad . \tag{7C}$$

Similarly, the second term in (7B) would represent the reverse wave and may be expressed as

$$\phi^{R_2} \quad = \quad \xi_{o}\;\; e^{-iE\left(x'-vt'\right)/\hbar c\,+\,i\theta}\;\; e^{-i\hbar^{-1}\left(Et-\text{þx}\right)}\quad . \tag{7D}$$

Instead of the standing wave with positive helicity, had we started with one with negative helicity, then also the form of the wave would remain the same (see annexure 1). That is to say

$$\phi^{L} \quad = \quad \xi_{o}\left[\,e^{iE\left(x'-vt'\right)/\hbar c\,-\,i\theta}+e^{-iE\left(x'-vt'\right)/\hbar c\,+\,i\theta}\right]e^{-i\hbar^{-1}\left(Et-\text{þx}\right)} \quad = \quad \phi^{R}\;. \tag{8}$$

Actually, the difference between the right handed and the left handed helicity states would have become explicit had retained the phase angle φ in the exponential term. But since we have dropped it, the distinguishing term between ϕ^{R} and ϕ^{L} got erased making the two functions look alike. But we shall reintroduce the phase angle φ whenever ϕ^{R} and ϕ^{L} have to be distinguished. The forward wave and the reverse wave of the left handed helicity state would be given by

$$\phi^{L_1} \quad = \quad \xi_{o}\;\; e^{iE\left(x'-vt'\right)/\hbar c\,-\,i\theta}\;\; e^{-i\hbar^{-1}\left(Et-\text{þx}\right)}\quad . \tag{8A}$$

$$\phi^{L_2} \quad = \quad \xi_{o}\;\; e^{-iE\left(x'-vt'\right)/\hbar c\,+\,i\theta}\;\; e^{-i\hbar^{-1}\left(Et-\text{þx}\right)}\,. \tag{8B}$$

3 Staphon and its Spin

In quantum mechanics, spin is taken as an intrinsic property of the electron. The idea of the electron spinning around its own axis does not make sense as it is taken as a point particle without any internal structure. Another reason is that the direction of the spin of a particle like electron will be perceived by two observers differently depending upon the direction of their motion relative to the particle. Recall that the spin up state of a particle is when its spin is in the direction of its velocity and spin down state when it is in the reverse direction. This makes the direction of the spin of a particle observer dependent. This is so different from the macroscopic situation where the direction of the spin of a body is observer independent. Therefore, it was believed that the spin of a micro-particle like electron cannot be understood in terms of the classical analogues of rotational motion in the three dimensional space. It is treated as a property defined in the internal space (of a particle) that cannot be analyzed further. But in our approach, the plane wave representing the electron is attributed an internal structure of the standing photino. This means that spin may be traced to some property of the photino. We shall see how this may be done.

We know that the accepted practice in quantum mechanics is to represent the angular momentum component, M_z along the z-axis by the relation

$$M_{z} \quad = \quad i\hbar\,\partial\phi\big/\partial\theta \qquad where \qquad \phi \quad = \quad \xi\,e^{i\hbar^{-1}A}\;, \tag{9}$$

where $\partial\theta$ represents an infinitesimal rotation around z-axis in the x-y plane and while A represents the action function. We shall now deviate from this generally accepted convention and use this relation for the angular momentum along the x-axis. Accordingly we have

$$M_x \quad = \quad i\hbar\, \partial\phi / \partial\theta \qquad where \qquad \phi \quad = \quad \xi\, e^{i\hbar^{-1}A} \quad . \tag{9A}$$

Here $\partial\theta$ represents an infinitesimal rotation around the x-axis in the y-z plane. We should keep in mind that if the angular momentum is to have a non-zero value, it should progress along a curved path. But we know that the spin angular momentum is independent of the path of the particle. This leaves us with only the rotation of the amplitude of the wave to represent the spin. We shall see if the amplitude wave forming part of ϕ^{R_1} in (7C) given by

$$\phi_A^{R_1} \quad = \quad \xi_o\, e^{-i\theta}\, e^{i\hbar^{-1}E(x'-vt')/c} \tag{10}$$

meets the requirements. Since $\phi_A^{R_1}$ is defined in the internal coordinates it appears to be the ideal candidate for representing the spin.

We know that the energy and the momentum of the plane wave is given by [1]

$$E \quad = \quad \tfrac{1}{2}\left(E_1 + E_2\right) \quad and \quad þ \quad = \quad \tfrac{1}{2}\left(p_1 - p_2\right) \quad = \quad \tfrac{1}{2}\left(E_1 - E_2\right)/c \ , \tag{11}$$

where E_1 and p_1 are the magnitude of the energy and the momentum of the forward wave while E_2 and p_2 are those of the reverse wave. We may now express the phase of the amplitude wave using (11) as

$$\begin{aligned} E\left(x' - vt'\right)/\hbar c \quad &= \quad \left(E\, x'/c - þct'\right)/\hbar \quad = \quad \left[\tfrac{1}{2}\left(E_1 + E_2\right)x'/c - \tfrac{1}{2}\left(E_1 - E_2\right)t'\right]/\hbar \\ &= \quad -\tfrac{1}{2}\hbar^{-1}E_1\left(t' - x'/c\right) + \tfrac{1}{2}\hbar^{-1}E_2\left(t' + x'/c\right) \quad . \end{aligned} \tag{12}$$

Therefore, the amplitude wave in (10) may be expressed as

$$\phi_A^{R_1} \quad = \quad \xi_o\, e^{-i\theta}\, e^{\frac{1}{2}i\hbar^{-1}A_1}\, e^{-\frac{1}{2}i\hbar^{-1}A_2} . \tag{13}$$

where A_1 and A_2 are respectively the action of the forward and the reverse waves in terms of the internal coordinates. This shows that the amplitude of the plane wave can be shown as a product of two CPL waves existing in the internal coordinates, one moving forward with a frequency $\tfrac{1}{2}E_1/\hbar = \tfrac{1}{2}\omega_1$ and the other one, the reverse wave with a frequency, $-\tfrac{1}{2}E_2/\hbar = -\tfrac{1}{2}\omega_2$. In other words, $\phi_A^{R_1}$ may be expressed as

$$\phi_A^{R_1} \quad = \quad u^1 u^{2'} \quad , \tag{14}$$

$$where \quad u^1 \quad = \quad \rho_o\, e^{-i\frac{1}{2}\left(\left(\omega_1 t' - k_1 x'\right) + \theta\right)} \quad and \quad u^{2'} \quad = \quad \rho_o\, e^{i\frac{1}{2}\left(\left(\omega_2 t' + k_2 x'\right) - \theta\right)} \ . \tag{14A}$$

and $\rho_o^2 = \xi_o$. Note that the phase of the forward and the reverse luminal waves constituting u^1 and $u^{2'}$ are relativistic invariants for obvious reasons. The prime in $u^{2'}$ is introduced to represent the conjugation involved in it. Here the product of u^1 and $u^{2'}$ forms the amplitude of the plane wave. `

We have to keep in mind that the angular motion appearing in u^1 is the same as what appears in $u^{2'}$. To prove this let us take $\phi_A^{R_1}$ shown in (10), to the rest frame of reference of the standing wave. Then we have

$$\phi_{Ao}^{R_1} \quad = \quad \xi_o\, e^{ik_o x_o} \quad = \quad \rho_o\, e^{-\frac{1}{2}i\left(\omega_o t_o - k_o x_o\right)}\, \rho_o\, e^{\frac{1}{2}i\left(\omega_o t_o + k_o x_o\right)} \quad . \tag{14B}$$

This means that the angular motion $\frac{1}{2}\omega_o t_o$ which appears in u^1 appears with the opposite sign in $u^{2'}$. This is so because the same angular rotation in the positive direction for the forward wave will appear as angular rotation in the negative direction for the reverse wave. Therefore, for the purpose of assessing the spin of the system, we have to take into account only one of the two waves. We may choose u^1 for the purpose as it represents the forward component of the standing wave. As discussed earlier, keeping in mind that the internal coordinates can be replaced by the external coordinates in the amplitude wave, this can be easily verified on ϕ^{R_1} given in (7C)..

$$\phi^{R_1} \;=\; \xi_o e^{-i\theta}\, e^{iE(x'-vt')/\hbar c}\, e^{-i\hbar^{-1}(Et-\text{þ}x)} \;=\; \xi_o e^{-i\theta}\, e^{iE(x-vt)/\hbar c}\, e^{-i\hbar^{-1}(Et-\text{þ}x)}$$

$$=\; \xi_o e^{-i\theta}\, e^{-i\hbar^{-1}(E_1 t_1 - p_1 x)}\;. \qquad (15)$$

Note that here we have used the relations $E = \frac{1}{2}(E_1+E_2)$ and $\text{þ} = \frac{1}{2}(p_1-p_2)$. Therefore, the spin angular momentum of the standing wave, S is given by

$$S\phi_A^{R_1} \;=\; i\hbar\frac{\partial\phi_A^{R_1}}{\partial(\omega_1 t')} \;=\; i\hbar\frac{\partial u^1}{\partial(\omega_1 t')}u^2 \;=\; \tfrac{1}{2}\hbar\phi_A^{R_1}. \qquad (16)$$

Note that by the time the phase of the forward component of the standing wave rotates by an angle $\omega_1 t$, u^1 would have rotated through only half of that. It is obvious from the above exposition that so far as we are dealing with the spin of the forward wave, we may ignore the existence of $u^{2'}$ for all practical purpose. This would mean that we may represent the forward wave given in (15) as

$$\phi^{R_1} \;=\; \rho_o\, e^{-i\frac{1}{2}\theta}\, e^{-\frac{i}{2}(\omega_1 t - k_1 x)}\, e^{-i\hbar^{-1}(Et-\text{þ}x)} \;=\; u^1\, e^{-i\hbar^{-1}(Et-\text{þ}x)}\;. \qquad (16A)$$

Note that u^1 does represent the true picture of the inner structure of the particle. But since we are interested only in the spin of the system that emerges from the inner structure, u^1 is adequate for the purpose. In the above discussion we have taken the CPL wave to have positive helicity which has been represented by ϕ^R. If the CPL wave had negative helicity the corresponding function would have been ϕ^L and then the spin obtained in (16) would have been $-\frac{1}{2}\hbar$.

In (7B), had we chosen the second term in the bracket then the function would have been

$$\phi^{R_2} \;=\; \xi_o e^{i\theta}\, e^{-\frac{1}{2}i\hbar^{-1}A_1}\, e^{\frac{1}{2}i\hbar^{-1}A_2}\, e^{-i\hbar^{-1}(Et-\text{þ}x)} \;=\; u^{1'}u^2\, e^{-i\hbar^{-1}(Et-\text{þ}x)}\;, \qquad (17)$$

where $u^{1'} = \rho_o\, e^{i\frac{1}{2}[(\omega_1 t - k_1 x)+\theta]}$ *and* $u^2 = \rho_o\, e^{-i\frac{1}{2}[(\omega_2 t + k_2 x)-\theta]}$. Following the same steps as taken from (12) to (16), we would obtain

$$S\phi_A^{R_2} \;=\; i\hbar\frac{\partial\phi_A^{R_2}}{\partial(\omega_2 t')} \;=\; i\hbar u^{1'}\frac{\partial u^2}{\partial(\omega_2 t')} \;=\; \tfrac{1}{2}\hbar\phi_A^{R_2}. \qquad (17A)$$

Note that by the time the phase of the reverse component of the standing wave rotates by an angle $\omega_2 t$, u^2 would have rotated through only half of that. As already explained, the rotation undergone by $u^{1'}$ has already been accounted by u^2 and therefore we might as well ignore it so far as the spin of the system is concerned. This would mean that we may represent the reverse wave given in (17) as

$$\phi^{R_2} \;=\; \rho_o\, e^{-\frac{i}{2}\left[\hbar^{-1}(E_2 t + p_2 x)-\theta\right]}\, e^{-i\hbar^{-1}(Et-\text{þ}x)} \;=\; u^2\, e^{-i\hbar^{-1}(Et-\text{þ}x)}\;. \qquad (17B)$$

Although the helicity of the reverse wave is negative, we have to keep in mind that it is aligned in the same direction as that of the forward wave ϕ^{R_1}. In other words, the spin of the forward component and the reverse component of the standing wave are in the same direction with regard to the translational velocity of the standing wave or that of the particle represented by it. Therefore, taking the average of the spin of the forward wave, S_1 and that of the reverse wave, S_2 , we obtain

$$S \quad = \quad \tfrac{1}{2}\left[S_1 + S_2\right] \quad = \quad \tfrac{1}{2}\left[\tfrac{1}{2}\hbar + \tfrac{1}{2}\hbar\right] \quad = \quad \tfrac{1}{2}\hbar \quad . \tag{18}$$

Here we observe from (15) and (16) that the spin of the forward wave is equal to the spin of the particle itself. The same holds good for the spin of the reverse wave also as can be seen from (17A). Therefore, the spin state of a particle can be represented just as well by either the forward half wave or by the reverse half wave by itself. This concept will be useful when we deal with the Dirac spinors in the next section.

4 The Standing Helical Half Wave and the Dirac Equation

We know that for a free particle, the Dirac equation is written as [4]

$$\left(\gamma_\mu \,\partial/\partial x_\mu + mc/\hbar\right)\psi \quad = \quad 0 \quad , \tag{19}$$

where $\gamma_k = \begin{bmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{bmatrix}$, $k = 1,2,3.$; $\gamma_4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Note that here σ_k represents Pauli's spin matrices. If we define a new matrix $\Sigma_k = \begin{bmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{bmatrix}$,

then, we know from the algebra of the gamma matrices that

$$\gamma_i^2 = \gamma_4^2 = I \,,\; \gamma_1\gamma_2 = i\Sigma_3 = -\gamma_2\gamma_1 \,,\; \gamma_2\gamma_3 = i\Sigma_1 = -\gamma_3\gamma_2 \,,\; \gamma_3\gamma_1 = i\Sigma_2 = -\gamma_1\gamma_3 \,.$$

Let us now introduce α_k such that

$$\gamma_k\gamma_4 = -\gamma_4\gamma_k = i\alpha_k \quad . \tag{20}$$

If we now denote the four components of the four momentum by $þ_\nu$ and those of the space-time by x_μ , where μ and ν take on values 1,2,3 and 4, then we have

$$þ_\nu \quad = \quad (\mathbf{þ}, \mathrm{i\,E/c}) \quad ; \quad x_\mu \quad = \quad (\mathbf{r}, ict) \quad and$$

$$(\gamma_\nu þ_\nu)(\gamma_\mu x_\mu) = \mathbf{þ}.\mathbf{r} - \mathrm{Et} - \mathrm{i}\Sigma\cdot(\mathbf{r}\times\mathbf{þ}) - \alpha_k(\mathbf{þ}_k ct - E\,\mathbf{x}_k/c) \,. \tag{21}$$

Here, we should keep in mind that since the particle is moving freely, its angular momentum represented by **(rxþ)** will be zero. Besides, since the particle is assumed to move along the x-axis, we may simplify (21) to obtain

$$(\gamma_\nu þ_\nu)(\gamma_\mu x_\mu) \quad = \quad A + \alpha_x E\,(x' - vt')/c \;, \tag{21A}$$

where A = -Et + þx, is the action function of a free particle. We have introduced the internal coordinates in the second term on the right hand side as we know from the previous section that it is defined in the internal coordinates and represents the spin up state. Let us now define a function Ψ given by

$$\psi \quad = \quad \xi\, e^{i\hbar^{-1}(\gamma_\nu þ_\nu)(\gamma_\mu x_\mu)} \quad . \tag{22}$$

Let us now assume that

$$\xi \quad = \quad \xi_o \, e^{-i\alpha_x \theta} \; , \tag{22A}$$

We have introduced the matrix α_x in the exponential intentionally as we observe from (7A) and (8) that the relativistic transformation of ξ manifests as a change in the phase of the amplitude wave "E(x-vt)/ħc". Note that the function "exp(-i$\alpha_x\theta$) is a real function as θ is imaginary. Let us verify if Ψ satisfies the Dirac equation. On substituting for Ψ in (19) from (22) we obtain

$$(i/\hbar)\gamma_k \left(\partial \mathrm{A}/\partial \mathrm{x}_k\right)\psi \; + \; (i/\hbar)\gamma_4 \left(\partial A/\partial x_4\right)\psi \; + (mc/\hbar)\psi \quad = \quad 0 \quad . \tag{23}$$

Note that the last term on the right hand side of (21A) is defined in the internal coordinates and therefore, the differential operator does not operate on it. Keeping in mind γ_4 = -iβα and γ_4 = β, the above equation can be simplified to give

$$E \quad = \quad \alpha_k \text{þ}_k + \beta \, mc \quad . \tag{24}$$

This is the linear form of the relativistic energy-momentum equation and therefore confirms that the function Ψ satisfies the Dirac equation.

We may express Ψ as a matrix given by

$$\psi^{\,r} \quad = \quad \xi_o \, e^{i\alpha_x\left(E\frac{x'-\mathrm{v}t'}{\hbar c} - \theta\right)} \; e^{-i\hbar^{-1}(Et-\text{þ}\mathrm{x})}$$

$$= \quad \rho_o \, e^{-i\frac{1}{2}\alpha_x\left(\frac{E_1 t' - \mathrm{p}_1 x'}{\hbar} + \theta\right)} \; \rho_o \, e^{i\frac{1}{2}\alpha_x\left[\frac{E_2 t' + \mathrm{p}_2 x'}{\hbar} - \theta\right]} \; e^{-i\hbar^{-1}(Et-\text{þ}\mathrm{x})} \quad . \tag{25}$$

Here r can take values from 1 to 4 and represents the four solutions of the Dirac equation with which we are familiar. Ignoring the second spinor defined in the internal coordinates as discussed in the previous section, we may write

$$\psi^{\,r} \quad = \quad \rho_o \, e^{-i\frac{1}{2}\alpha_x\left(\frac{E_1 t' - \mathrm{p}_1 x'}{\hbar} + \theta\right)} \; e^{-i\hbar^{-1}(Et-\text{þ}\mathrm{x})} \quad = \quad u^r \, e^{-i\hbar^{-1}(Et-\text{þ}\mathrm{x})} \quad , \tag{26}$$

where $$u^r \quad = \quad \rho_o \, e^{-i\frac{1}{2}\alpha_x\theta \, -i\frac{1}{2}\alpha_x\left(E_1 t' - \mathrm{p}_1 x'\right)/\hbar} \quad = \quad u_o^r \, e^{-i\frac{1}{2}\alpha_x\theta} \, .$$

Here we should keep in mind that the phase term "$\frac{1}{2}$(E_1t′-p_1x′)/ħ" is a relativistic invariant since it represents the phase of a luminal wave. Therefore, we may switch over to the rest frame and the phase function becomes "[-$\frac{1}{2}$α($E_o t_o$′- $p_o x_o$′)/ħ]" and this can be equated to zero by choosing the origin of coordinates suitably so that x_o = ct_o. Note that p_o = E_o/c. Therefore,

$$u_o^r \quad = \quad \rho_o \, e^{-i\frac{1}{2}\alpha_x\left(E_o t_o' - \text{þ}_o x_o'\right)/\hbar}$$

$$= \; I \; \rho_o \cos\!\left[\tfrac{1}{2}\left(E_o t_o' - \text{þ}_o' x'\right)/\hbar\right] - i\alpha_x \sin\!\left[\tfrac{1}{2}\left(E_o t_o' - \text{þ}_o' x'\right)/\hbar\right] \; . \tag{26A}$$

Here we have utilized the property $e^{i\alpha_x\varphi} = I \cos \varphi + i\alpha_x \sin \varphi$. This equation can be proved by expanding the exponential term into the familiar series keeping in mind α_x^2 = I. But since the phase of the cosine and sine functions is zero, we may represent u_o^r by a 4x4 unit matrix as follows, assuming ρ_o to be unity for the sake of convenience.

$$u_o^r \quad = \quad I \quad = \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; where \;\; u_o^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, u_o^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, u_o^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, u_o^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} . \quad (26B)$$

Now we may express the transformation of the spinor by the equation

$$u^r \quad = \quad e^{-i\frac{1}{2}\alpha_x\theta} u_o^r \quad = \quad \left[I \cos \tfrac{1}{2}\theta - i\alpha_x \sin \tfrac{1}{2}\theta \right] u_o^r \; .$$

Here let us introduce a phase function θ′ given by θ′ = v/c. This means θ = iθ′. Therefore, u^r now be expressed as

$$u^r \quad = \quad e^{\frac{1}{2}\alpha_x\theta'} u_o^r \quad = \quad \left[I \cosh \tfrac{1}{2}\theta' + \alpha_x \sinh \tfrac{1}{2}\theta' \right] u_o^r \; . \qquad (27)$$

Here cosh $\frac{1}{2}\theta'$ = √[(E/c + mc)/2mc], sinh $\frac{1}{2}\theta'$ = √[(E/c –mc)/2mc] so that tanh $\frac{1}{2}\theta'$ = þ/(E/c+mc). Ψ^r given in (26) may now be expressed as

$$ie; \quad \psi^r \quad = \quad \cosh \tfrac{1}{2}\theta \begin{bmatrix} 1 & 0 & 0 & b \\ 0 & 1 & b & 0 \\ 0 & b & 1 & 0 \\ b & 0 & 0 & 1 \end{bmatrix} u_o^r \, e^{-i\hbar^{-1}\varepsilon_r(Et - \text{þx})} , \qquad (28)$$

where b = tanh $\frac{1}{2}\theta'$ = þ/(E/c+mc) and $\varepsilon_r = 1$ for r = 1,2 and ε = -1 for r = 3,4. This is the familiar expression for the transformation of the spinor to a moving frame of reference along the x-axis [5]. It is interesting to note that the transformation matrix of the spinor emerges of relativistic transformation of the electric and magnetic fields of the electromagnetic wave. This is the Dirac spinor.

Let us now reverse the order of the matrices given in (21A) which could be expressed as

$$(\gamma_\mu x_\mu)(\gamma_\nu \text{þ}_\nu) \quad = \quad A - \alpha E\,(x' - vt')/c \; . \qquad (29)$$

Note that when we reverse the order of the products of the matrices on the left hand side, the terms representing angular momentum and spin get reversed while action A remains unchanged. This leads us to conclude that the reversal of the order of the matrices results in the reversal of the helicity of the waves constituting the system. Now following the same steps which were taken in arriving at Ψ given in (25), and replacing the function, exp(-i$\alpha_x\theta$) by exp(i$\alpha_x\theta$), we obtain $\Psi^{r'}$ given by

$$\psi^{r'} \quad = \quad \xi_o \, e^{i\alpha_x\theta} \, e^{-i\alpha E(x'-vt')/\hbar c} \quad e^{-i\hbar^{-1}\varepsilon_r(Et - \text{þx})} \; . \qquad (30)$$

Note that the relativistic transformation of ξ manifests as a change in the phase of the amplitude wave "E(x′-vt′)/ħc". Therefore, a change in the sign of the phase of the amplitude wave has to be accounted in θ also. If we now take Ψ' to be the linear combination of Ψ^r and $\Psi^{r'}$, then we have

$$\psi' \quad = \quad \xi_o \left[\, e^{i\alpha_x[E(x'-vt')/\hbar c - \theta]} + \, e^{-i\alpha_x[E(x'-vt')/\hbar c - \theta]} \right] e^{-i\hbar^{-1}\varepsilon_r(Et - \text{þx})}$$

$$= \quad \xi_o \left[e^{i\alpha_x\left(E\frac{x'-vt'}{\hbar c} - \theta\right)} + e^{-i\alpha_x\left(E\frac{x'-vt'}{\hbar c} - \theta\right)} \right] e^{-i\hbar^{-1}\varepsilon_r(Et - \text{þx})}$$

$$= \quad 2I\xi_o \cos\left[E\left(x' - vt'\right)/\hbar c - \theta\right] e^{-i\hbar^{-1}\varepsilon_r (Et - \text{þz})} \quad . \tag{31}$$

The matrix nature of Ψ' is expressed by showing the unit matrix on the right hand side of the equation explicitly. We observe that (31) is exactly the same function as represented by (7A). When we identify (7A) with (31) we cannot overlook the fact that (31) represents a matrix equation, while (7A) is a simple scalar equation. Technically this does not pose any problem for us. We need multiply both sides of (7A) by a unit matrix to make it identical to (31). But actually the difference between (7A) and (31) is that while (7A) represents a single standing wave, (31) represents four standing waves. If the first column on the right hand side of (31) represents a standing wave with positive helicity (+1), the second column would represent one with negative helicity (-1). The third and the fourth columns would represent the corresponding helicity states of waves with negative energy. In that sense (31) includes all possible situations. This shows that the standing electromagnetic half wave representation or the staphon structure of the particle is consistent with the Dirac equation.

Let us now examine what each of the states in Ψ^r and $\Psi^{r'}$ represents. For the sake of clarity, we shall use the term helicity in the case of the luminal waves only. In the case of the standing wave and the particle it represents, we shall use the term spin to identify its intrinsic rotation. We know by convention, the first spinor u^1 represents the spin $\frac{1}{2}$ state while the second spinor u^2 represents the spin -$\frac{1}{2}$ state. Now looking to the direction of rotation of the spinor in (28), Ψ^1, the first component of Ψ^r should be representing the forward half wave. Therefore, the first component $\Psi^{1'}$ of $\Psi^{r'}$ should be the reverse half wave such that they represent the forward and the reverse waves of the same standing wave representing the spin up electron. Similarly, if we take Ψ^2 and $\Psi^{2'}$, they should be representing respectively the reverse component and the forward component of another standing wave representing a spin down electron. Now we have a clear picture of what Ψ^r stands for. The first component Ψ^1 represents the forward half wave of the standing wave with positive helicity while the second component Ψ^2 represents the reverse wave of the standing wave having negative helicity. Since the spin of the forward half wave is the same as that of the particle, treating Ψ^1 as representing the particle as a whole does not create any problem (same is the case with Ψ^2 also) except that it does not give a clear explanation why the eigen value of the velocity operator "cα" is $\pm$ c and why the average velocity of the wave packet formed by the two positive energy solutions gives us the particle velocity. On the other hand, once we treat Ψ^1 and Ψ^2 as the forward and the reverse wave states, this issue gets easily explained. Note that here the identification of the spinor components Ψ^r with the spin states conforms to the universally accepted interpretation of the Dirac spinor.

5 Staphon and the van der Waerden Equation

We shall now examine the van der Waerden equation given below in the light of the standing electromagnetic half wave representation of the electron.

$$\left(i\hbar\frac{\partial}{\partial x_o} + i\hbar\boldsymbol{\sigma}.\nabla\right)\left(i\hbar\frac{\partial}{\partial x_o} - i\hbar\boldsymbol{\sigma}.\nabla\right)\phi \quad = \quad (mc)^2\phi \quad . \tag{32}$$

where ϕ is a two component wave function. This equation can be split into two first order equations each of which acts on two separate functions as given below [6].

$$i\hbar\left[\sigma.\nabla - (\partial/\partial\tau)\right]\phi^L \quad = \quad -mc\,\phi^R \quad , \tag{32A}$$

$$-i\hbar\left[\sigma.\nabla + (\partial/\partial\tau)\right]\phi^R \quad = \quad -mc\,\phi^L \quad . \tag{32B}$$

Here ϕ^R and ϕ^L are conventionally taken to describe a right handed (spin parallel to the momentum direction) and a left handed (spin anti-parallel to the momentum direction) state of the spin $\frac{1}{2}$ particle respectively. But in the approach proposed in this paper, we shall show that they represent the forward and the reverse components of the same standing CPL wave. We know that when a right handed CPL wave undergoes reflection, its helicity undergoes a reversal and it becomes a left handed CPL wave. This means that ϕ^R describes the case where the spin of the forward wave is parallel to ***the momentum of the forward wave*** while ϕ^L describes the case where the spin of the reverse wave is anti-parallel to ***the momentum of the reverse wave.*** But, if we take the standing wave as a whole moving with a translational velocity v, then the spin of both the forward and the reverse components of the waves will be ***directed parallel to the momentum of the particle***. Observe the difference in the interpretation with the conventional one given earlier where the momentum against which the direction of the spin was measured was that of the particle.

We shall now take ϕ^R to represent the right handed forward wave whose helicity can be taken to be positive. We know from (27) that the behavior of ϕ^R in a velocity transformation can be represented by

$$\phi^R \quad = \quad I \cosh \tfrac{1}{2}\theta'\left[1 + \alpha \tanh \tfrac{1}{2}\theta'\right] u_o^r \; e^{-i\hbar^{-1}(Et - þx)} \tag{33A}$$

If we take ϕ^L to be a two component matrix, we may express (33A) as

$$\phi^R \quad = \quad I \cosh \tfrac{1}{2}\theta'\left[1 + \sigma_x \tanh \tfrac{1}{2}\theta'\right] u_o^r \; e^{-i\hbar^{-1}(Et - þx)} \tag{33B}$$

It can be easily shown that $\cosh \frac{1}{2}\theta' = \sqrt{[(E/c + mc)/2mc]}$ and $\tanh \frac{1}{2}\theta' = þ/(E/c + mc)$. Accordingly, we may express (33B) as

$$\phi^R \quad = \quad I \cosh \tfrac{1}{2}\theta'\left[1 + \sigma . þ/(E/c + mc)\right] u_o^r \; e^{-i\hbar^{-1}(Et - þx)} \tag{34}$$

In a similar manner, keeping in mind that the rotation undergone by the reverse wave in a relativistic transformation is in the opposite direction, we obtain

$$\phi^L \quad = \quad I \cosh \tfrac{1}{2}\theta'\left[1 - \sigma . þ/(E/c + mc)\right] u_o^r \; e^{-i\hbar^{-1}(Et - þx)} \tag{34A}$$

Now it can be easily seen on substitution that ϕ^R and ϕ^L given in (34) and (34A) respectively satisfy the equations given in (32) and (32A). This confirms our assumption that a particle like electron has a standing half CPL wave structure.

The picture that emerges here is similar to the one proposed by Penrose in his 2-spinor formalism. According to Penrose, the Dirac electrons can be thought of as being composed a pair of 2-spinors [7]. He calls the state denoted by one of the 2-spinors as the 'zig' particle and that by the other as the 'zag' particle. He treats these as massless particles traveling with the speed of light, more like 'jiggling' backwards and forwards where the forward motion of the zig is continuously being converted to the backward motion of the zag and vice versa. He uses this picture of zig and zag particles to explain what is termed as 'zitterbewegung' of the electron. According to him each ingredient has a spin about its direction of motion with a magnitude of $\frac{1}{2}\hbar$. The spin is left handed in the case of zig and right handed for the zag. Although the velocity keeps reversing, the spin direction remains constant in the electron's rest frame. He proposes that the zig particle acts as the source for the zag particle and the zag particle as the source of the zig particle, the coupling strength being determined by M which is the rest mass of the particle. He observes that the average rate at which this zig-zag motion takes place is equal to the de Broglie frequency of the electron. Note that the picture that emerges from Penrose's 2-spinor formalism coincides with the one proposed here based on the standing wave representation of the electron except that he has chosen the negative helicity wave as the forward wave instead of the positive helicity one we had taken in our approach.

Penrose wonders whether the zig and zag particles are 'real' or if they are the artifacts of a particular mathematical formalism that he has been adopting for the description of the Dirac equation for the electron. He comments " So are these zigs and zags (particles) real? For my own part, I would say so: they are as real as the 'Dirac electron is itself real- as an idealized mathematical description of one of the most fundamental ingredients of the universe" [7]. We now know that the zig and zag particle picture (which is same as the standing wave picture with the forward wave playing the role of the zig particle and the reverse wave playing the role of the zag particle) represents the basic structure of the electron. In fact, the Dirac spinor turns out to be just a convenient mathematical artifact that represents a spin $\frac{1}{2}$ particle. In that sense, what Penrose assumed to be two mutually exclusive approaches turns out to be just two ways of looking at the most basic standing wave structure of the particle. The elegance of this basic structure will be clear in the next section when we obtain a simple explanation for the Pauli's exclusion principle. From the above discussion we should not have the notion that a particle is represented by a single standing wave. Actual state of a particle like electron is composed of a vast number of such individual standing waves existing in all directions due to what is known as quantum superposition. This also would provide the spatial symmetry which is the basic requisite for any structure that represents a particle. Therefore, the observed electron will be some sort of the average of these states.

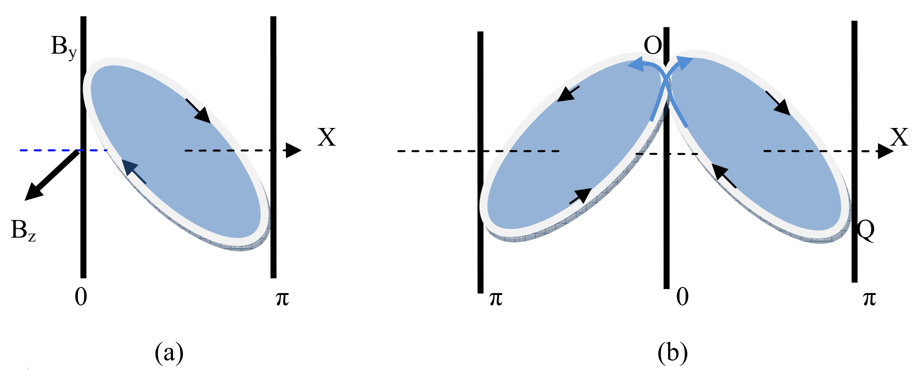
6 The Structure of a Particle and Pauli's Exclusion Principle

We observe that the Dirac spinors which are solutions to the Dirac equation are well placed to represent the state of the electron. But its construction is such that it misses out in providing a clear picture of the internal structure of the particle. We found that the standing wave picture of the electron explains what exactly the Dirac spinors stand for. Earlier, it was assumed that the first component represents the spin-up state of the electron. Now we know that what it represents is the forward component of the standing wave which also has the same spin state as the electron. Similarly the second component represents the reverse component of another standing wave which has a spin-down state. In other words, the Dirac spinors are constituted by mixing up the forward and the reverse waves from two different standing waves each of which represent two different spin states of the particle. This is the reason why the Dirac spinors could not provide a clear picture of the internal structure the electron.

In the light of the above discussion, it is quite clear that the electron could be represented by a standing circularly polarized half wave. Let us take the case of the electron in the spin up state which can be represented by ϕ^R in (1). To understand the physical picture of the standing wave, it is better to transform ϕ^R to its rest frame of reference which gives us

$$\phi_o^R \quad = \quad 2\left[\xi_y \cos\left(k_o x_o'\right) - \xi_z \sin\left(k_o x_o'\right)\right] e^{-i\hbar^{-1} E_o t_o} \tag{29}$$

Note that the terms within the bracket which represent the amplitude of the plane wave have



(a)This represents the standing helical half wave formed between two reflecting mirror. (b) This shows that two standing helical half waves can be joined only when their spins are oriented in the opposite directions.

Figure 1

the structure of the standing helical half wave which is time independent. Recall that this is defined in the internal coordinates. This is the actual inner structure of the electron which is described by a plane wave. Here we should note that the amplitude given by ξ_y and ξ_z do not represent oscillations in the physical space. They just represent the intensity of the electrical field at each point on the x-axis. It goes without saying that the magnitude of the electrical field is represented along the y and z axes just for convenience. On confinement when the basic harmonic is formed, we obtain a standing helical half wave as shown in figure 1(a). Needless to say, in the physical space, the form of the standing helical half wave will be a line segment between the mirrors on the x-axis.

On the basis of this standing helical half wave structure of the particle, it is possible to understand the Pauli's exclusion principle. It should be noted that two such standing waves could be joined together only at O as shown in figure 1(b). Joining these waves at Q with another wave is not possible as the wave gets completed only at O. But to be in phase at the point O, the other standing waves will have to have the opposite helicity (spin). Once one standing wave couples with another one with the opposite helicity (spin), then it is obvious that no further connections could be made with any other standing wave. This explains the Pauli's exclusion principle.

If a fermion is accorded the standing helical half wave structure, it is obvious that a boson having unit spin should be attributed the structure of standing helical full wave. On the basis of such a structure, it is quite clear that a boson could be attached to other bosons on either side, and a chain of bosons could be formed without limit.

7 Conclusion

From the above discussion, the representation of a particle like electron by a standing photino appears to be a viable proposition particularly as it is found to satisfy the Dirac equation. Besides, it provides us with a new insight into the spin and the structure of the elementary particles. We are now able to get a clear physical picture of the spinor in terms of the forward and reverse half waves having frequency half that of the plane wave. The implications of interpreting the Dirac spinors as representing standing CPL waves have to be investigated further. Needless to say one important advantage of this approach is that it gives a simple explanation for the Pauli's exclusion principle and the non-classical behavior of the spin angular momentum of the particles.

An interesting idea that emerges from this concept is regarding the cross products in the four dimensional Minkowski space. Using the expression given in (21) let us re-express the wave function given in (22) as

$$\psi \quad = \quad \xi\, e^{i\hbar^{-1}\left[\mathbf{þ}.\mathbf{r}\text{- Et- i}\Sigma\cdot(\mathbf{r}\times\mathbf{þ})-\alpha_k\left(\mathbf{þ}_{\mathrm{k}}ct-E\,\mathbf{x}_k/c\right)\right]} \tag{30}$$

Here the first two terms in the exponential represents the action of a free particle. The third term which is a cross product between space and momentum represents the angular momentum of the particle. These are classical concepts. The last term represents the cross product with the fourth coordinates by the three dimensional vectors of space and momentum. We saw that the fourth term can be treated as belonging to the internal coordinates and we saw that it represents the spin of the particle in the direction of x_k. This takes us to a very interesting insight. We know that the invariance of the angular momentum could be attributed to the directional isotropy of the space. Or in other words, the physical property of a system will not change whatever be its direction in space. Now a rotation between the x_k and the x_4^th coordinates could be effected only by introducing a translational velocity to the system. Therefore, the invariance of the spin of a particle could be attributed to the invariance of the physical properties of the system by introducing velocity transformations. In other words, spin may be the invariant quantity that emerges out of the relativity principle which is a fundamental symmetry. This would explain why spin is an intrinsic part of the Dirac equation. Note that the idea that the last term in (30) represents spin could not have been possible had we treated the coordinates to be external ones. The concept of the internal coordinates is essential to show that this term represents spin angular momentum of the system.

In quantum mechanics the state of a particle is represented by a plane wave which is an eigen state of the four-momentum in the coordinate representation. Note that the eigen state is the ultimate level of reality in quantum mechanics, beyond which no measurement is assumed to be possible. In relativistic quantum mechanics spin is introduced as an internal degree of freedom and is not directly related to the plane wave representation of the particle. But in the approach followed in this paper, it is observed that the confinement of the photino (electromagnetic wave) leads to the vector nature of the spatial component of the standing wave getting compacted into the inner coordinates where the spin of the particle is defined while the time dependent component which is defined in the external coordinates becomes the plane wave. Therefore, we are effectively assuming that the eigen state (plane wave) of a particle is not the dead end for the investigation into the nature of the particle. We are attributing an inner structure to the eigen state represented by the plane wave. In that sense, the present approach could be termed as a kind of a hidden variable theory.

Here the confinement of the photino is shown to create mass as well as the spin of the particle. If such a confined wave is to represent a particle like electron, it will be necessary to show that the electric charge also arises out of a similar confinement. This will be attempted in the next paper. The staphon (standing photino) representation of a particle promises to be an exciting new way of understanding the basic structure of matter and therefore this approach promises to open the doors for a deeper understanding of quantum mechanics.

Acknowledgements

I acknowledge the suggestions and support given by Prof. M.Rajendran, Cochin College, Kochi, Kerala, India on the subject.

References

[1] V.A.Induchoodan Menon, vixra: 0909.0035,(2009)

[2] Paul A La Violette, Int.J. of General Systems, vol.37(6)(2008), 649-676.

[3] Richard Feynman, The Feynman Lectures on Physics, Vol.II, Dorling Kindersley (India), (2005), p.26-9

[4] James D. Bjorken and Sidney D. Drell: Relativistic Quantum Mechanics, McGraw-Hill Book Company, New York (1964), 6-7.

[5] James D. Bjorken and Sidney D. Drell: Relativistic Quantum Mechanics, McGraw-Hill Book Company, New York (1964), 29-30.

[6] J.J.Sakurai, Addison-Wesley Longman Inc.,India (1967) p-79-80.

[7] Roger Penrose: The Road to Reality, Jonathan Cape, London (2004),629-31.

.

<u>Annexure-1</u>

We know that the relativistic transformation of the electric and the magnetic field is given by the relation

$$\boldsymbol{\xi}' = \gamma(\boldsymbol{\xi} - \boldsymbol{\beta}\times\mathbf{B}) \; ; \quad \mathbf{B}' = \gamma(\mathbf{B} + \boldsymbol{\beta}\times\boldsymbol{\xi}) \tag{I}$$

where $\boldsymbol{\xi}$ and $\mathbf{B}$ are the electric and the magnetic fields in one frame of reference while $\boldsymbol{\xi}'$ and $\mathbf{B}'$ are the corresponding fields as measured from another frame of reference having a relative velocity -v with regard to the first one. If we now take the case of the standing wave, we may identify the first frame of reference with its rest frame and the second frame as one with which the standing wave has a velocity **v**. Let us now take the case of the forward and the reverse components of the standing wave and write down their transformation equations for electric field separately.

$$\xi_1 = \gamma(\boldsymbol{\xi}_{\mathbf{o}} - \boldsymbol{\beta}\times\mathbf{B}_o) \tag{II}$$

$$\xi_2 = \gamma(\boldsymbol{\xi}_{\mathbf{o}} + \boldsymbol{\beta}\times\mathbf{B}_o) \tag{IIA}$$

Here ξ_o and B_o are the electric and the magnetic fields of the forward and the reverse waves in the rest frame of reference while ξ_1 and ξ_2 are the electric fields of the forward wave and the reverse waves as observed from the second frame of reference.

Let us now consider the case where the standing wave is formed along the x-axis and the plane of polarization is in the y-x plane. Let $\boldsymbol{\xi}_{\mathbf{yo}}$ be the electrical vector along the y-axis forming the amplitude of the forward and reverse waves. Let us represent the standing wave by $\phi(\boldsymbol{\xi}_{\mathbf{yo}})$ where

$$\phi(\xi_{yo}) = \boldsymbol{\xi}_{\mathbf{yo}}\cos\left[(E_o t_o - p_o x_o)/\hbar\right] + \boldsymbol{\xi}_{\mathbf{yo}}\cos\left[(E_o t_o + p_o x_o)/\hbar\right] \tag{III}$$

Introducing a translational velocity v to the standing wave along the x-axis, and transforming $\boldsymbol{\xi}_{\mathbf{yo}}$ using (II) and (IIA), and using the complex form for the wave function, we obtain

$$\phi(\boldsymbol{\xi}_{\mathbf{y}}) = \gamma\,\boldsymbol{\xi}_{\mathbf{yo}}(1+\beta)\,e^{-i\hbar^{-1}(E_1 t - p_1 x)} + \gamma\,\boldsymbol{\xi}_{\mathbf{yo}}(1-\beta)\,e^{-i\hbar^{-1}(E_2 t + p_2 x)} \tag{IIIA}$$

Here we have taken advantage of the fact $|\boldsymbol{\xi}_{\mathbf{yo}}|$ and $|\mathbf{B}_{\mathbf{yo}}|$ are equal for the electromagnetic wave. Note that the direction of the vector ($\boldsymbol{\beta}$x$\mathbf{B}$) reverse to that of $\boldsymbol{\xi}$. Taking $\gamma = \cos\theta$ and $i\beta\gamma = \sin\theta$ and simplifying further, keeping in mind that the amplitude wave is defined in the internal coordinates, we obtain

$$\begin{aligned}
\phi(\boldsymbol{\xi}_{\mathbf{y}}) &= \gamma\boldsymbol{\xi}_{\mathbf{yo}}\left[e^{-i\hbar^{-1}(E_1 t - p_1 x)} + e^{-i\hbar^{-1}(E_2 t + p_2 x)}\right] + \gamma\beta\,\boldsymbol{\xi}_{\mathbf{yo}}\left[e^{-i\hbar^{-1}(E_1 t - p_1 x)} - e^{-i\hbar^{-1}(E_2 t + p_2 x)}\right] \\
&= \left\{2\gamma\boldsymbol{\xi}_{\mathbf{yo}}\cos\left[\text{þc}\,(t' - x'/v)/\hbar\right] - 2i\gamma\beta\,\boldsymbol{\xi}_{\mathbf{yo}}\sin\left[\text{þc}\,(t' - x'/v)/\hbar\right]\right\}e^{-i\hbar^{-1}(Et - \text{þx})} \\
&= 2\boldsymbol{\xi}_{\mathbf{yo}}\cos\left[\text{þc}\,(t' - x'/v)/\hbar + \theta\right]\,e^{-i\hbar^{-1}(Et - \text{þx})} \\
&= 2\boldsymbol{\xi}_{\mathbf{yo}}\cos\left[\text{E}\,(x' - vt')/\hbar c - \theta\right]\,e^{-i\hbar^{-1}(Et - \text{þx})}
\end{aligned} \tag{IIIB}$$

If we take the standing wave along the z-x plane in its rest frame having a phase difference of $\pi/2$ by $\phi(\boldsymbol{\xi}_{\mathbf{zo}})$, then we have

$$\phi(\xi_{zo}) = \boldsymbol{\xi}_{z\mathbf{o}}\sin\left[(E_o t_o - p_o x_o)/\hbar\right] - \boldsymbol{\xi}_{z\mathbf{o}}\sin\left[(E_o t_o + p_o x_o)/\hbar\right] \tag{IV}$$

Introducing a translational velocity v to the standing wave along the x-axis, and transforming $\boldsymbol{\xi}_{\mathbf{zo}}$ using (II) and (IIA), and using the complex form for the wave function, we obtain

$$\phi(\xi_z) \quad = \quad i\gamma\, \xi_{\mathbf{zo}}(1+\beta)\, e^{-i\hbar^{-1}(E_1 t - p_1 x)} - i\gamma\, \xi_{zo}(1-\beta)\, e^{-i\hbar^{-1}(E_2 t + p_2 x)} \qquad \text{(IVA)}$$

Remember that (IVA) is the complex form of the real function with which we started. (IVA) can be further simplified to yield

$$\begin{aligned} \phi(\xi_{\mathbf{z}}) \quad &= \quad \left\{ 2\gamma\, \xi_{\mathbf{zo}} \sin\left[\text{þ}c\left(t' - x'/v\right)/\hbar\right] + 2i\gamma\,\beta\, \xi_{\mathbf{zo}} \cos\left[\text{þ}c\left(t' - x'/v\right)/\hbar\right]\right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} \\ &= \quad 2\xi_{\mathbf{zo}} \sin\left[\text{þ}c\left(t' - x'/v\right)/\hbar + \theta\right] e^{-i\hbar^{-1}(Et - \text{þ}x)} \\ &= \quad -2\xi_{\mathbf{zo}} \sin\left[\mathrm{E}\left(x' - vt'\right)/\hbar c - \theta\right] e^{-i\hbar^{-1}(Et - \text{þ}x)} \end{aligned}$$

Now let us represent by ϕ^R the sum of $\phi(\xi_y)$ and $\phi(\xi_z)$. Then we know that ϕ^R would represent a standing wave with positive spin.

$$\phi^R \quad = \quad 2\left\{ \xi_{\mathbf{yo}} \cos\left[\text{þ}c\left(t' - x'/v\right)/\hbar + \theta\right] + \xi_{\mathbf{zo}} \sin\left[\text{þ}c\left(t' - x'/v\right)/\hbar + \theta\right]\right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} \qquad \text{(V)}$$

The terms within the bracket represent a circularly polarized wave with positive helicity and we know that this is defined in the internal coordinates. This may be expressed in a slightly different form as

$$\phi^R \quad = \quad 2\left\{ \xi_{\mathbf{yo}} \cos\left[\mathrm{E}\left(x' - vt'\right)/\hbar c - \theta\right] - \xi_{\mathbf{zo}} \sin\left[\mathrm{E}\left(x' - vt'\right)/\hbar c - \theta\right]\right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} \qquad \text{(VA)}$$

Although (V) represents a right handed rotation of the amplitude vector, in the rest frame it gives an interesting result. Let us denote ϕ^R in the rest frame by ϕ_o^R given by

$$\phi_o^R = \quad 2\left\{ \xi_{\mathbf{yo}} \cos\, k_o x_o' - \xi_{\mathbf{zo}} \sin k_o x_o' \right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} \qquad \text{(VB)}$$

Here the spatial component of the standing wave forms a standing helical structure in the internal coordinates.

Now taking $\boldsymbol{\xi}_{\mathbf{yo}} = \xi_o \cos\varphi$ and $\boldsymbol{\xi}_{\mathbf{zo}} = \xi_o \sin\varphi$, we have

$$\phi^R \quad = \quad 2\xi_o \cos\left[E\left(x' - vt'\right)/\hbar c - \theta + \varphi\right] e^{-i\hbar^{-1}(Et - \text{þ}x)} \qquad \text{(VI)}$$

Since the angle φ can be chosen arbitrarily, we may drop it to obtain

$$\phi^R \quad = \quad 2\xi_o \cos\left[E\left(x' - vt'\right)/\hbar c - \theta\right] e^{-i\hbar^{-1}(Et - \text{þ}x)} \qquad \text{(VIA)}$$

If instead of taking the sum of the two plane polarized standing waves $\phi(\xi_y)$ and $\phi(\xi_z)$ had we taken the difference, we would have obtained the standing wave in the spin down state and could be represented by ϕ^L given by

$$\begin{aligned} \phi^L \quad &= \quad 2\left\{ \xi_{\mathbf{yo}} \cos\left[\text{þ}c\left(t' - x'/v\right)/\hbar + \theta\right] - \xi_{\mathbf{zo}} \sin\left[\text{þ}c\left(t' - x'/v\right)/\hbar + \theta\right]\right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} && \text{(VII)} \\ &= \quad 2\left\{ \xi_{\mathbf{yo}} \cos\left[\mathrm{E}\left(x' - vt'\right)/\hbar c - \theta\right] + \xi_{\mathbf{zo}} \sin\left[\mathrm{E}\left(x' - vt'\right)/\hbar c - \theta\right]\right\} e^{-i\hbar^{-1}(Et - \text{þ}x)} && \text{(VIIA)} \\ &= \quad 2\xi_o \cos\left[E\left(x' - vt'\right)/\hbar c - \theta - \varphi\right] e^{-i\hbar^{-1}(Et - \text{þ}x)} && \text{(VIIB)} \end{aligned}$$

Note that for the spin up state and the spin down state, only φ changes sign, while rest of the expression remain unchanged.