On Representing a Particle By a Standing Luminal Wave- Revisiting de Broglie's Phase Wave

V.A.Induchoodan Menon,
9, Readers' Row Houses, Gujarat University
Campus, Navrangpura, Ahmedabad-380009,
Gujarat, India. e-mail: induchoodanmenon@yahoo.co.in

Abstract

De Broglie when he introduced the concept of the phase wave to represent a particle, assumed that in the rest frame of reference the particle will have the form of a standing vibration. According to the author, this was a serious mistake. He shows that instead, had de Broglie assumed a standing luminal wave structure for the particle, it would have led him to very exciting insights. The author shows that in a relativistic transformation the average energy and the momentum of the forward and the reverse waves forming the standing wave transform exactly like the energy and momentum of a particle. Besides, the plane wave expansion which is used to represent a particle in quantum mechanics is found to emerge directly from this standing wave structure. He proposes to extend the approach to incorporate the spin of the particle and also provide a simple explanation for the Pauli's exclusion principle.

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1 Introduction

We know that the beginning of the wave mechanics could be traced to the wave particle hypothesis put forward by de Broglie [1]. According to de Broglie's hypothesis, each particle can be associated with a wave called the phase wave which defines its wave nature. He strongly believed that the phase wave exists in the real space. He observed that there were two equations which were central to the new physics. One introduced by Planck, given by E = hν, that connected energy with frequency of a wave and the other introduced by Einstein, given by $E = mc^2$, that related energy with mass. He related rest energy and frequency by proposing that a particle has a wave nature inherent in it and represented a stationary particle in terms of standing vibration

$$\phi_o \quad = \quad \xi_o e^{-i\hbar^{-1}E_o t_o} \quad . \tag{1}$$

Obviously, when the particle is observed from a second frame of reference with which it has a uniform velocity v, then, the above equation would undergo a relativistic transformation to give

$$\phi \quad = \quad \xi_o e^{-i\hbar^{-1}E\left(t - \mathrm{v}x/c^2\right)} \quad . \tag{2}$$

Here ξ_o is assumed to a relativistic invariant for the sake of convenience.

We know that the phase of the wave $E_o t_o/\hbar$ is conserved in a relativistic transformation. Therefore ϕ_o and ϕ will always be in phase. This was the reason why de Broglie called this wave the "phase wave". Equation (2) can be modified introducing momentum þ, where þ = $E\mathrm{v}/c^2$ and we obtain

$$\phi \quad = \quad \xi_o e^{-i\hbar^{-1}\left(Et - \text{þ}x\right)} \quad . \tag{3}$$

The main problem with the de Broglie's phase wave is that it is a plane progressive wave which is not localized and therefore ill-suited to represent a particle. Besides, the phase velocity of the plane wave is c^2/v which is more than the velocity of light and therefore could not have any physical meaning according to the theory of relativity.

To overcome this problem, it was necessary to take a group of waves instead of a single wave to represent a particle. The wave packet can be expressed as

$$\psi \quad = \quad \int_{E-\Delta E}^{E+\Delta E} \xi(E')\, e^{-i\hbar^{-1}\left(E't-þ'x\right)} dE', \tag{4}$$

where the region of integration is in the narrow band $(E-\Delta E) \le E' \le (E+\Delta E)$. In that region, we may take $þ' = þ(E') + (E-E')(dþ'/dE')$. Now taking $\xi(E')$ to be a constant in the narrow region denoted by ξ and on carrying out the integration we obtain

$$\psi \quad = \quad \frac{2\xi\,\hbar \sin\left[\Delta E \hbar^{-1}\left(t - x/\mathrm{v}\right)\right]}{\left(t - x/\mathrm{v}\right)} e^{-i\hbar^{-1}(Et - þx)} \quad . \tag{5}$$

Here we have taken $dE'/dþ' = v$. The above equation represents a plane wave with energy E and momentum þ having a very sharp maximum in its amplitude at the point x = vt while being negligible elsewhere. This allows us to identify the location of the particle at that point where the amplitude of the resultant wave has a very sharp maximum. This also allows us to identify v with the velocity of the particle. But assuming the momentum of the waves follows the Gaussian distribution, it was found that the width of the wave packet is given by the relation [2]

$$\Delta x(t) \quad = \quad \left(\hbar/\Delta þ\right)\left\{1 + \left[\frac{(\Delta þ)^4}{m^2\hbar^2}\right] t^2\right\}^{1/2} \quad . \tag{6}$$

This shows that as t increases, Δx becomes very large and the wave packet becomes highly dispersed. In the early years of quantum mechanics this spreading of the wave packet forced the physicists to discard it as a viable representation for a particle. Remember that in the early years these waves were taken as real waves existing in real space-time that carried with it energy and momentum.

When Schrödinger solved the problem of hydrogen spectra using the equation named after him, it became obvious that each wave which is a solution to the equation, represents a particle. There was no need to introduce the idea of wave packet in solving the problem. Gradually, the idea that the wave representing a particle does not represent a physical wave started gaining ground. Ultimately, when Max Born came out with the idea that each of the waves actually represents not a physical wave, but a probability wave with the square of the amplitude representing the probability to occupy that particular state, his idea received wide acceptance and the experimental results confirmed it. In the light of these developments the idea that the de Broglie wave represented not a physical wave, but rather a probability wave, came to be accepted widely after the successful application of Born's interpretation on various problems in atomic physics [3].

With the acceptance of the idea that the wave function is essentially a probability wave, the search for an inner structure to the particle became unimportant. Everything worth knowing about the particle is assumed to be contained in the probability wave. Apparently, such self limiting approach seems to have killed any new initiative for a deeper understanding of the nature of matter. We shall now investigate the implications of treating a particle as a standing luminal wave.

2 Standing Wave Structure of the Plane Wave

We shall now show that de Broglie might have been mistaken in assuming that the particle in the stationary state is represented by a standing vibration given in (1). Instead, had he taken a standing wave formed by a luminal wave to represent a

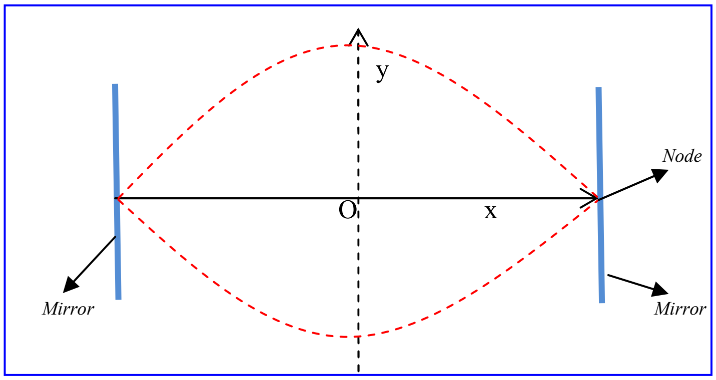

FIGURE 1

The dotted lines in the diagram represent a standing wave with nodes at the reflecting surfaces

particle, the situation would have been quite different. It would have been possible to show a real confined luminal wave structure to a particle. We shall later show that such structure would still end up in the plane wave representation of a particle. For the sake of convenience we shall take the case of the standing half wave formed by the reflection of the luminal wave between two mirrors kept facing each other (figure 1). We may represent the standing wave in the rest frame of reference as a linear combination of a forward wave and a reverse wave given by

$$\begin{aligned}\phi_o &= \xi\left(e^{-i(E_o t_o - p_o x_o)/\hbar} + e^{-i(E_o t_o + p_o x_o)/\hbar}\right)\\ &= 2\xi \cos\left(p_o\, x_o/\hbar\right)\, e^{-i\hbar^{-1}E_o t_o}\ . \end{aligned} \tag{7}$$

If we now observe the system of the standing wave from a frame of reference with regard to which it has a velocity v, then the forward and reverse waves will undergo Doppler shift in their energy and momentum values and we obtain

$$\phi = \xi\left(e^{-i(E_1 t - p_1 x)/\hbar} + e^{-i(E_2 t + p_2 x)/\hbar}\right)\ , \tag{8}$$

$$where \quad E_1 = \gamma E_o(1 + \mathrm{v}/c),\ E_2 = \gamma E_o(1 - \mathrm{v}/c) \ \ and \ \ \gamma = \left(1 - \mathrm{v}^2/c^2\right)^{-\frac{1}{2}} \tag{8A}$$

If we now denote the average value of the energy and momentum of the forward and the reverse waves taken together as E and þ respectively, then we have

$$E = \tfrac{1}{2}(E_1 + E_2) = \gamma E_o \ \ and \ \ þ = \tfrac{1}{2}(p_1 - p_2) = \tfrac{1}{2}(E_1 - E_2)/c = \gamma E_o \mathrm{v}/c^2 \tag{9}$$

Note that this is exactly how the energy and momentum of a particle transform under relativistic transformation. Therefore, it appears that the standing luminal wave could be well suited to represent a particle.

Now if we can show that ϕ given in (8) has the same form as the de Broglie wave, then it would mean that the standing wave structure offers a viable representation of a particle. Here we propose to confine to the one dimensional case only. On carrying out some manipulations on (8), keeping in mind the relation

$$(p_1 - þ)/(E_1 - E) = (p_2 + þ)/(E - E_2) = 1/v\ ,\quad E_1/p_1 = E_2/p_2 = c$$

and $(E_1 - E) = (E - E_2) = pc = E\,v/c$, we obtain

$$\phi = 2\xi \cos\left[þc\,(t - x/v)/\hbar\right] e^{-i\hbar^{-1}(Et - þx)}$$

$$= 2\xi \cos\left[E\,(x - vt)/\hbar c\right] e^{-i\hbar^{-1}(Et - þx)} \quad . \qquad (10)$$

Here the amplitude factor "$2\xi \cos\left[E\,(x - vt)/\hbar c\right]$" represents a wave moving with a velocity v. If we take x = vt, the cosine factor in (10) will become unity and ϕ acquires the form of the de Broglie wave (plane wave).

It is very interesting to note that we started with the luminal wave which when confined split into forward and reverse waves having energy E_1 and E_2 respectively. But the standing wave formed by the confinement is seen to be represented by the product of the amplitude wave and the de Broglie wave. The energy of the de Broglie wave is given by (9) which transforms like the energy of a particle. This crystallization of the rest mass by the confinement of the luminal wave is quite surprising and seem to have profound implications.

Note that the standing wave with which we started in (8) has nodes at the ends. If we take a system with anti-nodes at the ends, then the standing wave formed will be given by

$$\phi' = -2\xi \sin\left[E\,(x - vt)/\hbar c\right] e^{-i(Et - þx)} \quad . \qquad (11)$$

If we now assume that ϕ as given by (10) has its amplitude along the z-x plane while the ϕ' given by (11) has its amplitude along the y-x plane, then we may take the linear combination of ϕ and ϕ' to denote a standing circularly polarized wave. Denoting such a standing wave by ϕ'_c, we have

$$\phi'_c = 2\left\{ \boldsymbol{\xi}_\mathbf{y} \cos\left[E(x - vt)/\hbar c\right] - \boldsymbol{\xi}_\mathbf{z} \sin\left[E(x - vt)/\hbar c\right] \right\} e^{-i\hbar^{-1}(Et - þx)} . \qquad (12)$$

3 Localizing the de Broglie Wave

Note that the de Broglie wave that can be obtained from (12) by taking x = vt is not a localized one as the sine and cosine functions have a series of maxima. Therefore, for localizing the particle, we may take a group of waves close to the average value. If we represent the wave packet by ψ, then we have

$$\psi(x,t) = \sum 2\xi \cos\left[E'(x - vt)/\hbar c\right] e^{-i(E't - þ'x)} . \qquad (13)$$

Assuming that the values E' lie very close to the average value E of the group of waves, we may replace the summation by integration over a small region (E-ΔE) ≤ E′ ≤ (E+ΔE) to obtain

$$\psi(x,t) = \int_{E-\Delta E}^{E+\Delta E} 2\xi \cos\left[E'(x - vt)/\hbar c\right] e^{-i(E't - þ'x)}\, dE' \quad . \qquad (14)$$

This right hand side of the above equation can be simplified to give

$$\psi(x,t) = 2\xi\,\Delta E\, f(x - vt) \cos\left[E(x - vt/\hbar c)\right] e^{-i\hbar^{-1}(Et - þx)} . \qquad (15)$$

where $$f(x - vt) = f'(x - vt) + f''(x - vt) \qquad (15A)$$

$$f'(x - vt) = \frac{\sin\left[\hbar^{-1}\Delta E\left(1 + \mathrm{v}/c\right)\left(t - \mathrm{v}\right)\right]}{\hbar^{-1}\Delta E\left(1 + \mathrm{v}/c\right)\left(t - x/\mathrm{v}\right)}$$

and $$f''(x - vt) = \frac{\sin\left[\hbar^{-1}\Delta E\left(1 - \mathrm{v}/c\right)\left(t - x/\mathrm{v}\right)\right]}{\hbar^{-1}\Delta E\left(1 - \mathrm{v}/c\right)\left(t - x/\mathrm{v}\right)} \qquad (15B)$$

The presence of $\triangle E = \mathrm{v}\Delta þ$ in the amplitude in the above equation has interesting implications. The amplitude of the wave in (15) becomes zero as v becomes zero and the wave disappears.

Let us now examine the function f(x-vt). Taking $\Delta E = \mathrm{v}\ \Delta þ$ and $\Delta x = (x\text{-}vt)$ we have

$$f(x - vt) = \frac{\sin\left[\hbar^{-1}\Delta þ\ \Delta \mathrm{x}\left(1 + \mathrm{v}/c\right)\right]}{\hbar^{-1}\Delta þ\,\Delta \mathrm{x}\left(1 + \mathrm{v}/c\right)} + \frac{\sin\left[\hbar^{-1}\Delta þ\,\Delta \mathrm{x}\left(1 - \mathrm{v}/c\right)\right]}{\hbar^{-1}\Delta þ\,\Delta \mathrm{x}\left(1 - \mathrm{v}/c\right)} \quad . \qquad (15C)$$

Here Δx denotes the variation in x from the average value which is vt. We know that for the particle to be localized, Ψ should have a sharp maximum at x = vt. This means that f(x-vt) should be unity as $\Delta x \rightarrow 0$. This condition for maximum for Ψ leads us to very interesting results. We observe that the sharp maximum of the amplitude at $\Delta x = 0$ or x = vt is possible only when $\Delta þ$ is quite large. Thus we observe that the uncertainty principle satisfies the requirement to obtain a sharp maximum for the amplitude. We may take f(x-vt) = 1 at x = vt for all practical purposes. Since the amplitude of the wave in (15) is almost zero everywhere compared to that at x = vt, we may write

$$\psi(x,t) \quad = \quad 4\xi\ \mathbf{v}\ \Delta \mathbf{þ}\ \mathrm{e}^{-\mathrm{i}\hbar^{-1}(Et - þ\mathrm{x})} \quad . \qquad (16)$$

This is the localized de Broglie wave (LDB) that represents a particle located at the point x = vt. In the light of the later development, we know that the problem of the dispersion of the wave packet arises out of the assumption that the waves forming the wave packet are real waves. Quantum mechanics treats these waves as virtual waves.

Here it will be interesting to examine the case where the translational velocity is along the x-axis, while the standing wave is formed along the y-axis. We shall start with ϕ_o given in (7) with standing wave along the y-axis given by

$$\phi_o \quad = \quad 2\xi\ \cos\left(p_o y_o/\hbar\right) e^{-i\hbar^{-1}E_o t_o} \quad . \qquad (17)$$

If this standing wave is given a translational velocity v along the x-axis, then we have

$$\phi \quad = \quad 2\xi\ \cos\left(p_o y_o/\hbar\right) e^{-i\hbar^{-1}\left(t - \mathrm{v}x/c^2\right)} \quad . \qquad (17A)$$

It can be easily shown that if we take a group of waves having energy in the narrow region $(E - \Delta E) \leq E' \leq (E + \Delta E)$, then it will not be possible to obtain a sharp maximum for the amplitude at the point x = vt. Therefore, only standing waves formed along the direction of the translational velocity will get selected to represent the particle. We shall discuss this issue in detail in a separate paper when we shall deal with the three dimensional situation and the spin.

In (14) we note that $|\psi(x,t)|$ is largest in the vicinity of $E' = E$ as the phase function $(E't - þ'x)/\hbar$ is nearly a constant in that region. However, the function "exp[-i(E't-þ'x)/ħ]" would oscillate rapidly in the region outside the narrow range of integration making the contribution of the integral zero in that region. Thus the value of $|\psi(x,t)|$ would be significant only in the narrow range between (E-$\triangle$E) and (E+ΔE). Its maximum value occurs when the stationary phase condition

$$\frac{d}{dþ'}\left(E't - þ'\mathrm{x}\right)_{E'=E} \quad = \quad 0 \;\cdot \tag{18}$$

is satisfied. We observe that this condition also determines the center of the wave packet and is given by

$$x \quad = \quad \mathrm{v}_g\, t, \quad where \quad \mathrm{v}_g \quad = \quad \left[dE'/dþ'\right]_{E'=E} \;, \tag{18B}$$

where $\mathrm{v_g}$ is the group velocity which we could identify with the translational velocity v of the trapped wave.

Because of the sharp maximum of the integral around the average value, we may as well express (14) as

$$\psi\,(x,t) \quad = \quad \int_0^\infty 2\xi\;\cos\;\left[E'\left(x - \mathrm{v}t\right)/\hbar c\right] e^{-i\hbar^{-1}(E't - þ'\mathrm{x})}\; dE' \;\cdot \tag{19}$$

Here $\psi(x,t)$ could also be expressed as a function of þ and the integration could be carried over the values of þ to get

$$\psi\,(x,t) \quad = \quad \int_{-\infty}^\infty \varphi(þ')\;\mathrm{e}^{-\mathrm{i}\hbar^{-1}(E't - þ'\mathrm{x})}\; dþ' \;, \tag{19B}$$

where $\varphi(þ') = 2\xi\;\mathrm{v}\cos\;\left[þ'\mathrm{c}\left(\mathrm{t} - \mathrm{x}/\mathrm{v}\right)/\hbar\right]$. Note that 'vt' is the average spatial location of the system. This allows us to take (x-vt) as the spatial dispersion from the average value which may be denoted by Δx. In a field free state it is logical to take Δx to be independent of x and t. In other words, we may treat $\varphi(þ')$ as independent of x and t. We shall later show that φ (þ') could be expressed in terms of internal coordinates. The steps we have followed from (18) to (19B) are the same as those followed in standard texts books in quantum mechanics [4]. The basic proposition here is that the interpretation of a localized wave packet in terms of a forward and reverse luminal wave is quite amenable to the generally accepted approach to quantum mechanics.

It is evident from (15) that Ψ which may represent a particle can be expressed as a localized wave. The main objection against taking Ψ as a physical wave was that its phase velocity, given by c^2/v, is higher than the velocity of light. But in terms of the confined luminal wave picture we see that the trapped luminal wave packet may be identified with a particle and the phase velocity given by c^2/v has no physical meaning.

The conclusion that a trapped luminal wave packet acts like a particle with mass is quite surprising. It should be noted that a particle with a non-zero rest mass follows the Klein-Gordon equation (we shall not consider here the Dirac equation as the spin of the particle is not incorporated as yet into the scheme) given by

$$\nabla^2\psi \quad - \quad \left(\frac{1}{c^2}\right)\frac{\partial^2\psi}{\partial t^2} \quad = \quad \left(m^2c^2/\hbar^2\right)\psi \quad , \tag{20}$$

while the luminal wave follows the wave equation

$$\nabla^2\psi \quad - \quad \left(\frac{1}{c^2}\right)\frac{\partial^2\psi}{\partial t^2} \quad = \quad 0 \quad \cdot \tag{21}$$

Since the standing wave is composed of luminal waves, one may be tempted to presume that it should satisfy only the wave equation (21). But we observe that the standing wave surprisingly satisfies the Klein Gordon equation (20) also. This is a paradox. This paradox can be resolved only after we introduce the concept of the internal coordinates. It will be shown in the next section that the wave equation (21) applies to the internal coordinates where the luminal wave is confined whereas the Klein Gordon equation (20) applies to the external coordinates where the plane wave state is defined. It will be shown that the creation of the internal coordinates is a direct consequence of the confinement of the luminal wave. Needless to say, the same confinement also resulted in the creation of mass.

4 More on the Amplitude wave and the Phase Wave

Let us now go back to the plane wave given in (10) and express it as

$$\phi \quad = \quad \xi \left\{ e^{iE(x-\mathrm{v}t)/\hbar c} + e^{-iE(x-\mathrm{v}t)/\hbar c} \right\} e^{-i\hbar^{-1}(Et-\text{þ}x)} \quad . \qquad (22)$$

Of the two terms in the bracket representing the amplitude, we shall confine ourselves with only one term and express the wave as

$$\phi \quad = \quad \xi\, e^{iE(x-\mathrm{v}t)/\hbar c}\; e^{-i\hbar^{-1}(Et-\text{þ}x)} \quad = \quad \phi_A \phi_P \quad , \qquad (22A)$$

where $\phi_A \quad = \quad \xi\, e^{iE(x-\mathrm{v}t)/\hbar c} \qquad and \qquad \phi_P \quad = \quad e^{-i\hbar^{-1}(Et-\text{þ}x)}$, (22B)

ϕ_A represents the amplitude wave while ϕ_P stands for the phase wave or the plane wave. Note that while the frequency of the plane wave is given by $\omega_P = E/\hbar$, that of the amplitude wave is given by $\omega_A = \beta\omega/\hbar$, where $\beta = \mathrm{v}/c$. It can be easily shown that the wavelength of the amplitude wave is given by

$$\lambda_A \quad = \quad \lambda_o \sqrt{\left(1 - \mathrm{v}^2/c^2\right)} \quad , \qquad (23)$$

where λ_o is the wavelength of the standing luminal wave in the rest frame of reference. This means that the amplitude wave has got the same wave length as the trapped luminal wave since the factor $\sqrt{(1-\mathrm{v}^2/c^2)}$ represents the relativistic contraction in length. In the case of the phase wave, the wavelength is given by

$$\lambda_P \quad = \quad \frac{c^2/\mathrm{v}}{\omega/2\pi} \quad = \quad (c/\mathrm{v})\lambda_A \quad . \qquad (24)$$

Since $v < c$, it is obvious that $\lambda_P > \lambda_A$. Both waves will coincide only when v = c. Note that only the amplitude wave can represent the particle as its velocity is v. The phase wave has a velocity c^2/v which is superluminal and therefore cannot represent the movement of any physical entity.

We have to keep in mind that it is the to and fro motion of the luminal wave between the mirrors that created the amplitude wave and the phase wave. Interestingly, the time taken for the luminal wave to complete one to and fro motion is exactly equal to the period of one oscillation of the phase wave. To prove this, let us express the period of the phase wave T_P as

$$T_P \quad = \quad 2\pi/\omega \quad = \quad 2\pi/\gamma\omega_o \qquad (24A)$$

where $\omega = E/\hbar$ and $\omega_o = E_o/\hbar$. We know that in the rest frame of reference the time taken by the luminal wave to make one to and fro motion between the mirrors (figure 1) is equal to T_o where

$$T_o \quad = \quad 2\pi/\omega_o \quad .$$

Let us now introduce an observer with regard to whom the system of mirrors is having translational velocity v. The observer would notice relativistic time dilation of the period taken for the to and fro motion which will be given by

$$T = T_o\sqrt{(1 - \mathrm{v}^2/c^2)} = T_o/\gamma = 2\pi/\gamma\omega_o \, . \tag{24B}$$

Since T and T_P are equal, it is obvious that by the time the luminal wave completes one to and fro motion or traverses one wave length, the luminal wave would have completed one oscillation. In other words, the phase of the luminal wave and the plane wave will always be in phase. In fact, that is the reason why the plane wave was initially called the phase wave.

From the preceding discussion, we observe that a plane luminal wave when confined between two mirrors forms a complex wave which has two components, one amplitude wave which travels at a velocity v while the other one is the phase wave that travels at a velocity c^2/v. Let us now confine the particle represented by this complex wave by placing it in a box of length L and examine the case. Since we saw that the wave length of the amplitude wave is same as that of the luminal wave, it becomes reasonable to identify the amplitude wave with the particle. But this is true only at one level. The plane wave represents the particle at a different level. This is so because if we confine the moving particle within a larger box with totally reflecting walls, then the standing wave will be formed based on the wave length of the plane wave, and not that of the amplitude wave. The amplitude wave pertains to the confinement at the most basic level.

To clarify the picture further, let us take the case the electron orbiting in an atom. Here the electron chooses only such paths where the circumference of the orbit is an integral multiple of its wave length. The wave length here is that of the plane wave and not of the amplitude wave. Therefore, we come to the same conclusion that the amplitude wave represents only the inner structure of the particle while the plane wave determines how it interacts with the external objects. If we shift the wave ϕ given in (22A) back to the rest frame of reference, then we obtain

$$\phi_A(x_o) = \xi\, e^{iE_o x_o/\hbar c} = \xi\, e^{ip_o x_o/\hbar} \tag{25}$$

and

$$\phi_P(t_o) = e^{-iE_o t_o/\hbar} \; . \tag{25A}$$

This shows that the amplitude wave is nothing but the spatial component of the standing wave which is given a translational velocity. Likewise, the plane wave is the time dependent part of the standing wave which is given a translational velocity. We already saw that de Broglie used (1) which is same as (25A), to arrive at the concept of the phase wave. In the process, he seems to have missed out the spatial component represented by (25). This basic oversight seems to have led to the notion that the de Broglie wave or the plane wave is defined in a configuration space and not in the real space. On the other hand the above analysis seems to suggest that the standing wave exists in real space and therefore, the de Broglie wave or the plane wave which is nothing but a standing wave in motion, should also exist in the real space-time. We shall shortly show that the truth is more complicated.

5 Compacting and the Creation of the Inner Spaces

We shall show shortly how the plane wave gets defined in the external coordinates while the amplitude wave gets compacted into the internal coordinates of the particle. It will be shown that except for the interactions which involve the internal structure of a particle, for all other interactions where the particle is to be taken as a single point mass, the phase wave or the plane wave is a viable representation. Although the concept of the internal and external coordinates is convenient to explain the behavior of the two waves, we have to still explain how the single coordinate system with which we started gets split up into two separate systems.

Let us take the case of a point on the surface of a sphere. Let the centre of the sphere, P be at a distance $\hat{x}$ from the origin of axes and let a point on the surface of the sphere, P′ be at a distance x′ from the centre of the sphere. If we now treat the spherical body as a point particle, it is obvious that P′ will coincide with P and therefore x′ will have to be taken as zero. But such an assumption will cause problems if P' is spinning around P. If P' is made to coincide with P, then it would become impossible to account for the angular momentum of the sphere. In other words, the moment we treat the spherical particle as a point particle, the only viable option is to treat P' as existing in the internal space of the point particle at P. Thus, when we treat a particle with spatial spread as a point particle, we implicitly compact the spatial spread of the particle into its internal space. In the process we are forced to treat the angular momentum of the sphere as defined in its internal space. Note that the internal space introduced here is a mathematical construct which enables us to treat the sphere as a point particle.

We shall now apply this concept of the internal space in the case of the confined luminal wave. We may take $\hat{x}$ and $\hat{t}$ as the spatial and time coordinates of the confined wave assuming it to be a point particle. In that case x′ and t′ may be taken as the coordinates of a point on the confined luminal wave. Let the space and time coordinates of the point P′ on the confined luminal wave be denoted by x and t where

$$x \quad = \quad \hat{x} + x' \quad \text{and} \quad t \quad = \quad \hat{t} + t' \qquad (26)$$

We shall now see how the amplitude wave becomes a wave defined in the internal coordinates only, while the plane wave becomes defined in the external coordinates

Let us substitute x and t from (26) into (15) to obtain

$$\psi \quad = \quad 4\xi \, \mathrm{v} \, \Delta \, \text{þ} f(x' - \mathrm{v}t') \, e^{iE(x'-vt')/\hbar c} \; e^{-i\hbar^{-1}(E\hat{t} - \text{þ}\hat{x})} \, . \qquad (27)$$

Here we have taken ($\hat{x}$ -v$\hat{t}$) = 0 because the system is localized in the external coordinates at $\hat{x}$ = v$\hat{t}$. Further, we should remember that x′ << $\hat{x}$ and t′ << $\hat{t}$. Therefore in (27) in the phase of the plane wave, x and t has been replaced by $\hat{x}$ and $\hat{t}$. Now let x′-vt′ = ε, where ε is very small. Note that ε denotes the spatial spread of the particle. The function f(x′-vt′) will show a maximum in the small region where x′-vt′ = ε. The peak will be sharp as far as the external coordinates are concerned. For the sake of notational convenience we may express equation (27) by replacing $\hat{x}$ and $\hat{t}$ by x and t with the implicit understanding that x and t now onwards represent the external coordinates. Further, we may drop the function f(x'-vt') with the understanding that the wave function is localized at the point x = vt in the external coordinates. Accordingly, (27) may be written as

$$\psi \quad = \quad 4\xi \, \mathrm{v} \, \Delta \text{þ} \; \mathrm{e}^{iE(x'-vt')/\hbar c} \; e^{-i\hbar^{-1}E\left(t - \mathrm{v}x/c^2\right)}. \qquad (27\text{A})$$

It is interesting to see how the external coordinates gets erased out from the amplitude wave while the internal coordinates get approximated out in the plane wave. Thus we may say that the amplitude wave is defined in the internal coordinate system while the plane wave is defined in the external coordinate system.

In the above discussion, we have taken the case of the wave packet to introduce the concept of the internal coordinates. The fact that the plane (phase) wave is defined in the external coordinates while the amplitude wave is defined in the internal coordinates would hold good even when we take the case of the wave given in (10). The situation could be clearly understood by taking the time dependent component of the standing wave denoted by $\phi_P(t_o)$ in (25A). Here $\phi_P(t_o)$ represents oscillations at a certain fixed point in space. In fact, this fixed point at which the oscillations are defined by $\phi_P(t_o)$ is nothing but the spatial part of the standing wave given by $\phi_A(x_o)$ compacted into a point. Note that this oscillation when observed by a moving frame of reference provides us with the plane wave. Therefore, the point which oscillates in $\phi_P(t_o)$ has to be taken as a point representing the spatial component of the standing wave as a whole. This involves the same sort of compacting as was observed in the case of the sphere and a point on its surface discussed above. Therefore, it becomes clear that the splitting of the standing wave in terms of the internal coordinates and the external coordinates applies to each of the waves in the wave packet. Accordingly, (10) may be expressed introducing the internal coordinates as

$$\phi \quad = \quad 2\xi \ \cos\ \left[E\left(x' - \mathrm{v}t'\right)/\hbar c\right] e^{-i\hbar^{-1}(Et - \mathrm{p}x)} \ . \qquad (27B)$$

We may now modify (12) also by introducing the internal coordinates and express it as

$$\phi_c' = 2\left\{\xi_y \cos\left[E\left(x' - \mathrm{v}t'\right)/\hbar c\right] - \xi_z \sin\left[E\left(x' - \mathrm{v}t'\right)/\hbar c\right]\right\} e^{-i\hbar^{-1}(Et - \mathrm{p}x)} \ . \qquad (28)$$

This shows that the circularly polarized wave represented by the terms within the square bracket is defined in the internal coordinates. This is a very interesting outcome. It is presumed that a plane wave which represents a particle like electron cannot in any way be related to the electromagnetic waves. But here we observe that when we confine the electromagnetic wave, the amplitude wave formed as a result gets compacted into the internal coordinates while the phase wave acquires the plane wave structure in the external coordinates. Note that the vector nature of the amplitude of the electromagnetic wave gets pushed into the internal coordinates.

Let us now try to understand the relationship between the particle and the phase wave represented by (7). Here we should keep in mind that in the rest frame of reference, the particle is represented by standing wave in space which oscillates E_o/h times in a second. When we take the amplitude wave to be defined in the internal coordinates, implicitly the entire standing wave gets compacted to a point and the oscillation of the standing wave gets replaced by a point oscillating up and down in time. In other words, the particle gets represented by a standing vibration of the type assumed by de Broglie which is given in (1). We know that this vibration becomes the phase wave when viewed from a moving frame of reference. Therefore, each point on the phase wave represents the particle. But we should keep in mind that the phase wave may progress with a phase velocity c^2/v. But it will have a non zero amplitude only at the point x = vt. The phase wave would remain a virtual one at all other points. De Broglie proposed the concept of the pilot wave assuming the phase wave to be a real wave just like the electromagnetic wave. But now we observe that the situation is not as simple as that.

We see that the concept of the internal coordinates originates when we treat some entity having certain definite though small dimension as localized at a point. We should keep in mind that the internal coordinates and external coordinates are intimately related in the sense that the direction of the axes of the internal coordinates will align with that of the external coordinates. Note that the operators ∇ and $\partial/\partial t$ are defined in the external coordinates and therefore these operators leave the functions defined on the internal coordinates unchanged. It is now clear that the luminal wave is defined in the internal coordinates. Therefore, the wave equation (21) applies to the internal space only. On the other hand, the Klein Gordon equation (20) is defined in the external coordinates. To put it differently, we may say that the luminal wave has no internal structure while the plane wave has an internal structure created by the confined luminal wave. This is also the reason why in contrast to the luminal wave, the plane wave can be attributed rest mass. This result leads us to conclude that mass is localized energy. Such a definition has very profound implications.

In this context it is worthwhile to note that the idea of photons acquiring mass was discussed in detail by Einstein in his gedanken experiment where the case of a photon being exchanged by two walls of a closed cavity was examined in depth [5]. The idea which comes out of this is that while a free photon moves at the velocity of light evidencing absence of inertia, the instant the photon is trapped or localized by a reflecting system or any such device, the inertia of energy manifests itself as mass. In fact, Hasenohrl, as early as 1904 (ie; even before the advent of the Special Theory of Relativity) had shown that the electromagnetic energy E, enclosed in an empty box with perfectly reflecting walls, behaves when the box is set in motion as if it had a mass proportional to E [6].

The idea of a luminal wave getting reflected back and forth forming a standing wave immediately reminds us of the idea of the “zitterbewegung” which was introduced to explain the results obtained from the Dirac equation. We know that the velocity operator in terms of the Dirac equation turns out to be $c\alpha_i$ and the eigen values of this operator are $\pm$ c [7][8]. However, the expectation value of this operator with respect to the positive-energy wave packet represented by $< c\alpha_i >_+$ is the group velocity v of the positive energy packet. This contradiction has no explanation in the relativistic quantum mechanics. But the proposed confined luminal wave structure of particle gives a satisfactory explanation to the above problem. This would explain why the eigen values of the velocity operator $c\alpha_i$ actually stand for the velocity of the circularly polarized luminal wave which gets trapped forming the particle. Thus, prima facie, the phenomenon of "zitterbewegung" seems understandable in terms of the proposed approach.

6 Concluding Remarks

To sum up, we saw that a particle can be represented by a standing luminal wave. However, we are still to account for the spin and the electric charge of the particle. Although we did not relate this luminal wave to any physical wave, it is obvious that it has to be identified with the circularly polarized electromagnetic wave. Note that as the standing wave is formed, the electric and magnetic components of the amplitude of the electromagnetic wave get compacted into the internal coordinates and the phase wave takes on the role of the plane wave. Therefore, the electromagnetic nature of the luminal wave does not show up in the plane wave which is defined in the external coordinates. This is a prime requirement if this representation of a particle is to be consistent with the interpretation based on quantum mechanics. We shall show in a separate paper that the amplitude part which is compacted into the internal coordinates determines the spin of the particle and it will be shown that the Pauli’s exclusion principle emerges logically from the standing wave structure of the particle. In this approach we have introduced of two reflecting mirrors to confine the luminal wave. Needless to say this is an artificial construct and will have to be replaced by interactions with some field. This will be attempted in forthcoming papers.

The standing wave representation of a particle may appear to be similar to the concept of the pilot wave proposed by de Broglie in the early years of wave mechanics. Here also a point on the phase wave (plane wave) is supposed to represent a particle just as in the case of de Broglie's pilot wave. However, there are some major differences. De Broglie in his theory of double solitions treated the particle as a singularity which foreclosed any further study of the structure of the particle whereas in the present approach we are attributing it a standing luminal wave structure [9]. Secondly, de Broglie seems to have held on to the classical concepts and believed that the system evolved in a causal manner and the randomness is a direct result of the measurement process. In that sense, he was a "realist". We shall in a separate paper show that the standing wave representation of a particle is compatible with the basic principles of quantum mechanics.

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