## **Particle mass levels**

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Particles occupy mass levels and sublevels within two sequences that descend in geometric progression from the Planck Mass. The common ratios of the sequences are  $r_1=1/\pi$  and  $r_2=2/\pi$ . Sublevels are arranged in subsequences of common ratio  $r^{1/p}$ , where p is a power of 2. Concise graphical evidence is provided for the sequences. Atomic nuclei with  $2 \le A \le 10$ ,  $\alpha$ -particle nuclei with  $A \le 20$ , all nuclei of double magic number, many ground state hadrons, the weak gauge bosons and the charged leptons are shown to occupy mass levels and sublevels characterised by small p. The mass levels and sublevels constitute a framework upon which broken symmetries are displayed.

The scale of a unified theory that incorporates gravity is the Planck scale. Particles are shown to occupy mass levels that descend in geometric progression from Planck scale.

Sequence 1 descends from the Planck Mass  $M_{Planck}$  (1.220892 x 10<sup>19</sup> GeV [1]) with common ratio 1/ $\pi$ . Sequence 2 descends from the Planck Mass with common ratio 2/ $\pi$ . Mass levels and sublevels within Sequence 1 and Sequence 2 are characterised by the numbers  $n_1$  and  $n_2$ , respectively. For example, a level within Sequence 1 for which  $n_1$ =40 is of mass  $(1/\pi)^{40} M_{Planck}$ , and a level within Sequence 2 for which  $n_2$ =100 is of mass  $(2/\pi)^{100} M_{Planck}$ . Fundamental levels are of integer  $n_1$  and  $n_2$ . Sublevels are of fractional  $n_1$  and  $n_2$ . Half-integer, quarter-integer, eighth-integer, etc, values of  $n_1$  and  $n_2$  refer to first order, second order, third order, etc, sublevels. For example, a sublevel for which  $n_1 = 35.25$  is of second order, and a sublevel for which  $n_2 = 91.875$  is of third order. A particle of mass  $m_{particle}$  occupies levels or sublevels in Sequence 1 and Sequence 2 for which

$$n_1 = \ln\left(M_{Planck} / m_{particle}\right) / \ln(\pi) \tag{1}$$

and

$$n_2 = \ln(M_{Planck} / m_{particle}) / \ln(\pi / 2)$$
<sup>(2)</sup>

Nuclear masses have been calculated from tables of nuclide mass [2] by subtracting the mass of Z electrons from each value. Values of  $n_1$  range from 33.792 for <sup>208</sup>Pb to 37.843 for <sup>2</sup>H; values of  $n_2$  range from 85.660 for <sup>208</sup>Pb to 95.929 for <sup>2</sup>H. From each number  $n_1$  and  $n_2$ , the integer part is subtracted resulting in the fractions  $n_1'$  and  $n_2'$  which may be presented graphically more conveniently than  $n_1$  and  $n_2$ . In Figure 1, atomic nuclei, represented by points  $(n_1', n_2')$ , are shown to lie upon or close to fifth and lower order mass sublevels within Sequences 1 and 2. Included here are the nuclei <sup>2</sup>H, <sup>3</sup>H, <sup>4</sup>He, <sup>5</sup>He, <sup>6</sup>Li, <sup>7</sup>Li, <sup>8</sup>Be, <sup>9</sup>Be and <sup>10</sup>B, the  $\alpha$ -particle nuclei <sup>12</sup>C, <sup>16</sup>O and <sup>20</sup>Ne, and the double magic number nuclei <sup>40</sup>Ca, <sup>48</sup>Ca, <sup>56</sup>Ni, <sup>132</sup>Sn and <sup>208</sup>Pb.

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The evaluations of the Particle Data Group [3] have been used to calculate  $n_1$  and  $n_2$  for the hadrons, weak gauge bosons and charged leptons. The proton-neutron isospin doublet is arranged symmetrically about a sublevel of seventh order in Sequence 2, as shown in Figure 2. The geometric mean of the two masses is coincident with the sublevel that lies between the points accorded to the two baryons. Also shown in Figure 2 is the  $\eta'$  meson, upon a sublevel of seventh order within Sequence 1. The arrangement upon the sublevels of the proton, neutron and  $\eta'$  meson illustrates a consistent feature of mass level occupation: singlet states lie upon levels, while doublets, and multiplets in general, are arranged about levels. Normally the term multiplet will refer to an isospin multiplet but further types of multiplet are suggested by the data. In Figure 3, the  $J^P = \frac{1}{2} t^+$  isospin singlet states  $\Lambda$  (uds),  $\Lambda_c^+$  (udc) and  $\Lambda_b^0$  (udb) are shown to occupy the fundamental and first order sublevels of Sequence 2. A shares Level 97 with  $\Sigma^0$ , which is also of uds composition: the two baryons are arranged symmetrically about the level, in a doublet configuration.

The pseudoscalar meson multiplets<sup>2</sup>  $\pi^{\pm}$ - $\pi^{0}$ ,  $K^{\pm}$ - $K^{0}$ ,  $D^{\pm}$ - $D^{0}$  and  $B^{\pm}$ - $B^{0}$  are each arranged symmetrically about low order sublevels. In Figure 4, the  $\pi^{\pm}$ - $\pi^{0}$  and  $K^{\pm}$ - $K^{0}$  multiplets are represented by the geometric mean of the two masses, as are the multiplets  $\Lambda$ - $\Sigma^{0}$ ,  $W^{\pm}$ - $Z^{0}$  and  $\rho$ - $\omega$ , which are also arranged symmetrically about levels and sublevels. The singlet states  $\phi$ ,  $B_{c}^{+}$ ,  $\Lambda_{b}^{0}$  and  $\Omega_{c}^{0}$  lie upon low order sublevels. The Higgs field vacuum expectation value (246.22 GeV) coincides with a third order sublevel within Sequence 2. Couplings with the Higgs field generate mass in the Standard Model: the values of the coupling constants seem to be incorporated within a unified model. In Figure 5, the  $D^{\pm}$ - $D^{0}$  and  $B^{\pm}$ - $B^{0}$  multiplets, each represented by the geometric mean of the two masses, are shown to be arranged symmetrically about sublevels. The singlet states  $\eta$ ,  $\eta_{c}$ , J/ $\Psi$  and  $\Upsilon$  all lie upon sublevels.

The charged leptons and the pseudoscalar mesons  $\pi^{\pm}$ ,  $K^{\pm}$  and  $D^{\pm}$  all occupy fundamental levels within one or other of the two mass sequences, as shown in Figure 6. The electron and the  $K^{\pm}$  mesons lie upon fundamental levels within Sequence 1. The other two charged leptons, the muon and the tau, are arranged as multiplets with the  $\pi^{\pm}$  mesons and the  $D^{\pm}$  mesons, respectively, about fundamental levels within Sequence 2. Curiously, the characteristic number of each occupied level within Figure 6 is a multiple of 3.

Exceptionally, the quark doublets up-down, strange-charm and bottom-top are arranged about fundamental levels and low order sublevels within both mass sequences. Fundamental levels within Sequence 1 and Sequence 2 coincide to an extraordinary degree at the scale (~5 MeV) of the up-down quark doublet. Level 43 in Sequence 1 is of value 5.120 MeV, while Level 109 in Sequence 2 is of value 5.124 MeV. Using the evaluations of the Particle Data Group [3], the geometric mean of the strange and charm quark masses is 365 MeV, and the geometric mean of the bottom and top quark masses is 26.8 GeV. These two values coincide closely with the scales at which the coincident levels ( $n_1$ ,  $n_2$ ) = (39.25, 99.5) and (35.5, 90) occur: 374 MeV and 27.3 GeV, respectively. The quark doublet scales lie within a geometric sequence

<sup>&</sup>lt;sup>2</sup> For convenience, the multiplet  $\pi^{\pm} \cdot \pi^{0}$  refers to the isospin triplet; the multiplet K<sup>±</sup>-K<sup>0</sup> refers to the pair of isospin doublets; etc.

of scales of common ratio  $\pi^{3.75} \approx (\pi/2)^{9.5} \approx 73$ , as shown in Figure 7. The 'sequence of scales' extends from 5 MeV to both larger and smaller scales, and includes the MSSM GUT scale (2x10<sup>16</sup> GeV) but not the Planck scale. Nevertheless, the existence of the sequence of scales depends upon the coincidence at Planck scale of fundamental levels within Sequence 1 and Sequence 2.

## References

- [1] CODATA 2006
- [2] Kaye & Laby Tables of Physical & Chemical Constants, NPL (2009) (URL: http://www.kayelaby.npl.co.uk
- [3] C. Amsler et al. (Particle Data Group), Physics Letters **B667**, 1 (2008) and 2009 partial update for the 2010 edition



Figure 1: Occupation by atomic nuclei of mass sublevels in Sequence 1 and Sequence 2



Figure 2: Occupation by the proton, neutron and  $\eta'$  meson of mass sublevels in Sequence 1 and Sequence 2



Figure 3: Occupation by the ground state uds, udc and udb baryons of mass levels and sublevels in Sequence 2



Figure 4: Hadron and weak gauge boson occupation of mass levels and low order sublevels in Sequence 1 and Sequence 2



Figure 5: Hadron occupation of third, fourth and fifth order mass sublevels in Sequence 1 and Sequence 2  $\,$ 



Figure 6: Occupation by the charged leptons and the  $\pi^{\pm}$ ,  $K^{\pm}$  and  $D^{\pm}$  mesons of fundamental mass levels in Sequence 1 and Sequence 2



Figure 7: Occupation by the quark doublets of coincident levels and sublevels in Sequence 1 and Sequence 2. The up-down scale shown is 5 MeV. The strange-charm and bottom-top scales shown (365 MeV and 26.8 GeV) have been calculated from Particle Data Group evaluations.