A proof for Goldbach's conjecture

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Abstract

For a large even number there are a large number of pairs of odd numbers sum of the members of each being the even number. We eliminate those pairs that none of the members of each of them is prime and show that the number of the remaining pairs is still large. The process of proof shows that there can be no drop to zero in the function of the number of the mentioned prime pairs.

1 The proof

The purpose of this paper is proving that each even number (greater than 2) is sum of two prime numbers. For this, we first should prove that the number of pairs of prime numbers whose sum equals a given even number goes towards infinity as this even number goes towards infinity. Let's write the series 3, 5, 7,... in a row named "A" and just under it write the series of the even numbers 6, 8, 10,... in a row named "B":

$A:$	3	5	7	9	11	13	15	17	$\cdots$
$B:$	6	8	10	12	14	16	18	20	$\cdots$
$C_3:$	1	1	1	1	1	1	1	1	$\cdots$
$C_5:$			1	1	1	1	1	1	$\cdots$
$C_7:$					1	1	1	1	$\cdots$
$C_9:$							1	1	$\cdots$
$D:$	1	1	2	2	3	3	4	4	$\cdots$

There is a relation between the numbers in the row A and in the row B: 3+3 from A is 6 from B, 3+5 from A is 8 from B, 3+7 from A is 10 from B, ... We show this relation by the numbers of 1 in the row C_3. Also 5+5 from A is 10 from B, 5+7 from A is 12 from B, 5+9 from A is 14 from B, ... We show this relation by the numbers of 1 in the row C_5. Also 7+7 from A is 14 from B, 7+9 from A is 16 from B, 7+11 from A is

18 from B, ... We show this relation by the numbers of 1 in the row C_7. We can continue this procedure.

Each item in a column of the row D is the sum of all the items in the same column of the rows C_n. In this manner each item in a column of the row D is the number of all the pairs of odd numbers that the sum of the members of each pair is the even number of this column (of the row B).

As we can easily see, in the row A the horizontal distance between each odd number and the first next odd number in the column of which there is an even number which is sum of this odd number and itself, is as large as the horizontal distance between this odd number and the first number in the row A. (For example the horizontal distance between 9 and the column containing 18(=9+9) is three units, and the horizontal distance between 9 and the first number in the row A, ie 3, is also three units.) Let's call such a distance as a "triangular distance" related to the odd number (eg the triangular distance related to 9 in the row A is (9 11 13 15) the length of which is three units).

It is clear that the series 1, 1, 2, 2, 3, 3, ... approaches infinity. Let's choose an odd number and make the triangular distance of it and as soon as such making subtract one unit from each number in the row D which is in the same column in which a member of this triangular distance is, and let's substitute the obtained series for the old series (here the series D). By this work we are subtracting the number of all the pairs of odd numbers, the bigger member of each being our chosen odd number, from the numbers of pairs (of odd numbers) written in the row D. Now let's choose a bigger odd number and perform the same above procedure to obtain a new series. For instance for the odd numbers 9, 11, 15, 19 and 23 we get the following sets of series D_n from the initial series D:

$A:$	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45 $\cdots$
$B:$	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48 $\cdots$
$D:$	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11 $\cdots$
D_9:				(1	2	2	3)	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11 $\cdots$
D_{11}:					(1	1	2	3	4)	5	6	6	7	7	8	8	9	9	10	10	11	11 $\cdots$
D_{15}:							(1	2	3	4	5	5	6)	7	8	8	9	9	10	10	11	11 $\cdots$
D_{19}:									(2	3	4	4	5	6	7	7	8)	9	10	10	11	11 $\cdots$
D_{23}:											(3	3	4	5	6	6	7	8	9	9	10)	11 $\cdots$

As it is seen, the first number in each series which is right-adjacent to the triangular distance is equal to the number of the members of the triangular distance. It's evident that for an infinite odd number this first number, right-adjacent to the triangular distance of this odd number, is an infinitely large number. If the triangular distances of the successive odd numbers bigger than our odd number are to subtract one by one from this infinite number through one-by-one rightward advance of the triangular distances (in the successive stages ie in the successive rows caused by the next triangular distances, just as eg the second 4 in the row D has been changed into 2 in the row D_{15} after three times rightward advancing of the triangular distances), we can say that the advance speed of the left border of the triangular distance (ie eg the mark "(" in the above rows) toward this infinite number is equal to the rate of subtraction from infinity, and finally, when this border reaches this infinite number, there will be no infinite number but a finite number (because as we saw, the number of the members of the triangular distance is equal to that number which is immediately right-adjacent to this distance).

Now suppose that instead of the above successive triangular distances of successive odd numbers we have only triangular distances of those odd numbers that are not prime. In such a case since we know the number of primes is infinite, the row by row triangular subtractions are not one by one, ie this is not the case that the left border of the triangular distance (ie "(") in a row is always only one unit farther than this border in the previous row (towards right), but the advance of this left border towards right is sometimes impulsive (not one by one), because it is to consist only of the (eliminating) triangular distances of the odd numbers which are not prime and we know we have infinite prime (odd) numbers up to infinity. Thus, when this left border reaches our infinite number, it will have subtracted an amount from our infinite number smaller than the largeness of this infinite number. It's clear that the amount remained of this infinite number (after these subtractions) is equal to the total number of the primes existent in an infinite continuous distance of natural numbers. And we know this number is itself an infinite number. Thus in fact when all the possible triangular distances have done their subtractions from the infinite number during their advance towards this number the remaining number is still infinitely large. (As the number of primes in the distance $(1,\infty)$ is infinite, the number of primes in the distance $(\infty/2,\infty)$ is also infinite, because otherwise we must accept that there is an upper bound for the primes in this distance which is not smaller than the infinity existent in $(\infty/2,\infty)$, and consequently it must be also an upper bound (for the primes) in $(1,\infty)$ (since the infinity existent in $(1,\infty)$ is the same infinity existent in $(\infty/2,\infty)$), but we know existence of any upper bound for the primes in $(1,\infty)$ is not possible.) This means that for an infinite even number the number of pairs of odd numbers, the sum of the numbers of each being the even number and the bigger number of each pair being a prime, is infinite. Let's call each pair of odd numbers the sum of the numbers of which is an even number as a constructive pair of the mentioned even number. Therefore, if an even number approaches infinity the number of its constructive pairs, the bigger number of each of which is a prime, will also approaches infinity. (Notice that in the eliminating mechanism presented above, pairs like (9,11) or (15,19) won't be eliminated, because in each pair while the smaller number is not a prime the bigger number is a prime.)

In addition, as it is evident in the process of the proof, for an even number there are no prime numbers other than those constituting the set of the bigger numbers of its constructive pairs between the smallest and biggest numbers of these bigger numbers of these pairs. This means that the mentioned set is a continuous set of primes. Is it possible that for an infinitely large even number none of the smaller numbers of its constructive pairs is prime? No, it is not possible, because $\infty/2$ is not a finite number. Explanation: Suppose that the statement P is true for each member of the infinite distance (1,∞). In such a case we can consider infinity as a member of this distance and say that P is true for it, and of course this means that P is true for every number of this distance which is greater than a chosen number. It is possible that several statements are true separately for different members of this distance, for instance P_1 for the distance (1,100) and P_2 for (100,1000000) and P_3 for (1000000,∞) in which again we can consider infinity as a member for which the statement P_3 is true. What is important in dealing with infinity as a member is that the statement for infinity must be clear and determined. Now suppose that for the finite distance (1,n) the statement is that firstly n/2 is determined and secondly (after performing the above eliminating mechanism) all the bigger numbers of the constructive pairs of the number which is immediately right-adjacent to this distance construct all the members of the distance (n/2,n) and none of the smaller numbers of these pairs belongs to the distance (1,n/2). In which case can this statement be true for the infinite distance (1,∞)? When at least $\infty/2$ is determined, but we know this is not the case. In a similar manner we must conclude that in principle the set of the prime numbers of these smaller numbers of the pairs is not a finite set since otherwise we can eliminate the pairs related to them and again have an infinite set the center of which must be determined.

In this manner we showed that when the even number approaches infinity the number of its constructive pairs, both numbers of which being primes, approaches infinity. Now consider the function of the number of those constructive pairs of even numbers both members of each of which are primes. We proved that this function has an (infinite) limit in infinity; then it is a well-behaved function (ie its general tendency on sufficiently large continuous distances of its domain is not unpredictable). On the other hand, as initial values, we know that on a large continuous distance of the first portion of the domain of this function this function takes positive values which are ascending on average. Since this function is well-behaved and is positive and ascending on average at the first and has a tendency towards infinity at the end, we conclude that it cannot have an abrupt drop toward zero at any other point of its domain. (If through some way other than that the function has an (infinite) limit in infinity we accepted that the function was well-behaved, could we conclude only from its general ascending tendency at the first that, up to infinity, it would have an ascending tendency? Obviously not, because in such a case we couldn't be certain that our well-behaved function not to start falling towards zero at a point.) In this manner we have shown that every even number is sum of (the members of at least one pair consisting of) two primes.

We proved that the above-mentioned function is infinity in infinity and we inferred that then it is well-behaved. What do we mean by well-behaving? In simple words it means that as this function is infinity in infinity then by closing limitedly to zero from infinity it is still infinity. If this becoming closer to zero continues infinitely, although it generally decreases yet it will be very large (and definitely). With becoming closer to zero its general bigness decreases, but now it is moving in a realm that we know it doesn't drop to zero. Also this is an independent function that has shown that in its realm in infinity does not have any tendency to drop abruptly and so, since it is independent, it must not have this tendency in any region of its domain. In fact in the process of proof we showed that the number of pairs the sum of the members of each being equal to a given even number is proportional to the number of primes (in a half of the interval) smaller than this even number, and since we know on average every larger interval beginning from zero contains more primes we conclude that our function is ascending on average. (Although in an interval there may be less primes than in another interval which is a little greater than it, on average in bigger intervals there are bigger number of primes (if it is not another unsolved problem)).