Entropy growth in the early universe, and the search for determining if gravity is classical or quantum , part III

(Is gravity a classical or quantum phenomenon at its genesis 13.7 billion years ago?)

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Abstract

In the 12th Marcel Grossmann Meeting, July 9th, 2009, the author raised the issue of whether early graviton production could affect non-Gaussian contributions to DM density profiles. Non gaussianity of evolving cosmological states is akin to asking if there is a way to get quantum contributions due to squeezed initial vacuum states which act highly non classscially. If particle counting algorithms in graviton production is important as for entropy, and if entropy perturbations affects the density profile of dark matter clumping prifiles, then there is room to ask to what degree initial perturbations affecting structure formation are due to classical/ non linear processes, or more quantum theoretic states. If squeezing of the initial vacuum states is essential in the relic conditions, then quantization is unavoidable. If squeezing is not essential, then coherent initial vacuum states may contribute in semi classical ways to GW production . The end result of this stated inquiry may be answering if or not gravity in the onset of inflation is a quantized field. Or if a highly non linear set of complex initial conditions for gravity can be stated using purely classical models, as T'Hooft, Corda, and others believe. Note, also that Bojowald as of 2008 has left the degree of squeezing of initial vacuum states in the region of space as an open problem. In Bojowald's model of a cosmological bounce, the degree of squeezing is a measure of what strength the 'bounce' from an initial configuration of the universe takes, and how strongly quantum effects contribute to the evolution of the LQG cosmos, after inflation commences. Similar questions are being raised as to the necessity of squeezing of initial vacuum states and if or not coherency of initial states is initially largely achievable, before the rapid expansion of the universe commences. Finally, and not least is a series of questions as to what conditions which would either require high or low frequencies as to relic signals from the big bang. As it is, large spatial dimensions which could induce far lower initial frequencies for relic signals are popular in many string theory models. The author views this assumption as of debatable validity, as well as the assumption made by Arkani Hamid that largely does away with coherency of initial vacuum states and specifies highly quantum , low frequency generation of relic GW.

Introduction

The over riding question to ask is , if there is a tendency toward either high, or low frequencies as far as relic gravitational signals. The easiest analogy to use is thinking of a bowl of sticky fluid which is slapped via ones hands**.** If one has a high degree of transferal of imput energy into the big bang from a prior universe being transferred to todays 4 dimensional universe, then the total energy from a prior universe, call it energy E will be correlated to a high frequency ω ,of a signal which propagates from the origin of the big bang

What could lead to modulation and reduction of the frequency ω ? Several things. One is that there is a dispersion of a prior universe's energy into large higher dimensions. Arkani – Hamid, and others , e.g. Enqvist, K., Mazumdar, A., Perez--Lorenzana, A in their article about inflation energy being 'dumped out of this world'., have used this view to argue that there are essentially NO gravitational waves from the big bang ,and/or they must be very weak and of low frequency. It is inappropriate to state though that such low frequencies are mandated by all string theory models. As noted by the author, Beckwith, the authors R. Brustein, M. Gasperini, M. Giovannini, and G. Veneziano (1995) came up with a model of string theory with compact higher dimensions of very small size which has none of the bleed off into higher other dimensions as speculated by Enqvist, K., Mazumdar, A., Perez--Lorenzana, A , or Arkani- Hamid. Hence their 1995 paper had NO bias toward low frequency relic gravitational waves. There are variants of string theory which are embedded in large extra dimensions which do tend to lower frequencies. . But that does not mean they are necessarily true.

Contrast this with loop quantum gravity, which is four dimensional, and which makes no assumptions as to a hierarchy of additional dimensions. Also Christian Corda, and T'Hooft among others strongly doubt the existence of the need for highly quantized models as to the existence of gravity/ gravitational waves, which puts into question what Arkani-Hamid and others have worked with as far as a multi verse and string theory with very large additional dimensions.

In addition, there is the issue of dispersion of energy frpm a prior universe which comes up again and again. A question to ask is as follows. There are physics analogies to high and low pass band filters as far as initial frequency of signals which come up again and again. If one has a high frequency band pass filter in cosmology, there will be a high degree of focusing of energy E into a four 'dimensional' gate, with practically no dispersion of that incoming energy. Furthermore, there would also be,for **initial** states., at least in the beginning not an immediately picking of squeezed states, which would be a way of having states with a minimum dispersion of energy, as the states evolve. In terms of phase space arguments about squeezed states is the representation, again and again , of having a circle in phase space compressed to an oval, or even close to a straight line. If the compression occurs, especially with initial vacuum nucleation states, single vacuum states will become coupled with each other, thereby increasing non linear coupling. I.e. think of Van Der Poll's oscillator. As the non linearity increases, there will be less energy transferred to particles or similar initial vacuum states, with a net lowering of ω. I.e. the more complex and non linear the initial states of gravitons, as = vacuum energy nucleation constructions from a prior universe become, the lower the frequency, ω, as the available energy per graviton drops.

If as an example, there is one single universe, with expansion and compression cycles of the universe, if there is a corresponding low frequency for initial relic states, then there would be a huge degree of hysteresis in the system. Engineering hysterisis processes imply that there is little chance that a system once deformed or changed, can restore itself to its original form. In the case of cosmology, this also ties in with the question of information exchange from a prior to our present universe. If one has a high degree of information exchange, from prior to present universes, there may be ways to determine if that also implies if or not states are squeezed, or un squeezed. Furthermore, if there is a universe with lots of entropy generation due to gravitational waves, then if lots of entropy is generated, and if coherent states are dominant in the beginning of the universe, even if they are scattered/ dispersed later on, that would have measurable consequences as to how classical gravity is, as a force and/or the relative coherency existing in gravitational wave states at the onset/ beginning of inflation.

The higher the degree of classical, not too non linear processes occur, which is akin to minimization of position to momentum uncertainty, and correspondingly, energy to time uncertainty, the more chance one gets a close over lap between classical physics models of states, and quantum versions of those states. This is seen, in terms of how coherent states of SHO occurs, as noted by Glauber (1963) The same sort of process, up to a point happens in cosmology. Furthermore, the measurement of entropy, as akin to information transfer is also relevant to how much similarities exist in the prior to the present universe states of matter and energy. This measurement of entropy production, as a measure of information exchange would correspond to a large degree with

. Gravitons are stated conceptually to be akin to photons in light waves. In simple physics analogies. But this simple quantum generalization breaks down, since gravitons are spin two particles with a complex set of interactions not only with themselves, but with evolving space time geometry. We mention that gravitons may be important to initial entropy generation. Entropy generation and entropy perturbations affect the gaussianity of evolving wave functions of matter and energy evolving in space time. If there is a large deviation of the initially Gaussian states of space time wave functions , there is likely a break from classical physics due to the complexity of evolving wave function states influenced increasingly by non Gaussian perturbations. This non Gaussian process is reflected by marked deviation from planar wave state approximations used in the evolution of wave functions Hence the issue of apparently combined sources of planar wave generation of gravitational waves is a precursor to what would happen if squeezed states occurred at the onset of the big bang. I.e., what would happen with multiple superpositions of different coherent states?. A good reference as to coherent states in cosmology, as in this example, Bianchi I universes, was given by Brett Bolen, Luca Bombelli, Alejandro Corichi (2004) In particular, look at their equation **3.1**. If states are largely coherent, such a small variation/ smoothness of observables will have observational consequences as to relic gravitational wave signals seen in the onset of inflation.

In the case of gravitons, as coherent states, once squeezing of coherent states occurs, the ,mere act of squeezing of the initial states destroys the initial classical super position of graviton states which would contribute to a GW. How and what particular mix of squeezed versus un squeezed relic states one can expect is important for determining frequencies to look for which are from relic conditions.

The basic reason for making such an examination of the relative importance of squeezing/ lack of squeezing is to determine if or not relic GW are due to classical versus quantum Gravitational processes. The answer to if or not relic GW are due to classical versus quantum processes has huge consequences as to the dominant GW harmonics in terms of what are the most important frequencies researchers need to look for , for relic GW identification, with instrumentation. The problem facing GW researchers is how to find dominant sub harmonics, in GW signals, i.e. how to use pattern recognition , and updated advanced fourier analysis in order to identify dominant frequency ranges of GW signals which are of interest and which carry the most relevant physics information for cosmologists to review and learn from. Relic GW are messy , and the most dominant/ important frequencies identified can if properly analyzed confirm/ falsify many of our early universe cosmology theories as far as relic conditions. How does one actually know about first or second order phase transitions, due to GW. Does one see , as an example classically based non linear super position of GW, which have consequences as to admissible spectrum of GW frequencies to detect ? Finally, can one correlate an identified frequency spectrum for incoming GW with different points of time in the evolution of the big bang itself. ,

I.e. **the template, in terms of instrumentation to use is the lowly bolometer**, and also to identify which / what are the significant frequencies. Secondly, the random background of relic gravitational wave production means that if phase transitions / GW shock waves occur , then to identify what are the dominant harmonics/ sub harmonics which can be connected with the shock wave/ phase transition.

One final datum to consider, before commencing with the article itself. Many LIGO researchers have concluded by 'necessity' relic GW are LOW frequency. One of the jobs of this article is to falsify this prediction, and to explain how higher frequency GW may emerge as significant from relic conditions.

Proper investigation of the GW background of relic conditions is smart band pass and pattern recognition engineering. The coming discussions of if or not the processes for relic GW production are dominated by either classical vs quantum processes will help identify what are the most important frequencies to look for, as well as how to , with GW to eventually produce the GW equivalent of the WMAP instrumentation protocols, to move relic GW detection matched with theory into an empirical/ experimentally falsifiable science. In the end, asking appropriate frequency ranges to look for, and producing the equivalent of a WMAP survey , for either classical/ quantum models of GW is the only way this field will survive and become an experimental science.

Entropy is extremely important in this inquiry for the following reason. A rapid build up of entropy, as initially stated as due to Ng's particle count algorithm, would lead to at least moderately high levels of non gaussianity, which would affect the density profiles of clumping of DM. One of the current observational puzzles of cosmology is how and why galaxies form earlier than expected, i.e. up to 5 billion years ago clumping of Galactic structures started in earnest, with highly significant galazy formation between red shift values of $z \sim 1-.5$ years ago. I.e. the hierarchy picture of galaxy formation needs amendment . If or not DM clumping is affected by GW/ gravitons, or something else will be important to understand. And, in addition, identification of the relative role/ importance of DM/DE/GW interactions, if all three interact will be of clear importance to resolving this problem, experimentally. So now let us examine our analytical/instrumentation tools which may allow such questions to be answered. Note that Li et al's PRD article assumes a detector that measures the influx of gravitons from these astrophysical sources directly, as opposed to inferences from CMB anisotropies. The reproduced table 1 presents a generally accepted BY WHOM? range of GW frequencies,

Table 1: magnitude, sources, and top frequency values for HFGW (from Li et al. 2009)

Sources	Amplitude	frequency	Characteristics
HFGW in Quintessence inflationary models	$h_{rms} \sim 10^{-30} - 10^{-32} / \sqrt{Hz}$	$\nu \sim 10^{9} - 10^{10}\, Hz$	**Random background**
HFGW in some string theory scenarios	$h_{rms} \sim 10^{-30} - 10^{-34} / \sqrt{Hz}$	$\nu \sim 10^{8} - 10^{11}\, Hz$	**Random background**
Solar Plasma	$h_{rms} \sim 10^{-39} / \sqrt{Hz}$	$\nu \sim 10^{15}\, Hz$	**On the Earth**
High energy particles, e.g. Fermi Ring	$h_{rms} \sim 10^{-39} - 10^{-41} / \sqrt{Hz}$	$\nu \sim 10^{4} - 10^{5}\, Hz$	**On the center the frequency depends upon the rotational frequency of particles in the Fermi Ring**
Stanford Linear Accelerator	$h_{rms} \sim 10^{-39} / \sqrt{Hz}$	$\nu \sim 10^{23}\, Hz$	**On the collision center, the frequency depends upon the self energy and the Lorentz factor of high energy e^+e^- beams**
LHC- Large Hadron collider			**Spectra of high energy gravitons**
Nano-piezo electric crystal array, with size of about 100 nanometers	$h_{rms} \sim 10^{-28} - 10^{-31} / \sqrt{Hz}$	$\nu \sim 10^{9} - 10^{10}\, Hz$	**On the wave WHAT IS A "WAVE ZONE"? with an effective cross section of or less than .01 meters squared, for gravitational radiation**

This re produced PRD table is important, since it suggests that higher frequency relic GWs, if detected, may be easier to analyze/ identify due the relative size of $h_{rms} \sim 10^{-28} - 10^{-31} / \sqrt{Hz}$, as opposed to far smaller values of h_{rms} from other physical processes generating GW. Since the table gives a listing of potentially verifiable HFGW sources, it is important to ask if or not the GW from the relic big bang itself are primarily low or high frequency. The next page summarizes some of the arguments, for low and higher frequencies for relic GW, and concludes , with what the author , Beckwith, thinks, that the many studies purporting to find an optimal GW cluster from relic conditions , of, $10Hz < f < 100Hz$ are debatable.

Brief review of the reasons why some string theorists think Relic GW must be low frequency

Consider now how spacetime was created at the onset of the big bang.. The universe was "really small" compared what it is today, and all that matter and energy were crammed in very small volume.. The energy dispersed and matter began to form from the energy. Everything was still beyond the temperature in stars. But the matter-emergy plasma mix was cooling Corda (2008) has modeled adiabatically-amplified zero-point fluctuations processes in order to show how the standard inflationary scenario for the early universe can provide a distinctive spectrum of relic gravitational waves. De Laurentis, Mariafelicia, and Capozziello, Salvatore(2009) have further extended this idea to give a qualified estimate of GW from relic conditions which will be re produced here. Begin with De Laurentis's idea of a gravitational wave spectrum

$$\Omega_{sgw} = \frac{16}{9} \cdot \left(\frac{\rho_{dS}}{\rho_{Planck}} \Big/ 1 + z_{eq} \right) \xrightarrow[f \to low-value]{} f^{-2} \Leftrightarrow f\big|_{present=era} > (1+z_{eq})^{1/2} \cdot H_0 \qquad \textbf{(0.0)}$$

Here, H_0 is today's Hubble parameter, while f is GW frequency, and z_{eq} is the red shift value of when the universe became matter dominated. I.e. redshift z = 1.55 with an estimated age of 3.5 Gyr, or larger, would be a good starting point. I.e. this is for larger than 3.5 giga years for when matter domination became most prominent.. This border value for redshift z, as the dividing line for when matter domination was brought up by Lawrence Krauss (1996) as to what times could matter formation become significant. I.e. the further back z_{eq} goes the larger the upper bound for frequency f. The upper range for f appears to be about 100 Hertz. Needless to state, though, if z_{eq} drifted to a value of $z_{eq} \sim 10$ then the upper bound to $f \sim 1000$ Hertz. Note that there are string theory based calculations predicting relic GW at or lower than 1 Hertz. **,** as suggested by B Lamine**.** A Lambrecht, M T Jaekel and S Reynaud ***(2004)*** As a dominant GW frequency. Their article states, that **"**relic gravitational wave background is expected to be statistically isotropic, but the actual value of the associated spacetime metric should break isotropy. We propose to detect the resulting anisotropy by using an optical interferometer mounted on a rotating platformIs**"** this the last word ? Not necessarily**.** Grishchuck, (2008) from a non string theory perspective, predicted a dominant relic GW frequency range of up to 10^{10} Hertz, while as early as 1995, R. Brustein, M. Gasperini, M. Giovannini, and G. Veneziano (1995) predicted, on the basis of string theory, ultra high GW from relic big bang processes. Effectively with NO limitations as to HFGW from relic inflationary processes.To be very blunt, there has been an understandable pressure to try to obtain GW from relic conditions which would be within the sensitivity peak effectiveness of LIGO, as an example within 10 to 100 Hertz , i.e. $10Hz < f < 100Hz$ .One of the studies doing just that was a well done contribution: Buonanno, A.; Ungarelli, ***C.(2008). It would be considered definitive, if the following did not exist, i.e.*** **http://www.ba.infn.it/~gasperin/** ***which is a compendium of different string cosmology predictions.***I.e. the predictions which are on this Gasperini supplied link as to the relative import of either high or low frequency contributions to the GW production at or near the 'big bang" are all over the block. It is time for some definitive measurements to be taken and to end this problem once and for all. In addition, the author states unequivocally, that LIGO, as far as relic condition GW detection has been a failure. The possibility of very high GW frequencies cannot be dismissed, in lieu of the null results obtained by LIGO. Secondly, if or not the low frequency, to high frequency regimes of GW are dominant will also be impacted upon to the degree of the classical nature of GW/ gravity itself. Note that Christian Corda (2007) wrote that "The investigation of the transverse effect of gravitational waves (GW's) could constitute a further tool to discriminate among several relativistic theories of gravity on the ground.. Realistic tests of this issue, as well as non LIGO alternatives as to gravity should be investigated. I.e. the jury is still out on this issue of which frequency of GW is most important for relic conditions.

Next a summary of alternative counting algorithms which purport to show how GW models affect entropy generation. Since the affects of GW generation may induced non gaussianity processes, which affect both DM density profiles, and entropy, as either thermally, or non thermally based, the next section is included as part of a motivation to obtain models which would allow for experimentally falsifiable testing of GW and entropy generation theories. The String based entropy algorithm, is purely quantum gravity, whereas the WDW approach is at heart a semi classical, WKB approach. Ie. Can the two approaches be reconciled, up to a point ? Note that J. Martin, (2008) discussed as to how quantum perturbations can have pronounced classical model similarities. I.e. the Wigner function of a free particle can be localized. This localization phenomenon gives us a chance to , for certain quantum constructions obtain behavior very close to the semi classical, a.k,a. in the case of WDM theory, wave functionals which are WKB, in practice. is similar to the later analysis of squeezed versus un squeezed states. I.e. squeezed states no longer act classically, where as un squeezed coherent states are very close to classical wave functions in behavior.

Review of simple models as to gravitons as produced either by (Quantum gravity) strings , LQG,(or by processes which may not be Quantum Gravity based?)

We wish now to review what may be some of the counting algorithms appropriate for entropy generation, and which may contribute to answering if or not GW are mandated to be, from the beginning either a classical versus a quantum processes. IN part this next page is due to concepts A.Beckwith presented in Rencontres De Blois, 2009, and is a starting point for our inquiry as to the necessity, or lack of , of modeling Gravity as either classical / quantum based in relic conditions.

Introduction w.r.t. the NG paradigm

We wish to present two alternative routes to generation of entropy. The first, is a counting algorithm, is an adaptation of Y.J. Ng's infinite quantum (modified Boltzmann's) statistics; the second references A. Glinka's research presentation on "graviton gas" as a way to provide a perspective? as to how to get a partition function for gravitons that is congruent with the Wheeler De Witt equation. Here are a few questions which are posed for the reader.

1. Is each "particle count unit" as suggested by Ng equivalent to a brane-antibrane unit in brane treatments of entropy?

. 2. Is the change of entropy $\Delta S \approx \Delta N_{gravitons}$?

3. Is this graviton production scheme comparable to Glinka's quantum gas , from the Wheeler De Witt equation?

. Entropy generation via Ng's infinite quantum statistics (short review)

This discussion is motivated to show a purely string theory approach and to see if its predictions may over lap with semi classical WDM (semi classical) treatments of cosmology.. The contention being advanced is that if there is an over lap between these two methods, that it may aid in obtaining experimentally falsifiable data sets for GW from relic conditions.

We wish to understand the linkage between dark matter and gravitons. how relic gravitational waves relate to relic gravitons"?, To consider just that, we look at the "size" of the nucleation space, V for dark matter, DM. V for nucleation is HUGE. Graviton space V for nucleation is tiny , well inside inflation. Therefore, the log factor drops OUT of entropy S if V chosen properly for both eqn 1 and eqn 2. Ng's result begins with a modification of the entropy/ partition function Ng used the following approximation of temperature and its variation with respect to a spatial parameter, starting with temperature $T \approx R_H^{-1}$ (R_H can be thought of as a representation of the region of space where we take statistics of the particles in question). Furthermore, assume that the volume of space to be analyzed is of the form $V \approx R_H^3$ and look at a preliminary numerical factor we shall call $N \sim (R_H / l_P)^2$, where the denominator is Planck's length (on the order of 10^{-35} centimeters). We also specify a "wavelength" parameter $\lambda \approx T^{-1}$. So the value of $\lambda \approx T^{-1}$ and of R_H are approximately the same order of magnitude. Now this is how Jack Ng changes conventional statistics: he outlines how to get $S \approx N$, which with additional arguments we refine to be $S \approx < n >$ (where <n> is graviton density). Begin with a partition function

$$Z_N \sim \left(\frac{1}{N!}\right) \cdot \left(\frac{V}{\lambda^3}\right)^N \tag{0.1}$$

This, according to Ng, leads to entropy of the limiting value of, if $S = (\log[Z_N])$

$$S \approx N \cdot (\log[V/N\lambda^3] + 5/2) \xrightarrow[Ng-\inf inite-Quantum-Statistics]{} N \cdot (\log[V/\lambda^3] + 5/2) \approx N \tag{0.2}$$

But $V \approx R_H^3 \approx \lambda^3$, so unless N in Eqn (0.2) above is about 1, S (entropy) would be < 0, which is a contradiction. Now this is where Jack Ng introduces removing the N! term in Eqn (1) above , i.e., inside the Log expression we remove the expression of N in Eqn. (0.2) above. The modification of Ng's entropy expression is in the region of space time for which the general temperature dependent entropy Kolb and Turner expression breaks down. In particular, the evaluation of entropy we do via the modified Ng argument above is in regions of space time where g before re heat is an unknown, unmeasurable number of degrees of freedom The Kolb and Turner entropy expression (1991(has a temperature T related entropy density which leads to that we are able to state total entropy as the entropy density time's space time volume V_4 with $g_{re-heat} \approx 1000$, according to De Vega, while dropping to $g_{electro-weakt} \approx 100$ in the electro weak era. This value of the space time degrees of freedom, according to de Vega has reached a low of $g_{today} \approx 2-3$ today. We assert that Eqn (0.2) above occurs in a region of space time before $g_{re-heat} \approx 1000$, so after re heating Eqn (0.2) no longer holds, and we instead can look at

$$S_{total} \equiv s_{Density} \cdot V_4 = \frac{2\pi^2}{45} \cdot g_{\bullet} \cdot T^3 \cdot V_4 \tag{0.3}$$

Where $T < 10^{32} K$. We can compare eqn (0.1) and (0.2) , as how they stack up with Glinka's (2007) quantum gas, if we

identify $\Omega = \frac{1}{2|u|^2 - 1}$ as a partition function (with u part of a Bogoliubov transformation) due to a graviton-quintessence gas, to get information theory based entropy

$$S \equiv \ln \Omega \tag{0.4}$$

Such a linkage would open up the possibility that the density of primordial gravitational waves could be examined, and linked to modeling gravity as an effective theory. The details of linking what is done with (0.2) and bridging it to (0.3) await additional theoretical development , and are probably conceptually understandable if the following is used to link the two regimes. I.e. we can use the number of space time operations used to create (0.2), via Seth Lloyds

$$I = S_{total} / k_B \ln 2 = [\# operations]^{3/4} = \left[\rho \cdot c^5 \cdot t^4 / \hbar\right]^{3/4} \tag{0.5}$$

Essentially, what will be done is to use 0.5 to show linkage between a largely thermally based production of entropy, as implied by (0.3) and a particle counting algorithm, as given by (0.2). This due to the problems inherent in making connections between a particle count generation of entropy, and thermal contributions. I.e two different processes are involved.

Connection between gravitons and GWs

The first topic to raise is whether or not there is a way to make a connection between gravitons and GWs. In perturbative string theory, a graviton is a closed string in a very particular low-energy vibrational state. And in string theory, a graviton can be connected to a gravitational wave by linking the graviton particle to the curvature of the space-time continuum and calculating the gravitational force exerted. Unfortunately, for string theory, the only way to link gravitons to GWs is by obtaining the coherent state of many gravitons, i.e., looking at Gaussian states with minimum uncertainty, which would be stationary However, as Grishchuck showed, as reported by Allen, Flanagan, and Papa (1999) relic GW generation is Gaussian, but NOT stationary,... Now can a standard planar approximation optimally work for detecting GWs? Probably not, based on Ming-Lei Tong and Yang Zhang (2007), using GW spectra and numerical simulations, which gave a null result for detector (circular waveguides). This result has already been established by Ingley and Criuse (2001). So, let us see how the inputs gravitational waves OF WHAT? to the circular wave guide via numerical representation of planar waves for GW/ Gravitons MEANING? was initiated.

To do this, we need to consider the behavior of relic GW, as suggested by Tong and Zhang (2007) : $h_{ij}(x,\tau)$ in $dS^2 \equiv a^2(\tau) \cdot \left[-d^2\tau + (\delta^{ij} + h_{ij}) \cdot dx^i dx^j\right]$, in flat space (neglecting curvature),

$$\partial_\mu \left(\sqrt{g} \cdot \partial^\mu h_{ij}(x,\tau)\right) = 0 \tag{0.6}$$

This has the very simple solution, with a mean average for the approximate square of $h_{ij}(x,\tau)$

$$< h^{ij}(x,\tau) h_{ij}(x,\tau) > = \int_0^\infty h^2(k,\tau_H) \cdot \frac{dk}{k} \tag{0.6a}$$

Where the spectrum $h(k,\tau_H)$ may be given via $k_H \equiv 2\pi\gamma$, γ as an "accelerating parameter," $k_H \equiv 2\pi\gamma$, $k_S \cong 10^{26} k_H$, $\in \equiv (1+\beta)\cdot(1-\gamma)/\gamma$, with β an inflation parameter, and β_S a re-heating parameter, so that

$$h(k,\tau_H) \cong A_0 \cdot \left(\frac{k_S}{k_H}\right)^{\beta_S} \cdot \frac{k_H}{k_2} \cdot \left(\frac{k}{k_H}\right)^{\beta-\beta_S+1} \cdot \frac{1}{(1+z_E)^{3+\in}} \tag{0.6b}$$

This is where one can write $k = \nu \cdot 2\pi \cdot a(\tau_H)$, where ν is a physical frequency, β as an inflation parameter, β_S as a re-heating parameter, $1+z_E = \frac{a(\tau_H)}{a(\tau_E)} \sim \left[\frac{\Omega_\Lambda}{\Omega_E}\right]^{1/3} \sim`1.5$, where the red shift z_E is defined roughly via the time of equality between dark energy and matter density, with $z_E \approx .5 - .55$ as given by Eric Linder (2003) : in PRL (rapid communications) in his figure 1. We need to ask ourselves though whether or not the representation OF WHAT? make sense, because in the regime of billions of years after a big bang, it restricts us to making use of planar approximations to GW and attendant gravitons. Note that the approximation of gravitational spectra given by (0.6b) leads to a null result in simulations of detectable relic GW by Tong and Zhang (2007). So probably a more refined version of GW representation needs to be given. Now, can we connect the wavelength of a graviton and the GW frequency? Clifford Will (1997) wrote the wavelength of a graviton as, $\lambda_{Graviton} \equiv \left(h/m_{graviton} \cdot c\right)$ while other authors have suggested $m_{graviton} \approx 10^{-65}$ grams. As Will observes, if f is the frequency, and the wavelength $\lambda_{graviton}$ may play a role in modified gravity, via an effective Newtonian (gravitational) potential of $V \propto \left[\exp\left(-r/\lambda_{graviton}\right)\right]/r$. , i.e. are there observational ways to obtain a graviton wave length Will in (1997) experimentally estimated the magnitude of $\lambda_{graviton} \sim 6\times 10^{12}$ kilometers, which would make modified gravitational measurements a near impossibility. I.e. one of the challenges would be to see if or not experimental protocol exists that would allow tests of

$$[velocity]_{graviton} \equiv c \cdot \sqrt{\left(1-\left(c^2/f^2\lambda^2_{graviton}\right)\right)} \tag{0.6c}$$

Obviously, the issue of whether or not a graviton has a mass will impact how realistic the approximation given by (0.0a) and (0.0b) really are, as well as be important to the issue which Leszek M. Sokolowski, Andrzej Staruszkiewicz (2006) raised: "The graviton must have features different from those of the photon and these cannot be predicted from classical general relativity." This will impact strongly upon how to analyze the relationship between wavelength $\lambda_{graviton}$ and frequency, f, as mentioned in (0.0c). Note that Sokolowski , et al (2006) state that there is a decisive break down of application of MEANING? the formula $E = \hbar \cdot \omega$, which means in order to make sense out of the graviton andgravitational wave connection, one really needs to investigate the space-time constraints in which relic gravitons/ gravitational waves arose.. Note, it is possible that as much as up to 2/3rds of the initial relic "matter"-energy initial states used in the construction of the early universe was DM, with *no* dark energy , . What would be beneficial would be to delineate whether or not the graviton/ gravitational wave, at its genesis is linkable to DM, to understand the original space time in which the universe evolved in.. Note that Bert Janssen, Yolanda Lozano in hep-th/0207199 describe a so called massive/'giant' graviton in terms of study, from the microscopical point of view, via a giant graviton configurations where the gravitons expand into an M2-brane, with the topology of a fuzzy 2-sphere. Fine, but stating that AdS5×S5 background is used for embedding will not yield experimental confirmation. So in pursuit of experimental confirmation, it is appropriate to examine whether or not gravitons/ GW can tie in with DE and/or DM, which have measurable consequences as far as observational cosmology and astrophysics.

Linkage of DM to gravitons and gravitational waves?

Let us state that the object of early universe GW astronomy would be to begin with confirmation of whether or not relic GW were obtainable , and then from there to ascertain is there is linkage which can be made to DM production... Durrer, Massimiliano Rinaldi (2009) , state that there would be probably negligible for this case (practically non existent) graviton production in cosmological eras after the big bang.. In fact, they state that they investigate the creation of massless particles in a Universe which transits from a radiation-dominated era to any other (via an) expansion law . "We calculate in detail the generation of gravitons during the transition to a matter dominated era. We show that the resulting gravitons generated in the standard radiation/matter transition are negligible" This indicated to the author, Beckwith, that it is appropriate to look at the onset of relic GW/ Graviton production.. Note also that Ruth Durrer, Massimiliano Rinaldi state furthermore in their conclusions : " a graviton spectrum present at the beginning of the radiation era can become significantly amplified and modified by intermediate, non standard evolution of the universe". This is in part what will be suggested. A non standard evolution protocol which delivers One of the cruder ways of delineating the evolution of GW is the super adiabatic approximation, done for when $k^2 << \left|a''/a\right|$ as given by M. Giovannini (page 138) of the form, when $\mu_k \equiv a \cdot h_k$ is a solution to

$$\mu_k'' + \left[k^2 - \frac{a''}{a}\right]\mu_k = 0 \,. \tag{0.7}$$

Which to first order when $k^2 << \left|a''/a\right|$ leads to a GW solution

$$h_k(\tau) \cong A_k + B_K \cdot \int_0^\tau \frac{dx}{a(x)} \tag{0.7a}$$

This will be contrasted with a very similar evolution equation for gravitons, of the form (i.e. KK gravitons in higher dimensions)

$$h'' - \left[4k^2 + \frac{m^2}{a^2(z)}\right]h \equiv 0 \tag{0.7b}$$

One of the most frequently appealed to models of linkage between gravitons, and DM is the so called KK graviton, i.e. as a DM candidate. KK gravitons. Note that usual Randal Sundrum brane theory has a production rate of $\Gamma \sim T^6 \big/ M_{Planck}^2$ as the number of Kaluza Klein gravitons per unit time per unit volumeNote that this production rate is for a formula assuming mass for which $T_* > M_X$, and that we are assuming that the temperature $T \sim T_*$. Furthermore, we also are looking at a de facto total production rate of KK gravitons of the form

$$\frac{dn}{dt} \sim \frac{T^6}{M_{Planck}^2} \cdot (T \cdot R)^d \sim T^4 \cdot \left(\frac{T}{M_X}\right)^{2+d} \tag{0.7c}$$

Where R is the assumed higher dimension 'size' and , d is the number of dimensions above 4, and typically we obtain T >>1/R. I.e. we can typically assume tiny higher dimensional 'dimensions', very high temperatures, and also a wave length for the resulting KK graviton for a DM candidate looking like

$$\lambda_{KK-Graviton} \sim T^{-1} \tag{0.8}$$

If KK gravitons have the same wavelength as DM, this will support Jack Ng's treatment of DM. All that needs to put this on firmer ground will be to make a de facto linkage of KK Gravitons, as a DM candidate , and more traditional treatments of gravitons, which would assume a steady drop in temperature from $T \sim T^*$, to eventually much lower temperature scales. . Note that in a time interval based as proportional to the inverse of the Hubble parameter, we have the total numerical density of KK gravitons (on a brane?) as $n(T) \sim T^2 M^*_{Planck} \cdot (T/M)^{2+d}$, where $M^*_{Planck} \sim 10^{18} GeV$ give or take an order of magnitude. This number density $n(T)$ needs to be fully reconciled to $\lambda_{KK-Graviton} \sim T^{-1}$ and can be conflated with the dimensionality 'radius' value $R \sim 10^{\frac{32}{d}} \cdot 10^{-17}$ centimeters for dimensions above 4 space time GR values, with this value of R being unmanageable for d < 2 . V.A. Rubakov , and others also (2009) makes the claim of the KK graviton obeying the general Yukawa style potential

$$V(r) = -\frac{G_4}{r} \cdot \left(1 + \frac{const}{k^2 r^2}\right) \tag{0.8a}$$

As well as being related to an overall wave functional which can be derived from a line element

$$dS^2 \equiv \left[a^2(z) \cdot \eta_{uv} + h_{uv}(x,z)\right] \cdot dx^u dx^v + dz^2 \tag{0.9}$$

With $h'' - \left[4k^2 + \frac{m^2}{a^2(z)}\right] h \equiv 0$ (suppressing the u,v coefficients) . This evolution equation for the KK gravitons is very smilar to work done by Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi , Keitaro (2007) with similar assumptions, with the result that KK gravitons are a linear combination of Bessel functions. Note that one has for gravitions.

$$h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}} \tag{0.10}$$

Ruth Gregory, Valery A. Rubakov and Sergei M. Sibiryakov (2000) make the additional claim that for large z (the higher dimensions get significant) that there are marked oscillatory behaviors , ie. Rapid oscillations as one goes into the space for branes for massive graviton expansion.

$$h \equiv h_m(z \neq 0) \approx const \cdot \sqrt{a(z)} \cdot \sin\left(\frac{m}{k} \cdot esp(kz) + \varphi_m\right) \tag{0.11}$$

This is similar to what Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi , Keitaro (2007) for GW, in a relic setting, with the one difference being that the representation for a graviton is in the z (additional dimension) space, as opposed to what Bauman et al did for their evolution of GW, with an emphasis upon generation in over all GR space time.. Furthermore, the equation given in $h'' - \left[4k^2 + \frac{m^2}{a^2(z)}\right] h \equiv 0$ for massive graviton evolution as KK gravitons along dS branes is similar to evolution of GW in more standard cosmology that the author, Beckwith, thinks that the main challenge in clarifying this picture will be in defining the relationship of dS geometry, in overall Randall Sundrum brane world to that of standard 4 space,. We need though, now to look at whether or not higher dimensions are even relevant to GR itself.

How would DM be influenced by gravitons, in 4 dimensions

We will also discuss the inter relationship of structure of DM, with challenges to Gaussianity. The formula as given by

$$\delta \equiv -\left[\frac{3}{2} \cdot \Omega_m \cdot H^2\right]^{-1} \cdot \nabla^2 \Phi \tag{1.0}$$

will be gone into. The variation, so alluded to which we will link to a statement about the relative contribution of Gaussianity, via looking at the gravitational potential

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle \right] + g_{NL} \cdot \Phi_L^3 \tag{1.1}$$

Here the expression $f_{NL} =$ variations from Gaussianity, while the statements as to what contributes, or does not contribute will be stated in our presentation. Furthermore, $\Phi_L \equiv$ is a linear Gaussian potential, and the over all gravitational potential is altered by inputs from the term, presented, f_{NL} . The author discussed inputs into variations from Gaussianity, which were admittedly done from a highly theoretical perspective with Sabino Matarre, on July 10, with his contributions to non Guassianity being constricted to a reported range of $-4 < f_{NL} < 80$, as given to Matarre, by Senatore, et al, 2009. The author, Beckwith, prefers a narrower range along the lines of $.5 < f_{NL} < 20$ for reasons which will be gone into, in the text. . Needless to state, though, dealing with what we can and cannot measure, what is ascertained as far as DM , via a density profile variation needs to have it reconciled with DM detection values

$$\sigma_{DM-dectecion} \leq 3 \times 10^{-8} \quad \text{pb (pico barns)} \tag{1.2}$$

It is note worthy to note that the question of DM/ KK gravitons, and also the mass of the graviton not only has relevance to whether or not, higher dimensions are necessary/ advisable in space time models , but also may be relevant to if massive gravitons may solve / partly fulfill the DE puzzle. To whit, \ KK gravitons would have a combined sum of Bessel equations as a wave functional representation. In fact V. A Rubasov (2009) writes that KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$,where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \tag{1.3}$$

This allegedly is for KK gravitons having an order of TeV magnitude mass $M_Z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as a possible initial starting value for the KK graviton mass, before the KK graviton, as a 'massive' graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK gravition, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass many times lower in value, i.e. of at least $m_{graviton}(4 - Dim \quad GR) \sim 10^{-48} eV$, as opposed to $M_X \sim M_{KK-Graviton} \sim .5 \times 10^9 eV$. Whatever the range of the graviton mass , it may be a way to make sense of what was presented by Dubovsky, Flauger, Starobinsky, and Thackev (2009) who argue for graviton mass using CMBR measurements, of up to $m_{graviton}(4 - Dim \quad GR) \sim 10^{-20} eV$. This can be conflated with Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo's results arguing that non zero graviton mass may lead to acceleration of our present universe, in a manner usually conflated with DE , i.e. their graviton mass would be about

$m_{graviton}(4-Dim \quad GR) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$ grams, leading to a possible explanation for when the universe accelerated, i.e. the de-acceleration parameter, due to changes in the scale factor, written as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{1.4}$$

In the case of working with a simpler version of the Friedman equation with no graviton mass, but with pressure and density factored in, we can obtain

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \cdot \left[\left(-3p/c^2\right) - \rho\right] \tag{1.4a}$$

This will lead to a very simple de celebration parameter value of

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \left(\frac{4\pi G}{3c^2 H^2}\right) \cdot \left[3p + \rho\right] \tag{1.4b}$$

The article will see what happens to insure whether or not the sign of 1.4, and 1.4b goes from positive to negative.

Needless to say, if one has a graviton mass $m_{graviton} \neq 0$, then (1.4a) changes, and there will be a way forward to consider whether or not

Using a modification of GR, with scale factor evolution of , with non zero graviton mass terms added in to obtain

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{m_g^2 c^4}{2\hbar^2} \cdot \left(1 - a^2\right) \equiv \frac{8\pi G}{3c^2} \cdot \rho \tag{1.5}$$

and

$$\frac{\ddot{a}}{a} + .5 \cdot \left(\frac{\dot{a}^2}{a^2}\right) + \frac{m_g^2 c^4}{4\hbar^2} a^2 \cdot \left(a^2 - 1\right) = \frac{8\pi G}{3c^2} \cdot p \,. \tag{1.6}$$

For the matter dominated era, it is important to note that the R.H.S. of (1.6) is zero. This leads to (1.4) having increasingly positive acceleration values as would be definitely be given for masses of $m_{graviton}(4-Dim \quad GR) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$ grams for red shift values $z \sim .3$ for (1.4) just becoming > 0 to maximum values of (1.4) today, with $z = 0$, all at mass of the order of 10^{65} grams. This increase of (1.4) then leads us to consider how to configure (1.5) and (1.6) and for RS brane world values. there are terms which are added to the first Friedman equation. i.e.. when using ultra low graviton mass , where $r_c = \frac{M_P^2}{2M_{(5)}^3}$ and , often $\varepsilon = 1$ and r_C is usually though of as the separation between branes. I.e. if $r_C \to \infty$, we recover the usual first Friedman equation. For now we write the first Friedman equation for a brane system as.

$$\frac{\dot{a}}{a} \equiv H = \frac{\epsilon}{2r_c} + \sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} \tag{1.7}$$

As can be related to, if we wish to look at string theory versions of the FRW equation , in Friedman-Roberson – Walker metric space, we can do the following de compostion , with different limiting values of the mass, and other expressions, e.g. as a function of an existing cosmological constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{1.8}$$

As well as

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{\left(\rho_{Total} + 3p_{Total}\right)}{6M_{Planck}^2} + \frac{\Lambda}{3} \tag{1.9}$$

Not only this, if looking at the brane theory Friedman equations as presented by / for Randall Sundrum theory, it would be prudent working with

$$\dot{a}^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2} \right) a^2 - \kappa + \frac{C}{a^2} \right] \quad (1.10)$$

For the purpose of Randal Sundrum brane worlds, (1.10) is what will be differentiated with respect to $d/d\tau$, and then terms from (1.5) will be used, and put into a derivable equation which will be for a RS brane world version of $q = -\frac{\ddot{a}a}{\dot{a}^2}$. Several different versions of what q should be will be offered as far as what the time dependence of terms in 1.10 actually is. Note that Roy Maartens has written as of 2004 that KK modes (graviton) satisfy a 4 Dimensional Klein – Gordon equation, with an effective 4 dim mass, $m_n(Graviton) = \frac{n}{L}$, with $m_0(Graviton) = 0$, and L as the stated 'dimensional value' of higher dimensions. The value $m_0(Graviton) \sim 10^{-65} - 10^{-60}$ gram in value picked is very small, but ALMOST zero. Grossing has shown how the Schrodinger and Klein Gordon equations can be derived from classical lagrangians, i.e. using a version of the relativistic Hamilton-Jacobi- Bohm equation, with a wave functional $\psi \sim \exp(-iS/\hbar)$, with S the action, so as to obtain working values of for a tier of purported masses of a graviton from the equation , for 4 D of $\left[g^{\alpha\beta} \partial_\alpha \partial_\beta \xrightarrow[FLAT-SPACE]{} \nabla^2 - \partial_\tau^2 \right]$, and $\left[\nabla^2 - \partial_\tau^2 \right] \cdot \psi_n = m_n^2(graviton) \cdot \psi_n$ If one is adding , instead the small mass of $m_n(Graviton) = \frac{n}{L} + 10^{-65}$ grams, with $m_0(Graviton) \approx 10^{-65}$ grams, then the problem being worked with is a source term problem of the form given by Peskins as of the type

$$\psi_n(x) \equiv \int d^3p \cdot \frac{1}{(2\pi)^3} \cdot \frac{1}{\sqrt{2E_P}} \cdot \left\{ \left(a_P + \frac{i}{\sqrt{2E_P}} \cdot FT(m_0(graviton)) \right) \exp(-ipx) + H.C. \right\} \quad (1.11)$$

This is, using the language V.A. Rubakob (2009) put up equivalent to writing, using (1.3)

$$\psi_m(x) \approx h_m(x) + \int d^3p \cdot \frac{1}{(2\pi)^3} \cdot \left(\frac{1}{\sqrt{2E_P}} \right)^2 \cdot \left\{ (i \cdot FT(m_0(graviton))) \exp(-ipx) + H.C. \right\} \quad (1.12)$$

I.e. how to inteprept the quantity $FT(m_0(graviton)$ being the issue which will be covered in this document. If $m_0(graviton)$ is a constant, then the expression (1.12) has delta functions. This goes into evaluating, then, momentum, appropriately,.We will do a time differentiation of *(1.10) in this document, and compare it term by term with what arises if there is a sutiable graviton mass, and comment as to what would be needed to have graviton mass in a brane version of *(1.7) , and its time derivative, and do a similar analysis as to what was done to recover the positive acceleration , for *(1.4) using brane equivalents to (1.5) as well as imputs from (1.6) .Now why is this important ? This datum may especially show up about modification of the typical galaxy models, as follows

Time for the headache pills. Not everyone buys dark energy . I.e. Controversies of DM/ DE applications to cosmology. How HFGW may help resolve them.

The following is meant as a travelogue as to current problems in cosmology which will require significant revision of our models. Exhibit A as to what to consider is The cosmic void hypothesis'. See Timothy Clifton, Pedro G. Ferreira and Kate Land . I.e. Clifton raises the following question- can HFGW and detectors permit cosmologist to get to the bottom of this ? "Solving Einstein's equations for an averaged matter distribution is NOT the same as solving for the real matter distribution and then averaging the resultant geometry"("We average, then solve when in effect we should solve, then average") .

Next, let us look at a recently emerging conundrum of DM feeding into the structure of new galaxies and their far earlier than expected development, i.e. 5 billion years after the big bang. Galaxy formation issues…. Hierarchical Galaxy Formation theory at a glance usually proceeds as follows. I.e. what happens when the following diagram of simple addition of new structure no longer holds ? This is very significant, since when the significant formation of galaxies occurs, as of about $z \sim .2$ is before the turn up in the expansion rate for the universe, which will be referenced as of occurring about $z \sim .5 - .55$. What do we do if, as an example, find that the initial start of galaxy formation occurred five billion years ago, at, say $z \sim .5$. What could cause the earlier clumping?

$$\delta \equiv -\left[\frac{3}{2}\cdot \Omega_m \cdot H^2\right]^{-1} \cdot \nabla^2 \Phi$$

Several scenarios which will be investigated. First of all, note the formula of variation of DM density which exists has, among other things a Hubble parameter *H*, and also the 2nd derivative of the gravitational potential $\nabla^2 \Phi$, where ρ_0 , a_0 are today's values for density and 'distance' .Note that if the

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2}\right) - \frac{\kappa}{a^2} + \frac{C}{a^4}\right] \xrightarrow[\Lambda_4 \to 0]{\kappa \to 0,} \left[\frac{\rho}{3M_4^2} + \frac{\rho^2}{36M_{Planck}^2} + \frac{C}{a^4}\right],$$

$$\rho \to \rho(z) \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g}{8\pi G}\right] \cdot \left(\frac{a_0^4}{14\cdot(1+z)^4} + \frac{2a_0^2}{5\cdot(1+z)^2} - \frac{1}{2}\right), \text{ and } 1 + z = a_0 / a \text{ , then the}$$

contribution of large z , i.e. large contributions from red shift, that a significant early contributions will be for non zero contributions from $1/\rho^{\beta}$ terms , for **[large number]** $> \beta \geq 1$ in the DM density variation parameters. So long as $m_{graviton} \neq 0$, even if $m_{graviton}$ is very small. In addition, if the following is true

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle\right] + g_{NL} \cdot \Phi_L^3 ,$$ then there are contributions from terms to be considered.

When using the formula, $\nabla^2 \Phi$ consider the contributions to the expression f_{NL}. To do this consider first what Licia Verde (2000) put up about Φ considered to be the gravitational potential, and Φ_L its linear Gaussian contribution. This has been improved upon by , recently, P. Chingabam, C. Park (2009) improved upon the simulation done by Verde (2003) , who worked with f_{NL} bounded as follows: $10^{-4} < f_{NL} < 10^{-2}$, whereas the Chingabam, Park (2009) publication considered $-4 < f_{NL} < 80$ at a confidence level of 95%. One of the simpler suppositions a person could use is what would be involved if, $\lambda_{Graviton} = (h/m_{graviton} \cdot c) \equiv 1/m_{graviton}$

$$\left(1 + \frac{f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L \right\rangle^2\right]}{\Phi_L}\right) \propto \left[1 - \frac{r}{\lambda_{graviton}}\right] \sim \exp\left(-r/\lambda_{graviton}\right) \tag{1.12a}$$

Alternately, if the brane theory model of a gravitational potential were used, with KK graviton modes, then

$$\left(1 + \frac{f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L \right\rangle^2\right]}{\Phi_L}\right) \propto \left[1 + \frac{const.}{r^2 k^2}\right] \tag{1.12b}$$

Now for some sort of bounds as to what may be acceptable bounds in error, based upon CMB data

$$\left| f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle\right] \right| \leq 10^{-5} \cdot \left| f_{NL} \right| < ?up \quad to \quad 10^{-3} \tag{1.12c}$$

Depending upon which model is used for describing Φ_L i.e. as a perturbation of a gravitational potential, this eqn. (1.12c) may allow us to obtain a good guess as to what dimensions are crucial for the formation of a graviton, i.e. how much spread may be permitted. Note that Φ_L , is a linear approximation to the gravitational potential given in eqn.(1.1). In addition, one can, as a crude approximation write to first order $\Phi_L \sim 1/r$. Also the parameter f_{NL} is usually, often with partly sinusoidal variation, taken from primordial non gaussianity traces taken from the CMBR itself.

Non-linear dynamics at recombination will lead to, for CMBR a treatment similar to what was given by Jean-Luc Lehners, Paul J. Steinhardt, (2009), i.e. , if $\tilde{\Phi}_H$ is the Bardeen Space-Space metric perturbation, and $\tilde{\Phi}_L$ a Bardeen Space-Space perturbation to linear order. . Here, Bardeen Space-Space metric perturbation is defined by James M. Bardeen (1982) *

$$\tilde{\Phi}_H = \tilde{\Phi}_L + f_{NL} \cdot \tilde{\Phi}_L^2 \qquad (1.12d)$$

Also, White and Hu(1996), also have a convenient way to link the gravitational potential Φ to temperature fluctuations, and do it as , when assuming Φ_{final} is ignorable

$$\left.\frac{\Delta T}{T}\right|_{Final} - \left.\frac{\Delta T}{T}\right|_{Initial} = -\Phi_{Initial} \qquad (1.12e)$$

A simple way to understand what is said by equation (1.12e) is to consider if or not it is linkable to the Sach-Wolfe effect. Here, the Sachs–Wolfe effect (ISW) occurs when the Universe is dominated in its density by something other than matter. If the Universe is dominated by matter, then large-scale gravitational potential wells and hills do not evolve significantly. If the Universe is dominated by radiation, or by dark energy, though, those potentials do evolve, subtly changing the energy of photons passing through them. If there is a major difference in the initial and final ratios $\Delta T/T$ of temperature variations are for different red shift values, and for the Friedman model, to good approximation, $\delta T/T = \phi/3(c^2=1)$. If the approximations for the Friedman eqn. are valid, then one has , say

$$(\delta T/T) \cong (1/3) \cdot \left[\Phi_L + f_{NL} \cdot \left(\Phi_L^2 - \langle \Phi_L \rangle^2\right)\right] \qquad (1.12f)$$

Eqn. (1.12f) has its counter part in what Daniel Babich, Paolo Creminelli, Matias Zaldarriaga(2004) about the influence of curvature 'perturbations' with some of them being linear, and the others showing a slight perturbative effect. I.e. look at

$$\varsigma(x) \equiv \varsigma_{Gaussian}(x) - \frac{3}{5} f_{NL} \cdot \left[\varsigma_{Gaussian}^2(x) - \langle \varsigma_{Gaussian}(x) \rangle^2\right] \qquad (1.12g)$$

Here, $\varsigma_{Gaussian}(x)$ is with regards to a Gaussian perturbation of curvature. Where, $\varsigma_{Gaussian}(x)$, is as defined by David H. Lyth(2005)

$$\varsigma_{Non-Gaussian}(x) \equiv \sum_i N_i \delta\phi_i + \frac{1}{2} \cdot \sum_{ij} N_{ij} \delta\phi_i \delta\phi_j \xrightarrow[Single-field]{} N_\sigma \delta\sigma + N_{\sigma\sigma} (\delta\sigma)^2 \qquad (1.12h)$$

$$N_\sigma \equiv \partial N / \partial \sigma, N_{\sigma\sigma} \equiv \partial^2 N / \partial^2 \sigma, \text{ where } N = \int H d\tau$$

It is possible to construct good semiclassical physical states by such a procedure in this model; we also discuss the sense in which the original kinematical states may be a good approximation to the physical ones, and the situations in which this is the case. In addition, these models can be deparametrized in a natural way, and we study the effect of time evolution on an "intrinsic" coherent state in the reduced phase space

The upshot is that we intend to examine how this is linkable to entropy variations in Eqn. (1.12h) in future numerical simulations of CMBR irregularities

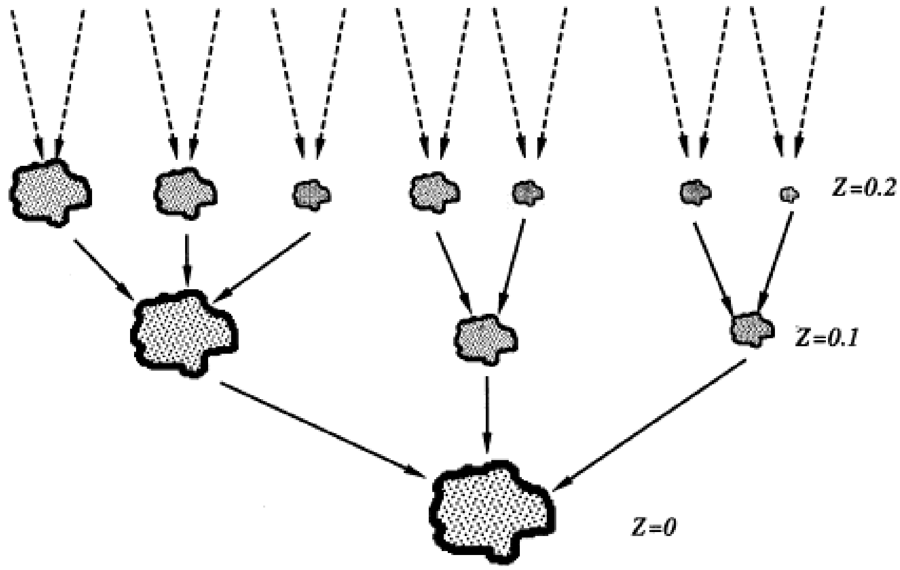

Figure 1. A schematic representation of a halo merging history 'tree'.

Figure 1. I.e. **how we obtain from the 'bottom up' development of galactic super structure.**

What is actually observed, contradicts this halo emerging history 'tree', i.e. Although this 'story' for DM seems to be well established. i.e. Just ONE little problem: DM appears to be fattening up young galaxies, allowing for far-earlier-than-expected creation of early galaxies. "A clutch of massive galaxies that seem to be almost fully-formed just 5 billion years after the big bang challenge models that suggest galaxies can only form slowly. Tendrils of dark matter that fed the young galaxies on gas could be to blame (NASA/CXC/ESO/P Rosati et al)"
http://www.newscientist.com/article/dn16912-overweight-galaxies-forcefed-by-dark-matter-tendrils.html.
Needless, to say though, an analysis of the influence of DM on structure formation would have to take into consideration the datum presented by G. Hinsaw and others as to the relative super abundance of DM in early universe conditions. I.e. considering the following. The relative imprecision of graviton measurement, can be given as follows. This is a measurement in particle physics, and if the KK graviton is linkable to DM, it means that we will have to have very good ways to test for production rates, as will be argued later. The following below is a typical representation of the KK tower model for gravitons, with the zeroth KK mode being approximately the 4 dimensional graviton. From scattering data, the relative mass contributions show up, as follows for KK gravitons, as modeled below. These representation are for leading up to investigating if or not one needs KK gravitons, as either a semi classical, or a brane theory/ string theory construct, as a was of determining if either

1). Gravitons have mass, i.e. $m_{graviton} \sim 10^{-65}$ grams $\sim 10^{-48}$ electron volts

2). DE style expansion at $z \sim .55$ can be seen as a consequences of 1) above. Does this necessitate or imply that KK gravitons should be presented as a quantum gravity theory ?

3). DM may be connected with KK variants of higher dimensional generalizations of $m_{graviton}$ mass

4). The issue of if or not gravitons/entropy/ constituent DM may be linked to instaton physics models of gravitons

Kaluza Klein modes in detector simulations for / as a DM candidate.

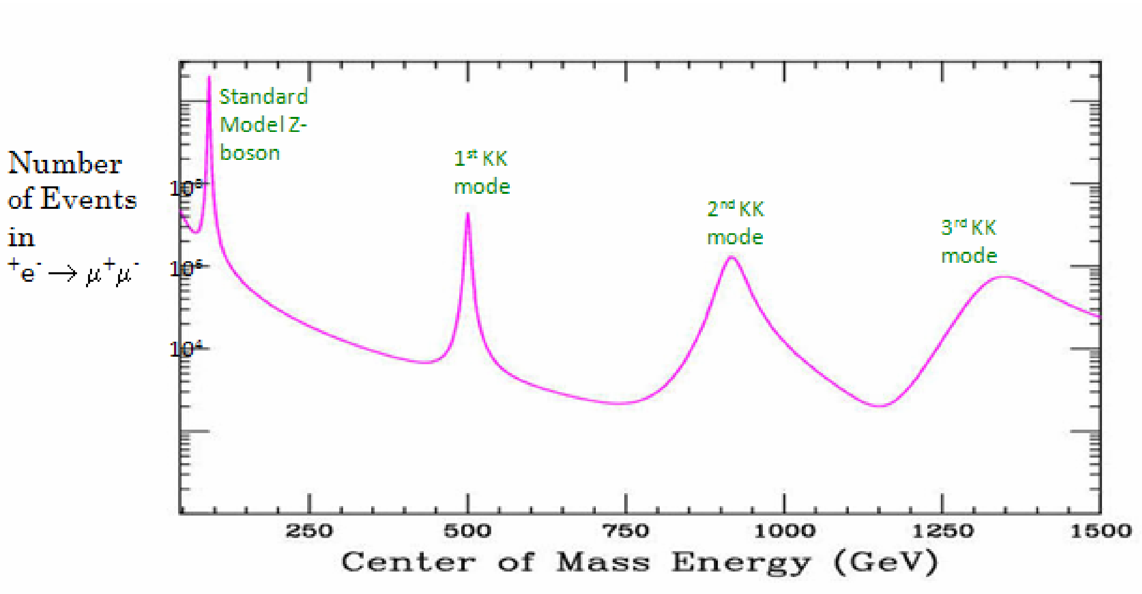

Figure 2a: **Example: Number of Events in e+e- → μ+μ- For a conventional braneworld model with a single curved extra dimension of size ~ 10-17 cm Numbers range from 10^4 to about 10^8 for the number of events in scattering. First peak is for KK zero mode , a.k.a. the standard Z boson, ending with the 4th peak for the 3rd KK mode ,**

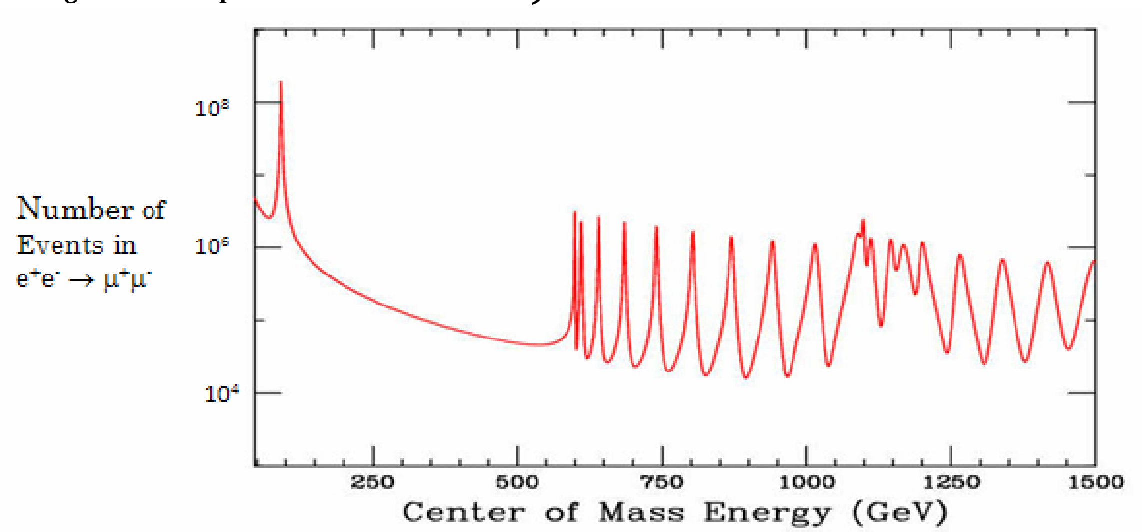

Figure 2b: **Example: Number of Events in e+e- → μ+μ- scattering experiment AGAIN. Thisfor a non conventional braneworld model with a single curved extra dimension of again of size ~ 10-17 cm. This is for the same model as given for figure 2a, but this time embedded in a string theory . Numbers range from 10^4 to about 10^8 for the number of events in scattering. First peak is for KK zero mode , a.k.a. the standard Z boson.**

Production rate for e+e- $\rightarrow \gamma$ + Graviton

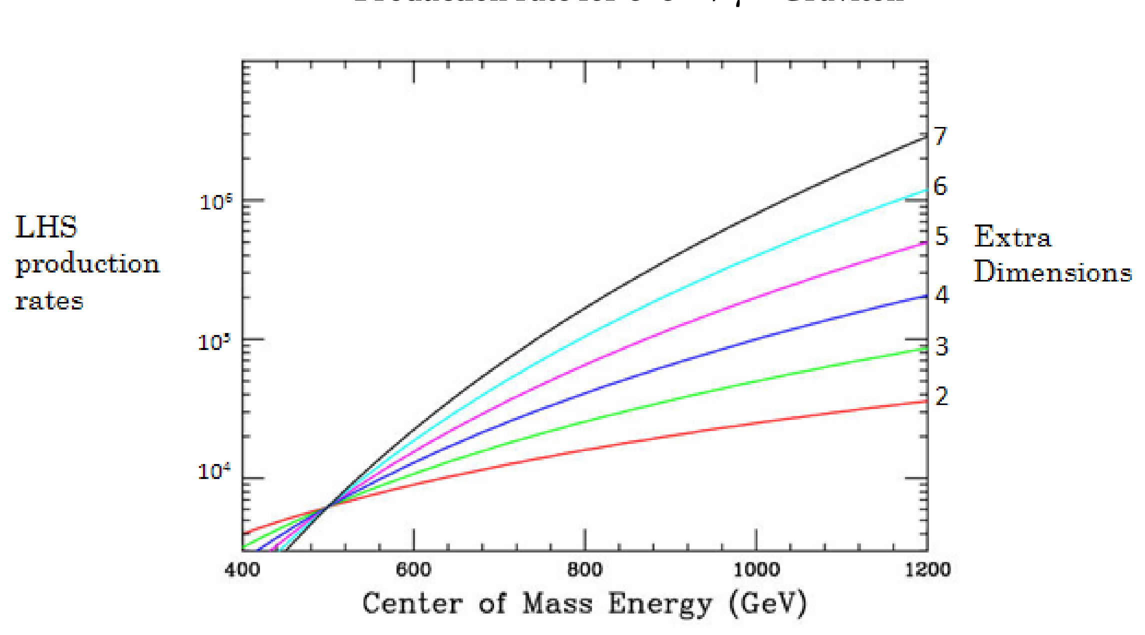

Figure 2c: Example: Production of Graviton Kaluza-Klein modes in flat extra dimensions, probes gravity at distances of ~ 10 to the -18 power cm . The LHS of the graph have production rates ranging from a low point of 10^4 at about 600 GeV , to values of about 10^6 at about 1000 GeV.

Understanding the KK gravitons as a DM candidate may permit us to understand how DM and DE are inter related. See as given below. The discussion of such will involve coherent state of gravitons as contributors to GW.

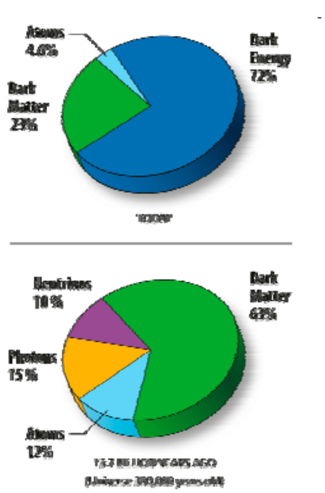

Figure 3 : From G. Hingsaw presentation, in COMO, Italy, July 2009 at the ISAPP
How DM, and other constituent parts of the early 380 thousand year old universe evolved to have connections with KK gravitons is connected closely with the following

Issues about Coherent state of Gravitons (linking gravitons with GW)

In the quantum theory of light (quantum electrodynamics) and other bosonic quantum field theories , coherent states were introduced by the work of Roy J. Glauber in 1963 Now, it is well appreciated that Gravitons are NOT similar to light. So what is appropriate for presenting gravitons as coherent states ? Coherent states , to first approximation are retrievable as minimum uncertainty states. If one takes string theory as a reference, the minimum value of uncertainty becomes part of a minimum uncertainty which can be written as given by Venziano (1993), where $l_S \cong 10^{\alpha} \cdot l_{Planck}$, with $\alpha > 0$, and $l_{Planck} \approx 10^{-33}$ centimeters

$$\Delta x > \frac{\hbar}{\Delta p} + \frac{l_S^2}{\hbar} \cdot [\Delta p] \tag{1.13}$$

To put it mildly, if we are looking at a solution to minimize graviton position uncertainty, we will likely be out of luck if string theory is the only tool we have for early universe conditions. Mainly, the momentum will not be small, and uncertainty in momentum will not be small either. Either way, most likely, $\Delta x > l_S \cong 10^{\alpha} \cdot l_{Planck}$ In addition, it is likely, as Klaus Kieffer in the book " Quantum Gravity" on page 290 of that book that if gravitons are excitations of closed strings, then one will have to look for conditions for which a coherent state of gravitons, as stated by Mohaupt (2003) occurs. What Mohaupt is referring to is a string theory way to re produce what Ford gave in 1995, i.e. conditions for how Gravitons in a squeezed vacuum state, the natural result of quantum creation in the early universe will introduce metric fluctuations. Ford's (1995) treatment is to have a metric averaged retarded Green's function for a massless field becoming a Gaussian. The condition of Gaussianity is how to obtain semi classical , minimal uncertainty wave states, in this case de rigor for coherent wave function states to form. Ford uses gravitons in a so called 'squeezed vacuum state' as a natural template for relic gravitons. I.e. the squeezed vacuum state (a **squeezed coherent state**) is any state such that the uncertainty principle is saturated.: In QM coherence would be when $\Delta x \Delta p = \hbar/2$. In the case of string theory it would have to be $\Delta x \Delta p = \frac{\hbar}{2} + \frac{l_S^2}{2 \cdot \hbar} \cdot [\Delta p]^2$. Putting it mildly, the string theory case is far more difficult. And that is the problem, with regards to string theory, what is an appropriate vacuum expectation value for treating a template of how to nucleate gravitons into a coherent state with respect to relic conditions. Ford, in 1994, wrote a squeezed state operation S ( ) via $|\varsigma\rangle = S(\varsigma) \cdot |0\rangle$, Here, the operator . $|0\rangle$ is a ground state, and frequently, as Ford did, in 1994, there is a definition of a root mean squared fluctuation of a graviton / gravitational wave state via use of an average scalar field Φ , where

$$h^2 = \frac{1}{30} \cdot \langle h_{ij} h^{ij} \rangle = \frac{1}{15} \cdot \langle \Phi^2 \rangle \equiv \frac{1}{180} \cdot T^2 \Big|_{Thermai-bath} \tag{1.14}$$

Here, the value $T \,\Big|_{Thermai-bath}$ has yet to be specified, and that actually for energy values approximately of the order of $10^{15}\, GeV$ which may be the mean temperature for the expanding universe mid way, to the end of inflation, which does not equal current even smaller string theory estimates as presented by Li et al $h \,\Big|_{THERMAL-BATH} \approx 10^{-18} / \sqrt{Hz} \neq h_{rms} \sim 10^{-30} - 10^{-34} / \sqrt{Hz} \equiv$ string theory values for inflationary Gravitational amplitudes. I.e. the more modern treatments are predicting almost infinitesimal GW fluctuations. It is not clear from Ford's 1995 treatment of gravitons, and fluctuations, if he is visualizing fluctuation of gravitons/ GW, but if one takes literally (1.14) as a base line, and then considering what would be the optimal way to obtain a way to obtain coherent states of gravitons, going to the Li stated value of $h_{rms} \sim 10^{-39} / \sqrt{Hz}$ for solar plasma from the sun as a graviton source, would be a way of obtaining fluctuations $10^{-5} - 10^{-9}$ times weaker , i.e. going to h_{rms} values so small that the requirement for a minimum fluctuation , in line with not contradicting $\Delta x \Delta p = \frac{\hbar}{2} + \frac{l_S^2}{2 \cdot \hbar} \cdot [\Delta p]^2$, if we consider experimental conditions for obtaining $\Delta x \approx h_{rms} \sim 10^{-39} / \sqrt{Hz}$. Note that this would put severe restrictions upon the variations in momentum. A subject which will be referenced in whether or not the Li-Baker detector can suitably obtain such small values of $\Delta x \approx h_{rms} \sim 10^{-39} / \sqrt{Hz}$ in detection capacity. To do so will require an investigation into extreme sensitivity requirements, for this very low value of h_{rms}. Fanguy Li. et al (2009) reports in their PRD document $h_{rms} \sim 10^{-26} - 10^{-30} \big/ \sqrt{Hz}$ would require up to 10^5 seconds in evaluative time for a clean signal, for GW. What will be asked in further sections is if or not the 10^5 seconds in evaluative time for a clean signal can evaluate additional data. I.e. what if one would have to do to distinguish if or not coherent states of gravitons which merge to form GW may be measured via the protocols brought up by Li et al (2009) for relic GW ?

Can any detector measure $h_{rms} \sim 10^{-39} \big/ \sqrt{Hz}$? How squeezed state conditions at the onset of inflation affects usual attempts at measurement of coherent relic graviton states.

Not now. Current limits would be, as Gary Stephenson said in private communications for $h_{rms} \sim 10^{-32} \big/ \sqrt{Hz}$ as a de facto limit for sensitivity. Now what could be said about forming states close to classical representations of gravitons? Venkatartnam, and Suresh, 2007 built up a coherent state via use of a displacement operator $D(\alpha) \equiv \exp(\alpha \cdot a^+ - \alpha^* \cdot a)$, applied to a vacuum state , where α is a complex number, and a, a^+ as annihilation, and creation operations $[a, a^+] = 1$, where one has

$$|\alpha\rangle = D(\alpha) \cdot |0\rangle \qquad (1.15)$$

However, what one sees in string theory, is a situation where a vacuum state as a template for graviton nucleation is built out of an initial vacuum state, $|0\rangle$. To do this though, as Venkatartnam, and Suresh did, involved using a squeezing operator $Z[r, \vartheta]$ defining via use of a squeezing parameter r as a strength of squeezing interaction term , with $0 \le r \le \infty$, and also an angle of squeezing, $-\pi \le \vartheta \le \pi$ as used in $Z[r, \vartheta] = \exp\left[\frac{r}{2} \cdot \left([\exp(-i\vartheta)] \cdot a^2 - [\exp(i\vartheta)] \cdot a^{+2}\right)\right]$, where combining the $Z[r, \vartheta]$ with (1.15) leads to a single mode squeezed coherent state, as they define it via

$$|\varsigma\rangle = Z[r, \vartheta]|\alpha\rangle = Z[r, \vartheta] D(\alpha) \cdot |0\rangle \xrightarrow[\alpha \to 0]{} Z[r, \vartheta] \cdot |0\rangle \qquad (1.16)$$

The right hand side . of eqn. (1.16) given above becomes a highly non classical operator, i.e. in the limit that the super position of states $|\varsigma\rangle \xrightarrow[\alpha \to 0]{} Z[r, \vartheta] \cdot |0\rangle$ occurs, there is a many particle version of a 'vacuum state' which has highly non classical properties. Squeezed states, for what it is worth, are thought to occur at the onset of vacuum nucleation , but what is noted for $|\varsigma\rangle \xrightarrow[\alpha \to 0]{} Z[r, \vartheta] \cdot |0\rangle$ being a super position of vacuum states, means that classical analog is extremely difficult to recover in the case of squeezing, and general non classical behavior of squeezed states. Can one, in any case, faced with $|\alpha\rangle = D(\alpha) \cdot |0\rangle \neq Z[r, \vartheta] \cdot |0\rangle$ do a better job of constructing coherent graviton states, in relic conditions, which may not involve squeezing ?. Note L. Grishchuk wrote in (1989) in "On the quantum state of relic gravitons", where he claimed in his abstract that 'It is shown that relic gravitons created from zero-point quantum fluctuations in the course of cosmological expansion should now exist in the squeezed quantum state. The authors have determined the parameters of the squeezed state generated in a simple cosmological model which includes a stage of inflationary expansion. It is pointed out that, in principle, these parameters can be measured experimentally'. Grishchuk , et al, (1989) reference their version of a cosmological perturbation h_{nlm} via the following argument. How we work with the argument will affect what is said about the necessity, or lack of, of squeezed states in early universe cosmology. From Class. Quantum Gravity: 6 (1989), L 161-L165, where h_{nlm} has a component $\mu_{nlm}(\eta)$ obeying a parametric oscillator equation, where K is a measure of curvature which is $= \pm 1,0$, $a(\eta)$ is a scale factor of a FRW metric, and $n = 2\pi \cdot [a(\eta)/\lambda]$ is a way to scale a wavelength, λ, with n, and with $a(\eta)$

$$h_{nlm} \equiv \frac{l_{Planck}}{a(\eta)} \cdot \mu_{nlm}(\eta) \cdot G_{nlm}(x) \tag{1.17}$$

$$\mu''_{nlm}(\eta) + \left(n^2 - K - \frac{a''}{a} \right) \cdot \mu_{nlm}(\eta) \equiv 0 \tag{1.18}$$

If $y(\eta) = \dfrac{\mu(\eta)}{a(\eta)}$ is picked, and a Schrodinger equation is made out of the Lagrangian used to formulate (1.18) above, with $\hat{P}_y = \dfrac{-i}{\partial y}$, and $M = a^3(\eta)$, $\Omega = \dfrac{\sqrt{n^2 - K^2}}{a(\eta)}$, $\breve{a} = [a(\eta)/l_{Planck}] \cdot \sigma$, and $F(\eta)$ an arbitrary function. $y' = \partial y / \partial \eta$. Also, we have a finite volume $V_{finite} = \int \sqrt{{}^{(3)}g}\, d^3x$

Then the Lagrangian for deriving (1.18) is (and leads to a Hamiltonian which can be **also** derived from the Wheeler De Witt equation), with $\varsigma = 1$ for zero point subtraction of energy

$$L = \frac{M \cdot y'^2}{2a(\eta)} - \frac{M^2 \cdot \Omega^2 a \cdot y^2}{2} + a \cdot F(\eta) \tag{1.19}$$

$$\frac{-1}{i} \cdot \frac{\partial \psi}{a \cdot \partial \eta} \equiv \hat{H}\psi \equiv \left[\frac{\hat{P}^2{}_y}{2M} + \frac{1}{2} \cdot M\Omega^2 \hat{y}^2 - \frac{1}{2} \cdot \varsigma \cdot \Omega \right] \cdot \psi \tag{1.20}$$

then there are two possible solutions to the S.E. Grushchuk created in 1989, one a non squeezed state, and another a squeezed state. So in general we work with

$$y(\eta) = \frac{\mu(\eta)}{a(\eta)} \equiv C(\eta) \cdot \exp(-B \cdot y) \tag{1.21}$$

The **non squeezed state** has a parameter $B|_\eta \xrightarrow[\eta \to \eta_b]{} B(\eta_b) \equiv \omega_b / 2$ where η_b is an initial time, for which the Hamiltonian given in (1.20) in terms of raising/ lowering operators is 'diagnonal', and then the rest of the time for $\eta \neq \eta_b$, the **squeezed state** for $y(\eta)$ is given via a parameter B for squeezing which when looking at a squeeze parameter r, for which $0 \le r \le \infty$, then (1.21) has, instead of $B(\eta_b) \equiv \omega_b / 2$

$$B|_\eta \xrightarrow[\eta \neq \eta_b]{} B(\omega, \eta \neq \eta_b) \equiv \frac{i}{2} \cdot \frac{(\mu/a(\eta))'}{(\mu/a(\eta))} \equiv \frac{\omega}{2} \cdot \frac{\cosh r + [\exp(2i\vartheta)] \cdot \sinh r}{\cosh r - [\exp(2i\vartheta)] \cdot \sinh r} \tag{1.22}$$

Taking Grishchuck's formalism literally, a state for a graviton/ GW is not affected by squeezing when we are looking at an initial frequency, so that $\omega \equiv \omega_b$ initially corresponds to a non squeezed state which may have coherence, but then right afterwards, if $\omega \neq \omega_b$ which appears to occur whenever the time evolution, $\eta \neq \eta_b \Rightarrow \omega \neq \omega_b \Rightarrow B(\omega, \eta \neq \eta_b) \equiv \dfrac{i}{2} \cdot \dfrac{(\mu/a(\eta))'}{(\mu/a(\eta))} \neq \dfrac{\omega_b}{2}$ A reasonable research task would be to determine, whether or not $B(\omega, \eta \neq \eta_b) \neq \dfrac{\omega_b}{2}$ would correspond to a vacuum state being initially formed right after the point of nucleation, with $\omega \equiv \omega_b$ at time $\eta \equiv \eta_b$ with an initial cosmological time some order of magnitude of a Planck interval of time $t \approx t_{Planck} \propto 10^{-44}$ seconds The next section will be to answer whether or not there could be a point of no squeezing, as Grishchuck implied, for initial times, and initial frequencies, and an immediate transition to times, and frequencies afterwards, where squeezing was mandatory. Note that in 1993, Grischchuk further extended his analysis, with respect to the same point of departure, ie. What to do with when $|\alpha\rangle = D(\alpha)\cdot|0\rangle \neq Z[r,\vartheta]\cdot|0\rangle$. Having $|\alpha\rangle = D(\alpha)\cdot|0\rangle$ with $D(\alpha)$ a possible displacement operator, seems to be in common with $B(\eta_b) \equiv \omega_b/2$, whereas $|\alpha\rangle = Z[r,\vartheta]\cdot|0\rangle$ which is highly non classical seems to be in common with a solution for which $B(\omega_b) \neq (\omega_b/2)$ This leads us to the next section, i.e. does $B(\eta_b) \equiv \omega_b/2$ when of time $t \approx t_{Planck} \propto 10^{-44}$ seconds, and then what are the initial conditions for forming 'frequency' $\omega \equiv \omega_b$?

Necessary and sufficient conditions for String/ Brane theory graviton coherent states?

A curved spacetime is a coherent background of gravitons, and therefore in string theory is a coherent state Joseph Gerard Polchinski starting with the typical small deviation from flat space times as can be written up by $G_{uv}(X) = \eta_{uv} + h_{uv}(X)$, with η_{uv} flat space time, and the Polyakov action, is generalized as follows, the S_σ Polyakov action is computed and compared with exponentiated values

$$S_\sigma = \frac{1}{4\pi\alpha'}\cdot\int_M d^2\sigma\cdot\sqrt{g}\cdot g^{ab}\cdot G_{uv}(X)\cdot\partial_a X^\mu \partial_b X^\nu \tag{1.23}$$

becomes

$$\exp(-S_\sigma) = [\exp(-S_P)]\cdot\left[1 - \frac{1}{4\pi\alpha'}\cdot\int_M d^2\sigma\cdot\sqrt{g}\cdot g^{ab}\cdot h_{uv}(X)\cdot\partial_a X^\mu \partial_b X^\nu + \ldots\right] \tag{1.24}$$

Polochinski writes that the term of order h in equation (1.24) is the vertex operator for the graviton state of the string, with $h_{uv}(X) \equiv -4\pi g_c \cdot \exp[-ikX_{S_{uv}}]$, and the action of S_σ a coherent state of a graviton. Now the important question to ask, is if this coherent state of a graviton, as mentioned by Polochinski can hold up in relic, early universe conditions. Rainer Dick, in 2001, argued as stating that the "graviton multiplet as one particular dark matter source in heterotic string theory. In particular, it is pointed out that an appreciable fraction of dark matter from the graviton multiplet requires a mass generating phase transition around $T_c \simeq 10^8$ GeV, where the symmetry partners of the graviton would evolve from an ultrahard fluid to pressureless dark matter. indicates $m \simeq 10$ MeV for the massive components of the graviton multiplet". This has a counter part in a presentation made by Berkenstein (2004) with regards to BPS states, and SHO models for $AdS_5 \times S^5$ geometry. The upsot is that string theory appears to construct coherent graviton states, but it has no answer to the problem that Ford (1995), Grishchuck, wrote on if the existing graviton coherent states would be squeezed into non classical configurations in relic conditions.

Does LQG give us more direct arguments as to coherent states, squeezed states, and the break down of classical behavior at the onset of inflation ?

Carlo Rovelli, in 2006, in a PRL article states that a vertex amplitude that contributes to a coherent graviton state is the exponential of the Regge action: the other terms, that have raised doubts on the physical viability of the model, are suppressed by the phase of the vacuum state, and Rovelli writes a coherent vacuum state as given by a Gaussian peaked on parts of the boundary Σ_d of a four dimensional sphere.

$$\Psi_q[s] = \Psi_q(\Gamma, j_{m,n}) \tag{1.25}$$

Rovelli states that "bad" contributions to the behavior of Eqn. (1.25) are cancelled out by an appropriate (Gaussian?) vacuum wave functional which has 'appropriately' chosen contributions from the boundary

Σ_d of a four dimensional sphere. This is to avoid trouble with “bad terms” from what is known as the Barret – Crane vertex amplitude contributions, which are can be iminized by an appropriate choice of vacuum state amplitude being picked. Rovelli calculated some components of the graviton two-point function and found that the Barrett-Crane vertex yields a wrong long-distance limit. A problem, as stated by Lubos Motel (2007), that there are infinitely many other components of the correlators in the LQG that are guaranteed not to work unless an infinite number of adjustments are made. The criticism is harsh, but until one really knows admissible early universe geometry one cannot rule out the Rovelli approach, or confirm it. In addition, Jakub Mielczarek (2009) considered tensor perturbations produced at a bounce phase in presence of the holonomy corrections. Here bounce phase and holonomy corrections originate from Loop Quantum Cosmology What comes to the fore are corrections due to what is called quantum holonomy, l.. A comment about the quantum bounce. i.e. what is given by Dah-Wei Chiou, Li-Fang Li,(2009) is that there is a branch match up between a prior to a present set of Wheeler De Witt equations for a prior to present universe, as far as modeling how the quantum bounce links the two Wheeler De Witt solution branches, i.e. one Wheeler De Witt wave function for a prior univers, and another wave function for a present universe.Furthermore, Abhay Ashtekar (2006) wrote a simple treatment of the Bounce causing Wheeler De Witt equation along the lines of , for $\rho_* \approx const \cdot (1/8\pi G\Delta)$ as a critical density, and Δ the eignvalue of a minimum area operator. Small values of Δ imply that gravity is a repulsive force, leading to a bounce effect.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{8\pi G}{3}\cdot \rho \cdot \left(1-\left(\rho/\rho_*\right)\right) + H.O.T. \tag{1.26}$$

Furthermore, Bojowald (2008) specified a criteria as to how to use an updated version of Δ and $\rho_* \approx const \cdot (1/8\pi G\Delta)$ in his GRG manuscript on what could constitute grounds for the existence of generalized squeezed initial (graviton ?) states. Bojowald (2008) was referring to the existence of squeezed states, as either being necessarily, or NOT necessarily a consequence of the quantum bounce. As Bojowald (2008) wrote it up, in both his equation (26) which has a quantum Hamiltonian $\langle \hat{V} \rangle \approx H$,with

$$\left.\frac{d\langle \hat{V}\rangle}{d\phi}\right|_{\phi \approx 0} \xrightarrow[existence-of-un-squeezed-states \Leftrightarrow \phi \approx 0]{} 0 \tag{1.27}$$

, and $\hat{V}$ is a ‘volume’ operator where the ‘ volume’ is set as V , Note also, that Bojowald has , in his initial Friedman equation, density values $\rho \equiv \frac{H_{matter}(a)}{a^3}$, so that when the Friedman equation is quantized, with an initial internal time given by ϕ , with ϕ becoming a more general evolution of state variable than ‘internal time’. If so, Bojowald (2008) writes, when there are squeezed states

$$\left.\frac{d\langle \hat{V}\rangle}{d\phi}\right|_{\phi \neq 0} \xrightarrow[existence-of-squeezed-states]{} N(value) \neq 0 \tag{1.28}$$

for his equation (26), which is incidently when links to classical behavior break down , and when the bounce from a universe contracting goes to an expanding present universe,. Bojowald also writes that if one is looking at an isotropic universe, that as the large matter ‘H’ increases, that in certain cases, one observes more classical behavior, and a reduction in the strength of a quantum bounce.. Bojowalds states that “Especially the role of squeezed states is highlighted. The presence of a bounce is proven for uncorrelated states, but as squeezing is a dynamical property and may change in time” The upshot is that although it is likely in a quantum bounce state that the states should be squeezed, it is not a pre requisite for the states to always start off as being squeezed states. .So a physics researcher can ,look at if an embedding of the present universe in a higher dimensional structure which could have lead to a worm hole from a prior universe to our present for re introduction of inflationary growth

2. **Other models. Do worm hole bridges between different universes allow for initial un squeezed states? Wheeler De Witt solution with pseudo time component added in.**

This discussion is to present a not so well known but useful derivation of how instanton structure from a prior universe may be transferred from a prior to the present universe..

1. The solution as taken from L. Crowell's (2005) book, and re produced here, as referenced by Beckwith (2008,2009) has many similarities with the WKB method. I.e. it is semi CLASSICAL.
2. left unsaid is what embedding structure is assumed
3. A final exercise for the reader. Would a WKB style solution as far as transfer of 'material' from a prior to a present universe constitute procedural injection of non compressed states from a prior to a present universe ? Also if uncompressed, coherent states are possible, how long would they last in introduction to a new universe?

This is the Wheeler-De-Witt equation with pseudo time component added. From Crowell

$$-\frac{1}{\eta r}\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{\eta r^2}\cdot\frac{\partial\Psi}{\partial r}+rR^{(3)}\Psi=\left(r\eta\phi-r\ddot{\phi}\right)\cdot\Psi \tag{1.29}$$

This has when we do it $\phi \approx \cos(\omega \cdot t)$, and frequently $R^{(3)} \approx$ constant, so then we can consider

$$\phi \cong \int_0^\infty d\omega\left[a(\omega)\cdot e^{ik_\omega x^\mu}-a^+(\omega)\cdot e^{-ik_\omega x^\mu}\right] \tag{1.30}$$

In order to do this, we can write out the following for the solutions to Eqn (1) above.

$$C_1=\eta^2\cdot\left(4\cdot\sqrt{\pi}\cdot\frac{t}{2\omega^5}\cdot J_1(\omega\cdot r)+\frac{4}{\omega^5}\cdot\sin(\omega\cdot r)+(\omega\cdot r)\cdot\cos(\omega\cdot r)\right)$$
$$+\frac{15}{\omega^5}\cos(\omega\cdot r)-\frac{6}{\omega^5}Si(\omega\cdot r) \tag{1.31}$$

And

$$C_2=\frac{3}{2\cdot\omega^4}\cdot(1-\cos(\omega\cdot r))-4e^{-\omega\cdot r}+\frac{6}{\omega^4}\cdot Ci(\omega\cdot r) \tag{1.32}$$

This is where $Si(\omega\cdot r)$ and $Ci(\omega\cdot r)$ refer to integrals of the form $\int_{-\infty}^{x}\frac{\sin(x')}{x'}dx'$ and $\int_{-\infty}^{x}\frac{\cos(x')}{x'}dx'$. .

Next, we should consider whether or not the instanton so formed is stable under evolution of space-time leading up to inflation. To model this, we use results from Crowell (2005) on quantum fluctuations in space-time, which gives a model from a pseudo time component version of the Wheeler-De-Witt equation, with use of the Reinssner-Nordstrom metric to help us obtain a solution that passes through a thin shell separating two space-times. The radius of the shell $r_0(t)$ separating the two space-times is of length l_P in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom metric

$$dS^2=-F(r)\cdot dt^2+\frac{dr^2}{F(r)}+d\Omega^2 \tag{1.33}$$

This has:

$$F(r)=1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{\Lambda}{3}\cdot r^2\xrightarrow[T\to 10^{32}\,Kelvin\sim\infty]{}-\frac{\Lambda}{3}\cdot\left(r=l_P\right)^2 \tag{1.34}$$

This assumes that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park (2003), leading to

$$\frac{\partial F}{\partial r} \sim -2 \cdot \frac{\Lambda}{3} \cdot \left(r \approx l_P\right) \equiv \eta(T) \cdot \left(r \approx l_P\right) \tag{1.35}$$

as a wave functional solution to a Wheeler-De-Witt equation bridging two space-times, similar to two space-times with “instantaneous” transfer of thermal heat, as given by Crowell (2005)

$$\Psi(T) \propto -A \cdot \left\{\eta^2 \cdot C_1\right\} + A \cdot \eta \cdot \omega^2 \cdot C_2 \tag{1.36}$$

This has $C_1 = C_1(\omega, t, r)$ as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function. This also applies to the second cyclical wave function $C_2 = C_2(\omega, t, r)$, where $C_1 =$ Eqn (1.31) and $C_2 =$ Eqn. (1.32) Eqn. (1.36) is a solution to the pseudo time WDM equation.

The question which will be investigated is if eqn (1.36) is a way to present either a squeezed or un squeezed state. A way forward is to note that Prado Martin-Moruno, Pedro F. Gonzalez-Diaz in July (2009) wrote up about thermal phantom-like radiation process coming from the wormhole throat. Note that the Crowell construction of a worm hole bridge is in some ways similar to Carco Cavaglià’s (1994) treatment of use of conjugate momentum π^{ij} of h_{ij} generalized momentum variables, also known as conjugate momenta $\hat{\pi}^{ij} \equiv \frac{\hbar}{i} \cdot \frac{\partial}{\partial \cdot h_{ij}}$, leading to the sort of formalism as attributed to Luis J. Garay’s (1991) article , of

$$\Psi\left(h_{ij}\right) \approx \left\lfloor \exp \int d^3 x \cdot \pi^{ij} \cdot h_{ij} \right\rfloor_T \tag{1.37}$$

Now in the case of what can be done with the worm hole used by Crowell, with, if $\hbar \equiv 1$, $\hat{\pi}^{ij} \equiv -i \frac{\delta}{\delta \cdot g_{ij}}$, $\hat{\pi}^{\theta\theta} \equiv -\frac{i}{2r} \frac{\partial}{\partial \cdot r}$, $\hat{\pi}^{tt} \equiv -i \cdot \left(\frac{\partial F(r)}{\partial r}\right)^{-1} \cdot \frac{\partial}{\partial \cdot r}$, and a kinetic energy value as given of the form $\hat{\pi}^{\theta\theta} \hat{\pi}^{tt} + \hat{\pi}^{tt} \hat{\pi}^{\theta\theta}$. The supposition which we have the worm hole wave functional may be like, so, use the wave functional looking like $\Psi\left(g_{ij}\right) \approx \left\lfloor \exp \int d^3 x \cdot \left[\pi^{ij}\right] \cdot g_{ij} \right\rfloor_T$ where the g_{ij} for the Weiner-Nordstrom metric will be

o $dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \equiv g^{ij} dx^i dx^j \equiv \frac{\Lambda}{3} \cdot \left(r = l_P\right)^2 \cdot dt^2 - \frac{dr^2}{\frac{\Lambda}{3} \cdot \left(r = l_P\right)^2} + d\Omega^2$

2. Creating an analysis of how graviton mass, assuming branes, can influence expansion of the universe

Following development of *(1.13) as mentioned above, with inputs from Friedman eqns. To do this,, the following normalizations will be used, i.e. $\hbar = c = 1$, so then

$$q = A1 + A2 + A3 + A4 \tag{2.1}$$

Where

$$A1 = \frac{C}{a^3} \cdot \left[1 \Big/ \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \right] \tag{2.2}$$

$$A2 = -\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right] \tag{2.3}$$

$$A3 = -\frac{1}{2} \cdot \left[\frac{(d\rho/d\tau)}{3M_4^2} + \frac{(d\Lambda_4/d\tau)}{3} + \frac{1}{18} \cdot \frac{\rho \cdot (d\rho/d\tau)}{M_P^2} \right] \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{3/2} \tag{2.4}$$

$$A4 = \frac{\kappa}{a^3} \cdot \left[\frac{(da/d\tau)}{3} \right] \Bigg/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{3/2} \tag{2.5}$$

Furthermore, if we are using density according to whether or not 4 dimensional graviton mass is used, then

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2} \right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right) \tag{2.6}$$

So, then one can look at $d\rho/d\tau$ obtaining

$$d\rho/d\tau = -\left(\frac{\dot{a}}{a} \right) \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_9}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g c^6}{8\pi G \hbar^2} \right) \right] \tag{2.7}$$

Here, use, $\left(\frac{\dot{a}}{a} \right) = \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)}$, and assume eqn. (2.6) covers ρ, then

If $\hbar \equiv c \equiv 1$,

$$d\rho/d\tau = -\sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_9}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \tag{2.8}$$

Now, if , to first order, $d\Lambda_4/d\tau \sim 0$, and , also, we neglect Λ_4 as of being not a major contributor

$$d\rho/d\tau \cong -\sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \cdot \left[3 \cdot \rho_9 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \tag{2.9}$$

$$A3 \cong \frac{1}{2} \left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho \cdot}{M_P^6} \right] \Bigg/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{1/2} \right) \cdot$$

$$\left[3 \cdot \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \cdot \tag{2.10}$$

Also, then , set the curvature equal to zero. i.e. $\kappa = 0$, so then $A4 = 0$, and

$$A3 \cong \frac{1}{2} \left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho \cdot}{M_P^6} \right] \Bigg/ \left[\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{1/2} \right) \cdot$$

$$\left[3 \cdot \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \cdot \tag{2.11}$$

Then

$$A2 \cong -\left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \Bigg/ \left[\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right] \tag{2.12}$$

$$A1 \cong \frac{C}{a^3} \cdot \left[1 \Bigg/ \sqrt{\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \right] \tag{2.13}$$

Pick, here, $\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 - \left[\frac{m_g}{8\pi G} \right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right)$, after $\hbar = c = 1$, and also set

$$\Phi(\rho, a, C) = \frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \tag{2.14}$$

$$A3 \cong \frac{1}{2}\left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho \cdot}{M_P^6} \right] \Big/ [\Phi(\rho, a, C)]^{1/2} \right) \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \tag{2.15}$$

$$A2 \cong -\left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \Big/ [\Phi(\rho, a, C)] \tag{2.16}$$

$$A1 \cong \frac{C}{a^3} \cdot \left[1 \Big/ \sqrt{\Phi(\rho, a, C)} \right] \tag{2.17}$$

For what it is worth, the above can have the shift to red shift put in by the following substitution. I.e. use $1 + z = a_0 / a$ .. Assume also that C is the dark radiation term which in the brane version of the Friedman equation scales as a^{-4} and has no relationship to the speed of light. a_0 is the value of the scale factor in the present era, when red shift z =0, and $a \equiv a(\tau)$ in the past era, where τ is an interval of time after the onset of the big bang. $(a_0 / a)^3 = (1+z)^3$, and $a \equiv a_0 / (1+z)$, Then

$$\rho(z) \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g}{8\pi G} \right] \cdot \left(\frac{a_0^4}{14 \cdot (1+z)^4} + \frac{2a_0^2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \tag{2.18}$$

$$A1(z) \cong \frac{C \cdot (1+z)^3}{a_0^3} \cdot \left[1 \Big/ \sqrt{\Phi(\rho(z), a_0/(1+z), C)} \right] \tag{2.19}$$

$$A2(z) \cong -\left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho(z)^2}{M_P^6} \right) \Big/ [\Phi(\rho(z), a_0/(1+z), C)] \tag{2.20}$$

$$\begin{aligned} A3(z) \cong & \frac{1}{2}\left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho(z) \cdot}{M_P^6} \right] \Big/ [\Phi(\rho(z), a_0/(1+z), C)]^{1/2} \right) \cdot \\ & \left[3 \cdot \rho_0 \cdot (1+z)^3 + 4 \cdot \left(\frac{a_0^4/(1+z)^4}{14} + \frac{a_0^2/(1+z)^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \end{aligned} \tag{2.21}$$

$$\Phi(\rho(z), a_0/(1+z)), C) = \frac{C \cdot (1+z)^4}{a_0^4} + \left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho(z)^2}{M_P^6} \right) \tag{2.22}$$

So, for $4 < z \leq 0$, i.e. not for the range, say $z \sim 1100$ 380 thousand years after the big bang, it would be possible to model, here

$$q(z) = A1(z) + A2(z) + A3(z) \tag{2.23}$$

Easy to see though, that to first order, $q(z) = A1(z) + A2(z) + A3(z)$ would be enormous when $z \sim 1100$, and also that for Z =0, $q(0) = A1(0) + A2(0) + A3(0) > 0$ .Negative values for Eqn. (2.23) appear probable at about $z \sim 1.5$, when Eqn. (2.20) would dominate, leading to $q(z \sim 1.5))$ with a negative expression/ value . The positive value conditions rely upon , the C dark radiation term,

And here are the results! Assume X is red shift, Z. q(X) is De - Celebration

De celebration parameter due to small $m_{graviton} \propto 10^{-65}$ **grams, with one additional dimension added**

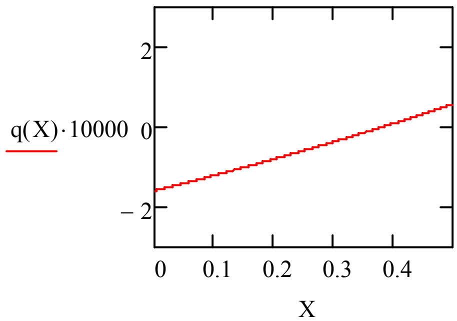

Figure 4 a: re duplication of basic results of Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009, using their parameter values, with an additional term of C for Dark flow added, corresponding to one KK additional dimensions

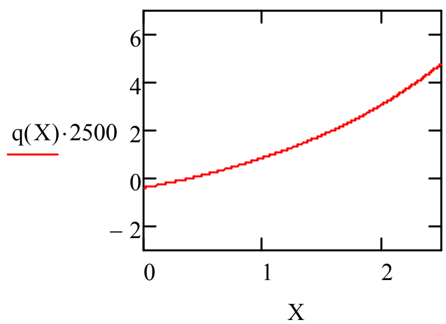

Figure 4 b: re duplication of basic results of Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009, using their parameter values, with an additional term of C for'Dark flow' added, corresponding to one KK additional dimensions. Results show asymptotic 'collapse' of de celebration parameter as one comes away from the red shift Z =1100 of the CMBR 'turn on ' regime for de coupling of photons from 'matter' , in end of 'dark ages'

Figures **4a,** and **4b** suggest that additional dimensions are permissible. They do not state that the initial states of GW/ initial vaccum states have to form explicitly due to either quatum or semi classical processes.

3. Unanswered questions, and what this suggests for future research endeavors

As far.back as 1982, Linde, when analyzing a potential of the form

$$V(\phi) = \frac{m^2 \phi^2}{2} + \lambda \phi^4 + V(0) \tag{3.1}$$

This is when the 'mass' has the form, (here M is the bare mass term of the field ϕ in de Sitter space, which does not take into account quantum fluctuations)

$$m^2(t) = M^2 + \frac{3\lambda H^3}{4\pi} \cdot (t - t_0) \tag{3.1a}$$

Specified non linearity of $< \phi^2 >$ at a time from the big bang, of the form

$$\Delta t_1 \approx \frac{3H}{2M} \tag{3.2}$$

The question raised repeatedly in whether or not i) if higher dimensions are necessary, and whether or not ii) mass gravitons are playing a role as far as the introduction of DE speed up of cosmological expansion may lead to an improvement over what was specified for density fluctuations and structure formation (the galaxy hierarchy problem) of density fluctuations given as

$$\frac{\delta\rho}{\rho} \sim 10^{-4} \Leftrightarrow \lambda \le 10^{-10} \tag{3.3}$$

Eqn (3.3) is for four space, a defining moment as to what sort of model would lead to density fluctuations. It totally fails as to give useful information as to the galaxy hierarchy problem as given in figure 1, above. Secondly, to what degree is the relative speed up of the q (z) function is impacted by various inter plays between , say a MODIFIED version of, say a KK DM model, using a MODIFIED mass hierarchy to get suitable DM masses of the order of 100 GeV or more. Giving a suitable definition as to q(z) as well as the inter play between DM values, 4 Dim Graviton mass issues, and /or what really contributes to the speed up of the universe will in the end dramatically improve the very crude estimate given by (3.3) above which says next to nothing about how the problems illustrated by the break down of the galaxy mass formation/ hierarchy can be fixed. Furthermore is considering the spectral index problem, where the spectral index is

$$n_S - 1 \cong -\frac{3}{8\pi}\cdot\left(\frac{V_\phi}{V}\right)^2 + \frac{1}{4\pi}\cdot\left(\frac{V_{\phi\phi}}{V}\right)^2 \tag{3.4}$$

Usual experimental values of density fluctuations experimentally are $\frac{\delta\rho}{\rho} \sim 10^{-5}$, instead of $\frac{\delta\rho}{\rho} \sim 10^{-4}$, and this is assuming that λ is extremely small. In addition, Linde (1982) had $\frac{d}{d\phi^2}V = m^2 \le \frac{H}{40} = \frac{1}{40}\cdot\frac{\dot{a}}{a}$ inside a false vacuum bubble. If something other than the Klein Gordon relationship $\frac{\dot{a}}{a} \Rightarrow \ddot{\phi} + 3H\dot{\phi} + m^2\varphi = 0$ occurs, then different models of how density fluctuation may have to be devised. A popular model of density fluctuations with regards to the horizon is

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \frac{k^{3/2}\left|\delta_k\right|}{\sqrt{2}\pi} \propto \frac{k^{(3/2)+3\alpha-3/2}}{\sqrt{2}\pi} \approx \left(1/\sqrt{2}\pi\right)\cdot k^{3\alpha} \tag{3.5}$$

, where $-.1 < \alpha < 0.2$, and $\alpha \equiv 0 \Leftrightarrow n_s \equiv 1$ and to first order, $k \cong Ha$. The values, typically of $n_s \ne 1$ If working with $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\rho^2}{36M_{Planck}^2}\right) + \frac{C}{a^4}\right]$, and with a density value $\rho \equiv \rho_0\cdot\left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2}\right]\cdot\left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$ where $m_g \approx 10^{-65}$ grams, and $\alpha < 0.2$ is usually picked to avoid over production of black holes, a very complex picture emerges. Furthermore, if working with $\alpha < 0.2$ and $\alpha \ne 0$

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \left(1/\sqrt{2}\pi\right)\cdot k^{3\alpha} \sim \frac{H^2}{\dot{\phi}} \propto 10^{-4} - 10^{-5} \tag{3.6}$$

The above equation gives inter relationships between the time evolution of a pop up inflaton field ϕ, and a Hubble expansion parameter H , and a wave length parameter $\lambda = (2\pi/k)\cdot a(t)$ for a mode given as δ_k .

What should be considered is the inter relation ship of the constituent components of (3.6) and $\lambda \leq H^{-1}$. What the author thinks is of particular import is to look at whether or not the more general expression, as given by Steinhardt

$$\left(\frac{\delta\rho}{\rho}\right) \cong Ak^{\left(\frac{n_S-1}{2}\right)} \propto 10^{-4}-10^{-5} \tag{3.7}$$

To first order, variations of $\alpha < 0.2$ and $\alpha \neq 0$, should be compared with admissible values of $([n_S - 1]/2)$ which would closely correspond to $\alpha \neq 0$ and $0 < \alpha << 0.2$. I.e. the precise values of this may help us out in determining how to unravel what is going on in the galaxy formation picture as given in Figure 1 on page 6, break down. I.e. how can we have earlier than expected galaxy formation ?.

Finally , we should note the two very startling results given in figures 4a and 4b of 28 of this document. We have managed to recover the basic results of Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009 as to small graviton mass leading to de celebration q(Z) becoming negative, i.e. indicating re acceleration of the universe, as of Z~ .42 to .5 , i.e. as of almost a billion years ago. I.e. one additional dimension does not necessarily destroy cosmology as we know it. There is, though, questions as to if additional dimensions are a classical or Quantum phenomenon.

The model used to obtain this , i.e. the two figure 4a, and 4b results , is assuming one additional dimension. Note, it does not mandate the sort of string theory results as indicated , in figure 2b, of page 17, but may permit some over lap with the brane structure of figure 2a, page 17. As noted beforehand, Bojowald does not rule out compression, and the over lap of results, possible between the counting algorithm, of Ng, as modified by Beckwith, 2008,2009, and Glinka's (2007) WdM graviton gas idea may be a way of arguing for initial coherency , i.e. non squeezed nature of vacuum states. I.e. before a bounce, or right during the beginning of inflation, there may be initial coherency, and the path from coherency may engender a measurable relic shock wave, of the sort written up in "Gravitational solitions" as written by Vladimir Belunski, and Enric Verdaguer (2001). Such a transition would have measurable consequences. I.e. note that Kourosh Nozari and Tahereh Azizi (2005) wrote up how Gaussian coherent states could form with respect to a generalized uncertainty principle. What could be investigated would be if inputs into the generalized uncertainty principle , due to a transition from coherent states, to squeezing at the onset of inflation may lead to shock fronts in GW generation.

We also argue that proper investigation of this phenomenon will also aid in settling once and for all if or not relic GW are high or low frequency. If it is high frequency, the gravitional wave detector modeled up by Li et al, 2009 in PRD, will be able to get to the bottom of this issue.

If weaker GW are de rigor and/or the lower frequency range is de rigor, it also opens up for investigation a model which Penrose(2007) talked about in the inaugural meeting of the Penn state gravitational research center , which is of an infinitely expanding universe, with infinite numbers of black holes gathering up the reminents of expansion and re conflating them back to the origins of a new bang. A qualitative test of such a proposal , from a model perspective will be offered as an alternative to string theory by the author, in a later document Beckwith already published a short document on this topic in 2008 and will revisit this issue in the new future. Furthermore the classical versus quantum nature of gravitons/ GW at their origin re opens up the appropriateness of the formalism given by Kuchiev, M. Yu as to derivation of graviton structure as a consequence of SO(4) gauge theory,which Beckwith has referenced in another (2009) document as to if or not gravitons could be a derivative of instanton structures which may have partial KdV characteristics

BIBLIOGRAPHY

B. Allen, E. Flanagan, and M.A. Papa, Is the squeezing of relic gravitational waves produced by inflation detectable?, Phys.Rev.D61:024024,2000., http://arxiv.org/PS_cache/gr-qc/pdf/9906/9906054v2.pdf ...

Marcio E. S. Alves, Oswaldo D. Miranda and Jose C. N. de Araujo," Can Massive Gravitons be an Alternative to Dark Energy?", arXiv:0907.5190 (July 2009)

Andrić, I, Jonke , L. and Jurman , D."Solitons and giants in matrix models", JHEP12(2006)006

Abhay Ashtekar (2006), solvaynet.pdf

Daniel Babich, Paolo Creminelli, Matias Zaldarriaga;"The shape of non – Gaussianities", CAP 0408:009,2004; http://arxiv.org/abs/astro-ph/0405356
Baker, R.M.L, Stephenson, G. V. , and Li Fangyu (2008), "Proposed Ultra-High Sensitivity HFGW Detector,"in the proceedings of Space Technology and Applications International Forum (STAIF-2008), edited by M.S. El-Genk, American Institute of Physics Conference Proceedings, Melville , NY 969, pp. 1045-54 http://www.gravwave.com/docs/ProposedUltraHighSensitivityHFGWDetector2005-15-08.pdf
Carlos Barcelo, Luis J. Garay,"Quantum fields in Anti-De Sitter Wordholes", http://arxiv.org/PS_cache/gr-qc/pdf/9703/9703028v2.pdf
James M. Bardeen * Gauge-invariant cosmological perturbations', Phys. Rev. D 22, 1882 - 1905 (1980);
Nicola Bartolo;
Barvinsky, A. "Quantum geometrodynamics: The Wheeler-DeWitt equations for the wave function of the universe"Physics letters B, Volume 175, issue 4, August 14, 1986, pp 401-
Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi , Keitaro ," Gravitational Wave Spectrum Induced by Primordial Scalar Perturbations", arXiv:hep-th/0703290
Beckwith,A.W. "Entropy growth in the early universe and confirmation of initial big bang conditions (Wheeler De Witt eqn. results vs. String theory ?)", http://vixra.org/abs/0908.0109
Beckwith, A.W. "Instanton formation of vacuum energy via the Reissner-Nordstrom geometry of a wormhole bridge between a prior to our present universe.", arXiv:0710.3788, October 2007
Beckwith, A.W., "Implications for the Cosmological Landscape: Can Thermal Inputs from a Prior Universe Account for Relic Graviton Production?" AIP Conference Proceedings 969, (2008), pp. 1091-1102
Beckwith, A. W. "Relic High Frequency Gravitational waves from the Big Bang, and How to Detect Them", http://arxiv.org/abs/0809.1454 (2009), AIP Conf.Proc.1103:571-581,2009 (SPESIF)
Berkenstein, (2004), http://uw.physics.wisc.edu/~strings/group/slides.04.fall/berenstein.pdf
Martin Bojowald,(2008), "Quantum nature of cosmological bounces "General Relativity and Gravitation, pp 2659-2683, Volume 40, Number 12, December, 2008; http://arxiv.org/abs/0801.4001
Vladimir Belunski, and Enric Verdaguer ," Gravitational Solitons", Cambridge Monographs on Mathematical physics, Cambridge, UK, (2001)
Brax, Ph., Davis, A., Davis, S., Jeannerot, R., Postman, M., " D-term Uplifted Racetrack Inflation", Journal of Cosmology and Astroparticle Physics, JCAP01(2008) 08
Brett Bolen, Luca Bombelli, Alejandro Corichi "Semiclassical States in Quantum Cosmology: Bianchi I Coherent States", Class.Quant.Grav. 21 (2004) 4087-4106; http://arxiv.org/abs/gr-qc/0404004
R. Brustein, M. Gasperini, M. Giovannini, and G. Veneziano,"Relic Gravitational waves from String Cosmology", http://cdsweb.cern.ch/record/284281/files/9507017.pdf; Physics Letters B 361(1995) 45-51
Buonanno, A.; Ungarelli, C."Gravitons and String Pre Big Bang cosmology", String Theory and Fundamental Interactions, Lecture Notes in Physics, Volume 737. ISBN 978-3-540-74232-6. Springer-Verlag Berlin Heidelberg, 2008, p. 845-851
Camp, J. , and Cornish, N., "Gravitational wave Astronomy",pp 525-577, as part of " Annual Review of Nuclear and Particle Science", Volume 54, with Kayser, B., Holstein, B., and Jawahery, A. as editors. Menlo Park, California, 2004
Capozziello, S., Feoli, A., Lambiase, G., Papini, G., ' Thin shell quantization in Weyl Space – time', Physics Letters A, 273 (2000), pp 25-30
Marco Cavaglià, "Quantum Electromagnetic Wormholes and Geometrical Description of the Electric Charge", Phys.Rev. D50 (1994) 5087-5092; http://arxiv.org/abs/gr-qc/9407030
Ceresole, A., D' Auria, R., Ferrara, S., and van Poeyen, A., " Duality transformations in super symmetric Yang-Mills theories, coupled to supergravity," Nucl. Phys. B444 (1995), pp 92-124, hep-th/ 9502072
P. Chingabam, C. Park;"Statistical nature of Non Gaussianity from cubic order Primoridal perturbations : CMB map simulations and genus statistics" http://arxiv.org/PS_cache/arxiv/pdf/0908/0908.1696v1.pdf
Dah-Wei Chiou, Li-Fang Li," Loop quantum cosmology with higher order holonomy corrections", Phys. Rev. D 80, 043512 (2009); http://arxiv.org/abs/0907.0640
Christian Corda,"Analysis of the transverse effects of Einsteins Gravitational waves", International Journal of Modern Physics A, Volume 22, Issue 26, pp. 4859-4881 (2007); http://adsabs.harvard.edu/abs/2007arXiv0707.1943C
Christian Corda," Primordial production of massive relic gravitational waves from a weak modification of General Relativity", Astropart.Phys.30:209-215,2008, http://arxiv.org/abs/0812.0483
Crowell, L., "Quantum Fluctuations of Space Time ", World Scientific Series in Contemporary Chemical Physics, Vol 25, World Scientific, PTE, LTD, 2005, Singapore
Dadhich, N., "Derivation of the Raychaudhuri Equation", http://arxiv.org/abs/gr-qc/0511123,
De Laurentis, Mariafelicia, and Capozziello, Salvatore,," Stochastic Background of Relic Scalar Gravitational Waves tuned by Extended Gravity", http://arxiv.org/abs/0906.3689
Rainer Dick," Heterotic string dark matter from the graviton multiplet", Physics Letters B, Volume 510, Issues 1-4, 21 June 2001, Pages 10-16
Dowker, H.F. "Causal sets and the deep structure of spacetime", arXIV gr-qc/0508109v1 26 Aug 2005; in 100 Years of Relativity Space-Time Structure: Einstein and Beyond, edited by Ashtekar, A., World Press Scientific, Singapore, 2005

Durrer, R., " Cosmological Pertubation theory", From the book "Physics of the Early Universe", in Lecture Notes in Physics, 653", with Papantonopoulos, E., as editor and printed by Springer- Verlag 2004
Ruth Durrer, Massimiliano Rinaldi , "Graviton production in non-inflationary cosmology", Phys.Rev.D79:063507,2009, http://arxiv.org/abs/0901.0650
Enqvist, K., Mazumdar, A., Perez--Lorenzana, A., " Dumping inflaton energy out of this world",Phys Rev D, Volume 70, 103508
Feng, Jonathan L. and Kumar, Jason "Dark-Matter Particles without Weak-Scale Masses or Weak Interactions." Physical Review Letters (2008). Available online: http://link.aps.org/doi/10.1103/PhysRevLett.101.231301.
Feinstein,A , Vazquez-Mozo, M.A, "M-theory resolution of four-dimensional cosmological singularities via U-duality" , http://arxiv.org/abs/hep-th/9906006 , Nucl.Phys. B568 (2000) 405-420
L.H. Ford," Gravitons and Lightcone Fluctuations", 10.1103/PhysRevD.51.1692, http://arxiv.org/abs/gr-qc/9410047
Hunt , Peter, and Sarkar, Subir "Constraints on large scale voids from WMAP-5 and SDSS" http://arxiv.org/PS_cache/arxiv/pdf/0807/0807.4508v2.pdf;
http://meghnad.iucaa.ernet.in/~ctscmb/talks/sarkar_lecture.pdf
Luis J. Garay ," Quantum state of wormholes and path integral" Phys. Rev. D 44, 1059 – 1066;
Giovannini, Massimo, " A Primer on the Physics of the Cosmic Microwave Background", World Press Scientific, 2008, Singapore, Republic of Singapore, (2008)
Glinka, L. "Quantum Information from Graviton-Matter Gas ", Sigma 3, (2007), 087, 13 pages
Roy Glauber, "Coherent and Incoherent States of the Radiation Field", Phys. Rev. 131, 2766 - 2788 (1963)
Leonid P Grishchuk"The problem of relic gravitational radiation", Sov. Phys. Usp. 23 705-706

L P Grishchuk and Yu Sidorov," On the quantum state of relic gravitons", 1989 Class. Quantum Grav. 6 L161-1
L.P. Grishchuk," Quantum Effects In Cosmology" Class.Quant.Grav. 10 (1993) 2449-2478, http://arxiv.org/abs/gr-qc/9302036
Lenoid P, Grushchuk,"The Detectability of Relic (Squeezed) Gravitational Waves by Laser Interferometers", http://arxiv.org/abs/gr-qc/9810055 , 1998
Grishchuk,L. "Discovering Relic Gravitational Waves in Cosmic Microwave Background Radiation", http://arxiv.org/abs/0707.3319 (2008)
H artle, J., and Hawking, S.W., "Wave function of the universe", Phys. Rev. D28, 2960 (1983)
Bert Janssen, Yolanda Lozano, "A Microscopical Description of Giant Gravitons" Nucl.Phys. B658 (2003) 281-299, , http://arxiv.org/abs/hep-th/0207199
Kahniashvili ,.Tina"Relic Gravitational Waves as a Test of the Early Universe",arXiv:0705.1733 [astro-ph], (2007)
Claus Kiefer," Quantum gravity", (International Series of Monographs on Physics) (Hardcover),Oxford Science Publications, Oxford University Press, 2007
Kolb, E., and Turner, S. "The Early Universe", Westview Press, Chicago, USA, 1994
Lawrence Krauss (1996); "Old Galaxies at high Red shifts, and the Cosmological constant", THE ASTROPHYSICAL JOURNAL, 480:466–469, 1997 May 10 http://www.iop.org/EJ/article/0004-637X/480/2/466/35523.text.html
Kuchiev, M. Yu, "Can gravity appear due to polarization of instantons in SO(4)gauge theory?", Class. Quantum Grav. 15 1895-1913 doi: 10.1088/0264-9381/15/7/008
Dawood Kothawala, T. Padmanabhan , Sudipta Sarkar, "Is gravitational entropy quantized ?" Phys. Rev., D78 , 104018 (2008) [arXiv:0807.1481]
Kuchiev, M. Yu, "Can gravity appear due to polarization of instantons in SO(4)gauge theory?", Class. Quantum Grav. 15 1895-1913 doi: 10.1088/0264-9381/15/7/008
Kumar, Jason, "A Brief Comment on Instanton-like Singularities and Cosmological Horizons", JHEP 0301 (2003) 010, arXiv arXiv:hep-th/0204119v2, 27 Apr 2002
Kurita, Y, and Ishihara, H.," Thermodynamics of squashed Kaluza–Klein black holes and black strings: a comparison of reference backgrounds", 2008 Class. Quantum Grav. 25 085006 (19pp)
B Lamine, A Lambrecht, M T Jaekel and S Reynaud," Relic gravitational wave background and the isotropy of space", *Class. Quantum Grav.* **21** S1209-S1214
Jean-Luc Lehners, Paul J. Steinhardt, 'Non-Gaussian Density Fluctuations from Entropically Generated Curvature Perturbations in Ekpyrotic Models', Phys.Rev.D77:063533,2008; http://arxiv.org/abs/0712.3779
Li, Fangyu, Baker, Robert. M.L., and Chen , Zhenya (2006), "Perturbative photon flux generated by high frequency relic gravitational waves and utilization of them for their detection," http://arxiv.org/abs/gr-qc/0604109
Li, Fangyu, Baker, Robert M L., Fang, Zhenyun, Stephenson, Gary V., and Chen, Zhenya, "Perturbative Photon Fluxes Generated by High-Frequency Gravitational Waves and Their Physical Effects," [Li-Baker Chinese HFGW Detector], European Physical Journal C., 56, 407- 423 (2008) ,
http://www.gravwave.com/docs/Li-Baker%206-22-08.pdf
A.D. Linde, " The New inflationary Universe Scenario", pp 205-249, from the volume, "The Very Early Universe", editied by G.W.Gibbons, S.W. Hawking, S.T.C. Siklos, Cambridge University press, Cambridge, UK, 1982.
Eric Linder , "Exploring the Expansion History of the Universe", Phys. Rev. Lett. 90, 091301 (2003)
Lifschytz, G., " Black Hole thermalization rate from brane antibrane model ", arXIV hep-th/ /0406203 v1 23 june, 2004

Linde in article in Physics 738, Inflationary cosmology , editors M. Lemoine, J. Martin, P. Peter, as Editors, Springer Verlag, 2008 Berlin, Federal Republic of Germany

Lloyd , Seth "Computational capacity of the universe", Phys. Rev. Lett. 88, 237901 (2002)

David H. Lyth;"Generating curvature perturbations at the end of inflation", JCAP 0511 (2005) 006; http://arxiv.org/abs/astro-ph/0510443

Roy Maartens , "Brane-World Gravity (2004)", http://www.livingreviews.org/lrr-2004-7

J. Martin, "Inflationary Perturbations:The cosmological Schwinger effect",pages 193-241, Lecture notes in Physics, 738, editors M. Lemonine, J. Martin, P. Peters, Springer - Verlag (2008), Berlin

Mathur, S., and Chowdhury, B., " Fractional Brane States in the Early Universe", Classical and Quantum Gravity, 24 (2007), pp. 2689-2720

Jakub Mielczarek," Tensor power spectrum with holonomy corrections in loop quantum cosmology", Phys. Rev. D 79, 123520 (2009); http://arxiv.org/abs/0902.2490

Mina, A., Takuya, O., and Hirosi, O, "Quantum entanglement of baby universes", Nuclear Physics B, Volume 778, Issue 1-2, p. 36-68. (2007)

Ali Mostafazadeh," Wave function of the universe, and its meaning", Czech.J.Phys. 54 (2004) 93-99, http://arxiv.org/abs/gr-qc/0308029

Lubos Montl, (2007) , http://motls.blogspot.com/2007/08/thiemann-dittrich-discreteness-of-lqg.html

Prado Martin-Moruno, Pedro F. Gonzalez-Diaz," Thermal radiation from Lorentzian traversable wormholes", Phys. Rev. D 80, 024007 (2009); http://arxiv.org/abs/0907.4055

Ng, Y.Jack,"Article: Spacetime Foam: From Entropy and Holography to Infinite Statistics and Nonlocality" Entropy 2008, 10(4), 441-461; DOI: 10.3390/e10040441

Ng, Y. Jack ," Quantum Foam and Dark Energy", International work shop on the Dark Side of the Universe, http://ctp.bue.edu.eg/workshops/Talks/Monday/QuntumFoamAndDarkEnergy.pdf

Kourosh Nozari and Tahereh Azizi,"Quantum Mechanical Coherent States of the Harmonic Oscillator and the Generalized Uncertainty principle", International Journal of Quantum Computing, Vol 3,No 4, (2005),pp623-632

Penrose, Roger," Conformal Cyclic Cosmology, Dark Matter, and Black Hole Evaporation", August 11, 2007 ,IGC Inaugural Conference. State College, PA. Penn State University ... August 7 – August 11, 2007.

Peskins, M. ,Schroeder, D. " An introduction to Quantum Field Theory", Westview Press, (1995)

Polarski D. and Ranquet, A, "On the equation of state of dark energy". Physics Letters B, Volume 627, Issues 1-4, 27 October 2005, Pages 1-8

Joseph Gerard Polchinski," String Theory: An introduction to the bosonic string", Cambridge University Press, 1999.

Rothman, Tony and Boughnm S., "Can Gravitons be Detected?" Foundations of Physics, 36(12), December (2006), pp. 1801-1825.

Carlo Rovelli," Graviton Propagator from Background-Independent Quantum Gravity", Phys. Rev. Lett. 97, 151301 (2006); http://arxiv.org/abs/gr-qc/0508124

Leszek M. Sokolowski, Andrzej Staruszkiewicz ,"On the issue of Gravitons", Class.Quant.Grav. 23 (2006) 5907-5918, http://arxiv.org/abs/gr-qc/0606111.

Steinhardt, Paul J., Turok, Neil,"Endless Universe, Beyond the Big Bang" Doubleday Broadway Publishing Group, a division of Random House, Inc., New York (2007); http://map.gsfc.nasa.gov/media/060915/index.html

Gary Stephenson, (Private discussions, August 20, 2009), w.r.t. Li-Baker detector sensitivity.

Ming-Lei Tong and Yang Zhang (2007), "Detecting very-high-frequency relic gravitational waves by electromagnetic wave polarizations in a waveguide", Chin. J. Astron. Astrophys. 8 (2008) 314-328, http://arxiv.org/abs/0711.4909

K.K.Venkataratnam, P.K.Suresh, "Density Fluctuations in the Oscillatory Phase of Nonclassical Inflaton in FRW Universe", Int.J.Mod.Phys.D17:1991-2005,2008, http://arxiv.org/abs/0712.3129

Licia Verde,"Non Gaussianity versus non linearity of cosmological perturbations"; proc. of the 15th Florida workshop in Nonlinear Astronomy and Physics. Annals of The New York Academy of Sciences as well as http://arxiv.org/abs/astro-ph/0004341

Visser, Matt, " Mass of the graviton", Gen.Rel.Grav. 30 (1998) 1717-1728, http://arxiv.org/abs/gr-qc/9705051 (1998)

Clifford Will," Bounding the mass of the graviton using gravitional-wave observations of inspiralling compact binaries", http://arxiv.org/abs/gr-qc/9709011

M.White and W. Hu;"The Sach-Wolfe Effect", http://arxiv.org/PS_cache/astro-ph/pdf/9609/9609105v1.pdf