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# There is No Einstein-Podolsky-Rosen Paradox in Clifford-Spaces

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**Abstract**

It is shown how one can attain the desired *locally causal* property of QM in Clifford-spaces despite the *spacelike* separation of two massive spin-$\frac{1}{2}$ particles involved in the Einstein-Podolksy-Rosen (EPR) experiment. This is achieved by proving why the addition laws of the particles poly-vector-valued momentum in Clifford-space is *null*-like. This is the key reason why it is possible to implement a *locally causal* QM theory in Clifford-spaces despite that QM has a non-local character in ordinary spacetime. The two particles can exchange signals *in* Clifford-space encoding their respective spin measurement values. Consequently, there is *no* EPR paradox in the Clifford space associated with the Clifford algebra $Cl(3,1)$ of the underlying $4D$ spacetime.

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Not so long ago Borchsenius [1] argued that when the quantum measurement principle is adapted to the generating space of Clifford algebras the transition probabilities for two-fold degenerate paths in space-time equals the transition amplitudes for the underlying paths in Clifford space. This property was used to show that the apparent non-locality of quantum mechanics in a double slit experiment and in an EPR type of measurement is resolved when analyzed in terms of the full paths in the underlying Clifford space.

Most recently, Christian [2] has shown that Bell's theorem fails for Clifford-algebra-valued local realistic variables. This was shown by exactly reproducing the Quantum Mechanical expectation values for the EPR-Bohm type spin correlations observables by means of local, deterministic, Clifford algebra valued variables, and without necessitating either remote contextuality or backward causation. Since the Clifford product of poly-vector (multivector) variables is non-commutative in general, the spin correlations values derived within such a locally causal model violate the CHSH (Clauser-Horne-Shimonov-Holt) inequality just as strongly as their quantum mechanical counterparts.

In a lengthy review of Clifford algebras, Varlamov [3] has given very convincing arguments as to why the division of physical fields into "gauge" fields (bosons) and "matter" fields (spinors) has an artificial character that disappears when one formulates the basic physical theories within the framework of Clifford algebras; i.e. there should not be a division of wavelike phenomenon (like Electromagnetism) and material one as envisioned many years ago in the de Broglie-Jordan neutrino (spinorial) theory of light [4] and references in [3].

Using our results over the past years on the Extended Relativity Theory in C-spaces (Clifford spaces) [7] we will prove that there is *no* EPR paradox in the Clifford-space associated with the spacetime Clifford algebra $Cl(3,1)$. C-space is a Clifford manifold in which there are coordinates $x^\mu, x^{\mu_1\mu_2}, x^{\mu_1\mu_2\mu_3}, ...$ linked to the basis vector $\gamma^\mu$ generator and to each element of the Clifford algebra : the bi-vectors $\gamma_\mu \wedge \gamma_\nu$, tri-vectors $\gamma_{\mu_1} \wedge \gamma_{\mu_2} \wedge \gamma_{\mu_3}$ ,....... including the Clifford algebra unit element (associated to a scalar coordinate). These poly-vector valued coordinates can be interpreted as the quenched-degrees of freedom of an ensemble of $p$-loops associated with the dynamics of closed $p$-branes, for $p = 0, 1, 2, ...., D-1$, embedded in a target $D$-dim spacetime background.

The $x^\mu$ coordinates represent the coordinates of a point (or the center of mass of an extended object); $x^{\mu_1\mu_2}$ are the area-coordinates enclosed [5] by the projections of a closed loop onto the coordinates planes ("holographic" screens) of the $D$-dim spacetime. $x^{\mu_1\mu_2\mu_3}$ are the volume-coordinates enclosed by the projections of a closed membrane ( a 2-loop ) onto the coordinates planes of the $D$-dim spacetime, ..... and so forth. Since a $p$-loop space is infinite dimensional, it is only in the quenched-approximation, by freezing the degrees of freedom, that one can match the zero modes of these $p$-loop configurations with the poly-vector valued coordinates of C-space. Further details on the Extended Relativity Theory in curved C-spaces can be found in [6], [7], [9] where the C-space metric $\mathbf{G}_{MN}$ with poly-vector valued indices $M, N$ represents lines, areas, volumes, .... hyper-volumes metrics and has a generalized curvature which admits an expansion in powers of the ordinary spacetime curvature and torsion and permits a construction of a master action encompassing the unified dynamics of an ensemble of $p$-branes for different values of $p = 0, 1, 2, ...., D-1$. For references pertaining Clifford algebras see [10].

We begin by writing the C-space poly-vector-valued momentum in the form described in [9]

$$\mathbf{P} \ = \ \pi \ \mathbf{1} \ + \ p^\mu \ \gamma_\mu \ + \ p^{\mu\nu} \ \gamma_\mu \wedge \gamma_\nu \ + \ \pi^\mu \ \gamma_5 \ \gamma_\mu \ + \ p^{0123} \ \gamma_5. \qquad (1)$$

where $(\gamma_5)^2 = -\mathbf{1}$, $\{\gamma^\mu, \gamma^5\} = 0$, the C-space invariant norm-squared of a momentum poly-vector is defined by the scalar part of the Clifford geometric product of $< \ \mathbf{P}^\sim \ \mathbf{P} \ >$ where $\mathbf{P}^\sim$ is the reversal-conjugate of $\mathbf{P}$ obtained by reversing the order of the gamma factors in the decomposition of the poly-vector $\mathbf{P}$ [10]. The norm-squared is

$$|| \mathbf{P} ||^2 = \pi^2 + p_\mu p^\mu + \frac{1}{2} p_{\mu\nu} p^{\mu\nu} + \frac{1}{3!} p_{\mu\nu\rho} p^{\mu\nu\rho} + \frac{1}{4!} p_{\mu\nu\rho\tau} p^{\mu\nu\rho\tau} =$$

$$\pi^2 + p_\mu p^\mu + \frac{1}{2} p^{\mu\nu} p_{\mu\nu} + \pi_\mu \pi^\mu - (p^{0123})^2. \quad (2)$$

it is necessary to introduce suitable powers of the Planck mass (that is set to unity) in order to match the units in the terms of eqs-(1-2). The norm can also be recast as

$$|| \mathbf{P} ||^2 = \pi^2 + p_\mu p^\mu + \frac{1}{2} S^{\mu\nu} S_{\mu\nu} + \pi_\mu \pi^\mu - (p^{0123})^2 \quad (3)$$

by *identifying* the spin bi-vector $S^{\mu\nu}$ with the momentum bi-vector $p^{\mu\nu}$. The physical motivation why the spin bi-vector $S^{\mu\nu}$ can be represented by the momentum bi-vector $p^{\mu\nu}$ (up to a power of $m^2_{Planck} = 1$) was explained by [8]. A natural coupling of the classical spin ( spin bi-vector $S^{\mu\nu}$ ) to the linear motion of the particle providing a new derivation of the Papapetrou equations can be found in [8] .

As explained in detail in chapter 2 of the monograph by [9], the ordinary momentum $p_\mu$ is *timelike* :

$$p_\mu p^\mu - (p^{0123})^2 = 0 \Rightarrow p_\mu p^\mu = (p^0)^2 - (\vec{p})^2 = (p^{0123})^2 = m^2 > 0,. \quad (4)$$

since the $p^{0123}$ pseudo-scalar component of the poly-momentum ( that is dual to a scalar ) is identified with the standard mass $m$ of the particle; whereas the axial-vector is spacelike resulting from the condition

$$\pi^2 + \pi_\mu \pi^\mu = 0 \Rightarrow \pi^2 = - \pi_\mu \pi^\mu \Rightarrow \pi_\mu \pi^\mu < 0, \ \textit{spacelike}. \quad (5)$$

The key to show why there is no EPR paradox in Clifford spaces (C-spaces) relies in the crucial observation that one must *not* impose the transversality constraints [8] $S^{\mu\nu} p_\nu = 0$ and studied in [9] but instead recur to the self-duality conditions of the spin bi-vector components (in a flat C-space) given by

$$S_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} S_{\rho\sigma}. \quad (6)$$

The self-duality conditions admit a simple solution in the center of mass frame when the two massive spin-$\frac{1}{2}$ particles involved in the EPR experiment are moving along the $z$-axis ( in opposite directions ) given by

$$S_{01} = S_{02} = S_{13} = S_{23} = 0. \quad (7)$$

and the remaining non-vanishing spin bi-vector components are $S_{03}, S_{12}$ such that

$$S_{03} = \frac{1}{2}\ \epsilon_{0312}\ S_{12}\ +\ \frac{1}{2}\ \epsilon_{0321}\ S_{21}\ =\ \epsilon_{0312}\ S_{12} =$$

$$\epsilon_{0123}\ S_{12}\ =\ S_{12}\ =\ S_{03}; \quad \epsilon_{0123}\ =\ -\ \epsilon^{0123}\ =\ 1. \tag{8}$$

where the signature is chosen to be $(+,-,-,-)$. Hence the solutions to the self-duality conditions (6) in the center of mass frame amount to a null-like condition on the spin bi-vector

$$S^{\mu\nu}\ S_{\mu\nu}\ =\ (S^{03})^2\ -(S^{12})^2\ =\ 0. \tag{9}$$

The quadratic Casimir is defined in terms of the spatial components of the spin bi-vector (in units of $\hbar = 1$) as

$$(S^{ij})^2\ =\ (S^{12})^2\ =\ s(s+1)\ \ when\ \ S^{13}\ =\ S^{23}\ =\ 0. \tag{10a}$$

where $s$ is the spin of the particles. Thus one arrives at

$$(S^{03})^2\ -(S^{12})^2 =\ 0\ \Rightarrow\ (S^{03})^2\ =\ (S^{12})^2\ =\ s(s+1). \tag{10b}$$

and the poly-momentum corresponding to the particles is itself null-like if the on-shell conditions eqs-(4-5) are satisfied :

$$if\quad S^{\mu\nu}\ S_{\mu\nu}\ = 0\ \Rightarrow ||\ \mathbf{P}\ ||^2\ =\ 0 \tag{11}$$

despite the fact that $m^2 \neq 0,\ \pi^2 \neq 0$. The axial-vector $\pi_\mu$ is just *proportional* to the Pauli-Lubansky vector

$$W_\sigma\ =\ S^{\mu\nu}\ p^\rho\ \epsilon_{\mu\nu\rho\sigma}. \tag{12}$$

obeying the orthogonality condition $W_\sigma\ p^\sigma\ =\ S^{\mu\nu}\ p^\rho\ p^\sigma\ \epsilon_{\mu\nu\rho\sigma} = 0$ due to the symmetry of the product $p^\rho\ p^\sigma = p^\sigma\ p^\rho$. If $p^\mu$ is timelike (spacelike), the Pauli-Lubansky vector $W_\mu$ is spacelike (timelike).

For the solutions (7-8), the non-vanishing components of $W_\mu$ are

$$W_0\ =\ \lambda\ \pi_0\ =\ S^{12}\ p^3\ \epsilon_{1230},\quad W_3\ =\ \lambda\ \pi_3\ =\ S^{12}\ p^0\ \epsilon_{1203}. \tag{13}$$

such that $W_\mu$ is *spacelike*

$$W_\mu W^\mu\ =\ \lambda^2\ [\ (\pi_0)^2\ -\ (\pi_3)^2\ ]\ =\ -\ \lambda^2\ \pi^2\ =$$

$$(S^{12})^2\ [\ (p_3)^2\ -\ (p_0)^2\ ]\ =\ -\ \ m^2\ s(s+1)\ <\ 0. \tag{14}$$

with the aid of the on-shell condition $(p_0)^2 - (p_3)^2\ =\ m^2$ and $(S^{12})^2 = s(s+1)$.

The normal addition of the timelike four-momentum of the two particles in the center of mass frame is

$$p^{(1)}_{\mu} \ + \ p^{(2)}_{\mu} \ = \ ( \ 2p_0, \ 0 \ ); \quad p^{(1)}_{\mu} \ - \ p^{(2)}_{\mu} \ = \ ( \ 0, \ 2\vec{p} \ ). \tag{15}$$

The addition of the spacelike axial-four-vectors differs from the addition of timelike four-vectors in the center of mass frame. Taking the $z$-axis as the direction of propagation, the non-vanishing components $\pi_{\mu}$ for the particle 1 moving upwards are

$$\pi^{(1)}_0 = \pi^{(1)}_0 = \epsilon_{0123} \ p^{123}_{(1)}, \quad \pi^{(1)}_z = \pi^{(1)}_3 = \epsilon_{0123} \ p^{012}_{(1)}. \tag{16}$$

whereas the non-vanishing components of the particle 2 moving downwards are

$$\pi^{(2)}_0 = \pi^{(2)}_0 = \epsilon_{0123} \ p^{123}_{(2)} \ = \ - \ \epsilon_{0123} \ p^{123}_{(1)} \ = \ - \ \pi^{(1)}_0. \tag{17}$$

due to the fact that $p^{123}_{(2)} = -p^{123}_{(1)}$ resulting from the relative *change* of sign of the $z$-component of the tri-vector. The other component is

$$\pi^{(2)}_z = \pi^{(2)}_3 = \epsilon_{0123} \ p^{012}_{(2)} \ = \ \epsilon_{0123} \ p^{012}_{(1)} \ = \ \pi^{(1)}_3 \tag{18}$$

there is no sign change in this case for the tri-vector $p^{012}$. Consequently, the addition law of the axial-four-momentum (spacelike) has a "dual" behavior compared to the addition law of the ordinary four-momentum (timelike)

$$\pi^{(1)}_{\mu} \ + \ \pi^{(2)}_{\mu} \ = \ ( \ 0, \ 2\vec{\pi} \ ) \ = \ ( \ 0, \ 0, \ 0, \ 2\pi_3 \ ); \quad \pi^{(1)}_{\mu} \ - \ \pi^{(2)}_{\mu} \ = \ ( \ 2\pi_0, \ 0, \ 0, \ 0 \ ). \tag{19}$$

The addition laws of the spin bivectors in the center of mass frame are

$$S^{(1)}_{\mu\nu} \ + \ S^{(2)}_{\mu\nu} \ = \ ( \ 0, \ 0 \ ), \quad S^{(1)}_{\mu\nu} \ - \ S^{(2)}_{\mu\nu} \ = \ ( \ 2S_{0i}, \ 2S_{ij} \ ), \ i, j = 1, 2, 3.. \tag{20}$$

consistent with the fact that the combined net spin of the two particles in the EPR experiment is *zero*. Therefore, one has that the addition law of the spin bi-vectors preserves the null-like condition

$$(S^{\mu\nu})^2_{(1\pm 2)} \ = \ 4(S_{0i})^2_{(1\pm 2)} \ - \ 4(S_{ij})^2_{(1\pm 2)} \ = 0. \tag{21}$$

resulting from the solutions (7-8) to the self-duality equations (6).

The addition laws of the scalar and pseudoscalar components of the poly-momentum are

$$\pi_{(1)} \ + \ \pi_{(2)} \ = \ 2\pi; \quad \pi_{(1)} \ - \ \pi_{(2)} \ = \ 0. \tag{22}$$

$$m_{(1)} \ + \ m_{(2)} \ = \ 2m \ = \ 2 \ p^{0123}; \quad ; \quad m_{(1)} \ - \ m_{(2)} \ = \ 0. \tag{23}$$

since we assumed the particles to be identical $m_{(1)} = m_{(2)} = m$ and $\pi_{(1)} = \pi_{(2)} = \pi$.

After this detailed analysis, from the above equations one can construct the poly-momentum addition laws in C-space so that the norm

$$(\mathbf{P}_1 + \mathbf{P}_2)^2 = 4 \, ( \, (p_0)^2 - (p^{0123})^2 \, ) + 4 \, ( \, \pi^2 - (\pi_3)^2 \, ) =$$
$$4 \, (p_3)^2 - 4 \, (\pi_0)^2 \geq 0 \tag{24}$$

as a result of the on-shell conditions

$$(p_0)^2 - (p_3)^2 = m^2 = (p^{0123})^2; \quad (\pi_0)^2 - (\pi_3)^2 = - \pi^2. \tag{25}$$

Similarly one can infer that

$$(\mathbf{P}_1 - \mathbf{P}_2)^2 = - 4 \, (p_3)^2 + 4 \, (\pi_0)^2 \leq 0 \tag{26}$$

as expected, eq-(26) has an overall sign change with respect to eq-(24). One can deduce now from eqs-(24,26) that in the very special case $p_3 = \pi_0$ we have

$$if \quad p_3 = \pi_0 \Rightarrow (\mathbf{P}_1 \pm \mathbf{P}_2)^2 = 0. \tag{27}$$

despite the fact that the addition law of the ordinary four-momentum obeys

$$(p^{(1)}_\mu + p^{(2)}_\mu)^2 > 0, \quad (p^{(1)}_\mu - p^{(2)}_\mu)^2 < 0. \tag{28}$$

Thus, despite that the addition of two momenta in ordinary spacetime remains *timelike* and the difference of the momenta is *spacelike*, consistent with the *spacelike* separation of the two particles 1, 2 moving along the $z$-axis in opposite directions, the addition laws of the poly-momentum in C-space is *null*-like ! This is the key reason why it is possible to implement a *locally causal* QM theory in C-spaces despite that QM has a non-local character in ordinary spacetime. The particles 1, 2 can exchange signals in C-space encoding their spin measurement values. The analog of "photons" or "light" signals in C-space are *tensionless* strings ($p$-branes) and a thorough discussion can be found in [7].

From the orthogonality condition

$$W_\mu \, p^\mu = \lambda \, \pi_\mu \, p^\mu = \lambda \, (\pi_0 \, p_0 - \pi_3 \, p_3 \,) = 0. \tag{29}$$

when $\pi_0 = p_3$, resulting in the sought-after relation $(\mathbf{P}_1 \pm \mathbf{P}_2)^2 = 0$, one learns also that $\pi_3 = p_0$, which in turn leads to the important finding that $m = \pi$ and $\lambda^2 = s(s+1)$ due to the on-shell conditions (25) and the Pauli-Lubansky relation (14) .

Concluding, after this detailed discussion involving the solutions to the self-duality conditions on the spin bi-vector $S^{\mu\nu}$ which is consistent with the null like $S^{\mu\nu}S_{\mu\nu}$ behavior, and which leads to the important results that the value of the scalar part of the Clifford-valued poly-momentum $\pi$ coincides with the value of the pseudo-scalar component $p^{0123} = m$, and the constant of proportionality $\lambda$ between the Pauli-Lubansky and axial vector $W_\mu = \lambda\ \pi_\mu$ obeys the relation $\lambda^2 = s(s+1)$, we can attain the desired *locally causal* property of QM in C-spaces.

This can be readily seen by noticing that if the particles are not subject to any external fields (forces) and one neglects the gravitational, electromagnetic ... interactions between the two particles, one has that the poly-vector valued coordinates in C-space are

$$\mathbf{X}_1 = \frac{1}{m}\mathbf{P}_1\,\tau;\ \mathbf{X}_2 = \frac{1}{m}\mathbf{P}_2\,\tau \Rightarrow (\mathbf{X}_1 - \mathbf{X}_2)^2 = \frac{1}{m^2}\,(\mathbf{P}_1 - \mathbf{P}_2)^2\,\tau^2 = 0. \quad (30)$$

where the proper-time $\tau$ elapsed for each of the particles is $\tau^2 = t^2 - z^2 \neq 0$ despite that their coresponding C-space norm is null $\mathbf{X}_1^2 = \mathbf{X}_2^2 = 0$. Since the interval in C-space $(\mathbf{X}_1 - \mathbf{X}_2)^2$ is *null* one can exchange signals from the locations $1, 2$ in C-space. As mentioned earlier, "light" signals in C-space correspond to *tensionless* strings ( branes ) [7]. Concluding, under these physical conditions there is *no* longer an Einstein-Podolsky-Rosen paradox in the Clifford-space associated with the spacetime Clifford algebra $Cl(3,1)$. The generalization of QM in Clifford spaces has been studied in [6] extending the results of [9].

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