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Chaotic Dynamics of the Renormalization Group Flow and Standard Model Parameters

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Abstract: Bringing closure to the host of open questions posed by the current Standard Model for particle physics (SM) continues to be a major challenge for theoretical physics community. Motivated by recent advances in the study of complex systems, our work suggests that the pattern of particle masses and gauge couplings emerges from the critical dynamics of renormalization group equations. Using the ε-expansion method along with the universal path to chaos in unimodal maps, we find that the observed hierarchies of SM parameters amount to a series of scaling ratios depending on the Feigenbaum constant.

Key words: Renormalization Group; Critical phenomena; Feigenbaum scaling; Standard Model

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1 Introduction and motivation

As of today, predictions inferred from the Standard Model of elementary particles (SM) — a body of knowledge discovered in the early 1970's — agree with all the experiments that have been conducted to date. Nevertheless, the majority of particle theorists feel that SM is not a complete framework, but rather an "effective field theory" that needs to be amended by new physics at some higher energy scale reaching in the TeV region. Despite years of research on multiple fronts, there is currently no compelling and universally accepted resolution to the challenges posed by SM. Expanding on a series of recent investigations centered on the contribution of chaos and complexity in field theory [1-11, 13-14], the present work suggests that chaotic dynamics of the renormalization group (RG) flow may explain some of the many unsettled questions regarding SM. Specifically, we argue that the physical parameters of SM emerge from the universal period-doubling scenario of the RG flow near its critical manifold.

We emphasize from the outset the introductory nature of our work. As such, our contribution is not aimed to be a rigorous and formally complete solution for the host of complex challenges posed by SM. We have chosen to proceed from a less conventional standpoint, outline a new research avenue and point out its merits and limitations.

It is well known that critical behavior is characterized by an unbounded growth of the correlation length [15, 19]. It is also known that the momentum cutoff Λ and the dimensional parameter of the regularization program $\varepsilon = d_c - d$ are dependent entities [15, 28, 32 and Appendix A]. As a result, the approach to criticality is understood as an iterative process whose limit corresponds to $\Lambda \to \infty$ or $\varepsilon \to 0$. Starting from these observations, we develop our work from two premises:

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1.1 Critical phenomena in continuous dimension [22-23,Appendix A]

This recently formulated framework is based on the idea that the space-time dimension assumes the role of a continuous control parameter. This is consistent with the celebrated RG theory of Wilson and Fisher according to which the problem of computing the partition function and critical exponents can be analytically continued to arbitrary dimension d [12].

1.2 Fully developed deterministic chaos is a critical phenomenon [24, Appendix B]

If the beta function of the coupling flow represents a one-dimensional unimodal map, fully developed chaos is expected to occur according to the Feigenbaum scenario [7, 26]. Appendix B outlines the close connection between the approach to criticality in field theory and the universal transition to chaos driven by period-doubling bifurcations.

For the sake of concision and simplicity, we restrict our analysis to the infrared limit of self-interacting scalar field theory. It is in this limit where the Landau-Ginzburg (LG) action provides a unified description of the long-wavelength behavior associated with many dynamical systems [15, 19, 27]. Despite the fact that LG model is not a substitute for relativistic quantum field theories, it gives insights into how dynamics evolves near criticality. With these considerations in mind, the LG action offers a convenient baseline for understanding the primary attributes of infrared quantum electrodynamics (QED), of ultraviolet quantum chromodynamics (QCD) and asymptotically free theories.

The paper is organized as follows: The implementation of the ε-expansion method in gauge field theory of is detailed in section 2. Critical dynamics as a possible foundation for the electroweak model forms the topic of section 3. Section 4 links the universal Feigenbaum scenario to gauge field theory and formulates predictions on SM parameters. Scaling corrections to the Feigenbaum constant are briefly introduced in section 5. The last two sections are devoted to a listing of open questions, future challenges and concluding remarks.

2 RG flow analysis in the Wilson-Fisher picture

We start from the standard LG action for the $O(N)$ field theory in d dimensions, driven by external sources [30-31]

$$S[\sigma] = \int d^dx\{\frac{1}{2}\sigma_\alpha(x)[r - \nabla^2]\sigma_\alpha(x) + \frac{u}{4}[\sigma_\alpha(x)\sigma_\alpha(x)]^2 - j_\alpha(x)\sigma_\alpha(x)\} + S_{J_0} \tag{1}$$

Here, $\sigma(x) = (\sigma_\alpha(x))$ represents the Yang-Mills field having $\alpha = 1, 2, ...N$ independent components, $j = (j_\alpha(x))$ is the external fermion current (whose contribution to the action in the absence of interactions is denoted by S_{J_0}) and the summation convention is implied. To make the ensuing derivation more transparent and without a significant loss of generality, we proceed with the following set of simplifying assumptions:

a1) space-time dimension is near four, that is $d = 4 - \varepsilon$. As previously pointed out, according to the philosophy of critical phenomena in continuous dimension, ε is regarded as the sole control parameter driving the LG dynamics [22-23]. Following (A3), fine-tuning ε is formally equivalent to continuous changes of momentum cutoff Λ. By definition, for a given reference mass μ_0, the momentum cutoff is $\Lambda^2 > \mu_0^2$. Then the passage to the physical limit $\varepsilon \to 0$ can be approached in two distinct ways: a) $\Lambda \to \infty$ and $\mu_0 < \infty$; b) $\Lambda < \infty$ and $\mu_0 \to 0$. The latter condition matches the infrared behavior of field theory, i.e. its long-wavelength properties $|\mathrm{Q}| \propto \mu_0 \to 0$, in which $|Q|$ represents the magnitude of momentum transfer. We exclusively focus below on this asymptotic regime of field theory, whereby $\mu_0^2 \to 0, \mu_0^2 > 0$.

a2) the discussion is limited to the O (1) model, i.e. the gauge field $\sigma(x)$ is treated as a scalar.

a3) the overall fermion current contains two terms,

$$J(x) = j(x) + J_0(x) \tag{2}$$

where $j(x)$ represents the component that couples to $\sigma(x)$ and $J_0(x)$ the free (non-interacting) component.

If $j(x)$ is uniform and couples only to the $k = 0$ mode of $\sigma(x)$, its contribution to the action reads

$$S_j = -j \int \sigma(x) d^d x = -j\sigma_0 \tag{3}$$

in which

$$\sigma_0 = \sigma(k = 0) \tag{4}$$

These assumptions imply that j is not affected by momentum shell integration and its normalization follows the prescription of the Gaussian model [30]. It is instructive to note that, since dimensional analysis requires $[j] = mass^3$, the energy carried by the overall current $J(x)$ for a given reference mass μ_0(cf. (A2)) may be expressed as

$$E_J = \mu_0 \int J(x) d^3x = \mu_0 \int [j + J_0(x)] d^3x \tag{5}$$

If we take $J_0(x)$ to be uniform as well, its contribution to the action amounts to an additive constant that may be represented as $S_{J_0} = J_0 \int d^3x \cong J_0 \mu_0^{-3}$.

Under these circumstances, the action reads

$$S[\sigma] = \int d^4x \frac{1}{2}\{\sigma(x)[r - \nabla^2]\sigma(x) + \frac{u}{2}\sigma(x)^4\} - j\sigma_0 + S_{J_0} \tag{6}$$

The nontrivial expectation of the field vacuum v results from minimizing the LG potential, that is

$$\mathrm{v} = \pm\sqrt{\frac{(-r)}{u}} \Rightarrow r = -u\mathrm{v}^2 \tag{7}$$

The source term is typically treated as a relevant perturbation that vanishes on the critical manifold. Accordingly, the flow equations in the $\varepsilon-$expansion taken to quadratic order assume the form [27, 30]

$$\begin{aligned} \beta(u) &= \frac{\partial u}{\partial t} \approx \varepsilon u - 3bu^2 \\ \beta(r) &= \frac{\partial r}{\partial t} \approx r(2 + bu) + au \\ \beta(j) &= \frac{\partial j}{\partial t} \approx (\frac{d}{2} + 1)j = (\frac{4-\varepsilon}{2} + 1)j \end{aligned} \tag{8}$$

where

$$a = 3K_4\Lambda^2, \quad b = 3K_4 \tag{9}$$

Here, $K_4 = (8\pi^2)^{-1}$ in $d = 4$ dimensions [27-28]. With these notations, the Wilson-Fisher fixed point of (8) is given by

$$u^* = \frac{1}{3b}\varepsilon, \quad r^* = -\frac{a}{6b}\varepsilon \tag{10}$$

and the critical value of the field vacuum (7) becomes

$$\mathrm{v}^* = \pm\left(\frac{3K_4}{2}\right)^{1/2}\Lambda \tag{11}$$

On the other hand, starting from (A3) and assuming $\varepsilon \ll 1$, a leading order calculation of the momentum cutoff corresponding to a non-vanishing reference mass μ_0 yields roughly the approximation

$$\Lambda^2 \approx \frac{\mu_0^2}{\varepsilon} \tag{12}$$

From (8), (11) and (12) we obtain

$$u^*(\mathrm{v}^*)^2 = \frac{1}{6}\mu_0^2 = const. \tag{13}$$

Returning to (8), the equation of the fermion flow may be written as

$$\beta(j) = \frac{\partial j}{\partial t} \approx 3(1 - \frac{\varepsilon}{6})j \tag{14}$$

(14) is solved in closed form by

$$j \approx j_0 \exp[3(1 - \frac{\varepsilon}{6})(t - t_0)] \tag{15}$$

with $j_0 = j(t_0)$. Expanding (15) and keeping only the leading-order term yields the normalized fermion current

$$(j_0^*)_f \approx \frac{j}{[1 + 3(t - t_0)]j_0} = 1 + \mathrm{A}\varepsilon \tag{16}$$

where $\mathrm{A} = -\frac{1}{2}(t-t_0)[1+3(t-t_0)]^{-1}$ is a factor independent of ε. Finally, to get a consistent representation of parameters described by (10) and (16), we proceed by shifting the origin in (16) and introducing the definition

$$\triangle j_f^* = A^{-1}[(j_0^*)_f - 1] \approx \varepsilon \tag{17}$$

With the help of (13), the set of critical parameters $(r^*, u^*, \Delta j_f^*)$ may be translated in field theoretic language using the identification

$$\begin{gathered} r^* \Leftrightarrow (g^*)^2(\mathrm{v}^*)^2 = (g^*)^2 M^2 = \frac{1}{6}\mu_0^2 \\ u^* \Leftrightarrow (g^*)^2 \propto \varepsilon \\ \Delta j_f^* \Leftrightarrow m_f^* \propto \varepsilon \end{gathered} \tag{18}$$

in which M stands for the mass of the gauge boson, g^* for the critical gauge coupling and m_f^* for the normalized fermion mass. We note that the first relation in (18) represents an approximation consistent with a1). The last relation in (18) is a consequence of (5), which makes explicit connection between the fermion current and its energy content.

We also note that, following (11) and (13), the critical value of the gauge field vacuum v^*, momentum cutoff Λ and the gauge boson mass M are linearly related to each other.

To summarize this section, near criticality and under the assumptions previously introduced, a) relation (13) holds for any arbitrarily small but non-vanishing reference mass μ_0; b) coupling strengths and fermion masses scale linearly with ε. In vector format, combined use of (8), (13), (17) and (18) yields

$$P = \begin{bmatrix} M^{-2} & (g^*)^2 & m_f^* \end{bmatrix} \approx \varepsilon \tag{19}$$

3 Critical dynamics and the electroweak model

Using (18), we may express (13) in an alternative way, namely as

$$(M_n g_n^*)^2 = (M_{n+1} g_{n+1}^*)^2 = ... = (M_{n+q} g_{n+q}^*)^2 = ... = const. \tag{20}$$

where $n, q \gg 1$ denote iteration indices that follow the arguments of section 4. For the first two terms of this infinite series we obtain

$$\frac{M_Z^2}{M_W^2} = \frac{g_2^2 + e^2}{g_2^2} = 1 + \frac{\alpha_{EM}}{\alpha_2} \tag{21}$$

in which $\alpha_{EM} = e^2/_{4\pi}$ and $\alpha_2 = g_2^2/_{4\pi}$. The rationale for (21) lies in the fact that the charged vector boson $W^{\pm}$ carries a superposition of weak and electromagnetic charges, whereas the neutral vector boson Z^0 carries only the weak isospin charge. Inverting the above and taking into account (24a)-(26), (25) and Tab. 2 from section 4, leads to

$$\frac{M_W^2}{M_Z^2} = \frac{1}{1 + \frac{\alpha_{EM}}{\alpha_2}} = \frac{1}{1 + \frac{1}{\delta}} \approx 1 - \frac{1}{\delta} = \cos^2\theta_W \tag{22}$$

(22) suggests a natural explanation for the Weinberg angle θ_W. Likewise, we may write (20) and (21) as

$$\frac{g_2^2}{M_W^2} = \frac{g_2^2 + e^2}{M_Z^2} = const \tag{23a}$$

This relation offers a straightforward interpretation for both Fermi constant and the mass of the "would-be" Higgs boson. Indeed, in SM we have [28, 32]

$$\frac{g_2^2}{M_W^2} = 4\sqrt{2}G_F \tag{23b}$$

and

$$\mathrm{v}(\varphi^0) \propto \sqrt{\frac{1}{G_F\sqrt{2}}} \approx 246.22GeV \tag{23c}$$

Here, G_F is the Fermi constant and $\mathrm{v}(\varphi^0)$ denotes the vacuum expectation value for the neutral component of the Higgs doublet. Relations ((23a)-(23c)) lead to the intriguing conclusion that G_F and $\mathrm{v}(\varphi^0)$ are not determined by a symmetry breaking mechanism in the electroweak sector but rather by the intrinsic attributes of the RG flow near criticality.

Relations (18) and (19) indicate that, near criticality, fermion masses scale linearly with the coupling strength $\alpha = {g^2}/{4\pi}$. Gauge bosons acquire mass as a result of self-interaction, whereas fermions acquire supplemental mass through coupling to gauge bosons. According to (5), the interaction energy absorbed by fermions adds to the mass carried by the free fermion current. In the physical limit $\varepsilon \to 0$ the gauge field becomes massless* and the fermion current decouples from the gauge field.

4 Fully developed chaos and parameter scaling

The object of this section is to analyze the dynamics of (8) with the standard methods employed in the study of nonlinear maps [18, 27]. To this end, we first note that the last equation in (8) is uncoupled to the first two. This enables us to reduce the dimensionality of (8) to a planar system of differential equations. The next step is to transform (8) into the corresponding two-dimensional map

$$u_{n+1} \approx (1 + \varepsilon\Delta t)u_n - 3b\Delta t u_n^2 \tag{24a}$$

$$r_{n+1} \approx r_n(2\Delta t + 1 + bu_n\Delta t) + au_n\Delta t \tag{24b}$$

where Δt denotes the increment of the dimensionless sliding scale (A2) and n is the iteration index. Linearizing (24) and computing its Jacobian J,we find

$$J = 1 + \Delta t(\varepsilon + 2) \tag{24c}$$

It follows that map (24a,b) is dissipative for $\Delta t \neq 0$ and becomes asymptotically conservative in the physical limit $\Delta t = 0$. Invoking universality arguments [27] we conclude that, near criticality, (24a,b) shares the same universality class with the quadratic map (24a). Furthermore, near the Feigenbaum attractor, the control parameter ε approaches the critical value $\varepsilon_\infty = 0$ according to the geometric progression:

$$\varepsilon_n - \varepsilon_\infty \approx a_n \cdot \delta^{-n} \tag{25}$$

Here, $n \gg 1$ is the index defining the number of iteration steps, $\delta = 4.669$...is the Feigenbaum constant for the quadratic map and a_n is a coefficient which becomes asymptotically independent of n, that is, $a_\infty = a$ [18, 26, 29]. Direct substitution of (25) in (19) yields the series

$$P(n) = \left[\ M_n^{-2} \quad (g^*)_n^2 \quad (m_f^*)_n\ \right] \propto \delta^{-n} \quad \texttt{if}\ \ n \gg 1 \tag{26}$$

*Here we refer to the dressed gauge boson mass $m = g^* M \propto \mu_0$ (see also (A6) and (A7)).

Period-doubling bifurcations are characterized by $n = 2^p$, with $p \geq 1$. The ratio of two consecutive terms in the parameter series (26) is given by

$$\frac{P_j(p+1)}{P_j(p)} \propto \delta^{-(2^p)} \tag{27}$$

It is important to emphasize that (27) is only valid up to a leading-order approximation considering that, a) no higher order corrections are accounted for, b) (27) is expected to be much less accurate if the iteration index is not large enough, that is, if $p \approx O(1)$, c) there is a fair amount of uncertainty involved in determining the quark mass spectrum [37].

Numerical results derived from (27) are displayed in Tab. 2. This table contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks and a similar comparison of gauge coupling ratios. Tab. 1 contains the set of known quark and vector boson masses as well as the SM coupling strengths. All quark masses are reported at the energy scale given by the top quark mass and are averaged using the most recent reports issued by the Particle Data Group [37]. Vector boson masses are evaluated at the scale of the electroweak interaction $(G_F^{-1/2} \approx 293 GeV)$and the coupling strengths at the scale set by the mass of the Z^0 boson.

Table 1: Actual values of selected SM parameters

Parameter	Value	Units
m_u	2.12	MeV
m_d	4.22	MeV
m_s	80.90	MeV
m_c	630	MeV
m_b	2847	MeV
m_t	170,800	MeV
M_W	80.46	GeV
M_Z	91.19	GeV
α_{EM}	1/128	-
α_W	0.0338	-
α_{QCD}	0.123	-

Data on the horizontal axis is partitioned in ascending order, i.e. $1 = 1 - (M_W/M_Z)^2$; $2 = m_\mu/m_\tau$; $3 = m_d/m_s$; $4 = (\alpha_{EM}/\alpha_W)^2$; $5 = m_s/m_b$; $6 = m_e/m_\mu$; $7 = m_c/m_t$; $8 = m_u/m_c$; $9 = (\alpha_{EM}/\alpha_s)^2$. The scaling sequence of charged leptons and quarks may be graphically summarized with the help of the following diagrams:

$$\overbrace{d \quad s}^{\delta^2} \quad \overbrace{s \quad b}^{\delta^2}$$

$$\overbrace{u \quad c}^{\delta^4} \quad \overbrace{c \quad t}^{\delta^4}$$

$$\overbrace{e \quad \mu}^{\delta^4} \quad \overbrace{\mu \quad \tau}^{\delta^2}$$

$$\overbrace{\alpha_{EM}^2 \quad \alpha_W^2}^{\delta^2} \quad \overbrace{\alpha_W^2 \quad \alpha_s^2}^{\delta^2}$$

Based on the above scheme, one may infer that one of the possible patterns for neutrino masses is represented by:

$$\overbrace{\nu_e \quad \nu_\mu}^{\delta^{16}} \quad \overbrace{\nu_\mu \quad \nu_\tau}^{\delta^4}$$

It is also instructive to note that quarks and charged leptons follow a different period doubling pattern. To this end, let us organize the charged lepton and quark masses in a collection of triplets, that is

$$M_{lepton} = [m_{electron}, m_{muon}, m_{taon}]; M_{quark} = [(m_u, m_c, m_t), (m_d, m_s, m_b)] \tag{28}$$

It can be seen that the mass scaling for adjacent quarks stays constant within either one of the triplets (u, c, t) or (d, s, b), whereas the mass scaling for charged leptons varies as a geometric series in δ_2^2 within the triplet (e, μ, τ). This finding points out toward a symmetry breaking mechanism that segregates lepton and quark phases during cooling from the far ultraviolet region of field theory to the low-energy region of SM.

5 Scaling corrections

The standard Feigenbaum scaling may be generalized as follows [17]:

$$\varepsilon_n - \varepsilon_\infty = \sum_{i=\varepsilon 0}^{\infty} a_i \delta_i^{-n} \quad \texttt{for} \quad n \gg 1 \tag{29}$$

This entails that the ratio of two arbitrary observables amounts to a more complex expression than a simple scaling. Further studies are needed to evaluate if corrections (29) can close the numerical gap between predictions and actual data.

6 Open questions

As pointed out in section 1, the analysis we have undertaken is far from being either entirely rigorous or formally complete. Although leading-order predictions match reasonably well the existing experimental database, follow-up efforts are required to provide all the necessary clarifications. The list of open questions includes (but is not limited to) the following items:
a) how does the Higgs mechanism of generating masses fit into the picture? Is the Higgs doublet still necessary in view of results discussed in sections 3 and 4?
b) what is the dynamic role of scaling corrections (29)?
c) can the hierarchy of mixing angles be consistently derived from this approach? [7]
d) is there experimental evidence for additional fermion and gauge boson states that fit the hierarchical patterns implied by Feigenbaum universality?
e) what can be gained if one starts from the non-perturbative renormalization group instead of the traditionalε-expansion? [25]

7 Summary and concluding remarks

Inspired by recent advances in the study of complex systems, our investigation has shown that the pattern of particle masses and gauge couplings may emerge from the chaotic dynamics of RG flow equations. Combining the ε-expansion method with the period-doubling path to chaos in unimodal maps, we have found that the observed hierarchies of SM parameters amount to a series of scaling ratios depending on the Feigenbaum constant.

It is instructive to note that our results are consistent with the recently developed Feigenbaum-Sharkovskii-Magnitskii (FSM) paradigm for the universal onset of chaos in multidimensional systems of nonlinear autonomous differential equations [38].

Although we have exclusively treated the infrared limit of field theory, the dual perturbation model may be used to link the two asymptotic regions of field theory [20-21]. To the extent that the perturbation expansion is still considered reliable near criticality, the ultraviolet regime of strong coupling and short wavelength modes can be treated on equal footing with the infrared regime of opposite features. This observation expands, in principle, the applicability of our study.

In closing, it is worth emphasizing that critical dynamics in continuous dimension is deeply related to the novel frameworks of fractional dynamics and non-extensive statistical physics. It is reasonable to expect that these analytic tools will play a key role in modeling the rich array of complex phenomena that may arise beyond SM [33-36].

Table 2: Actual versus predicted SM parameters

Parameter ratio	**Behavior**	**Actual**	**Predicted**
m_u/m_c	δ^{-4}	3.365×10^{-3}	2.104×10^{-3}
m_c/m_t	δ^{-4}	3.689×10^{-3}	2.104×10^{-3}
m_d/m_s	δ^{-2}	0.052	0.046
m_s/m_b	δ^{-2}	0.028	0.046
m_e/m_μ	δ^{-4}	4.745×10^{-3}	2.104×10^{-3}
m_μ/m_τ	δ^{-2}	0.061	0.046
$(\alpha_{EM}/\alpha_W)^2$	δ^{-2}	0.045	0.046
$(\alpha_{EM}/\alpha_s)^2$	δ^{-4}	2.368×10^{-3}	2.104×10^{-3}
$1-(M_W/M_Z)^2$	δ^{-1}	0.2215	0.2142

APPENDIX A: Critical dynamics in continuous dimension

It is well known that the dependence of the coupling charge on the energy scale is a basic outcome of RG in quantum field theory. According to this viewpoint, the beta-function of a generic field model determines how a coupling g runs with the sliding energy scale μ, that is

$$\beta(g) \doteq \mu \frac{\partial g(\mu,\varepsilon)}{\partial \mu} = \frac{\partial g(t,\varepsilon)}{\partial t} \tag{A1}$$

Here, the derivative is taken at fixed ε and the dimensionless sliding scale

$$t \doteq \ln\left(\frac{\mu}{\mu_0}\right) \tag{A2}$$

is considered in relation to an arbitrary reference mass μ_0. The dimensional parameter is given by [32]

$$\varepsilon \approx [\log(\frac{\Lambda^2}{\mu_0^2})]^{-1} \tag{A3}$$

In the basin of attraction of the critical point t_c the field correlation length scales as [15, 19]

$$\xi \approx |t-t_c|^{\nu} \tag{A4}$$

Here, critical exponent ν is related to the first order derivative of the beta function through [22-23]

$$\nu^{-1} = -\frac{\partial \beta}{\partial (g)}\bigg|_{g=g^*} \tag{A5}$$

and g^* stands for a fixed point of the beta-function, $\beta(g^*) = 0$. As previously stated, the continuity of the beta-function with respect to $\varepsilon = d_c - d$ makes it possible to interpret ε as a control parameter in the same way the energy scale acts as a control parameter in RG. According to this philosophy, the sliding scale t and ε play an interchangeable role [22-23]. The mass of the underlying field is known to be inversely proportional to the divergent correlation length and identically vanishes at the fixed point

$$m[g^*(d_c), d_c] = 0 \tag{A6}$$

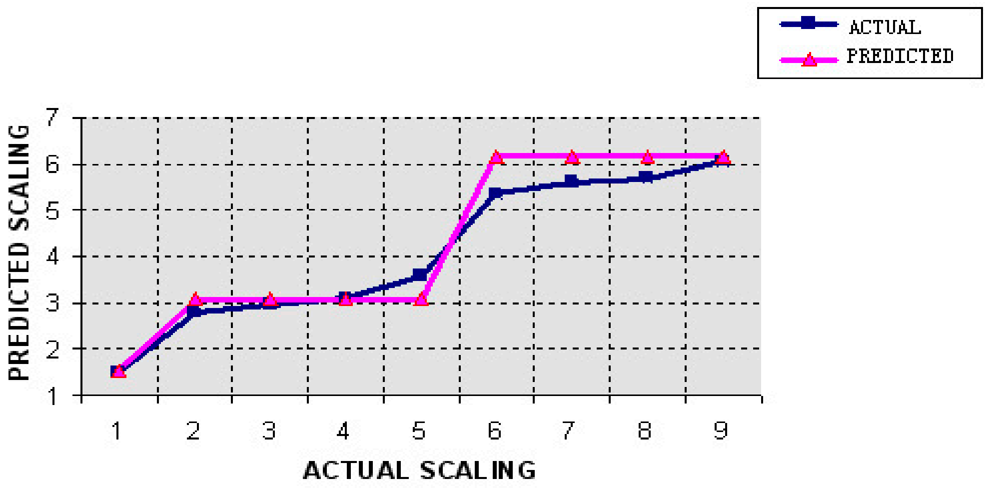

Figure 1: Predicted vs. actual SM parameters (abs. log. scale)

In the basin of attraction of g^* the field develops mass according to the power law

$$m[g^*(d_c), d_c - d] \approx |d_c - d|^{\nu(d_c)} = \varepsilon^{\nu(d_c)} \tag{A7}$$

where, by analogy with (A5)

$$\nu^{-1}(d_c) = -\frac{\partial\beta}{\partial(g)}\bigg|_{g^*(d_c)} \tag{A8}$$

One is led to conclude that, as the fixed point is asymptotically approached and the continuous space-time dimensionality collapses to $d \to d_c = 4$, the underlying field theory becomes massless, in agreement with the current formulation of quantum field theory [28, 32]. Numerical analysis yields $\nu(d_c) = 0.5$ for $d_c = 4$ which is found to match well the value reported in the literature [22-23].

APPENDIX B: Chaotic dynamics as a critical phenomenon

A very close analogy is known to exist between criticality in statistical physics and field theory, on the one hand, and the onset of deterministic chaos in nonlinear dynamical systems, on the other [16, 24]. Specifically, consider a field theory defined on a linear space lattice $z_l = la$, $l \in Z$ characterized by a microscopic length scale $\Delta z = a \propto \Lambda^{-1}$ and a macroscopic length scale $L = Na$. Here, $N \gg 1$ represents the number of occupied lattice sites. The short-range coupling between neighboring sites is described by the pair hamiltonian

$$h_l(\lambda) = -\lambda\sigma_l\sigma_{l+1} \tag{B1}$$

where $\lambda = \lambda(t)$ is the neighbor-to-neighbor coupling. Assuming dimensionless units, criticality is defined by the occurrence of a second-order phase transition in the thermodynamic limit $N \to \infty$ and strong coupling regime $K \to K_c = \infty$, where $K = \lambda t^{-1}$ denotes the reduced coupling constant. Here, we account for (A2) and take $t_c = 0$ when $\mu \to \mu_c = \mu_0$.

Likewise, consider a generic dynamical system described by the unimodal evolution map

$$x_{l+1} = f_\eta(x_l) \tag{B2}$$

Here, the time variable is discretized through $t_l = l\tau$, $l \in Z$ with the microscopic time scale $\delta t = \tau \propto \Lambda^{-1}$ and η is a control parameter. The coordinate at a given time is $x_l = x(t_l) \in [-1, +1]$ and the asymptotic time limit corresponds to a large number of iteration steps $N \gg 1$ or, equivalently, to a large macroscopic time scale $T = N\tau$. The onset of deterministic chaos in this limit is identical to driving the field dynamics towards criticality by continuously tuning the control parameter ($\eta \rightarrow \eta_c$).

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