A deliberation on the limits of the validity of Newton's third law in Electromagnetism

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Abstract

It is shown that as Newton's third law holds in static discussions of Electromagnetism it holds also in dynamic discussions of it, ie where the motion of electric charge in the magnetic field arising from the magnetic static (or magnetostatic) poles is under consideration. In addition, it is shown that in this recent case the forces of action and reaction are not collinear. In other words as we know there is some force exerted on a current-carrying wire in the magnetic field of a magnet. It is shown that this magnet is in fact the same enlarged magnetic needle in Oersted's experiment, and then there exists also some force exerted on the magnet due to the electric current in the wire. These two action and reaction forces are in opposite directions but are not collinear.

KEYWORDS: Newton's ; third ; law ; action ; reaction ; electromagnetism ; electrostatics ; magnetostatics ; electrodynamics ; magnetodynamics ; Biot ; Savart ; Oersted's

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1 Introduction

What are stated about the action-reaction law are the following three matters: 1. The magnitudes of action and reaction forces are equal to each other. 2. Their directions are opposite to each other to the extent of 180°. 3. They are exerted on the same line, or in other words these two forces lie on the line passing through the points of exertion of the forces. (To this third matter is not pointed explicitly in many sources while it is accepted implicitly, and in other sources this matter is pointed explicitly, and anyhow no counterexample is presented to it. For example note the phrase of one of the famous textbooks [1] in this respect: "The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line)." Also, in addition to some of the existent textbooks (eg see [2]), earlier editions of some of the famous textbooks have explicit pointings to this third matter[3, 4].)

It is certain that Newton's third law, as expressed through the above three matters, holds totally in Mechanics. To what extent do the above three matters hold in Electromagnetism? We try to answer this question section by section:

First question: Do they hold in Electrostatics? Answer: Yes. Coulomb's law proves this.

Second question: Do they hold in Magnetostatics? Answer: Since, contrary to electric charges (or electric poles), we cannot isolate magnetic poles in the lab in order to do experiments over them to find their influences on each others, reply to this question is not as straightforward as the previous one. But, clearly, the influences of the magnetic dipoles (or, in fact, magnets) on each others in the lab indicates that some law similar to Coulomb's law governs the form of forces between the magnetic poles and then Newton's third law, as expressed through the above three matters, holds among them.

Third question: Do they hold in Electrodynamics and Magnetodynamics? (The first is about the influences exerted on electric charges due to their motion in the magnetostatic fields (like the Lorentz force) and the second is about the influences exerted on magnetic poles due to the motion of electric charges in their magnetostatic field (like the force exerted on the poles of the magnet in Oersted's experiment).) Answer: The following reasoning shows that in this case only the first two out of the above three matters hold:

2 Analysis

"We know that there is a force exerted on a wire carrying electric current in a magnetic field, which is inward in the experiment shown in Fig. 1. It seems that its reaction force must be a force exerted on the poles N and S of the magnet in this figure toward the direction opposite to the direction of the force exerted on the wire. If we show existence of such a force, then it will have been proven that in the action-reaction law collinearity of the action and reaction forces is not necessary.

With some attention we realize that this experiment is in fact enlargement of the same Oersted's experiment shown in Fig. 2. In this experiment flowing of the electric current I in the direction shown in Fig. 2 will exert a force in the direction ON' on the pole N of the magnetic needle and a force in the direction OS' on the pole S of this needle. Reaction of the first force is a force in the direction OS' exerted on the wire carrying current I, and reaction of the second force is a force in the direction ON' exerted on this wire, and these two reaction forces cancel each other out and the wire will remain motionless. (Of course, if eg the pole N of the needle is toward the wire, since this pole will be in the region of intenser magnetodynamic field produced around the wire, resultant of forces exerted on the dipole SN (ie the magnetic needle) will be in the direction ON' and naturally its reaction will be a resultant force exerted on the wire in the direction OS'; it is evident that these two resultant forces are equal in magnitude and opposite in direction but are not collinear (they won't lie on the same line).)

Now consider Fig. 3 as an extended figure of the magnetic needle (or magnetic dipole). In this state the force exerted on the pole N is in the direction ON' and the force exerted on the pole S is in the direction OW'. Resultant of these two forces is a force exerted on the center of mass of the magnet toward the wire. (Reaction of the force exerted on N will be a force in the direction OS' exerted on the wire and reaction of the force exerted on S will be a force in the direction OE' exerted on the wire, and the resultant of them will be a force exerted on the wire toward the center of mass of the magnet.)

Now extend the magnetic dipole still more to obtain Fig. 4. In this state the force exerted on N is in the direction ON' (and its reaction will be a force exerted on the wire toward OS'), and the force exerted on S is also in the direction ON' (and its reaction will be a force exerted on the wire still toward OS'). Fig. 4 is the same Fig. 1. In this manner we have shown that firstly reactions of the forces exerted on the wire in Fig. 1 (which their resultant is F) exist and secondly they don't lie on the same line of F."

The above three questions and their presented answers are based on the supposition of existence of ((isolated) electric and (pole) magnetic) static charges and their influences on each other in which as we saw Newton's third law is not violated. The reasoning leading to Biot and Savart law is that a first electric charge which is moving creates some magnetic field which exerts force on a second electric charge which is moving in this magnetic field; and, conversely, the second electric charge which is moving creates some magnetic field which exerts force on the first electric charge which is moving in this magnetic field. The form of these mutual forces that these two charges exert on each other, as can be seen in the textbooks of electromagnetism, is not symmetric. For instance at the beginning of the chapter 8 of [5] we can see the following expression: "If the charges q and q_1 are uniformly moving, with velocities $\mathbf{v}$ and $\mathbf{v}_1$, respectively, there is a magnetic force $\mathbf{F}_m$ exerted on q by q_1:

$$\mathbf{F}_m = \frac{\mu_0}{4\pi}\frac{qq_1}{r^2}\mathbf{v} \times (\mathbf{v}_1 \times \frac{\mathbf{r}}{r}) \quad "$$

Since charges have no preference relative to each other, there is also a magnetic force $\mathbf{F}_{m1}$ exerted on q_1 by q:

$$\mathbf{F}_{m1} = \frac{\mu_0}{4\pi}\frac{q_1 q}{r^2}\mathbf{v}_1 \times (\mathbf{v} \times \frac{-\mathbf{r}}{r})$$

It is natural that $\mathbf{F}_m$ and $\mathbf{F}_{m1}$ must be a couple of action-reaction forces. But, does we have really $\mathbf{F}_m = -\mathbf{F}_{m1}$? This is not the case since the expression $\mathbf{v}\times(\mathbf{v}_1\times\hat{r}) = \mathbf{v}_1\times(\mathbf{v}\times\hat{r})$ (in which $\mathbf{r}/r = \hat{r}$) is not an identity. Namely, the law of action-reaction is violated easily (even without any limit condition). Certainly this is not true to say that for magnetic forces between electric charges we must not consider two electric point charges (which are moving) (and we must only consider the whole charges in closed circuits), because this is not the case for electric forces between electric charges, and in principle the basis of obtaining Coulomb's law is this same influences two point charges have on each other.

Now, considering that the situation related to the above-mentioned third question, as we saw, establishes Newton's third law, we conclude that we must distinguish between the magnetic field arising from magnetic static poles (eg the magnetic field due to the poles of a magnet) and a magnetic field which is not arising from the existence of any magnetic static poles (ie the magnetic field arising from the motion of the electric charge described above in the reasoning leading to Biot and Savart law) because in the first magnetic field Newton's third law holds while in the second one it does not hold.

Now, a question shows itself: We know there is a manner which is sometimes chosen in which it is tried to deny in principle the existence of any magnetic static poles, and instead of it, we only hear of the magnetic fields arising from atomic or molecular electric currents (eg see [6]). Is this manner true or not? If it is true, we must expect Newton's third law not to hold generally in the cases related to the above-mentioned second and third questions, because firstly it becomes necessary that, in the case related to the second question, the atomic currents in the magnets to exert force on each other according to the Biot and Savart law and according to this same law Newton's third law not to hold for them generally while we know in practice Newton's third law holds always about the force between the poles of the magnets, and secondly it becomes necessary that, in the case related to the third question, the microscopic (atomic) currents of the magnet and the macroscopic current due to the flow or motion of electrons in the wire (or in other words the microscopic circuits of the magnet and the macroscopic circuit carrying the electric current) to exert force on each other according to the Biot and Savart law and according to this same law Newton's third law not to hold for them generally while as we saw in the analysis presented above Newton's third law holds in Electro-Magnetdynamics which is about the influence that an electric current and a magnetostatic pole have on each other. (We should notice that in many of the preliminary textbooks we can find some expressions similar to the following: "As electric current exerts force on a magnet (Oersted's experiment) it is predictable that the magnet too will exert an equal force in the opposite direction on the wire carrying the current according to the action-reaction law. So if the magnet is motionless and the wire is free, the wire will move due to this force.")

References

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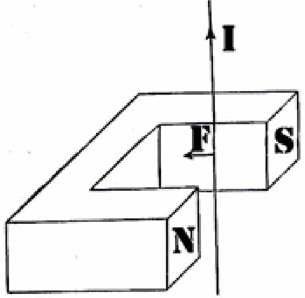

Fig 1

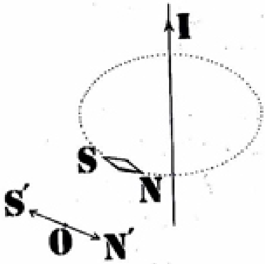

Fig 2

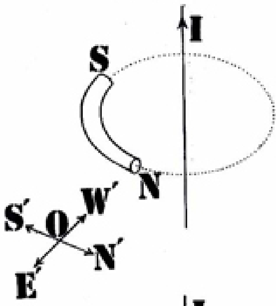

Fig 3

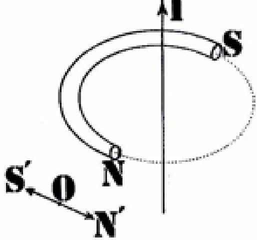

Fig 4