Renormalizable Yang-Mills Tetrad Gravity

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Abstract
Although Yang-Mills theory was developed for non-universal compact internal symmetry groups of subsets of matter fields, it should also work for the universal non-compact symmetry groups of *all* matter fields implied by the classical Einstein local equivalence principle. We introduce a new class of direct gravity couplings of rotating matter to the electromagnetic field that can be tested in principle especially in rotating superconductors.

On the Physical Meaning of Einstein's 1915 Theory of Gravity
First we review the Yang-Mills formalism[i] based on Lie algebra. Given a locally gauged Lie *frame transformation group* with elements $\Omega(\{x\})$ at Einstein's "local coincidence" (aka "gauge orbit") of local reference frame detectors.[ii] The $T^a, a = 1,2,...N_V$ matrices represent the "charges" of the Lie algebra. The infinitesimal frame transformation is

$$\Omega(x) = \mathrm{I} + i\sum_{a}^{N_V} \Lambda^a(x) T_a \tag{1.1}$$

The Lie algebra structure constants f_c^{ab} are defined by charge commutators

$$[T_a, T_b] = f_{ab}^c T_c \tag{1.2}$$

The matter fields $\Psi(x)$ for an *internal symmetry* group infinitesimally transform as

$$\begin{aligned}\Psi(x) &\to \Psi(x)' \equiv \Omega(x)\Psi(x) \\ &\approx \Psi(x) + i\Lambda^a T_a \Psi(x) + ...\end{aligned} \tag{1.3}$$

This is for a fixed spacetime event x where the Lie group is internal. For the space-time Lie group in Einstein's *globally flat* 1905 Special Relativity without gravity

$$\begin{aligned}\Psi(x) &\to \Psi(gx)' \equiv \Omega_g(\Delta x)\Psi(x) \\ &\approx \Psi(gx) + i\Lambda^a(gx) T_a \Psi(gx) + ... \\ &= \Psi(x+\Delta x) + i\Lambda^a(x+\Delta x) T_a \Psi(x+\Delta x) + ...\end{aligned} \tag{1.4}$$

where g is an element of the *globally rigid* translation subgroup T4 of the Poincare group P10, and $\Omega_g(\Delta x)$ is the matrix representation of that group element in the fiber space of local matter field second-quantized creation and destruction operators. This means that all point events x of non-dynamical 4D Minkowski space-time are displaced by the same amount Δx. From Noether's theorem[iii] this leads to conservation of the total 4-momentum of the matter field in the stack of spacelike 3D hypersurfaces for a given arbitrary global slicing of 4D space-time. The transition from 1905 SR to 1915 GR is a profound paradigm shift in Einstein's thinking in which we replace the above equation with

$$\begin{aligned}\Psi(\{x\}) &\to \Psi(\{x\})' \equiv \Omega(\{x\})\Psi(\{x\}) \\ &\approx \Psi(\{x\}) + i\Lambda^a(\{x\})T_a\Psi(\{x\}) + \ldots\end{aligned} \tag{1.5}$$

formally this is exactly the same as the case for internal symmetry groups with the change from the bare manifold point x to the gauge orbit equivalence class $\{x\}$ of all bare manifold points connected to each other by *active diffeomorphisms* (aka "General Coordinate Transformations" GCT). For example, for first rank tensors

$$\begin{aligned}A^{\mu'}(\{x\}) &= \frac{\partial x^{\mu'}}{\partial x^{\mu}}(\{x\})A^{\mu}(\{x\}) \\ A_{\mu'}(\{x\}) &= \frac{\partial x^{\mu}}{\partial x^{\mu'}}(\{x\})A_{\mu}(\{x\})\end{aligned} \tag{1.6}$$

This is the space-time gauge invariant description in which physically redundant representations are eliminated from the actual measurement theory in accord with what experimental physicists actually do in the laboratory and the observatory. It is important to eliminate excess formal baggage and stick as close to experimental physics as is possible. It is this realization that allows us to think of gravity as simply another "internal symmetry" Yang-Mills gauge theory *once the immediate significance of coordinates is renounced* in the transition $x \to \{x\}$. I will usually not use this cumbersome $\{x\}$ notation below, but it should be understood tacitly. One other prerequisite point, there are four distinct groups of frame transformations to carefully distinguish.[iv]

i	Internal Yang-Mills	Change of gauge frame
ii	SO(1,3) 1905 SR	LIF{x}→LIF{x}' zero g-force both frames
iii	GCT	LNIF{x}→LNIF{x}' non-zero g-force both frames
iv	4 Cartan Tetrad 1-forms	Zero g-forceLIF{x}→non-zero g-forceLNIF{x} non-zero g-forceLNIF{x}→ Zero g-forceLIF{x}

$$\begin{aligned}&(ii)\,LIF(\{x\}) \leftrightarrow LIF(\{x\})' \\ &(iv)\updownarrow \\ &(iii)\,LNIF(\{x\}) \leftrightarrow LNIF(\{x\})'\end{aligned} \tag{1.7}$$

The Yang-Mills formalism
The electro-weak-strong gauge force partial covariant derivative on the matter fields in 1905 SR without gravity is

$$D_I\Psi \equiv \left(\partial_I + igA_I^a T_a\right)\Psi \tag{1.8}$$

This gauge covariant partial derivate transforms as

$$\left(D_I\Psi\right)' = \Omega D_I\Psi \approx D_I\Psi + i\Lambda^a T_a\Psi + \ldots \tag{1.9}$$

Therefore, the compensating internal symmetry gauge potential connection fields[v] must transform in *globally flat* Minkowski Space-Time as

$$\begin{aligned} igA'^a_I T_a &= \Omega\left(\partial_I + igA_I^a T_a\right)\Omega^{-1} \\ &\approx igA_I^a T_a - i\partial_I\Lambda^a T_a + g\left[T_a, T_b\right]\Lambda^a A_I^b + \mathrm{O}\left(\Lambda^2\right) + \ldots \\ A'^a_I &\approx A_I^a - \frac{1}{g}\partial_I\Lambda^a + f^a_{bc}\Lambda^b A_I^c + \ldots \end{aligned} \tag{1.10}$$

The *gauge functions* $\Lambda^a\left(\{x\}\right)$ are actually macro-quantum coherent post-inflation Goldstone phases of generalized Higgs fields in what Frank Wilczek calls the "multi-layered multi-colored vacuum superconductor"[vi], the gauge potentials $A_I^a\left(\{x\}\right)$ with non-vanishing curl correspond to *Goldstone phase singularities* in $\Lambda^a\left(\{x\}\right)$ which are *multi-valued* [vii] with finite jumps $2\pi n$ across the "branch cuts" corresponding to disclination curvature and dislocation torsion. See Hagen Kleinert's papers and books for details beyond the scope of this short paper.

It must be remembered that 1905 SR is only good for the transformations between Global Inertial Frames (GIFs), which in 1915 GR are demoted in ontic status to LIFs, for which I use the capital Latin indices $I,J,K,\ldots = 0,1,2,3$ I,J,K … for *both* GIF and LIF with 0 timelike inside the local light cone and 1,2,3 spacelike outside the classical local lightcone not yet smeared by quantum zero point vacuum fluctuations of the tetradic gravitational fields. Similarly, small Greek indices $\mu,\nu,\lambda,\sigma = 0,1,2,3$ are for the locally coincident LNIFs. Finally, the internal electro-weak-strong charge indices are the small Latin $a,b,c,\ldots = 1,2,\ldots,N_V$. However, from our above discussion of the solution of the Einstein hole problem using the active diffeomorphic GCT LNIF{x} → LNIF{x}' gauge orbit equivalence classes $\{x\}$ of physically equivalent bare manifold points with their equivalent "gauge redundant" local coordinate charts $x^\mu(x)$, the LIF indices play the same role for the universal space-time symmetry local gauge potentials $A^I_\mu\left(\{x\}\right)$ as do the non-universal Yang-Mills indices for the electro-weak-strong force internal symmetry gauge potentials $A_\mu^a(\{x\})$.

The Lie algebra for the *globally rigid* Poincare group P_{10} in the absence of gravity is[viii]

$$\begin{aligned} &[P_I, P_J] = 0 \\ &[M_{IJ}, P_K] = \eta_{IK} P_J - \eta_{JK} P_I \\ &[M_{IJ}, M_{KL}] = \eta_{IK} M_{JL} - \eta_{IL} M_{JK} - \eta_{JK} M_{IL} + \eta_{JL} M_{IK} \end{aligned} \tag{1.11}$$

Where the P's are the total 4-momenta (total energy and total linear momentum) of the matter fields integrals of the local current densities over 3D spacelike slices. Similarly the M's are the total angular momenta and boosts of the matter fields generating the six rigid spacetime rotations. The three boosts connect different GIF descriptions of the matter fields. This all seriously breaks down if we want to include gravity. We need to locally gauge the Poincare group. The induced gravity gauge potentials for *curvature without torsion* are the four tetrad Cartan 1-forms $A^I = A_\mu^I e_{LNIF}^\mu$ that pair with the four P_I generators of the T_4 translation subgroup to give the first rank GCT (Rovelli's "iii") LNIF tensor

$$A_\mu = A_\mu^I P_I \tag{1.12}$$

When there is torsion in addition to curvature we have the six additional dynamically independent antisymmetric spin connection 1-forms $\varpi^{IJ} = -\varpi^{JI} = \varpi_\mu^{IJ} e_{LNIF}^\mu$. We can now think of the LIF indices $I, J...$ *as if* they were internal indices $a, b...$ and the LNIF indices as if they were the GIF indices in the SRQFT case. Therefore, the *space-time gravity partial covariant derivative* on the matter fields sans the electro-weak-strong force terms must be

$$D_{\mu(G)}\Psi \equiv \left(\partial_\mu + igA_\mu^I P_I + ig'\left(\omega_{\mu(T_4)}^{IJ} + \varpi_{\mu(SO_{1,3}))}^{IJ}\right) M_{IJ}\right)\Psi \tag{1.13}$$

When there is zero torsion $\varpi_{SO_{1,3}}^{IJ} = 0$ and the 24 components of $\omega_{T_4}^{IJ}$ are given by Rovelli's (2.89)

$$\omega[e]_\mu^{IJ} = 2\, e^{\nu[I} \partial_{[\mu} e_{\nu]}{}^{J]} + e_{\mu K} e^{\nu I} e^{\sigma J} \partial_{[\sigma} e_{\nu]}{}^K. \tag{2.89}$$

where

$$e_\mu^I = \delta_\mu^I + A_\mu^I \tag{1.14}$$

Coupling of gravity to the quarks and leptons

The usual minimal coupling rule only uses the tetrads and ignores the spin connection. It is manifestly consistent with Einstein's Equivalence Principle (EEP). It is clearly sufficient for the EEP, but is it also necessary? We investigate this question here.

Given any rigid Poincare group local tensor field of any rank. Start with a free-float weightless zero g-force LIF (Alice) where Einstein's 1905 SR holds in a small region about its center of mass. Let's take a first rank tensor $T_{I(LIF)}\left(\{x\}\right)$ keeping the exposition as simple as possible without being simpler than is possible.[1] The minimal coupling rule for tensor is simply in this case

$$\begin{aligned}&T_{I(LIF)}\left(\{x\}\right) \to T_{\mu(LNIF)}\left(\{x\}\right) \equiv e_{\mu}^{I}\left(\{x\}\right)T_{I(LIF)}\left(\{x\}\right)\\&=\left(\delta_{\mu}^{I}+A_{\mu}^{I}\left(\{x\}\right)\right)T_{I(LIF)}\left(\{x\}\right)\end{aligned} \tag{1.15}$$

This is not perturbation theory. That is, this is not the first two terms of a power series expansion in the 16 tetrad components A_{μ}^{I} that need not be a small warping of Minkowski spacetime. In fact, A_{μ}^{I} is really an "acceleration field" that vanishes in the LIF. Newton's "gravity force" is 100% an "inertial force" telling us firstly about the local frame of reference that the detector is rigidly attached to. Only when there is a non-vanishing curl of the spin-connection do we have intrinsic gravity curvature. In the case of Einstein's 1915 we need Rovelli's (2.89) above in terms of the tetrads alone to compute the intrinsic curvature 2-form R^{IJ}

$$\begin{aligned}&R^{IJ} = D\omega^{IJ} = d\omega^{IJ} + \omega_{K}^{I} \wedge \omega^{KJ} = R_{\mu\nu}^{IJ}e^{\mu} \wedge e^{\nu}\\&d^{2} = 0\end{aligned} \tag{1.16}$$

Where $\left\{e^{\mu}\right\}$ is a set of four LNIF 1-form base co-vectors for a particular non-zero g-force LNIF that is locally coincident with the set $\left\{e^{I}\right\}$ of 1-form base co-vectors for a particular zero g-force LIF.

$$\begin{aligned}&(ii)e^{I} \to e^{I'} \equiv \left[\!\left[SO_{1,3}\right]\!\right]_{I}^{I'} e^{I}\\&(iii)e^{\mu} \to e^{\mu'} \equiv \frac{\partial x^{\mu'}}{\partial x^{\mu}}e^{\mu}\\&(iv)e^{\mu} = e_{I}^{\mu}e^{I}\end{aligned} \tag{1.17}$$

[1] Paraphrase of a famous Einstein quote.

$$
\begin{aligned}
ds^2 &= \eta_{IJ(LNIF)} e^I e^J = g_{\mu\nu(LNIF)} e^\mu e^\nu \\
g_{\mu\nu(LNIF)} &= \eta_{\mu\nu} + \eta_{IJ}\delta^I_\mu A^J_\nu + \eta_{IJ} A^I_\mu \delta^J_\nu + A^I_\mu A^J_\nu
\end{aligned} \quad (1.18)
$$

The torsion field 2-form is

$$
\begin{aligned}
&\mathrm{T}^I \equiv De^I = de^I + \left(\omega^I_J + \varpi^I_J\right)\wedge e^J = \varpi^I_J \wedge \left(\mathrm{I}^J + A^J\right) \\
&\mathrm{I}^J \equiv \delta^J_\mu e^\mu \neq e^J
\end{aligned} \quad (1.19)
$$

The key spinor field invariant differential operator on Dirac 4-spinor fields in SRQFT is

$$
\not{D}_{SR} \equiv \gamma^I \partial_I \quad (1.20)
$$

If we use the minimal coupling rule, then

$$
\begin{aligned}
&\gamma^I \to \gamma^\mu \equiv e^\mu_I \gamma^I \\
&\partial_I \to \partial_\mu \equiv e^K_\mu \partial_K \\
&\gamma^\mu \partial_\mu = e^\mu_I \gamma^I e^K_\mu \partial_K = e^\mu_I e^K_\mu \gamma^I \partial_K = \delta^K_I \gamma^I \partial_K = \gamma^I \partial_I = \not{D}
\end{aligned} \quad (1.21)
$$

This is no change at all. However, it's not that simple. Keeping to 1915 GR, i.e. $\varpi^{IJ} = 0$, then

$$
\gamma^\mu D_{\mu(G)} \Psi \equiv \gamma^\mu \left(\partial_\mu + igA^I_\mu P_I + ig'\omega^{IJ}_\mu M_{IJ}\right)\Psi \quad (1.22)
$$

What shall we use for γ^μ? One possibility is

$$
\begin{aligned}
&\gamma^\mu \overset{?}{=} e^\mu_I \gamma^I + \omega^\mu_{IJ}\left[\gamma^I, \gamma^J\right] \\
&\left[\gamma^I, \gamma^J\right] = i\varepsilon^{IJ}_K \gamma^K \\
&\gamma^\mu \overset{?}{=} e^\mu_I \gamma^I + \omega^\mu_{JK} i\varepsilon^{JK}_I \gamma^I = \left(\delta^\mu_I + A^\mu_I + \omega^\mu_{JK} i\varepsilon^{JK}_I\right)\gamma^I
\end{aligned} \quad (1.23)
$$

This is an empirical question. When the spin connection vanishes, sufficient though not necessary for zero curvature, we get back a sensible result. The tetrads describe translational g-forces and the spin-connections obviously describe rotational g-forces. The important thing is that we seem to get what we may call anomalous *"Shipov-Podkletnov couplings"* of the electro-weak-strong fields to the orbital angular momentum of rotating matter not merely to its quantum spins. This even is prior to additional torsion fields. Therefore, the purely gravitational "invariant" differential operator on Dirac spinor fields may be quite complicated, i.e.

$$
\not{D}_{\mu(G)}\Psi = \gamma^\mu D_{\mu(G)}\Psi \equiv \left(\delta^\mu_I + A^\mu_I + \omega^\mu_{JK} i\varepsilon^{JK}_I\right)\gamma^I \left(\partial_\mu + igA^I_\mu P_I + ig'\omega^{IJ}_\mu M_{IJ}\right)\Psi \quad (1.24)
$$

We can now do the whole shmear and include the 12 electro-weak-strong internal charges T^{a} [ix]

$$\begin{aligned} \not{D}_{\mu(G-YM)}\Psi &= \gamma^{\mu} D_{\mu(G-YM)}\Psi \\ &\equiv \left(\delta_{I}^{\mu} + A_{I}^{\mu} + \omega_{JK}^{\mu} i\varepsilon_{I}^{JK}\right)\gamma^{I}\left(\partial_{\mu} + igA_{\mu}^{I}P_{I} + ig'\omega_{\mu}^{IJ}M_{IJ} + e_{\mu}^{I} ig'' A_{I}^{a}T_{a}\right)\Psi \end{aligned} \tag{1.25}$$

Note the direct rotational g-force "Shipov-Podkletnov" couplings to the U_1 electromagnetic field ieA, e.g $\left(\omega_{JK}^{\mu} i\varepsilon_{I}^{JK}\right)\gamma^{I}\left(ieA_{I}^{em}\right)$. The effect is even more dramatic in a macro-quantum coherent superconductor. We now have a giant quantum Landau-Ginzburg order parameter Φ corresponding to a many-particle ground state boson condensate. The basic differential operator on Φ is now the quadratic Klein-Gordon operator $D^{\mu(G-YM)}D_{\mu(G-YM)}$.

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[i] Conceptual Basis of Quantum Field Theory, G. t'Hooft

[ii] In 1905 Einstein Special Relativity (SR), 4D Minkowski Space-Time (ST) is a fixed non-dynamical stage on which the matter fields perform among themselves. In this case the bare manifold points x have individual meaning. This changes qualitatively in Einstein 1915 General Relativity. Indeed, Einstein explicitly wrote about his years long struggle to free himself from the prejudice of the immediate physical meaning of coordinates that works even in modern day SR Quantum Field Theory (QFT) without gravity as in the Standard Model (SM) of leptons, quarks and electro-weak-strong gauge bosons with the Higgs mechanism in the weak sector, but no gravity. The solution of the "Hole Problem" showed that physically the local coincidence $\{x\}$ is a "gauge orbit" equivalence class $x \sim x'$ of all bare manifold points connected to each other by so-called active diffeomorphisms $x^{\mu} \to x^{\mu'}\left(x^{\mu}\right)$. Indeed, the classical Principle of General Relativity is the measurement of the same actual events by two locally coincident observers Alice and Bob. What makes the principle "general" is that Alice and Bob can be in arbitrary relative motion. Both can feel g-force, neither can, or only one of them can. In contrast, 1905 SR only works when neither Alice nor Bob feel any g-force. The local detection of g-force is the operational definition of absolute acceleration. The

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Principle of General Relativity means that the laws of physics do not depend on whether or not the detector is absolutely accelerating. What is meant by "coincident" is that Alice and Bob are not spatio-temporally separated by more than the characteristic locally variable radii of curvature and indeed more stringently much less. Alice and Bob may be looking at distant events like a supernova along their past warped light cones, but what matters is the local coincidence of their irreversible detections (pointer movements, digital readouts) in accord with John A. Wheeler's "no phenomenon is a phenomenon until it is recorded" (paraphrase). In addition, we need the Einstein principle of equivalence that Newton's "gravity force" is entirely an inertial g-force contingent accident of which local frame is used to measure phenomena. This is the essential physical meaning of Einstein's 1915 theory of gravity in its entirety. The passive diffeomorphism is of more immediate operational meaning. The gauge orbit construction in this paper projects the gauge orbit active diffeomorphisms $x \to x'$ to a single passive diffeomorphism $\{x\} \equiv x \sim x' \sim x'' \sim ...$ in which all possible non-zero g-force LNIF observers measure the same process up to Heisenberg uncertainty quantum limits.
See also John Norton's
http://www.springerlink.com/content/lm46773xk1304t27/fulltext.pdf
http://philsci-archive.pitt.edu/archive/00000380/01/NortonGCGTKO.ps

[iii] http://math.ucr.edu/home/baez/noether.html
http://en.wikipedia.org/wiki/Noether's_theorem

[iv] Rovelli "Quantum Gravity" Ch. 2
[v] "Connection field" refers to "parallel transport" of tensor/spinor fields. For the case of the electro-weak-strong forces that are induced from localizing the internal symmetry groups, the parallel transport is in the internal fiber space with a fiber above each space-time local coincidence equivalence class {x}. In the case the spacetime symmetry groups the parallel transport is in the physical quotient space of non-overlapping gauge orbit equivalence classes {x} =/= {x}'. If there is no torsion, then tiny mutually generated parallelograms $\Delta \equiv \delta x dx - dx \delta x$ of non-collinear infinitesimal parallel displacements of one by the other δx & dx close to 2nd order, i.e. the *torsion dislocation defect* gap vanishes $\Delta \approx 0$. Parallel transport a third independent vector A around the tiny parallelogram, the *disclination defect* angular difference
$\Theta \equiv \arccos A_{initial} \cdot A_{final} / |A_{initial}||A_{final}| \equiv \lim_{area \to 0} C \times area$ where C is the *sectional curvature* associated with the parallelogram.
[vi] "Lightness of Being"
[vii] "Multivalued fields in condensed matter, electromagnetism, and gravitation," Hagen Kleinert
[viii] http://en.wikipedia.org/wiki/Poincaré_group
[ix] 1 electric, 3 weak, 8 strong in the Standard Model (SM) we assume here.