TOPOLOGICAL GEOMETRODYNAMICS: AN OVERVIEW

Ph.D. Matti Pitkänen

TOPOLOGICAL GEOMETRODYNAMICS: AN OVERVIEW

Matti Pitkänen

Köydenpunojankatu D 11, 10900, Hanko, Finland

Preface

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole contact defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy.

The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

Matti Pitkänen

Hanko,
September 16, 2014

Acknowledgements

Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 32 years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published

result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Matti Pitkänen

Hanko,
September 15, 2010

Contents

III HYPER-FINITE FACTORS OF TYPE II_1 AND HIERARCHY OF PLANCK CONSTANTS 685

List of Figures

Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. The CMAP files at my homepage provide an overview about ideas and evolution of TGD and make easier to understand what TGD and its applications are about (`http://www.tgdtheory.fi/cmaphtml.html` [L21]).

1.1.1 Basic vision very briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections ofCP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and

lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogenuity in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.1.2 Two manners to see TGD and their fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A117, A80, A99, A73].

The identification of the space-time as a sub-manifold [A65, A114] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.1.3 Basic objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 p-Adic variants of space-time surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.1.5 The threads in the development of quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name 'TGD as a generalized number theory'. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K96, K73, K60, K111, K85, K110, K109, K82] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K89, K12, K66, K10, K37, K46, K49, K81, K104] are warmly recommended to the interested reader.

Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH ("world of classical worlds",WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A94, A120, A122]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory [1]. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

   M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian

[1] There are four kinds of Dirac operators in TGD. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators

square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The modified gamma matrices define as anticommutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.

5. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak

form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worldss", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that

clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

The notion of p-adic manifold [K115] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheets as chart maps provides a possible solution of the basic challenge. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, "thought bubbles"). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution. The braids actually co-emerge with string world sheets implied by the condition that em charge is well-defined for spinor modes.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K88]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.

2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K88]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained

by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

3. M^8-H duality can be generalized to a duality $H \to H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.

4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K88] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the "free" gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.

2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.

3. For gamma matrix option induced rather than modified gamma matrices must be in question since modified gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially interesting is that p-adic and real regions of the space-time surface might aso emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.1.6 Hierarchy of Planck constants and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E27] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K79] .

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K67, K68, K103]) support the view that dark matter might be a key player in living matter.

Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant $\hbar_0$ is assigned with a multiple singular covering of the imbedding space [K27]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invariance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n.

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of biochemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.2 Bird's eye of view about the topics of the book

This book tries to give an overall view about quantum TGD as it stands now. The topics of this book are following.

1. In the first part of the book I will try to give an overall view about the evolution of TGD and about quantum TGD in its recent form. I cannot avoid the use of various concepts without detailed definitions and my hope is that reader only gets a bird's eye of view about TGD. Two visions about physics are discussed. According to the first vision physical states of the Universe correspond to classical spinor fields in the world of the classical worlds identified as 3-surfaces or equivalently as corresponding 4-surfaces analogous to Bohr orbits and identified as special extrema of Kähler action. TGD as a generalized number theory vision leading naturally also to the emergence of p-adic physics as physics of cognitive representations is the second vision.

2. The second part of the book is devoted to the vision about physics as infinite-dimensional configuration space geometry. The basic idea is that classical spinor fields in infinite-dimensional "world of classical worlds", space of 3-surfaces in $M^4 \times CP_2$ describe the quantum states of the Universe. Quantum jump remains the only purely quantal aspect of quantum theory in this approach since there is no quantization at the level of the configuration space. Space-time surfaces correspond to special extremals of the Kähler action analogous to Bohr orbits and define what might be called classical TGD discussed in the first chapter. The construction of the configuration space geometry and spinor structure are discussed in remaining chapters.

3. The third part of the book describes physics as generalized number theory vision. Number theoretical vision involves three loosely related approaches: fusion of real and various p-adic physics to a larger whole as algebraic continuations of what might be called rational physics; space-time as a hyper-quaternionic surface of hyper-octonion space, and space-time surfaces as a representations of infinite primes.

4. The first chapter in fourth part of the book summarizes the basic ideas related to Neumann algebras known as hyper-finite factors of type II_1 about which configuration space Clifford algebra represents canonical example. Second chapter is devoted to the basic ideas related to the hierarchy of Planck constants and related generalization of the notion of imbedding space to a book like structure.

5. The physical applications of TGD are the topic of the fifth part of the book. The cosmological and astrophysical applications of the many-sheeted space-time are summarized and the applications to elementary particle physics are discussed at the general level. TGD explains particle families in terms of generation genus correspondences (particle families correspond to 2-dimensional topologies labelled by genus). The notion of elementary particle vacuum functional is developed leading to an argument that the number of light particle families is three is discussed. The general theory for particle massivation based on p-adic thermodynamics is discussed at the general level. The detailed calculations of elementary particle masses are not however carried out in this book.

1.3 Sources

The eight online books about TGD [K96, K73, K111, K85, K60, K110, K109, K82] and nine online books about TGD inspired theory of consciousness and quantum biology [K89, K12, K66, K10, K37, K46, K49, K81, K104] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://www.tgdtheory.com/curri.html`) contains a lot of material about TGD. In particular, there is summary about TGD and its applications using CMAP representation serving also as a TGD glossary [L21, L22] (see `http://www.tgdtheory.fi/cmaphtml.html` and `http://www.tgdtheory.fi/tgdglossary.pdf`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://journals.sfu.ca/jnonlocality/index.php/jnonlocality` founded by Lian Sidorov and in *Prespacetime Journal* (`http://prespacetime.com`), *Journal of Consciousness Research and Exploration* (`https://www.createspace.com/4185546`), and *DNA Decipher Journal* (`http://dnadecipher.com`), all of them founded by Huping Hu. One can find the list about the articles published at `http://www.tgdtheory.com/curri.html`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 PART I: General Overview

Why TGD and What TGD is?

This piece of text was written as an attempt to provide a popular summary about TGD. This is of course mission impossible since TGD is something at the top of centuries of evolution which has led from Newton to standard model. This means that there is a background of highly refined conceptual thinking about Universe so that even the best computer graphics and animations fail to help. One can still try to create some inspiring impressions at least. This chapter approaches the challenge by answering the most frequently asked questions. Why TGD? How TGD could help to solve the problems of recent day theoretical physics? What are the basic princples of TGD? What are the basic guidelines in the construction of TGD?

These are examples of this kind of questions which I try to answer in using the only language that I can talk. This language is a dialect of the language used by elementary particle physicists, quantum field theorists, and other people applying modern physics. At the level of practice involves technically heavy mathematics but since it relies on very beautiful and simple basic concepts, one can do with a minimum of formulas, and reader can always to to Wikipedia if it seems that more details are needed. I hope that reader could catch the basic principles and concepts: technical details are not important. And I almost forgot: problems! TGD itself and almost every new idea in the development of TGD has been inspired by a problem.

Topological Geometrodynamics: Three Visions

In this chapter I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics.

2. Second vision is number theoretical and involves three threads. The first thread relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept. Second thread is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically. The third threadrelies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units.

3. The third vision is based on TGD inspired theory of consciousness, which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics.

TGD Inspired Theory of Consciousness

The basic ideas and implications of TGD inspired theory of consciousness are briefly summarized. The notions of quantum jump and self can be unified in the recent formulation of TGD relying on dark matter hierarchy characterized by increasing values of Planck constant. Negentropy Maximization Principle serves as a basic variational principle for the dynamics of quantum jump. The new view about the relation of geometric and subjective time leads to a new view about memory and intentional action. The quantum measurement theory based on finite measurement resolution and realized in terms of hyper-finite factors of type II_1 justifies the notions of sharing of mental images and stereo-consciousness deduced earlier on basis of quantum classical correspondence. Qualia reduce to quantum number increments associated with quantum jump. Self-referentiality of consciousness can be understood from quantum classical correspondence implying a symbolic representation of contents of consciousness at space-time level updated in each quantum jump. p-Adic physics provides space-time correlates for cognition and intentionality.

Overall View About Evolution of TGD

This chapter provides a bird's eye view about evolution of TGD with the hope that this kind of summary might make it easier to follow the more technical representation provided by subsequent chapters. The geometrization of fundamental interactions assuming that space-times are representable as 4-surfaces of $H = M^4_+ \times CP_2$ is wherefrom everything began. The two manners to understand TGD is TGD as a Poincare invariant theory of gravitation obtained by fusing special and general relativities, and TGD as a generalization of string model obtained my replacing 1-dimensional strings with 3-surfaces. The fusion of these approaches leads to the notion of the many-sheeted space-time.

The evolution of quantum TGD involve five threads which have become more and more entangled with each other. The first great vision was the reduction of the entire quantum physics (apart from quantum jump) to the geometry of classical spinor fields of the infinite-dimensional space of 3-surfaces in H, the great idea being that infinite-dimensional Kähler geometric existence and thus physics is unique from the requirement that it is free of infinities. The outcome is geometrization and generalization of the known structures of the quantum field theory and of string models.

The second thread is p-adic physics. p-Adic physics was initiated by more or less accidental observations about reduction of basic mass scale ratios to the ratios of square roots of Mersenne primes and leading to the p-adic thermodynamics explaining elementary particle mass scales and masses with an unexpected success. p-Adic physics turned eventually to be the physics of cognition and intentionality. Consciousness theory based ideas have led to a generalization of the notion of number obtained by gluing real numbers and various p-adic number fields along common rationals to a more general structure and implies that many-sheeted space-time contains also p-adic space-time sheets serving as space-time correlates of cognition and intentionality. The hypothesis that real and p-adic physics can be regarded as algebraic continuation of rational number based physics provides extremely strong constraints on the general structure of quantum TGD.

TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory replacing the notion of observer as an outsider with the notion of self. The detailed analysis of what happens in quantum jump have brought considerable understanding about the basic structure of quantum TGD itself. It seems that even quantum jump itself could be seen as a number theoretical necessity in the sense that state function reduction and state preparation by self measurements are necessary in order to reduce the generalized quantum state which is a formal superposition over components in different number fields to a state which contains only rational or finitely-extended rational entanglement identifiable as bound state entanglement. The number theoretical information measures generalizing Shannon entropy (always non-negative) are one of the important outcomes of consciousness theory combined with p-adic physics.

Physics as a generalized number theory is the fourth thread. The key idea is that the notion of divisibility could make sense also for literally infinite numbers and perhaps make them useful from the point of view of physicist. The great surprise was that the construction of infinite primes corresponds to the repeated quantization of a super-symmetric arithmetic quantum field theory. This led to the vision about physics as a generalized number theory involving infinite primes, integers, rationals and reals, as well as their quaternionic and octonionic counterparts. A further generalization is based on the generalization of the number concept already mentioned. Space-time surfaces could be regarded in this framework as concrete representations for infinite primes and integers, whereas the dimensions 8 and 4 for imbedding space and space-time surface could be seen as reflecting the dimensions of octonions and quaternions and their hyper counterparts obtained by multiplying imaginary units by $\sqrt{-1}$. Also the dimension 2 emerges naturally as the maximal dimension of commutative sub-number field and relates to the ordinary conformal invariance central also for string models.

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

This chapter represents a overall view about evolution of classical TGD and of p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, the vision about physics as a generalized number theory.

Overall View About Quantum TGD: Part I

This chapter is the first one of the two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision in this chapter

In this chapter the discussion is mostly concentrated on general ideas whereas the topics related to the construction of M-matrix are discussed on the second chapter. TGD relies heavily on geometric ideas and number theoretical ideas, which have gradually generalized during the years.

1. The basic vision is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

3. p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

4. The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. This leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. This leads to an explicit formula for the Dirac determinant. What is remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like

orbits of partonic 2-surfaces: they should correspond to preferred extremals of Kähler action. Thus hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality.

5. HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory.

Overall View About Quantum TGD: Part II

This chapter is the second one of two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development. In this chapter ideas related to the construction of S-matrix and coupling constant evolution are discussed.

The construction of S-matrix involves several ideas that have emerged during last years.

1. Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2. The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree.

3. Zero energy ontology leads to profoundly new view about the notion of virtual particle allowing to prove that the M-matrix is finite and that the number of Feynman diagrams contributing to given reaction is finite if particles have p-adic thermal mass.

4. The symmetric space property of world of classical worlds (WCW) allows to reduce WCW functional integral to Fourier analysis in WCW having a direct generalization to p-adic context so that the great dream about algebraic universality can be realized.

TGD and M-Theory

In this chapter a critical comparison of M-theory and TGD as two competing theories is carried out. Dualities and black hole physics are regarded as basic victories of M-theory.

a) The counterpart of electric magnetic duality plays an important role also in TGD and it has become clear that it might change the sign of Kähler coupling strength rather than leaving it invariant. The different signs would be related to different time orientations of the space-time sheets. This option is favored also by TGD inspired cosmology but unitarity seems to exclude it.

b) The AdS/CFT duality of Maldacena involved with the quantum gravitational holography has a direct counterpart in TGD with 3-dimensional causal determinants serving as holograms so that the construction of absolute minima of Kähler action reduces to a local problem.

c) The attempts to develop further the nebulous idea about space-time surfaces as quaternionic sub-manifolds of an octonionic imbedding space led to the realization of duality which could be called number theoretical spontaneous compactification. Space-time can be regarded equivalently as a hyper-quaternionic 4-surface in M^8 with hyper-octonionic structure or as a 4-surface in $M^4 \times CP_2$.

d) The duality of string models relating Kaluza-Klein quantum numbers with YM quantum numbers could generalize to a duality between 7-dimensional light like causal determinants of the imbedding space (analogs of "big bang") and 3-dimensional light like causal determinants of space-time surface (analogs of black hole horizons).

e) The notion of cotangent bundle of configuration space of 3-surfaces suggests the interpretation of the number-theoretical compactification as a wave-particle duality in infinite-dimensional context. Also the duality of hyper-quaternionic and co-hyper-quaternionic 4-surfaces could be understood analogously. These ideas generalize at the formal level also to the M-theory assuming that stringy configuration space is introduced. The existence of Kähler metric very probably does not allow dynamical target space.

In TGD framework black holes are possible but putting black holes and particles in the same basket seems to be mixing of apples with oranges. The role of black hole horizons is taken in TGD by 3-D light like causal determinants, which are much more general objects. Black hole-elementary particle correspondence and p-adic length scale hypothesis have already earlier led to a formula for the entropy associated with elementary particle horizon.

The recent findings from RHIC have led to the realization that TGD predicts black hole like objects in all length scales. They are identifiable as highly tangled magnetic flux tubes in Hagedorn temperature and containing conformally confined matter with a large Planck constant and behaving like dark matter in a macroscopic quantum phase. The fact that string like structures in macroscopic quantum states are ideal for topological quantum computation modifies dramatically the traditional view about black holes as information destroyers.

The discussion of the basic weaknesses of M-theory is motivated by the fact that the few predictions of the theory are wrong which has led to the introduction of anthropic principle to save the theory. The mouse as a tailor history of M-theory, the lack of a precise problem to which M-theory would be a solution, the hard nosed reductionism, and the censorship in Los Alamos archives preventing the interaction with competing theories could be seen as the basic reasons for the recent blind alley in M-theory.

1.4.2 PART II: Physics as Infinite-dimensional Geometry and Generalized Number Theory: Basic Visions

The geometry of the world of classical worlds

The topics of this chapter are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of the "world of classical worlds", with " classical world" identified either as 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surfaces so that unions of space-like surfaces with time like separations must be allowed. The considerations are restricted mostly to real context and the problems related to the p-adicization are discussed later.

There are two separate tasks involved.

1. Provide configuration space of 3-surfaces with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General coordinate invariance implies that the definition of metric must assign to a give 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.

2. Provide the configuration space with a spinor structure. The great idea is to identify configuration space gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

From the experience with loop spaces one can expect that there is no hope about existence of well-defined Riemann connection unless this space is union of infinite-dimensional symmetric spaces with constant curvature metric and simple considerations requires that Einstein equations are satisfied by each component in the union. The coordinates labeling these symmetric spaces are zero modes having interpretation as genuinely classical variables which do not quantum fluctuate since they do not contribute to the line element of the configuration space.

The construction of the Kähler structure involves also the identification of complex structure.

1. Direct construction of Kähler function as action associated with a preferred Bohr orbit like extremal for some physically motivated action action leads to a unique result.

2. Second approach is group theoretical and is based on a direct guess of isometries of the infinite-dimensional symmetric space formed by 3-surfaces with fixed values of zero modes. The group of isometries is generalization of Kac-Moody group obtained by replacing finite-dimensional Lie group with the group of symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone.

3. Third approach is based on the conjecture that yhr vacuum functional of the theory identifiable as an exponent of Kähler function is expressible as a Dirac determinant. This approach leads to an explicit expression of configuration space metric in terms of finite number of eigenvalues assignable to the modified Dirac operator defined by Kähler action. The notion of number theoretical compactification and the known properties of extremals of Kähler action play key role in this approach.

Classical TGD

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. Years later the understanding of quantum TGD at fundamental level deepened the understanding.

1. Preferred extremals and quantum criticality

The identification of preferred extremals of Kähler action defining counterparts of Bohr orbits has been one of the basic challenges of quantum TGD. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. It should also represent space-time correlate for quantum criticality.

The solution of the problem through the understanding of the implications number theoretical compactification and the realization of quantum TGD at fundamental level in terms of second quantization of induced spinor fields assigned to the modified Dirac action defined by Kähler action. Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open. The vanishing of second variations of preferred extremals suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context.

The space-time representation for dissipation comes from the interpretation of regions of space-time surface with Euclidian signature of induced metric as generalized Feynman diagrams (or equivalently the light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions). Dissipation would be represented in terms of Feynman graphs representing irreversible dynamics and expressed in the structure of zero energy state in which positive energy part corresponds to the initial state and negative energy part to the final state. Outside Euclidian regions classical dissipation should be absent and this indeed the case for the known extremals.

2. Hamilton-Jacobi structure

Most known extremals share very general properties. One of them is Hamilton-Jacobi structure meaning the possibility to assign to the extremal so called Hamilton-Jacobi coordinates. This means dual slicings of M^4 by string world sheets and partonic 2-surfaces. Number theoretic compactification led years later to the same condition. This slicing allows a dimensional reduction of quantum TGD to Minkowksian and Euclidian variants of string model and allows to understand how Equivalence Principle is realized at space-time level. Also holography in the sense that the dynamics of 3-dimensional space-time surfaces reduces to that for 2-D partonic surfaces in a given measurement resolution follows. The construction of quantum TGD relies in essential manner to this property. CP_2 type vacuum extremals do not possess Hamilton-Jaboci structure but this can be understood in the picture provided by number theoretical compactification.

3. Physical interpretation of extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium and dissipation is absent except in the sense that the super-position of generalized Feynman graphs representing the zero energy state represents dissipation. Lorentz 4-force vanishes for all known solutions of field equations.

1. The vanishing of the Lorentz 4-force in turn implies local covariant conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity.

2. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

3. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Kähler current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

4. Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

5. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context as it should by number theoretic universality.

6. As a consequence field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

4. The dimension of CP_2 projection as classifier for the fundamental phases of matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter.

1. For $D_{CP_2} = 4$ empty space Maxwell equations would hold true. This phase is chaotic and analogous to de-magnetized phase. There is also a CP breaking associated with this phase. At least CP_2 type vacuum extremals and their deformations represent this phase.

2. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. In fact, this phase as such does not correspond to preferred extremals but only their small deformations obtained by topological condensation of CP_2 type vacuum extremals representing elementary fermions at these extremals and by topological condensation of these extremals at larger space-time sheets creating wormhole contacts representing elementary bosons.

3. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. Also these extremals would represents ground states whose small deformations represent the phase. This phase is the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

5. Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

   (b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

   (c) In the so called Maxwell's phase, ordinary Maxwell equations for the induced Kähler field are satisfied in an excellent approximation. A special case is provided by a radially symmetric extremal having an interpretation as the space-time exterior to a topologically condensed particle. The sign of the gravitational mass correlates with that of the Kähler charge and one can understand the generation of the matter antimatter asymmetry from the basic properties of this extremal. The possibility to understand the generation of the matter antimatter asymmetry directly from the basic equations of the theory gives strong support in favor of TGD in comparison to the ordinary EYM theories, where the generation of the matter antimatter asymmetry is still poorly understood.

Physics as a generalized number theory

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry for the "world of classical worlds" identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question. The second vision is the identification of physics as a generalized number theory. This program involves three threads: various p-adic physics and their fusion together with real number based physics to a larger structure, the attempt to understand basic physics in terms of classical number fields (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. In this article brief summaries of physics as infinite-dimensional geometry and generalized number theory are given to be followed by more detailed articles.

1. p-Adic physics and their fusion with real physics

The basic technical problems of the fusion of real physics and various p-adic physics to single coherent whole relate to the notion of definite integral both at space-time level, imbedding space level and the level of WCW (the "world of classical worlds"). The expressibility of WCW as a union of symmetric spacesleads to a proposal that harmonic analysis of symmetric spaces can be used to define various integrals as sums over Fourier components. This leads to the proposal the p-adic variant of symmetric space is obtained by a algebraic continuation through a common intersection of these spaces, which basically reduces to an algebraic variant of coset space involving algebraic extension of rationals by roots of unity. This brings in the notion of angle measurement resolution coming as $\Delta\phi = 2\pi/p^n$ for given p-adic prime p. Also a proposal how one can complete the discrete version of symmetric space to a continuous p-adic versions emerges and means that each point is effectively replaced with the p-adic variant of the symmetric space identifiable as a p-adic counterpart of the real discretization volume so that a fractal p-adic variant of symmetric space results.

If the Kähler geometry of WCW is expressible in terms of rational or algebraic functions, it can in principle be continued the p-adic context. One can however consider the possibility that that the integrals over partonic 2-surfaces defining flux Hamiltonians exist p-adically as Riemann sums. This requires that the geometries of the partonic 2-surfaces effectively reduce to finite sub-manifold geometries in the discretized version of $\delta M^4_+ \times CP_2$. If Kähler action is required to exist p-adically same kind of condition applies to the space-time surfaces themselves. These strong conditions might make sense in the intersection of the real and p-adic worlds assumed to characterized living matter.

2. TGD and classical number fields

The basis vision is that the geometry of the infinite-dimensional WCW ("world of classical worlds") is unique from its mere existence. This leads to its identification as union of symmetric spaces whose Kähler geometries are fixed by generalized conformal symmetries. This fixes space-time dimension and the decomposition $M^4 \times S$ and the idea is that the symmetries of the Kähler manifold S make it somehow unique. The motivating observations are that the dimensions of classical number fields are the dimensions of partonic 2-surfaces, space-time surfaces, and imbedding space and M^8 can be identified as hyper-octonions- a sub-space of complexified octonions obtained by adding a commuting imaginary unit. This stimulates some questions.

Could one understand $S = CP_2$ number theoretically in the sense that M^8 and $H = M^4 \times CP_2$ be in some deep sense equivalent ("number theoretical compactification" or $M^8 - H$ duality)? Could associativity define the fundamental dynamical principle so that space-time surfaces could be regarded as associative or co-associative (defined properly) sub-manifolds of M^8 or equivalently of H.

One can indeed define the associativite (co-associative) 4-surfaces using octonionic representation of gamma matrices of 8-D spaces as surfaces for which the modified gamma matrices span an associate (co-associative) sub-space at each point of space-time surface. Also $M^8 - H$ duality holds true if one assumes that this associative sub-space at each point contains preferred plane of M^8 identifiable as a preferred commutative or co-commutative plane (this condition generalizes to an integral distribution of commutative planes in M^8). These planes are parametrized by CP_2 and this leads to $M^8 - H$ duality.

WCW itself can be identified as the space of 4-D local sub-algebras of the local Clifford algebra of M^8 or H which are associative or co-associative. An open conjecture is that this characterization of the space-time surfaces is equivalent with the preferred extremal property of Kähler action with preferred extremal identified as a critical extremal allowing infinite-dimensional algebra of vanishing second variations.

3. Infinite primes

The construction of infinite primes is formally analogous to a repeated second quantization of an arithmetic quantum field theory by taking the many particle states of previous level elementary particles at the new level. Besides free many particle states also the analogs of bound states appear. In the representation in terms of polynomials the free states correspond to products of first order polynomials with rational zeros. Bound states correspond to n^{th} order polynomials with non-rational but algebraic zeros.

The construction can be generalized to classical number fields and their complexifications obtained by adding a commuting imaginary unit. Special class corresponds to hyper-octonionic primes for which the imaginary part of ordinary octonion is multiplied by the commuting imaginary unit so that one obtains a sub-space M^8 with Minkowski signature of metric. Also in this case the basic construction reduces to that for rational or complex rational primes and more complex primes are obtained by acting using elements of the octonionic automorphism group which preserve the complex octonionic integer property.

Can one map infinite primes/integers/rationals to quantum states? Do they have space-time surfaces as correlates? Quantum classical correspondence realized in terms of modified Dirac operator implies that if infinite rationals can be mapped to quantum states then the mapping of quantum states to space-time surfaces automatically gives the map to space-time surfaces. The question is therefore whether the mapping to quantum states defined by WCW spinor fields is possible. A natural hypothesis is that number theoretic fermions can be mapped to real fermions and number theoretic bosons to WCW ("world of classical worlds") Hamiltonians. The crucial observation is that one can construct infinite hierarchy of hyper-octonionic units by forming ratios of infinite integers such that their ratio equals to one in real sense: the integers have interpretation as positive and negative energy parts of zero energy states. One can construct also sums of these units with complex coefficients using commuting imaginary unit and these sums can be normalized to unity and have interpretation as states in Hilbert space. These units can be assumed to possess well defined standard model quantum numbers. It is possible to map the quantum number combinations of WCW spinor fields to these states. Hence the points of M^8 can be said to have infinitely complex number theoretic anatomy so that quantum states of the universe can be mapped to this anatomy. One could talk about algebraic holography or number theoretic Brahman=Atman identity.

One can also ask how infinite primes relate to the p-adicization program and to the hierarchy of Planck constants. The key observation is that infinite primes are in one-one correspondence with rational numbers at the lower level of hierarchy. At the first level of hierarchy the p-adic norm with respect to p-adic prime for this rational gives power p^{-n} so that one has two powers of p - p^{n_+} andp^{n_-}- since two infinite primes corresponding to fermionic vacua $X \pm 1$, where X is the product of all primes at given level of hierarchy, characterize the partonic 2-surface. The proposal inspired by the p-adicization program is that $\Delta\phi = 2\pi/p^n$ defines angle measurement resolution crucial in the construction of p-adic variants of WCW ("world of classical world") as a union of symmetric coset spaces by starting from discrete variants of the real counterpart of symmetric space having common points with tis p-adic variant. The two measurement resolutions correspond to CD and CP_2 degrees of freedom. The hierarchy of Planck constants generalizes imbedding space to a book like structure with pages identified in terms of singular coverings and factor spaces of CD and CP_2. There are good arguments suggesting that only coverings characterized by integers n_a and n_b are realized. The identifications $n_a = n_+$ and $n_b = n_-$ lead to highly non-trivial physical predictions and allow sharpen the view about the hierarchy of Planck constants. Therefore the notion of finite measurement resolution becomes the common denominator for the three threads of the number theoretic vision and give also a connection with the physics as infinite-dimensional geometry program and with the inclusions of hyper-finite factors defined by WCW spinor fields and proposed to characterize finite measurement resolution at quantum level.

1.4.3 Unified Number Theoretical Vision

An updated view about M^8-H duality is discussed. M^8-H duality allows to deduce $M^4\times CP_2$ via number theoretical compactification. One important correction is that octonionic spinor structure makes sense only for M^8 whereas for $M^4\times CP_2$ complefixied quaternions characterized the spinor structure.

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized.

There is a beautiful pattern present suggesting that $H = M^4 \times CP_2$ is completely unique on number theoretical grounds. Consider only the following facts. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. Octonionic projective space OP_2 appears as octonionic twistor space (there are no higher-dimensional octonionic projective spaces). Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing OP_2 to to 12-D $G_2/U(1)\times U(1)$ having same dimension as the the twistor space $CP_3\times SU(3)/U(1)\times U(1)$ of H assignable to complexified quaternionic representation of gamma matrices.

A further fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically six-sphere. Also the analogy of quaternionicity of preferred extremals in TGD with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. M^8-H correspondence in turn would map the space-time surface in M^8 to $M^4\times CP_2$.

1.4.4 PART III: Hyperfinite factors of type II_1 and hierarchy of Planck constants

Was von Neumann right after all?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology. In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD.

1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic

Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [?] Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the modified gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality [?]his leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

4. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator

type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which modified Dirac action containing measurement interaction term defines stringy propagators. This M-matrix should be consistent with the above proposal.

5. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

6. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD since 2005. The basic idea was stimulated by the finding of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, where the velocity parameter v_0 has the approximate value $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. It was who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge

couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

1.4.5 PART IV: Some Applications

Cosmology and Astrophysics in Many-Sheeted Space-Time

This chapter is devoted to the applications of TGD to astrophysics and cosmology are discussed. In a well-defined sense classical TGD defined as the dynamics of space-time surface determining them as kind of generalized Bohr orbits can be regarded as an exact part of quantum theory and assuming quantum classical correspondence has served as an extremely valuable guideline in the attempts to interpret TGD, to form a view about what TGD really predicts, and to to guess what the underlying quantum theory could be and how it deviates from standard quantum theory. Also TGD inspired cosmology and astrophysics relies on this general picture.

1. Many-sheeted cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

The most important differences are following.

a) Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.

b) TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.

c) The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.

d) Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.

e) Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

2. Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

3. Dark matter and quantization of gravitational Planck constant

The notion of gravitational Planck constant having gigantic value is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

Overall View About TGD from Particle Physics Perspective

Topological Geometrodynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.

- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the imbedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces and one can understand the generalization of Equivalence Principle in terms of coset representations for the two super Virasoro algebras associated with lightlike boundaries of so called causal diamonds defined as intersections of future and past directed lightcones (CDs) and with light-like 3-surfaces. Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.

- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves three Dirac operators. The Kähler Dirac equation holds true in the interior of space-time surface and its solutions havea natural interpretation in terms of description of matter, in particular condensed matter. Chern-Simons Dirac action is associated with the light-like 3-surfaces and space-like 3-surfaces at ends of space-time surface at light-like boundaries of CD. One can assign to it a generalized eigenvalue equation and the matrix valued eigenvalues correspond to the the action of Dirac operator on momentum eigenstates. Momenta are however not usual momenta but pseudo-momenta very much analogous to region momenta of twistor approach. The third Dirac operator is associated with super Virasoro generators and super Virasoro conditions define Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S-matrix to a collection of what I call M-matrices defining the rows of unitary U-matrix defining unitary process.

- Twistor approach has inspired several ideas in quantum TGD during the last years and it seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness. Twistor approach also inspires the conjecture that quantum TGD allows also formulation in terms of 6-dimensional holomorphic surfaces in the product $CP_3 \times CP_3$ of two twistor spaces and general arguments allow to identify the partial different equations satisfied by these surfaces.

Particle Massivation in TGD Universe

This chapter represents the most recent view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters. In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the modified Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected.

During 2010 a further progress took place. These steps of progress relate closely to zero energy ontology, bosonic emergence, the realization of the importance of twistors in TGD, and to the discovery of the weak form of electric-magnetic duality. Twistor approach and the understanding of the Chern-Simons Dirac operator served as a midwife in the process giving rise to the birth of the idea that all particles at fundamental level are massless and that both ordinary elementary particles and string like objects emerge from them. Even more, one can interpret virtual particles as being composed of these massless on mass shell particles assignable to wormhole throats so that four-momentum conservation poses extremely powerful constraints on loop integrals and makes them manifestly finite.

The weak form of electric-magnetic duality led to the realization that elementary particles correspond to bound states of two wormhole throats with opposite Kähler magnetic charges with second throat carrying weak isospin compensating that of the fermion state at second wormhole throat. Both fermions and bosons correspond to wormhole contacts: in the case of fermions topological condensation generates the second wormhole throat. This means that altogether four wormhole throats are involved with both fermions, gauge bosons, and gravitons (for gravitons this is unavoidable in any case). For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM. Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

1. Physical states as representations of super-symplectic and Super Kac-Moody algebras

Physical states are assumed to belong to the representation of super-symplectic algebra and Super Kac-Moody algebra assignable $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ associated with the 2-D surfaces X^2 defined by the intersections of light-like 3-surfaces with $\delta M^4_\pm \times CP_2$. These 2-surfaces have interpretation as partons.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [**?**] The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [**?**]

2. Particle massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

2. By Equivalence Principle the thermal average of mass squared can be calculated either in terms of thermodynamics for either super-symplectic of Super Kac-Moody Virasoro algebra and p-adic thermodynamics is consistent with conformal invariance.

3. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

4. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses via a generation of Higgs vacuum expectation having possibly interpretation

in terms of a coherent state. The deviation Δh of the total ground state conformal weight from negative integer gives rise to Higgs type contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. It is natural to relate Δh to the generalized eigenvalues of Chern-Simons Dirac operator.

5. A natural identification of the non-integer contribution to the conformal weight is as Higgsy and stringy contributions to the vacuum conformal weight (strings are now "weak strings"). In twistor approach the generalized eigenvalues of Chern-Simons Dirac operator for external particles indeed correspond to light-like momenta and when the three-momenta are opposite this gives rise to non-vanishing mass. Higgs is necessary to give longitudinal polarizations for gauge bosons and also gauge bosons usually regarded as exactly massless particles would naturally receive small mass in this manner so that Higgs would disappear completely from the spectrum. The theoretetical motivation for a small mass would be exact Yangian symmetry. Higgs vacuum expectation assignable to coherent state of Higgs bosons is not needed to explain the boson masses. Twistorial consideration suggest that Higgs disappears completely from the spectrum and this might happen also for its super counterpart.

6. Hadron massivation requires the understanding of the CKM mixing of quarks reducing to different topological mixing of U and D type quarks. Number theoretic vision suggests that the mixing matrices are rational or algebraic and this together with other constraints gives strong constraints on both mixing and masses of the mixed quarks.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

New Physics Predicted by TGD

TGD predicts a lot of new physics and it is quite possible that this new physics becomes visible at LHC. Although the calculational formalism is still lacking, p-adic length scale hypothesis allows to make precise quantitative predictions for particle masses by using simple scaling arguments.

The basic elements of quantum TGD responsible for new physics are following.

1. The new view about particles relies on their identification as partonic 2-surfaces (plus 4-D tangent space data to be precise). This effective metric 2-dimensionality implies generalizaton of the notion of Feynman diagram and holography in strong sense. One implication is the notion of field identity or field body making sense also for elementary particles and the Lamb shift anomaly of muonic hydrogen could be explained in terms of field bodies of quarks.

2. The topological explanation for family replication phenomenon implies genus generation correspondence and predicts in principle infinite number of fermion families. One can however develop a rather general argument based on the notion of conformal symmetry known as hyper-ellipticity stating that only the genera $g = 0, 1, 2$ are light. What "light" means is however an open question. If light means something below CP_2 mass there is no hope of observing new fermion families at LHC. If it means weak mass scale situation changes.

   For bosons the implications of family replication phenomenon can be understood from the fact that they can be regarded as pairs of fermion and antifermion assignable to the opposite wormhole throats of wormhole throat. This means that bosons formally belong to octet and singlet representations of dynamical SU(3) for which 3 fermion families define 3-D representation. Singlet would correspond to ordinary gauge bosons. Also interacting fermions suffer topological condensation and correspond to wormhole contact. One can either assume that the resulting wormhole throat has the topology of sphere or that the genus is same for both throats.

3. The view about space-time supersymmetry differs from the standard view in many respects. First of all, the super symmetries are not associated with Majorana spinors. Super generators correspond to the fermionic oscillator operators assignable to leptonic and quark-like induced spinors and there is in principle infinite number of them so that formally one would have $\mathcal{N} = \infty$ SUSY. I have discussed the required modification of the formalism of SUSY theories and it turns out that effectively one obtains just $\mathcal{N} = 1$ SUSY required by experimental constraints. The reason is that the fermion states with higher fermion number define only short range interactions analogous to van der Waals forces. Right handed neutrino generates this super-symmetry broken by the mixing of the M^4 chiralities implied by the mixing of M^4 and CP_2 gamma matrices for induced gamma matrices. The simplest assumption is that particles and their superpartners obey the same mass formula but that the p-adic length scale can be different for them.

4. The new view about particle massivation involves besides p-adic thermodynamics also Higgs but there is no need to assume that Higgs vacuum expectation plays any role. The most natural option favored by the assumption that elementary bosons are bound states of massless elementary fermions, by twistorial considerations, and by the fact that both gauge bosons and Higgs form SU(2) triplet and singlet, predicts that also photon and other massless gauge bosons develop small mass so that all Higgs particles and their colored variants would disappear from spectrum. Same could happen for Higgsinos.

5. One of the basic distinctions between TGD and standard model is the new view about color.

   (a) The first implication is separate conservation of quark and lepton quantum numbers implying the stability of proton against the decay via the channels predicted by GUTs. This does not mean that proton would be absolutely stable. p-Adic and dark length scale hierarchies indeed predict the existence of scale variants of quarks and leptons and proton could decay to hadons of some zoomed up copy of hadrons physics. These decays should be slow and presumably they would involve phase transition changing the value of Planck constant characterizing proton. It might be that the simultaneous increase of Planck constant for all quarks occurs with very low rate.

   (b) Also color excitations of leptons and quarks are in principle possible. Detailed calculations would be required to see whether their mass scale is given by CP_2 mass scale. The so called leptohadron physics proposed to explain certain anomalies associated with both electron, muon, and τ lepton could be understood in terms of color octet excitations of leptons.

6. Fractal hierarchies of weak and hadronic physics labelled by p-adic primes and by the levels of dark matter hierarchy are highly suggestive. Ordinary hadron physics corresponds to $M_{107} = 2^{107} - 1$ One especially interesting candidate would be scaled up hadronic physics which would correspond to $M_{89} = 2^{89} - 1$ defining the p-adic prime of weak bosons. The corresponding string tension is about 512 GeV and it might be possible to see the first signatures of this physics at LHC. Nuclear string model in turn predicts that nuclei correspond to nuclear strings of nucleons connected by colored flux tubes having light quarks at their ends. The interpretation might be in terms of M_{127} hadron physics. In biologically most interesting length scale range 10 nm-2.5 μm there are four Gaussian Mersennes and the conjecture is that these and other Gaussian Mersennes are associated with zoomed up variants of hadron physics relevant for living matter. Cosmic rays might also reveal copies of hadron physics corresponding to M_{61} and M_{31}

7. Weak form of electric magnetic duality implies that the fermions and antifermions associated with both leptons and bosons are Kähler magnetic monopoles accompanied by monopoles of opposite magnetic charge and with opposite weak isospin. For quarks Kähler magnetic charge need not cancel and cancellation might occur only in hadronic length scale. The

magnetic flux tubes behave like string like objects and if the string tension is determined by weak length scale, these string aspects should become visible at LHC. If the string tension is 512 GeV the situation becomes less promising.

In this chapter the predicted new physics and possible indications for it are discussed.

Part I

GENERAL OVERVIEW

Chapter 2

Why TGD and What TGD is?

2.1 Introduction

This text was written as an attempt to provide a popular summary about TGD. This is of course mission impossible as such since TGD is something at the top of centuries of evolution which has led from Newton to standard model. This means that there is a background of highly refined conceptual thinking about Universe so that even the best computer graphics and animations do not help much. One can still try - at least to create some inspiring impressions. This chapter approaches the challenge by answering the most frequently asked questions. Why TGD? How TGD could help to solve the problems of recent day theoretical physics? What are the basic principles of TGD? What are the basic guidelines in the construction of TGD?

These are examples of this kind of questions which I try to answer in this chapter using the only language that I can talk. This language is a dialect used by elementary particle physicists, quantum field theorists, and other people applying modern physics. At the level of practice involves technically heavy mathematics but since it relies on very beautiful and simple basic concepts, one can do with a minimum of formulas, and reader can always to to Wikipedia if it seems that more details are needed. I hope that reader could catch the basic idea: technical details are not important, it is principles and concepts which really matter. And I almost forgot: problems! TGD itself and almost every new idea in the development of TGD has been inspired by a problem.

2.1.1 Why TGD?

The first question is "Why TGD?". The attempt to answer this question requires overall view about the recent state of theoretical physics.

Obviously standard physics plagued by some problems. These problems are deeply rooted in basic philosophical - one might even say ideological - assumptions which boil down to -isms like reductionism, materialism, determinism, and locality.

Thermodynamics, special relativity, and general relativity involve also postulates, which can be questioned. In thermodynamics second law in its recent form and the assumption about fixed arrow of thermodynamical time can be questions since it is hard to understand biological evolution in this framework. Clearly, the relationship between the geometric time of physics and experienced time is poorly understood. In general relativity the beautiful symmetries of special relativity are in principle lost and by Noether's theorem this means also the loss of classical conservation laws, even the definitions of energy and momentum are in principle lost. In quantum physics the basic problem is that the non-determinism of quantum measurement theory is in conflict with the determinism of Schrödinger equation.

Standard model is believed to summarize the recent understanding of physics. The attempts to extrapolate physics beyond standard model are based on naive length scale reductionism and have products Grand Unified Theories (GUTs), supersymmetric gauge theories (SUSYs). The attempts to include gravitation under same theoretical umbrella with electroweak and strong interactions has led to super-string models and M-theory. These programs have not been successful, and the recent dead end culminating in the landscape problem of super string theories and M-theory could have its origins in the basic ontological assumptions about the nature of space-time and quantum.

2.1.2 How could TGD help?

The second question is "Could TGD provide a way out of the dead alley and how?". The claim is that is the case. The new view about space-time as 4-D surface in certain fixed 8-D space-time is the starting point motivated by the energy problem of general relativity and means in certain sense fusion of the basic ideas of special and general relativities.

This basic idea has gradually led to several other ideas. Consider only the identification of dark matter as phases of ordinary matter characterized by non-standard value of Planck constant, extension of physics by including physics in p-adic number fields and assumed to describe correlates of cognition and intentionality, and zero energy ontology (ZEO) in which quantum states are identified as counterparts of physical events. These new elements generalize considerably the view about space-time and quantum and give good hopes about possibility to understand living systems and consciousness in the framework of physics.

2.1.3 Two basic visions about TGD

There are two basic visions about TGD as a mathematical theory. The first vision is a generalization of Einstein's geometrization program from space-time level to the level of "world of classical worlds" identified as space of 4-surfaces. There are good reasons to expect that the mere mathematical existence of this infinite-dimensional geometry fixes it highly uniquely and therefore also physics. This hope inspired also string model enthusiasts before the landscape problem forcing to give up hopes about predictability.

Second vision corresponds to a vision about TGD as a generalized number theory having three separate threads.

1. The inspiration for the first thread came from the need to fuse various p-adic physics and real physics to single coherent whole in terms of principle that might be called number theoretical universality.

2. Second thread was based on the observation that classical number fields (reals, complex numbers, quaternions, and octonions) have dimensions which correspond to those appearing in TGD. This led to the vision that basic laws of both classical and quantum physics could reduce to the requirements of associativity and commutativity.

3. Third thread emerged from the observation that the notion of prime (and integer, rational, and algebraic number) can be generalized so that infinite primes are possible. One ends up to a construction principle allowing to construct infinite hierarchy of infinite primes using the primes of the previous level as building bricks at new level. Rather surprisingly, this procedure is structurally identical with a repeated second quantization of supersymmetric arithmetic quantum field theory for which elementary bosons and fermions are labelled by primes. Besides free many-particle states also the analogs of bound states are obtained and this means the situation really fascinating since it raises the hope that the really hard part of quantum field theories - understanding of bound states - could have number theoretical solution.

It is not yet clear whether both great visions are needed or whether either of them is in principle enough. In any case their combination has provided a lot of insights about what quantum TGD could be.

2.1.4 Guidelines in the construction of TGD

The construction of new physical theory is slow and painful task but leads gradually to an identification of basic guiding principles helping to make quicker progress. There are many such guiding principles.

- "Physics is uniquely determined by the existence of WCW" is is a conjecture but motivates highly interesting questions. For instance: "Why $M^4 \times CP_2$ a unique choice for the imbedding space?", "Why space-time dimension must be 4?", etc...

- Number theoretical Universality is a guiding principle in attempts to realize number theoretical vision, in particular the fusion of real physics and various p-adic physics to single structure.
- The construction of physical theories is nowadays to a high degree guesses about the symmetries of the theory and deduction of consequences. The very notion of symmetry has been generalized in this process. Super-conformal symmetries play even more powerful role in TGD than in super-string models and gigantic symmetries of WCW in fact guarantee its existence.
- Quantum classical correspondence is of special importance in TGD. The reason is that where classical theory is not anymore an approximation but in well-defined sense exact part of quantum theory.

There are also more technical guidelines.

- Strong form of General Coordinate invariance (GCI) is very strong assumption. Already GCI leads to the assumption that Kähler function is Kähler action for a preferred extremal defining the counterpart of Bohr orbit. Even in a form allowing the failure of strict determinism this assumption is very powerful. Strong form of general coordinate invariance requires that the light-like 3-surfaces representing partonic orbits and space-like 3-surfaces at the ends of causal diamonds are physically equivalent. This implies effective 2-dimensionality: the intersections of these two kinds of 3-surfaces and 4-D tangent space data at them should code for quantum states.
- Quantum criticality states that Universe is analogous to a critical system meaning that it has maximal structural richness. One could also say that Universe is at the boundary line between chaos and order. The original motivation was that quantum criticality fixes the basic coupling constant dictating quantum dynamics essentially uniquely.
- The notion of finite measurement resolution has also become an important guide-line. Usually this notion is regarded as ugly duckling of theoretical physics which must be tolerated but the mathematics of von Neumann algebras seems to raise its status to that of beautiful swan.
- What I have used to call weak form of electric-magnetic duality is a TGD version of electric-magnetic duality discovered by Olive and Montonen [B7]. It makes it possible to realize strong form of holography implied actually by strong for of General Coordinate Invariance. Weak form of electric magnetic duality in turn encourages the conjecture that TGD reduces to almost topological QFT. This would mean enormous mathematical simplification.
- TGD leads to a realization of counterparts of Feynman diagrams at the level of space-time geometry and topology: I talk about generalized Feynman diagrams. The highly non-trivial challenge is to give them precise mathematical content. Twistor revolution has made possible a considerable progress in this respect and led to a vision about twistor Grassmannian description of stringy variants of Feynman diagrams. In TGD context string like objects are not something emerging in Planck length scale but already in scales of elementary particle physics. The irony is that although TGD is not string theory, string like objects and genuine string world sheets emerge naturally from TGD in all length scales. Even TGD view about nuclear physics predicts string like objects.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L74]
- Physics as generalized number theory [L58]

- Quantum physics as generalized number theory [L64]
- Hyperfinite factors and TGD [L42]
- Weak form of electric-magnetic duality [L83]
- Generalized Feynman diagrams [L36]
- The unique role of twistors in TGD [L79]
- Twistors and TGD [L81]

2.2 The great narrative of standard physics

Narratives allow a simplified understanding of very complex situations. This is why they are so powerful and this is why we love narratives. Unfortunately, narrative can also lead to the wrong track when one forgets that only a rough simplification of something very complex is in question.

2.2.1 Philosophy

In the basic philosophy of physics reductionism, materialism, determinism, and locality are four basic dogmas forming to which the great narrative relies.

Reductionism

Reductionism can be understood in many manners. One can imagine reduction of physics to few very general principles, which is of course just the very idea of science as an attempt to understand rather than only measure. This reductionism is naive length scale reductionism. Physical systems consist of smaller building bricks which consist of even smaller building bricks... The entire physics would reduce to the dance of quarks and this would reduce to the dynamics of super strings in the scale of Planck length. The brief summary about the reductionistic story would describe physics as a march from macroscopic to increasingly microscopic length scales involving a series of invasions:

Biology $\rightarrow$ biochemistry $\rightarrow$ chemistry $\rightarrow$ atomic physics as electrodynamics for nuclei and electrons. Nuclear physics for nuclei $\rightarrow$ hadronic physics for nuclei and their excitations $\rightarrow$ strong and weak interactions for quarks and and leptons.

One can of course be skeptic about the first steps in the sequence of conquests. Is biology really in possession? Physicists cannot give definition of life and can say even less about consciousness. Even the physics based definition of the notion of information central for living systems is lacking and only entropy has physics based definition. Do we really understand the extreme effectiveness of bio-catalysts and miracle like replication of DNA, transcription of DNA to mRNA, and translation of mRNA to aminoacids. It is yet impossible to test numerically whether phenomenological notions like chemical bond really emerge from Schrödinger equation.

The reduction step from nuclear physics to hadron physics is purely understand as is the reduction step from hadron physics to the physics of quarks and gluons. Here one can blame mathematics: the perturbative approach to quantum chromodynamics fails at low energies and one cannot realize deduce hadrons from basic principle by analytical calculations and must resort to non-perturbative approaches like QCD involving dramatic approximations.

The standard model is regarded as the recent form of reductionism. The generalization of standard model: Grand Unified Theories (GUTs), Supersymmetric gauge theories (SUSYs), and super string models and M-theory are attempts to continue reductionistic program beyond standard model making an enormous step in terms of length scales directly to GUT scale or Planck scale. These approaches have been followed during last forty years and one must admit that they have not been very successful. This point will be discussed in detail later.

Therefore reductionistic dogma involves many bridges assumed to exist but about whose existence we do not really know. Further, reductionistic dogma cannot be tested. This untestability might be the secret of its success besides the natural human laziness and temptations of groupthink, which could quite generally explain the amazing success of great narratives even when they have been obviously wrong.

Materialism

Materialism is another big chunk in the great narrative of physics. What it states is that only the physically measurable properties matter. One cannot measure the weight of the soul, so that there is no such thing as soul. The physical state of the brain at given moment determines completely the contents of conscious experience. In principle all sensory qualia, say experience of redness, must have precise correlates at the level of brain state.

At what level does life and consciousness appear. What makes matter conscious and behaving as if would have goals and intentions and need to survive? This is difficult question for the materialistic approach one postulates the fuzzy notion of emergence. When the system becomes complex enough, something genuinely new - be it consciousness or life - emerges. The notion of emergence seems to be in obvious conflict with that of naive length scale reductionism and a lot of handwaving is needed to get rid of unpleasant questions. What this something new really is is very difficult or even impossible to define in in the framework reductionistic physics.

The problems culminate in neuroscience and consciousness theory which has become a legitimate field of science during last decade. The hard problem is the coding of the properties of the physical state of the brain to conscious experience. Recent day physics does not provide a slightest clue regarding this correspondence. One has of course a lot of correlations. Light with certain wavelength creates the sensation of red but a blow in the head can produce the same sensation. EEG and nerve pulse activity correlate with the contents of conscious experience and EEG seems to even code for contents of conscious experience. Only correlates are however in question. It is also temporal patterns of EEG rather than EEG at given moment of time which matters from the point of view of conscious experience. This relates closely to another dogma of standard quantum physics stating that time=constant slice of time evolution contains all information about the state of the system.

Determinism

The successes of Newtonian mechanism were probable the main reason for why determinism became a basic dogma of physics. Determinism implies a romantic vision: theoretician working with mere paper and pencil can predict the future. This leads also to the idea that Nature can be governed: this idea has dominated western thinking for centuries and led to the various crises that human kind is suffering. Ironically, this idea is actually in conflict with the belief in strict determinism! Also the narrative provided by Darwinism assumes survival as a goal, which means that organisms behave like intentional agents: something in conflict with strict determinism predicting clockwork Universe. On the other hand, genetic determinism assumes that genes determine everything. The great narrative is by no means free of contradictions. They are present and one must simply put them under the rug in order to keep the faith. The situation is same as in religions: everyone realizes that Bible is full of internal contradictions and one must just forget them in to not lose the great narrative provided by it.

In quantum theory one is forced to give up the notion of strict determinism at the level of individual systems. The outcome of state function reduction occurring in quantum measurement is not predictable at the level of individual systems. For ensembles one can predict probabilities of various outcomes so that classical determinism is replaced with statistical determinism, which of course involves the idealized notion of ensemble consisting of large number of identical copies of the system under consideration.

In consciousness theory strict determinism means denial of free will. One could ask whether the non-determinism of state function reduction could be interpreted in terms of free will so that even elementary particles would be conscious systems. It seems that this identification cannot explain intentional goal directed free will. State function reductions produce entropy and this provides deeper justification for the second law and quantum mechanism makes it possible to calculate various parameters like viscosity and diffusion constants needed in the phenomenological description of macroscopic systems. Living systems however produce and store information and experience it consciously. Quantum theory in its recent form does not have the descriptive power to describe this. Something more is needed: one should bring the notion of information to physics.

Locality

Locality is fourth basic piece of great narrative. What locality says that physical systems can be split into basic units and that understanding the behavior of this units and the interaction between them is enough to understand the system. This is very much akin to naive length scale reductionism stating that everything can be reduced to the level of elementary particles or even to the level of superstrings.

Already in quantum theory one must give up the notion of locality although Schrödinger equation is still local. Standard quantum theory tells that in macroscopic scales entanglement has no implications. Quantum entanglement is now experimentally demonstrated to be possible between systems with macroscopic distance and even between macroscopic and microscopic systems. What does this mean: is the standard quantum theory really all that is needed or should we try to generalize it?

Locality dogma becomes especially problematic in living systems. Living systems behave as coherent units behaving very "quantally" and it is very difficult to understand how sacks of water containing some chemicals could climb in trees and even compose symphonies. The attempts to produce something which would look like living from a soup of chemicals have not been successful.

The proposed cure is macroscopic quantum coherence and macroscopic entanglement. There exist macroscopically quantum coherent systems such as suprafluids and super-conductors but these systems are very simple all particles are in same state- Bose Einstein condensate and quite different from living matter. Standard quantum theory is also unable to explain macroscopic quantum coherence and preservation of entanglement at physical temperatures.

Evidence for quantum coherence in cell scales and at physiological temperatures is however accumulating. Photosynthesis, navigation behavior of some birds and fishes, and olfaction represent examples of this kind. The recent finding that microtubules carry quantum waves should be also mentioned. Does this mean that something is missing from standard quantum theory. The small value of Planck constant characterizes the sizes of quantum effects and tells that spatial and temporal scales of quantum coherence are typically rather short. Is Planck constant really constant. One can of course ask whether this problem could relate to another mystery of recent day physics: the dark matter. We know that it exists but there is no generally accepted idea about what it is. Could living systems involve dark matter in an essential manner and could it be that Planck constant does not have only its standard value?

Locality postulate has far reaching implications for science policy. There is a lot of anecdotal evidence for various remote mental interactions such as telepathy, clairvoyance, psychokinesis of various kinds, remote healing, etc... The common feature of these phenomena is non-locality so that standard science denies them as impossible. Fr this reason people trying to study these phenomena have automatically earned the label of crackpot. Therefore experimental demonstration of these phenomena is very difficult since we do not have any theory of consciousness. Situation is not helped by the fact that skeptics deny in reflex like manner all evidence.

2.2.2 Classical physics

Classical physics began with the advent of Newton's mechanics and brought the dogma of determinism to physics. In the following only thermodynamics and special and general relativities are discussed as examples about classical physics because they are most relevant from the TGD viewpoint.

Thermodynamics

Second law is the basic pillar of thermodynamics. It states that the entropy of a closed system tends to increase and achieve maximum in thermodynamical equilibrium. This law does not tell about the detailed evolution but only poses the eventual goal of evolution. This means irreversibility: one cannot reverse the arrow of thermodynamical time. For instance, one one can live life in the reverse direction of time.

The physical justification for the second law comes from quantum theory. Again one must however make clear that the basic assumption that that time characteristic time scale for interactions involved is short as compared to the time scale one monitors the system. In time scales shorter the quantum coherence time the situation changes. If quantum coherence is possible in macroscopic time scales, one cannot apply thermodynamics.

The thermodynamical time has a definite arrow and is believe to be the same always. Living matter might form exception to this belief and Fantappie has proposed that this is indeed the case and proposed the notion of syntropy to characterize systems which seem to have non-standard arrow of time. Also phase conjugate laser rays seem to dissipate in wrong direction of time so that entropy seems to decrease from them when they are viewed in standard time direction.

The basic equations of physics are not believed posses arrow of time. Therefore the relationship between thermodynamical time and the geometric time of Einstein is problematic. Thermodynamical arrow of time relates closely to that of experienced/psychological arrow of time. Is the identification of experienced time and geometric time really acceptable? They certainly look different notions: experienced time has not future unlike geometric time, and experienced time is irreversible unlike geometric time. Certainly the notion of geometric time is well-understood. The notion of experienced time is not. Are we hiding ourselves behind the back of Einstein when we identify these two times. Should we bravely face the reality and ask what experienced time really is? Is it something different from geometric time and why these two times have also many common aspects - so many that we have identified them.

Second law provides a rather pessimistic view about future: Universe is unavoidably approaching heat death as it approaches thermodynamical equilibrium. Thermodynamics provides a measure for entropy but not for information. Is biological evolution really a mere thermodynamical fluctuation in which entropy in some space-time volume is reduced? Can one really understand information created and stored by living matter as a mere thermodynamical fluctuation? The attempt to achieve this has been formulated as non-equilibrium thermodynamics for open systems. One can however wonder whether could go wrong in the basic premises of thermodynamics?

Special Relativity

Relativity principle is the basic pillar of special relativity. It states that all system with respect to each other with constant relativity are physically equivalent: in other worlds the physics looks the same in these systems. Light velocity is absolute upper limit for signal velocity.

This kind of principle holds true also in Newton's mechanics and is known as Galilean relativity. Now there is however not upper bound for signal velocity. The difference between these principles follows from different meaning for what it is to move with constant relativity velocity. In special relativity time is not absolute anymore but the time shown by the clocks of two systems are different: time and spatial coordinates are mixed by the transformation between the systems.

Maxwell's electrodynamics satisfied the Relavity Principle and in modern terminology Poincare group generated by rotations, Lorentz transformations (between systems moving with respect to each other with constant velocity), translations in spatial and time drections act as symmetries of Maxwell's equations. In particle physics and quantum theory the formulation of relativity principle in terms of symmetries has become indispensable.

The essence of Special theory of relativity is geometric. Minkowski space is four-dimensional analog of Riemannian geometry with metric which characterizes what length and angle measurement mean mathematically. The metric is characterized in terms of generalization of the law of Pythagoras stating $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$ in Minkowski coordinates. What is special is that time and space are in different positions in this infinitesimal expression for line element telling the length of the diameter of 4-dimensional infinitesimal cube.

Time dilation and Lorentz contraction are two effects predicted by special relativity. Time dilation day-to-day phenomenon in particle physics: particles moving with high velocity live longer in the laboratory system. Lorentz contraction must be also taken into account. Lorentz himself believed for long that Lorentz contraction is a physical rather than purely geometric effect but finally admitted that Einstein was right.

There are some pseudo paradoxes associated with Special Relativity and regularly some-one comes and claims that is some horrible logical error in the formulation of the theory. One paradox is twin paradox. One consider twins. Second goes for a long space-time travel moving very near to light-velocity and experiences time dilation. When he arrives at home he finds that his twin brother is very old. One can however argue that by relativity principle it is the second twin who has made the travel and should look older. The solution of the paradox is trivial. The situation is not symmetric since the second brother is not entire time in motion with constant velocity since he must turn around during the travel and spend this period in accelerated motion.

General Relativity

Einstein based his theories on general principles and maybe this is why they have survived all the tests. The theoretical physics has become very technical since the time of Einstein and the formulation of theories in terms of principles has not been in fashion. Instead, concrete equations and detailed models have replaced this approach. Super string models provide a good example. Maybe this explains why the modest success.

In general relativity there are two basic principles. General Coordinate Invariance and Equivalence Principle.

General Coordinate Invariance (GCI) states that the formulation of physics must be such that the basic equations are same in all coordinate systems. This is very powerful principle when formulated in terms of space-time geometry which is assumed to be generalization of Riemannian geometry from that for the Minkowski space of special relativity. Now line element is expressed as $ds^2 = g_{ij}dx^idx^j$ and it can be reduced to Minkowskian form only in vacuum regions far enough from massive bodies. Another new element is curvature of space-time which can be concretized in terms of spherical geometry. For triangles at the surface of sphere having as sides pieces of big circles (geodesic lines, which now represent the analog of free rectilinear motion) the sum of angles is larger than 180 degrees. For geodesic triangles at the surface of saddle like surface the sum is smaller than 180 degrees. This holds for arbitrarily small geodesic triangles and is therefore a local property of Riemann geometry.

Quite often one encounters the belief that GCI is generalization of Relativity Principle. This is not the case. Relativity Principle states that the isometries of Minkowski space consisting of Poincare transformations leave the physics invariant. General Coordinate transformations are not in general isometries of space-time and in the case of general space-time there are not isometries. Therefore GCI is only a constraint on the form of field equations: they just remain invariant under general coordinate transformations. Tensor analysis is the mathematical tool making it possible to expresss this universality. Tensor analysis allows to express the space-time geometry algebraically in terms of metric tensor, curvature tensor, Ricci tensor and Einstein tensor, and Ricci scalar associated with it. In particular, the notion of angle defect can be expressed in terms of curvature tensor.

In the case of Equivalence Principle (EP) the starting point is the famous thought experiment involving lift. In stationary elevator material objects fall down with accelerated velocity. One can however study the situation in freely falling life and in this case the material objects remain stationary as if there were not gravitational force. The idea is therefore that gravitational force is not a genuine force but only apparent coordinate forces which vanishes locally in suitable coordinates known as geodesic coordinates for which coordinate lines are geodesic lines. Gravitational force would be analogous to apparent forces like centripetal forces and Coriolis force appearing in rotating coordinate systems already in Newton's mechanics. The characteristic signature is that the associated acceleration does not depend on the mass of the particle. This leads to the postulate that the motion of particles occurs along geodesic lines in absence of other than gravitational interactions. Equivalence Principle is already present in Newton's theory of gravitation and states that inertial masses appearing in $F = ma$ can be chosen to be same as the gravitational mass appearing in the expression of gravitational forces $F_{gr} = GmM/r^2$ between bodies with gravitational masses m and M. Equivalence Principle looks rather innocent and almost trivial but its formulation in competing theories is surprisingly difficult and the situation is not made easier by the fact that the mathematics involved is highly non-linear.

Tensor analysis allows the tools to deduce the implications of EP. The starting point is the equality of inertial and gravitational masses but made a local statement for the corresponding mass densities or more generally corresponding tensors. For inertial mass energy momentum tensor characterizing the density and currents of four-momentum components is the notion needed. For gravitational energy the only tensor quantities to be considered are Einstein tensor and metric tensor because they satisfy the conservation of energy and momentum locally in the sense that their covariant divergence is vanishing. Also energy momentum tensor should be conserved and thus have vanishing divergence. The manner to achieve this is to assume that the two tensor are proportional to each other. This identification actually realizes EP and gives Einstein's equations. Cosmological term proportional to the metric tensor can be present and Einstein consider also this possibility since otherwise cosmology was predicted to be expanding and this did not fit with the prevailing wisdom. The cosmological expansion was observed and Einstein regarded his proposal as the worst blunder of this professional life. Ironically, the recently observed acceleration of cosmic expansion might be understood if cosmological term is present after all albeit with sign different than in Einstein's proposal. Einstein's equations state that matter serves as a source of gravitational fields and gravitational fields tell for matter how to move in presence of gravitational interaction. These equations have been amazingly successful.

There is however a problem relating to the difference between GCI and Principle of Relativity already mentioned. Noether's theorem states that symmetries and conservation laws correspond to each other. In quantum theory this theorem has become the guiding principle and construction of new theories is to high degree postulation of various kinds of symmetries and deducing the consequences. In generic curved space-time the presence of massive bodies makes space-time curved and Poincare symmetries of empty Minkowski space are lost. This does not imply not only non-conservation of otherwise conserved quantities. These quantities do not even exist mathematically. This is a very serious conceptual drawback and the only manner to circumvent the problem is to make an appeal to the extreme weakness of gravitational interaction and say that gravitational four-momentum can be assigned to a system in regions very far from it because gravitational field is very weak.

This difficulty might explain why the quantization of gravitation by starting from Einstein's equations has been so difficult. It must be however noticed that the perturbative quantization of super-symmetric variant of Einstein's equation works amazingly well in flat Minkowski background and it has been even conjectured that divergences which plague practically every quantum field theory might be absent. Here the twistor Grassmann approach has allowed to overcome the formidable technical difficulties due to the extreme non-linearity the action principle involved. Still the question remains: could it be possible to modify general relativity in such a manner that the symmetries of special relativity would not be lost?

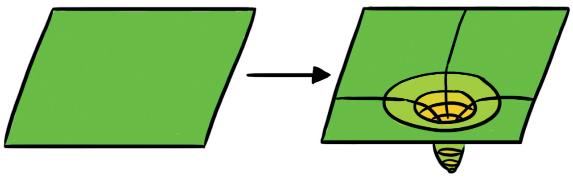

Figure 2.1: Matter makes space-time curved and leads to the loss of Poincare invariance so that momentum and energy are not well-defined notions in GRT.

2.2.3 Quantum physics

Quantum physics forces to change both the ontology and epistemology of classical physics dramatically.

Quantum theory

In the following I just list the basic aspects of quantum theory which distinguish it from classical physics.

1. Point like particle is replaced in quantum physics by wave function. This is rather radical abstraction in ontology. For mathematician this looks almost trivial transition from space to function space: the 3-D configuration for particle is replaced by the space of complex valued

functions in this space - Schrödinger amplitudes. From the point of view of physical interpretation this is big step since wave function means abstraction which cannot be visualized in terms of sensory experience. This transition is repeated in second quantization whether the function space is replaced with functional space consisting of functions defined classical fields. Also the proper interpretation of Schrödinger amplitudes is found to be in terms of classical fields. The new exotic elements are spinor fields, which are anti-commuting already at the classical level. They are introduced to describe fermions: this element is however not absolutely necessary.

The interpretation is as probability amplitudes - square roots of probability densities familiar from probability theory applied in kinetic theory.

2. Schrödinger amplitude is mathematically analogous to a classical field, say classical electromagnetic fields fields appearing in Maxwell's theory. Interference for probability amplitudes leads to completely analogous effects such as interference and diffraction. The classical experiment demonstrating diffraction is double slit experiment in which electron bream travels along double slit system and is made visible at screen behind it. What one observes a distribution reflecting interference pattern for Schrödinger waves from the two slits just as for classical electromagnetic fields. The modulus square for probability amplitude inhibits the interference pattern. As the other slit is closed, interference pattern disappears. One cannot explain the interference pattern using ordinary probability theory: in this case electrons of the beam would not "know" which slits are open and destructive interference would be impossible. In quantum world they "know" and behave accordingly. Physics is not anymore completely local.

3. The model of electrons in atoms relies on Schrödinger amplitude and this might suggests that Schrödinger amplitude is classical field. This is however not the case. To understand what is involved one must introduce the notion of state function reduction and Uncertainty Principle.

   It was learned basically by doing experiments that quantum measurements differ from classical ones. First of all, even ideal quantum measurement typically changes the system, which does not happen in ideal classical measurement. The outcome of the measurement is non-deterministic and there are several outcomes, whose number is typically finite. One can predict only the probability of particular outcome and it is dictated by the state of the system and the measured observables.

   Uncertainty Principle is a further new element and dramatic restriction to ontology. For instance, one cannot measure momentum and position of the particle simultaneously in arbitrary accuracy. Ideal momentum measurement delocalizes the particle completely and vice versa. This is very difficult to understand in the framework of classical mechanics were particle is point of space. If one accepts the mathematician's view that particle states are elements of function space, Uncertainty Principle can be understood and is present already in Fourier analysis. One also can get rid of ontological un-easiness created by statements like "electron can exist simultaneously in many places". Also the construction of more complex systems using simpler ones as building bricks (second quantization) is easy to understand in this framework: in classical particle picture second quantization looks rather mysterious procedure. It is however not at all easy for even mathematical physicist to think that function space could be something completely real rather than only a figment of mathematical imagination.

4. What remains something irreducibly quantal is the occurrence of the non-deterministic state function reduction. This seems to be the core of quantum physics. The rest might reduce to deterministic physics in some function space characterizing physical states.

   The real problem is that the non-determinism of state function is not consistent with the determinism of Schrödinger equation. It seems that the laws of physics cease to hold temporarily and this has motivated the statements about craziness of quantum theory. More plausible view is that something in our view about time - or more precisely, about the relation between the geometric time of physicist and experienced time is wrong. These times are identified but we know that they are different: geometric time as no intrinsic arrow whereas subjective time has and future does not exist for subjective time but for geometric time it exists.

There have been several attempts to reduce also state function reduction to deterministic classical physics or change the ontology so that it does not exist, but these attempts have not been successful. Ironically the core of quantum physics has remained also the taboo of quantum physics. The formulation is as "shut and calculate" paradigm which has dominated academic theoretical physics for century. One can only imagine where we could be without this professional taboo.

5. Quantum entanglement is a phenomenon without any classical counterpart. Schrödinger cat has become the standard manner to illustrate what is involved. One considers cat and bottle of poison which can be either open or closed. Classically one has two states: cat alive-bottle closed and cat dead-bottle open. Quantum mechanically also the superposition of these two states is possible and this obviously does not make sense in classical ontology. We cannot however observe quantum entanglement. When we want to know whether cat is dead or alive we induce state function reduction selecting either of these two states and the situation become completely classical. This suggests epistemological restriction: the character of conscious experience is that it produces always classical world as an outcome. One should of course not take this as dogma. The so called interaction free measurement allows to get information about system without destroying entanglement.

Standard model

Standard model summarizes our recent official understanding about physics. The attribute "official" is important here: there exists a lot of claims for anomalies, which are simply denied by the mainstream as impossible. Reductionists believe standard model to summarize even physics accessible to us. Standard model has been extremely successful in elementary particle physics. Even Higgs particle was found at LCH with predicted properties.

There are however issues related to the Higgs mechanism. Higgs particle has mass that it should not have and SUSY particles are too heavy to help in the problem. Stabilization of Higgs mass by cancelling radiative corrections to Higgs mass from heavy particles was one of the basic motivations for postulating SUSY in TeV energy scaled studied at LHC. Therefore one has what is called fine tuning problem for the parameters characterizing the interactions of Higgs and theory loses its predictivity.

Even worse, RHIC and LCH provide data telling that perturbative QCD does not seem to work at high energies where it should work. What was though to be quark gluon plasma - something behaving in very simple manner - was something different and one cannot exclude that there is some new physics there.

Neutrinos are the black sheep of the standard model. Each of the three leptons is accompanied by neutrino and in the most standard standard model they are massless. This has turned out to be not the case. Neutrinos also mix with each other as do also quarks. This phenomenon relates closely to the massivation. There are also indications that neutrinos could have several states with different mass values. The experimental neutrino physics is however extremely difficult since neutrinos are so weakly interaction so that the experimental progress is slow and plagued by uncertainties.

Therefore there are excellent reasons to be skeptical about standard model: one should continue to ask questions about the basics of the standard model. The attempt to answer this kind of fundamental questions concerning standard model could lead to re-awakening of particle physics from its recent stagnation. In particular, one could wonder what might be the the origin of standard model quantum numbers and what is the origin of quark and gluon color. Standard model gauge group has very special and apparently un-elegant structure - something not suggested by GUT ideology. Why this Could this reflect some deeper principles?

This kind of questions were possible at sixties, and they led to the amazingly fast evolution of standard model. This hippie era in theoretical physics continued to the beginning of eighties but then the super string revolution around 1984 changed suddenly everything. Comparison with the revolution leading to birth of Soviet Union might be very rewarding. For me hippie era meant the possibility to make my thesis at Helsinki Technological University receiving even little salary:

officially the goal was to make me a citizen able to take care of myself. Nowadays the idea about a person writing thesis about his own theory of everything is something totally unthinkable.

Grand Unified Theories

According to the great narrative the next step was huge: something like 13 orders of magnitude from the length scale of electroweak bosons (10^{-17} meters) to the length scale of extremely have gauge bosons of GUTs. At the time when I was preparing my thesis, GUTs were the highest fashion and every graduate student in particle physics had the opportunity to become the new Einstein and pick up his/her own gauge group and build up the GUT. All the needed formulas could be found easily and there was even a thick article containing all the recipes ranging from formulas for tensor products of group representations to beta functions for given group.

Both leptons and quarks form single family belonging to same multiplet of the big GUT gauge symmetry. The new gauge interactions predicted that and lepton and baryon number are not separately conserved so that proton is not stable. The theory allowed to predict its lifetime. The disappointing fact has been that no decays of proton have been however observed and this has led to a continual fine tuning of coupling parameters to keep proton alive for long time enough. This of course should put bells ringing since the stability of proton is extremely powerful guideline in theory building would suggest totally different track to follow based on question "Can one imagine any scenario in which B and L are separately conserved?"

The mass splittings between different fermions (quarks and leptons) believed to be related by gauge symmetries are huge: the mass ratio for top quark and neutrinos would be of the order 10^{12}, which is a huge number. Quite generally, the mass scales between symmetry related particles would be huge, which suggests that the notion of mass scale is part of physics. Also could serve as extremely powerful hint for a theory builder who is not afraid for becoming kicked out from the academic community.

GUT approach predicts a huge desert without any new physics ranging from electroweak scale to GUT length scale! So many orders of magnitude without any new physics looks like an incredible prediction when one recalls that 2 orders of magnitude separating electron and nuclei is the record hitherto. This assumption is of course just a scaled up variant of the child's assumption that the world ends at the backyard, and its basic virtue is that it makes theorist's life simple. There is nothing bad in this kind of assumption when taken as simplifying working hypothesis. The problem is that people have forgot that GUT hypothesis is only a pragmatic working hypothesis and believe that it represent an established piece of physics. Nothing could be farther from truth.

Super Symmetric Yang Mills theories

GUTs were followed by supersymmetric Yang-Mills theories - briefly SUSYs. The ambitious idea was to extend the unification program even further. Also fermions and bosons - particles with different statistics - would belong to same multiplet of some big symmetry group replaced with something even more general- super symmetry group. This required generalization of the very notion of symmetry by extending the notion of infinitesimal symmetry. One manner to achieve this is to replace space-time with a more general structure - superspace - possessing fermionic dimensions. This is however not necessarily and many mathematicians would regard this structure highly artificial. As a mathematical idea the generalization of symmetry is however extremely beautiful and shows how powerful just the need to identify bigger patterns is. One can indeed generalize of the various GUTs to supersymmetric gauge theories.

The number $\mathcal{N}$ of independent super-symmetries characterizes SUSY, and there are arguments suggesting that physically $\mathcal{N} = 1$ theories are the only possible ones. Certainly they are the simplest ones, and it is mostly these theories that particle phenomwnologists have studied. $\mathcal{N} = 4$ SUSYs possesses in certain sense maximal SUSY in four-dimensions. It is unrealistic as a physical model but because of its exceptional simplicity has led to a mathematical breakthrough in theoretical physics. The twistor Grassmannian approach has been applied to these theories and led to a totally new view about how to calculate in quantum field theory. The earlier approach based on Feynman diagrams suffered from combinatorial explosion so that only few lowest orders could be calculated numerically. The new approach strongly advocated by Nima Arkani Hamed and his coworkers allows to sum up huge numbers of Feynman diagrams and write the answer which took earlier ten pages with few lines. Also a lot of new mathematics developed by leading Russian mathematicians has been introduced.

$\mathcal{N} = 1$ SUSY, whose particles would have mass scale of order TeV, the energy scale studied at LHC, was motivated by several reasons. One reason was that in that ideal situation that all particles remain massless the contributions of ordinary and supersymmetric particles to many kinds of radiative corrections in particle reactions cancel each other. In the case of Higgs this would mean stability of the parameters characterizing the interactions of Higgs with other particles. In particular, Higgs vacuum expectation value determining the masses of leptons and quarks and gauge bosons would be stable. All this depends sensitively on precise values of particle masses and unfortunately it happens that the mechanism does not stabilize the parameters of Higgs.

Second motivation was that SUSY might provide solution to the dark matter mystery. The called lightest super-symmetric particle is predicted to be stable by so called R-parity symmetry which naturally accompanies SUSY but can be also broken. This particle is fermion and super partner of photon or weak boson Z^0 or mixture of these. This particle would provide an explanation for the mysterious dark matter about which we recently know only its existence. Dark matter would be a remnant from early cosmology - those lightest supersymmetric particles which failed annihilate with their antiparticles to bosons because cosmic expansion reduced their densities and made annihilation rate too small.

The results from LCH were however a catastrophic event in the life of SUSY phenomenologists. Not a slightest shred of evidence for SUSY has been found. There is still hope that some fine tuned SUSY scenarios might survive but if SUSY is there it cannot satisfy the basic hopes put on it. The results from LHC arriving during 2005 will be decisive for the fate of SUSY.

The results of LHC do not of course exclude the notion of supersymmetry. There are lots of variants of supersymmetry and $\mathcal{N} =$ SUSYs represents only one particular, especially simple variant in some respects and involving ad hoc assumptions such as straightforward generalization of Higgs mechanism as origin of particle massivation, which can be questioned already in standard model context. Furthermore, $\mathcal{N} = 1$ SUSY forces to give up separate conservation of lepton and baryon numbers for which there is no experimental evidence. For higher values of $\mathcal{N}$ this is not necessary.

Superstrings and M-theory

Super-strings mean a further extension for the notion of symmetry and thus reductionism at conceptual level. Conformal symmetries define infinite-dimensional symmetries and were first discovered in attempts to understand 2-dimensional critical systems. Critical system is a system in phase transition. There are two phases present that and the regions of given phase can have arbitrary large sizes. This means scale invariance and long range fluctuations: system does not behave as if it would consist of billiard balls having only contact interactions. The discovery was that the notion of scale invariance generalizes to local scale invariance. The transformations of plane (or sphere or any 2-D space) known as conformal transformations preserve the angle between two curves and introduce local scaling of distances. These transformations appear in complex analysis as holomorphic maps.

In string model which emerged first as hadronic string model, hadrons are identified as strings. Their orbits define 2-D surfaces and conformal transformations for these surfaces appear as symmetries of the theory. One could say that strings physics resembles that of 2-D critical systems. Hadronic string model did not evolve to a real theory of hadrons: for instance, the critical dimension in which worked was 26 for bosonic strings and 10 for their super counterparts. Therefore hadronic string model was largely given up as quantum chromodynamics trying to reduce hadronic physics to that of point-like quarks and gluons emerged. This approach worked nicely at high energies but at low energies the problem is that perturbative approach fails. The already mentioned unexpected behavior of what was expected to be quark gluon plasma challenges also QCD.

String model contained also graviton like states possessing spin 2 and the description for their interactions resemble that for the description of gravitons with matter according to the lowest order predictions of quantized general relativity. This eventually led to the idea that maybe supersymmetric variants of string might provide the long sough solution to the problem of quantizing gravitation. Perhaps even more: maybe they could allow to unify all known fundamental interactions with framework of single notion: super string.

In superstring approach the last step in the reductionistic sequence of conquests would be directly to the Planck length scale making about 16 orders of magnitudes. The first superstring revolution shook physics world around 1984. During the first years gurus believed that proton mass would be calculated within few years and first Nobels would be received within decade. Gradually the optimism began to fade as it turned out that superstring theory is not so unique as it was believed to be. Also the building or the bridge to the particle phenomenology was not at all so easy as was believed first.

Superstring exists in mathematically acceptable manner only in dimension $D = 10$ and this was of course a big problem. The notion of spontaneous compactification was needed and brought in an ugly ad hoc trick to the otherwise so beautiful vision. This mechanism would compactify 6 large dimensions of the 10-D Minkowski space so that they would become very small - the scale would be of the order of Planck length. For all practical purposes the 10-D space would look 4-dimensional. The 6 large dimensions would curl up to so called Calabi-Yau space and the finding of the correct Calabi-Yau was thought to be a simple procedure.

This was not the case. It turned out that there are very many Calabi-Yau manifolds [?] to begin with: the number 10^{500} was introduced to give some idea about how many of them are - the number could be quite well infinite. The simple Calabi-Yau spaces did not produce the standard model physics at low energies. This problem became known as landscape problem. Landscape inspired in cosmology to the notion of multiverse: universe would split to regions which can have practically any imaginable laws of physics. There is no empirical support for this vision but this has not bothered the gurus.

Gradually it became clear that landscape problem spoils the predictivity of the theory and eventually many leading gurus turned they coat. The original idea was that string models are so wonderful because they predict unique physics. Now they were so beautiful because they force us to give up completely the belief that physical theories can predict something. In this framework antropic principle remains the only guideline in attempts to relate theory to the real world. This means that we can deduce the properties of the particular physics we happen to live from our own existence and by scanning through this huge repertoire of possible physics.

Around 1995 so called second superstring revolution took place. Five very different looking super string models had emerged. The great vision advocated especially by Witten was that they are limiting cases of one theory christened as M-theory. The 10-D target space for superstrings was replaced with 11-dimensional one. Besides this higher dimensional objects - branes- of varying dimension entered the picture and made it even more complex. This gave of course and enormous flexibility. For instance, the 4-D observed space-time could be understood as brane rather than the effectively 4-D target space obtained by spontaneous compactification. This gave for particle phenomenologists wanting to reproduce standard model an endless number of alternatives and the theory degenerated to endless variety of attempts to reproduce standard model by suitable configurations of branes. Around 2005 the situation in M-theory began to become public and so called string wars began. At this moment the funding of super-strings has reduced dramatically and the talks in string conferences hardly mention superstrings.

One can conclude that the forty years of unification based on naive length scale reductionism was a failure. What was thought to become the brightest jewel in the crown of reductionistic vision was a complete failure. If history could teach something, it should teach us that we should perhaps follow Einstein and his co-temporaries and be asking questions about fundamentals. The shut-up and calculate approach forbidding all discussion about the basic assumptions has leads nowhere during these four decades.

As one looks this process in the light of after wisdom, one realizes that there are two kinds of reductionisms involved. The naive length scale reductionism has not been successful. Time might be ripe for its replacement with the notion of fractality which postulates that similar looking structures appear in all length scales. Fractality is also a central aspect of the renormalization group approach to quantum field theory.

A second kind of reductionistic sequence has been realized at conceptual level. The notion of symmetry has evolved from ordinary symmetry to supersymmetry to super-conformal symmetry and even created new mathematical notions. The size of the postulated symmetry groups has steadily increased: note that already Einstein initiated this trend by postulating general coordinate invariance as a symmetry analogous to gauge symmetry. In superstring type approaches one can ask whether one should put all particles to same symmetry multiplet in the ultimate theory.

Symmetry breaking is what remains poorly understood in gauge theories and GUTS. Conformal field theories however provide a very profound and deep mechanism involving now ad hoc elements as Higgs mechanism does. Maybe one should try to understand particle massivation in terms of breaking of superconformal symmetries rather than blindly following the reductionistic approach and trying to reproduce SUSY and GUT approaches and Higgs mechanism as intermediate steps in the imagined reductionistic ladder leading from standard model to the ultimate theory. Maybe we should try to understand symmetry breaking as reflecting the limitations of the observer. For instance, in thermodynamical systems we can observe only thermodynamical averages of the properties of particles, such as energy.

2.2.4 Summary of the problems in nutshell

New theory must solve the problems of the old theory. The old theory indeed has an impressive list of problems. The last 30 or 40 years have been an Odysseia in theoretical physics. When did this Odysseia begin?

Did the discovery of super strings initiate the misery for thirty years ago? Or can we blame SUSY approach? Was the SUSY perhaps too simple - or perhaps better to say, too simplistic? Did already the invention of GUTs lead to a side track: is it too simplistic to force quarks and leptons to multiplets of single symmetry group? This forcing of the right leg to the left hand shoe predicts proton decay, which has not been observed?

Or is there something badly wrong even with the cherished standard model: do particles really get their masses through Higgs mechanism: is the fact that Higgs is too light indication that something went wrong? Do we really understand quark and gluon color and neutrinos? What about family replication and standard model quantum numbers in general? What about dark matter and dark energy? The only thing we know is that they exist and naive identifications for dark matter have turned out to be wrong. There is also the energy problem of General Relativity. Did we go choose a wrong track already almost century ago?

And even at the level of the basic theory - quantum mechanics - taken usually as granted we have the same problem that we had almost century ago.

2.3 Could TGD provide a way out of the dead end?

The following gives a concise summary of the basic ontology and epistemology of TGD followed by a more detailed discussion of the basic ideas.

2.3.1 What new ontology and epistemology TGD brings in?

TGD based ontology and epistemology involves several elements, which might help to solve the listed problems.

1. The new view about space-time as 4-D surface in certain 8-D imbedding space leads to the notion of many-sheeted space-time and to geometrization and topological quantization of classical fields replacing the notion of superposition for fields with superposition for their effect.

2. Zero energy ontology means new view about quantum state. Quantum states as states with positive energy are replaced with zero energy states which are pairs of states with opposite quantum numbers and "live" at opposite boundaries of causal diamond (CD) which could be seen as spotlight of consciousness at the level of 8-D imbedding space.

3. Zero energy ontology leads to a new view about state function reduction identified as moment of consciousness. Consciousness is not anymore property of physical states but something between two physical states, in the moment of recreation. One ends up to ask difficult questions: how the experience flow of time experience in this picture, how the arrow of geometric time emerges from that of subjective time, is the arrow of geometric time same always, etc...

4. Hierarchy of Planck constants is also a new element in ontology and means extension of quantum theory. It is somewhat matter of taste whether one speaks about hierarchy of effective or real Planck constants and whether one introduces only coverings of space-time surface or also those of imbedding space to describe what is involved. What however seems clear that hierarchy of Planck constants follows from fundamental TGD naturally. The matter forms phases with different values of $h_{eff}(h = n$ and for large values of n this means macroscopic quantum coherence so that application to living matter is obvious challenge. The identification of these new phases as dark matter is the natural first working hypothesis.

5. p-Adic physics is a further new ontological and epistemological element. p-Adic numbers fields are completions of rational numbers in many respects analogous to reals and one can ask whether the notion of p-adic physics might make sense. The first success comes from elementary particle mass calculations based on p-adic thermodynamics combined with very general symmetry arguments. It turned out that the most natural interpretation of p-adic physics is as physics describing correlates of cognition and intentionality. This brings to the vocabulary p-adic space-time sheets, p-adic counterparts of field equations, p-adic quantum theory, etc.. The need to fuse real and various p-adic physics to gain by number-theoretical universality becomes a powerful constraint on the theory.

   The notion of negentropic entanglement is natural outcome of p-adic physics. This entanglement is very special: all entanglement probabilities are identical and unitary entanglement matrix gives rise to this kind of entanglement automatically. The U-matrix characterizing interactions indeed consists of unitary building blocks giving rise to negentropic entanglement. Negentropic entangelment tends to be respected by Negentropy Maximization Principle (NMP) which defines the basic variational principle of TGD inspired theory of consciousness and negentropic entanglement defines kind of Akaschic records which are approximate quantum invariants. They form kind of universal potentially conscious data basis, universal library. This obviously represents new epistemology.

2.3.2 Space-time as 4-surface

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity, which has been already mentioned. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its rotational, translational and Lorentz symmetries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant. Noether's theorem states that symmetries and conservation laws correspond to each other so that conservations laws are lost: energy, momentum, and angular momentum are not only non-conserved but even ill-defined. The mathematical expression for this is that the energy momentum tensor is 2-tensor so that it is impossible to assign with it any conserved energy and momentum mathematically except in empty Minkowski space. Usually it is argued that this is not a practical problem since gravitation is so weak interaction. When one however tries to quantized general relativity, this kind of sloppiness cannot be allowed, and the problem reason for the continual failure of the attempts to build a theory of quantum gravity might be tracked down to this kind of conceptual sloppiness.

The way out of the problem is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [L19] , is the unique choice. Symmetries as isometries of space-time are lifted to those of imbedding space. Symmetry transformation does not move point of space-time along it but moves entire space-time surface. Space-time surface is like rigid body rotated, translated, and Lorentz boosted by symmetries. This means that Noether's theorem predicts the classical conserved charges once general coordinate action principle is written down.

Also now the curvature of space-time codes for gravitation. Now however the number of solutions to field equations is dramatically smaller than in Einstein's theory. An unexpected bonus was that a geometrization classical fields of standard model for $S = CP_2$. Later it turned out that also the counterparts for field quanta emerge naturally but this requires profound generalization of the notion of space-time so that topological inhomogenities of space-time surface are identified as particles. This meant a further huge reduction in dynamical field like variables. By general coordinate invariance only four imbedding space coordinates appear as variables analogous to classical fields: in a typical gut their number is hundreds.

CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry : the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers which correspond to the isometries of CP_2 and are unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma. The conservation of baryon and lepton number follows as a prediction. Leptons and quarks correspond to opposite chiralities for fermions at the level of imbedding space.

What remains to be explained is family replication phenomenon for leptons and quarks which means that both quarks and leptons appear as three families which are identical except that they have different masses. Here the identification of particles as 2-D boundary components of 3-D surface inspired the conjecture that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surfae: sphere, torus, ... The identification as boundary component turned out to be too simplistic but can be replaced with partonic 2-surface assignable to light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. This 3-D surfaces replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 3-surface.

The problem was that only three lowest genera are observed experimentally. Are the genera $g > 2$ very heavy or don't they exist. One ends up with a possible explanation in terms of conformal symmetries: the genera $g \leq 2$ allow always two element group as subgroup of conformal symmetries (this is called hyper-ellipticity) whereas higher genera in general do not. Observed 3 particle families would have especially high conformal symmetries. This could explain why higher genera are very massive or not realized as elementary particles in the manner one would expect.

The surprising outcome is that $M^4 \times CP_2$ codes for the standard model. Much later further arguments in favor of this choice have emerged. The latest one relates to twistorialization. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a big problem in attempts to introduce twistors to General Relativity Theory (GRT) and very serious obstacle in quantization based on twistor Grassmann approach which has demonstrate its enormous power in the quantization of gauge theories. The obvious idea in TGD framework is whether one could lift also the twistor structure to the level of imbedding space $M^4 \times CP_2$. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space which is also Kähler manifold!

It soon became clear that TGD can be seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales. Also strictly 2-D string world sheets pop up in the formulation of quantum TGD so that one can say that string model in 4-D space-time is part of TGD.

One can say that TGD generalizes standard model symmetries and provides a proposal for a dynamics which is incredibly simple as compared to the competing theories: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of afterwisdom. One loses linear superposition of fields which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem relies on the notion many-sheeted space-time to be discussed below.

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the space-time in this framework is much more richer than in general relativity.

1. Space-time decomposes into space-time sheets with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. We can directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   What does one mean with space-time sheet? Originally it was identified as a piece of slightly deformed M^4 in $M^4 \times CP_2$ having boundary. It however became gradually clear that boundaries are probably not allowed since boundary conditions cannot be satisfied. Rather, it seems that sheet in this sense must be glued along its boundaries together with its deformed copy to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

   So: what happens to the identification of family replication in terms of genus of boundary of 3-surface and to the interpretation of the boundaries of physical objects as space-time boundaries? Do they correspond to the surfaces at which the gluing occurs? Or do they correspond to 3-D light-like surfaces at which the signature of the induced metric changes. My educated guess is that the latter option is correct but one must keep mind open since TGD is not an experimentally tested theory.

2. Elementary particles are roughly speaking identified as topological inhomogenities glued to these space-time sheets using topological sum contacts. This means roughly drilling a hole to both sheets and connecting with a cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy-momentum currents reduce to space-time curvature in general relativity.

   This view has gradually evolved to much more detailed picture. Without going to details one can say that particles have wormhole contacts as basic building bricks. Wormhole contact is very small Euclidian connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon is not affected and nothing new is predicted. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with superposition of their effects. Particle can topologically condensed simultaneously to several space-time sheets by generating topological sum contacts. Particle experiences the superposition of the *effects* of the classical fields at various space-time sheets rather than the superposition of the fields. It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four fields since only four primary field like variables are present. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly

to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales and there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced field means that one induces electroweak gauge potentials defining so called spinor connection to space-time surface. Induction means locally a projection for the imbedding space vectors representing the spinor connection locally. This is essentially dynamics of shadows! The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns.

   Induced fields are not however equivalent with ordinary free fields. In particular, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology. Quite generally the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of general coordinate invariance (GCI).

   One might of course argue that this kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Particles experience superposition of effects from the archetypal field configurations. Basic field patterns are simple but effects are complex!

   The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K61] . One can speak about field body or magnetic body of the system.

3. Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C3] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K52] .

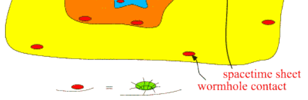

Figure 2.2: Many-sheeted space-time.

2.3.3 The hierarchy of Planck constants

The motivations for the hierarchy of Planck constants come from both astrophysics and biology [K70, K24] . In astrophysics the observation of Nottale [E27] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K79, K62] .

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the imbedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite.

A convenient technical manner to treat the situation is to replace imbedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $\hbar = n\hbar_0$. This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/\hbar$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [D2] is almost obvious. At the more detailed level one finds that the spectrum of Planck constants would be given by $\hbar = n_a n_b \hbar_0$ [K27] .

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoome up version of quantum world at elementary particle length scales.

1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.

2. Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J3] . Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.

3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.

4. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and $\hbar$ changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [K30] . The braiding of the magnetic

flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K26] .

5. p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like $\hbar$. This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

   Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C11] . What is required is that classical Z^0 and W fields responsible for parity breaking effects are present in cellular length scale. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

   For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I5] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K70] .

2.3.4 p-Adic physics and number theoretic universality

p-Adic physics [K54, K88] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition and intentionality. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

p-Adic number fields

p-Adic number fields Q_p [A50] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x,y) = d(x-y)$ depends on the lowest pinary digit of $x-y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x-y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultrametricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x-y) \leq Min\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics -not for energy but infinitesimal scaling generator of so called super-conformal algebra [A30] acting as symmetries of quantum TGD [K73] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K54] . The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1+i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5 μm contains as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ assignable to Gaussian Mersennes $M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$, [K70] . This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition and intention [K58] . Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes. One can also consider p-adic space-time sheets as correlates of intentions. The quantum jump in which p-adic space-time sheet is replaced with a real one could serve as a quantum correlate of intentional action. This process is forbidden by conservation laws in standard ontology: one cannot even compare real and p-adic variants of the conserved quantities like energy in the general case. In zero energy ontology the net values of conserved quantities for zero energy states vanish so that conservation laws allow these transitions.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

1. The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.

2. The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $log(p_n)$ with $-log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely.

Negentropy Maximization Principle (NMP) [K51] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I3] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [K30] .

2.3.5 Zero energy ontology

Zero energy state as counterpart of physical event

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

Zero energy ontology conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. Free will is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for the spot-light of consciousness: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets restricted to the interior of CD. Whether the sheets can continue outside CD is still unclear.

2. p-Adic length scale hypothesis [K55] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves $T = 2^n T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K7] . This means a direct coupling between microscopic and macroscopic scales.

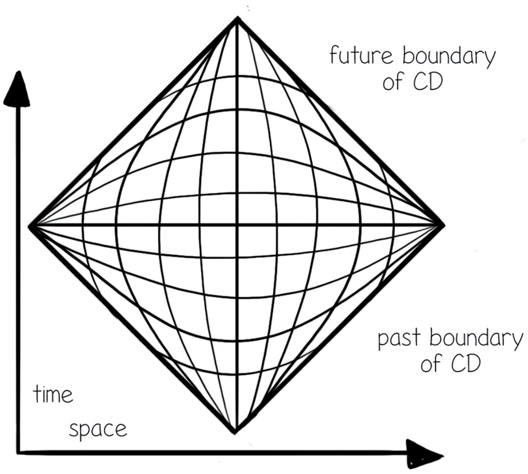

Figure 2.3: The 2-D variant of CD is equivalent with Penrose diagram in empty Minkowski space although interpretation is different.

Quantum theory as square root of thermodynamics

Quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump.

The fact that M-matrices are products of Hermitian square roots (operator analog for real variable) of Hermitian density matrix multiplied by a unitary S-matrix S with they commutte implies that possible U-matrices for an algebra generalizing Kac-Moody algebra defining Kac-Moody type symmetries of the the S-matrix. This might mean final step in the reduction of theories to their symmetries since the states predicted by the theory would generate its symmetries!

State function reduction,arrow of time in ZEO, and Akaschic records

From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K51] as the fundamental variational principle.

CD as two light-like boundaries corresponding to the positive and negative energy parts of zero energy states which correspond to initial and final states of physical event. State function reduction can occur to both of these boundaries.

1. If state function reductions occur alternately- one at time- then it is very difficult to understand why we experience same arrow of time continually: why not continual flip-flop at the level of perceptions. Some people claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. Why we experience the arrow of time as constant?

2. One possible way to solve this problem - perhaps the simplest one - is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately! I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs could change in these reductions. Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions don't affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time?

3. This option would allow the size scale of CD associated with human consciousness be rather short, say .1 seconds. It would also allow to understand why we do not observe continual change of arrow of time. Maybe living systems are working hardly to keep the personal arrow of time changed and that it would be extremely difficult to live against the collective arrow of time.

NMP implies that negentropic entanglement generated in state function reductions tends to increase. This tendency is mirror image of entropy growth for ensembles and would provide a natural explanation for evolution as something real rather than just thermodynamical fluctuation as standard thermodynamics suggests. Quantum Universe is building kind of Akashic records. The history would be recorded in a huge library and these books could might be read by interaction free quantum measurements giving conscious information about negentropically entangled states and without changing them: as a matter fact, this is an idealization. Conscious information would require also now state function reduction but it would occur for another system. Elitzur-Vaidman bomb tester(`http://en.wikipedia.org/wiki/ElitzurVaidman_bomb-testing_problem`) is a down-to-earth representation for what is involved.

2.4 Different visions about TGD as mathematical theory

There are two basic vision about Quantum TGD: physics as infinite-dimensional geometry and physics as generalized number theory.

2.4.1 Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the WCW CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A94, A120, A122]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading

from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "WCW". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices which form orthonormal rows of what I call U-matrix. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

   M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. This modified gamma matrices define as anticommutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. The conjecture is that Dirac determinant for the modified Dirac action gives the exponent of Kähler action for a preferred extremal as vacuum functional so that one might talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and antifermion.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids wold replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the

signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory. Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulombic contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

3. A further quite recent hypothesis inspired by effective 2-dimensionality is that Chern-Simons terms reduce to a sum of two 2-dimensional terms. An imaginary term proportional to the total area of Minkowskian string world sheets and a real term proportional to the total area of partonic 2-surfaces or equivalently strings world sheets in Euclidian space-time regions. Also the equality of the total areas of strings world sheets and partonic 2-surfaces is highly suggestive and would realize a duality between these two kinds of objects. String world sheets indeed emerge naturally for the proposed ansatz defining preferred extremals. Therefore Kähler action would have very stringy character apart from effects due to the failure of the strict determinism meaning that radiative corrections break the effective 2-dimensionality.

The definition of spinor structure - in practice definition of so called gamma matrices of WCW- and WCW Kähler metric define by their anti-commutators has been also a very slow process. The progress in the physical understanding of the theory and the wisdom that has emerged about preferred extremals of Kähler action and about general solution of the field equations for modified Dirac operator during last decade have led to a considerable progress in this respect quite recently.

1. Preferred extremals of Kähler action [K9] seem to have slicing to string world sheets and partonic 2-surfaces such that points of partonic 2-surface slice parametrize different world sheets. I have christened this slicing as Hamilton-Jacobi structure. This slicing brings strongly in mind string models.

2. The modes of the modified Dirac action - fixed uniquely by Kähler action by the requirement of super-conformal symmetry and internal consistency - must be localized to 2-dimensional string world sheets with one exception: the modes of right handed neutrino which do not mix with left handed neutrino, which are delocalized into entire space-time sheet. The localization follows from the condition that modes have well-defined em charge in presence of classical W boson fields. This implies that string model in 4-D space-time becomes part of TGD.

This input leads to a modification of the earlier construction allowing to overcome its features vulnerable to critics. The earlier proposal forced strong form of holography in sense which looked too strong. The data about WCW geometry was localized at partonic 2-surfaces rather than 3-surfaces. The new formulations uses data also from interior of 3-surfaces and this is due to replacement of point-like particle with string: point of partonic 2-surface -wormhole throat- is replaced with a string connecting it to another wormhole throat. The earlier approach used only single mode of induced spinor field: right-handed neutrino. Now all modes of induced spinor field are used and one obtains very concrete connection between elementary particle quantum numbers and WCW geometry.

2.4.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the WCW of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics grew steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) central for the applications to living matter.

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution.

The proposal for a concrete realization of this program at space-time level is in terms of the notion of p-adic manifold [K115] generalising the notion of real manifold. Chart maps of p-adic manifold are however not p-adic but real and mediated by a variant of canonical correspondence between real and p-adic numbers. This modification of the notion of chart map allows to circumvent the grave difficulties caused by p-adic topology. Also p-adic manifolds can serve as charts for real manifolds and now the interpretation is as cognitive representation. The coordinate maps are characterized by finite measurement/cognitive resolution and are not completely unique. The basic principle reducing part of the non-uniqueness is the condition that preferred extremals are mapped to preferred extremals: actually this requires finite measurement resolution (see fig. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg` or fig. 15 in the appendix of this book).

The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K88] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^8$ [K113]. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

1. The first meaning for associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces are quaternionic in some sense and thus associative. This can be formulated in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$ and states that induced gamma matrices generate quaternionic sub-algebra at each space-time point. It seems that induced rather than modified gamma matrices must be in question.

2. Second meaning for associative (co-associativity) would be following. In the case of complex numbers the vanishing of the real part of real-analytic function defines a 1-D curve. In octnionic case one can decompose octonion to sum of quaternion and quaternion multiplied by an octonionic imaginary unit. Quaternionicity could mean that space-time surfaces correspond to the vanishing of the imaginary part of the octonion real-analytic function. Co-quaternionicity would be defined in an obvious manner. Octonionic real analytic functions form a function field closed also with respect to the composition of functions. Space-time surfaces would form the analog of function field with the composition of functions with all operations realized as algebraic operations for space-time surfaces. Co-associativity could be perhaps seen as an additional feature making the algebra in question also co-algebra.

3. The third conjecture is that these conjectures are equivalent.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-odic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably an extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2.5 Guiding Principles

2.5.1 Physics is unique from the mathematical existence of WCW

1. The conjecture inspired by the geometry of loop spaces [A71] is that H is fixed from the mere requirement that the infinite-dimensional Kähler geometry exists. WCW must reduce to a union of symmetric spaces having infinite-dimensional isometry groups and labeled by zero modes having interpretation as classical dynamical variables.

   This requires infinite-dimensional symmetry groups. At space-time level super-conformal symmetries are possible only if the basic dynamical objects can be identified as light-like or space-like 3-surfaces. At imbedding space level there are extended super-conformal symmetries assignable to the light-cone of H if the Minkowski space factor is four-dimensional.

2. The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface. This means geometrization of Fermi statistics usually regarded as one of the purely quantal features of quantum theory.

2.5.2 Number theoretical Universality

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K85].

Fusion of real and padic physics to single coherent whole

Fusion of real and p-adic physics to single coherent whole is the first part in the program aiming to realize number theoretical universality.

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger structure (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also).

2. At the level of imbedding space this gluing corresponds to a gluing of real and p-adic variants of the imbedding space together along rational and common algebraic points (the number of which depends on algebraic extension of p-adic numbers used) to what could be seen as a book like structure. General Coordinate Invariance restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices. Whether this can be seen as an objection against the approach is not clear.

3. The analogous gluing of real and various p-adic physics to a larger structure forces to ask what are the common points of WCWs associated with real and various p-adic worlds. What it is to be a partonic 2-surface belonging to the intersection of real and p-adic variants of WCW? The natural answer is that partonic 2-surfaces which have a mathematical representation making sense both for real numbers and p-adic numbers or their algebraic extensions can be regarded as "common points" or identifiable points of p-adicity and reality. This of course applies also to partonic 2-surfaces corresponding to two different p-adic number fields. This mathematical property means a representability in terms of ratios of polynomials with rational (or possibly even algebraic) coefficients in the preferred imbedding space coordinates.

4. The intersections of WCWs and partonic 2-surfaces in different number fields are involved. An attractive idea is that only the information about common points of surfaces belonging to different number fields code for physics so that number-theoretically universal part of physics

is number theoretical physics relying only on rationals and their algebraic extensions. For instance, the transition amplitudes between p-adic and real variants of partonic 2-surface can involve only the data at these points. This suggests the existence of what might be called number theoretical QFT. At space-time level this extension of introduce a discretization at space-time level in terms of rational and algebraic points common to real space-time sheets and their p-adic variants. The number of these points is in general finite for a given CD and the proposed interpretation is in terms of cognitive representations. The discrete intersections would define the initial and final points of number theoretical braids central for the formulation of the theory in finite measurement resolution.

5. Much later came the realization that living matter or what makes living matter living could be interpreted as something in this intersection of real and p-adic worlds so that number theoretic QFT might apply to crucial aspects of living matter.

Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A43]) are involved [K88] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K88]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of modified Dirac action the identification of space-time surface as an associative (co-associative) submanifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a associative (co-associative) sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K28, K98].

2.5.3 Symmetries

Magic properties of light cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM^4_+, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are parameterized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

   Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of the WCW.

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations would correspond to zero modes having zero norm in the Kähler metric of WCW. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphisms of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of the WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labelling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2

Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

Attempts to identify WCW Hamiltonians

The construction of WCW geometry reduces to that for complexified WCW gamma matrices expressible in terms of fermionic oscillator operators for second quantized induced spinor fields. The contractions of the gamma matrices with isometry generators are the natural operators. The hypothesis is that the isometry generators at the level of WCW correspond to the symplectic algebra at the boundary of CD that is at $\delta M^4 \pm \times CP_2$ defining WCW Hamiltonians acting as isometries. Therefore WCW gamma matrices have interpretation as super charges of infinite-D conformal supersymmetry. The matrix elements of Kähler metric in the basis defined by isometry generators are obtained as anti-commutators of the gamma matrices and reduce to Poisson brackets of corresponding Hamiltonians. A direct connection with physics results since Hamiltonians correspond to irreducible representations of the rotation group $SO(3)$ and color group $SU(3)$.

I have made several attempts to identify explicit representations of WCW Hamiltonians acting as isometries [?] The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage: this approach is discussed in this chapter. Magnetic option is the simplest one but plagued by what looks too strong form of effective 2-dimensionality (inspired by strong form of holography) and lacking connection with the dynamics of Kähler action and modified Dirac action.

The most recent approach identifies super Hamiltonians as Noether super charges and is motivated by the QFT analogy. This proposal discussed in detail in [K116] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the modified Dirac action: in particular, about their localization to string worlds sheets (right handed neutrino is an exception).

The basic formulas however generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Covariantly constant right handed neutrino spinor is replaced with any mode of the modified Dirac operator localized at string world sheet in the case of Kac-Moodysubalgebra of super-symplectic algebra. In the case of right- handed neutrino one obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

2.5.4 Quantum classical correspondence

Quantum classical correspondence (QCC) has been the basic guiding principle in the construction of TGD. Below are some basic examples about its application.

1. QCC led to the idea that Kähler function for poitn X^3 of WCW must have interpretation as classical action for a preferred extremal $X^4(X^3)$ assignable to Kähler action assumed to be unique: this assumption can of course be criticized because the dynamics is not strictly deterministic. This criticism led to ZEO. The interpretation of preferred extremal is as analog of Bohr orbit so that Bohr orbitology usually believed to be an outcome of stationary phase approximations would be an exact part of quantum TGD.

2. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

   The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

One aspect of quantum criticality is the condition that the eigenvalues of quantal Noether charges in Cartan algebra associated with the Kähler Dirac action have correspond to the Noether charges for Kähler action in the sense that for given eigenvalue the space-time surfaces have same Kähler Noether charge.

3. A stronger form of QCC requires that classical correlation functions for general coordinate invariance observables as functions of two points of imbedding space are equal to the quantal ones - at least in the length cale resolution considered. This would give a very powerful - maybe too powerful - constraint on the zero energy states.

4. Measurement interaction term can be added also to the modified Dirac action as a term associated with 3-D space-like 3-surfaces at the boundaries of CDs and at light-like 3-surfaces at which the induced metric changes its signature from Minkowskian to Euclidian. This term implies that Chern-Simons Dirac operator is effectively equivalent with the Dirac operator of M^4 at the lines defining boundaries of string world sheets so that a connection with ordinary Dirac equation would result. This would simplify enormously perturbation theory since fermion propagator would be simply the ordinary massless Dirac propagator for massless particle as indeed assumed in the twistor Grassmann approach [K78].

2.5.5 Quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question about how to make TGD unique if Kähler function $K(X^3)$ in WCW is defined by the Kähler action for a preferred extremal $X^4(X^3)$ assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obvious idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength α_K is analogous to critical temperature so that TGD would be more or less uniquely defined. One cannot exclude the possibility that α_K has several values, and the doomsday scenario is that there is infinite number of critical values converging towards $\alpha_K = 0$, which corresponds to vanishing temperature).

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface in such a manner that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface X^3 and perhaps also at light-like 3-surfaces representing partonic orbits. The correspondence between X^3 and $X^4(X^3)$ would not be unique. Actually this is suggested by that the non-deterministic dynamics characteristic for critical systems. Also the failure of the strict classical determinism implying spin glass type vacuum degeneracy forces to consider this possibility. This criticality would correspond to criticality of Kähler action at X^3 but not that of Kähler function. Note that the original working hypothesis was that $X^4(X^3)$ is unique.

2. The variation could act on zero modes which do not affect Kähler metric, which corresponds to (1,1) part of Hessian in complex coordinates for WCW. Only the zero modes characterizing 3-surface appearing as parameters in the metric of WCW would be affected, and the result would be a generalization of modification of conformal scaling factor. Kähler function would change but only due to the change in zero modes. These transformations do not correspond to critical transformations since Kähler function changes.

3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that

(1,1) part of Hessian is degenerate. This would mean that in the vicinity of X^3 the Hessian has non-definite signature: same could be true also for the (1,1) part. Physically this is unacceptable since the inner product in Hilbert space should be positive definite.

Consider now critical deformations (the first option). Critical deformations are expected to relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. Critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical deformations then critical they would allow to extend WCW to a bundle for which 3-surfaces X^3 would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

2. Critical deformations could correspond to H or sub-group of H (which depends on X^3). For other 3-surfaces than X^3 the action of H is non-trivial: to see this consider the simple finite-dimensional case $CP_2 = SU(3)/U(2)$. The groups $H(X^3)$ are symplectic conjugates of each other for given values of zero modes which are symplectic invariants.

3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies $= km$, $k \in Z$, of given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1.

   For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical [K18, K28]. Basic example of criticality is the bifurcation diagram for cusp catastrophe [A4]. Quantum criticality realized as the vanishing of the second variation gives hopes about identification of preferred extremals. One must however give up hopes about uniqueness. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$. I have considered also alternative identifications such as absolute minimization of Kähler action. which is just the opposite of criticality (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

One must also remember that space-time surface decomposes to regions with Euclidian and Minkowskian signature of the induced metric and it is not quite clear whether the conformal symmetries giving rise to quantum criticality appear in both regions.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to the deformations of four-surface and second for the Kähler function itself with respect to the deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals (G/H option) would make 4-D criticality a property of all physical systems. Conformal symmetry breaking would however break criticality below some scale.

2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler

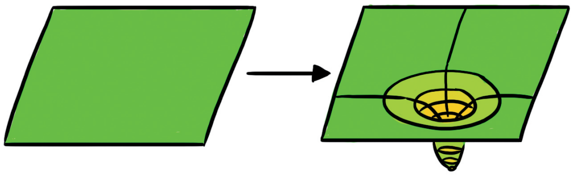

Figure 2.4: Matter makes space-time curved and leads to the loss of Poincare invariance so that momentum and energy are not well-defined notions in GRT.

metric defined by (1,1) part of Hessian in complex coordinates must be positive definite. Thus criticality might therefore imply problems.

This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no troubles with Gaussian approximation to the functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could however be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function. Kähler function would make WCW integral mathematically existing and Morse function would imply the typical quantal interference effects.

3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded as almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW. These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interpretation seems to be that these degrees of freedom correspond to deformations in zero modes.

This is a brief summary about quantum criticality in bosonic degrees of freedom. One must formulate quantum criticality for the modified Dirac action [K28]. The new element is that critical deformations with vanishing second variation of Kähler action define vanishing first variation of Kähler Dirac action so that second order Noether charges correspond to first order Noether charges in fermionic sector. It seems that the formulation in terms of hierarchy of broken conformal symmetries is the most promising one mathematically and also correspond to physical intuition.

2.5.6 The notion of finite measurement resolution

Finite measurement resolution has become one of the basic principles of quantum TGD. Finite measurement resolution has two realizations: the quantal realization in terms of inclusions of von Neumann algebras and the classical realization in terms of discretization having a nice description in number theoretic approach. The notion of p-adic manifold (see the appendix of the book) relying on the canonical correspondence between real and p-adic physics forces finite cognitive and measurement resolution automatically and implies that p-adic preferred extremals are cognitive representations for real preferred extremals in finite cognitive representations [K115].

Von Neumann introduced three types of algebras as candidates for the mathematics of quantum theory. These algebras are known as von Neumann algebras and the three factors (kind of basic building bricks) are known as factors of type I,II, and III. The factors of type I are simplest and apply in wave mechanics where classical system has finite number of degrees of freedom. Factors of type III apply to quantum field theory where the number of degrees of freedom is infinite. Von Neumann himself regarded factors of type III somehow pathological.

Factors of type II contains as sub-class hyper-finite factors of type II_1 (HFFs). The naive definition of trace of unit matrix as infinite dimension of the Hilbert space involved is replaced with a definition in which unit matrix has finite trace equal to 1 in suitable normalization. One cannot anymore select single ray of Hilbert space but one must always consider infinite-dimensional sub-space. The interpretation is in terms of finite measurement resolution: the sub-Hilbert space representing non-detectable degrees of freedom is always infinite-dimensional and the inclusion to larger Hilbert space is accompanied by inclusion of corresponding von Neumann algebras.

HFFs are between factors of type I and III in the sense that approximation of the system as a finite-dimensional system can be made arbitrary good: this motivates the term hyper-finite.

The realization that HFFs [K99] re tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

HFF has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the imbedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the imbedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in zero energy ontology states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.

2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.

3. The inclusions of HFFs are closely related to quantum groups studied in recent modern physics but interpreted in terms of Planck length scale exotics formulated in terms of non-commutative space-time. The formulation in terms of finite measurement resolution brings this mathematics to physics in all scales.

   For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.

4. The realization for quantum measurement theory modulo finite measurement resolution is in terms of M-matrices defined in terms of Connes tensor product which essentially means that the included hyper-finite factor N takes the role of complex numbers.

Discretization defines a classical space-time correlate for the finite measurement resolution.

1. The dynamics of TGD itself might realize finite measurement resolution automatically in the sense that the quantum states at partonic 2-surfaces are always defined in terms of fermions localized at discrete points defined the ends of braids defined as the ends of string world sheets.

2. The condition that these selected points are common to reals and some algebraic extension of p-adic numbers for some p allows only algebraic points. General coordinate invariance requires the special coordinates and natural coordinate systems are possible thanks to the symmetries of WCW. A restriction of general coordinate invariance to discrete subgroup might well occur and have interpretation in terms of the constraints from the presence of cognition. One might say that the world in which mathematician uses Cartesian coordinates is different from the world in mathematician uses spherical coordinates.

3. The realization at the level of WCW would be number theoretical. In given resolution all parameters characterizing the mathematical representation of partonic 2-surfaces would belong to some algebraic extension of rational numbers. Same would hold for their 4-D tangent space data. This would imply that WCW would be effectively discrete space so that finite measurement resolution would be realized.

2.5.7 Weak form of electric magnetic duality

The notion of electric-magnetic duality [B7] was proposed first by Olive and Montonen and is central in $\mathcal{N}=4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge.

The notion of electric-magnetic self-duality is more natural in TGD since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant.

In TGD framework one must adopt a weaker form of the self-duality applying at partonic 2-surfaces [K28]. The principle is statement about boundary values of the induced Kähler form analogous to Maxwell field at the light-like 3-surfaces, at which the situation is singular since the induced metric for four-surface has a vanishing determinant because the signature of the the induced metric changes from Minkowskian to Euclidian. What the principle says is that Kähler electric field in the normal space is the dual of Kähler magnetic field in the 4-D tangent space of the light-like 3-surface. One can consider even weaker formulation assuming this only at partonic 2-surfaces at the intersection of light-like 3-surfaces and space-like 3-surfaces at the boundaries of CD.

Every new idea must be taken with a grain of salt but the good sign is that this concept leads to precise predictions.

1. Elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes. The string picture was later found to emerge naturally from Kähler Dirac action.

2. Second implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

3. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

4. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies. Stringy character is manifested in two manners: as string like objects defined by Kähler magnetic flux tubes and 2-D string world sheets.

5. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

6. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

7. The general solution ansatz works for Chern-Simons Dirac equation and reduces it to ordinary differential equation along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equation along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

2.5.8 TGD as almost topological QFT

Topological QFTs (TQFTs) represent examples of the very few quantum field theories which exist in mathematically rigorous manner. TQFTs are of course physically non-realistic since the notion of distance is lacking and one cannot assign to the particles observables like mass. This raises the hope that TGD could be as near as possible to TQFT.

The vision about TGD as almost topological QFT is very attractive. Almost topological QFT property would naturally correspond to the reduction of Kähler action for preferred extremals to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. This is achieved if weak form of em duality vanishes and $j \cdot A$ term in the decomposition of Kähler action to 4-D integral and 3-D boundary term vanishes. Almost topological QFT would suggests conformal field theory at partonic 2-surface or at their light-like orbits. Strong form of holography states that also conformal field theory associated with space-like 3-surfaces at the ends of CDs describes the physics. These facts suggest that almost 2-dimensional QFT coded by data given at partonic 2-surfaces and their 4-D tangent space is enough to code for physics.

Topological QFT property would mean description in terms of braids. Braids would correspond to the orbits of fermions at partonic 2-surfaces identifiable as ends of string world sheets at which the modes of induced spinor field are localized with one exception: right-handed neutrino. This follows from well-definedness of electromagnetic charge in presence of induce W boson fields. The first guess is that induced W boson field must vanish at string world sheet. "Almost" could mean the replacement of the ends of strings defining braids with strings and duality for the descriptions based on string world sheets resp. partonic 2-surfaces analogous to AdS/CFT duality.

2.5.9 Twistor revolution and TGD

During last decade so called twistor revolution has revived theoretical physics and has also had strong impact on TGD.

Twistor revolution

There are classical papers by several authors such as Witten and Nima Arkani-Hamed who is one of the leading theoreticians driving the twistor revolution [B31, B60, B25, B26] .

1. The notion of twistor is due to Penrose and is very convenient notion in theories describing massless particles and therefore possessing conformal invariance extending Poincare symmetries by the inclusion of scalings and so called special conformal transformation which is analogous to reflections in spherical mirror.

2. Twistor kinematics means that one can express massless four-momentum and helicity in terms of two massless spinors combining to a twistor living in 4-D complex space which reduces to CP_3 because of projective invariance of the description. The beauty of the twistor kinematics is that non-linear action of special conformal transformations linearizes in CP_3. What one does is to replace light-like geodesic in Minkowski space with points in the twistor space CP_3 whereas the complex lines of CP_3 correspond to points of Minkowski space. To be honest, complexified Minkowski space is in question and this is one of the technical difficulties involved.

3. Gauge theories without fermions and scalar fields are such theories and the applications of twistorial methods to $\mathcal{N} = 4$ super-symmetric Yang-Mills theory has produced amazingly strong results demonstrating that Feynman diagrams sum up to stunningly single twistorial expressions. The key idea is that four-dimensional integrals over loop momenta are interpreted as residue integrals in the complexified space of four-momenta so that they reduce to residues from poles. The surprising discovery is that using Yangian invariants one can express the planar loop amplitudes for given number of external states with given helicities and momenta in terms of on mass shell amplitudes for smaller number of particles and with smaller number of loops by using recursion formulas.

4. Twistor revolution has led to a discovery of what is known as dual twistors. The massless momenta associated with incoming states of twistor diagram and expressible in terms of ordinary twistors, can be also expressed as differences of so called region momenta propagating in the edges of polygon characterizing twistor diagram. The massless momenta correspond to intersections of complex lines in what is called momentum twistor space so that one diagram has interpretation also as a diagram in momentum twistor space. The theory possesses conformal invariance also in the momentum twistor space and the two conformal symmetries combine to form a large infinite-dimensional symmetry known as Yangian symmetry [A36] associated with the conformal group of Minkowski space.

5. The work of Nima Arkani-Hamed and others [B26] has revealed that the integrands for the twistor amplitudes for planar diagrams can be expressed as residue integrals over Grassmannians $G(n, k)$ where n is the number of massless external particles (gluons or gluinos) and k is the number of negative (say) helicities. The integrands appearing in these integrals are Yangian invariants and there are recipes for their construction. The generalization of BCFW formula gives a recursion formula allowing to deduce the l-loop construction to the scattering of n particles with k negative helicities. The vision of Arkani-Hamed is that this approach allows to get rid of space-time altogether.

The latest view about twistors in TGD

The TGD view about twistors has evolved gradually anbd I have written three chapters about twistors and TGD [K98, K101, K78] giving a view about development of ideas. Some of the conjectures have turned out to be wrong and the original idea about QFT type twistorial description of scattering amplitudes has been replaced by its stringy variant so that also problems caused by non-planar diagrams disappear. Below I summarize only the latest view about the situation [K78].

First the deep result making TGD Universe unique from twistorial point of view. Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg

M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1)\times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthmore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via super-conformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for $M^4 \times CP_2$ serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of CP_2 twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor $1/p^2$ assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of $\mathcal{N} = 4$ SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.

2. This as also the fact that physical particles are necessarily pairs of wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book) connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators $G, G^\dagger$ and L bringing in CP¡sub¿2¡/sub¿ scale appear in propagator lines: most importantly, the fact that G and $G^\dagger$ carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has $1/G$ and $1/G^\dagger$ at its ends. Twistorialization applies because all fermion lines are light-like.

3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to CP_2 scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting CP_2 scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

Chapter 3

Topological Geometrodynamics: Three Visions

3.1 Introduction

Originally Topological Geometrodynamics (TGD) was proposed as a solution of the problems related to the definition of conserved four-momentum in General Relativity. It was assumed that physical space-times are representable as 4-D surfaces in certain higher-dimensional space-time having symmetries of the empty Minkowski space of Special Relativity. This is guaranteed by the decomposition $H = M^4 \times S$, where S is some compact internal space. It turned out that the choice $S = CP_2$ is unique in the sense that it predicts the symmetries of the standard model and provides a realization for Einstein's dream of geometrizing of fundamental interactions at classical level. TGD can be also regarded as a generalization of super string models obtained by replacing strings with light-like 3-surfaces or equivalently with space-like 3-surfaces: the equivalence of these identification implies quantum holography.

The construction of quantum TGD turned out to be much more than mere technical problem of deriving S-matrix from path integral formalism. A new ontology of physics (many-sheeted space-time, zero energy ontology, generalization of the notion of number, and generalization of quantum theory based on spectrum of Planck constants giving hopes to understand what dark matter and dark energy are) and also a generalization of quantum measurement theory leading to a theory of consciousness and model for quantum biology providing new insights to the mysterious ability of living matter to circumvent the constraints posed by the second law of thermodynamics were needed. The construction of quantum TGD involves a handful of different approaches consistent with a similar overall view, and one can say that the construction of M-matrix, which generalizes the S-matrix of quantum field theories, is understood to a satisfactory degree although it is not possible to write even in principle explicit Feynman rules except at quantum field theory limit [K64, K29] .

In this chapter I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics [K73] .

2. Second vision is number theoretical [K85] and involves three threads.

   (a) The first thread [K87] relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept.

   (b) Second thread [K88] is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically.

(c) The third thread [K86] relies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units. This implies number theoretical Brahman=Atman identity or number theoretical holography when one consider hyper-octonionic infinite primes.

(d) The third vision is based on TGD inspired theory of consciousness [K89] , which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics. The basic notions at quantum jump identified as as a moment of consciousness and self. Negentropy Maximization Principle (NMP) defines the fundamental variational principle and reproduces standard quantum measurement theory and predicts second law but also some totally new physics in the intersection of real and p-adic worlds where it is possible to define a hierarchy of number theoretical variants of Shannon entropy which can be also negative. In this case NMP favors the generation of entanglement and state function reduction does not mean generation of randomness anymore. This vision has obvious almost applications to biological self-organization.

My aim is to provide a bird's eye of view and my hope is that reader would take the attitude that details which cannot be explained in this kind of representation are not essential for the purpose of getting a feeling about the great dream behind TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L74]
- Geometry of WCW [L38]
- Physics as generalized number theory [L58]
- Quantum physics as generalized number theory [L64]
- TGD inspired theory of consciousness [L77]
- Negentropy Maximization Principle [L54]
- Zero Energy Ontology (ZEO) [L85]

3.2 Quantum physics as infinite-dimensional geometry

The first vision in its original form is a the generalization of Einstein's program for the geometrization of physics by replacing space-time with the WCW identified roughly as the space of 4-surfaces in $H = M^4 \times CP_2$. Later generalization due to replacement of H with book like structures from by real and p-adic variants of H emerged. A further book like structure of imbedding space emerged via the introduction of the hierarchy of Planck constants. These generalizations do not however add anything new to the basic geometric vision.

3.2.1 World of the classical worlds as the arena of quantum physics

Physics as the classical spinor field geometry of WCW consisting of light-like 3-surfaces in 8-D imbedding space $H = M^4 \times CP_2$ (to be referred as WCW CH or WCW in the sequel) is the oldest and best developed approach to TGD and means generalization of Einstein's program of geometrizing classical physics so that it applies to entire quantum physics [K73]. There are two natural identifications for the 3-surfaces.

1. By general coordinate invariance light-like 3-surfaces can be identified as orbits of wormhole throats at which the signature of the induced metric changes from a Minkowskian signature of space-time sheet to that of deformed CP_2 type vacuum extremal representing elementary particle. One can interpret so called CP_2 type vacuum extremals as lines of generalized Feynman diagrams so that geometrization and generalization of the notion of Feynman diagram emerges.

2. In zero energy ontology causal diamonds (CDs, (see fig. 2.3.5 in thttp://www.tgdtheory.fi/appfigures/heappendixofthisbook)ofM^4defined.jpg, which is also as intersection of future and past directed light-cones become define basic building bricks of WCW. The space-time surfaces belonging to CD having their 3-D future and past ends at the light-like boundaries of CD become the basic objects. The ends are 3-surfaces are space-like and come in pairs. WCW decomposes into a union over sub-WCWs associated with various CDs and their unions and the space-like ends of the space-time sheets at future and past boundaries of CD become very natural fundamental objects.

The condition that the two identifications of 3-surfaces are equivalent implies that all information about the geometry of WCW and quantum physics is coded by the 2-dimensional intersections of the space-like and light-like 3-surfaces at the boundaries of CDs plus the information about the distribution of 4-D tangent spaces of the space-time sheet at these surfaces. I have christened partonic 2-surfaces since they are carriers of various quantum numbers. Therefore 4-D General Coordinate invariance implies effective 2-dimensionality and quantum holography. The effective two-dimensionality is implies also by general consistency conditions related to conformal symmetries: this became obvious much before the emergence of zero energy ontology and led to interpretational difficulties at that time. The non-determinism of Kähler action defining space-time dynamics in the standard sense of the word implies that effective 2-dimensionality holds only locally.

WCW is endowed with Kähler metric guaranteeing the geometrization of hermitian conjugation of quantum theory.

1. The conjecture inspired by the geometry of loop spaces [A71] is that H is fixed from the mere requirement that the infinite-dimensional Kähler geometry exists. WCW must reduce to a union of symmetric spaces having infinite-dimensional isometry groups and labeled by zero modes having interpretation as classical dynamical variables. This requires infinite-dimensional symmetry groups. At space-time level super-conformal symmetries are possible only if the basic dynamical objects can be identified as light-like or space-like 3-surfaces. At imbedding space level there are extended super-conformal symmetries assignable to the light-cone of H if the Minkowski space factor is four-dimensional.

   Equivalence Principle seems obvious in the stringy picture provided by recent view about TGD and might indeed reduce to a mere tautology. My belief has been however that the EP is non-trivial. The question whether it is possible to define gravitational and inertial masses and understand their equivalence has been however a continual headache in TGD. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance [K93].

   One can consider also the possibility that their generalization holds true for preferred extremals in all scales. Generalization would involve cosmological constant or even two of them and depending on position. A string model type description emerges in a finite measurement resolution when light-like 3-surfaces are replaced by braids. This means also quantum holography. In positive energy ontology General Coordinate Invariance implies that classical space-time physics becomes an exact part of quantum theory in the sense that space-time sheets are analogous to Bohr orbits. In zero energy ontology (ZEO) 3-surfaces are untions of the space-like ends of space-time surface at the opposite boundaries of CD and in this kind of situation it might be that there is no need to talk about preferred extremals. The non-determinism of Kähler action might however change the situation.

2. The condition that the symmetries of standard model are realized geometrically and that one can understand the known quantum numbers characterizing elementary particles in terms of the geometry of the imbedding space, leads to a unique choice for the imbedding space as $H = M^4 \times CP_2$. The challenge is to understand what makes this choice so special and number theoretic approach based on classical number fields allows to interpret this choice number theoretically so that the standard model symmetries find a number theoretical interpretation.

3.2.2 Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model [B64] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   WCW spinor fields can have arbitrary fermion number and there are good hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field. One non-trivial implication is bosonic emergence: elementary bosons correspond to fermion anti-fermion bound states associated with the wormhole contacts (pieces of CP_2 type vacuum extremals) with throats carrying fermion and anti-fermion numbers. Fermions correspond to single throats associated with topologically condensed CP_2 type vacuum extremals.

2. The classical theory for the bosonic fields is an essential part of WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ ,$$

where J_{AB} denotes the matrix elements of the Kähler form of WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

3.2.3 Construction of WCW Clifford algebra in terms of second quantized induced spinor fields

The construction of WCW spinor structure must have a direct relationship to quantum physics as it is usually understood. The second quantization of the space-time spinor fields is needed to define the anti-commutative gamma matrices of WCW: this means a geometrization of Fermi statistics [K17] in the sense that free fermionic quantum fields at space-time surface correspond to purely classical Clifford algebra of WCW. This is in accordance with the idea that physics at WCW level is purely classical apart from the notion of quantum jump.

The identification of the correct variational principle for the dynamics of space-time spinor fields identified as induced spinor fields has involved many trials. Ironically, the final outcome was almost the most obvious guess. The so called Kähler-Dirac action with Chern-Simons-Dirac term at partonic orbits and corresponding Chern-Simons term in Kähler action cancelling the partonic Chern-Simons terms coming from Kähler action is the starting point. C-S-D action defines fermionic propagators as massless propagators if the spinor modes are eigenstates of the C-S-D operator with generalized eigenvalues $p^k\gamma_k$ representing virtual momenta.

Quantum classical correspondence suggests with measurement interaction term defined as Lagrange multiplier terms stating that classical charges belonging to Cartan algebra are equal to their quantal counterparts after state function reduction for space-time surfaces appearing in quantum superposition [K28]. This makes sense if classical charges parametrize zero modes. State function reduction would mean state function collapse in zero modes.

Kähler function which equals to the real part of Kähler action coming from Euclidian space-time regions for a preferred extremal. The conjecture is that preferred extremals by internal consistency conditions is critical in the sense that it allows infinite number of vanishing second variations having interpretation as conformal deformations respecting light-likeness of the partonic orbits. This realizes the notion of quantum criticality-one of guiding principles of quantum TGD-at space-time level.

Number theoretical approach in turn leads to the conclusion that space-time surfaces are either associative or co-associative in the sense that the modified gamma matrices at each point of space-time surface in their octonionic representation reduces to a quaternionic or co-quaternionic algebra and therefore have matrix representation. The conjecture is that these identifications of space-time dynamics are consistent or even equivalent.

The recent understanding of the modified Dirac action has emerged through a painful process and has strong physical implications.

1. Stringy propagators and emerge naturally thanks to the measurement interaction term in the modified Dirac action coupling to four-momentum and color hyper-charge and isospin.

2. The space-time super-symmetry generalizes to what might be called $\mathcal{N} = \infty$ supersymmetry which however effectively reduces to $\mathcal{N} = 1$ broken super-symmetry [K29]. The generators of the super-symmetry correspond to the modes of the induced spinor field at space-time sheet. Bosonic emergence means dramatic simplifications in the formulation of QFT limit of TGD. This formulation should generalize also to the level of the fundamental theory.

3. It is also possible to generalize the twistor program to TGD framework if one accepts the use of octonionic representation of the gamma matrices of imbedding space and hyper-quaternionicity of space-time surfaces [K98].

3.2.4 Zero energy ontology and WCW geometry

In the zero energy ontology quantum states have vanishing net values of conserved quantum numbers and decompose to superposition of pairs of positive and negative energy states defining counterparts of initial and final states of a physical event in standard ontology.

Zero energy ontology

Zero energy ontology was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. The most feasible resolution of the conflict comes from the realization that GRT space-time is obtained by lumping the sheets of many-sheeted space-time to M^4 endowed with effective metric. Vacuum extremals could however serve as models for GRT space-times such that the effective metric is identified with the induced metric [K93]. This is true if space-time is genuinely single-sheeted. In the models of astrophysical objects and cosmology vacuum extremals have been used [K80].

In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology. It is possible to speak about the energy of the system if one identifies it as the average positive energy for the positive energy part of the system. Same applies to other quantum numbers.

The matrix ("M-matrix") representing time-like entanglement coefficients between positive and negative energy states unifies the notions of S-matrix and density matrix since it can be regarded as a complex square root of density matrix expressible as a product of real squared of density matrix and unitary S-matrix. The system can be also in thermal equilibrium so that thermodynamics becomes a genuine part of quantum theory and thermodynamical ensembles cease to be practical fictions of the theorist. In this case M-matrix represents a superposition of zero energy states for which positive energy state has thermal density matrix.

Zero energy ontology combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. Zero energy ontology leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

Causal diamonds

The imbedding space correlates for zero energy ontology are causal diamonds (CDs) CD serves as the correlate zero energy state at imbedding space-level whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates of the system at the level of 4-D space-time. Zero energy state can be regarded as a quantum superposition of space-time sheets with fermionic and other quantum numbers assignable to the partonic 2-surfaces at the ends of the space-time sheets.

1. The basic construct in the zero energy ontology is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4. In zero energy ontology physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.

2. CDs form a fractal hierarchy and one can glue smaller CDs within larger CDs. Also unions of CDs are possible.

3. Without any restrictions CDs would be parametrized by the position of say lower tip of CD and by the relative M^4 coordinates of the upper tip with respect to the lower one so that

the moduli space would be $M^4 \times M^4_+$. p-Adic length scale hypothesis follows if the values of temporal distance T between tips of CD come in powers of 2^n: $T = 2^n T_0$. This would reduce the future light-cone M^4_+ reduces to a union of hyperboloids with quantized value of light-cone proper time. A possible interpretation of this distance is as a quantized cosmic time. Also the quantization of the hyperboloids to a lattices of discrete points classified by discrete sub-groups of Lorentz group is an attractive proposal and the quantization of cosmic redshifts provides some support for it.

Zero energy ontology forces to replaced the original WCW by a union of WCWs associated with CDs and their unions. This does not however mean any problems of principle since Clifford algebras are simply tensor products of the Clifford algebras of CDs for the unions of CDs.

Generalization of S-matrix in ZEO

ZEO forces the generalization of S-matrix with a triplet formed by U-matrix, M-matrix, and S-matrix. The basic vision is that quantum theory is at mathematical level a complex square roots of thermodynamics. What happens in quantum jump was already discussed.

1. U-matrix as has its rows M-matrices , which are matrices between positive and negative energy parts of the zero energy state and correspond to the ordinary S-matrix. M-matrix is a product of a hermitian square root - call it H - of density matrix ρ and universal S-matrix S commuting with H: $[S, H] = 0$. There is infinite number of different Hermitian square roots H_i of density matrices which are assumed to define orthogonal matrices with respect to the inner product defined by the trace: $Tr(H_i H_j) = 0$. Also the columns of U-matrix are orthogonal. One can interpret square roots of the density matrices as a Lie algebra acting as symmetries of the S-matrix.

2. One can consider generalization of M-matrices so that they would be analogous to the elements of Kac-Moody algebra. These M-matrices would involve all powers of S.

   (a) The orthogonality with respect to the inner product defined by $\langle A|B\rangle = Tr(AB)$ requires the conditions $Tr(H_1 H_2 S^n) = 0$ for $n \neq 0$ and H_i are Hermitian matrices appearing as square root of density matrix. $H_1 H_2$ is hermitian if the commutator $[H_1, H_2]$ vanishes. It would be natural to assign n:th power of S to the CD for which the scale is n times the CP_2 scale.

   (b) Trace - possibly quantum trace for hyper-finite factors of type II_1) is the analog of integration and the formula would be a non-commutative analog of the identity $\int_{S^1} exp(in\phi)d\phi = 0$ and pose an additional condition to the algebra of M-matrices.

   (c) It might be that one must restrict M matrices to a Cartan algebra for a given U-matrix and also this choice would be a process analogous to state function reduction. Since density matrix becomes an observable in TGD Universe, this choice could be seen as a direct counterpart for the choice of a maximal number of commuting observables which would be now hermitian square roots of density matrices. Therefore ZEO gives good hopes of reducing basic quantum measurement theory to infinite-dimensional Lie-algebra.

1. The first guess is wrong

The definition of U-matrix elements as a matrix inducing a change of basis requires two natural state basis. The first guess is that the following two state basis are natural and unitarily related.

1. The pairs of positive and negative energy states with same quantum numbers.

2. The states obtained by entangling positive and negative energy states with various M-matrices.

If these state basics are in one-one correspondence then the orthogonality of the rows of U-matrix means that different M-matrices are orthogonal. The orthogonality of columns of U-matrix means that for the pair $|m_1\rangle_+|n_1\rangle_-$ and $|m_2\rangle_+|n_2\rangle_-$ of zero energy states gives

$$\sum_K M^K_{m_1n_1}\overline{M^K}_{m_2n_2} = \delta_{m_1,m_2}\delta_{n_1,n_2} \quad .$$

The first guess is however not physically acceptable. The assumption that all pairs $|m_1\rangle_+|n_1\rangle_-$ are allowed as a complete set of states would mean complete non-determinism since correlations between the counterparts of initial and finals states would be absent apart from those induced by zero energy property.

2. Second guess for the two state basis

A better guess is that the collections of M-matrices defined as time reversals of each other define the sought for two natural state basis.

1. As for ordinary S-matrix, one can construct the states in such a manner that either positive or negative energy part of the state has well defined particle numbers, spin, etc... resulting in state function preparation. Therefore one has two kinds of M-matrices: $M^{\pm}_K$ and for both of these the above orthogonality relations hold true. This implies also two kinds of U-matrices call them $U^{\pm}$. The natural assumption is that the two M-matrices differ only by Hermitian conjugation so that one would have $M^-_K = (M^+_K)^{\dagger}$.

   One can assign opposite arrows of geometric time to these states and the proposal is that the arrow of time is a result of a process analogous to spontaneous magnetization. The possibility that the arrow of geometric time could change in quantum jump has been already discussed.

2. Unitary U-matrix $U^{\pm}$ is induced from a projector to the zero energy state basis $|K^{\pm}\rangle$ acting on the state basis $|K^{\mp}\rangle$ and the matrix elements of U-matrix are obtained by acting with the representation of identity matrix in the space of zero energy states as $I = \sum_K |K^+\rangle\langle K^+|$ on the zero energy state $|K^-\rangle$ (the action on K^+ is trivial!) and gives

$$U^+_{KL} = Tr(M^+_K M^+_L) \quad .$$

   Note that finite measurement resolution requires that the trace operation is q-trace rather than ordinary trace.

3. As the detailed discussion of the anatomy of quantum jump demonstrated, the first step in state function reduction is the choice of $M^{\pm}_K$ meaning the choice of the hermitian square root of a density matrix. A quantal selection of the measured observable takes place. This step is followed by a choice of "initial" state analogous to state function preparation and a choice of the "final state" analogous to state function reduction. The net outcome is the transition $|K^{\pm}\rangle \rightarrow |L^{\pm}\rangle$. It could also happen that instead of state function reduction as third step unitary process $U^{\mp}$ (note the change of the sign factor!) takes place and induces the change of the arrow of geometric time.

4. As noticed, one can imagine even higher level choices and this would correspond to the choice of the commuting set of hermitian matrices H defining the allowed square roots of density matrices as a set of mutually commuting observables. This would fix the choices of U.

3.2.5 Hierarchy of Planck constants and WCW geometry

The motivations for introducing the hierarchy of Planck constants interpreted in terms of phases of dark matter came from astrophysics [E27] [K79, K62] and biology [K70] and led to a generalization of the imbedding space to a book like structure [K27]. This implies additional richness of structure at the level of geometry of WCW. In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years [K27, K65, K79].

1. The starting point was the proposal of Nottale [E27] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K79] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K80]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K91].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K65]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E27] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L6, K91].

The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric

is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and multiplies YM action action by n_an_b which is equivalent with the $\hbar = n_an_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_an_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	n_an_b	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_an_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K24].

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants [K27]. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

Implications for the construction WCW geometry

1. In the realization of the hierarchy of Planck constants $CD \times CP_2$ is replaced with a Cartesian product of book like structures formed by almost copies of CDs and CP_2s defined by singular coverings and factors spaces of CD and CP_2 with singularities corresponding to intersection $M^2 \cap CD$ and homologically trivial geodesic sphere S^2 of CP_2 for which the induced Kähler form vanishes. The coverings and factor spaces of CDs are glued together along common $M^2 \cap CD$. The coverings and factors spaces of CP_2 are glued together along common homologically non-trivial geodesic sphere S^2. The choice of preferred M^2 as subspace of tangent space of X^4 at all its points and interpreted as space of non-physical polarizations, brings M^2 into the theory also in different manner. S^2 in turn defines a subspace of the much larger space of vacuum extremals as surfaces inside $M^4 \times S^2$.

2. WCW (the world of classical worlds, WCW) decomposes into a union of sub-WCWs corresponding to different choices of M^2 and S^2 and also to different choices of the quantization axes of spin and energy, color isospin and hyper-charge for each choice of this kind. This means breaking down of the isometries to a subgroup. This can be compensated by the fact that the union can be taken over the different choices of this subgroup.

3. This means extension of the moduli space of CDs from $M^4 \times X$, where $X \subset M^4_+$ is suggested to be identifiable as a discrete lattice for the relative positions of the tips of CD. What is added is the space characterizing the choice of the quantization axes for energy and spin on one hand and color hypercharge and isospin on the other hand. This choice is part of a statefunction reduction process and means localization in this space. In the case of color charges the moduli space is the flag-manifold $SU(3)/U(1) \times U(1)$.

3.2.6 Hyper-finite factors and the notion of measurement resolution

The work with TGD inspired model [K97, K26] for topological quantum computation [K97] led to the realization that von Neumann algebras [A63], in particular so called hyper-finite factors of type II_1 [A84], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. Later came the realization that the Clifford algebra of WCW defines a canonical representation of hyper-finite factors of type II_1 and that WCW spinor fields give rise to HFFs of type III_1 encountered also in relativistically invariant quantum field theories [K99].

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A84].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [K28] based on the notion of delta function, plus the emergence of generalized Feynman graphs [K35], the possibility to formulate the notion of delta function rigorously in terms of distributions [A112, A83], and the emergence of path integral approach [A102] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A125, A109] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A87, A85] relate closely to type II_1 factors. In topological quantum computation [K97] based on braid groups [A126] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B34] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B63, B19].

Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type II_1 and III_1- the latter appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of WCW is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K80]. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix [K99].

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem [A118], which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which modified Dirac action containing measurement interaction term defines stringy propagators [K20]. This M-matrix should be consistent with the above proposal.

Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product [A58] but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

Number theoretical braids as space-time correlates for finite measurement resolution

Finite measurement resolution has discretization as a space-time counterpart. In the intersection of real and p-adic worlds defines as partonic 2-surfaces with a mathematical representation allowing interpretation in terms of real or p-adic number fields one can identify points common to real and p-adic worlds as rational points and common algebraic points (in preferred coordinates dictated by symmetries of imbedding space). Quite generally, one can identify rational points and algebraic points in some extension of rationals as points defining the initial points of what might be called number theoretical braid beginning from the partonic 2-surface at the past boundary of CD and connecting it with the future boundary of CD. The detailed definition of the braid inside light-like 3-surface is not relevant if only the information at partonic 2-surface is relevant for quantum physics.

Number theoretical braids are especially relevant for topological QFT aspect of quantum TGD. The topological QFT associated with braids accompanying light-like 3-surfaces having interpretation as lines of generalied Feynman diagrams should be important part of the definition of amplitudes assigned to generalized Feynman diagrams. The number theoretic braids relate also closely to a symplectic variant of conformal field theory emerges very naturally in TGD framework (symplectic symmetries acting on $\delta M^4_\pm \times CP_2$ are in question) and this leads to a concrete proposal for how to to construct n-point functions needed to calculate M-matrix [K20]. The mechanism guaranteeing the predicted absence of divergences in M-matrix elements can be understood in terms of vanishing of symplectic invariants as two arguments of n-point function coincide.

Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities [K99]. For quantum spinors state function reduction to spin eigenstates cannot be performed unless quantum deformation parameter $q = exp(i2\pi/n)$ equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. Therefore the probability for either spin state becomes a quantized observable. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

3.2.7 Twistor revolution and TGD

During last decade so called twistor revolution has revived theoretical physics and has also had strong impact on TGD.

Twistor revolution

There are classical papers by several authors such as Witten and Nima Arkani-Hamed who is one of the leading theoreticians driving the twistor revolution [B31, B60, B25, B26] .

1. The notion of twistor is due to Penrose and is very convenient notion in theories describing massless particles and therefore possessing conformal invariance extending Poincare symmetries by the inclusion of scalings and so called special conformal transformation which is analogous to reflections in spherical mirror.

2. Twistor kinematics means that one can express massless four-momentum and helicity in terms of two massless spinors combining to a twistor living in 4-D complex space which reduces to CP_3 because of projective invariance of the description. The beauty of the twistor kinematics is that non-linear action of special conformal transformations linearizes in CP_3. What one does is to replace light-like geodesic in Minkowski space with points in the twistor space CP_3 whereas the complex lines of CP_3 correspond to points of Minkowski space. To be honest, complexified Minkowski space is in question and this is one of the technical difficulties involved.

3. Gauge theories without fermions and scalar fields are such theories and the applications of twistorial methods to $\mathcal{N} = 4$ super-symmetric Yang-Mills theory has produced amazingly strong results demonstrating that Feynman diagrams sum up to stunningly single twistorial expressions. The key idea is that four-dimensional integrals over loop momenta are interpreted as residue integrals in the complexified space of four-momenta so that they reduce to residues from poles. The surprising discovery is that using Yangian invariants one can express the planar loop amplitudes for given number of external states with given helicities and momenta in terms of on mass shell amplitudes for smaller number of particles and with smaller number of loops by using recursion formulas.

4. Twistor revolution has led to a discovery of what is known as dual twistors. The massless momenta associated with incoming states of twistor diagram and expressible in terms of ordinary twistors, can be also expressed as differences of so called region momenta propagating in the edges of polygon characterizing twistor diagram. The massless momenta correspond to intersections of complex lines in what is called momentum twistor space so that one diagram has interpretation also as a diagram in momentum twistor space. The theory possesses conformal invariance also in the momentum twistor space and the two conformal symmetries

combine to form a large infinite-dimensional symmetry known as Yangian symmetry [A36] associated with the conformal group of Minkowski space.

5. The work of Nima Arkani-Hamed and others [B26] has revealed that the integrands for the twistor amplitudes for planar diagrams can be expressed as residue integrals over Grassmannians $G(n,k)$ where n is the number of massless external particles (gluons or gluinos) and k is the number of negative (say) helicities. The integrands appearing in these integrals are Yangian invariants and there are recipes for their construction. The generalization of BCFW formula gives a recursion formula allowing to deduce the l-loop construction to the scattering of n particles with k negative helicities. The vision of Arkani-Hamed is that this approach allows to get rid of space-time altogether.

The impact of twistor approach on TGD

Twistor revolution has had also a strong impact on TGD [K98, K101, K36].

1. The study of the modified Dirac equations for induced spinors [K28] motivated the proposal that the fermions associated with the braid strands accompanying wormhole throats are massless and on mass shell always - even for the internal lines of generalized Feynman diagrams. Since the sign of the energy of internal line can be also negative, wormhole contacts identified as building bricks of virtual bosons can carry space-like virtual momentum. The condition that all wormhole throats appearing in the loops of generalized Feynman diagrams are on mass shell and massless poses enormously strong constraints on loops and with additional restrictions coming from the geometric picture and Uncertainty Principle one can expect that only finite number of diagrams contributes to a given scattering amplitude so that the diagrammatics should be extremely simple, perhaps much simpler than in twistorial diagrammatics involving infinite number of diagrams labelled by the number of loops.

2. The masslessness of all fermionic braid strands appearing in the diagram makes twistorial approach extremely natural. The problem is only the treatment of massive external particles expressible as bound states of massless fermions and anti-fermions at wormhole throats. The idea is that the vertices of generalized Feynman diagrams can be interpreted as 3-gons of twistorial diagrams with region momenta describing momentum exchanges between the throats of wormhole contacts. This picture allows to consider the possibility that 3-vertices could be expressible using the same general formulas as used in Grassmannian approach to $\mathcal{N}=4$ SYM. Kinematical constraints would imply that only a finite number of diagrams would contribute to a give reaction meaning also upper bound on the number of loops.

3. One might of course argue that the massless propagators on mass shell braid strands make the amplitudes infinite. This is not the case since the momentum appearing in the propagators is M^2 projection of the momentum for a preferred $M^2 \subset M^4$.

   Generalized Feynman diagrammatics - or rather given CD - involve however a selection of preferred $M^2 \subset M^4$. M^2 is forced by number theoretical vision as complex and thus commutative subspace of octonions. Physically M^2 is forced by the construction of massless states as the sub-space containing non-physical polarizations. M^2 fixes the quantization axes of energy and spin. In generalized Feynman diagrams the propagators for on mass shell states contain only the M^2 projection of four-momentum so that the propagators are finite. The modification of gauge conditions to statement that M^2 projection of momentum is orthogonal to polarization vector allows massive states for gauge bosons. Lorentz invariance is not broken since one must integrate over all the choices of M^2.

Could TGD circumvent the difficulties of twistor approach?

The twistor approach has also some problems and TGD allows also to consider solutions to the difficulties of twistorial approach.

1. Twistor approach to $\mathcal{N}=4$ SYM applies only to planar Feynman diagrams containing no intersecting lines and this restriction interpreted as approximation becomes exact only at the limit when the gauge group becomes infinite-dimensional. One could argue that in TGD

string like objects defined by the Kähler magnetic flux tubes are the basic objects and the stringy character makes planar approximation exact. One could also argue that infinite-dimensional symplectic group takes the role of gauge group in TGD. A more convincing argument is that non-planar diagrams are possible but that generalized Feynman diagrams can be regarded as generalizations of knot diagrams. The crossings of the lines are the problem and for knot diagrams there is a recipe for removing the crossings gradually completely and in this manner obtain an expression for knot invariant. Similar un-knotting procedure could make sense also in TGD framework.

2. $\mathcal{N}=4$ SYM is believed to be ultraviolet finite but twistor approach does not remove the infrared divergences. In TGD framework external particles are bound states of massless particles and this brings in IR cutoff naturally. The upper bound for the size for CDs brings this cutoff in the case of photons, gluons, and gravitons. One implication is that Higgs like states are not needed in TGD framework.

   The recent considerations [K36] suggest a more refined view about particle masses. It seem that it is M^2 mass squared which is given by stringy mass formula fixed by conformal invariance. If so, p-adic thermodynamics allows to calculate thermal M^2 mass squared and this mass squared wold defining the observed mass of the elementary fermion. Fermionic braid strands would be massless in M^4 sense. The situation is obviously tricky and the understanding of the role of M^2 in TGD framework is one of the basic challenges of the theory.

3. In twistor approach there are also problems with the understanding of renormalization group, which involves momentum scale: this is understandable since conformal invariance does not allow a preferred scale. One should be able to to bring in massive particles without losing the conformal invariance. The fractal hierarchy of CDs within CDs with quantized size scales leads to a detailed proposal for how the vision about p-adic coupling constant evolution is realized for generalized Feynman diagrams [K36].

4. The beauty of Feynman graphs is their ability to code unitarity in an elegant manner by using analyticity. Unitarity conditions are obtained simply by considering the discontinuities of the amplitude at cuts associated with on mass shell configurations of momenta. These discontinuities are expressible by putting internal lines on mass shell so that integral over intermediate on mass shell states is obtained. Unitarity is however not manifest in twistor approach.

   The situation is more complex in TGD framework. M-matrices are not unitary and S-matrix is analogous to the phase of a complex number where as the hermitian square root of the density matrix is analogous to its modulus. Therefore unitary is not required at this level. Different M-matrices (allowing also integer powers of S) must form an infinite-dimensional Kac Moody type algebra and this gives strong constraints on the amplitudes. How to take into account these constraints should be understood.

5. Locality is not manifest in twistor approach. In other words, for Grassmannian amplitudes the poles do not correspond to single particle propagator poles associated with internal lines as they do in Feynman graphs. TGD is manifestly non-local theory since zero energy states involve partonic 2-surfaces at both light-like boundaries of CD. Also partonic 2-surfaces and braids are non-local objects. Yangian symmetry is manifestly non-local since the generators are multilocal objects and in TGD framework this multilocality generalized since the n points of multilocal generator of Yangian algebra are replaced by n partonic 2-surfaces. Also this aspect should be understand in detail.

Twistor approach combined with the requirement of number theoretic universality realized in terms of quantum arithmetics leads to a rather detailed view about generalized Feynman diagrams [K36].

1. One can understand how p-adic length scale hypothesis (stating that primes near powers of two are physically preferred) emerges. This has been one of the main challenges of quantum TGD since 1995 when I performed p-adic mass calculations for the first time.

2. A deep connection with adeles used in Langlands program emerges since the amplitudes can be understood as having values in the tensor product of quantum rationals corresponding to various values of p-adic prime and real amplitudes are obtained by using canonical identification mapping powers of p with their inverses.

3. One implication is that for large primes assignable to elementary particles the convergence in powers of p is extremely fast (one has $p = M_{127} = 2^{127} - 1$ for electron and $p = M_{89}$ for weak gauge bosons!).

3.3 Physics as a generalized number theory

Physics as a generalized number theory vision involves actually three threads: p-adic ideas [K87], the ideas related to classical number fields [K88], and the ideas related to the notion of infinite prime [K86].

3.3.1 Fusion of real and p-adic physics to a coherent whole

p-Adic number fields were not present in the original approach to TGD. The success of the p-adic mass calculations (summarized in the first part of [K54]) made however clear that one must generalize the notion of topology also at the infinitesimal level from that defined by real numbers so that the attribute "topological" in TGD gains much more profound meaning than intended originally. It took a decade to get convinced that the identification of p-adic physics as a correlate of cognition and intentionality is the most plausible interpretation discovered hitherto [K58], and that p-adic topology of p-adic space-time sheets somehow induces the effective p-adic topology of real space-time sheets. The discovery of the properties of number theoretic variants of Shannon entropy led to the idea that living matter could be seen as as something in the intersection of real and p-adic worlds and gave additional support for this interpretation. If even elementary particles reside in this intersection and effective p-adic topology applies for real partonic 2-surfaces, the success of p-adic mass calculations can be understood.

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K85].

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger structure (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also).

2. At the level of imbedding space this gluing corresponds to a gluing of real and p-adic variants of the imbedding space together along rational and common algebraic points (the number of which depends on algebraic extension of p-adic numbers used) to what could be seen as a book like structure. General Coordinate Invariance restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices. Whether this can be seen as an objection against the approach is not clear.

3. The analogous gluing of real and various p-adic physics to a larger structure forces to ask what are the common points of WCWs associated with real and various p-adic worlds. What it is to be a partonic 2-surface belonging to the intersection of real and p-adic variants of WCW? The natural answer is that partonic 2-surfaces which have a mathematical representation making sense both for real numbers and p-adic numbers or their algebraic extensions can be regarded as "common points" or identifiable points of p-adicity and reality. This of course applies also to partonic 2-surfaces corresponding to two different p-adic number fields. This mathematical property means a representability in terms of ratios of polynomials with rational (or possibly even algebraic) coefficients in the preferred imbedding space coordinates.

4. The intersections of WCWs and partonic 2-surfaces in different number fields are involved. An attractive idea is that only the information about common points of surfaces belonging to different number fields code for physics so that number-theoretically universal part of physics is number theoretical physics relying only on rationals and their algebraic extensions. For instance, the transition amplitudes between p-adic and real variants of partonic 2-surface can involve only the data at these points. This suggests the existence of what might be called number theoretical QFT. At space-time level this extension of introduce a discretization at space-time level in terms of rational and algebraic points common to real space-time sheets and their p-adic variants. The number of these points is in general finite for a given CD and the proposed interpretation is in terms of cognitive representations. The discrete intersections would define the initial and final points of number theoretical braids central for the formulation of the theory in finite measurement resolution.

5. Much later came the realization that living matter or what makes living matter living could be interpreted as something in this intersection of real and p-adic worlds so that number theoretic QFT might apply to crucial aspects of living matter.

The interpretation for discretization could be in terms of cognitive, sensory, and measurement resolutions rather than fundamental discreteness of the space-time. What looks rather counter intuitive first is that transcendental points of p-adic space-time sheets are at spatiotemporal infinity in real sense so that the correlates of cognition and intentionality cannot be localized to any finite spatiotemporal volume unlike those of sensory experience. The description of intentionality and cognition in this manner predicts p-adic fractality of real physics meaning chaos in short scales combined with long range correlations: p-adic mass calculations represent one example of p-adic fractality.

The realization of this program at the level of WCW is far from trivial. Modified Dirac equation and classical field equations make sense but quantities expressible as space-time integrals - in particular Kähler action- do not make sense p-adically. Therefore one can ask whether only the partonic surfaces in the intersection of real and p-adic worlds should be allowed. Also this restricted theory would be highly non-trivial physically.

3.3.2 Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A43]) are involved [K88] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K88]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of modified Dirac action the identification of space-time surface as a hyper-quaternionic sub-manifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K28, K98].

3.3.3 Infinite primes and quantum physics

The hierarchy of infinite primes (and of integers and rationals) [K86] was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many-particle states of previous level become the elementary particles of new level. At a given level there are free many particles states plus counterparts of many particle states. There is strong structural analogy with polynomial primes. For polynomials with rational coefficients free many-particle states would correspond to products of first order polynomials and bound states to irreducible polynomials with non-rational roots.

The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe. Also the hierarchy of n:th order logics are possible correlates for this hierarchy.

This construction leads also to a number theoretic generalization of space-time point since a given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest a kind of number theoretical Brahman=Atman identity stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of WCW or maybe even quantum states assignable to causal diamond. One could also speak about algebraic holography.

The correspondence between the quantum states defined by WCW spinor fields and wave functions in the infinite-dimensional discrete space of hyper-octonionic units can be made more concrete [K86]. These wave functions must transforming irreducibly under discrete subgroup SU(3) of octonion automorpisms transforming ordinary hyper-octonionic prime to a new hyper-octonionic prime. SU(3) has interpretation as color group. One can assign standard model quantum numbers to these wave functions and prime property in principle fixes the spectrum of possible quantum states- in particular the spectrum of masses. Therefore the extremely esoteric looking notion of infinite prime might turn out to be very practical calculational tool.

3.3.4 Quantum Mathematics and Quantum Mechanics

Quantum Mathematics replaces numbers with Hilbert spaces and arithmetic operations $+$ and $\times$ with direct sum $\oplus$ and tensor product $\otimes$.

1. The original motivation comes from quantum TGD where direct sum and tensor product are naturally assigned with the two basic vertices analogous to stringy 3-vertex and 3-vertex of Feynman graph. This suggests that generalized Feynman graphs could be analogous to sequences of arithmetic operations allowing also co-operations of $\oplus$ and $\otimes$.

2. One can assign to natural numbers, integers, rationals, algebraic numbers, transcendentals and their p-adic counterparts for various prime p Hilbert spaces with formal dimension given by the number in question. Typically the dimension of these Hilbert spaces in the ordinary sense is infinite. Von Neuman algebras known as hyper-finite factors of type II_1 assume as a convention that the dimension of basic Hilbert space is one although it is infinite in the standard sense of the word. Therefore this Hilbert space has sub-spaces with dimension which can be any number in the unit interval. Now however also negative and even complex, quaternionic and octonionic values of Hilbert space dimension become possible.

3. The decomposition to a direct sum matters unlike for abstract Hilbert space as it does also in the case of physical systems where the decomposition to a direct sum of representations of symmetries is standard procedure with deep physical significance. Therefore abstract Hilbert space is replaced with a more structured objects. For instance, the expansion $\sum_n x_n p^n$ of a p-adic number in powers of p defines decomposition of infinite-dimensional Hilbert space to a direct sum $\oplus_n x_n \otimes p^n$ of the tensor products $x_n \otimes p^n$. It seems that one must modify the notion of General Coordinate Invariance since number theoretic anatomy distinguishes between the representations of space-time point in various coordinates. The interpretation would be in terms of cognition. For instance, the representation of Neper number requires infinite number of pinary digits whereas finite integer requires only a finite number of them so that at the level of cognitive representations general coordinate invariance is broken.

   Note that the number of elements of the state basis in p^n factor is p^n and $m \in \{0, ..., p-1\}$ in the factor x_n. Therefore the Hilbert space with dimension $p^n > x_n$ is analogous to the Hilbert space of a large effectively classical system entangled with the microscopic system characterized by x_n. p-Adicity of this Hilbert space in this example is for the purpose of simplicity but raises the question whether the state function reduction is directly related to cognition.

4. On can generalize the concept of real numbers, the notions of manifold, matrix group, etc... by replacing points with Hilbert spaces. For instance, the point $(x_1, .., x_n)$ of E^n is replaced with Cartesian product of corresponding Hilbert spaces. What is of utmost importance for the idea about possible connection with the multiverse idea is that also this process can be also repeated indefinitely. This process is analogous to a repeated second quantization since intuitively the replacement means replacing Hilbert space with Hilbert space of wave functions in Hilbert space. The finite dimension and its continuity as function of space-time point must mean that there are strong constraints on these wave functions. What does this decomposition to a direct sum mean at the level of states? Does one have super-selection rules stating that quantum interference is possible only inside the direct summands?

5. Could one find a number theoretical counterpart for state function reduction and preparation and unitary time evolution? Could zero energy ontology have a formulation at the level of the number theory as earlier experience with infinite primes suggest? The proposal was that zero energy states correspond to ratios of infinite integers which as real numbers reduce to real unit. Could zero energy states correspond to states in the tensor product of Hilbert spaces for which formal dimensions are inverses of each other so that the total space has dimension 1?

The fractal character of the Quantum Mathematics is what makes it a good candidate for understanding the self-referentiality of consciousness. The replacement of the Hilbert space with the direct sum of Hilbert spaces defined by its points would be the basic step and could be repeated endlessly corresponding to a hierarchy of statements about statements or hierarchy of n^{th} order logics. The construction of infinite primes leads to a similar structure.

What about the step leading to a deeper level in hierarchy and involving the replacement of each point of Hilbert space with Hilbert space characterizing it number theoretically? What could it correspond at the level of states?

1. Suppose that state function reduction selects one point for each Hilbert space $x_n \times p^n$. The key step is to replace this direct sum of points of these Hilbert spaces with direct sum of Hilbert spaces defined by the points of these Hilbert spaces. After this one would select point from this very big Hilbert space. Could this point be in some sense the image of the Hilbert space state at previous level? Should one imbed Hilbert space $x_n \times p^n$ isometrically to the Hilbert space defined by the preferred state $x_n \times p^n$ so that one would have a realization of holography: part would represent the whole at the new level. It seems that there is a canonical manner to achieve this. The interpretation as the analog of second quantization suggest the identification of the imbedding map as the identification of the many particle states of previous level as single particle states of the new level.

2. Could topological condensation be the counterpart of this process in many-sheeted space-time of TGD? The states of previous level would be assigned to the space-time sheets topologically

condensed to a larger space-time sheet representing the new level and the many-particle states of previous level would be the elementary particles of the new level.

3. If this vision is correct, second quantization performed by theoreticians would not be a mere theoretical operation but a fundamental physical process necessary for cognition! The above proposed unitary imbedding would imbed the states of the previous level as single particle states to the new level. It would seem that the process of second quantization, which is indeed very much like self-reference, is completely independent from state function reduction and unitary process. This picture would conform with the fact that in TGD Universe the theory about the Universe is the Universe and mathematician is in the quantum jumps between different solutions of this theory.

Unitary process and state function reduction in ZEO

The minimal view about unitary process and state function reduction is provided by ZEO [K6].

1. Zero energy states correspond to a superposition of pairs of positive and negative energy states. The M-matrix defining the entanglement coefficients is product of Hermitian square root of density matrix and unitary S-matrix, and various M-matrices are orthogonal and form rows of a unitary U-matrix. Quantum theory is square root of thermodynamics. This is true even at single particle level. The square root of the density matrix could be also interpreted in terms of finite measurement resolution.

2. It is natural to assume that zero energy states have well-defined single particle quantum numbers at the either end of CD as in particle physics experiment. This means that state preparation has taken place and the prepared end represents the initial state of a physical event. Since either end of CD can be in question, both arrows of geometric time identifiable as the Minkowski time defined by the tips of CD are possible.

3. The simplest identification of the U-matrix is as the unitary U-matrix relating to each other the state basis for which M-matrices correspond to prepared states at two opposite ends of CD. Let us assume that the preparation has taken place at the "lower" end, the initial state. State function reduction for the final state means that one measures the single particle observables for the "upper" end of CD. This necessarily induces the loss of this property at the "lower" end. Next preparation in turn induces localization in the "lower" end. One has a kind of time flip-flop and the breaking of time reversal invariance would be absolutely essential for the non-triviality of the process.

The basic idea of Quantum Mathematics is that M-matrix is characterized by Feynman diagrams representing sequences of arithmetic operations and their co-arithmetic counterparts. The latter ones give rise to a superposition of pairs of direct summands (factors of tensor product) giving rise to same direct sum (tensor product). This vision would reduce quantum physics to generalized number theory. Universe would be calculating and the consciousness of the mathematician would be in the quantum jumps performing the state function reductions to which preparations reduce.

Note that direct sum, tensor product, and the counterpart of second quantization for Hilbert spaces in the proposed sense would be quantum mathematics counterpart for set theoretic operations, Cartesian product and formation of the power set in set theory.

ZEO, state function reduction, unitary process, and quantum mathematics

State function reduction acts in a tensor product of Hilbert spaces. In the p-adic context to be discussed n the following $x_n \otimes p^n$ is the natural candidate for this tensor product. One can assign a density matrix to a given entangled state of this system and calculate the Shannon entropy. One can also assign to it a number theoretical entropy if entanglement coefficients are rationals or even algebraic numbers, and this entropy can be negative. One can apply Negentropy Maximization Principle to identify the preferred states basis as eigenstates of the density matrix. For negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) the state function reduction tends to preserve the entanglement.

Could the state function reduction take place separately for each subspace $x_n \otimes p^n$ in the direct sum $\oplus_n x_n \otimes p^n$ so that one would have quantum parallel state function reductions? This is an old proposal motivated by the many-sheeted space-time. The direct summands in this case would correspond to the contributions to the states localizable at various space-time sheets assigned to different powers of p defing a scale hierarchy. The powers p^n would be associated with zero modes by the previous argument so that the assumption about independent reduction would reflect the super-selection rule for zero modes. Also different values of p-adic prime are present and tensor product between them is possible if the entanglement coefficients are rationals or even algebraics. In the formulation using adeles the needed generalization could be formulated in a straightforward manner.

How can one select the entangled states in the summands $x_n \otimes p^n$? Is there some unique choice? How do unitary process and state function reduction relate to this choice? Could the dynamics of Quantum Mathematics be a structural analog for a sequence of state function reductions taking place at the opposite ends of CD with unitary matrix U relating the state basis for which single particle states have well defined quantum numbers either at the upper or lower end of CD? Could the unitary process and state function reduction be identified solely from the requirement that zero energy states correspond to tensor products Hilbert spaces, which correspond to inverses of each other as numbers? Could the extension of arithmetics to include co-arithmetics make the dynamics in question unique?

3.4 Physics as extension of quantum measurement theory to a theory of consciousness

TGD inspired theory of consciousness could be seen as a generalization of quantum measurement theory to make observer, which in standard quantum measurement theory remains an outsider, a genuine part of physical system subject to laws of quantum physics. The basic notions are quantum jump identified as moment of consciousness and the notion of self [K50]: in zero energy ontology these notions might however reduce to each other. Negentropy Maximization Principle [K51] defines the dynamics of consciousness and as a special case reproduces standard quantum measurement theory.

3.4.1 Quantum jump as moment of consciousness

TGD suggests that the quantum jump between quantum histories could identified as moment of consciousness and could therefore be for consciousness theory what elementary particle is for physics [K50].

This means that subjective time evolution corresponds to the sequence of quantum jumps $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$ consisting of unitary process followed by state function process. Originally U was thought to be the TGD counterpart of the unitary time evolution operator $U(-t, t)$, $t \rightarrow \infty$, associated with the scattering solutions of Schrödinger equation. It seems however impossible to assign any real Schrödinger time evolution with U. In zero energy ontology U defines a unitary matrix between zero energy states and is naturally assignable to intentional actions whereas the ordinary S-matrix telling what happens in particle physics experiment (for instance) generalizes to M-matrix defining time-like entanglement between positive and negative energy parts of zero energy states. One might say that U process corresponds to a fundamental act of creation creating a quantum superposition of possibilities and the remaining steps generalizing state function reduction process select between them.

3.4.2 Negentropy Maximization Principle and the notion of self

U-process is followed by a sequence of state function reductions. Negentropy Maximization Principle (NMP [K51]) states that in a given quantum state the most quantum entangled subsystem-complement pair can perform the quantum jump. More precisely: the reduction of the entanglement entropy in the quantum jump is as large as possible. This selects the pair in question and in case of ordinary entanglement entropy leads the selected pair to a product state. The interpretation of the reduction of the entanglement entropy as conscious information gain makes sense. The sequence of state function reductions decomposes at first step the entire system to two parts in such a manner that the reduction entanglement entropy is maximal. This process repeats itself for subsystems. If the subsystem in question cannot be divided into a pair of entangled free system the process stops since energy conservation does not allow it to occur (binding energy).

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. Everything is consciousness but consciousness can be lost if self develops bound state entanglement during U process so that state function reduction to smaller un-entangled pieces is impossible.

The existence of number theoretical entanglement entropies in the intersection of real and various p-adic worlds force to modify this picture. The reduction process can stop also if the self in question allows only decompositions to pairs systems with negentropic entanglement. This does not require that that the system forms a bound state for any pair of subsystems so that the systems decomposing it can be free (no binding energy). This defines a new kind of bound state not describable as a jail defined by the bottom of a potential well. Subsystems are free but remain correlated by negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

The ordinary state function reductions imply dissipation crucial for self organization and quantum jump could be regarded as the basic step of an iteration like process leading to the asympotic self-organization patterns. One could regard dissipation as a Darwinian selector as in standard theories of self-organization. NMP thus predicts that self organization and hence presumably also fractalization can occur inside selves. NMP would favor the generation of negentropic entanglement. This notion is highly attractive since it could allow to understand how quantum self-organization generates larger coherent structures. Note that state function reduction for negentropic entanglement is highly deterministic since the number of degenerate states with same negative entanglement entropy is expected to be small. This could allow to understand how living matter is able to develop almost deterministic cellular automaton like behaviors.

A further implication of NMP is that Universe generates information about itself represented in terms of negentropic enanglement: if one is not afraid of esoteric associations one could call this information Akashich records. This is not in conflict with second law since the entropy in the case of second law is ensemble entropy assignable to single particle in thermodynamical description.

3.4.3 Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

The observation that Shannon entropy allows an infinite number of number theoretic variants for which the entropy can be negative in the case that probabilities are algebraic numbers leads to the idea that living matter in a well-defined sense corresponds to the intersection of real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

This picture has also a direct connection with consciousness [K51].

1. Algebraic entanglement is a prerequisite for the realization of intentions as transformations of p-adic space-time sheets to real space-time sheets representing actions. Essentially a leakage between p-adic and real worlds is in question and makes sense only in zero energy ontology. Since various quantum numbers in real and p-adic sectors are not in general comparable in positive energy ontology so that conservation laws would be broken in positive energy ontology. Algebraic entanglement could be also called cognitive since it is something between real and p-adic worlds. The transformation can occur if the partonic 2-surfaces and their 4-D tangent space-distributions are representable using rational functions with rational coefficients in preferred coordinates for the imbedding space dictated by symmetry considerations. Intentional systems must live in the intersection of real and p-adic worlds. For the minimal option life would be also effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests.

2. What happens that the Universe corresponding to given CD decomposes to two un-entangled subsystems, which in turn decompose, and the process continues until all subsystems have only entropic bound state entanglement or negentropic algebraic entanglement with the external world. If the sub-system generates entropic bound state entanglement in the process, it loses consciousness. The generation of negentropic entanglement means expansion of consciousness.

3. One can ask whether the entanglement entropy of the sub-system should be defined as a sum over entanglement entropies over all subsystems involved or whether the levels are independent. This hierarchy of subsystems corresponds to the hierarchy if sub-CDs so that for the first option the survival without a loss of consciousness depends on what happens at all levels below the highest level for a given self is this is assumed. In more concrete terms, ability to stay conscious depends on what happens at cellular level too. For the first option the stable evolution of systems having algebraic entanglement is expected to be a process proceeding from short to long length scales as the evolution of life indeed is.

4. U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. This would suggest that the choice of the type of entanglement is a volitional selection. A possible interpretation is as a choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices.

5. This formulation means a sharpening of the earlier statement "Everything is conscious and consciousness can be only lost" with the additional statement "This happens when non-algebraic bound state entanglement is generated". Clearly, the quantum criticality of TGD Universe seems to have many aspects and life as a critical phenomenon in the number theoretical sense is only one of them besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K75].

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua.

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

3.4.4 Two times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [K50]. This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subject future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schrödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined by the mental images assignable to sub-selves (having sub-CDs as imbedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images. The hypothesis is that self experiences these sequences of mental images as a continuous time flow. In absence of mental images self would have experience of "timelessness" in accordance with the reports of practitioners of various spiritual practices. Self would lose consciousness in quantum jump generating entropic entanglement and experience expansion of consciousness if the resulting entanglement is negentropic. The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

Thus the measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

3.4.5 General view about psychological time and intentionality

The recent TGD inspired attempts to understand the arrow of psychological time and the localization of the contents of conscious sensory experience and experienced volition to a rather narrow time interval of .1 seconds rely on zero energy ontology. The most argument below summarizes the most recent view [K6].

Why sensory experience is about so short time interval?

The picture based on CDs implies automatically the 4-D character of conscious experience and memories form part of conscious experience even at elementary particle level. Amazingly, the secondary p-adic time scale of electron characterizing the time scale of electronic CD is $T = 0.1$ seconds defining a fundamental time scale in living matter. The problem is to understand why the sensory experience is about a short time interval of geometric time rather than about the entire personal CD with temporal size of order life-time. The explanation would be that sensory input corresponds to sub-selves (mental images) with $T \simeq .1$ s at the upper light-like boundary of CD in question. This requires a strong asymmetry between upper and lower light-like boundaries of CDs.

The localization of the contents of the sensory experience to the upper light-cone boundary and local arrow of time could emerge as a consequence of self-organization process involving conscious intentional action. Sub-CDs would be in the interior of CD and self-organization process would lead to a distribution of CDs concentrated near the upper or lower boundary of CD. The local arrow of geometric time would depend on CD and even differ for CD and sub-CDs.

1. The localization of contents of sensory experience to a narrow time interval would be due to the concentration of sub-CDs representing mental images near the either boundary of CD representing self.

2. Phase conjugate signals identifiable as negative energy signals to geometric past are important when the arrow of time differs from the standard one in some time scale. If the arrow of time establishes itself as a phase transition, this kind of situations are rare. Negative energy signals as a basic mechanism of intentional action and transfer of metabolic energy would explain why living matter is so special.

3. Geometric memories would correspond to sub-selves in the interior of CD, the oldest of them to the regions near "lower" boundaries of CD. Since the density of sub-CDs is small there geometric memories would be rare and not sharp. A temporal sequence of mental images, say the sequence of digits of a phone number, would correspond to a temporal sequence of sub-CDs.

4. Sharing of mental images corresponds to a fusion of sub-selves/mental images to single sub-self by quantum entanglement: the space-time correlate could be flux tubes connecting space-time sheets associated with sub-selves represented also by space-time sheets inside their CDs.

Arrow of time

TGD forces a new view about the relationship between experienced and geometric time. Although the basic paradox of quantum measurement theory disappears the question about the arrow of geometric time remains. There are actually two times involved. The geometric time assignable to the space-time sheets and the M^4 time assignable to the imbedding space.

Consider first the the geometric time assignable to the space-time sheets.

1. Selves correspond to CDs. The CDs and their projections to the imbedding space do not move anywhere. Therefore the standard explanation for the arrow of geometric time cannot work.

2. The only plausible interpretation at classical level relies on quantum classical correspondence and the fact that space-times are 4-surfaces of the imbedding space. If quantum jump corresponds to a shift for a quantum superposition of space-time sheets towards geometric past in the first approximation (as quantum classical correspondence suggests), one can understand the arrow of time. Space-time surfaces simply shift backwards with respect to the geometric time of the imbedding space and therefore to the 8-D perceptive field defined by the CD. This creates in the materialistic mind a temporal variant of train illusion. Space-time as 4-surface and macroscopic and macro-temporal quantum coherence are absolutely essential for this interpretation to make sense.

Why this shifting should always take place to the direction of geometric past of the imbedding space? Does it so always? The proposed mechanism for the localization of sensory experience to a short time interval suggests an explanation in terms of intentional action.

1. CD defines the perceptive field for self. Selves are curious about the space-time sheets outside their perceptive field and perform quantum jumps tending to shift the superposition of the space-time sheets so that unknown regions of space-time sheets emerge to the perceptive field. Either the upper or lower boundary of CD wins in the competition and the arrow of time results as a spontaneous symmetry breaking. The arrow of time can depend on CD but tends to be the same for CD and its sub-CDs. Global arrow of time could establish itself by a phase transitions establishing the same arrow of time globally by a mechanism analogous to percolation phase transition.

2. Since the news come from the upper boundary of CD, self concentrates its attention to this region and improves the resolution of sensory experience. The sub-CDs generated in this manner correspond to mental images with contents about this region. Hence the contents of conscious experience, in particular sensory experience, tends to be about the region near the upper boundary.

The emergence of the arrow of time at the level of imbedding space reduces to a modification of the oldest TGD based argument for the arrow of time which is wrong as such. If physical objects correspond to 3-surfaces inside future directed light-cone then the sequence of quantum jumps implies a diffusion to the direction of increasing value of light-cone proper time. The challenge has been to give a precise formulation for this. In zero energy ontology the most recent formulation goes like this.

1. What seems clear now is the decisive role of ZEO and hierarchy of CDs, and the fact that the quantum arrow of geometric time is coded into the structure of zero energy states to a high extent. The still questionable but attractively simple hypothesis is that U matrix two basis with opposite quantum arrows of geometric time: is this assumption really consistent with what we know about the arrow of time? If this is the case, the question is how the relatively well-defined quantum arrow of geometric time implies the experienced arrow of geometric time. Should one assume the arrow of geometric time separately as a basic property of the state function reduction cascade or more economically- does it follow from the arrow of time for zero energy states?

2. The state function reductions occur alternately at the two boundaries of CD. If the reduction occurs at given boundary is immediately followed by a reduction at the opposite boundary, the arrow of time alternates: this does not conform with intuitive expectations: for instance, this would imply that there are two selves assignable to the opposite boundaries!

   Zero energy states are however de-localized in the moduli space CDs (size of CD plus discrete subgroup of Lorentz group defining boosts of CD leaving second tip invariant). One has quantum superposition of CDs with difference scales but with fixed upper or lower boundary belonging to the same light-cone boundary after state function reduction. In standard quantum measurement theory the repetition of state function reduction does not change the state but now it would give rise to the experienced flow of time. Zeno effect indeed requires that state function reductions can occur repeatedly at the same boundary. In these reductions the wave function in moduli degrees of freedom of CD changes. This implies "dispersion" in the moduli space of CDs experienced as flow of time with definite arrow.

3. This approach codes also the arrow of time at the space-time level: the average space-time sheet in quantum superposition increases in size as the average position of the "upper boundary" of CDs drifts towards future state function reduction by state function reduction.

4. In principle the arrow of time can temporarily change but it would seem that this can occur in very special circumstances and probably takes place in living matter. Phase conjugate laser beam is a non-biological example in this respect.

Chapter 4

TGD Inspired Theory of Consciousness

4.1 Introduction

The conflict between the non-determinism of state function reduction and determinism of time evolution of Schrödinger equation is serious enough a problem to motivate the attempt to extend physics to a theory of consciousness by raising the observer from an outsider to a key notion also at the level of physical theory. Further motivations come from the failure of the materialistic and reductionistic dogmas in attempts to understand consciousness in neuroscience context. There are reasons to doubt that standard quantum physics could be enough to achieve this goal and the new physics predicted by TGD is indeed central in the proposed theory.

4.1.1 Quantum jump as moment of consciousness and the notion of self

If quantum jump occurs between two different time evolutions of Schrödinger equation (understood here in very metaphorical sense) rather than interfering with single deterministic Schrödinger evolution, the basic problem of quantum measurement theory finds a resolution. The interpretation of quantum jump as a moment of consciousness means that volition and conscious experience are outside space-time and state space and that quantum states and space-time surfaces are "zombies". Quantum jump would have actually a complex anatomy corresponding to unitary process U, state function reduction and state preparation at least.

Quantum jump is expected to have a complex anatomy since it must include state preparation, state function reduction, and also unitary process characterized by U-matrix. Zero energy ontology means that one must distinguish between M-matrix and U-matrix. M-matrix characterizes the time like entanglement between positive and negative energy parts of zero energy state and is measured in particle scattering experiments. M-matrix need not be unitary and can be identified as a "complex" square root of density matrix representable as a product of its real and positive square root and of unitary S-matrix so that thermodynamics becomes part of quantum theory with thermodynamical ensemble being replaced with a zero energy state. The unitary U-matrix describes quantum transitions between zero energy states and is therefore something genuinely new. It is natural to assign the statistical description of intentional action with U-matrix since quantum jump occurs between zero energy states.

Negentropy Maximization Principle (NMP) codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to maximal reduction of entanglement entropy at each step. In the generic case this implies decomposition of the system to unique unentangled systems and the process repeats itself for these systems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states.

Intuitively self corresponds to a sequence of quantum jumps which somehow integrates to a larger unit much like many-particle bound state is formed from more elementary building blocks.

It also seems natural to assume that self stays conscious as long as it can avoid bound state entanglement with the environment in which case the reduction of entanglement is energetically impossible. One could say that everything is conscious and consciousness can be only lost when the system forms bound state entanglement with environment. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

The precise definition of self has remained a long standing problem and I have been even ready to identify self with quantum jump. Zero energy ontology allows what looks like a final solution of the problem. Self indeed corresponds to a sequence of quantum jumps integrating to single unit, but these quantum jumps correspond state function reductions to a fixed boundary of CD leaving the corresponding parts of zero energy states invariant. In positive energy ontology these repeated state function reductions would have no effect on the state but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and gives rise to self. The first quantum jump to the opposite boundary corresponds to the act of free will or wake-up of self. I would be forced by NMP since the increase of ordinary entropy inside self probably also means reduction of negentropy gain in state function reduction and eventually reduction to opposite boundary of CD is unavoidable by NMP.

Negentropy Maximization Principle (NMP) states that entanglement entropy tends to be reduced in state function reduction. In standard quantum measurement this would mean that reduction reduces the enganglement between the system and its complement. There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement, which need not be bound state entanglement in standard sense. Negentropic entanglement might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble.

The generation of negentropic entanglement means that the outcome of the reduction is not random: the prediction is that second law is not universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. Negentropic entanglement need not involved binding energy. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism [K30] . Generation of negentropic entanglement gives rise to what could be called Akashic records read consciously via interaction free quantum measurement: the Universe would be increasing its information resources.

The consistency with ordinary measurement theory requires that negentropic entanglement corresponds to a density matrix proportional to a unit matrix: this correspond to unitary entanglement characterizing quantum computation. The negentropic entanglement of this kind corresponds naturally to the hierarchy of Planck constants made possible by the non-determinism of Kähler action. There is also a connection with quantum criticality.

Self is assumed to experience sub-selves as mental images identifiable as "averages" of their mental images. This implies the notion of ageing of mental images as being due to the growth of ensemble entropy as the ensemble consisting of quantum jumps (sub-sub-sub-selves) increases. That sequence of sub-selves are experienced as separate mental images explains why we can distinguish between digits of phone number. The irreducible component of self (pure awareness) would correspond to the highest level in the "personal" hierarchy of quantum jumps and the sequence of lower level quantum jumps would be responsible for the experience of time flow. Entire life cycle would correspond to self at the highest(?) level of the personal self hierarchy and pure awareness would prevail during sleep: this would make it possible to experience directly that I existed yesterday.

4.1.2 Sharing and fusion of mental images

The standard dogma about consciousness is that it is completely private. It seems that this cannot be the case in TGD Universe. Von Neumann algebras known as hyper-finite factors of type II_1 (HFF) [K99, K27] provide the basic mathematical framework for quantum TGD and this suggests important modifications of the standard measurement theory besides those implied by the zero energy ontology predicting that all physical states have vanishing net quantum numbers and are creatable from vacuum. The notion of measurement resolution characterized in terms of Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs implies that entanglement is defined always modulo some resolution characterized by infinite-dimensional sub-Clifford algebra $\mathcal{N}$ playing a role analogous to that of gauge algebra.

This modification has also important implications for consciousness. For ordinary quantum measurement theory separate selves are by definition unentangled and the same applies to their sub-selves so that they cannot entangle and thus fuse and shared mental images are impossible: consciousness would be completely private.

Space-time sheets as correlates for selves however suggests that space-time sheets topologically condensed at larger space-time sheets and serving as space-time correlates for mental images can be connected by join along boundaries bonds so that mental images could fuse and be shared.

HFFs allow to realize mathematically this intuitive picture. The entanglement in $\mathcal{N}$ degrees of freedom between selves corresponding to $\mathcal{M}$ is below the measurement resolution so that these selves can be regarded as separate conscious entities. These selves can be said to be unentangled although their sub-selves corresponding to $\mathcal{N}$ (mental images at upper level) can entangle. Fusion and sharing of mental images becomes possible. For instance, in stereo vision right and left visual fields would fuse together. More generally, a pool of shared stereo mental images might be fundamental for evolution of social structures and development of social and moral rules and language (shared mental images make possible common meaning for symbols of language). A concrete realization for this would be in terms of hyper-genome making possible collective gene expression [K38, K47] .

4.1.3 Qualia

Since physical states are labeled by quantum numbers, various qualia correspond naturally to the increments of quantum numbers in quantum jump which leads to a general classification of qualia in terms of the fundamental symmetries [K34] . One can speak also about geometric qualia assignable to the increments of zero modes which correspond to the classical variables in ordinary quantum measurement theory and non-quantum fluctuating degrees of freedom which do not contribute to the metric of world of classical worlds (WCW) in TGD framework. Dark matter hierarchy suggests that also qualia form a hierarchy with larger values of Planck constant identifiable as more refined qualia. Rather amusingly, visual colors would correspond to increments of color quantum numbers assignable to quarks and gluons in standard model physics. The term "color", originally introduced as an algebraic joke, would directly relate to visual color.

4.1.4 Self-referentiality of consciousness

Quantum classical correspondence is the basic guiding principle of quantum TGD. Thanks to the failure of a complete determinism of classical dynamics, space-time surface can provide symbolic representations not only for quantum states (as maximal deterministic regions) but also for quantum jump sequences (sequences of quantum states) and thus for the contents of consciousness. These representations are regenerated in each quantum jump, and make possible the self referentiality of consciousness: self can be conscious of what it *was* conscious of.

The "Akashic records" realized in terms of negentropic entanglement are a natural candidate for self model.

4.1.5 Hierarchy of Planck constants and consciousness

The hierarchy of Planck constants is realized in terms of a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. $CD \times CP_2$ is generalized by gluing singular coverings and factor spaces of both CD and CP_2 together like pages of book along common back, which is 2-D sub-manifold which is M^2 for CD and homologically trivial geodesic sphere S^2 for CP_2 [K27] . The value of the Planck constant characterizes partially given page and arbitrary large values of $\hbar$ are predicted so that macroscopic quantum phases are possible since the fundamental quantum scales scale like $\hbar$. All particles in the vertices of Feynman diagrams have the same value of Planck constant so that particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to $\hbar$ changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory.

Dark matter hierarchy and p-adic length scale hierarchy would provide a quantitative formulation for the self hierarchy. To a given p-adic length scale one can assign a secondary p-adic time scale as the temporal distance between the tips of the causal diamond (pair of future and past directed light-cones in $H = M^4 \times CP_2$). For electron this time scale is .1 second, the fundamental biorhythm. For a given p-adic length scale dark matter hierarchy gives rise to additional time scales coming as $\hbar/\hbar_0$ multiples of this time scale. These two hierarchies could allow to get rid of the notion of self as a primary concept by reducing it to a quantum jump at higher level of hierarchy. Self would in general consists of quantum jumps inside quantum jumps inside... and thus experience the flow of time through sub-quantum jumps.

As already mentioned, it is possible to reduce the hierarchy of Planck constant to quantum criticality made possible by the non-determinism of Kähler action.

4.1.6 Zero energy ontology and consciousness

Zero energy ontology was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology.

Zero energy ontology combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. Zero energy ontology leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

At the imbedding space-level CD is the correlate of self whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates at the level of 4-D space-time. The hierarchy of CDs within CDs corresponds to the hierarchy of selves.

ZEO forces to generalize the quantum measurement theory since state function reduction is possible at either boundary of CD. This leads to a precise definition of self and allows to understand the arrow of time and the localization of the contents of sensory consciousness to such a narrow time interval (located near the future boundary of CD). Volition corresponds to the first quantum jump to opposite boundary of CD and thus reverses the arrow of time at some level of the self hierarchy.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD inspired theory of consciousness [L77]
- Negentropy Maximization Principle [L54]
- Quantum consciousness [L60]
- Zero Energy Ontology (ZEO) [L85]
- Quantum model of qualia [L63]
- Nature of time [L53]
- Quantum intelligence [L62]
- Intelligence and hierarchy of Planck constants [L44]

4.2 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP [K51]) stating that the reduction of entanglement entropy is maximal at a given step of state function reduction process following U-process is the basic variational principle for TGD inspired theory of consciousness and says that the information contents of conscious experience is maximal. Although this principle is diametrically opposite to the second law of thermodynamics it is structurally similar to the second law. NMP does not dictate the dynamics completely since in state function reduction any eigen state of the density matrix is allowed as final state. NMP need not be in contradiction with second law of thermodynamics which might relate as much to the ageing of mental images as to physical reality.

4.2.1 Number theoretic Shannon entropy as information

The notion of number theoretic entropy obtained by can be defined by replacing in Shannon entropy the logarithms of probabilities p_n by the logarithms of their p-adic norms $|p_n|_p$. This replacement makes sense for algebraic entanglement probabilities if appropriate algebraic extension of p-adic numbers is used. What is new that entanglement entropy can be negative, so that algebraic entanglement can carry information and NMP can force the generation of bound state entanglement so that evolution could lead to the generation of larger coherent bound states rather than only reducing entanglement. A possible interpretation for algebraic entanglement is in terms of experience of understanding or some positive emotion like love.

Standard formalism of physics lacks a genuine notion of information and one can speak only about increase of information as a local reduction entropy. It seems strange that a system gaining wisdom should increase the entropy of the environment. Hence number theoretic information measures could have highly non-trivial applications also outside the theory consciousness.

NMP combined with number theoretic entropies leads to an important exception to the rule that the generation of bound state entanglement between system and its environment during U process leads to a loss of consciousness. When entanglement probabilities are rational (or even algebraic) numbers, the entanglement entropy defined as a number theoretic variant of Shannon entropy can be non-positive (actually is) so that entanglement carries information. NMP favors the generation of algebraic entanglement. The attractive interpretation is that the generation of algebraic entanglement leads to an expansion of consciousness ("fusion into the ocean of consciousness") instead of its loss.

State function reduction period of the quantum jumps involves much more than in wave mechanics. For instance, the choice of quantization axes realized at the level of geometric delicacies related to CDs is involved. U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. If state function reduction involves also a choice between generic and negentropic entanglement (between real world, a particular p-adic world, or their intersection) it might be possible to identify a candidate for the physical correlate for the choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices. Note that if the total entanglement negentropy defined as sum of contributions from various levels of CD hierarchy up to the highest matters in NMP then also sub-selves should develop negentropic entanglement. For instance, the generation of entropic entanglement at cell level can lead to a loss of consciousness also at higher levels. Life would evolve from short to long scales.

4.2.2 About NMP and quantum jump

NMP is assumed to be the variational principle telling what can happen in quantum jump and says that the information content of conscious experience for the entire system is maximized. In zero energy ontology (ZEO) the definition of NMP is far from trivial and the recent progress - as I believe - in the understanding of structure of quantum jump forces to check carefully the details related to NMP. A very intimate connection between quantum criticality, life as something in the intersection of realities and p-adicities, hierarchy of effective values of Planck constant, negentropic entanglement (NE), and p-adic view about cognition emerges. One ends up also with an argument why p-adic sector is necessary if one wants to speak about conscious information. I will proceed by making questions.

What happens in single state function reduction?

State function reduction is a measurement of density matrix. The condition that a measurement of density matrix takes place implies standard measurement theory on both real and p-adic sectors: system ends to an *eigen-space* of density matrix. This is true in both real and p-adic sectors. NMP is stronger principle at the real side and implies state function reduction to 1-D subspace - its eigenstate.

The resulting N-dimensional space has however rational entanglement probabilities $p = 1/N$ so that one can say that it is the intersection of realities and p-adicities. If the number theoretic variant of entanglement entropy is used as a measure for the amount of entropy carried by entanglement rather than either entangled system, the state carries genuine information and is stable with respect to NMP if the p-adic prime p divides N. NMP allows only single p-adic prime for real $\rightarrow$ p-adic transition: the power of this prime appears is the largest power of prime appearing in the prime decomposition of N. Degeneracy means also criticality so that that ordinary quantum measurement theory for the density matrix favors criticality and NMP fixes the p-adic prime uniquely.

If one - contrary to the above conclusion - assumes that NMP holds true in the entire p-adic sector, NMP gives in p-adic sector rise to a *reduction* of the negentropy in state function reduction if the original situation is negentropic and the eigen-spaces of the density matrix are 1-dimensional. This situation is avoided if one assumes that state function reduction cascade in real or genuinely p-adic sector occurs first (without NMP) and gives therefore rise to N-dimensional eigen spaces. The state is negentropic and stable if the p-adic prime p divides N. Negentropy is generated.

The real state can be transformed to a p-adic one in quantum jump (defining cognitive map) if the entanglement coefficients are rational or belong to an algebraic extension of p-adic numbers in the case that algebraic extension of p-adic numbers is allowed (number theoretic evolution gradually generates them). The density matrix can be expressed as sum of projection operators multiplied by probabilities for the projection to the corresponding sub-spaces. After state function reduction cascade the probabilities are rational numbers of form $p = 1/N$.

Number theoretic entanglement entropy also allows to avoid some objections related to fermionic and bosonic statistics. Fermionic and bosonic statistics require complete anti-symmetrization/symmetrization. This implies entanglement which cannot be reduced away. By looking for symmetrized or antisymmetrized 2-particle state consisting of spin 1/2 fermions as the simplest example one finds that the density matrix for either particle is the simply unit 2×2 matrix. This is stable under NMP based on number theoretic negentropy. One expects that the same result holds true in the general case. The interpretation would be that particle symmetrization/antisymmetrization carries negentropy.

The degeneracy of the density matrix is of course not a generic phenomenon and one can argue that it corresponds to some very special kind of physics. The identification of space-time correlates for the hierarchy for the effective values $\hbar_{eff} = n\hbar$ of Planck constant as n-furcations of space-time sheet suggests strongly the identification of this physics in terms of this hierarchy. Hence quantum criticality, the essence of life as something in the rational intersection of realities and p-adicities, the hierarchy of effective values of $\hbar$, negentropic quantum entanglement, and the possibility to make real-p-adic transitions and thus cognition and intentionality would be very intimately related. This is a highly satisfactory outcome, since these ideas have been rather loosely related hitherto.

What happens in quantum jump?

Suppose that everything can be reduced to what happens for a given CD characterized by a scale. There are at least two questions to be answered.

1. There are two processes involved. State function reduction and quantum jump transforming real state to p-adic state (matter to cognition) and vice versa (intention to action). Do these transitions occur independently or not? Does the ordering of the processes matter? The proposed view about state function reduction strongly suggests that the p-adic $\leftrightarrow$real transition (if possible at all) can occur any time without affecting the outcome of the state function reduction.

2. State function reduction cascade in turn consists of two different kinds of state function reductions. The M-matrix characterizing the zero energy state is product of square root of density matrix and of unitary S-matrix and the first step means the measurement of the projection operator. It defines a density matrix for both upper and lower boundary of CD and these density matrices are essentially same.

   (a) At the first step a measurement of the density matrix between positive and negative energy parts of the quantum state takes place for CD. One can regard both the lower and upper boundary as an eigenstate of density matrix in absence of NE. The measurement is thus completely symmetric with respect to the boundaries of CDs. At the real sector this leads to a 1-D eigen-space of density matrix if NMP holds true. In the intersection of real and p-adic sectors this need not be the case if the eigenvalues of the density matrix have degeneracy. Zero energy state becomes stable against further state function reductions! The interactions with the external world can of course destroy the stability sooner or later. An interesting question is whether so called higher states of consciousness relate to this kind of states.

   (b) If the first step gave rise to 1-D eigen-space of the density matrix, a state function reduction cascade at either upper of lower boundary of CD proceeding from long to short scales. At given step divides the sub-system into two systems and the sub-system-complement pair which produces maximum negentropy gain is subject to quantum measurement maximizing negentropy gain. The process stops at given subsystem resulting in the process if the resulting eigen-space is 1-D or has NE (p-adic prime p divides the dimension N of eigenspace in the intersection of reality and p-adicity).

4.2.3 Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that life and conscious intelligence reside in the intersection of the real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

The identification of intentionality as the basic aspect of life seems to be consistent with this idea.

1. The proposed realization of the intentional action has been as a transformation of p-adic space-time sheet to a real one. Also transformations of real space-time sheets to p-adic space-time sheets identifiable as cognitions are possible. Algebraic entanglement is a prerequisite for the realization of intentions in this manner. Essentially a leakage between p-adic and real worlds is in question and makes sense only in zero energy ontology. The reason is that various quantum numbers in real and p-adic sectors are not in general comparable in positive energy ontology so that conservation laws would be broken or even cease to make sense.

2. The transformation of intention to action can occur if the partonic 2-surfaces and their 4-D tangent space-distributions are representable using rational functions with rational (or even algebraic) coefficients in preferred coordinates for the imbedding space dictated by symmetry considerations. Intentional systems must live in the intersection of real and p-adic worlds.

3. For the minimal option life would be also effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic partonic two-surfaces appears in U-matrix so that only the data from rational and some algebraic points of the partonic 2-surface dictate U-matrix. This means discretization at parton level and something which might be called number theoretic quantum field theory should emerge as a description of intentional action.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K26] . Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K75] .

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

4.2.4 Hyper-finite factors of type II_1 and NMP

Hyper-finite factors of type II_1 bring in additional delicacies to NMP. The basic implication of finite measurement resolution characterized by Jones inclusion is that state function reduction can never reduce entanglement completely so that entire universe can be regarded as an infinite living organism. It would seem that entanglement coefficients become $\mathcal{N}$ valued and the same is true for eigen states of density matrix. For quantum spinors associated with $\mathcal{M}/\mathcal{N}$ entanglement probabilities must be defined as traces of the operators $\mathcal{N}$. An open question is whether entanglement probabilities defined in this manner are algebraic numbers always (as required by the notion of number theoretic entanglement entropy) or only in special cases.

4.3 Time, memory, and realization of intentional action

Quantum classical correspondence requires that the flow of subjective time identified as a sequence of quantum jumps should have the flow of geometric time as a space-time correlate. The understanding of the detailed relationship between these two times has however remained a long standing problem, and only the emergence of zero energy ontology allows an ad hoc free model for how the flow and arrow of geometric time emerge, and answers why the relationship between geometric past and future is so asymmetric and why sensory experience is about so narrow interval of geometric time. Also the notion of self reduces in well-defined sense to the notion of quantum jump with fractal structure.

4.3.1 Two times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [K50] . This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subject future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined by the mental images assignable to subselves (having sub-CDs as imbedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images. The hypothesis is that self experiences these sequences of mental images as a continuous time flow. In absence of mental images self would have experience of "timelessness" in accordance with the reports of practitioners of various spiritual practices. Self would lose consciousness in quantum jump generating entropic entangelement and experience expansion of consciousness if the resulting entanglement is negentropic. The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

Thus the measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

The fact that the contents of conscious experience is about 4-D rather than 3-D space-time region, motivates the notions of 4-D brain, body, and even society. In particular, conscious existence continues after biological death since 4-D body and brain continue to exist.

4.3.2 About the arrow of psychological time

Quantum classical correspondence predicts that the arrow of subjective time is somehow mapped to that for the geometric time. The detailed mechanism for how the arrow of psychological time emerges has however remained open. Also the notion of self is problematic.

Two earlier views about how the arrow of psychological time emerges

The basic question how the arrow of subjective time is mapped to that of geometric time. The common assumption of all models is that quantum jump sequence corresponds to evolution and that by quantum classical correspondence this evolution must have a correlate at space-time level so that each quantum jump replaces typical space-time surface with a more evolved one.

1. The earliest model assumes that the space-time sheet assignable to observer ("self") drifts along a larger space-time sheet towards geometric future quantum jump by quantum jump: this is like driving car in a landscape but in the direction of geometric time and seeing the changing landscape. There are several objections.

   i) Why this drifting?

   ii) If one has a large number of space-time sheets (the number is actually infinite) as one has in the hierarchy the drifting velocity of the smallest space-time sheet with respect to the largest one can be arbitrarily large (infinite).

   iii) It is alarming that the evolution of the background space-time sheet by quantum jumps, which must be the quintessence of quantum classical correspondence, is not needed at all in the model.

2. Second model relies on the idea that intentional action -understood as p-adic-to-real phase transition for space-time sheets and generating zero energy states and corresponding real space-time sheets - proceeds as a kind of wave front towards geometric future quantum jump by quantum jump. Also sensory input would be concentrated on this kind of wave front. The difficult problem is to understand why the contents of sensory input and intentional action are localized so strongly to this wave front and rather than coming from entire life cycle.

There are also other models but these two are the ones which represent basic types for them.

The third option

The third explanation for the arrow of psychological time - which I have considered earlier but only half-seriously - looks to me the most elegant at this moment. This option is actually favored by Occam's razor since it uses only the assumption that space-time sheets are replaced by more evolved ones in each quantum jump. Also the model of topological quantum computation favors it. A more detailed discussion of this option can be found in [K6] . Here only a rough summary of the basic ideas is given.

1. In standard picture the attention would gradually shift towards geometric future and space-time in 4-D sense would remain fixed. Now however the fact that quantum state is quantum superposition of space-time surfaces allows to assume that the attention of the conscious observer is directed to a fixed volume of 8-D imbedding space. Quantum classical correspondence is achieved if the evolution in a reasonable approximation means shifting of the space-time sheets and corresponding field patterns backwards backwards in geometric time by some amount per quantum jump so that the perceiver finds the geometric future in 4-D sense to enter to the perceptive field. This makes sense since the shift with respect to M^4 time coordinate is an exact symmetry of extremals of Kähler action. It is also an excellent approximate symmetry for the preferred extremals of Kähler action and thus for maxima of Kähler function spoiled only by the presence of light-cone boundaries. This shift occurs for both the space-time sheet that perceiver identifies itself and perceived space-time sheet representing external world: both perceiver and percept change.

2. Both the landscape and observer space-time sheet remain in the same position in imbedding space but both are modified by this shift in each quantum jump. The perceiver experiences this as a motion in 4-D landscape. Perceiver (Mohammed) would not drift to the geometric future (the mountain) but geometric future (the mountain) would effectively come to the perceiver (Mohammed)!

3. There is an obvious analogy with Turing machine: what is however new is that the tape effectively comes from the geometric future and Turing machine can modify the entire incoming tape by intentional action. This analogy might be more than accidental and could provide a model for quantum Turing machine operating in TGD Universe. This Turing machine would be able to change its own program as a whole by using the outcomes of the computation already performed.

4. The concentration of the sensory input and the effects of conscious motor action to a narrow interval of time (.1 seconds typically, secondary p-adic time scale associated with the largest Mersenne M_{127} defining p-adic length scale which is not completely super-astronomical) can be understood as a concentration of sensory/motor attention to an interval with this duration: the space-time sheet representing sensory "me" would have this temporal length and "me" definitely corresponds to a zero energy state.

5. The fractal view about topological quantum computation strongly suggests an ensemble of almost copies of sensory "me" scattered along my entire life cycle and each of them experiencing my life as a separate almost copy.

6. The model of geometric and subjective memories would not be modified in an essential manner: memories would result when "me" is connected with my almost copy in the geometric past by braid strands or massless extremals (MEs) or their combinations (ME parallel to magnetic flux tube is the analog of Alfwen wave in TGD).

This argument leaves many questions open. What is the precise definition for the volume of attention? Is the attention of self doomed to be directed to a fixed volume or can quantum jumps change the volume of attention? What distinguishes between geometric future and past as far as contents of conscious experience are considered? How this picture relates to p-adic and dark matter hierarchies? Does this framework allow to formulate more precisely the notion of self? Zero energy ontology allows to give tentative answers to these questions.

4.3.3 Questions related to the notion of self

I have proposed two alternative notions of self and have not been able to choose between them. A further question is what happens during sleep: do we lose consciousness or is it that we cannot remember anything about this period? The work with the model of topological quantum computation has led to an overall view allowing to select the most plausible answer to these questions. But let us be cautious!

Can one choose between the two variants for the notion of self or are they equivalent?

I have considered two different notions of "self" and it is interesting to see whether the new view about time might allow to choose between them or to show that they are actually equivalent.

1. In the original variant of the theory "self" corresponds to a sequence of quantum jumps. "Self" would result through a binding of quantum jumps to single "string" in close analogy and actually in a concrete correspondence with the formation of bound states. Each quantum jump has a fractal structure: unitary process is followed by a sequence of state function reductions and preparations proceeding from long to short scales. Selves can have sub-selves and one has self hierarchy. The questionable assumption is that self remains conscious only as long as it is able to avoid entanglement with environment.

   Even slightest entanglement would destroy self unless on introduces the notion of finite measurement resolution applying also to entanglement. This notion is indeed central for entire quantum TGD also leads to the notion of sharing of mental images: selves unentangled in the given measurement resolution can experience shared mental images resulting as fusion of sub-selves by entanglement not visible in the resolution used.

2. According to the newer variant of theory, quantum jump has a fractal structure so that there are quantum jumps within quantum jumps: this hierarchy of quantum jumps within quantum jumps would correspond to the hierarchy of dark matters labeled by the values of Planck constant. Each fractal structure of this kind would have highest level (largest Planck constant) and this level would corresponds to the self. What might be called irreducible self would corresponds to a quantum jump without any sub-quantum jumps (no mental images). The quantum jump sequence for lower levels of dark matter hierarchy would create the experience of flow of subjective time.

It would be nice to reduce the original notion of self hierarchy to the hierarchy defined by quantum jumps. There are some objections against this idea. One can argue that fractality is a purely geometric notion and since subjective experience does not reduce to the geometry it might be that the notion of fractal quantum jump does not make sense. It is also not quite clear whether the reasonable looking idea about the role of entanglement as destroyer of self can be kept in the fractal picture.

These objections fail if one can construct a well-defined mathematical scheme allowing to understand what fractality of quantum jump at the level of space-time correlates means and showing that the two views about self are equivalent. The following argument represents such a proposal. Let us start from the causal diamond model as a lowest approximation for a model of zero energy states and for the space-time region defining the contents of sensory experience.

Let us make the following assumptions.

1. Assume the hierarchy of causal diamonds within causal diamonds in a sense to be specified more precisely below. Causal diamonds would represent the volumes of attention. Assume that the highest level in this hierarchy defines the quantum jump containing sequences of lower level quantum jumps in some sense to be specified. Assume that these quantum jumps integrate to single continuous stream of consciousness as long as the sub...-sub-self in question remains unentangled and that entangling means loss of consciousness or at least that it is not possible to remember anything about contents of consciousness during entangled state.

2. Assume that the contents of conscious experience come from the interior of the causal diamond. A stronger condition would be that the contents come from the boundaries of the two light-cones involved since physical states are defined at these in the simplest picture. In this case one could identify the lower light-cone boundary as giving rise to memory.

3. The time span characterizing the contents of conscious experience associated with a given quantum jump would correspond to the temporal distance T between the tips of the causal diamond. T would also characterize the average and approximate shift of the superposition of space-time surfaces backwards in geometric time in single quantum jump at a given level of hierarchy. This time scale naturally scales as $T_n = 2^n T_{CP_2}$ so that p-adic length scale hypothesis follows as a consequence. T would be essentially the secondary p-adic time scale $T_{2,p} = \sqrt{p} T_p$ for $p \simeq 2^k$. This assumption - absolutely essential for the hierarchy of quantum jumps within quantum jumps - would differentiate the model from the model in which T corresponds to either CP_2 time scale or p-adic time scale T_p. One would have hierarchy of quantum jumps with increasingly longer time span for memory and with increasing duration of geometric chronon at the highest level of fractal quantum jump. Without additional restrictions, the quantum jump at n^{th} level would contain 2^n quantum jumps at the lowest level of hierarchy. Note that in the case of sub-self - and without further assumptions which will be discussed next - one would have just two quantum jumps: mental image appears, disappears or exists all the time. At the level of sub-sub-selves 4 quantum jumps and so on. Maybe this kind of simple predictions might be testable.

4. We know that the contents of sensory experience comes from a rather narrow time interval of duration about .1 seconds, which corresponds to the time scale T_{127} associated with electron. We also know that there is asymmetry between positive and negative energy parts of zero energy states both physically and at the level of conscious experience. This asymmetry must have some space-time correlate. The simplest correlate for the asymmetry between positive and negative energy states would be that the upper light-like boundaries in the structure formed by light-cones within light-cones intersect along light-like radial geodesic. No condition of this kind would be posed on lower light-cone boundaries. The scaling invariance of this condition makes it attractive mathematically and would mean that arbitrarily long time scales T_n can be present in the fractal hierarchy of light cones. At all levels of the hierarchy all contribution from upper boundary of the causal diamond to the conscious experience would come from boundary of the same past directed light-cone so that the conscious experience would be sharply localized in time in the manner as we know it to be. The new element would be that content of conscious experience would come from arbitrarily large region of Universe and seing Milky Way would mean direct sensory contact with it.

5. These assumptions relate the hierarchy of quantum jumps to p-adic hierarchy. One can also include also dark matter hierarchy into the picture. For dark matter hierarchy the time scale hierarchy $\{T_n\}$ is scaled by the factor $r = \hbar/\hbar_0$ which can be also rational number. For $r = 2^k$ the hierarchy of causal diamonds generalizes without difficulty and there is a kind of resonance involved which might relate to the fact that the model of EEG favors the values of $k = 11n$, where $k = 11$ also corresponds in good approximation to proton-electron mass ratio. For more general values of $\hbar/\hbar_0$ the generalization is possible assuming that the position of the upper tip of causal diamond is chosen in such a manner that their positions are always the same whereas the position of the lower light-cone boundary would correspond to $\{rT_n\}$ for given value of Planck constant. Geometrically this picture generalizes the original idea about fractal hierarchy of quantum jumps so that it contains both p-adic hierarchy and hierarchy of Planck constants.

The contributions from lower the boundaries identifiable in terms of memories would correspond to different time scales and for a given value of time scale T the net contribution to conscious experience would be much weaker than the sensory input in general. The asymmetry between geometric now and geometric past would be present for all contributions to conscious experience, not only sensory ones. What is nice that the contents of conscious experience would rather literally come from the boundary of the past directed light-cone along which the classical signals arrive. Hence the mystic feeling about telepathic connection with a distant object at distance of billions of light years expressed by an astrophysicist, whose name I have unfortunately forgotten, would not be romantic self deception.

This framework explains also the sharp distinction between geometric future and past (not surprisingly since energy and time are dual): this distinction has also been a long standing problem of TGD inspired theory of consciousness. Precognition is not possible unless one assumes that communications and sharing of mental images between selves inside disjoint causal diamonds is possible. Physically there seems to be no good reason to exclude the interaction between zero energy states associated with disjoint causal diamonds.

The mathematical formulation of this intuition is however a non-trivial challenge and can be used to articulate more precisely the views about what WCW and configurations space spinor fields actually are mathematically.

1. Suppose that the causal diamonds with tips at different points of $H = M^4 \times CP_2$ and characterized by distance between tips T define sectors CH_i of the full WCW CH ("world of classical worlds"). Precognition would represent an interaction between zero energy states associated with different sectors CH_i in this scheme and tensor factor description is required.

2. Inside given sector CH_i it is not possible to speak about second quantization since every quantum state correspond to a single mode of a classical spinor field defined in that sector.

3. The question is thus whether the Clifford algebras and zero energy states associated with different sectors CH_i combine to form a tensor product so that these zero energy states can interact. Tensor product is required by the vision about zero energy insertions assignable to CH_i which correspond to causal diamonds inside causal diamonds. Also the assumption that zero energy states form an ensemble in 4-D sense - crucial for the deduction of scattering rates from M-matrix - requires tensor product.

4. The argument unifying the two definitions of self requires that the tensor product is restricted when CH_i correspond to causal diamonds inside each other. The tensor factors in shorter time scales are restricted to the causal diamonds hanging from a light-like radial ray at the upper end of the common past directed light-cone. If the causal diamonds are disjoint there is no obvious restriction to be posed, and this would mean the possibility of also precognition and sharing of mental images.

This scenario allows also to answers the questions related to a more precise definition of volume of attention. Causal diamond - or rather - the associated light-like boundaries containing positive and negative energy states define the primitive volume of attention. The obvious question whether the attention of a given self is doomed to be fixed to a fixed volume can be also answered. This is not the case. Selves can delocalize in the sense that there is a wave function associated with the position of the causal diamond and quantum jumps changing this position are possible. Also many-particle states assignable to a union of several causal diamonds are possible. Note that the identification of magnetic flux tubes as space-time correlates of directed attention in TGD inspired quantum biology makes sense if these flux tubes connect different causal diamonds. The directedness of attention in this sense should be also understood: it could be induced from the ordering of p-adic primes and Planck constant: directed attention would be always from longer to shorter scale.

What after biological death?

Could the new option allow to speculate about the course of events at the moment of death? Certainly this particular sensory "me" would effectively meet the geometro-temporal boundary of the biological body: sensory input would cease and there would be no biological body to use anymore. "Me" might lose its consciousness (if it can!). "Me" has also other mental images than sensory ones and these could begin to dominate the consciousness and "me" could direct its attention to space-time sheets corresponding to much longer time scale, perhaps even to that of life cycle, giving a summary about the life.

What after that? The Tibetan Book of Dead gives some inspiration. A western "me" might hope (and even try use its intentional powers to guarantee) that quantum Turing tape sooner later brings into the volume of attention (which might also change) a living organism, be it human or cat or dog or at least some little bug. If this "me" is lucky, it could direct its attention to it and become one of the very many sensory "me's" populating this particular 4-D biological body. There would be room for a newcomer unlike in the alternative models. A "me" with Eastern/New-Ageish traits could however direct its attention permanently to the dark space-time sheets and achieve what she might call enlightment.

Does sleep state involve a loss of consciousness?

The ability to avoid entropic entanglement with environment is essential for the original notion of self and in the case of sub-selves it would explain the finite life-time of mental images. Algebraic entanglement can be however negentropic and the idea that its generation does not lead to a loss of consciousness is attractive. If sleep really means a loss of consciousness it must lead to a generation of entropic entanglement. But does this really happen? Could sleep only lead to a loss of consciousness at those levels of self hiererachy responsible for conscious memories, which correspond to mental images and thus sub-CDs located in those space-time regions of CD, where the sleeping occurs?

Is the assumption about the loss of consciousness during sleep really necessary? Can one imagine good reasons for why we should remain conscious during sleep?

1. One could argue that if consciousness is really lost during sleep, we could not have the deep conviction that we existed yesterday.

2. Second argument is based on the assumption that brains are acting as topological quantum computers during sleep. During an ideal topological quantum computation the entanglement with the surrounding world is absent and thus topological quantum computation should correspond to a conscious experience with a vanishing entanglement entropy. Night time is the best time for topological quantum computation since sensory input and motor action do not take metabolic resources and we certainly do problem solving during sleep. Thus we should be conscious at some level during sleep and perform quite a long topological quantum computation. The problem with this argument is that the ideal topological quantum computation could be performed by a larger system than brain so that ability to perform topological quantum computation does not allow to conclude whether we are conscious during sleep or not. In fact, the idea that large number of brains entangle to a larger unit giving rise to a stereo consciousness about what it is to be human besides performing topological quantum computation like processes, is rather attractive.

Could it then be that we do not remember anything about the period of sleep because our attention is directed elsewhere and memory recall uses only copies of "me" assignable to brain manufacturing standardized mental images? Perhaps the communication link to the mental images during sleep experienced at dark matter levels of existence is lacking or sensory input and motor activities of busy westeners do not allow to use metabolic energy to build up this kind of communications. Hence one can at least half-seriously ask, whether self is actually eternal with respect to the subjective time and whether entangling with some system means only diving into the ocean of consciousness as someone has expressed it. Could we be Gods as also quantum classical correspondence in the reverse direction suggests (p-adic cognitive space-time sheets have literally infinite size in both temporal and spatial directions)?

4.3.4 Do declarative memories and intentional action involve communications with geometric past?

Communications with geometric past using time mirror mechanism (see fig. `http://www.tgdtheory.fi/appfigures/timemirror.jpg` or fig. 24 in the appendix of this book) in which phase conjugate photons propagating to the geometric past are reflected back as ordinary photons (typically dark photons with energies above thermal threshold) make possible realization of declarative memories in the brain of the geometric past [K72] .

This mechanism makes also possible realization of intentional actions as a process proceeding from longer to shorter time scales and inducing the desired action already in geometric past. This kind of realization would make living systems extremely flexible and able to react instantaneously to the changes in the environment. This model explains Libet's puzzling finding that neural activity seems to precede volition [J11] .

Also a mechanism of remote metabolism ("quantum credit card") based on sending of negative energy signals to geometric past becomes possible [K44] : this signal could also serve as a mere control signal inducing much larger positive energy flow from the geometric past. For instance, population inverted system in the geometric past could allow this kind of mechanism. Remote metabolism could also have technological implications.

4.3.5 Episodal memories as time-like entanglement

Time-like entanglement explains episodal memories as sharing of mental images with the brain of geometric past [K72] . An essential element is the notion of magnetic body which serves as an intentional agent "looking" the brain of geometric past by allowing phase conjugate dark photons with negative energies to reflect from it as ordinary photons. The findings of Libet about time delays related to the passive aspects of consciousness [J6] support the view that the part of the magnetic body corresponding to EEG time scale has the same size scale as Earth's magnetosphere. The unavoidable conclusion would be that our field/magnetic bodies contain layers with astrophysical sizes.

p-Adic length scale hierarchy and number theoretically preferred hierarchy of values of Planck constants, when combined with the condition that the frequencies f of photons involved with the communications in time scale T satisfy the condition $f \sim 1/T$ and have energies above thermal energy, lead to rather stringent predictions for the time scales of long term memory. The model for the hierarchy of EEGs relies on the assumption that these time scales come as powers $n = 2^{11k}$, $k = 0, 1, 2,,$ and predicts that the time scale corresponding to the duration of human life cycle is ~ 50 years and corresponds to $k = 7$ (amusingly, this corresponds to the highest level in chakra hierarchy).

4.4 Cognition and intentionality

4.4.1 Fermions and Boolean cognition

Fermionic Fock state basis defines naturally a quantum version of Boolean algebra. In zero energy ontology predicting that physical states have vanishing net quantum numbers, positive and negative energy components of zero energy states with opposite fermion numbers define realizations of Boolean functions via time-like quantum entanglement. One can also consider an interpretation of zero energy states in terms of rules of form $A \rightarrow B$ with the instances of A and B represented as elements Fock state basis fixed by the diagonalization of the density matrix defined by $M-$-matrix. Hence Boolean consciousness would be basic aspect of zero energy states. Physical states would be more like memes than matter. Note also that the fundamental super-symmetric duality between bosonic degrees of freedom (size and shape of the 3-surface) and fermionic degrees of freedom would correspond to the sensory-cognitive duality.

This would explain why Boolean and temporal causalities are so closely related. Note that zero energy ontology is certainly consistent with the usual positive energy ontology if unitary process U associated with the quantum jump is more or less trivial in the degrees of freedom usually assigned with the material world. There are arguments suggesting that U is tensor product of of factoring S-matrices associated with 2-D integrable QFT theories [K20] : these are indeed almost trivial in momentum degrees of freedom. This would also imply that our geometric past is rather stable so that quantum jump of geometric past does not suddenly change your profession from that of musician to that of physicist. The maximal diagonality of U-matrix for p-adic-to-real transitions would in turn favor precise realization of intentions as actions. One must however take this kind of arguments with extreme caution.

4.4.2 Fuzzy logic, quantum groups, and Jones inclusions

Matrix logic [A115] emerges naturally when one calculates expectation values of logical functions defined by the zero energy states with positive energy fermionic Fock states interpreted as inputs and corresponding negative energy states interpreted as outputs. Also the non-commutative version of the quantum logic, with spinor components representing amplitudes for truth values replaced with non-commutative operators, emerges naturally. The finite resolution of quantum measurement generalizes to a finite resolution of Boolean cognition and allows description in terms of Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of infinite-dimensional Clifford algebras of the world of classical worlds (WCW) identifiable in terms of fermionic oscillator algebras. $\mathcal{N}$ defines the resolution in the sense that quantum measurement and conscious experience does not distinguish between states differing from each other by the action of $\mathcal{N}$.

The finite-dimensional quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates the physical states modulo the resolution. This algebra is non-commutative which means that corresponding quantum spinors have non-commutative components. The non-commutativity codes for the that the spinor components are correlated: the quantized fractal dimension for quantum counterparts of 2-spinors satisfying $d = 2cos(\pi/n) \leq 2$ expresses this correlation as a reduction of effective dimension.

The moduli of spinor components however commute and have interpretation as eigenvalues of truth and false operators or probabilities that the statement is true/false. They have quantized spectrum having also interpretation as probabilities for truth values and this spectrum differs from the spectrum $\{1, 0\}$ for the ordinary logic so that fuzzy logic results from the finite resolution of Boolean cognition [K99] .

4.4.3 p-Adic physics as physics of cognition and intentionality

p-Adic physics as physics of cognition and intentionality provides a further element of TGD inspired theory of consciousness. At the fundamental level light-like 3-surfaces are basic dynamical objects in TGD Universe and have interpretation as orbits of partonic 2-surfaces. The generalization of the notion of number concept by fusing real numbers and various p-adic numbers to a more general structure makes possible to assign to real parton a p-adic prime p and corresponding p-adic partonic 3-surface obeying same algebraic equations. The almost topological QFT property of quantum TGD is an essential prerequisite for this. The intersection of real and p-adic 3-surfaces would consists of a discrete set of points with coordinates which are algebraic numbers. p-Adic partons would relate to both intentionality and cognition.

The transformation of p-adic variant of the partonic 3-surface with bosonic quantum numbers to its real counterpart in quantum jump would represent a transformation of intention to action and the unitary matrix U would govern this process. The larger the number of algebraic points in the intersection, the more precise the realization of intention as action would be.

Real fermion and its p-adic counterpart forming a pair would represent matter and its cognitive representation being analogous to a fermion-hole pair resulting when fermion is kicked out from Dirac sea. The larger the number of points in the intersection of real and p-adic surfaces, the better the resolution of the cognitive representation would be. This would explain why cognitive representations in the real world are always discrete (discreteness of numerical calculations represent the basic example about this fundamental limitation).

All transcendental p-adic integers are infinite as real numbers and one can say that most points of p-adic space-time sheets are at spatial and temporal infinity in the real sense so that intentionality and cognition would be literally cosmic phenomena. If the intersection of real and p-adic space-time sheet contains large number of points, the continuity and smoothness of p-adic physics should directly reflect itself as long range correlations of real physics realized as p-adic fractality. It would be possible to measure the correlates of cognition and intention and in the framework of zero energy ontology [K20] the success of p-adic mass calculations can be seen as a direct evidence for the role of intentionality and cognition even at elementary particle level: all matter would be basically created by intentional action as zero energy states.

4.4.4 Algebraic Brahman=Atman identity

The proposed view about cognition and intentionality emerges from the notion of infinite primes [K86] , which was actually the first genuinely new mathematical idea inspired by TGD inspired consciousness theorizing. Infinite primes, integers, and rationals have a precise number theoretic anatomy. For instance, the simplest infinite primes correspond to the numbers $P_{\pm} = X \pm 1$, where $X = \prod_k p_k$ is the product of all finite primes. Indeed, $P_{\pm}\ mod\ p = 1$ holds true for all finite primes. The construction of infinite primes at the first level of the hierarchy is structurally analogous to the quantization of super-symmetric arithmetic quantum field theory with finite primes playing the role of momenta associated with fermions and bosons. Also the counterparts of bound states emerge. This process can be iterated: at the second level the product of infinite primes constructed at the first level replaces X and so on.

The structural similarity with repeatedly second quantized quantum field theory strongly suggests that physics might in some sense reduce to a number theory for infinite rationals M/N and that second quantization could be followed by further quantizations. As a matter fact, the hierarchy of space-time sheets could realize this endless second quantization geometrically and have also a direct connection with the hierarchy of logics labeled by their order. This could have rather breathtaking implications.

1. One is forced to ask whether this hierarchy corresponds to a hierarchy of realities for which level below corresponds in a literal sense infinitesimals and the level next above to infinity.

2. Second implication is that there is an infinite number of infinite rationals behaving like real units ($M/N \equiv 1$ in real sense) so that space-time points could have infinitely rich number theoretical anatomy not detectable at the level of real physics. Infinite integers would correspond to positive energy many particle states and their inverses (infinitesimals with number theoretic structure) to negative energy many particle states and $M/N \equiv 1$ would be a counterpart for zero energy ontology to which oneness and emptiness are assigned in mysticism.

3. Single space-time point, which is usually regarded as the most primitive and completely irreducible structure of mathematics, would take the role of Platonia of mathematical ideas being able to represent in its number theoretical structure even the quantum state of entire Universe. Algebraic Brahman=Atman identity and algebraic holography would be realized in a rather literal sense.

This number theoretical anatomy should relate to mathematical consciousness in some manner. For instance, one can ask whether it makes sense to speak about quantum jumps changing the number theoretical anatomy of space-time points and whether these quantum jumps give rise to mathematical ideas. In fact, the identifications of Platonia as spinor fields in WCW on one hand and as the set number theoretical anatomies of point of imbedding space force the conclusion that WCW spinor fields (recall also the identification as correlates for logical mind) can be realized in terms of the space for number theoretic anatomies of imbedding space points. Therefore quantum jumps would be correspond to changes in anatomy of the space-time points. Imbedding space would be experiencing genuine number theoretical evolution. The whole physics would reduce to the anatomy of numbers. All mathematical notions which are more than mere human inventions would be imbeddable to the Platonia realized as the number theoretical anatomies of single imbedding space point.

In [K21, K86] a concrete realization of this vision is discussed by assuming hyper-octonionic infinite primes as a starting point. In this picture associativity and commutativity are assigned only to infinite integers representing many particle states but not necessarily to infinite primes themselves: this guarantees the well-definedness of the space-time surface assigned to the infinite rational. Quantum states are required to be associative in the sense that they correspond to quantum super-positions of all possible associations for the products of (infinite) primes (say $|A(BC)\rangle + |(AB)C\rangle$). The ground states of super conformal representations would correspond to infinite primes mappable to space-time surfaces (quantum classical correspondence). The excited states of super-conformal representations would be represented as quantum entangled states in the tensor product of state spaces $\mathcal{H}_{h_k}$ formed from Schrödinger amplitudes in discrete subsets of the space of 8 real units associated with imbedding space 8 coordinates at point h_k: the interpretation is in terms of a 8-fold tensor power of basic super-conformal representation. Although the representations are not completely local at the level of imbedding space, they involve only a discrete set of points identifiable as arguments of n-point function. The basic symmetries of the standard model reduce to number theory if hyper-octonionic infinite rationals are allowed. Color confinement reduces to rationality of infinite integers representing many particle states.

4.5 Quantum information processing in living matter

The notion of magnetic body leads to a dramatic modification of the views about functions of brain. In the following the discussion the the new vision about life as number theoretically critical phenomenon is not discussed separately.

4.5.1 Magnetic body as intentional agent and experiencer

In TGD Universe brain would be basically a builder of symbolic representations assigning a meaning to the sensory input by decomposing sensory field to objects and making possible effective motor control by magnetic body containing dark matter. A concrete model for how magnetic controls biological body and receives information from it is discussed in the model for the hierarchy of EEGs [K24] .

Also magnetic body could have sensory qualia, which should be in a well-defined sense more refined than ordinary sensory qualia [K34] . The quantum number increments associated with cyclotron phase transitions of dark ion cyclotron condensates at magnetic body could correspond to emotional and cognitive content of sensory input and would indeed have interpretation as higher level sensory qualia. Right brain sings – left brain talks metaphor would characterize this emotional-cognitive distinction for higher level qualia and would correspond to coding of sensory input from brain by frequency patterns *resp.* temporal patterns (analogs of phonemes). These qualia would be somatosensory qualia at the level of magnetic body.

Remote mental interactions between magnetic body and biological body are a key element of this picture. Remote mental interactions in the usual sense of the world would occur between magnetic body and some other, not necessary biological, body. This would include receival of sensory input from and motor control of other than own body. Also "dead" matter possesses magnetic bodies so that also psychokinesis would be based on the same mechanism. Magnetic body for which dissipation is much smaller than for ordinary matter (proportional to $1/\hbar$, would presumably continue its conscious existence after biological death and find another biological body and use it as a tool of sensory perception and intentional action.

4.5.2 Summary about the possible role of the magnetic body in living matter

The notion of magnetic/field body is probably the feature of TGD inspired theory of quantum biology which creates strongest irritation in standard model physicist. A ridicule as some kind of Mesmerism might be the probable reaction. The notion of magnetic/field body has however gradually gained more and more support and it is now an essential element of TGD based view about living matter. In the following I list the basic applications in the hope that the overall coherency of the picture might force some readers to take this notion seriously. I will talk only about magnetic body although it is clear that field body has also electric parts as well as radiative parts realized in terms of "massless extremals" or topological light rays.

In the following discussion the possible implications of the idea that living matter resides in the intersection of real and p-adic worlds is not taken into account. An attractive working hypothesis is that negentropic entanglement can be assigned to the magnetic bodies. For instance, the ends of the magnetic flux tubes connecting (say) biomolecules could be entangled negentropically. This idea has been already applied to explain the stability of high energy phosphate bond and of DNA polymers, which are highly charged [K30] .

Anatomy of magnetic body

Consider first the anatomy of the magnetic body.

1. Magnetic body has a fractal onion like structure with decreasing magnetic field strengths and the highest layers can have astrophysical sizes. Cyclotron wave length gives an estimate for the size of particular layer of magnetic body. $B = .2$ Gauss is the field strength associated with a particular layer of the magnetic body assignable to vertebrates and EEG. This value is not the same as the nominal value of the Earth's magnetic field equal to .5 Gauss. It is quite possible that the flux quanta of the magnetic body correspond to those of wormhole magnetic field and thus consist of two parallel flux quanta which have opposite time orientation. This is true for flux tubes assigned to DNA in the model of DNA as a topological quantum computer.

2. The layers of the magnetic body are characterized by the values of Planck constant and the matter at the flux quanta can be interpreted as macroscopically quantum coherent dark matter. This picture makes sense only if one accepts the generalization of the notion of imbedding space.

3. In the case of wormhole magnetic fields it is natural to assign a definite temporal duration to the flux quanta and the time scales defined by EEG frequencies are natural. In particular, the inherent time scale .1 seconds assignable to electron as a duration of zero energy space-time sheet having positive and negative energy electron at its ends would correspond to 10 Hz cyclotron frequency for ordinary value of Planck constant. For larger values of Planck constants the time scale scales as $\hbar$. Quite generally, a connection between p-adic time scales of EEG and those of electron and lightest quarks is highly suggestive since light quarks play key role in the model of DNA as topological quantum computer.

4. TGD predicts also hierarchy of scaled variants of electro-weak and color physics so that ZXG, QXG, and GXG corresponding to Z^0 boson, W boson, and gluons appearing effectively as massless particles below some biologically relevant length scale suggest themselves. In this phase quarks and gluons are unconfined and electroweak symmetries are unbroken so that gluons, weak bosons, quarks and even neutrinos might be relevant to the understanding of living matter. In particular, long ranged entanglement in charge and color degrees of freedom becomes possible. For instance, TGD based model of atomic nucleus as nuclear string suggests that biologically important fermionic could be actually chemically equivalent bosons and form cyclotron Bose-Einstein condensates.

Functions of the magnetic body

The list of possible functions of the magnetic body is already now rather impressive.

1. Magnetic body controls biological body and receives sensory data from it. Together with zero energy ontology and new view about time explains Libet's strange findings about time lapses of consciousness. EEG, or actually fractal hierarchy of EXGs assignable to various body parts makes possible communications to and control by the various layers of the magnetic body. WXG could induce charge density gradients by the exchange of W boson.

2. The flux sheets of the magnetic body traverse through DNA strands. The hierarchy of Planck constants and quantization of magnetic flux predicts that the flux sheets can have arbitrarily large width. This leads to the idea that there is hierarchy of genomes corresponding to ordinary genome, supergenome consisting of genomes of several cell nuclei arranged along flux sheet like lines of text, and hypergenomes involving genomes of several organisms arranged in a similar manner. The prediction is coherent gene expression at the level of organ, and even of population. In this picture the big jumps in evolution, in particular, the emergence of EEG, could be seen as the emergence of a new larger layer of magnetic body characterized by a larger value of Planck constant. For instance, this would allow to understand why the quantal effects of ELF em fields requiring so large value of Planck constant that cyclotron energies are above thermal energy at body temperature are observed for vertebrates only.

3. Magnetic body makes possible information process in a manner highly analogous to topological quantum computation. The model of DNA as topological quantum computer assumes that flux tubes of wormhole magnetic field connect DNA nucleotides with the lipids of the lipid layer of nuclear or cell membrane. The flux tubes would continue through the membrane and split during topological quantum computation. The time-like braiding of flux tubes makes possible topological quantum computation via time-like braiding and space-like braiding makes possible the representation of memories. The model allows general vision about the deeper meaning of the structure of cell and makes testable predictions about DNA.

   One prediction is the coloring of braid strands realized by an association of quark or antiquark to nucleotide. Color and spin of quarks and antiquarks would thus correspond to the quantum numbers assignable to braid ends. Color isospin could replace ordinary spin as a representation of qubit and quarks would naturally give rise to qutrit, with third quark would have interpretation as unspecified truth value. Fractionization of these quantum numbers takes place which increases the number of degrees of freedom. This prediction would relate closely to the discovery of topologist Barbara Shipman that the model for the honeybee dance suggests that quarks are in some manner involved with cognition. Also microtubules associated with axons connected to a space-time sheet outside axonal membrane via lipids could be involved with topological quantum computation and actually define an analog of a higher level programming language.

4. The strange findings about the behavior of cell membrane, in particular the finding that metabolic deprivation does not lead to the death of cell, the discovery that ionic currents through the cell membrane are quantal, and that these currents are essentially similar than those through an artificial membrane, suggest that the ionic currents are dark ionic Josephson currents along magnetic flux tubes. A high percent of biological ions would be dark and ionic channels and pumps would be responsible only for the control of the flow of ordinary ions through cell membrane.

5. These findings together with the discovery that also nerve pulse seems to involve only low dissipation lead to a model of nerve pulse in which dark ionic currents automatically return back as Josephson currents without any need for pumping. This does not exclude the possibility that ionic channels might be involved with the generation of nerve pulse so that the original view about quantal currents as controllers of the generation of nerve pulse would be turned upside down. Nerve pulse would result as a perturbation of kHz soliton sequence mathematically equivalent to a situation in which a sequence of gravitational penduli rotates with constant phase difference between neighbors except for one pendulum which oscillates and oscillation moves along the sequence with the same velocity as the kHz wave. The oscillation would be induced by a "kick" for which one can imagine several mechanisms.

   The model explains features of nerve pulse not explained by Hodkin-Huxley model. These include the mechanical changes associated with axon during nerve pulse, the outwards force generated by nerve pulse with a correct prediction for its order of magnitude, the adiabatic character of nerve pulse, and the small rise of temperature of membrane during pulse followed by a reduction slightly below the original temperature.

   The model predicts that the time taken to travel along any axon is a multiple of time dictated by the resting potential so that synchronization is an automatic prediction. Not only kHz waves but also a fractal hierarchy of EEG (and EXG) waves are induced as Josephson radiation by voltage waves along axons and microtubules and by standing waves assignable to neuronal (cell) soma. The value of Planck constant involved with flux tubes determines the frequency scale of EXG so that a fractal hierarchy results.

The model forces to challenge the existing interpretation of nerve pulse patterns and the function of neural transmitters. Neural transmitters need not represent actual/only) signal but could be more analogous to links in quantum web. The transmitter would coding the address of the receiver, which could be gene inside neuronal nucleus. Nerve pulses would build a connection line between sender and receiver of nerve pulse along which actual signals would propagate. Also quantum entanglement between receiver and sender can be considered.

6. Acupuncture points, meridians, and Chi are key notions of Eastern medicine and find a natural identification in terms of magnetic body lacking from the western medicine. Also a connection with well established notions of DC currents and potentials discovered by Becker and with TGD based view about universal metabolic currencies as differences of zero point energies for pairs of space-time sheets with different p-adic length scale emerges.

   Chi would correspond to these fundamental metabolic energy quanta to which ordinary chemically stored metabolic energy would be transformed. Meridians would most naturally correspond to flux tubes with large $\hbar$ along which dark supra currents flow without dissipation and transfer the metabolic energy between distant cells. Acupuncture points would correspond to points between which metabolic energy is transferred and their high conductivity and semiconductor like behavior would conform with the interpretation in terms of metabolic energy storages. The energy gained in the potential difference between the points would help to kick the charge carrier to a smaller space-time sheet. It is possible that the main contribution to the of charge at magnetic flux tube is magnetic energy and slightly below the metabolic energy quantum and that the voltage difference gives only the lacking small energy increment making the transfer possible. Also direct kicking of charge carriers to smaller space-time sheets by photons is possible and the observed action spectrum for IR and red photons corresponds to the predicted increments of zero point kinetic energies.

7. Magnetic flux tubes could also play key role in bio-catalysis and explain the magic ability of biomolecules to find each other. The model of DNA as topological quantum computer [K26] suggest that not only DNA and its conjugate but also some amino-acid sequences acting as catalysts could be connected to DNA and other amino-acids sequences or more general biomolecules by flux tubes acting as colored braid strands. The shortening of the flux tubes in a phase transition reducing the value of Planck constant would make possible extremely selective mechanisms of catalysis allowing precisely defined locations of reacting molecules to attach to each other. With recently discovered mechanism for programming sequences of biochemical reactions this would make possible to understand the miraculous looking feats of bio-catalysis.

8. The ability to construct "stories", temporally scaled down or possible also scaled up representations about the dynamical processes of external world, might be one of the key aspects of intelligence. There is direct empirical evidence for this activity in hippocampus. The phase transitions reducing or increasing the value of Planck constant would indeed allow to achieve this by scaling the time duration of the zero energy space-time sheets providing cognitive representations.

Direct experimental evidence for the notion of magnetic body carrying dark matter

The list of nice things made possible by the magnetic body is impressive and one can ask whether there is any experimental support for this notion. The findings of Peter Gariaev and collaborators give evidence for the representation of DNA sequences based on the coding of nucleotide to a rotation angle of the polarization direction as photon travels through the flux tube and for the decoding of this representation to gene activation [I8] , for the transformation of laser light to light at various radio-wave frequencies having interpretation in terms of phase transitions increasing $\hbar$ [I7, I1] , and even for the possibility to photograph magnetic flux tubes containing dark matter by using ordinary light in UV-IR range scattered from DNA [I13] .

4.5.3 Brain and consciousness

In the proposed vision the role of brain for consciousness is not so central than in neuroscience view. Brain is not the seat of sensory mental images but builder of symbolic representations and magnetic body replaces brain as an intentional agent and higher level experiencer. Furthermore, p-adic view about cognition means that only cognitive representations but not cognition itself can be localized in a finite space-time region.

The simplest sensory qualia would be realized at the level of sensory organs so that one can avoid the problematic assignment of sensory qualia to the sensory pathways. The new view about time would allow to resolve the objections against this view. For instance, phantom leg phenomenon would result by sharing of sensory mental images of the geometric past by time like quantum entanglement. For instance, visual colors would correspond to increments of color quantum numbers in quantum jumps at the level of retina. Our sensory mental images do not correspond to the sensory input as such. Rather, the feedback from brain (or from magnetic body via brain) to sensory organs is an essential element in the construction of sensory mental images. For instance, during REM sleep rapid eye movements would reflect the presence of this feedback. The feedback would be also very important in the case of hearing. Visual mental images in absence of eye movements could be interpreted as sharing of visual mental images by quantum entanglement (in particular, time-like entanglement giving rise to episodal memories).

Chapter 5

Overall View About Evolution of TGD

5.1 Introduction

Topological Geometrodynamics was born for 37years ago as an attempt to construct a Poincare invariant theory of gravitation by assuming that physically allowed space-times are representable as surfaces in the space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 is complex projective space having real dimension four (see the appendix of the book).Poincare group was identified as the isometry group of M^4 rather than of the space-time surface itself. The isometries of CP_2 were identified as color group and the geometrization of electro-weak gauge fields and elementary particle quantum numbers was achieved in terms of the spinor structure of CP_2. Rather remarkably, after 37 years one can still say that CP_2 codes the known elementary particle quantum numbers and interactions in its geometry. The construction of quantum theory suggests the replacement of M^4 with M^4_+, the interior of the future light cone of Minkowski space so that Poincare invariance is broken by the global geometry of the light cone but not locally.

It took almost half decade to develop the new view about space-time implied by the basic hypothesis: this is summarized in my PhD thesis [C26] . The construction of a mathematical theory around these physically very attractive ideas became the basic challenge and I have devoted my professional life to the realization of this dream. The great idea was that quantum physics reduces to the construction of Kähler metric and spinor structure for the infinite-dimensional space CH of all possible 3-surfaces of H. Physical states correspond to classical spinor fields in this space and a natural geometrization of fermionic statistics in terms of gamma matrices emerges [K40, K18] .

p-Adic number fields R_p [A50] (one number field for each prime obtained as a completion of the rational numbers) emerged for about ten years ago as a separate thread only loosely related to quantum TGD. What made them so attractive was that, with certain additional assumptions about physically favored p-adic primes, it became possible to understand the basic elementary particle mass scales number theoretically. This led to a successful calculation of the elementary particle masses using p-adic thermodynamics assuming that Super Virasoro algebra and related Kac Moody algebras, which are also basic algebraic structures of string models, act as symmetries of TGD [K48, K56, K57, K52] . The success of the mass calculations in turn forced the attempts to understand how Super Virasoro and related symmetries might emerge from basic TGD. Several trials led finally to the realization that these super algebras (or actually the proper generalizations of them) are the basic symmetries of quantum TGD. One of the most dramatic predictions is the uniqueness of the space H: quantum TGD exists mathematically (cancellation of various infinities occurs) only for the space $M^4_+ \times CP_2$, the choice which is forced also by the cosmological and symmetry considerations. One can say that infinite-dimensional Kähler geometric existence and thus physics is unique.

A third thread to the development emerged when I started systematic development of TGD inspired theory of consciousness [K89] . This work has led to dramatic increase of understanding also at the level of basic quantum TGD and allowed to develop quantum measurement theory in which conscious observer is not anymore Cartesian outsider but an essential part of quantum physics. The need to understand the mechanism making bio-systems macroscopic quantum systems led to a dramatic progress in the understanding of the new physics implied by the notion of many-sheeted space-time. Dramatic change in views about the relation between subjectively experienced and geometric time of physicist emerges and leads to the solution of the basic paradoxes of quantum physics. It became also clear that p-adic numbers are indeed an absolutely essential element of the mathematical formulation of quantum TGD proper and that the general properties of quantum TGD force the introduction of the p-adic numbers. One can say that physics involves both real and p-adic number fields with real numbers describing the topology of the real world and various p-adic number fields serving as correlates of cognition with the prime p labelling the p-adic topology serving as kind of intelligence quotient.

A further thread into the development of ideas came from the realization that physics might be basically number theory in generalized sense. TGD more or less forces the notion of infinite primes [K86] , and it turned out that their construction reduces to a repeated second quantization of arithmetic quantum field theory. Generalization of the concept of integer and real number emerges implying that the configuration space and state space of TGD could be imbedded into the field of generalized reals which is infinite-dimensional algebraic extension of ordinary reals. Physics could be basically theory of generalized reals! The dimensions of space-time *resp.* imbedding space correspond to the dimensions of quaternion *resp.* octonion fields as well as the dimensions of algebraic extensions of $p > 2$- *resp.* 2-adics allowing square root of ordinary p-adic number. The discussions with Tony Smith suggested that one can endow space-time and imbedding space with what might be called local quaternion and octonion structures.

This stimulated a development, which led to the notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic , and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [K88] . What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

Further important number theoretical ideas emerged from the attempt to construct a model for how intentions are transformed to actions. The process was interpreted as a quantum jump in which p-adic space-time sheet representing intention is transformed to a real one. This model led to a bundle of ideas and conjectures.

1. The core idea is the generalization of the notion of number obtained by gluing all number number fields together along rationals and algebraic numbers common to them. This means a generalization of the notion of manifold. In particular, imbedding space is obtained by gluing real and p-adic imbedding spaces together along rational points. This picture also justifies the decomposition of space-time surface to real and p-adic space-time sheets. Also finite-dimensional algebraic extensions, even extensions involving transcendentals like e are needed.

2. p-Adic space-time sheets are identified as correlates of intentionality and cognition. The differences between real and p-adic topologies (two rationals near to each other as p-adic numbers are very far in real sense) have deep implications concerning the understanding of cognitive consciousness. The evolution of cognition corresponds naturally to the increase of the p-adic prime and dimension of the extension of p-adic numbers.

3. Real physics and various p-adic physics are obtained from finitely extended rational physics by algebraic continuation to p-adic number fields and their extensions analogous to analytic continuation in complex analysis. This algebraic continuation is performed both at space-time level, state space level, and configuration space level. One can also generalize the notion of unitarity and the generalization poses extremely strong conditions on S-matrix.

This chapter represents a overall view of classical TGD, a discussion of the p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, and the vision about physics as generalized number theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Overall view about TGD [L55]
- What TGD is [L84]
- TGD as unified theory of fundamental interactions [L75]
- Basic TGD [L26]
- Basic TGD [L27]
- Space-time as 4-surface in $M^4 \times CP_2$ [L67]

5.2 Evolution of classical TGD

The TGD based space-time concept means a radical generalization of standard views already in the real context. Many-sheetedness means a hierarchy of space-time sheets of increasing size making possible to understand the emergence of structures in terms of the macroscopic space-time topology. The non-determinism of the Kähler action forces the notion of the association sequence defined as a union of space-like 3-surfaces with time-like separations: association sequence provides a geometric correlate for thought as simulation of the classical history. Non-determinism forces also the notion of mind like space-time sheet defined as a space-time sheet having finite temporal duration, which is an attractive candidate for the geometric correlate of self. Topological field quantization means that space-time topology provides classical correlates for the basic notions of the quantum field theory. The decomposition of space-time surface into real and p-adic regions brings in besides the matter also cognitive representations of material world.

5.2.1 Quantum classical correspondence and why classical TGD is so important?

In standard quantum physics classical theory is seen as a result of some kind of approximation procedure, say stationary phase approximation. In TGD framework classical physics is an exact part of quantum physics, and even more of WCW geometry since, apart from the complications caused by the classical non-determinism of the Kähler action, the definition of the Kähler geometry in terms of Kähler action assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$.

The evolution of TGD inspired theory of consciousness has gradually led to the notion of quantum classical correspondence which states that every quantum aspect of existence has space-time correlate. The correspondence is certainly not faithful but rather like the representation of contents of consciousness provided by spoken or written language. Space-time surface can be indeed seen as a symbolic representation, kind of written language. Not only the characteristics of quantum states, but also quantum jumps and their sequences defining the contents of conscious experience, have space-time correlates made possible by the classical determinism of the Kähler action, and the inherent p-adic non-determinism of p-adic counterparts of the field equations. In fact, there are reasons to believe that classical non-determinism of the Kähler action and a p-adic non-determinism have close relationship in the sense that the effective topology of the real space-time sheets is expected to correspond to p-adic topology in some length scale range.

5.2.2 Classical fields

In TGD framework the physics of classical fields are an essential part of the quantum theory and the study of classical fields has provided the easiest manner to get grasp about the physics of TGD Universe.

Geometrization of classical fields and of quantum numbers

The basic motivation for TGD was provided by the finding that known interactions at classical level and quantum number spectrum of known particles could be readily understood from the assumption that space-time is a 4-surface in $H = M^4 \times CP_2$.

The geometrization of classical gauge fields is based on the following identifications.

1. The classical gravitational field is identified as the induced metric. The still open question is whether the classical gravitational fields couple to matter with the gravitational constant $G \simeq kR^2$, $k \simeq 10^{-8}$, where R is CP_2 size (the length of CP_2 geodesic line). There is however an argument leading to a precise and correct prediction for k, and fixing the value of the Kähler coupling strength α_K at electron length scale to a value rather near to that of the fine structure constant.

2. The geometrization of electro-weak gauge fields reduces to the curvature of CP_2 just like the geometrization of gravitation reduces to the curvature of the space-time surface. Classical electro-weak fields are identified as components of CP_2 spinor connection projected to the space-time surface. The holonomy group of CP_2 spinor connection is $U(2)$ and naturally identifiable as electro-weak gauge group.

3. Color symmetries correspond to the isometries of CP_2 so that there is deep and unexpected connection between electro-weak and color interactions. Color gauge potentials are identified in the spirit of Kaluza-Klein theory as projections of the Killing vector fields of color isometries to the space-time surface. Color gauge fields are of form $F^A_{\alpha\beta} \propto H^A \times J_{\alpha\beta}$, where H^A is the Hamiltonian of the color isometry and J denotes the induced Kähler form. Therefore the vacuum extremals of Kähler action carry also non-vanishing color gauge fields.

Also elementary particle quantum numbers can be understood in terms of the induced spinor structure and simple 3-topology.

1. CP_2 does not allow ordinary spinor structure and it is necessary to couple CP_2 spinors to the Kähler potential of CP_2. The couplings are different for different H-chiralities identifiable as leptonic and quark like spinors. Baryon and lepton numbers are separately conserved for both the ordinary massless Dirac action and modified Dirac action. The modified Dirac action is fixed uniquely by requiring that it has the vacuum degeneracy of Kähler action. The modified Dirac action allows local super-symmetries generated by the right-handed neutrino.

2. At the fundamental level color quantum numbers are not spin like quantum numbers but can be said to correspond to the color partial waves in CP_2 center of mass degrees of freedom of the 3-surface representing the elementary particle. Ordinary Dirac equation for CP_2 predicts wrong correlations between electro-weak and color quantum numbers of the color partial waves associated with the spinor harmonics. This was a longstanding problem of TGD approach but the construction of physical states as representations of the Super Kac Moody algebra allows to obtain correct correlations and an interpretation in terms of electro-weak symmetry breaking coded already into the CP_2 geometry.

3. The first guess was that the genus of the two-dimensional boundary associated with the 3-surface representing particle explains family replication phenomenon. The identification of the super-conformal symmetries as symmetries associated with light like effectively 2-dimensional 3-surfaces X^3_l acting as causal determinants suggests a more concrete identification.

Quaternion conformal invariance allows to assign to X^3_l a highly unique 2-dimensional surface X^2 as a surface at which superconformal structure reduces to ordinary conformal structure and thus becomes Abelian. The genus of this surface telling whether the surface is sphere, torus, etc... determines the particle family. X^3_l could correspond to either a boundary of 3-surface or to an elementary particle horizon. Elementary particle horizon would surround the wormhole contact connecting CP_2 extremal with an Euclidian signature of the induced metric to a larger space-time sheet with a Minkowskian signature of metric. The induced metric is degenerate at the elementary particle horizon so that this surface is indeed metrically 2-dimensional.

More concretely, sphere, torus, and sphere with two handles would correspond to (e, ν_e), (μ, ν_μ), (τ, ν_τ) in the leptonic sector and and (u, d), (c, s), and (t, b) in the quark sector respectively. The experimental absence of heavier particle families would be most naturally due to the fact that they are extremely heavy. The 3 lowest particle families differ from the higher genera in the sense that 2-surfaces with genus $g < 3$ are always hyper-elliptic, that is they allow always Z_2 conformal symmetry, whereas higher genera generically do not allow any conformal symmetries. Hyper-ellipticity is an excellent candidate for an explanation of the lightness of $g < 3$ genera. The construction of elementary particle functionals as functionals in the conformal equivalence classes of the 2-surface X^2 associated with X_l^3 allows to formulate this argument more precisely.

The explanation of Cabibbo mixing as being due to the mixing of boundary topologies, and number theoretic arguments (complex rationality of CKM matrix) lead to a highly unique CKM matrix for quarks and also leptonic mixings can be fixed highly uniquely. Also bosons are predicted to possess family replication phenomenon.

The new physics associated with classical gauge fields

Long range electro-weak, in particular Z^0, vacuum gauge fields are unavoidable in TGD: this is a necessary outcome of the induced gauge field concept reducing the number of the primary bosonic field variables to four (CP_2 coordinates)! The interpretation of this puzzling prediction has been a long standing challenge of TGD. There are three alternative options to consider.

Option I: Classical gauge fields are space-time correlates for gauge bosons with mass scale determined by the p-adic length scale of the space-time sheet in question. The electro-weak charges of elementary particles are screened by vacuum gauge charges (possible in TGD) in a region of size L_W of order intermediate boson length scale. This option does not explain the presence of long range electro-weak gauge fields unavoidably present if the dimension of CP_2 projection of space-time sheet is higher than 2 nor classical color gauge fields present for non-vacuum extremals.

Option II: Electro-weak gauge charges are not screened in the length scale L_W and the gauge fluxes of elementary particles flow to larger space-time sheets via # throats within region of size L_W and elementary particles have the quantized values of em Z^0 charges. The problem for this option are anomalously large Rutherford cross sections in condensed matter and large parity breaking effects in hadronic, nuclear, and atomic length scales. Despite this I regarded this option as the most realistic one until the realization that the mysterious long ranged weak fields could be assigned to dark matter particles at various space-time sheets.

Option III: There is a hierarchy of color electro-weak physics such that weak bosons are massless below the p-adic length scale determining the mass scale of weak bosons. Classical long range gauge fields serve as space-time correlates for gauge bosons below the p-adic length scale in question.

The unavoidable long ranged electro-weak and color gauge fields are created by dark matter and dark particles can screen dark nuclear electro-weak charges below the weak scale above which vacuum screening occurs as for ordinary weak interactions. Dark gauge bosons are massless below the appropriate p-adic length scale but massive above it and $U(2)_{ew}$ is broken only in the fermionic sector. For dark copies of ordinary fermions masses are essentially identical with those of ordinary fermions.

This option is consistent with the standard elementary particle physics for visible matter apart from predictions such as the possibility of p-adically scaled up versions of ordinary quarks predicted to appear already in ordinary low energy hadron physics. The most interesting implications are seen in longer length scales. Dark quarks and gluons and a scaled up copy of ordinary gluons emerge already in ordinary nuclear physics [K84] and explain some recently discovered anomalies such as neutron halos and tetraneutron. The field bodies associated with are predicted to have sizes of order atom size. Also scaled down versions of weak bosons giving to interactions between exotic quarks with a range of order atomic length scale are predicted.

The new nuclear physics has deep implications for chemistry and condensed matter where color bonds between neighboring atoms might be part of the chemical bonding [K25] . Long ranged repulsive weak force behind exotic quarks compensated by color force would contribute to the repulsive force assumed in van der Waals equations of state for condensed matter. No strong isotopic dependence is predicted.

Classical long range weak and color forces become also key players at the level of molecular physics and biophysics. Chiral selection of bio-molecules can be seen as one direct signature of the long ranged weak force which suggests that non-broken $U(2)_{ew}$ symmetry and and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. The central role of the long ranged weak forces in bio-systems and in pre-biotic evolution is discussed in [K66, K37, K24] .

Classical em fields and Z^0 fields are not invariant under color rotations acting as exact symmetries and are accompanied by classical color gauge fields. This implies new physics potentially important for TGD inspired theory of consciousness. For instance, in TGD Universe the original joke like term "quark color" inspired by certain algebraic similarities ceases to be a joke since it is possible to reduce the 3+3 primary colors in color vision to the 3+3 different increments of color quantum numbers induced by the absorption or emission of color octet gluon.

5.2.3 Many-sheeted space-time concept

The detailed study of TGD led to a further generalization of the space-time concept and the end result is what I have used to call topological condensate or many-sheeted space-time (see fig. 2.3.2 in t`http://www.tgdtheory.fi/appfigures/heappendixofthisbook).The3-spaceis.jpg`, which is also many-sheeted such that the sheets of 3-space have finite size and outer boundary. The physical interpretation of a given space-time sheet of a finite size is as a 'particle'. Depending on their size, these particles correspond to elementary particles, nucleons, atomic nuclei, atoms, molecules, cells, ourselves, stars, galaxies, etc. For instance, my skin corresponds to the outer boundary of a 3-surface glued to a larger 3-surface identifiable as the room in which I sit! I am a small Universe glued to a larger one, the 3-space associated with me literally ends on my skin just as string ends at its end! The surface of earth, the outer surfaces of trees, etc...: everywhere I can see nontrivial 3-topology.

Important new physics is associated with the extremely tiny wormholes contacts with size of order CP_2 length needed to perform the gluing operation. Join along boundaries bonds serving as space-time correlates for the bound state formation is second important notion. The larger sheets of the many-sheeted space-time are ideal for carrying various macroscopic quantum phases. Topological field quantization allows to define precisely the notions of coherence and de-coherence and also means that one can assign to a given material system what might be called field body or magnetic body.

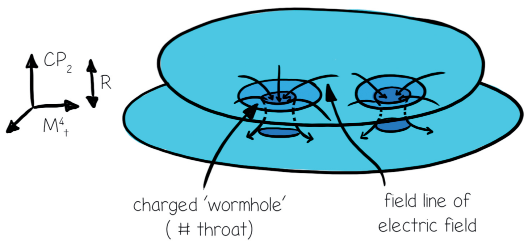

Figure 5.1: Charged wormholes feed the electromagnetic gauge flux to the 'lower' space-time sheet.

Obviously the outcome is a thorough-going generalization of the space-time concept and means that TGD has highly nontrivial consequences in all length scales rather than in particle physics only, as one might naively expect.

Join along boundaries contacts and join along boundaries condensate

The recipe for constructing the 3-space of TGD Universe is simple. Take 3-surfaces with boundaries, glue them by topological sum to larger 3-surfaces, glue these 3-surfaces in turn on even larger 3-surfaces, etc.. The smallest 3-surfaces correspond to CP_2 type extremals that is elementary particles and they are at the top of hierarchy. In this manner You get quarks, hadrons, nuclei, atoms, molecules,... cells, organs, ..., stars, ..,galaxies, etc...

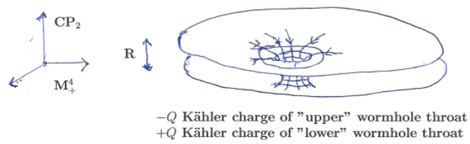

Figure 5.2: The two throats of wormhole behave as classical charges of opposite sign.

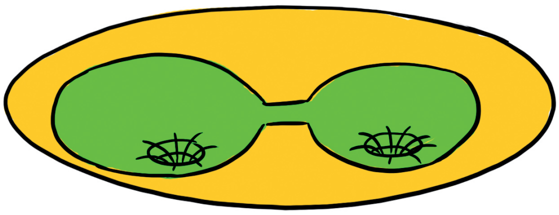

Figure 5.3: Many-sheeted space-time structure results from the requirement of gauge flux conservation.

Besides this, one can also glue different 3-surfaces together by tubes connecting their *boundaries*: this is just connected sum operation for boundaries. Take disks D^2 on the boundaries of two objects and connect these disks by cylinder D^2xD1 having D^2:s as its ends. Or more concretely: let the two 3-surfaces just touch each other.

Depending on the scale join along boundaries bonds are identified as color flux tubes connecting quarks, bonds giving rise to strong binding between nucleons inside nuclei, bonds connecting neutrons inside neutron star, chemical bonds between atoms and molecules, gap junctions connecting cells, the bond which is formed when You touch table with Your finger, etc.

One can construct from a group of nearby disjoint 3-surfaces so called join along boundaries condensate by allowing them to touch each other here and there.

The formation of join along boundaries condensates creates clearly strong correlation between two quantum systems and it is assumed that the formation of join along boundaries condensate is necessary prerequisite for the formation of *macroscopic quantum systems*. Crucially important examples in biology are gap junctions connecting cells and MAPs (micro-tubule associated proteins) connecting micro-tubules.

Quantum classical correspondence inspires the hypothesis that quite generally join along boundaries bonds are space-time correlates for the formation of the bound state entanglement. Since join along boundaries bonds between space-time sheets condensed on larger space-time sheets having no join along boundaries bonds between them is possible, one is forced to conclude that entanglement between sub-systems of un-entangled systems is possible in the many-sheeted space-time. The paradox disappears when entanglement is understood as a length scale dependent notion so that the bound state entanglement of sub-systems is not visible in the length and time scales of the systems.

Wormhole contacts

The gauge and gravitational fluxes at the boundary of a given space-time sheet must go somewhere by gauge flux conservation. This forces the existence of a larger space-time sheet and of tiny

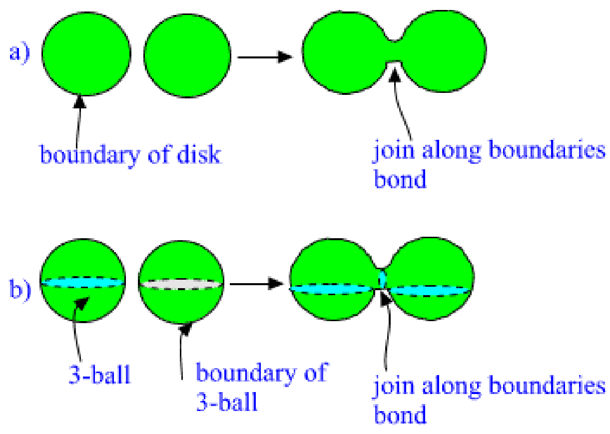

Figure 5.4: Join along boundaries bond a): in two dimensions and b): in 3-dimensions for solid balls.

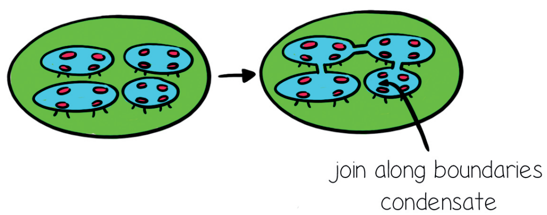

Figure 5.5: Join along boundaries condensate in 2 dimensions.

wormhole contacts connecting the two space-time sheets and feeding the gauge fluxes from the smaller sheet to the larger one. Wormhole contacts (# contacts) are elementary particle like objects (actually deformed pieces of so called CP_2 type extremals) having size of order CP_2 size about 10^4 Planck lengths and, being sources and sinks of gauge field lines, wormhole throats effectively like classical charges, the charges of throats at the two space-time sheets being of opposite sign. Hence wormhole contacts look like dipoles and couple to the difference of the classical gauge potentials associated with the two space-time sheets. Also the coupling to the difference of the gauge potentials serving as order parameters for the coherent states of photons is possible.

The crucial experiment would be the one demonstrating the existence of the wormholes.

1. There are good reasons to expect that wormhole gauge flux is quantized. The reason for quantization would be that the preferred extremals of the Kähler action are critical in the sense that they allow infinite number of vanishing second variations, which is mathematically a condition very similar to the Bohr's quantization condition. There are also other proposals for what "preferred" might mean and the conjecture is that various proposals are equivalent.

   In the usual initial value problem one would fix only the imbedding space coordinates of 4-surface for given value of time and allow their time derivatives be arbitrary. Now preferred exrtremal property fixes the values of the time derivatives just like Bohr's quantization rules fix the momenta. The most aesthetic possibility is that the unit of wormhole em charge is the smallest possible elementary particle charge 1/3 associated with d quarks but also integer charge could be considered.

   From the point of view of zero energy ontology, where one fixes the 3-surfaces at the ends of causal diamond (CD) the preferred extremal property means that not all pairs are allowed:

there is correlation between the 3-surfaces serving as space-time correlate for quantum correlations between positive and negative energy parts of the zero energy state.

To be an absolute minimum of Kähler action was the first identification for the preferred extremaly property. The notion of absolute minimization does not make sense in p-adic context unless one manages to reduce it to purely algebraic conditions. Therefore it is better to talk just about preferred extremals of Kähler action and accept as the fact that there are several proposals for what this notion could mean.

For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K9] defining also this kind of slicing and the approaches could be equivalent. A further interpretation for preferred extremal property is in terms of quantum criticality [K73], which at the level of space-time surfaces is something different from absolute minimum property since minimization implies complete stability against deformations.

2. If wormhole charge is quantized then the gauge flux of an external em field running from a larger space-time sheet to a smaller one is quantized. The experimental arrangement should demonstrate that this flux indeed can change by a multiple of the elementary flux only. One could also try to detect wormhole currents. It must be emphasized that wormhole current is a pseudo current in the sense that two space-time sheets carry opposite classical currents. These currents are created, when magnetic field penetrates from space-time sheet to another. The detection of charge 1/3 for the charge carriers of this current would be a triumph.

3. One cannot exclude the possibility that the recently found evidence for 1/3 charge in condensed matter systems (quantum Hall effect) could be interpreted in terms of an em gauge flux quantized in this manner. Electron current flowing inside a planar layer like structure is studied. Strong magnetic field, which could lead to a generation of wormhole currents is present! Evidence for some quasi particles in current flow possessing this charge has been found. The anyon interpretation of quasi particles as bound states of magnetic flux quanta and electrons explains the effect (McLaughlin wave function). The prediction is however that also fluxes of m/5, m/7,... , m integer, should be observed. Only 1/3 has been detected hitherto and it is not understood why higher charges have not been observed. The question is whether the quasi particles are actually wormholes created by the penetration of magnetic field and flowing along the boundaries of the arrangement.

One application of the new space-time concept is a model of brain. The basic idea is that brain can be regarded as a macroscopic quantum system and that our experiences of free will correspond to quantum jumps which are unpredictable as also is the end result of a free choice. The idea that quantum theory might provide some light in the problem of consciousness has become popular during the last years and a serious building of quantum theories of consciousness has begun. The bottleneck problem is how the brain can be a macroscopic quantum system. Some kind of super conductivity looks a promising idea but standard physics does not provide promising candidates for a super conductor like system. Womholes might provide one such system besides high T_c electronic and protonic superconductors and Bose-Einstein condensates of bosonic ions.

To see what is involved, consider in more precise manner how many sheeted 3-space is constructed. When one glues a sheet of 3-space to a larger sheet of 3-space one does it by constructing extremely tiny elementary particle sized wormholes connecting the two sheets of 3-space.

These wormholes serve important function. For instance, the flux of the electric field (usually it is unlucky space traveller) flows to this kind of wormhole on the smaller sheet of 3-space and and comes back from it to the larger sheet of 3-space. Since the field lines of the electric field flow to the wormhole on the smaller sheet of 3-space, the wormhole looks like a charge since it acts as a sink of field lines. Same applies on the larger sheet of 3-space except that the sign of the charge is opposite. Hence, on both space-time sheets wormhole looks classically like a charged particle. Shortly, wormholes behave like particles and represent a new exotic form of matter. More generally, it seems that many-sheeted nature of the space-time is crucial for the understanding of a bio-system as a macroscopic quantum system.

The interaction between space-time sheets is mediated by these wormholes having size of order CP_2 radius R and located near the boundaries of the smaller space-time sheet. Wormholes feed various gauge fluxes from the smaller space-time sheet to the larger one (say from the atomic sheet to some molecular sheet). p-Adic considerations suggest that wormholes are light having mass of order $1/L_p$: this implies that they suffer Bose-Einstein condensation on the ground state. One could even say that space-time sheets "perceive" the external world and act on it with the help of the charged wormhole BE condensates near their boundaries. Wormholes provide a very general mechanism making possible the transfer of classical electromagnetic fields and various quantum numbers such as energy, momentum and angular momentum, between different space-time sheets and bio-systems are especially promising as far as applications are considered.

Topological field quantization

Topological field quantization [K45] implies that various notions of quantum field theory have rather precise classical analogies. Topological field quantization is basically implied by the compactness of CP_2, which typically implies that a given Maxwell field allows only a partial imbedding as a space-time surface in H. One can say that magnetic fields, electric fields and radiation fields decompose into field quanta.

The energies and other classical charges of the topological field quanta are quantized by the criticality of the extremals of the Kähler action making classical space-time surfaces the counterparts of the Bohr orbits. Feynman diagrams become classical space-time surfaces with lines thickened to 4-manifolds. For instance, "massless extremals" representing topologically quantized classical radiation fields are the classical counterparts of gravitons and photons. Topologically quantized non-radiative nearby fields give rise to various geometric structures such as magnetic and electric flux tubes.

Topological field quantization provides the correspondence between the abstract Fock space description of elementary particles and the description of the elementary particles as concrete geometric objects detected in the laboratory. In standard quantum field theory this kind of correspondence is lacking since classical fields are regarded as a phenomenological concept only.

Topological field quanta define coherence regions for the classical gauge fields and induced spinor fields and classical coherence is the prerequisite of the quantum coherence. Whether and how macroscopic and macro-temporal quantum coherence are possible in living matter is the basic question of quantum consciousness theories and quantum biology. In TGD this question is even more difficult since the first estimate for de-coherence time is CP_2 time which is about 10^4 Planck times. The length scale hierarchy of space-time sheets allows immediately to understand at the level of space-time correlates how macroscopic and macro-temporal quantum coherence are possible. A good order of magnitude guess for the zero point energy of a particle at a space-time sheet of size L is given by $E = \pi^2/2mL^2$. $T \leq \pi^2/2mL^2$ gives an estimate for the temperature of the space-time sheet populated by particles of mass m: the larger the size of the space-time sheet, the lower the temperature. Superconductivity and various macroscopic phenomena become thus possible at larger space-time sheets. TGD based model of living matter is based on the hypothesis that large space-time sheets are responsible for quantum control.

The virtual particles of quantum field theory have also classical counterparts. In particular, the virtual particles of quantum field theory can have negative energies: this is true also for the TGD counterparts of the virtual particles. The fundamental difference between TGD and GRT is that in TGD the sign of energy depends on the time orientation of the space-time sheet: this is due to the fact that in TGD energy current is vector field rather than part of tensor field. Therefore space-time sheets with negative energies are possible.

One can criticize the notion of time orientation. A more precise definition of the time orientation requires the realization that configuration space of 3-surfaces ("world of classical worlds", WCW), call it CH, can be understood as a union of corresponding sub-WCWs associated with unions of arbitrary many light cones, both future and past light cones with positive/negative energies assignable to to future/past light-cones. This brings in in a natural manner also the super-symplectic symmetries associated with the boundaries of the light-cones.

Negative energies would have quite dramatic technological consequences: consider only the possibility of generating energy from vacuum and classical signalling backwards in time along negative energy space-time sheets [K8] . Also bio-systems might have invented negative energy space-time sheets: in fact, they define the basic mechanism for the realization of intentional action, long term memory, and metabolism [K63] .

Quantum classical correspondence suggests that quantum entanglement has the formation of the join along boundaries bonds as its geometric correlate. The superposition of the topologically quantized space-time surfaces in the state $U\Psi$ could be regarded as a geometric correlate for quantum fields: creation/annihilation operators would correspond to positive/negative energy space-time sheets. This hypothesis, together with the expansion of the interacting quantum field in terms of creation and annihilation operators, would make it possible to make quantitative estimates about the fraction of energy density carried by the negative energy space-time sheets, in particular, about the energy density associated with the massless extremals.

In TGD Universe topological field quanta serve as templates for the formation of the bio-structures. Thus topologically quantized classical electromagnetic fields associated with the material objects, field bodies or more concretely, magnetic bodies, could be equally important for the functioning of the living systems as the structures formed by the visible bio-matter and the visible part of bio-system might represent only a dip of an ice berg. For instance, in [K38] the implications of the notion of field body for the understanding of bio-systems and pre-biotic evolution are discussed in detail.

Negative energy space-time sheets and new view about energy

Negative energy space-time sheets represents an important distinction between TGD and standard physics. They are possible because energy momentum tensor is replaced by a collection of conserved currents associated with various components of four momentum. This resolves the energy problem of general relativity but, since the sign of the conserved charged depends on the time orientation of the space-time sheet, the sign of energy is not positive definite anymore.

Quantum classical correspondence implies that also elementary particles can have negative energies and this means a new kind of physics. It seems that this physics has been already discovered: the strange properties of phase conjugate laser waves can be understood if they consist of negative energy photons.

Negative energy space-time sheets have far reaching implications for TGD inspired theory of consciousness. What I have called time mirror mechanism (see fig. `http://www.tgdtheory.fi/appfigures/timemirror.jpg` or fig. 24 in the appendix of this book) involves the reflection of negative energy signals sent to the geometric past from population inverted lasers as amplified positive energy signals propagating to the geometric future. Time mirror mechanism provides the holy grail to the understanding of the mechanisms of brain functioning and also of the workings of the living matter. There are obvious implications for communication and energy technologies since negative energy signals could make possible instantaneous remote sensing and quantum control over arbitrarily long distances so that light velocity would cease to be a restriction forcing us to be habitants of 3-space instead of space-time.

If Kähler action were strictly deterministic, the only possible choice for H would be $M^4_+ \times CP_2$. Together with negative energies the classical non-determinism of the Kähler action it is possible to assume that imbedding space is $M^4 \times CP_2$ meaning exact Poincare invariance. The point is that generation of pairs of positive and negative energy space-time sheet at light-like 7-surfaces $X^3_l \times CP_2$ means emergence of new kind of causal determinants generalizing the light cone boundary $\delta M^4_+ \times CP_2$ as a fundamental causal determinant. All states of the Universe have vanishing net quantum numbers and everything in the Universe would have been pair-created from vacuum. Future light cones containing positive energy could also be created when negative energy radiation (in particular gravitons) is generated and propagates to the geometric past and leaks from the future light cone. This vision can be applied also to the second quantization of fermions by giving fermions and anti-fermions opposite energies. Depending on time orientation either fermions or anti-fermions have negative energy.

By crossing symmetry the assumption that the net quantum numbers of the Universe vanish is not in conflict with elementary particle physics. In macroscopic length scales the identification of the gravitational energy as the difference of inertial (Poincare) energies of positive and negative energy matter plus the possibility that negative and positive energy matter interact weakly allows to understand why western view about objective reality with conserved positive total energy is so good an approximation. The non-conservation of the gravitational energy can be understood, and vacuum extremals, of which Robertson-Walker cosmologies, are most important examples find interpretation. The non-determinism of Kähler action explains naturally the fact that Universe is to some extended an outcome of engineering. The notion of gravitational energy generalizes to that of gravitational quantum numbers and the inertial-gravitational dichotomy is a direct correlate for the geometric-subjective dichotomy for time discovered while developing TGD inspired theory of consciousness. Indeed, positive and negative energy space-time sheets correspond to initial and final states of quantum jump so that gravitational quantum numbers characterize changes.

This vision would resolve the unpleasant philosophical questions like "What is the total fermion number of the Universe". One could see entire universe as a result of intentional actions in which intentions represented by p-adic space-time sheets are transformed to actions represented by real space-time sheets. Everyone knows the anecdotes about yogis and gurus creating material objects from nothing and very few "scientifically thinking" westerner can take these stories really seriously. Whether or not these stories are true, they might however express a deep truth about reality.

More precise view about topological condensate

The challenge is to define precisely the concepts like classical gauge charge, gauge flux, wormhole contacts, join along boundaries bonds, topological condensation and evaporation, etc... Number theoretical vision allows to achieve this goal [K32, K32] .

The crucial ingredients in the model are so called CP_2 type vacuum extremals. The realization that # contacts (topological sum contacts and $\#_B$ contacts (join along boundaries bonds) are accompanied by causal horizons which carry quantum numbers and allow identification as partons leads to a more detailed articulation of these notions.

The partons associated with topologically condensed CP_2 type extremals carry elementary particle vacuum numbers whereas the parton pairs associated with # contacts connecting two space-time sheets with Minkowskian signature of induced metric define parton pairs. These parton pairs do not correspond to ordinary elementary particles. Gauge fluxes through # contacts can be identified as gauge charges of the partons. Gauge fluxes between space-time sheets can be transferred through # and $\#_B$ contacts concentrated near the boundaries of the smaller space-time sheet. The dynamics of topological condensation and evaporation can be formulated in terms of gauge interactions of partons and splitting and fusion of CP_2 type extremals. This picture generalizes to the case of gravitational flux which need not be well-defined purely classically.

Number theoretical vision and p-adic length scale hypothesis allow to quantify this picture and lead to an overall view about interactions of particles in many-sheeted space-time. A far reaching generalization of standard physics results predicting an infinite hierarchy of dark matters besides ordinary elementary particles of standard model. In particular, the partons associated with # and $\#_B$ contacts represent dark matter.

5.2.4 Classical non-determinism of Kähler action

The classical non-determinism of Kähler action has been deep source of inspiration and challenges and guided the evolution of TGD inspired theory of consciousness and finally also of quantum TGD proper. In nut-shell, classical non-determinism makes possible quantum-classical correspondence in the sense that space-time surface becomes a symbolic representation for the quantum states and quantum jump sequences defining conscious experience.

Matter-mind duality geometrically

The non-determinism of Kähler action implies huge vacuum degeneracy: any 4-surface whose projection belongs to $M^4_+ \times Y^2$, where Y^2 is so called Lagrange manifold of CP_2 (has vanishing induced Kähler form), is a vacuum extremal. This suggests that one must radically generalize the concept of space-time. It seems that the correct picture is roughly like follows. Space-time is many-sheeted. Each sheet can be regarded as a slightly deformed piece of M^4 in H containing smaller sheets glued to it and being itself glued to a larger space-time sheet. Gluing means the formation of topological sum contacts between the space-time sheets. There are reasons to believe that topological sum contacts, "wormhole contacts" are located near the boundaries of the smaller space-time sheet.

Material space-time sheets have infinitely long time duration if they possess non-vanishing energy (and provided that they do feed their energy to some other space-time sheets). "Mind like" space-time sheets can be regarded as obtained by gluing space-time sheets with finite time duration to material space-time sheets. The gluing operation implies that tiny amounts of energy and momenta and other conserved quantities flow to the mind like space-time sheet when it begins and back to the material space-time sheets when mind like space-time sheet ends.

Mind like space-time sheets are space-time correlates for contents of consciousness. In particular, they form symbolic representations for material space-time sheets. For instances, the frequencies of various oscillatory processes are mapped also to frequencies of processes occurring in mind like space-time sheets. The possibility of mind like space-time sheets implies that the absolute minima of Kähler action (or more general preferred extremals defining analogs of Bohr orbits [K88]) are degenerate: one can glue mind like space-time sheets to given absolute minimum to get new absolute minima. This conforms with the fact that contents of consciousness are defined by a sequence of non-deterministic quantum jumps.

This picture must of course be taken with strong reservations, and one should actually state more precisely what "mind like" means. The interpretation of p-adic space-time sheets as correlates of intentions and cognitions gives some ideas about what aspects of consciousness mind like space-time sheets correlate with. The model for how intentions are realized as actions in quantum jump assumes that p-adic "topological light rays" representing intentions are transformed to real topological light rays with negative energy serving as correlates of desires, which in turn induce the action initiated in the geometric past. Thus it would seem that real "mind like" space-time sheets with negative energy would serve as correlates for desires.

The precise definition of p-adic space-time sheets is a separate question and requires a precise vision about how real and various p-adic physics integrate to a coherent whole. This requires a generalization of the number concept based on the fusion of real and p-adic number number fields to a larger book like structure along common rationals. The precise definition of p-adic space-time sheets is discussed in [K88] . The surprising outcome, basically due to the difference between real and p-adic notions of distance, is that most points of p-adic space-time sheets can be said to reside at infinity of the real imbedding space and the projection to real space-time consists of a discrete set of rational points. Thus cognition can be said to look material cosmos from outside (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also).

Association sequence concept and a mind like space-time sheets

The vacuum degeneracy of the Kähler action defining quantum TGD solves the difficulty. The vacuum degeneracy implies spin glass analogy and strongly suggests that the Bohr orbit like space-time surface defined as a preferred extremal of Kähler action and going through a given space-like 3-surface, cannot be unique in general. To achieve uniqueness one must generalize the concept of 3-surface to what might be called association sequence . In order to specify uniquely one of the degenerate absolute minimum space-times going through a given 3-surface one must fix some minimum number, say N, of 3-surfaces on a given preferred extremal. These sequences of disjoint 3-surfaces with time-like separations can be regarded as a simulations of the classical time development and hence as a geometric correlate of conscious experience localized temporally. It seems that in real case geometric correlates of sensory experiences are in question whereas in p-adic case correlates of thoughts are in question.

Association sequences are very probably not all that is needed to overcome the complications caused by the non-determinism of Kähler action. The enormous vacuum degeneracy of Kähler action suggests strongly that the classical non-determinism does not reduce to simple sequences of bifurcations. Hence it seems that must give up the idea of identifying space-like 3-surfaces given value of geometric time as causal determinants which are possibly degenerate because of the

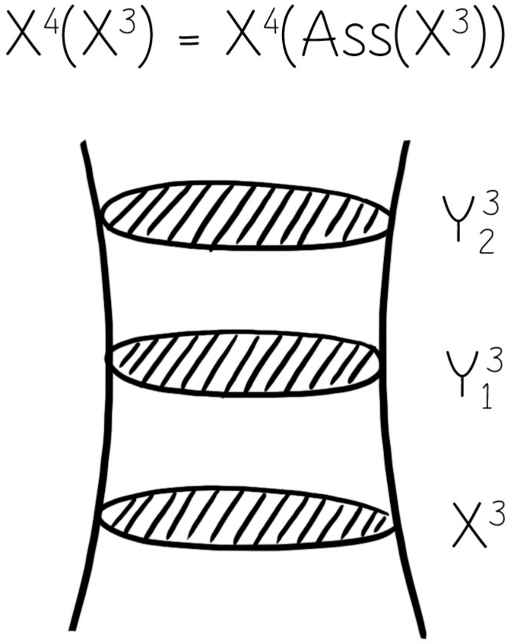

Figure 5.6: 'Association sequence': a geometric model for thought as a sequence of disjoint 3-surfaces with time-like separations.

bifurcations.

Vacuum degeneracy and spin glass analogy

Kähler action determines WCW geometry and is hence a cornerstone of quantum TGD. Kähler action can be regarded as a Maxwell action for the Kähler form of CP_2 induced to space-time surface and defining nonlinear Maxwell field. Kähler action possesses enormous vacuum degeneracy. Any space-time surface in $M^4_+ \times CP_2$, where Y^2 is so called Lagrange sub-manifold of CP_2 having by definition vanishing induced Kähler form, is vacuum extremal. In canonical coordinates (P_i, Q_i) for CP_2 Lagrange sub-manifolds correspond to functions

$$P_i = \nabla_i f(Q_j) \ .$$

This means that there is infinite number of vacuum sectors since all 4-surfaces in any six-dimensional space $M^4_+ \times Y^2$ are vacua.

Also non-vacuum configurations are almost degenerate. Only the gravitational effects caused by the presence of the induced metric in the Maxwell action for the induced Kähler form of CP_2 on space-time surface breaks the canonical invariance of the Kähler action. Canonical transformations of CP_2 act as $U(1)$ gauge transformations and in the absence of gravitation one would have ordinary $U(1)$ gauge invariance. Gravitation however changes the situation. Various canonically related configurations are physically *non-equivalent*. This means a characteristic degeneracy analogous to the degeneracy of the states for spin glass rather than to the physically uninteresting gauge degeneracy. The effective breaking of $U(1)$ gauge invariance makes possible vacuum charge densities, scalar wave pulses propagating with light velocity and carrying longitudinal electric field parallel to the propagation direction, and topological light rays carrying light like vacuum current and transversal electric and magnetic fields are predicted.

Contrary to the original beliefs, p-adic physics does not seem to follow from vacuum degeneracy alone. Rather, p-adic space-time topology is a genuine rather than only effective space-time topology and emerges independently from the vacuum degeneracy. p-Adic topology seems however to serve as effective topology for the real space-time sheets in the sense that the non-determinism implied by the vacuum degeneracy mimics the inherent non-determinism of p-adic field equations for some value of p so that one can indeed assign a definite p-adic prime to a given real space-time sheet. Vacuum degeneracy has a wide spectrum of implications. For instance, the spin glass degeneracy implied by it allows to understand at quantum level generation of macroscopic and macro-temporal quantum coherence. The same mechanism explains also color confinement.

The p-adic fractality of real space-time sheets is in turn implied by the fact that p-adic and real space-time sheets have common rational points which implies that the purely local p-adic physics sets constraints on the long ranged real physics because rational points close to each other p-adically are very distant in real sense.

Connection with catastrophe theory and Haken's theory of self-organization for spin glass

If the effects related to the induced metric (classical gravitation) are neglected, canonical transformations of CP_2 act as $U(1)$ gauge symmetries and all canonically related surfaces are physically equivalent. Classical gravitation however breaks this gauge invariance but due to the extreme weakness of the gravitational interaction one has good reasons to expect that the maxima of Kähler function for given values of the zero modes are highly degenerate. The hypothesis that single maximum of Kähler function with respect to fiber degrees of freedom is selected in quantum jump, means huge simplification of the mathematical theory.

Besides the degeneracy resulting from the non-determinism, there is also the spin glass degeneracy related to zero modes. The nonphysical $U(1)$ gauge degeneracy is transformed to physical spin glass degeneracy. The energies of various absolute minima differ only by the classical gravitational energy. Zero modes serve as coordinates for the "energy" landscape of quantum spin glass and the energy landscape of non-equilibrium thermodynamics is fractal containing valleys inside valleys...inside valleys.

One naturally ends up with a generalization of the catastrophe theory [A120] to the infinite-dimensional WCW context. Zero modes play the role of the control parameters forming master slave-hierarchy and non-zero modes characterizing various degenerate absolute minima of Kähler action correspond to the state variables [K75] . There is natural connection with the

non-equilibrium thermodynamics of Haken [B35] . Since time development by quantum jumps means hopping in the zero modes characterizing the macroscopic space-time surfaces associated with the final states of the quantum jumps, Haken's classical theory applies almost as such. Asymptotically the self-organizing quantum jumping system (self) ends up to a fixed point, limiting cycle, strange attractor, etc. near the bottom of some valley of the energy landscape. The bottom of a valley corresponds to a maximum of the Kähler function rather than minimum of free energy as in thermodynamics since vacuum functional is exponent of Kähler function. Self-organization in spin glass energy landscape by quantum jumps is extremely powerful notion allowing to understand general features of living systems.

5.2.5 Quantum classical correspondence as an interpretational guide

The overall view about interpretation of TGD can be deduced from the general properties of space-time surfaces, the notion of induced gauge field, the general properties of Kähler action, and the known extremals using quantum classical correspondence. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence.

The implications deriving from the topology of space-time surface and from the properties of induced gauge fields

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales. It took relatively long time to realize that perhaps the only manner to understand this is a generalization of the quantum theory itself by allowing Planck constant to be dynamical and quantized. TGD leads indeed to a "prediction" for the spectrum of Planck constants and macroscopic quantum phases with large value of Planck constant allow an identification as a dark matter hierarchy.

Also long ranged classical color and electro-weak fields are an unavoidable prediction and it took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labelled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy defines as $\hbar = \lambda^k \hbar_0$, $k_d = 0, 1, ...$ λ depends logarithmically on p-adic length scale $L_e(k)$ and satisfies $\lambda \simeq 2^{11}$ in atomic length scale $L_e(k = 137)$. Dark space-time sheets are identifiable as space-time sheets defining locally λ^k-fold covering of M^4 factor of imbedding space.

The new view about energy and time means that the sign of inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric past. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

A further important fact is that the holonomy group of induced color gauge field is Abelian. Together with quantum classical correspondences this suggests a weak form of color confinement in the sense that only color neutral states of color multiplets are realized as physical states. This would mean a weak form of color confinement.

5.3 Evolution of p-adic ideas

It took quite a long time to end up with the recent picture how p-adic numbers emerge as a basic aspect of quantum TGD and what p-adicization of TGD might mean. Of course, recent picture need not be the final yet and there are several unsolved problems. In the following the basic properties of the p-adic numbers are described shortly and then it is demonstrated how p-adic numbers might emerge from TGD and how one should formulate p-adic version of quantum TGD formalism.

5.3.1 p-Adic numbers

Like real numbers, p-adic numbers can be regarded as completions of the rational numbers to a larger number field allowing the generalization of differential calculus. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x, y)$ (the counterpart of $|x - y|$ in the real context) satisfies the inequality

$$d(x, z) \leq Max\{d(x, y), d(y, z)\} \ ,$$

(Max(a,b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x, z) \leq d(x, y) + d(y, z) \ .$$

p-Adic numbers have expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n \geq 0} x_n p^n \ ,$$

and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x = \sum_{n \geq 0} x_n p^n) = p^{-n_0} \ ,$$

and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.

p-Adic numbers allow the generalization of the differential calculus and of the concept of analytic function $f(x) = \sum f_n x^n$. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which depend on a finite number of positive pinary digits of x only so that one has

$$f_N(x = \sum_n x_n p^n) = f(x_N = \sum_{n<N} x_n p^n) \ .$$

In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic.

An essential element is the map of the p-adic numbers to the positive real numbers by the so called canonical identification I:

$$I : \sum x_n p^n \in R_p \rightarrow \sum_n x_n p^{-n} \in R \ .$$

Canonical identification has inverse, which is single valued for the real numbers having infinite number of pinary digits but two-valued for real numbers having finite number of pinary digits (the reason is that real number with finite number or pinary digits has two equivalent pinary expansions: ($x = 1 = .999999...$ in case of decimal expansion and $x = 1 = 0yyyy...$, $y = p-1$, in the case of pinary expansion).

Canonical identification in its basic form cannot map real space-time surface to p-adic ones or vice versa because it is not a general coordinate invariant notion. A variant of canonical identification, call it I_Q, maps defined only for rationals is given by $I(q = m/n) = I(m)/I(n)$, where $q = m/n$ is the unique representation of rational q in terms of integers [K87] .

I_Q can be applied to map rational points of p-adic CP_2 to their real counterparts whereas the points of p-adic M^4 are mapped as such to real points as such [K87] . General coordinate invariance is not lost since the projection of p-adic space-time sheet to real imbedding space is discrete and genuinely p-adic points are at infinite real distance, "outside the real cosmos". This means a deep number theoretic difference between M^4 and CP_2 and gives one reason for the product decomposition of the imbedding space. O_Q makes it also possible to map the predictions of the p-adic probability theory and thermodynamics to real numbers so that probability is conserved.

5.3.2 Evolution of physical ideas

In the sequel the evolution of physical ideas related to p-adic numbers is summarized.

p-Adic length scale hypothesis

p-Adic length scale hypothesis [K54] states that to a given p-adic prime p there corresponds a primary p-adic length scale $L_p = \sqrt{p}l$, $l \simeq 1.288 \times 10^4\sqrt{G}$ ($\sqrt{G}$ denotes Planck length) and that physically favored primes correspond to $p \simeq 2^k$, k power of prime. The corresponding p-adic time scale is obtained as $T_p = L_p/c$. The justification for the first part of the hypothesis comes from Uncertainty Principle and from the p-adic mass calculations [K54] predicting that the mass of elementary particle, resulting from the mixing of massless states with $10^{-4}m_{Planck}$ mass states described by p-adic thermodynamics, is of order $1/L_p$ for the light states.

The first principle explanation for p-adic length scale hypothesis derives from the fusion of real and p-adic physics to a single larger framework. The fact that real and p-adic space-time sheets can have common points implies that local p-adic physics give rise to p-adic fractality of real physics. Also multi-p p-adic fractality is possible. $p \simeq 2^k$ would reflect the presence of 2-adic fractality besides $p > 2$-adic fractality.

A heuristic justification for the preferred values of p comes from
elementary particle black hole analogy [K59] generalizing the
Bekenstein-Hawking area-entropy law to apply to the elementary particle horizon defined as the surface at which the Euclidian signature for the so called CP_2 type extremal describing elementary

particle changes to the Minkowskian signature of the background space-time at which elementary particle has suffered topological condensation.

The hypothesis is especially interesting above the elementary particle length scales $p > M_{127}$ and has testable implications in nuclear physics, atomic physics and condensed matter length scales. The most convincing support for this hypothesis are provided by the elementary particle mass calculations: if one assumes that the p-adic primes associated with elementary particles are primes near prime powers of two, one can predict lepton and gauge boson masses with accuracy better than one per cent. Also quark masses can be predicted but the calculation of the hadron masses requires some modelling (CKM matrix, color force, etc...). The existing empirical information about neutrino mass squared differences suggests that the allowed values of k are indeed *powers* of prime rather than primes.

It is natural to postulate that space-time sheets form a hierarchy with respect to p in the sense that the lower bound for the size of the space-time sheets at level p is of order L_p and that $p_1 < p_2$ sheets condensed on p_2 sheets behave like particles on sheet p_2.

The following table lists the p-adic length scales $L_e(k) = \sqrt{5}L_(k)$, $p \simeq 2^k$, k power or prime, which might be interesting as far as condensed matter is considered (the notation $L_e(k)$ will be used instead of L_p). It must be emphasized that the definition of the length scale is bound to contain some unknown numerical factor K: the requirement that the thickness of cell membrane corresponds to $L_e(151)$ fixes the proportionality coefficient K to $K \simeq 1.1$.

k	127	131	137	139	149
$L_e(k)/10^{-10}m$	.025	.1	.8	1.6	50
k	151	157	163	167	169
$L_e(k)/10^{-8}m$	1	8	64	256	512
k	173	179	181	191	193
$L_e(k)/10^{-4}m$	.2	1.6	3.2	100	200
k	197	199	211	223	227
$L_e(k)/m$	.08	.16	10	640	2560

Table 1. Primary p-adic length scales $L_e(k) = \sqrt{5}L_(k) = 2^{k-151}L_e(151)$, $p \simeq 2^k$, k prime, possibly relevant to bio physics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L_e(151)$ is take to be thickness of cell scale, which is 10^{-8} meters in good approximation.

The assumption that p-adic space-time regions provide cognitive representations of the real space-time regions forces to conclude that cognition is present in all length scales and that the properties of the p-adic space-time regions reflect those of the real space-time regions. p-adic–real phase transitions and identifiable as transformation of intentions to actions [K58] occurring even at elementary particle length scales would explain this elegantly.

Besides primary p-adic length scales also n-ary p-adic length scales defined as $L_p(n) = p^{(n-1)/2}L_p$ and corresponding time scales are possible and form a fractal hierarchy coming as powers of $\sqrt{p}$. Accepting these scales means that all length scales $L_e(n)$ coming as powers of $2^{n/2}$, n a positive integer, should have a preferred physical role. The TGD inspired model for living matter lends support for the hypothesis that biologically important length and time scales indeed appear as half octaves. A possible explanation for this is the existence of a hierarchy of cognitive codes associated with the time scales $T(n)$. Any prime power factor k^i in the decomposition of the integer n to a product of prime power factors defines a candidate for a cognitive code. The duration of code word would be $T(n)$ and the number of bits would be k^i. For prime values of n the information content of the code word is maximal so that one could understand why prime values of n are especially important.

CP_2 type extremals and elementary particle black hole analogy

CP_2 type extremals are vacuum extremals having a finite negative action so that one can lower the action of the ordinary vacuum extremals by gluing CP_2 type extremals to them. CP_2 type extremals have one-dimensional M^4_+ projection which is light like random curve. Light likeness condition leads to classical Virasoro algebra constraints. $M^4 \times SO(3,1) \times SU(3) \times SU(2)_{ew}$ Super-Kac-Moody algebra acts as symmetries and the spectrum of elementary particles is precisely known. The obvious interpretation of the CP_2 type extremals is as a model of elementary particle.

CP_2 type extremals are much like black holes in the sense that they possess elementary particle horizon: this is the surface at which the Euclidian signature of the metric of the CP_2 type extremal changes to the Minkowskian signature of the background space-time. One can indeed generalize Bekenstein-Hawking law to a statement saying that the real counterpart of the p-adic entropy predicted by the p-adic thermodynamics is proportional to the surface area of the elementary particle horizon. In particular, for primes $p \sim 2^k$, where k is power of prime, the radius of the elementary particle horizon is itself a p-adic length scale. This suggests a double p-adicization associated with p and k and an additional cognitive degeneracy due to the k-adic non-determinism, and hence also the dominance of the final states of quantum jump for which $p \simeq 2^k$ holds true: there would be simply very many physically equivalent physical states for these values of p.

p-Adic thermodynamics and particle massivation

The underlying idea of TGD based description of particle massivation is following. Due to the interaction of a topologically condensed 3-surface describing elementary particle with the background space-time, massless ground states are thermally mixed with the excitations with mass of order $m_0 \sim 1/R$ (R is CP_2 length scale, $1/R$ of order 10^{-4} Planck masses) created by the Super Virasoro generators. Instead of energy, the Virasoro generator L_0 (essentially mass squared) is thermalized. This guarantees Lorentz invariance automatically. p-Adic temperature is quantized by purely number theoretical constraints (the Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0 representing scaling!)).

Optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The calculations support the existence of massless gluons and electro-weak quanta associated with so called massless extremals (MEs). One important prediction is that p-adic thermodynamics cannot explain the masses of the intermediate gauge bosons although the predictions for the fermion masses are excellent. This observation led to the identification of the TGD counterpart of Higgs field whose vacuum expectation provides the dominating contribution to the bosonic masses and only shifts bosonic masses [K48] .

p-Adic coupling constant evolution

The original hypothesis was that Kähler coupling strength α_K is completely fixed by quantum criticality implying that α_K is analogous to critical temperature. p-Adic considerations led to the view that there is infinite number of critical values of α_K labelled by p-adic primes. In many-sheeted space-time one can indeed consider the possibility that α_K is not a universal constant. This would mean that space-time sheets joined only by wormhole contacts and surrounded by light like elementary particle horizons would be characterized by different values of Kähler coupling strength.

Since p-adic primes correspond to p-adic length scales this inspires the idea that the ordinary coupling constant evolution is replaced by a discrete coupling constant evolution. This view is also consistent with the criticality of the Kähler coupling constant. The assumption that gravitational constant is invariant under critical temperature-adic coupling constant evolution fixes highly unique the evolution of Kähler coupling strength. This picture makes sense if one can assign to a given 3-surface a unique p-adic prime and there are good reasons to believe that this is indeed the case.

The progress in the understanding of the spectrum of Planck constants predicted by TGD however forced to question the idea about p-adic evolution of the Kähler coupling strength and consider the possibility that the original vision is correct after all. Assume that gauge bosons and graviton correspond to Mersenne primes and that graviton, or more generally, the space-time sheets mediating gravitational interaction, corresponds to the largest Mersenne prime for which the p-adic length scale is non-super-astronomical. This Mersenne is M_{127} defining the p-adic length scale of electron. If only $p = M_{127}$ is experimentally relevant, one can tolerate the proportionality

$G = exp(S_K(CP_2))L_p^2$ following from simple dimensional considerations ($S_K(CP_2)$ denotes Kähler action for CP_2 type extremals representing elementary particles) and meaning a rapid increase of G as a function of L_p if α_K is RG invariant. This leads to a highly predictive scenario reproducing the basic features of electro-weak and color coupling constant evolution and also allowing to deduce the value of R^2/CP_2 with electro-weak coupling $\alpha_{U(1)}(M_{127})$.

Vacuum degeneracy of the Kähler action and spin glass analogy

The space of minima of free energy for spin glass is known to have ultra-metric topology. p-Adic topology is also ultra-metric and this motivated the hypothesis that quantum average space-time, 'topological condensate', defined as a maximum of Kähler function can be obtained by gluing together regions characterized by various values of the p-adic prime p. It must be emphasized that this hypothesis is just a guess and not even correct as such, and it seems that TGD as a generalized number theory vision gives the real justification for the p-adics. A good guess is however that the ultra-metric topology of the reduced WCW consisting of the maxima of the Kähler function is induced from the p-adic norm and that there is a close connection between the two p-adicities. The following arguments tries to make this idea more precise.

The unique feature of the Kähler action is its enormous vacuum degeneracy: any space-time surface, whose CP_2 projection is a so called Lagrange manifold (having dimension $D \leq 2$) is vacuum extremal. This is expected to imply a large degeneracy of the absolute minimum space-times: for instance, several absolute minima with the same action are possible for single 3-surface (this forces to a generalization of space-time concept obtained by introducing 'association sequences'). The degeneracy means an obvious analogy with the spin glass phase characterized by 'frustration' implying a large number of degenerate ground states. In the construction of WCW geometry the analogy between quantum TGD and spin glass becomes precise.

Spin glass consists of magnetized regions such that the direction of the magnetization varies randomly in the spatial degrees of freedom but is frozen in time. What is peculiar that, although there are large gradients on the boundaries of the regions with a definite direction of magnetization, no large surface energies are generated. An obvious p-adic explanation suggests itself: p-adic magnetization could be pseudo constant and hence piecewise constant with a vanishing derivative on the boundaries of the magnetized regions so that no p-adic surface energy would be generated.

In the description of the spin glass phase also ultra-metricity, which is the basic property of the p-adic topology, emerges in a natural manner. The energy landscape describing the free energy of spin glass as a function of various parameters characterizing spin glass, is fractal like function and there are infinite number of energy minima. In this case there is a standard manner to endow the space of the free energy minima with an ultra-metric topology [A106] .

The counterpart of the energy landscape in TGD can be constructed as follows. The WCW of TGD (the space of 3-surfaces in H) has fiber-space like structure deriving from the decomposition $CH = \cup_{zeromodes} G/H$. The fiber is the coset space G/H such that G is the group of the canonical transformation of the light cone boundary. In particular, the canonical transformations of CP_2 act in the fiber as isometries. The base space is the infinite-dimensional space of the zero modes characterizing the size and shape as well as the classical Kähler field at the 3-surface.

To calculate S-matrix element, one must form Fock space inner product as a functional of 3-surface X^3 multiplied with the vacuum functional $exp(K)$ and integrate it over the entire configuration space:

$$S_{i \rightarrow f} = \int \langle \Psi_f, \Psi_i \rangle (X^3) exp(K(X^3) \sqrt{G} DX^3 \quad .$$

The integration over the fiber degrees of freedom reduces to a Gaussian integration around the maxima of the Kähler function with respect to the fiber coordinates. The equally poorly defined Gaussian and metric determinants cancel each other in this integration and one obtains a well defined end result. Canonical transformations are 'almost gauge symmetries' since only classical gravitational fields destroy canonical symmetries acting as $U(1)$ gauge transformations. This means that the action for several canonically related configurations can be degenerate and several maxima are expected for given values of the zero modes. This means that the subset CH_0 of WCW consisting of the maxima of the Kähler function has many sheets parameterized by the zero modes and that generalized catastrophe theory is obtained.

If a localization in the zero modes occurs in the quantum jump, one can circumvent the integration over the zero modes in practice. The exponent for the maximum of the Kähler action is expected to have maxima as a function of the zero modes too. The maxima of $exp(K_{max})$ as function of zero modes define the counterpart of the energy landscape and $exp(K_{max})$ is the counterpart of the energy serving as a height function of the energy landscape. It could quite well be that this height function can be induced from a p-adic norm. If so, the allowed values of p define a decomposition of the space of zero modes to sectors D_p. For 'full' CP_2 type extremals representing virtual gravitons the exponent is indeed proportional to $1/p$ if one takes seriously the argument determining the possible values of the Kähler coupling strength. Thus cognitive p-adicity and spin glass p-adicity would be related to each other. The connection with gravitons is especially interesting since also classical gravitation is closely related to the spin glass degeneracy.

5.3.3 Evolution of mathematical ideas

The evolution of mathematical ideas has been driven by the following frequently asked questions.

1. Is p-adicity realized at space-time level or only at the level of p-adic thermodynamics which was the first application of p-adic numbers? If p-adic space-time regions really make sense, what is their physical interpretation?

2. Physics seems to require correspondence between p-adic and real numbers. What is the role of canonical identification: does it only map p-adic probabilities to their real counterparts or could it be applied also at space-time level despite the obvious difficulties with general coordinate invariance? What about correspondence defined by rational numbers which can be regarded as numbers common to all number fields. Is it possible to assign to a real space-time surface a p-adic counterpart by procedure respecting general coordinate invariance?

3. Does the notion of p-adicization of real physics make sense? How one might achieve the p-adicization in general coordinate invariance manner? What should one p-adicize: only probability calculus and thermodynamics? Or should one include also Hilbert space level? What about p-adicization at space-time level and perhaps even WCW-level?

4. What is the origin of p-adicity? What is the origin of p-adic length scale hypothesis? How it is possible to assign p-adic prime to a given real space-time sheet as required by the p-adic mass calculations?

5. There have been also technical problems. Besides differential calculus also integral calculus is basic element of classical physics since all variational principles involve integrals over space-time. Also the functional integral over WCW is needed in order to define S-matrix elements. How one could circumvent the difficulties caused by the non-existence of a p-adic valued define integral based on Riemann sum.

p-Adic physics as physics of cognition and intentionality and generalization of number concept

The identification of p-adic physics as physics of cognition and intention suggests strongly connections between cognition, intentionality, and number theory. The new idea is that also real transcendental numbers can appear in the extensions of p-adic numbers which must be assumed to be finite-dimensional at least in the case of human cognition.

The basic ingredient is the new view about numbers: real and p-adic number fields are glued together like pages of a book along common rationals representing the rim of the book. Also the rational multiples of algebraic numbers existing p-adically are shared in this manner so that the pages of the book can be stuck together along these lines. This generalizes to the extensions of p-adic number fields and the outcome is a complex fractal book like structure containing books within books.

This holds true also for manifolds and one ends up to the view about many-sheeted space-time realized as 4-surface in 8-D generalized imbedding space and containing both real and p-adic space-time sheets. The transformation of intention to action corresponds to a quantum jump in which p-adic space-time sheet is replaced with a real one.

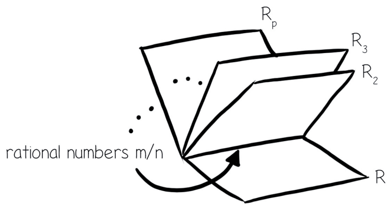

Figure 5.7: Various number fields combine to form a book like structure.

One implication is that the rationals having short distance p-adically are very far away in the real sense. This implies that p-adically short temporal and spatial distances correspond to long real distances and that the evolution of cognition proceeds from long to short temporal and spatial scales whereas material evolution proceeds from short to long scales. Together with p-adic non-determinism due the fact that the integration constants of p-adic differential equations are piecewise constant functions this explains the long range temporal correlations and apparent local randomness of intentional behavior. The failure of the real statistics and its replacement by p-adic fractal statistics for time series defined by varying number N of measurements performed during a fixed time interval T allows very general tests for whether the system is intentional and what is the p-adic prime p characterizing the "intelligence quotient" of the system. The replacement of $log(p_n)$ in the formula $S = -\sum_n p_n log(p_n)$ of Shannon entropy with the logarithm of the p-adic norm $|p_n|_p$ of the rational valued probability allows to define a hierarchy of number theoretic information measures which can have both negative and positive values.

Since p-adic numbers represent a highly number theoretical concept one might expect that there are deep connections between number theory and intentionality and cognition. The discussions with Uwe Kämpf in CASYS'2003 conference in Liege indeed stimulated a bundle of ideas allowing to develop a more detailed view about intention-to-action transformation and to disentangle these connections. These discussions made me aware of the fact that my recent views about the role of extensions of p-adic numbers are perhaps too limited. To see this consider the following arguments.

1. Pure p-adic numbers predict only p-adic length scales proportional to $p^{n/2}l$, l CP_2 length scale about 10^4 Planck lengths, $p \simeq 2^k$, k prime or power of prime. As a matter fact, all positive integer values of k are possible. This is however not enough to explain all known scale hierarchies. Fibonacci numbers $F_n : F_n + 1 = F_n + F_{n-1}$ behave asymptotically like $F_n = kF_{n-1}$, k solution of the equation $k^2 = k + 1$ given by $k = \Phi = (1 + \sqrt{5})/2 \simeq 1.6$. Living systems and self-organizing systems represent a lot of examples about scale hierarchies coming in powers of the Golden Mean $\Phi = (1 + \sqrt{5})/2$.

   By allowing the extensions of p-adics by algebraic numbers one ends up to the idea that also the length scales coming as powers of x, where x is a unit of algebraic extension analogous to imaginary unit, are possible. One would however expect that the generalization of the p-adic length scale hypothesis alone would predict only the powers $\sqrt{x}p^{n/2}$ rather than $x^k p^{n/2}$, $k = 1, 2, ...$ Perhaps the purely kinematical explanation of these scales is not possible and genuine dynamics is needed. For sinusoidal logarithmic plane waves the harmonics correspond to the scalings of the argument by powers of some scaling factor x. Thus the powers of Golden Mean might be associated with logarithmic sinusoidal plane waves.

2. Physicist Hartmuth Mueller has developed what he calls Global Scaling Theory [B4] based on the observation that powers of e (Neper number) define preferred length scales. These powers associate naturally with the nodes of logarithmic sinusoidal plane waves and correspond to various harmonics (matter tends to concentrate on the nodes of waves since force vanishes at the nodes). Mueller talks about physics of number line and there is great temptation to assume that deep number theory is indeed involved. What is troubling from TGD point of view that Neper number e is not algebraic. Perhaps a more general approach allowing also

transcendentals must be adopted.

3. Classical mathematics, such as the theory of elementary functions, involves few crucially important transcendentals such as e and π. This might reflect the evolution of cognition: these numbers should be cognitively and number theoretically very special. The numbers e and π appear also repeatedly in the basic formulas of physics. They however look p-adically very troublesome since it has been very difficult to imagine a physically acceptable generalization of such simple concepts as exponent function, trigonometric functions, and logarithm resembling its real counterpart by allowing only the extensions of p-adic numbers based on algebraic numbers.

These considerations stimulate the question whether, besides the extensions of p-adics by algebraic numbers, also the extensions of p-adic numbers involving π and e and other transcendentals might be needed. The intuitive expectation motivated by the finiteness of human intelligence is that these extensions should have finite algebraic dimensions, and it indeed turns out that this is possible under some conditions which can be formulated as very general number theoretical conjectures. Since e^p exists p-adically, the powers $e, ..., e^{p-1}$ define a p-dimensional extension as do also the roots of polynomials with coefficients which are in an extension of rationals containing e and its powers. Contrary to the original conjecture, π however cannot belong to a finite-dimensional extension of p-adics. It is an open question whether one should allow infinite-dimensional extension of p-adic numbers containing π. In any case, the special role of π however becomes an extremely strong constraint for the p-adicization of quantum TGD by algebraic continuation from the realm of rationals to real and p-adic number fields.

Second question is whether there might be some dynamical mechanism allowing to understand the hierarchy of scalings coming in powers of some preferred transcendentals and algebraic numbers like Golden Mean. Conformal invariance implying that the system is characterized by a universal spectrum of scaling momenta for the logarithmic counterparts of plane waves seems to provide this mechanism. This spectrum is determined by the requirement that it exists for both reals and all p-adic number fields assuming that finite-dimensional extensions are allowed in the latter case. The spectrum corresponds to the zeros of the Riemann Zeta if Zeta is required to exist for all number fields in the proposed sense, and a lot of new understanding related to Riemann hypothesis emerges and allows to develop further the previous TGD inspired ideas about how to prove Riemann hypothesis [L1] , [H1] .

Algebraic continuation as a basic principle

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function. Logarithm is also such a function provided that the above mentioned number theoretic conjecture holds true.

The definition of a definite integral for p-adic numbers has been the key challenge in attempts to construct p-adic physics and algebraic continuations seems to solve this problem. The first problem is that p-adic numbers are not well ordered and one cannot define what ordered integration interval $[a, b]$ means p-adically. The second problem is that Riemann sum gives identically vanishing p-adic integral if coordinate increments approach zero at the limit. One can however define the definite integral in terms of the integral function:

$$\int_a^b f(x)dx = F(b) - F(a) \ ; \ f(x) = \frac{dF(x)}{dx} \ .$$

Integral function $F(x)$ is obtained using the inverse of the derivation just as in the real context. If integration limits are restricted to be rational numbers or finitely extended rational numbers, they can be ordered using the ordering of real numbers. This would essentially mean that p-adic integration measure is an algebraic continuation of the real integration measure.

Also residy calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residy formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". This could mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Gaussian integration as a purely algebraic process gives hopes to define p-adic variants of WCW integrals but only in the case that the integral over WCW reduces effectively to the Gaussian integral of a free quantum field theory. If configuration space is indeed a union of symmetric spaces, there are good hopes for achieving this (Duistermaat-Hecke theorem).

p-Adic integration is not necessarily needed to define the p-adic counterpart for the field equations associated with Kähler action but the continuation of the physics from real WCW to the p-adic variants of WCWs requires the existence of the p-adic valued Kähler action. If it is possible to assign to a given real space-time surface a p-adic counterpart uniquely in a given resolution for rational numbers, one can define the p-adic Kähler action as the real action interpreted as p-adic number in case that the real action belongs to a finite extension of rationals.

5.3.4 Generalized Quantum Mechanics

One can consider two generalizations of quantum mechanics to a fusion of p-adic and real quantum mechanics.

1. For the first generalization the guiding principle for the generalization of quantum mechanics is that quantum mechanics in a given number field is obtained as an algebraic continuation of the quantum mechanics in the field of rational numbers common to all number fields or in finite-dimensional extensions of rational numbers. This means that U-matrices U_F for transitions from H_Q to H_F, where F refers to various completions of rationals, are obtained as algebraic continuations of the unitary U-matrix U_Q for H_Q. The generalization means enormously strong algebraic constraints on the form of the U-matrix.

2. A more radical option is that transitions from rational Hilbert space H_Q to the Hilbert spaces H_F associated with different number fields occur. This requires that U-process is followed by a process analogous to a state function reduction and preparation takes care that the resulting states become states in H_Q: this is what makes this generalization of a special interest. In this case one can speak about total scattering probability from H_Q to H_F. The U-matrices U_F are not anymore mere analytic continuations of U_Q. A possible interpretation of the unitary process $H_Q \rightarrow H_F$ is as generation of intention whereas the reduction and preparation means the transformation of the intention to action.

The assumption that H_Q allows an algebraic continuation to the spaces H_F is probably too strong an idealization in p-adic and even in the real case. For instance, one cannot allow all rational valued momenta in p-adic case for the simple reason that the continuation to the p-adic case involves always some momentum cutoff if the extension of p-adics remains finite. Even in the real case the summation over all rational momenta in the unitarity conditions of U-matrix fails to make sense and cutoff is needed. A hierarchy of cutoffs suggests itself and has a natural interpretation as number theoretical hierarchy of extensions of p-adics.

In order to avoid un-necessary complications the following formal discussion however uses H_Q as a universal Hilbert space contained by the various state spaces H_F.

Quantum mechanics in H_F as a algebraic continuation of quantum mechanics in H_Q

The rational Hilbert space H_Q is representable as the set of sequences of real or complex rationals of which only finite number are non-vanishing. Real and p-adic Hilbert spaces are obtained as the numbers in the sequences to become real or p-adic numbers and no limitations are posed to the number of non-vanishing elements. All these Hilbert spaces have rational Hilbert space H_Q as a common sub-space. Also momenta and other continuous quantum numbers are replaced by a discrete value set. Superposition principle holds true only in a restricted sense, and state function reduction and preparation leads always to a final state which corresponds to a state in H_Q. This picture differs from the earlier one in which p-adic and real Hilbert spaces were assumed to form a direct sum.

The notion of unitarity generalizes. Contrary to the earlier beliefs, U-matrix does not possess matrix elements between different number fields but between rational Hilbert space and Hilbert spaces associated with various completions of rationals. This makes sense since the final state of the quantum jump (and thus the initial state of the unitary process, is always in H_Q.

The U-matrix is a collection of matrices U_F having matrix elements in the number field F. U_F maps H_Q to H_F. Each of these U-matrices is unitary. Also U_Q is unitary and U_F is obtained by algebraic continuation in the quantum numbers labelling the states of U_Q to U_F.

Hermitian conjugation makes sense since the defining condition

$$\langle \alpha_F | U n_Q \rangle = \langle U^\dagger \alpha_F | n_Q \rangle \ . \tag{5.3.1}$$

allows to interpret $|n_Q\rangle$ also as an element of H_F. If U would map different completed number fields to each other, hermiticity conditions would not make sense.

The hermitian conjugate of U-matrix maps H_F to H_Q so that $UU^\dagger$ *resp.* $U^\dagger U$ maps H_F *resp.* H_Q to itself. This means that there are two independent unitarity conditions

$$\begin{aligned} U_F U_F^\dagger &= Id_F \ , \\ U_F^\dagger U_F &= Id_Q \ . \end{aligned} \tag{5.3.2}$$

One can write $U = P_Q + T_F$ and $U^\dagger = P_Q + T_F^\dagger$, where P_Q refers to the projection operator to H_Q. This gives

$$\begin{aligned} T_F + T_F^\dagger &= -T_F T_F^\dagger \ , \\ P_Q T_F + T_F^\dagger P_Q &= -T_F^\dagger T_F \ . \end{aligned} \tag{5.3.3}$$

It is convenient to introduce the notations $T_Q = P_Q T_F$ and $T_Q^\dagger = T_F^\dagger P_Q$ with analogous notations for U and $U^\dagger$. The first condition, when multiplied from both sides by P_Q, gives together with the second equation unitarity conditions for T_Q

$$\begin{aligned} T_Q + T_Q^\dagger &= -T_Q T_Q^\dagger \ , \\ T_Q + T_Q^\dagger &= -T_F^\dagger T_F \ . \end{aligned} \tag{5.3.4}$$

This means that the restriction of the U-matrix to H_Q is unitary.

The difference between the right hand sides of the equation should vanish. The understanding of how this happens requires more delicate considerations. For instance, in the case of $F = C$ continuous sum over indices appears at the right hand side coming from four-momenta labelling the states. The restrictions of quantum numbers to Q and its subsets could be a process analogous to the momentum cutoff of quantum field theories. The continuation from discrete integer valued labels of, say discrete momenta, to continuous values is performed routinely in various physical models routinely, and it would seem that this process has cognitive and physical counterparts. This picture conforms with the vision that the rational (or extended rational) U-matrix U_Q gives the U-matrices U_F by an algebraic continuation in the quantum numbers labelling the states (say 4-momenta).

Could U_F describe dispersion from H_Q to the spaces H_F?

One can also consider a more general situation in which the states in H_Q can be said to disperse to the sectors H_F. In this case one can write

$$T = \text{"}\sum_F\text{"} T_F \ . \tag{5.3.5}$$

Here the sum has only a symbolic meaning since different number fields are in question and an actual summation is not possible. The T-matrix T_Q is the sum of the restrictions of T_F to H_Q and is the sum of rational valued T-matrices: $T_Q = \sum_F P_Q T_F$.

The T-matrices T_F are not anymore obtainable by algebraic continuation from same T-matrix T_Q. The unitarity conditions

$$\sum_F (P_Q T_F + T_F^\dagger P_Q) = -\sum_F T_F^\dagger T_F \tag{5.3.6}$$

make sense only if they are satisfied separately for each T_F, exactly as in the previous case. T

The diagonal elements

$$T_F^{mm} + \overline{T}_F^{mm} = \sum_\alpha T_F^{m\alpha}\overline{T}_F^{m\alpha} = \sum_r T_F^{mr}\overline{T}_F^{mr}$$

give essentially total scattering probabilities from the state $|m\rangle$ of H_Q to the sector H_F, and must be rational (or extended rational) numbers. One can therefore say that each U-process leads with a definite probability to a particular sector of the state space.

The fact that states which are superpositions of states in different spaces H_F does not make sense mathematically, forces the occurrence of a process, which might be regarded as a number theoretical counterpart of state function reduction and preparation. First a sector H_F is selected with probability p_F. Then F-valued (in particular complex valued) entanglement in H_F is reduced by state reduction and preparation type processes to a rational or extended rational entanglement having interpretation as bound state entanglement. It would be natural to assume that Negentropy Maximization Principle governs this process. Obviously the possibility to reduce state function reduction to number theory forces to consider quite seriously the proposed option.

5.3.5 Do state function reduction and state-preparation have number theoretical origin?

The foregoing considerations support the view that state function reduction and state preparation are number theoretical necessities so that there would be a deep connection between number theory and free will. One could even say that free will is a number theoretic necessity. The resulting more unified view provides the reason why for state function reduction, and preparation and allows to generalize previous views developed gradually by physics and consciousness inspired educated guess work.

Negentropy Maximization Principle as variational principle of cognition

It is useful to discuss the original view about Negentropy Maximization Principle (NMP) before considering the possible generalization of NMP inspired by the number theoretic vision.

NMP was originally motivated by the need to construct a TGD based quantum measurement theory. Gradually it however became clear that standard quantum measurement theory more or less follows from the assumption that the world of conscious experience is classical: this meant that NMP became a principle governing only state preparation.

State function reduction is achieved if a localization in zero modes occurs in each quantum jump, and if U matrix in zero modes corresponds to a flow in some orthogonal basis for the WCW spinor fields in the quantum fluctuating fiber degrees of freedom of the WCW. The requirement that U-matrix induces effectively a flow in zero modes is consistent with the effective classicality of the zero modes requiring that quantum evolution causes no dispersion. The one-one correlation between preferred quantum state basis in quantum fluctuating degrees of freedom and zero modes implies nothing but a one-one correspondence between quantum states and classical variables crucial for the interpretation of quantum theory. It seems that number theoretical vision forces to generalize this view, and to raise NMP to a completely general principle applying also to the state function reduction as the original proposal indeed was.

In its original form NMP governs the dynamics of self measurements and thus applies to the quantum jumps reducing the entanglement between quantum fluctuating degrees of freedom for given values of zero modes. Self measurements reduce the entanglement only between sub-systems in quantum fluctuating degrees of freedom since they occur after the localization in the zero modes. Self measurement is repeated again and again for the unentangled sub-systems resulting in each self measurement. This cascade of self measurements leads to a state possessing only extended rational entanglement identifiable as bound state entanglement and having negative number theoretic entanglement entropy. This process should be equivalent with the state preparation process assumed to be performed by a conscious observer in standard quantum measurement theory.

NMP states that the self measurement can be regarded as a quantum measurement of the sub-system's density matrix reducing the counterpart of the entanglement entropy of some subsystem to a smaller value, and that this occurs for the sub-system for which the reduction of the entanglement entropy is largest among all sub-systems of the p-adic self. Inside each self NMP fixes some sub-system which is quantum measured in the quantum jump. One could perhaps say that self measurements make possible quantum level self repair since they allow the system in self state to fight against thermalization which results from the generation of unbound entanglement between sub-system-complement pairs.

NMP and number theory

The requirement the universe of conscious experience is classical is one manner to justify the notion of quantum jump. This hypothesis could be replaced by a postulate that state function reduction and preparation project quantum states to a definite number field and that only extended rational entanglement identifiable as bound state entanglement is stable. This is consistent with NMP since it is possible to assign to an extended rational entanglement a non-negative number theoretic negentropy as the maximum over entropies defined by various p-adic entropies $S_p = -\sum p_k log(|p_k|_p)$.

The unitary process U would thus start from a product of bound states for which entanglement coefficient are extended rationals, and would lead to a formal superposition of states belonging to different number fields. Both state function reduction and state preparation would begin with a localization to a definite number field. This localization would be followed by a self measurement cascade reducing the entanglement to extended rational entanglement.

This vision forces to challenge the earlier views about state function reduction.

1. There is no good reason for why NMP could not be applied to both state function reduction and preparation.

2. If the entanglement between zero modes and quantum fluctuating degrees of freedom involves only discrete values of zero modes, the problems caused by the fact that no well-defined functional integral measure over zero modes exists, find an automatic resolution. Since extended rational entanglement possesses negative entanglement entropy, it is stable also against reduction if NMP applies completely generally. A discrete entanglement involving transcendentals not contained to any *finite* extension of any p-adic number field is unstable and reduced.

3. The quantum measurement lasts for a time determined by the life-time of the bound state entanglement between zero modes and quantum fluctuating degrees of freedom. Physical considerations of course support the view that it takes more than single quantum jump (10^{-39} seconds of psychological time) for the state function reduction to take place. The notion of zero mode-zero mode bound state entanglement seems however to be self-contradictory. If join along boundaries bonds are space-time correlates for the bound state entanglement, their formation should transform roughly half of the zero modes associated with the two space-time sheets to quantum fluctuating degrees of freedom.

4. If p-adic length scale hierarchy has as its counterpart a hierarchy of state function reduction and preparation cascades, one must accept the quantum parallel occurrence of state function reduction and preparation processes in the parallel quantum universes corresponding to different p-adic length scales. This picture provides a justification for the modelling of hadron as a quantum system in long length and time scales and as a dissipative system consisting of quarks and gluons in shorter length and time scales. The bound state entanglement between sub-systems of entangled systems having as a space-time correlate join along boundaries bonds connecting sub-system space-time sheets, is a second important implication of the new sub-system concept, and plays a central role in TGD inspired theory of consciousness.

5.4 The boost from TGD inspired theory of consciousness

Quite generally, TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory. The identification of quantum jump as a moment of consciousness is analogous to the identification of elementary particles as basic building blocks of matter. The observer is an outsider in standard quantum measurement theory and is replaced by the notion of self in TGD inspired theory of consciousness. Selves identified as systems able to avoid bound state entanglement and identifiable as ensembles of quantum jumps, are analogous to many-particle states. The sensory and other qualia of self are determined as statistical averages over quantum number and zero mode increments for the increasing sequence of quantum jumps defining self. Especially important are selves, which are in a state of macro-temporal quantum coherence since for these selves the entropy of the ensemble defined by the quantum jumps does not increase and the qualia stay sharp. These selves are analogous to bound states of elementary particles and their formation actually corresponds to the generation of bound state entanglement.

5.4.1 The anatomy of the quantum jump

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories rather than to a non-deterministic behavior of a single quantum history. Therefore U-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of U-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

1. The classical time development determined by the criticality condition selecting preferred extremals as generalized Bohr orbits [K88] .

2. The unitary "time development" defined by U associated with each quantum jump

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

   and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator and this means that one must leave open the identification of U-matrix with S-matrix.

3. The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness. A crucial role is played by the classical non-determinism of Kähler action implying that the non-determinism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to

Schrödinger time evolution of infinite duration since there is *no* real time evolution or translation involved. It is not clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. Physically it seems obvious that U matrix should decompose to a cosmological U-matrix representing dispersion in WCW and U-matrix representing local dynamics: this indeed occurs thanks to the classical non-determinism of the Kähler action. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation. An important exception are the zero modes characterizing center of mass degrees of freedom of 3-surface which correspond to the isometries of $M^4_+ \times CP_2$. In these degrees of freedom localization does not occur. At the limit when 3-surfaces are regarded as point-like objects theory should obviously reduce to quantum field theory.

The three non-determinisms

Besides the non-determinism of quantum jump, TGD allows two other kinds of non-determinisms: the classical non-determinism basically due the vacuum degeneracy of the Kähler action and p-adic non-determinism of p-adic differential equations due to the fact that functions with vanishing p-adic derivative correspond to piecewise constant functions.

To achieve classical determinism in a generalized sense, one must generalize the definition of the 3-surfaces Y^3 (belonging to light cone boundary) by allowing also "association sequences", that is 3-surfaces which have, besides the component belonging to the light cone boundary, also disjoint components which do not belong to the light cone boundary and have mutual *time-like separations*. This means the introduction of additional, one might hope typically discrete, degrees of freedom (consider non-determinism based on bifurcations as an example). It is even possible to have quantum entanglement between the states corresponding to different values of time.

Without the classical and p-adic non-determinisms general coordinate invariance would reduce the theory to the light cone boundary and this would mean essentially the loss of time which occurs also in the quantization of general relativity as a consequence of general coordinate invariance. Classical and p-adic non-determinisms imply that one can have quantum jumps with non-determinism (in conventional sense) located to a finite time interval. If quantum jumps correspond to moments of consciousness, and if the contents of consciousness are determined by the locus of the non-determinism, then these quantum jumps must give rise to a conscious experience with contents located in a finite time interval.

Also p-adic space-time sheets obey their own quantum physics and are identifiable as seats of cognitive representations. p-Adic non-determinism is the basic prerequisite for imagination and simulation. The notion of cognitive space-time sheet as a space-time sheet having finite time duration is one aspect of the p-adic non-determinism and allows to understand how the notion of psychological time emerges. Cognitive space-time sheets simply drift quantum jump by quantum to the direction of geometric future since there is much more room there in the light cone cosmology.

The classical non-determinism is maximal for CP_2 type extremals for which the M^4_+ projection of the space-time surface is random light-like curve. In this case, basic objects are essentially four- rather than 3-dimensional. The basic implication of the classical non-determinism is that quantum theory does not reduce to the light cone boundary. Secondly, U-matrix reduces to a tensor product of a cosmologcal U-matrix and local U-matrices relevant for particle physics. As a matter fact, an entire hierarchy of U-matrices defined in various p-adic time scales is expected to appear in the hierarchy. Thirdly, the classical non-determinism of CP_2 type extremals allows a topologization of the Feynman diagrammatics of quantum field theories and string models. Although localization in zero modes characterizing zitterbewegung orbit occur in quantum jump, there is integral over the positions of vertices which correspond to cm degrees of freedom for imbedding space, and this gives rise to a sum over various Feynman diagrams.

How psychological time and its arrow emerge?

How psychological time and its arrow emerge is the basic challenge for the hypothesis that quantum jumps occur between quantum histories and are identifiable as moments of consciousness. Mind like space-time sheets provide a geometric model of unconscious mind in TGD framework and make it possible to solve the puzzle of psychological time. The first argument is following.

Mind like space-time sheets have well center of mass time coordinate and this coordinate is zero mode identifiable as psychological time. Localization in zero modes means that final states of quantum jumps correspond to quantum superpositions of space-time surfaces having same number of mind like space-time sheets such that given mind like space-time sheet possesses same value of psychological time for all space-time surfaces appearing in the superposition. The arrow of psychological time follows from the gradual drift of the mind like space-time sheets in future direction occurring quantum jump by quantum jump and is implied by the geometry of future light cone (there is more volume in the future of a given light cone point than in its future). The simplest assumption is that the average increment of psychological time in single quantum jump is of order CP_2 time, which is about 10^4 Planck times.

Besides classical non-determinism there is also p-adic non-determinism and one should keep mind open in the attempts to identify the roles of these two non-determinisms. The interpretation taken as a working hypothesis in the recent version of TGD inspired theory of consciousness is that p-adic space-time regions provide cognitive representations of the real regions and serve as correlates for intentions. Real regions are in turn symbolic representations for the material world in TGD sense of the word. This means that besides ordinary matter also higher level physical states associated with the real space-time sheets of a finite duration and having vanishing net energy are possible. The zero energy states representing pairs of incoming and outgoing states could make possible self-referential real physics representing the laws of physics in the structure of the higher level physical states. Real space-time sheets of finite temporal duration might be interpreted also as correlates of pure sensory experience as opposed to p-adic space-time sheets which can be identified as correlates of thoughts. Also volition could be assigned to the quantum jumps involving selection between various branches of multi-furcations implied by the classical non-determinism.

A more refined argument explaining the arrow of psychological time is based on the idea that psychological time correspond to the moment of geometric time which gives the dominant contribution to the conscious experience, and that it is the transformation of intentions to actions which provides this contribution. The transformation of intentions to actions corresponds to the transformation of p-adic space-time sheets to real ones, and one can identify psychological time as characterizing the position of the intention-to action phase transition front. In order to have consistency with the basic facts about everyday conscious experience one must assume that the geometric past remains unable to express intentions for a period of time longer than the life cycle since otherwise the decisions made in say my geometric youth subjectively now could induce dramatic changes in my recent life. This dead time would be analogous to the recovery time of neuron after the generation of nerve pulse.

Macro-temporal quantum coherence and spin glass degeneracy

At the space-time level the generation of macroscopic quantum coherence is easy to understand if one accepts the identification of the space-time sheets as coherence regions. Quantum criticality and the closely related spin glass degeneracy are essential for the fractal hierarchy of space-time sheets. The problem of understanding macro-temporal and macroscopic quantum coherence at the level of WCW (of 3-surfaces) is a more tricky challenge although quantum-classical correspondence strongly suggests that this is possible.

Concerning macro-temporal quantum coherence, the situation in quantum TGD seems at the first glance to be even worse than in standard physics. The problem is that simplest estimate for the increment in psychological time in single quantum jump is about 10^{-39} seconds derived from the idea that single quantum jump represent a kind of elementary particle of consciousness and thus corresponds to CP_2 time of about 10^{-39} seconds. If this time interval defines coherence time one ends up to a definite contradiction with the standard physics. Of course, the average increment of the geometric time during single quantum jump could vary and correspond to the de-coherence time. The idea of quantum jump as an elementary particle of consciousness does not support this assumption.

To understand how this naive conclusion is wrong, one must look more precisely the anatomy of quantum jump. The unitary process $\Psi_i \rightarrow U\Psi_i$, where Ψ_i is a prepared maximally unentangled state, corresponds to the quantum computation producing maximally entangled multi-verse state. Then follows the state function reduction and after this the state preparation involving a sequence of self measurements and given rise to a new maximally unentangled state Ψ_f.

1. What happens in the state function reduction is a localization in zero modes, which do not contribute to the line element of the WCW metric. They are non-quantum fluctuating degrees of freedom and TGD counterparts of the macroscopic, classical degrees of freedom. There are however also quantum-fluctuating degrees of freedom and the assumption that zero modes and quantum fluctuating degrees of freedom are correlated like the direction of a pointer of a measurement apparatus and quantum numbers of the quantum system, implies standard quantum measurement theory.

2. Bound state entanglement is assumed to be stable against state function reduction and preparation. Bound state formation has as a geometric correlate formation of join along boundaries bonds between space-time sheets representing free systems. Thus the members of a pair of disjoint space-time sheets are joined to single space-time sheet. Half of the zero modes is transformed to quantum fluctuating degrees of freedom and only overall center of mass zero modes remain zero modes. These new quantum fluctuating degrees of freedom represent macroscopic quantum fluctuating degrees of freedom. In these degrees of freedom localization does not occur since bound states are in question.

   Both state function reduction and state preparation stages leave this bound state entanglement intact, and in these degrees of freedom the system behaves effectively as a quantum coherent system. One can say that a sequence of quantum jumps binds to form a single long-lasting quantum jump effectively. This is in complete accordance with the fractality of consciousness. Quantum jumps represent moments of consciousness which are "elementary particles of consciousness" and in macro-temporal quantum coherent state these elementary particles bind to form atoms, molecules, etc. of consciousness.

3. The properties of the bound state plus its interaction with the environment allow to estimate the typical duration of the bound state. This time takes the role of coherence time. This suggests a connection with the standard approach to quantum computation. An essential element is spin glass degeneracy. The generation of join along boundaries bonds connecting the space-time sheets of the composite systems is the space-time correlate for the formation of the bound states. Spin glass degeneracy is much higher for the bound states because of the presence of the join along boundaries bonds. This together with the fact that these degenerate states are almost identical so that transition amplitudes between them are also almost identical, implies that the life-time of the majority of bound states is much longer than one might expect otherwise. The detailed argument is carried out in [K21] and can be applied to show that spin glass degeneracy for the color flux tubes explains color confinement [K32] .

4. The number theoretic notion of information relies on Shannon entropy in which the logarithms of probabilities are replaced by logarithms of their p-adic norms. This requires that the probabilities are rational or belong to an finite-dimensional extension of rationals. What is so important is that this entropy can have also negative values. If one assumes that bound states form a hierarchy such that the entanglement coefficients belong always to a finite-dimensional extension of rationals, one can define the entanglement entropy as a number theoretic entropy associated with some prime p. In p-adic context the prime is unique whereas in the real context the value of the prime can be selected in such a manner that the entropy is maximally negative. This prime would be naturally a maximal prime factor of the integer N defining the number of strictly deterministic regions of the space-time sheet in

question. If this assumption is made, NMP alone implies the stability of bound states against state preparation by self measurements. This generalization of the information concept has far reaching implications in TGD inspired theory consciousness.

5.4.2 Negentropy Maximization Principle and new information measures

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = -\sum_n p_n log(p_n) \ . \tag{5.4.1}$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

p-Adic entropies

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of x, where log_p denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) \ , \\ k(p_n) &= -log_p(|p_n|) \ . \end{aligned} \tag{5.4.2}$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n)log(p) + p^{k_n} log(p_n/p^{k_n}\right] . \quad (5.4.3)$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times log(p)) \ , \\ (\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ . \end{aligned} \quad (5.4.4)$$

The real counterpart of the p-adic entropy is non-negative.

Number theoretic entropies and bound states

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|)log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv Max\{-S_p, \ p \text{ prime}\} . \quad (5.4.5)$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP, and has a natural interpretation as bound state entanglement. The prediction would be that the bound states of real systems form a number theoretical hierarchy according to the prime p and and dimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes projecting the physical states from either real or p-adic sectors of the state space to their intersection. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the U-matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

Number theoretic information measures at the space-time level

Quantum classical correspondence suggests that the notion of entropy should have also space-time counterpart. Entropy requires ensemble and both the p-adic non-determinism and the non-determinism of Kähler action allow to define the required ensemble as the ensemble of strictly deterministic regions of the space-time sheet. One can measure various observables at these space-time regions, and the frequencies for the outcomes are rational numbers of form $p_k = n(k)/N$, where N is the number of strictly deterministic regions of the space-time sheet. The number theoretic entropies are well defined and negative if p divides the integer N. Maximum is expected to result for the largest prime power factor of N. This would mean the possibility to assign a unique prime to a given real space-time sheet and thus the solve the basic problem created already by p-adic mass calculations.

The classical non-determinism resembles p-adic non-determinism in the sense that the space-time sheet obeys effective p-adic topology in some length and time scale range is consistent with this idea since p-adic fractality suggests that N is power of p.

5.5 TGD as a generalized number theory

The vision about a number theoretic formulation of quantum TGD is based on the gradual accumulation of wisdom coming from different sources. The attempts to find a formulation allowing to understand real and p-adic physics as aspects of some more general scenario have been an important stimulus and generated a lot of, not necessarily mutually consistent ideas, some of which might serve as building blocks of the final formulation. The original chapter representing the number theoretic vision as a consistent narrative grew so massive that I decided to divide it to three parts.

The first part is devoted to the p-adicization program attempting to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also). Highly non-trivial number theoretic conjectures are an i outcome of the program.

Second part focuses on the idea that the tangent spaces of space-time and imbedding space can be regarded as 4- *resp.* 8-dimensional algebras such that space-time tangent space defines sub-algebra of imbedding space. The basic candidates for the pair of algebras are hyper-quaternions and hyper-octonions. The problems are caused by the Euclidian signature of the Euclidian norm.

The great idea is that space-time surfaces X^4 correspond to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of $HO = M^8$. The possibility to assign to X^4 a surface in $M^4 \times CP_2$ means a number theoretic analog for spontaneous compactification. Of course, nothing dynamical is involved: a dual relation between totally different descriptions of the physical world are in question. In the spirit of generalized algebraic geometry one can ask whether hyper-quaternionic space-time surfaces and their duals could be somehow assigned to hyper-octonion analytic maps $HO \rightarrow HO$, and there are good arguments suggesting that this is the case.

The third part is devoted to infinite primes. Infinite primes are in one-one correspondence with the states of super-symmetric arithmetic quantum field theories. The infinite-primes associated with hyper-quaternionic and hyper-octonionic numbers are the most natural ones physically because of the underlying Lorentz invariance, and the possibility to interpret them as momenta with mass squared equal to prime. Most importantly, the polynomials associated with hyper-octonionic infinite primes have automatically space-time surfaces as representatives so that space-time geometry becomes a representative for the quantum states.

5.5.1 The painting is the landscape

The work with TGD inspired theory of consciousness has led to a vision about the relationship of mathematics and physics. Physics is not in this view a model of reality but objective reality itself: painting is the landscape. One can also equate mathematics and physics in a well defined sense and the often implicitly assumed Cartesian theory-world division disappears. Physical realities are mathematical ideas represented by WCW spinor fields (quantum histories) and quantum jumps between quantum histories give rise to consciousness and to the subjective existence of mathematician.

The concrete realization for the notion algebraic hologram based on the notion of infinite prime is a second new element. The notion of infinite rationals leads to the generalization of also the notion of finite number since infinite-dimensional space of real units obtained from finite rational valued ratios q of infinite integers divided by q. These units are not units in p-adic sense. The generalization to the (hyper-)quaternionic and (hyper-)octonionic context means that ordinary space-time points become infinitely structured and space-time point is able to represent even the quantum physical state of the Universe in its algebraic structure. Single space-time point becomes the Platonia not visible at the level of real physics but essential for mathematical cognition.

In this view evolution becomes also evolution of mathematical structures, which become more and more self-conscious quantum jump by quantum jump. The notion of p-adic evolution is indeed a basic prediction of quantum TGD but even this vision might be generalized by allowing rational-adic topologies for which topology is defined by a ring with unit rather than number field.

5.5.2 p-Adic physics as physics of cognition

Real and p-adic regions of the space-time as geometric correlates of matter and mind

The solutions of the equations determining space-time surfaces are restricted by the requirement that imbedding space-coordinates are real. When this is not the case, one might apply instead of a real completion with some rational-adic or p-adic completion: this is how rational-adic p-adic physics could emerge from the basic equations of the theory. One could interpret the resulting rational-adic or p-adic regions as geometrical correlates for 'mind stuff'.

p-Adic non-determinism implies extreme flexibility and therefore makes the identification of the p-adic regions as seats of cognitive representations very natural. Unlike real completion, p-adic completions preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with 'mind like' regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of 'self' and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves!

The generalization of the notion of number and p-adicization program

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

For instance, residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition and intentionality. The basic stumbling block of this program is integration and algebraic continuation should allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. Two types of cutoffs are predicted: p-adic length scale cutoff and a cutoff due to phase resolution. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$ related closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra [K27] . This cutoff hierarchy seems to relate closely to the hierarchy of cutoffs defined by the hierarchy of subalgebras of the super-symplectic algebra defined by the hierarchy of sets $(z_1, ...z_n)$, where z_i are the first n non-trivial zeros of Riemann Zeta. Hence there are good hopes that the p-adicization program might unify apparently unrelated branches of mathematics.

5.5.3 Space-time-surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?

Second thread in the development of ideas has been present for only few years ideas inspired by the possibility that quaternions and octonions might allow a deeper understanding of TGD. This thread emerged from the discussions with Tony Smith which stimulated very general ideas about space-time surface as associative, quaternionic sub-manifold of octonionic 8-space. Also the observation that quaternionic and octonionic primes have norm squared equal to prime in complete accordance with p-adic length scale hypothesis, led to suspect that the notion of primeness for quaternions, and perhaps even for octonions, might be fundamental for the formulation of quantum TGD [K88] . It turned out that, much in spirit with transition from Riemannian to pseudo-Riemannian geometry, hyper-quaternins and hyper-octonions are forced by physical considerations.

Transition from string models to TGD as replacement of real/complex numbers with quaternions/octonions

One can fairly say, that quantum TGD results from string model with the pair of real and complex numbers replaced with the pair of hyper-quaternions and hyper-octonions. Hyper is necessary in order to take into the Minkowskian signature of the metric.

Space-time identified as a hyper-quaternionic sub-manifold of the hyper-octonionic space in the sense that the tangent space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point. One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered.

Some delicacies are caused by the question whether the induced algebra at X^4 is just the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If the normal part of the product is projected out, the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2. This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH--M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteeing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2. This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter familiy co-hyper-quaternionic space-time surfaces.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that the absolute minima of Kähler action in $H = M^4 \times CP_2$ correspond to the hyper-quaternion analytic surfaces in HO. The map $f : HO \rightarrow S^6$ would probably satisfy some constraints posed by the requirement that the resulting surfaces define solutions of field equations in $M^4 \times CP_2$ picture. This conjecture has several variants. It could be that only the asymptotic behavior corresponds to hyper-quaternion analytic function but that hyper-quaternionicity is a general property of absolute minima. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates coding for the local selection of the plane of non-physical polarizations, appear naturally also in the construction of general solutions of field equations [K9] .

Physics as a generalized algebraic number theory and Universe as algebraic hologram

The third stimulus encouraging to think that TGD might be reduced to algebraic number theory and algebraic geometry in some generalized sense, came from the work with Riemann hypothesis [K76] . One can assign to Riemann Zeta a super-conformal quantum field theory and identify Zeta as a Hermitian form in the state space possibly defining a Hilbert space metric. The proposed form of the Riemann hypothesis implies that the zeros of ζ code for infinite primes which in turn have interpretation as Fock states of a super-symmetric quantum field theory if the proposed vision is correct.

A further stimulus came from the realization that algebraic extensions of rationals, which make possible a generalization of the notion of prime, could provide enormous representative and information storage power in arithmetic quantum field theory. Algebraic symmetries defined as transformations preserving the algebraic norm represent new kind of symmetries commuting with ordinary quantum numbers. Fractal scalings and discrete symmetries are in question so that the notion of fractality emerges to the fundamental physics in this manner.

The basic observation, completely consistent with fractality, is that these symmetries make possible what might be called *algebraic hologram.* The algebraic quantum numbers associated with elementary particle depend on the environment of the particle. The only possible conclusion seems to be that these fractal quantum numbers provide some kind of 'cognitive representation' about external world. This kind of an algebraic hologram would be in complete accordance with fractality and would provide first principle realization for fractality observed everywhere in Nature but not properly understood in standard physics framework. A further basic idea which emerged was the principle of *algebraic democracy*: all possible algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions are possible and emerge dynamically as properties of physical systems in algebraic physics.

5.5.4 Infinite primes and physics in TGD Universe

The notion of infinite primes emerged originally from TGD inspired theory of consciousness [K49] but it soon turned out that the notion could be used to build a number theoretic interpretation of quantum TGD and relate quantum to classical. Also the notion of infinite-P p-adicity emerges naturally and could replaces real topology with something more refined and appropriate for description of the space-time correlates of cognition.

Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes was one important step in the development suggesting strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes so that it applies even in the case of hyper-quaternionic and hyper-octonionic primes. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

What is remarkable is that one has quite realistic possibilities to understand the quantum numbers of physical particles in terms of hyper-octonionic infinite primes. Also the TGD inspired model for $1/f$ noise [K63] based on thermal arithmetic quantum field theory encouraged also to consider the idea about hyper-quaternionic or hyper-octonionic arithmetic quantum field theory as an essential element of quantum TGD.

Infinite primes as a bridge between quantum and classical

The final stimulus came from the observation stimulated by algebraic number theory [A66] . Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes. Infinite primes allow nice interpretation as Fock states of a second quantized super-symmetric quantum field theory. Also bound states are included.

This in turn led to the observation that one can represent infinite primes (integers) geometrically as surfaces related to the polynomials associated with infinite primes (integers). Thus infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite numbers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

The original mapping to 4-surfaces inspired by algebraic geometry was essentially as zeros of polynomials. It however turned out that the mapping is more delicate and based on the idea that space-time surfaces correspond to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of imbedding space with hyper-octonionic structure. Also the attribute maximally associative or co-associate could be used. The assignment of a space-time surface to an infinite prime boils down to an assignment of a hyper-octonion analytic polynomial to infinite prime, which in turn defines a foliation of $M^4 \times CP_2$ by hyper-quaternionic space-time surfaces. The procedure generalizes also to the higher levels of the hierarchy and the natural interpretation is in terms of the hierarchical structure of the many-sheeted space-time.

The connection with the basic ideas of algebraic geometry from the possibility to order space-time surfaces according to the complexity of the polynomial involved (at higher levels rational coefficients of the polynomial are replaced with rational polynomials). In particular, the notions of degree and genus make sense for space-time surface.

Various equivalent characterizations of space-times as surfaces

The idea about space-times as associative, hyper-quaternionic surfaces of a hyper-octonionic imbedding space M^8 and the notion of infinite prime serving as a bridge between classical and quantum are the two basic tenets of the algebraic approach. This vision leads to an equivalence of quite different views about space-time: space-time as an associative/hyper-quaternionic or co-associative/cohyperquaterionic surface of an hyper-octonionic imbedding space $HO = M^8$; space-time as a surface in $H = M^4 \times CP_2$; space-time as a geometric counterpart of an infinite prime representing also Fock state identifiable as a particular ground state of super-symplectic representation; and finally, space-time surface as an absolute minimum of the Kähler action. The great challenge is to prove that the last characterization is equivalent with the others.

Infinite primes and quantum gravitational holography

Infinite primes emerge naturally in the realization of the quantum gravitational holography in terms of the modified Dirac operator and provide a deeper understanding of the basic aspects of the configuration space geometry.

1. Two types of infinite primes are predicted corresponding to the two types of fermionic vacua $X \pm 1$, where X is the product of all finite primes. The physical interpretation for the two types of infinite primes $X \pm 1$ is in terms of two quantizations for which creation and oscillator operators change role and which correspond to the two signs of inertial energy in TGD Universe. In particular, phase conjugate photons would be negative energy photons erratically believed to reduce to standard physics.

2. The new view about gravitational and inertial masses forced by TGD leads also the view that positive and negative energy space-time sheets are created pairwise at space-like 3-surfaces located at 7-D light-like causal determinants $X^7_\pm = \delta M^4_\pm \times CP_2$. The conjecture is that the ratio of Dirac determinants associated with the positive and negative energy space-time sheets, which is finite, equals to the exponent of Kähler function which would be thus determined completely by the data at 3-dimensional causal determinants and realizing quantum gravitational holography.

3. The spectra associated with the space-time sheets X^4_+ and X^4_- meeting at X^3 would correspond to the infinite primes built from the vacua corresponding to the infinite primes $X \pm 1$. The close analogy of the product of all finite hyper-octonionic primes with Dirac determinant suggest that the ratio of the determinants corresponds to the ratio of infinite primes defining X^4_+ and X^4_-. The theory predicts the dependence of the eigenvalues of the modified Dirac operator on the value of the Kähler action. Both Kähler coupling strength and gravitational coupling strength are expressible in terms of the finite primes characterizing the ratio of the infinite primes and this ratio depends on the p-adic prime characterizing X^4_+ and X^4_-.

4. Some modes of the spectrum of the modified Dirac operator at $X^4_\pm$ become zero modes, and by the resulting spectral asymmetry the ratio of the determinants differs from unity. Thus the spectral asymmetry or the infinite primes defining the space-time sheets X^4_+ and X^4_- is all that would be needed to deduce the value of the vacuum functional once causal determinants are known.

5.5.5 Infinite primes and more precise view about p-adic length scale hypothesis

Number theoretical considerations allow to develop more quantitative vision about the how p-adic length scale hypothesis relates to the ideas just described.

How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass squared. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles. This might apply also to photons. Infinite range might mean that the join along boundaries bonds mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [K86] .

1. If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$-adic topology is homeomorphic with p-adic topology).

2. One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89}, if this p-adic prime does not somehow characterize also the particle itself.

3. The formulation of quantum TGD based on the identification of light-like 3-surfaces as fundamental dynamical objects (supported by 4-D general coordinate invariance) suggests that light-like 3 surface identifiable as orbits of partons are characterized by p-adic primes and one can even characterize what this means at the level of the modified Dirac operator characterizing quantum dynamics at parton level [K21] . Space-time sheet itself would be characterized by a collection of p-adic primes so that multi-p-p-adicity would emerges naturally. Even q-adicity might make sense. In the lowest order approximation only partonic boundary components with same prime would interact. The hierarchy of space-time sheets would give rise to a hierarchy of infinite primes. This view leads also to a nice interpretation of infinite primes and fermion-boson dichotomy in terms of cognition and intentionality.

What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

1. The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

2. A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n

gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [K86] .

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [K86] , hierarchy of Jones inclusions [K99] , hierarchy of dark matters with increasing values of $\hbar$ [K25, K23] , the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [K86] . Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

1. The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

2. Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [K86] coding information too subtle to be caught by ordinary physical measurements.

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

Under what conditions space-time sheets can be connected by $\#_B$ contact?

Assume that particles are characterized by a p-adic prime determining it mass scale plus p-adic primes characterizing the gauge bosons to which they couple and assume that $\#_B$ contacts mediate gauge interactions. The question is what kind of space-time sheets can be connected by $\#_B$ contacts.

1. The first working hypothesis that comes in mind is that the p-adic primes associated with the two space-time sheets connected by $\#_B$ contact must be identical. This would require that particle is many-sheeted structure with no other than gravitational interactions between various sheets. The problem of the multi-sheeted option is that the characterization of events like electron-positron annihilation to a weak boson looks rather clumsy.

2. If the notion of multi-p p-adicity is accepted, space-time sheets are characterized by integers and the largest prime dividing the integer might characterize the mass of the particle. In this case a common prime factor p for the integers characterizing the two space-time sheets could be enough for the possibility of $\#_B$ contact and this contact would be characterized by this prime. If no common prime factors exist, only # contacts could connect the space-time sheets. This option conforms with the number theoretical vision. This option would predict that the transition to large $\hbar$ phase occurs simultaneously for all interactions.

What about the integer characterizing graviton?

If one accepts the hypothesis that graviton couples to both visible and dark matter, graviton should be characterized by an integer dividing the integers characterizing all particles. This leaves two options.

Option I: gravitational constant characterizes graviton number theoretically

The argument leading to an expression for gravitational constant in terms of CP_2 length scale led to the proposal that the product of primes $p \leq 23$ are common to all particles and one interpretation was in terms of multi-fractality. If so, graviton would be characterized by a product of some or all primes $p \leq 23$ and would thus correspond to a very small p-adic length scale. This might be also the case for photon although it would seem that photon cannot couple to dark matter always. $p = 23$ might characterize the transversal size of the massless extremal associated with the space-time sheet of graviton.

Option II: graviton behaves as a unit with respect to multiplication

One can also argue that if the largest prime assignable to a particle characterizes the size of the particle space-time sheet it does not make sense to assign any finite prime to a massless particle like graviton. Perhaps graviton corresponds to simplest possible infinite prime $P = X \pm 1$, X the product of all primes.

As found, one can assign to any infinite prime, integer, and rational a rational number $q = m/n$ to which one can assign a q-adic topology as effective space-time topology and as a special case effective p-adic topologies corresponding to prime factors of m and n.

In the case of $P = X \pm 1$ the rational number would be equal to ± 1. Graviton could thus correspond to $p = 1$-adic effective topology. The "prime" $p = 1$ indeed appears as a factor of any integer so that graviton would couple to any particle. Formally the 1-adic norm of any number would be 1 or 0 which would suggest that a discrete topology is in question.

The following observations help in attempts to interpret this.

1. CP_2 type extremals having interpretation as gravitational instantons are non-deterministic in the sense that M^4 projection is random light-like curve. This condition implies Virasoro conditions which suggests interpretation in terms topological quantum theory limit of gravitation involving vanishing four-momenta but non-vanishing color charges. This theory would represent gravitation at the ultimate CP_2 length scale limit without the effects of topological condensation. In longer length scales a hierarchy of effective theories of gravitation corresponds to the coupling of space-time sheets by join along boundaries bonds would emerge and could give rise to "strong gravities" with strong gravitational constant proportional to L_p^2. It is quite possible that the M-theory based vision about duality between gravitation and gauge interactions applies to electro-weak interactions and in these "strong gravities".

2. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, implies that $L_k \propto \sqrt{k}$ corresponds to the size scale of causal horizon associated with # contact. For $p = 1$ k would be zero and the causal horizon would contract to a point which would leave only generalized Feynman diagrams consisting of CP_2 type vacuum extremals moving along random light-like orbits and obeying Virasoro conditions so that interpretation as a kind of topological gravity suggests itself.

3. $p = 1$ effective topology can make marginally sense for vacuum extremals with vanishing Kähler form and carrying only gravitational charges. The induced Kähler form vanishes identically by the mere assumption that X^4, be it continuous or discontinuous, belongs to $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2.

Why topological graviton, or whatever the particle represented by CP_2 type vacuum extremals should be called, should correspond to the weakest possible notion of continuity? The most plausible answer is that discrete topology is *consistent* with any other topology, in particular with any p-adic topology. This would express the fact that CP_2 type extremals can couple to any p-adic prime. The vacuum property of CP_2 type extremals implies that the splitting off of CP_2 type extremal leaves the physical state invariant and means effectively multiplying integer by $p = 1$.

It seems that Option I suggested by the deduction of the value of gravitational constant looks more plausible as far as the interpretation of gravitation is considered. This does not however mean that CP_2 type vacuum extremals carrying color quantum numbers could not describe gravitational interactions in CP_2 length scale.

5.5.6 Infinite primes, cognition and intentionality

Somehow it is obvious that infinite primes must have some very deep role to play in quantum TGD and TGD inspired theory of consciousness. What this role precisely is has remained an enigma although I have considered several detailed interpretations, one of them above.

In the following an interpretation allowing to unify the views about fermionic Fock states as a representation of Boolean cognition and p-adic space-time sheets as correlates of cognition is discussed. Very briefly, real and p-adic partonic 3-surfaces serve as space-time correlates for the bosonic super algebra generators, and pairs of real partonic 3-surfaces and their algebraically continued p-adic variants as space-time correlates for the fermionic super generators. Intentions/actions are represented by p-adic/real bosonic partons and cognitions by pairs of real partons and their p-adic variants and the geometric form of Fermi statistics guarantees the stability of cognitions against intentional action. It must be emphasized that this interpretation is not identical with the one discussed above since it introduces different identification of the space-time correlates of infinite primes.

Infinite primes very briefly

Infinite primes have a decomposition to infinite and finite parts allowing an interpretation as a many-particle state of a super-symmetric arithmetic quantum field theory for which fermions and bosons are labelled by primes. There is actually an infinite hierarchy for which infinite primes of a given level define the building blocks of the infinite primes of the next level. One can map infinite primes to polynomials and these polynomials in turn could define space-time surfaces or at least light-like partonic 3-surfaces appearing as solutions of Chern-Simons action so that the classical dynamics would not pose too strong constraints.

The simplest infinite primes at the lowest level are of form $m_BX/s_F + n_Bs_F$, $X = \prod_i p_i$ (product of all finite primes). The simplest interpretation is that X represents Dirac sea with all states filled and $X/s_F + s_F$ represents a state obtained by creating holes in the Dirac sea. m_B, n_B, and s_F are defined as $m_B = \prod_i p_i^{m_i}$, $n_B = \prod_i q_i^{n_i}$, and $s_F = \prod_i q_i$, m_B and n_B have no common prime factors. The integers m_B and n_B characterize the occupation numbers of bosons in modes labelled by p_i and q_i and $s_F = \prod_i q_i$ characterizes the non-vanishing occupation numbers of fermions.

The simplest infinite primes at all levels of the hierarchy have this form. The notion of infinite prime generalizes to hyper-quaternionic and even hyper-octonionic context and one can consider the possibility that the quaternionic components represent some quantum numbers at least in the sense that one can map these quantum numbers to the quaternionic primes.

The obvious question is whether WCW degrees of freedom and WCW spinor (Fock state) of the quantum state could somehow correspond to the bosonic and fermionic parts of the hyper-quaternionic generalization of the infinite prime. That hyper-quaternionic (or possibly hyper-octonionic) primes would define as such the quantum numbers of fermionic super generators does not make sense. It is however possible to have a map from the quantum numbers labelling super-generators to the finite primes. One must also remember that the infinite primes considered are only the simplest ones at the given level of the hierarchy and that the number of levels is infinite.

Precise space-time correlates of cognition and intention

The best manner to end up with the proposal about how p-adic cognitive representations relate bosonic representations of intentions and actions and to fermionic cognitive representations is through the following arguments.

1. In TGD inspired theory of consciousness Boolean cognition is assigned with fermionic states. Cognition is also assigned with p-adic space-time sheets. Hence quantum classical correspondence suggests that the decomposition of the space-time into p-adic and real space-time sheets should relate to the decomposition of the infinite prime to bosonic and fermionic parts in turn relating to the above mention decomposition of physical states to bosonic and fermionic parts.

   If infinite prime defines an association of real and p-adic space-time sheets and this association could serve as a space-time correlate for the Fock state defined by WCW spinor for given 3-surface. Also spinor field as a map from real partonic 3-surface would have as a space-time correlate a cognitive representation mapping real partonic 3-surfaces to p-adic 3-surfaces obtained by algebraic continuation.

2. Consider first the concrete interpretation of integers m_B and n_B. The most natural guess is that the primes dividing $m_B = \prod_i p^{m_i}$ characterize the effective p-adicities possible for the real 3-surface. m_i could define the numbers of disjoint partonic 3-surfaces with effective p_i-adic topology and associated with with the same real space-time sheet. These boundary conditions would force the corresponding real 4-surface to have all these effective p-adicities implying multi-p-adic fractality so that particle and wave pictures about multi-p-adic fractality would be mutually consistent. It seems natural to assume that also the integer n_i appearing in $m_B = \prod_i q_i^{n_i}$ code for the number of real partonic 3-surfaces with effective q_i-adic topology.

3. Fermionic statistics allows only single genuinely q_i-adic 3-surface possibly forming a pair with its real counterpart from which it is obtained by algebraic continuation. Pairing would conform with the fact that n_F appears both in the finite and infinite parts of the infinite prime (something absolutely essential concerning the consistency of interpretation!).

   The interpretation could be as follows.

   (a) Cognitive representations must be stable against intentional action and fermionic statistics guarantees this. At space-time level this means that fermionic generators correspond to pairs of real effectively q_i-adic 3-surface and its algebraically continued q_i-adic counterpart. The quantum jump in which q_i-adic 3-surface is transformed to a real 3-surface is impossible since one would obtain two identical real 3-surfaces lying on top of each other, something very singular and not allowed by geometric exclusion principle for surfaces. The pairs of boson and fermion surfaces would thus form cognitive representations stable against intentional action.

   (b) Physical states are created by products of super algebra generators. Bosonic generators can have both real or p-adic partonic 3-surfaces as space-time correlates depending on whether they correspond to intention or action. More precisely, m_B and n_B code for collections of real and p-adic partonic 3-surfaces. What remains to be interpreted is why m_B and n_B cannot have common prime factors (this is possible if one allows also infinite integers obtained as products of finite integer and infinite primes).

   (c) Fermionic generators to the pairs of a real partonic 3-surface and its p-adic counterpart obtained by algebraic continuation and the pictorial interpretation is as fermion hole pair. Unrestricted quantum super-position of Boolean statements requires that many-fermion state is accompanied by a corresponding many-anti-fermion state. This is achieved very naturally if real and corresponding p-adic fermion have opposite fermion numbers so that the kicking of negative energy fermion from Dirac sea could be interpreted as creation of real-p-adic fermion pairs from vacuum.

   If p-adic space-time sheets obey same algebraic expressions as real sheets (rational functions with algebraic coefficients), the Chern-Simons Noether charges associated with real partons defined as integrals can be assigned also with the corresponding p-adic partons if they are rational or algebraic numbers. This would allow to circumvent the problems related to the p-adic integration. Therefore one can consider also the possibility that p-adic partons carry Noether charges opposite to those of corresponding real partons sheet and that pairs of real and p-adic fermions can be created from vacuum. This makes sense also for the classical charges associated with Kähler action in space-time interior if the real space-time sheet obeying multi-p p-adic effective topology has algebraic representation allowing interpretation also as p-adic surface for all primes involved.

   (d) This picture makes sense if the partonic 3-surfaces containing a state created by a product of super algebra generators are unstable against decay to this kind of 3-surfaces so that one could regard partonic 3-surfaces as a space-time representations for a configuration space spinor field.

4. Are alternative interpretations possible? For instance, could $q = m_B/n_B$ code for the effective q-adic topology assignable to the space-time sheet. That q-adic numbers form a ring but not a number field casts however doubts on this interpretation as does also the general physical picture.

5.5.7 Complete algebraic, topological, and dimensional democracy?

Without the notion of Platonia allowing realization of all imaginable algebraic structures cognitively but leaving no trace on the physics of matter, the idea about dimensional democracy would look almost compelling despite the fact that it might well be in conflict with the special role of the dimensions associated with the classical number fields. One can imagine several realizations of this idea.

1. The most (if not the only) plausible realization for the dimensional hierarchy would be following. Both fractal cosmology, non-determinism of Kähler action, and Poincare invariance favor the option in which WCW is a union of sectors characterized by unions of future and past light cones $M^4_\pm(a)$ where a characterizes the position a of the dip of the light-cone in M^4. Future/past dichotomy would correspond to positive/negative energy dichotomy and to the two kinds of infinite primes constructed from $X \pm 1$, X the product of all finite primes. Hence the cm degrees of freedom for the sectors of WCW would correspond to the union of the spaces $(M^4)^m \times (M^4)^n$ of dimension $D = 4(m+n)$, and the dimensional democracy would conform with the 8-dimensionality of the imbedding space.

2. The most plausible identification consistent with the p-adic length scale hierarchy is as unions of n disjoint 4-surfaces of H. This correspondence is completely analogous to that involved when the WCW of n point-like particles is identified as $(E^3)^n$ in wave mechanics.

3. One might also consider of assigning with hyper-octonionic infinite primes of level n $4n$-dimensional surfaces in $8n$-dimensional space $H^n = (M^4_+ \times CP_2)^n$. This would suggests a dimensional hierarchy of space-time surfaces and a complete dimensional and algebraic democracy: quite a considerable generalization of quantum TGD from its original formulation. This option does not however look physically plausible since it is not consistent with the hierarchical "abstractions about abstractions" structure of infinite primes and corresponding space-time representations.

Since quantum field theories are based on the notion of point like particles, the hierarchy of arithmetic quantum field theories associated with infinite primes cannot code entire quantum TGD but only the ground states of the super-symplectic representations. This might however be the crucial element needed to understand the construction S-matrix of quantum TGD at the general level.

One can imagine also a topological democracy and an evolution of algebraic topological structures. At the lowest, primordial level there are just algebraic surfaces allowing no completion to smooth ...-adic or real surfaces, and defined only in algebraic extensions of rationals by algebraic field equations. At higher levels rational-adic, p-adic and even infinite-P p-adic completions of infinite primes could appear and provide natural completions of function spaces. Of course, all these generalizations might make sense only as cognitive structures in Platonia and it is comforting to know that there is room in just a single point of TGD Universe for all this richness of imaginable structures!

The reader not familiar with the basic algebra of quaternions and octonions is encourated to study some background material: the homepage of Tony Smith provides among other things an excellent introduction to quaternions and octonions [A113] . String model builders are beginning to grasp the potential importance of octonions and quaternions and the articles about possible applications of octonions [A61, A110, A79] provide an introduction to octonions using the language of physicist.

Personally I found quite frustrating to realize that I had neglected totally learning of the basic ideas of algebraic geometry, despite its obvious potential importance for TGD and its applications in string models. This kind of losses are the price one must pay for working outside the scientific community. It is not easy for a physicist to find readable texts about algebraic geometry and algebraic number theory from the bookshelves of mathematical libraries. The book "Algebraic Geometry for Scientists and Engineers" by Abhyankar [A37] , which is not so elementary as the name would suggest, introduces in enjoyable manner the basic concepts of algebraic geometry and binds the basic ideas with the more recent developments in the field. "Problems in Algebraic Number Theory" by Esmonde and Murty [A66] in turn teaches algebraic number theory through exercises which concretize the abstract ideas. The book "Invitation to Algebraic Geometry" by K. E. Smith. L. Kahanpää, P. Kekäläinen and W. Traves is perhaps the easiest and most enjoyable introduction to the topic for a novice. It also contains references to the latest physics inspired work in the field.

Chapter 6

An Overview About Quantum TGD: Part I

6.1 Introduction

This chapter is the first one of two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development.

The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the WCW or "world of the classical worlds" identified as the infinite-dimensional WCW of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis this vision.

6.1.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years.

1. The basic dynamical objects of TGD are 3-surfaces of 8-D imbedding space fixed uniquely by the symmetries of particle physics and the structure of standard model. 4-D general coordinate invariance allows to assume that these surfaces are light-like and the interpretation is as random light-like orbits of 2-dimensional partons. This picture leads immediately to an understanding of the fundamental super-conformal symmetries of the theory and realization that TGD can be seen as an almost topological quantum field theory.

2. The basic vision is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

3. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW Clifford algebra defines a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

4. The reduction of the WCW geometrization to second quantization of induced spinor fields at light-like 3-surface is crucial for the practical progress made in the geometrization. The Dirac determinant defined as the product of generalized eigenvalues of the modified Dirac operator has identification as vacuum functional defined by Kähler function. By construction the generalized eigenvalues carry information about the preferred extremal of Kähler action, and their number for a given light-like 3-surface is finite so that finiteness of the theory is guaranteed and the notion of finite measurement resolution -forced originally by the properties of hyper-finite factors- emerges automatically.

5. p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

6. The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds.

7. HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

6.1.2 Ideas related to the construction of S-matrix

The construction of S-matrix has been the most difficult challenge of TGD and involves several ideas that have emerged during last years. It is not possible to represent explicit formulas yet but the general principles behind S-matrix, or rather its generalization to M-matrix, are reasonably well understood now.

1. Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory.

   One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2. The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor

product. Thus S-matrix is characterized by the measurement resolution analogous to length scale cutoff of quantum field theories. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree. The amazing conclusion interpreted in terms of asymptotic freedom is that at the never-reachable limit of infinite measurement resolution the S-matrix becomes trivial.

3. An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD: for instance, photons travelling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

4. Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory.

5. $M^8 - H$ duality or "number theoretical compactification" [K88] states that one can regard space-time surfaces X^4 either as associative (co-associative) surfaces in the space M^8 of hyper-octonions or as preferred extremals of Kähler action in $M^4 \times CP_2$. Associativity means that the tangent space of X^4 at each point is some hyperquaternionic subspace $HQ = M^4$ of HO. Besides this a preferred plane $M^2 \subset M^8$ identifiable as a plane of non-physical polarizations belongs to the tangent space at each point. This hypothesis provides a purely number theoretic interpretation of gauge conditions and implies a large number of "must-be-trues" of quantum TGD, and together with zero energy ontology leads to a precise view about the realization of zero energy states in terms of causal diamonds allowing to deduce p-adic length scale hypothesis and a general vision about coupling constant evolution in which time scales appear as power of 2 multiples of a basic length scale.

   One can ask whether this duality generalizes to H-H duality such that the image of associative (co-associative) surface in duality is associative (co-associative). If this were the case the dualities would make the space of space-time surfaces a category and one could iterate the duality to construct new preferred extremals of Kähler action.

   One important implication is a justification for the coset construction based on the lifting of Super Kac-Moody algebra (SKM) at a given light-like 3-surface to a sub-algebra of super-symplectic algebra (SC) lifted from $\delta M^{\pm} \times CP_2$ to algebra in H.

6. The outcome is a generalization of Feynman diagrammatics in which the lines of Feynman diagrams are replaced with 3-D light-like surfaces meeting at 2-D surfaces representing vertices. The contribution of a given Feynman diagram is calculated using the fusion rules of a generalized conformal field theory recursively rather than instead of the ordinary Feynman rules. A new element is symplectically invariant (invariant under symplectic/contact transformations of $\delta M^4_{\pm} \times CP_2$) factor of N-point function and thus expressible in terms of symplectic invariants constructed from the areas assignable to the geodesic triangles defined by the subsets of N points and satisfying fusion rules. Simple argument shows that this factor vanishes if any two arguments of N-point function are identical: this gives excellent hopes that infinities are avoided as general arguments indeed predict. The construction and classification of symplectic QFTs as analogs of conformal field theories becomes a basic mathematical challenge.

   The restriction of the arguments of N-point functions to a discrete set of points at partonic 2-surfaces and defining number theoretical braids is an essential ingredient of the approach making it possible the completion of the theory to real and various p-adic domains. These points correspond to the unique intersection of the hyper-quaternionic (and thus associative subset $M^4 \subset M^8$ with the partonic 2-surfaces, where M^4 is now a fixed associative plane of M^8 which should not be confused with the varying associative plane assignable to each point of X^4.

A structure resembling stringy perturbation theory involving fermionic propagators expressible as inverses of the super-generator G_0 is what one naively expects. The fact that G_0 must carry fermion number seems however to be a problem: the stringy propagator actually corresponds to $G{-}1p^k\gamma_k(G^{\dagger))^{-1}}$. There is thus no need for Majorana spinors leading to super string models and imbedding space dimension $D = 8$ works.

6.1.3 Some general predictions of quantum TGD

TGD is consistent with the symmetries of the standard model by construction although there are definite deviations from the symmetries of standard model. TGD however predicts also a lot of new physics. Below just some examples of the predictions of TGD.

1. Fractal hierarchies meaning the existence of scaled variants of standard model physics is the basic prediction of quantum TGD. p-Adic length scale hypothesis predicts the possibility that elementary particles can have scaled variants with mass scales related by power of $\sqrt{2}$. Dark matter hierarchy predicts the existence of infinite number of scaled variants with same mass spectrum with quantum scales like Compton length scaling like $\hbar$.

2. TGD predicts that standard model fermions and gauge bosons differ topologically in a profound manner. Free fermions correspond to light-like wormhole throats associated with topologically condensed CP_2 type extremals whereas gauge bosons correspond to fermion-anti-fermion states associated with the throats of wormhole contacts connecting two space-time sheets with opposite time orientation. The implication is that Higgs vacuum expectation value cannot contribute to fermion mass: this conforms with the results of p-adic mass calculations. TGD predicts also so called super-symplectic quanta and these give dominating contribution to most hadron masses. These degrees of freedom correspond to those of hadronic string and should not reduce to QCD.

3. The most fascinating applications of zero energy ontology are to quantum biology and TGD inspired theory of consciousness. Basic new element are negative energy photons making possible communications to the direction of geometric past. Here also dark matter hierarchy is involved in an essential manner.

4. In cosmology the mere imbeddability required for Robertson-Walker cosmology implies that critical and over-critical cosmologies are almost unique and characterized by their finite duration. The cosmological expansion is accelerating for them and there is no need to assume cosmological constant. Macroscopic quantum coherence of dark matter in astrophysical scales is a dramatic prediction and allows also to assign periods of accelerating expansion to quantum phase transition changing the value of gravitational Planck constant. The dark matter parts of astrophysical systems are predicted to be quantum systems.

5. The notion of hyper-finite factors suggesting the representation of finite measurement resolution as gauge symmetry suggests that the physics of TGD Universe is universal in the sense that it is possible to engineer a system able to mimic the physics of any consistent gauge theory. Kind of analog of Turing machine would be in question.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22].

6.2 Physics as geometry of WCW spinor fields

The construction of the geometry of WCW ("world of classical worlds" or simply WCW) has proceeded rather slowly. The experimentation with various ideas has however led to the identification of the basic constraints on WCW geometry. The most recent vision is described in [K116].

The basic philosophical motivation for the hypothesis that quantum physics could reduce to the construction of WCW Kähler metric and spinor structure, is that infinite-dimensional Kähler geometric existence could be unique not only in the sense that the geometry of the space of 3-surfaces could be unique but that also the dimension of the space-time is fixed to $D = 4$ by this requirement and $M^4_+ \times CP_2$ is the only possible choice of imbedding space. This optimistic vision derives from the work of Dan Freed with loops spaces demonstrating that they possess unique Kähler geometry and from the fact that in $D > 1$ case the existence of Riemann connection, finiteness of Ricci tensor, and general coordinate invariance poses even stronger constraints.

6.2.1 Constraints on WCW geometry

The detailed considerations of the constraints on WCW geometry suggests that it should possess at least the following properties.

1. Metric should be Kähler metric. This property is necessary if one wants to geometrize the oscillator algebra used in the construction of the physical states and to obtain a well defined divergence free functional integration in the configuration space.

2. Metric should allow Riemann connection, which, together with the Kähler property, very probably implies the existence of an infinite dimensional isometry group as the construction of Kähler geometry for the loop spaces demonstrates [A71] .

3. The so called symmetric spaces classified by Cartan [A81] are Cartesian products of the coset spaces G/H with maximal isometry group G. Symmetric spaces possess G invariant metric and curvature tensor is constant so that all points of the symmetric space are metrically equivalent. Symmetric space structure means that the Lie-algebra of G decomposes as

$$\begin{array}{l} g = h \oplus t \ , \\ [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ , \end{array}$$

   where g and h denote the Lie-algebras of G and H respectively and t denotes the complement of h in g. The existence of the $g = t + h$ decomposition poses an extremely strong constraint on the symmetry group G.

   In the infinite-dimensional context symmetric space property would mean a drastic calculational simplification. The most one can hope is that WCW is expressible as a union $\cup_i (G/H)_i$ of symmetric spaces. Reduction to a union of G/H is the best one can hope since 3-surface of Planck size cannot be metrically equivalent with a 3-surface having the size of galaxy! The coordinates labelling the symmetric spaces in this union do not appear as differentials in the line element of WCW and are thus zero modes. They correspond to non-quantum fluctuating degrees of freedom in a well defined sense and are identifiable as classical variables of quantum measurement theory.

4. Metric should be Diff^4 (not only $Diff^3$!) invariant and degenerate and the definition of the metric should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 to act on. This requirement is absolutely crucial for all developments.

5. Divergence cancellation requirement for the functional integral over WCW requires that the metric is Ricci flat and thus satisfies vacuum Einstein equations.

6.2.2 WCW as a union of symmetric spaces

In the finite-dimensional context, globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. Good guess is that same holds true in the infinite-dimensional context. The task is to identify the infinite-dimensional groups G and H. Only quite recently, more than seven years after the discovery of the candidate for Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy.

The crux of the matter is $Diff^4$: all 3-surfaces on the orbit of 3-surface X^3 must be physically equivalent so that one can effectively replace all 3-surfaces Z^3 on the orbit of X^3 with a suitably chosen surface Y^3 on the orbit of X^3. The Lorentz and $Diff^4$ invariant choice of Y^3 is as the intersection of the 4-surface with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light-cone: effectively the imbedding space can be replaced with the product $\delta M^4_+ \times CP_2$ as far as vibrational degrees of freedom are considered. More precisely: WCW has a fiber structure: the 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ correspond to the base space and the 3-surfaces on the orbit of given Y^3 and diffeomorphic with Y^3 correspond to the fiber and are separated by a zero distance from each other in WCW metric.

These observations lead to the identification of the isometry group as some subgroup G of the group of the diffeomorphisms of $\delta H = \delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of the 3-surfaces in δH. Therefore one can identify the WCW as the union of the coset spaces G/H, where H corresponds to the subgroup of G acting as diffeomorphisms for a given X^3. H depends on the topology of X^3 and since G does not change the topology of the 3-surface, each 3-topology defines a separate orbit of G. Therefore, the union involves the sum over all topologies of X^3 plus possibly other 'zero modes'.

The task is to identify correctly G as a sub-algebra of the diffeomorphisms of δH. The only possibility seems to be that the symplectic transformations of δH generated by the function algebra of δH act as isometries of WCW. The symplectic transformations act nontrivially also in δM^4_+ since δM^4_+ allows Kähler structure and thus also symplectic structure.

The magic properties of the light like 3-surfaces

In case of the Kähler metric, G- and H Lie-algebras must allow a complexification so that the isometries can act as holomorphic transformations. The unique feature of the δM^4_+, realized already seven years ago, is its metric degeneracy: the boundary of the light-cone is metrically 2-dimensional sphere although it is topologically 3-dimensional! This implies that light-cone boundary allows an infinite-dimensional group of conformal symmetries generated by an algebra, which is a generalization of the ordinary Virasoro algebra! There is actually also an infinite-dimensional group of isometries (!) isomorphic with the group of the conformal transformations! Even more, in case of δH the groups of the conformal symmetries and isometries are local with respect to CP_2. Furthermore, light-cone boundary allows infinite dimensional group of symplectic transformations as the symmetries of the symplectic structure automatically associated with the Kähler structure. Therefore 4-dimensional Minkowski space is in a unique position in TGD approach. δM^4_+ allows also complexification and Kähler structure unlike the boundaries of the higher-dimensional light-cones so that it becomes possible to define a complexification in the tangent space of the WCW , too.

The space of the vector fields on $\delta H = \delta M^4_+ \times CP_2$ inherits the complex structure of the light-cone boundary and CP_2. The complexification can be induced from the complex conjugation for the functions depending on the radial coordinate of the light-cone boundary playing the same role as the time coordinate associated with string space-time sheet. In M^4_+ degrees of freedom complexification works only provided that the radial vector fields posses zero norm as WCW vector fields (they have also zero norm as vector fields).

The effective two-dimensionality of the light-cone boundary allows also to circumvent the no-go theorems associated with the higher-dimensional Abelian extensions. First, in the dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite dimensional Abelian group rather than central extensions by the group $U(1)$. In the present case the extension is a symplectic extension analogous to the extension defined by the Poisson bracket $\{p, q\} = 1$ rather than the standard central extension but is indeed 1- dimensional and well defined provided that the configuration space metric is Kähler. Secondly, $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [A93] . The point is that light-cone boundary is metrically and conformally 2-sphere and therefore the gauge algebra is effectively the algebra associated with the 2-sphere and, as a consequence, also WCW metric is Kähler.

There is counter argument against complexifixation. The Kähler structure of the light-cone boundary is not unique: various complex structures are parameterized by $SO(3,1)/SO(3)$ (Lobatchewski space). The definition of the Kähler function as absolute minimum of Kähler action however makes it possible to assign unique space-time surface $X^4(Y^3)$ to any Y^3 on the light-cone boundary and the requirement that the group $SO(3)$ specifying the Kähler structure is isotropy group of the classical four-momentum associated with $X^4(Y^3)$, fixes the complex structure uniquely as a function of Y^3. Thus it seems that Kähler action is necessary ingredient of the group theoretical approach.

Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X_l^3 of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X_l^3 -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X_l^3)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

2. One could also say that light-like 3-surfaces X_l^3 and the space-like 3-surfaces X^3 in the intersections of $X^4(X_l^3) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M^4_\pm \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M^4_\pm \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X_l^3 \times \delta M^4_\pm \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M^4_\pm$ implies that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M^4_+ \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

Symmetric space property reduces to conformal and symplectic invariance

The idea about symmetric space is extremely beautiful but it millenium had to change before I was ripe to identify the precise form of the Cartan decomposition. The solution of the puzzle turned out to be amazingly simple.

The algebra is a direct sum $g = g_1 \oplus g_2$ such that g_1 has $h = n$ as conformal weights and g_2 has more general conformal weights. This motivates the guess that the ground state conformal weights are given by $h = i/2 + y$. It is actually possible to regard the imaginary part of h as a pseudo conformal weight, which can be eliminated by a natural choice of the light-like radial coordinate of δM^4_+. Conformal invariance suggests integer spectrum for y whereas Riemann hypothesis favors zeros of Riemann Zeta.

The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of WCW , which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories.

The light-cone conformal invariance differs in many respects from the conformal invariance of string theories. In particular, the finite-dimensional group defining Kac-Moody group is replaced by an infinite-dimensional symplectic group.

6.2.3 An educated guess for the Kähler function

The turning point in the attempts to construct WCW geometry was the realization that four-dimensional $Diff$ invariance (not only 3-dimensional $Diff$ invariance!) of General Relativity must have a counterpart in TGD. In order to realize this symmetry in the space of 3-surfaces, the definition of WCW metric should somehow associate to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$ for Diff^4 to act on. Physical considerations require that the metric should be, not only Diff^4 invariant, but also Diff^4 degenerate so that infinitesimal Diff^4 transformations should correspond to zero norm vector fields of WCW .

Since Kähler function determines Kähler geometry, the definition of the Kähler function should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3. The natural physical interpretation for this space-time surface is as the classical space-time associated with X^3 so that in TGD classical physics ($X^4(X^3)$) becomes a part of WCW geometry and of the quantum theory.

Kähler function as Kähler action for preferred extremal

One could try to construct WCW geometry by finding the metric for a single representative 3-surface at each orbit of G and extending it by left translations to the entire orbit of G. The metric for this representative should be $Diff^3$ invariant and somehow it should associate a unique space-time surface to the 3-surface in question. The original attempt was however more indirect and based on the realization that the construction of the Kähler geometry reduces to that of finding Kähler function $K(X^3)$ with the property that it associates a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 and possesses mathematically and physically acceptable properties. The guess for the Kähler function is the following one.

By Diff^4 invariance one can restrict the consideration on the set of 3-surfaces Y^3 on the 'light-cone boundary' $\delta H = \delta M^4_+ \times CP_2$ since one can define the space-time surface associated with $X^3 \subset X^4(Y^3)$ to be $X^4(X^3) = X^4(Y^3)$ in case that the initial value problem for X^3 has $X^4(Y^3)$ as its solution. This implies $K(X^3) = K(Y^3)$.

The value of the Kähler function K for a given 3-surface Y^3 on light-cone boundary is obtained in the following manner.

1. Consider all possible 4-surfaces $X^4 \subset M^4_+ \times CP_2$ having Y^3 as its sub-manifold: $Y^3 \subset X^4$. If Y^3 has boundary then it belongs to the boundary of X^4: $\delta Y^3 \subset \delta X^4$.

2. Associate to each four surface Kähler action as the Maxwell action for the Abelian gauge field defined by the projection of the CP_2 Kähler form to the four-surface. For a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density whereas for an Euclidian signature the action density is always non-positive.

3. Define the value of the Kähler function K for Y^3 as the absolute minimum of the Kähler action S_K over all possible four-surfaces having Y^3 as its sub-manifold: $K(Y^3) = Min\{S_K(X^4)|X^4 \supset Y^3\}$.

This definition of the Kähler function has several physically appealing features.

1. Kähler geometry associates with each X^3 a unique four-surface, which will be interpreted as the classical space-time associated with X^3. This means that the so called classical space time (and physics!) in TGD approach is not defined via some approximation procedure (stationary phase approximation of the functional integral) but is an essential part of not only quantum theory, but also of WCW geometry, which in turn might be determined by a mere mathematical consistency! Since quantum states are superpositions over these classical space-times, it is clear that the observed classical space-time is some kind of effective, quantum average space-time, presumably defined as an absolute minimum for the effective action of the theory.

2. The space-time surface associated with a given 3-surface is analogous to a Bohr orbit of the old fashioned quantum theory. The point is that the initial value problem in question differs from the ordinary initial value problem in that although the values of the H coordinates h^k as functions $h^k(x)$ of X^3 coordinates can be chosen arbitrarily, the time derivatives $\partial_t h^k(x)$ at X^3 are uniquely fixed by the principle selecting preferred extremals as generalized Bohr orbits (absolute minimization or probably something more delicate such as criticality [K28], existence of quaternionic tangent space structure [K88], or Hamilton-Jacobi structure [K9]) unlike in the ordinary variational problems encountered in the classical physics. This implies something closely analogous to the quantization of the symplectic momenta so that the space-time surface can be regarded as a generalized Bohr orbit. The classical quantization of electric charge and mass are possible consequences of the Bohr orbit property.

3. Kähler function is Diff^4 invariant in the sense that the value of the Kähler function is same for all 3-surfaces belonging to the orbit of a given 3-surface. As a consequence, WCW metric is Diff^4 degenerate. The implications of the Diff^4 invariance have turned out to be decisive, not only for the geometrization of WCW , but also for the construction of the quantum theory. For instance, time like vibrational modes tangential to the 4-surface imply tachyonic mass spectrum unless they correspond to the zero modes of WCW metric. Diff^4 invariance however guarantees the required kind of degeneracy of the metric.

4. The non-determinism of Kähler action means that the complete reduction to the light-cone boundary is not possible. This means a mathematical challenge but is physically a highly desirable feature since otherwise time would be lost as it is lost in the canonically quantized general relativity.

The most general expectation is that WCW can be regarded as a union of coset spaces: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is determined as absolute minimum of Kähler action.

It will be found that in the case of $M^4_+ \times CP_2$ Kähler geometry, or strictly speaking contact Kähler geometry, characterized by a degenerate Kähler form (Diff^4 degeneracy and plus possible other degeneracies) seems possible. Although it seems that this construction must be generalized by allowing all light like 7-surfaces $X^3_l \times CP_2$, at least those for which X^3_l is boundary of light-cone inside M^4_+ or M^4, with the physical interpretation differing dramatically from the original one, the original construction discussed in the sequel involves the most essential aspects of the problem.

How to identify preferred extremals of Kähler action?

The first guess for preferred extremals of Kähler action defining the Bohr orbits was that they correspond to absolute minima of Kähler action. One can criticize this assumption, and I have proposed several identifications of preferred extremals [K9, K116] and some of them could be equivalent.

The number theoretical vision discussed in [K88] would suggest the separate minimization of magnitudes of positive and negative contributions to the Kähler action. It must be emphasized that this option need not conform nicely with number theoretical universality since in p-adic context absolute minimization does not make sense and should be replaced by some algebraic notion. The non non-vanishing determinant for Hessian of Kähler action would be such a purely algebraic condition characterizing absolute minimum and maximum but would not be able to distinguish between then. This notion is not consistent with the idea that quantum criticality has criticality of preferred extremals as space-time correlate [K28] since at criticality the Hessian is degenerate.

For this option Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant: note that they would be only inertial vacua and carry non-vanishing density gravitational energy. The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself.

The 3-surfaces for which CP_2 projection is at least 2-dimensional and not a Lagrange manifold would correspond to non-vacua since conservation laws do not leave any other option. The variational principle would favor equally magnetic and electric configurations whereas absolute minimization of action based on S_K would favor electric configurations. The positive and negative contributions would be minimized for 4-surfaces in relative homology class since the boundary of X^4 defined by the intersections with 7-D light-like causal determinants would be fixed. Without this constraint only vacuum bubbles would result.

The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable solution to the construction of Kähler function.

It should be noticed that the considerations of this chapter relate only to the extremals of Kähler action which need not be absolute minima nor more general preferred extremals discussed in [K88] although this is suggested by the high symmetries. The number theoretic approach based on the properties of quaternions and octonions discussed in the chapter [K88] leads to a proposal for the general solution of field equations based on the generalization of the notion of calibration [A79] providing absolute minima of volume to that of Kähler calibration. This approach will not be discussed in this chapter.

6.2.4 The construction of WCW geometry from symmetry principles

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

The gigantic size of the isometry group suggests that it might be possible to deduce very detailed information about the metric of the WCW by group theoretical arguments. This turns out to be the case. In order to have a Kähler structure, one must define a complexification of WCW . Also one should identify the Lie algebra of the isometry group and try to derive explicit form of the Kähler metric using this information. One can indeed construct the metric in this manner but a rigorous proof that the corresponding Kähler function is the one defined by Kähler action does not exist yet although both approaches predict the same general qualitative properties for the metric. The argument stating the equivalence of the two approaches reduces to the hypothesis stating electric-magnetic duality of the theory. For the Bohr orbit like preferred extremals of Kähler action magnetic WCW Hamiltonians derivable from group theoretical approach are essentially identical with electric WCW Hamiltonians derivable from Kähler action.

General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of WCW consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as absolute minimum of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome.

This picture is however too simple.

1. The degeneracy of the absolute minima caused by the classical non-determinism of Kähler action however brings in additional delicacies, and it seems that the reduction to the light-cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving more general light like 7-surfaces $X^3_l \times CP_2$.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_+ \times CP_2$ manifest themselves as the properties of propagators and vertices, and that M^4 is favored over M^4_+. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of WCW s associated with various 7-D causal determinants. The minimum assumption is that all possible unions of future and past light-cone boundaries $\delta M^4_\pm \times CP_2 \subset M^4 \times CP_2$ label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^3_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers. One cannot exclude the possibility that even more general light like surfaces $X^3_l \times CP_2$ of M^4 are important as causal determinants.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3 and also a preferred light like 7-surface $X^3_l \times CP_2$.

This is indeed possible. The possibility of negative values of Poincare energy(or equivalently inertial energy) inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action in the set of 4-surfaces going through X^3. These space-time sheets should also define uniquely the light like 7-surface $X^3_l \times CP_2$, most naturally as the "earliest" surface of this kind. Note that this means that it become possible to assign a unique value of geometric time to the space-time sheet.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

The most natural option is that symplectic and Kac-Moody algebras together generate the isometry algebra and that the corresponding transformations leaving invariant the partonic 2-surfaces and their 4-D tangent space data act as gauge transformations and affect only zero modes.

Does the symmetric space property reduce to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{6.2.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

WCW geometry allows two super-conformal symmetries assignable the coset space decomposition G/H for a sector of WCW with fixed values of zero moes. One can assign to the tangent space algebras g resp. h of G resp. H analogous to Kac-Moody algebras super Virasoro algebras and construct super-conformal representation as a coset representation meaning that the differences of super Virasoro generators annihilate the physical states. This obviously generalizes Goddard-Olive-Kent construction Sugawara.

The original conjecture was that the four-momenta associated with the two representations are identical. The physical interpretation would be in terms of Equivalence Principle (EP). This need not to be the case and the four-momenta associated with H vanish naturally. Later a more feasible identification of quantal and classical variants of EP has emerged [K93].

The identification of the two algeras is not a mechanical task and has involved a lot of trial and error. The algebra g should be be spanned by the generators of super-symplectic algebra of light-cone boundary and by the Kac-Moody algebra acting on light-like orbits of partonic 2-surfaces. The sub-algebra h should be spanned by generators which vanish for a preferred point of WCW analogous to origin of $CP_2 = SU(3)/U(2)$. Now this point would correspond to maximum or minimum of Kähler function (no saddle points are allowed if the WCW metric has definite signature). In hindsight it is obvious that the generators of both symplectic and Kac-Moody algebras are needed to generate g and h: already the effective 2-dimensionality meaning that 4-D tangent space data of partonic surface matters requires this.

The maxima of Kähler function could correspond to this kind of points and could play also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories. It took quite a long time to realize that Kähler function must be identified as Kähler action for the Euclidian region of preferred extremal. Kähler action for Minkowskian regions gives imaginary contribution to the action exponential and has interpretation in terms of Morse function. This part of Kähler action can have and is expected to have saddle points and to define Hessian with signature which is not positive definite.

What effective 2-dimensionality and holography really mean?

Concerning the interpretation of Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with holography.

1. Holography suggests that light-like 3-surfaces with fixed ends give rise to same WCW metric and the deformations of these surfaces by Kac-Moody algebra correspond to zero modes just like the interior degrees of freedom for space-like 3-surface do. The same would be true for space-like 3-surfaces at the ends of space-time surface with respect to symplectic transformations.

2. The non-trivial action of Kac-Moody algebra in the interior of X_l^3 together with effective 2-dimensionality and holography would encourage the interpretation of Kac-Moody symmetries acting trivially at X^2 as gauge symmetries. Light-like 3-surfaces having fixed partonic 2-surfaces at their ends would be equivalent physically and effective 2-dimensionality and holography would be realized modulo gauge transformations. As a matter fact, the action on WCW metric would be a change of zero modes so that one could identify it as analog of conformal scaling. The action of symplectic transformations vanishing in the interior of space-like 3-surface at the end of space-time surface affects only zero modes.

3. Gauge symmetry property means that the Kähler metric of the WCW is same for all gauge equivalent choices of X_l^3 and Kac-Moody deformations correspond to zero modes. Kähler function could differ by a real part of a holomorphic function of configuration space coordinates representing now Kac-Moody transforms of X_l^3. If Dirac determinant gives the exponent of Kähler function, the eigenvalues of the modified Dirac action can differ only by scalings with are products of holomorphic function of WCW coordinates and its conjugates labeling different Kac-Moody transforms of X_l^3. This condition makes sense if one restricts the consideration to the finite number of eigenvalues λ_k assigned to D_K. The introduction of instanton term transforming the eigenvalues to $\lambda_k + \sqrt{n}$ would not allow his scaling.

   Either one must assume more general spectrum of form $\lambda_k + \sqrt{n}x_k$ with λ_k and x_k scaling in identical manner or that $n = 0$ modes are enough to define Kähler function. The latter option might be correct since the preferred extremal realizes effective 2-dimensionality at space-time level and conformal excitations break it so that they should not contribute to Kähler function. Also number theoretic universality favors this option. One cannot however exclude the first option. It must be admitted that the situation is not completely understood.

6.2.5 Attempts to identify WCW Hamiltonians

I have made several attempts to identify WCW Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate is based on the formulation of quantum TGD using 3-D light-like surfaces identified as orbits of partons. The proposal is out-of-date but the most recent proposal is obtained by a very straightforward generalization from the proposal for magnetic Hamiltonians discussed below.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structures of WCW. There is almost unique identification for the symplectic structure. WCW counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$\begin{aligned} Q_m(H_A, X^2) &= Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ , \\ Q^+_m(H_A, r_M) &= Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ , \\ J &\equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ . \end{aligned}$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q^+_m.

$$Q^{\alpha,\beta}_m(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q^+_m(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the WCW metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Generalization

The generalization for definition WCW super-Hamiltonians defining WCW gamma matrices is discussed in detail in [K116] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the modified Dirac action: in particular, about their localization at string worlds sheets (right handed neutrino could be an exception).

The basic formulas generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Right handed neutrino spinor is replaced with any mode of the modified Dirac operator localized at string world sheet in the case of Kac-Moody sub-algebra of super-symplectic algebra corresponding to symplectic isometries at light-cone boundary and CP_2. In the case of right- handed neutrino one obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

The resulting super Hamiltonians define WCW gamma matrices. They are labelled by two conformal weights. The first one is the conformal weight associated with the light-like coordinate of $\delta M^4_\pm \times CP_2$. Second conformal weight is associated with the spinor mode and the coordinate along stringy curve. One cannot exclude the possibility that the two conformal weights have same value. Effective 2-dimensionality and the fact that string coordinate cannot be always radial light-like coordinate would suggest that they are independent.

The presence of two conformal weights is in accordance with the idea that a generalization of conformal invariance to 4-D situation is in question. If Yangian extension of conformal symmetries is possible and would bring an additional integer n telling the degree of multilocality of Yangian generators defined as the number of partonic 2-surfaces at which the generator acts. For conformal algebra degree of multilocality equals to $n = 1$.

6.2.6 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states angular momentum (and possibly also of Lorentz boost), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{6.2.2}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{or} \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{6.2.3}$$

   In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to use the "half Poisson bracket"

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{6.2.4}$$

Here the subscript + and − refer to complex isometry current and its complex conjugate in terms of which the "half Poisson bracket" can be expressed.

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

6.2.7 WCW spinor structure

Quantum TGD should be reducible to the classical spinor geometry of WCW . In particular, physical states should correspond to the modes of WCW spinor fields. The immediate consequence is that WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of WCW spinor structure there are some important clues.

1. The classical bosonic physics is coded into the definition of WCW metric; therefore the classical physics associated with the spinors of the imbedding space should be coded into the definition of WCW spinor structure. This means that the generalized massless Dirac equation for the induced spinor fields on $X^4(X^3)$ should be closely related to the definition of WCW gamma matrices.
2. Complex probability amplitudes (scalar fields) in the WCW correspond to the second quantized boson fields in X^4. Hence the spinor fields of WCW should correspond to the second quantized, free, induced spinor fields on X^4. The space of WCW spinors should be just the Fock space of the second quantized fermions on X^4!
3. Symplectic algebra might generalize to a super symplectic algebra and that super generators should be linearly related to the gamma matrices of WCW . If this indeed is the case then the construction of WCW spinor structure becomes a purely group theoretical problem.

The realization of these ideas is simple in principle. Perform a second quantization for the free induced spinor field in X^4. Express WCW gamma matrices and symplectic super generators as superpositions of the fermionic oscillator operators. This means that WCW gamma matrices are analogous to spin 3/2 fields and can be regarded as a superpartner of the gravitational field of WCW . Deduce the anti-commutation relations of the spinor fields from the requirement of super symplectic invariance. Generalize the flux representation for the WCW Hamiltonians to a spinorial flux representation for their super partners.

WCW gamma matrices as super algebra generators

The basic idea is that the space of WCW spinors must correspond to the Fock space for the second quantized induced spinor fields. In accordance with this the gamma matrices of the configuration space must be expressible as superpositions of the fermionic oscillator operators for the second quantized induced free spinor fields in X^4 so that they are analogous to spin 3/2 fields. The Dirac equation is fixed from the requirement of super symmetry and has same vacuum degeneracy as Kähler action. A further assumption is that the contractions of the gamma matrices with isometry currents correspond to super charges of the group of isometries of WCW so that the construction reduces to group theory.

The super Kac Moody algebra was assigned originally with light like 3-D causal determinants but has a more natural identification as the Kac-Moody algebra of symplectic isometries. The corresponding gammma matrices (super Hamiltonians) are essentially inner products of the modes of induced spinor field with the second quantized spinor field and all modes of indued spinor fields with all possible charge states are allow. For the entire symplectic algebra only the inner products with right-handed neutrino spinors define the super-generators. This implies that super-generators are labelled by two conformal weights. The first conformal weight is associated with the imbedding space Hamiltonians and corresponds to the light-like radial coordinate of light-cone boundary. Second conformal weight labels the spinor modes localized at 2-D string world sheets. The super generators are integrals of the spinor modes localized at 1-D stringy curves so that one has formally a 3-D situation [**?**, K105, K116]. Holography implied by the strong form of general coordinate invariance however implies effective 2-dimensionality. Gamma matrices define the components of WCW metric as anti-commutators.

The modified Dirac equation and gamma matrices

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the vacuum functional of the theory is identifiable as Dirac determinant. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the modified Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique modified Dirac action, which is internally consistent and super-symmetric. Space-time geometry must carry information about conserved quantum charges assignable to partonic

2-surfaces and it took considerable to to realize that this is achieved via a measurement interaction terms which are Lagrangian multiplier terms expressing that conserved classical charges are identical with their quantum counterparts in Cartan algebra for the space-time surfaces in quantum superposition representing the outcome of measurement. This makes sense if classical charges parametrize zero modes.

Second key idea [K105, K116] is that the well-definedness of em charge eigenvalue for spinor modes requires their localization to 2-D string world sheets. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet. Due to the presence of classical W boson fields this is possible only if localization takes plce at 2-D string world sheets and partonic 2-surfaces. Therefore string theory like structure emerges as part of TGD. The super Hamiltoanians defined in terms fluxes of Hammiltonians over partonic 2-surfaces are modified: a super-Hamiltonian at point of partonic 2-surface is replaced with an integral over stringy curve connecting points of two partonic 2-surfaces.

6.2.8 What about infinities?

The construction of a divergence free and unitary inner product for the WCW spinor fields is one of the major challenges. In the sequel constraints on the geometry of WCW posed by the finiteness of the inner product are analyzed.

Inner product from divergence cancellation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of WCW over the reduced WCW containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('light-cone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{6.2.5}$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light-cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by WCW integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of WCW integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the non-compact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [B72]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \ . \tag{6.2.6}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancellation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the WCW into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha\rightarrow y,\beta) = \sum_{r,s} |S(r,\alpha\rightarrow s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m\rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since WCW metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in WCW integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero

modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems stating that semiclassical approximation is exact for certain systems (for example Duistermaat-Hecke theorem for integrable systems [A64]). Symmetric space property suggests that Kähler function might possess the properties guaranteeing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving WCW spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that WCW integrals are continuable to p-adic number fields requires this kind of reduction.

Divergence cancellation, Ricci flatness, and symmetric space and Hyper Kähler properties

In the case of the loop spaces left invariance implies that Ricci tensor is a multiple of the metric tensor so that Ricci scalar has an infinite value. Mathematical consistency (essentially the absence of the divergences in the integration over WCW) forces the geometry to be Ricci flat: in other words, vacuum Einstein's equations are satisfied. It can be shown that Hyper Kähler property guarantees Ricci flatness. The reason is that the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(\infty)$ generators instead of $U(\infty)$ generators as in case of loop spaces, so that the traces vanish.

Hyper Kähler property requires a quaternionic structure in the tangent space of WCW . Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper-Kähler property means the possibility to perform complexification in S^2-fold manners. An interesting possibility raised by the notion of visionb is that hyper Kähler structure could be replaced with what might be called "hyper-hyper-Kähler structure" resulting when quaternionic tangent space is replaced with its hyper-quaternionic variant. This would conform with the Minkowski signature of the space-time surface. In this framework also hyper-octonionic structure might be considered. An interesting question not yet even touched, is whether the conjectured $M^8 - -M^4 \times CP_2$ duality is realized also at the level of the WCW of 3-surfaces.

Consider now the arguments in favor of Ricci flatness of the WCW .

1. The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric

space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property. In the following argument reader can well consider replacing the attribute "quaternionic" with "hyper-quaternionic".

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of WCW in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of WCW . Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of WCW . First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at $X^2_+ \times CP_2$ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

3. One can see the super-symplectic conformal weights as points in a particular complex plane of the quaternionic space and the choice of this plane corresponds to a selection of one WCW Kähler structure which are parameterized by S^2. The necessity to restrict the conformal weights to a complex plane brings in mind the commutativity constraint on simultaneously measurable quantum observables.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of WCW and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

6.3 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B7] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K18] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak

scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

6.3.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{6.3.1}$$

A more general form of this duality is suggested by the considerations of [K40] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{6.3.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} \ , \tag{6.3.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L5] , [L5] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_ZF_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (6.3.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (6.3.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (6.3.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (6.3.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K65] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \to \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{6.3.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{6.3.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \qquad (6.3.10)$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L5]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K70]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K93]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

6.3.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D15] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K29] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy.

In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K51] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K52] .

6.3.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \to r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K9]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed

variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \quad (6.3.11)$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j^\alpha_K \partial_\alpha \phi = -j^\alpha A_\alpha \ . \quad (6.3.12)$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j^\alpha_K$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 \ . \quad (6.3.13)$$

j_K is a four-dimensional counterpart of Beltrami field [B44] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K9] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j^\alpha_I \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j^\alpha_K \phi$ and $j^\alpha_I \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha (j^\alpha \phi) = 0 \ . \quad (6.3.14)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

6.3.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered.

The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

1. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 . \tag{6.3.15}$$

   As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

2. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

   The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

3. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

1. The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

2. If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^{\alpha}_{Q_i} D_{\alpha})\Psi = 0 \ ,$$

where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

6.4 Von Neumann algebras and TGD

The work with TGD inspired model [K97] for topological quantum computation [K97] led to the realization that von Neumann algebras [A96, A121, A101, A63] , in particular so called hyper-finite factors of type II_1 [A84] , seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [A92] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter.

6.4.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A84] .

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

6.4.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [K28] based on the notion of delta function, plus the emergence of s [A70] , the possibility to formulate the notion of delta function rigorously in terms of distributions [A83, A112] , and the emergence of path integral approach [A102] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A109, A125] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A87] relate closely to type II_1 factors. In topological quantum computation [K97] based on braid groups [A126] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B34] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B63, B19] .

6.4.3 Factors of type II_1 and quantum TGD

For me personally the realization that TGD Universe is tailored for topological quantum computation [K97] led also to the realization that hyper-finite (ideal for numerical approximations) von Neumann algebras of type II_1 have a direct relevance for TGD.

The basic facts about hyper-finite von Neumann factors of type II_1 suggest a more concrete view about the general mathematical framework needed.

1. The effective 2-dimensionality of the construction of quantum states and WCW geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the generators of conformal algebras, the gamma matrices of WCW, and the modes of the induced spinor fields are labelled by discrete labels. Hence the tangent space of WCW is a separable Hilbert space and its Clifford algebra is a hyper-finite type II_1 factor. Super-symmetry requires that the bosonic algebra generated by WCW Hamiltonians and the Clifford algebra of WCW both correspond to hyper-finite type II_1 factors.

2. Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, WCW can be regarded as union of big-bang/big crunch type WCWs obtained as a union of light-cones parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4 are hyper-finite and of type III_1 in algebraic quantum field theory [B63] . The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

3. Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner to imbed interacting sub-systems to a universal II_1 factor which now naturally corresponds to the infinite Clifford algebra of the tangent space of WCW of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, the formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations might be described by Jones inclusion [A3, A68] characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and quantum group in case of $\mathcal{M} : \mathcal{N} < 4$ and conformal theory with $k = 1$ Kac Moody for $\mathcal{M} : \mathcal{N} = 4$ [B39] .

4. von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that dimensional regularization works only in $D = 4 - \epsilon$. Dimensional regularization with space-time dimension $D = 4 - \epsilon \rightarrow 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

6.4.4 Does quantum TGD emerge from local version of HFF?

There are reasons to hope that the entire quantum TGD emerges from a version of HFF made local with respect to $D \leq 8$ dimensional space H whose Clifford algebra $Cl(H)$ raised to an infinite tensor power defines the infinite-dimensional Clifford algebra. Bott periodicity meaning that Clifford algebras satisfy the periodicity $Cl(n + k8) \equiv Cl(n) \otimes Cl(8k)$is an essential notion here [K99, K27] . The points m of M^k can be mapped to elements $m^k\gamma_k$ of the finite-dimensional Clifford algebra $Cl(H)$ appearing as an additional tensor factor in the localized version of the algebra.

The requirement that the local version of HFF is not isomorphic with HFF itself is highly non-trivial. The only manner to achieve non-triviality is to multiply the algebra with a non-associative tensor factor representing the space of hyper-octonions M^8 identifiable as sub-space of complexified octonions with tangent space spanned by real unit and octonionic imaginary unit multiplied by commuting imaginary unit (for a good review about properties of octonions see [A43]) .

Space-times could be regarded equivalently as surfaces in M^8 or in $M^4 \times CP_2$ and the dynamics would reduce to associativity (hyper-quaternionicity) or co-associativity condition. It is rather remarkable that CP_2 forced by the standard model symmetries has also a purely number theoretic interpretation as parameterizing hyper-quaternionic four-planes containing a preferred hyper-octonionic imaginary unit defining hyper-complex structure in M^8. Physically this choice corresponds to a choice of Cartan algebra of Poincare algebra for which the system is at rest so that a connection with quantum measurement theory is suggestive. Color group is identifiable as a subgroup of octonionic automorphism group G_2 respecting this choice.

6.4.5 Quantum measurement theory with finite measurement resolution

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ [A3, A85] of these algebras lead to quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ [K99, K27] . Quantum Clifford algebra $\mathcal{M}/\mathcal{N}$

interpreted as $\mathcal{N}$-module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by $\mathcal{N}$-rays and the notions of unitarity, hermiticity, and eigenvalue generalize [K20, K27] .

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole.

At the level of conscious experience, the entanglement below measurement resolution would give rise to a pool of shared and fused mental images giving rise to "stereo consciousness" (say stereovision) [K50] so that contents of consciousness would not be something completely private as usually believed. Also fuzzy logic emerges naturally since ordinary spinors are replaced by quantum spinors for which the discrete spectrum of the eigenvalues of the moduli of its spinor components can be interpreted as probabilities that corresponding belief is true is [E19] [K99] .

6.4.6 Cognitive consciousness, quantum computations, and Jones inclusions

Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qubits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

6.4.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [J7] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J10] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i, j), where i, j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha, \beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha - \beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha - \beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J7] are not consistent with this prediction [J1] and this is identified as the anomaly.

2. The QM prediction $E(\alpha, \beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha - \beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J1] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly 2) but not anomaly a). A "mundane" explanation for anomaly 1) can be imagined [K99] .

6.5 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

6.5.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E27] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K79, K62] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K80] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K91] .

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E27] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic

2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L6, K91] , [L6] .

6.5.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2

or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6.5.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

6.5.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

6.5.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K24] .

6.5.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

6.5.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \rightarrow CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi^n_k = \partial L_K/\partial h^k_n$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi^n_k = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π^0_k and π^n_k. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

   (b) This picture is not quite correct yet. The fixing of π^0_k and π^n_k should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π^0_k and π^n_k must be fixed at X^3 and X^3_l in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X^3_l as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π^k_0 would give n_b branches in CP_2 degrees of freedom and the conditions for π^n_k would split each of these branches to n_a branches.

   (c) The existence of these two kinds of conserved charges (possibly vanishing for π^n_k) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g^2_K$ factor of the action with $\hbar/g^2_K$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = ng_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

   The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

6.5.8 Updated view about the hierarchy of Planck constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K92].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K65] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E27] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units

$c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K105] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value

of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N+1$:th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{6.5.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{6.5.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 \quad . \tag{6.5.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{6.5.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E17, E31] discussed in [K112] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter

v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

1. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

2. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

3. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K108, K107] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   (a) If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K24] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   (b) For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

   (c) Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.

   Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

   It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

   (d) The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0

to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

4. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

5. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

6. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K105]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K101] so that conformal dynamics represents conformal evolution) [K88].

6.5.9 Updated view about the hierarchy of Planck constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K92].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K65] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E27] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

   It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K105] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer

charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N+1$:th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta}p_\alpha p_\beta = g_{eff}^{\alpha\beta}\partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl}p_k p_l = n^2\frac{\hbar^2}{L^2} \quad . \tag{6.5.5}$$

Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2\hbar^2}{K^2L^2} \quad .$$

$\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{6.5.6}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta}L_\alpha L_\beta = g_{eff}^{kl}L_k L_l = l(l+1)\hbar^2 \quad . \tag{6.5.7}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{6.5.8}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E17, E31] discussed in [K112] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

1. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

2. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

3. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K108, K107] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   (a) If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K24] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   (b) For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

(c) Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.

Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

(d) The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

4. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

5. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

6. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times

stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K105]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K101] so that conformal dynamics represents conformal evolution) [K88].

6.6 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M^8_c using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M^8_c composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M^8_c. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCWs associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

2. One can formulate associativity in M^8 (or M^8_c) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest.

Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the modified gamma matrices need not span the entire tangent space. The space spanned by the modified gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the modified gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by modified gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

2. For modified gamma matrices the notion of co-associativity can produce problems since modified gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by modified gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the modified gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the modified gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

6.6.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M^8_c. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units

commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

3. Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \to S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \to M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition generalizes to the case of M_c^8: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M_c^8$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a manner to build quaternionic space-time surfaces properly.

2. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

3. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K9]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

4. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K28] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of modified Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

5. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in

H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

6. One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

6.6.2 Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K98]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H$.... This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and modified gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

6.6.3 Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not

make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

   Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

6.6.4 How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H...$ iteration generating new solutions from existing ones.

Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of

the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{6.6.1}$$

   The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{6.6.2}$$

   One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A43] expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product $e_1 e_2$ (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

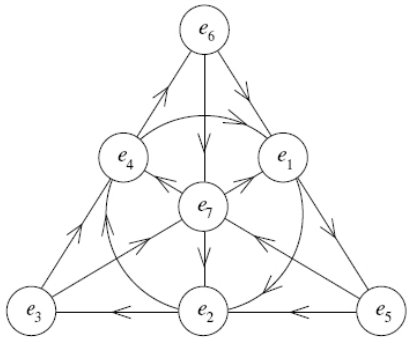

Figure 6.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

6.6.5 Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A43] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

6.6.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k\gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

   The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

2. The space spanned by the modified gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection modified gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence modified gamma matrices are flexible enough to realize associativity in H.

3. Modified gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

   This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

$M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K57] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

6.6.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \to H \to H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

6.7 Does modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action. One cannot however get rid of Kähler action since the gamma matrices appearing in Kähler-Dirac action are defined in terms of canonical momentum densities of Kähler action. The most one can hope is that Dirac determinant reduces to the exponent of Kähler action for preferred extremals.

6.7.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. An infinite hierarchy of these currents is expected and they would define fermionic counterparts for zero modes. In number theoretic vision space-time surfaces are proposed to be identifiable as associative (co-associative) surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking [?]

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the modified Dirac (Kähler-Dirac) equation. The requirement that there are deformations of the space-time surface -actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

3. The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [K28] , [L18] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [K28] , [L18] . If Kähler current defines Beltrami flow [B44] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

4. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

6.7.2 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a far reaching generalization of conformal symmetries. The development of the understanding of conservation laws has been slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

1. The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{6.7.1}$$

Here h^k_β denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{6.7.2}$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

3. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha \Psi \ . \tag{6.7.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [A27, A30] . Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha \Psi \ . \tag{6.7.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [A13] .

4. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

5. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P\times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

   (a) Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [A116] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [A13] . Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

   (b) The understanding of the contributions to Kähler-Dirac action has been slow. It seems that what is needed is Chern-Simons Dirac action assigned to partonic orbits: this was the original proposal. The condition that the action of C-S-D operator reduces to that of massless M^4 Dirac operator. $Gamma^n\Psi = p^k\gamma_k\Psi$ would be space-time counterpart for the massless Dirac equation at the level of imbedding space. I have called this condition earlier generalized eigenvalue condition.

   The assumption that C-S-D is present strongly suggests that also Kähler action contains C-S term meaning that the C-S terms from Kähler action are cancelled at partonic orbits for preferred extremals. If C-S term is present also at space-like ends of space-time surface Kähler action and therefore also Kähler function vanishes identically. At the ends of space-time surface one would therefore have $\Gamma^n\Psi = 0$ if C-S-D term is not present. Hence this assumption seems unphysical. One would have massless Dirac propagator at the fermionic lines defined by the partonic boundaries of Kähler-Dirac equation and on-mass-shell condition at the space-like ends of the space-time surface.

   If this is correct interpretation then the fermionic lines identified as boundaries of string world sheets correspond to massless fermion propagators and the stringy propagators $1/L_0$ could be associated with fermion fermion scattering at wormhole contacts (see fig. **??** in the appendix of this book) . The generalized Feynman diagrammatics would be a combination of stringy and Feynman diagrammatics. External fermion lines would carry massless on-shell momenta and wormhole contacts could be seen as massive bound states of massless fermions falling into representations of super-conformal algebras assignable to wormhole contacts. This would allow stringy variant of twistor approach.

2. The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

3. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

4. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

5. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective

2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. The conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires additional conditions to be satisfied and the holomorphy of string world sheets (partonic 2-surfaces) and associated Kähler-Dirac gamma matrices makes this possible [K105].

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K27] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the understanding of the relationship between criticality and hierarchy of Planck constants [K40] , [L11] . Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

The vanishing of second variations of preferred extremals - at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that WCW metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B10] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the

degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

6.7.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose modified Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

Why modified Dirac action?

1. Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K18, K88] .

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

2. Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_{\ \alpha}} L_K \ . \end{aligned} \qquad (6.7.5)$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \qquad (6.7.6)$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \qquad (6.7.7)$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \qquad (6.7.8)$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T^\alpha_l \Gamma^l \ . \end{aligned} \qquad (6.7.9)$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^\alpha D_\alpha \Psi \ . \qquad (6.7.10)$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \qquad (6.7.11)$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

3. How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T^k_\mu T_{\nu k} \ . \qquad (6.7.12)$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

4. Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K88] . In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

Overall view about Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

2. By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j\cdot A=0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j\cdot A=0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman

diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

2. If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{6.7.13}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Some details about Chern-Simons Dirac equation

To avoid confusion some general comments are in order. Only the Chern-Simons Dirac operator will be considered. Modified gamma matrices contain also the contribution from the Lagrange multiplier term stating weak form of electric-magnetic duality. At space-like 3-surface one has also the contribution coming from the Lagrange multiplier terms identifying classical and quantal charges in Cartan algebra.

When C-S-D action at partonic orbits is included, one obtains what I have called generalized eigenvalue equation introduced in ad hoc manner in order to define Dirac determinant. Now Dirac determinant at least formally reduces to the same expression as in massless gauge theories. Dirac determinant could be also defined directly as the product of generalized eigenvalues $p^k\gamma_k$ defining virtual momenta propagating in fermion lines. Also the identification as hyperquaternions makes sense and the outcome is by symmetries real number or perhaps complex number.

One can of course wonder whether the Dirac determinant has anything to do with the exponent of Kähler action! Measurement interaction term states that the action of D_{C-S} modified by the contribution from em-duality constraint is identical with that of the Dirac operator of M^4 regarded as algebraic multiplication with $p^k\gamma_k$, where p^k is the four-momentum associated with the propagator line defined by the light-like orbit of parton. This simplifies the formalism enormously and gives a direct connection with similar condition posed independently in twistorial approach [K78].

One can require that the modes annihilated by Kähler-Dirac operator are eigenstates of C-S-D oeprator with generalized eigenvalues $p^k\gamma_k$ giving rise to fermion propagator Consider now the properties of eigenmodes of D_{C-S}.

1. For $p^k = 0$ there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish.

2. If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action.

   $D(CP_2) \leq 2$ is inconsistent with the weak form of electric-magnetic duality. The extrema of Chern-Simons action have $D(CP_2) \leq 2$ and vanishing Chern-Simons density so that they would naturally represent on mass shell particles appearing as incoming and outgoing particles. This conforms with the interpretation of the basic extremals as free particles (massless extremals and cosmic strings with 2-D CP_2 projection). One could say that CP breaking is not present for free particles but unavoidably accompanies the propagator lines.

The explicit expression of D_{C-S} without constraint terms from the weak form of electric-magnetic duality is given by

$$\begin{aligned} D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\ \hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ D_\mu \hat{\Gamma}^\mu &= B^\alpha_K (J_{k\alpha} + \partial_\alpha A_k) \ , \\ B^\alpha_K &= \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ , \ \ J_{k\alpha} = J_{kl}\partial_\alpha s^l \ , \ \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \tag{6.7.14}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma}$ does not depend on the induced metric.

The extremals of Chern-Simons action satisfy

$$B^\alpha_K (J_{kl} + \partial_l A_k)\partial_\alpha h^l = 0 \ , \ \ B^\alpha_K = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ . \tag{6.7.15}$$

For non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional and S^2 projection is 1-dimensional or vice versa. This implies a vanishing of Chern-Simons action for both options. Consider for the simplicity the case when S^2 projection is 1-dimensional.

1. Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [K40] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

2. With this assumption the Chern-Simons Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{6.7.16}$$

3. Consider first the general solutions of the modified Dirac equation when M^4 Dirac operator $p^k\gamma_k$ annhilates the spinor so that on mass shell massless fermion is in question. The spinor is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{6.7.17}$$

   The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X^3_l is arbitrary.

4. For internal lines $p^k\gamma_k$ does not annihilate the spinor although four-momentum can be still on mass shell if the spinor has unphysical helicity. In this case the equation is modified. Again the modes can be localized to 1-D curves.

5. The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate.

The localization is of utmost importance since and is consistent with the localization of the modes (other than right-handed neutrino) of Kähler Dirac equation at string world sheets discussed in chapter [K105]. String ends would thus define braid strands. The absence of correlation between the behaviors with respect longitudinal coordinate and transversal coordinates looked very strange at first glance. System looked like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines.

A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

(b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^{\alpha} &= \overline{\Psi} O \hat{\Gamma}^{\alpha} \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB} J\} \ . \end{aligned} \tag{6.7.18}$$

Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

(c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0}$= $kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

   The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z) + \overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

How to calculate Dirac determinant?

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

1. Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

2. Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

3. The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

4. The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

5. Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu (dx^\mu/ds)ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

6. The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

   The formula would be

$$K \propto \oint J(x,y) J_T dx^1 \wedge dx^2 \ . \tag{6.7.19}$$

   This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

6.8 Identification of elementary particles and the role of Higgs in particle massivation

The development of the recent view about the identification of elementary particles and particle massivation has taken fifteen years since the discovery of p-adic thermodynamics around 1993. p-Adic thermodynamics worked excellently from the beginning for fermions. Only the understanding of gauge boson masses turned out to be problematic and group theoretical arguments led to the proposal that Higgs boson should be present and give the dominating contribution to the masses of gauge bosons whereas the contribution to fermion masses should be small and even negligible. The detailed understanding of quantum TGD at partonic level eventually led to the realization that the coupling to Higgs is not needed after all. The deviation Δh of the ground state conformal weight from negative integer has interpretation as effective Higgs contribution since Higgs vacuum expectation is naturally proportional to Δh but the coupling to Higgs does not cause massivation. In the following I summarize the basic identification of elementary particles and massivation. A more detailed discussion can be found in [K32] .

6.8.1 Identification of elementary particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [K21, K20] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [K56, K57, K52] leave a lot of freedom concerning the detailed identification of elementary particles.

Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [K20] . If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [K19] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R, which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).

2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [K17, K65] . This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.

3. The TGD view about coupling constant evolution [K5] predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127} - 1$ assignable to electron [K5] . I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.

4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact, the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [K84] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [K13, K14] . Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1) \times (3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet as also wormhole throats can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X^2_1 \times X^2_2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X^2_2 is a stationary surface of E^3. Both $X^2_1 \subset M^1 \times CP_2$ and X^2_2 have an Euclidian signature of metric except at light-like boundaries $X^1_a \times X^2_2$ and $X^1_b \times X^2_2$ defined by ends of X^2_1 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X^2_1 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

If one accepts the hierarchy of Planck constants [K27] leading to the generalization of the notion of imbedding space, the identification of anyonic phases in terms of macroscopic light-like surfaces emerges naturally. In this kind of states large fermion numbers are possible. Dark matter would correspond to this kind of phases and "partonic" 2-surfaces could have even astrophysical size. Also black holes can be identified as dark matter at light-like 3-surfaces analogous to black hole horizons and possessing gigantic value of Planck constant [K65] .

6.8.2 New view about the role of Higgs boson in massivation

The proposed identifications challenge the standard model view about particle massivation.

1. The standard model inspired interpretation would be that Higgs vacuum expectation associated with the coherent state of neutral Higgs wormhole contacts generates gauge boson mass. The TGD counterpart of Higgs would be however not H-scalar but complex CP_2 tangent vector. There are no covariantly constant vector fields in CP_2 so that the idea about Higgs vacuum expectation is not mathematically feasible. This led to the original exaggerated conclusion that TGD does not allow Higgs: it is however only Higgs vacuum expectation which does not look plausible. Fermionic mass would be solely due to p-adic thermodynamics. Also in the case gauge boson masses one encounters a problem: the natural guess for the p-adic prime as M_{89} represents too small gauge boson masses, and it is very difficult to understand Weinberg angle, which is essentially group theoretical notion.

2. The modified Dirac equation plus well-definedness of em charge requires that the spinor modes are restricted to stringy curves connecting the throats of two wormhole contacts associated with the elementary particles and carrying monopole fluxes. One can say that the wormhole throats are connected by flux tube behaving like string. The obvious idea is that the flux tube gives additional contribution to the mass squared, which can be interpreted as a contribution to the conformal weight of the ground state. If the string tension is proportional to gauge coupling strength for W and Z and to the counterpart of Higgs self coupling λ for Higgs one can explain the mass ratios of gauge bosons.

3. Besides the thermodynamical contribution to the particle mass there would be a small contribution from the ground state conformal weight unless this weight is not negative integer. Gauge boson mass would correspond to the ground state conformal weight present in both fermionic and bosonic states and in the case of gauge bosons this contribution would dominate due to the small value of p-adic temperature. For fermions p-adic thermodynamics for super Virasoro algebra would give the dominating contribution to the mass.

4. The remaining problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro

generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.

(a) Modified Dirac action gives for the solutions of Dirac action a boundary term which is essentially contraction of the normal component of the vector defined by Kähler-Dirac gamma matrices. In absence of measurement interaction terms the boundary condition for K-D equation states $\Gamma^n\Psi = 0$ at the stringy curves at the space-like ends of space-time surface. Γ^n must be lightlike and the assumption is that the spinor modes are generalized eigenmodes of Γ^n: $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$ where p^k is constant light-like four-momentum. This conforms with the idea that all fermions are massless and massive states of super-conformal representations emerges as bound states of fermions at wormhole throats. Elementary particles would correspond to pair of wormholes with magnetic flux flowing between the throats at the two space-time sheets involved. Massivation would be many-sheeted phenomenon. The string like objects would have string tension explaining the masses of weak bosons at microscopic level.

Very naively, $\Gamma^n\Psi = 0$ is possible only in the regions of space-like 3-surface which belong to Minkowskian space-time regions. Since Kähler-Dirac gamma matrices are in question it can however happen that the effective metric of string world sheet defined by Γ is degenerate. If CP_2 projection is 4-D as it is for CP_2 type extremals, one however ecpects that Γ^α is not degenerate inside wormhole contacts, and one can even question the localization of the spinor modes to 2-D string world sheets in these regions. The TGD based variant of stringy diagrammatics would indeed involve massless fermionic propagators only in the Minkowskian regions. The interaction of fermions at opposite throats of wormhole contacts would be described by stringy propagator $1/L_0$ or its non-local generalization to the product $(1/G)(1)\times(1/G)^\dagger(2)$ with supergenerators $G(i)$ assigned with the opposite wormhole throats.

(b) One can add to the Kähler action measurement interaction term fixing the space-time surfaces to have conserved classical identical to their quantum counterparts belonging to Cartan algebra of symmetries. This can be achieved by adding Lagrange multiplier terms. These terms contribute to the Kähler-Dirac action a term at space-like ends of 3-surface and this term modifies the TGD counterpart of massless Dirac equation. The original generalized massless generalized eigenvalue spectrum associated with $p^k\gamma_k\Psi = 0$ of Γ^n is modified to massive spectrum given by the condition

$$\Gamma^n\Psi = -\sum_i \lambda_i\Gamma^\alpha_{Q_i}D_\alpha\Psi = p^k\gamma_k\Psi \ ,$$

where Q_i refers to i:th conserved charge. Fermions are not massless anymore. This description is certainly over-simplified since several wormhole throats are involved. It is also only a formal description for the values of quantum numbers Q_i. One might say that $(\Gamma^n)^2$ serves as the analog of Higgs field vacuum expectation defined at the string curve.

(c) It is not clear whether the tachyonic value of mass squared for ground state of superconformal representations can emerge from this kind of description. This might be possible inside wormhole contacts which have Euclidian signature of induced metric and define the lines of generalized Feynman diagrams.

6.8.3 General mass formulas

In the following general view about p-adic mass formulas and related problems is discussed.

Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-symplectic

Virasoro algebra (SSV). Conformal invariance holds true only for the generators of the differences of $SKMV$ and SSV generators. In the case of SSV and $SKMV$ only the generators L_n, $n > 0$, annihilate the physical states. Obviously the actions of super-symplectic Virasoro (SSV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle. p-Adic mass expectation value is same irrespective of whether it is calculated for the excitations created by SSV or $KKMV$ generators and p-adic mass calculations are consisted with super-conformal invariance.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the ground state conformal weight corresponds to the square of the imaginary eigenvalue of the modified Dirac operator having dimensions of mass. If the value of ground state conformal weight is not negative integer, a contribution to mass squared analogous to Higgs expectation is obtained.

2. Massless state is thermalized with respect to SKMV (or SSV) with thermal excitations created by generators L_n, $n > 0$.

Under what conditions conformal weight is additive

The question whether four- momentum or conformal weight is additive in p-adic mass calculations becomes acute in hadronic mass calculations. Only the detailed understanding of quantum TGD at partonic level allowed to understand the situation. One can consider three options.

1. Conformal weight and thus mass squared is additive only inside the regions of X^3_l, which correspond to non-vanishing of induced Kähler magnetic field since these behave effectively as separate 3-surfaces as far as eigenmodes of the modified Dirac operator are considered. The spectrum of the ground state conformal weights is indeed different for these regions in the general case. The four-momenta associated with different regions would be additive. This makes sense since the tangent space of $X^4(X^3_l)$ contains at each point of X^3_l a subspace $M^2(x)) \subset M^4$ defining the plane of non-physical polarizations and the natural interpretation is that four-momentum is in this plane. Hence the problem of original mass calculations forcing to assign all partonic four-momenta to a fixed plane M^2 is avoided.

2. If assigns independent translational degrees of freedom only to disjoint partonic 2-surfaces, a separate mass formula for each X^2_i would result and four-momenta would be additive:

$$M_i^2 = \sum_i L_{0i}(SKM) \ . \tag{6.8.1}$$

   Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. Also vacuum conformal weight is included.

3. At the other extreme one has the option is based on the assignment of the mass squared with the total cm. This option looked the only reasonable one for 15 years ago. This would give

$$M^2 = (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i\cdot p_j = -\sum_i L_{0i}(SKM) \ . \tag{6.8.2}$$

   The additivity of mass squared is strong condition and p-adic mass calculations for hadrons suggest that it holds true for quarks of low lying hadrons. For this option the decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses.

Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{6.8.3}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{6.8.4}$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

Chapter 7

An Overview About Quantum TGD: Part II

7.1 Introduction

This chapter is the second one of two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development. In this chapter ideas related to the construction of S-matrix and coupling constant evolution are discussed.

The construction of S-matrix involves several ideas that have emerged during last years.

1. Zero energy ontology (ZEO) motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2. The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree.

3. ZEO leads to profoundly new view about the notion of virtual particle strongly suggeseting that the M-matrix is finite and that the number of Feynman diagrams contributing to given reaction is finite if particles have p-adic thermal mass. It has turned out that stringy generalization of twistor Grassmann approach is needed in order to avoid UV divergences. This approach emerges naturally in TGD framework.

4. The symmetric space property of world of classical worlds (WCW) allows to reduce WCW functional integral to Fourier analysis in WCW having a direct generalization to p-adic context so that the great dream about algebraic universality can be realized.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at http://www.tgdtheory.fi/cmaphtml.html [L21]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L22]. The topics relevant to this chapter are given by the following list.

- Overall view about TGD [L55]
- What TGD is [L84]
- TGD as unified theory of fundamental interactions [L75]
- Basic TGD [L26]
- Basic TGD [L27]
- Space-time as 4-surface in $M^4 \times CP_2$ [L67]

7.2 About the construction of S-matrix

During years I have proposed a long list of nice looking ideas concerning the construction of S-matrix. After the progress in understanding the role of hyper-finite factors of type II_1 it become clear that the basic problems have been more at the conceptual level rather than calculational. Thus the key questions seem to be following ones.

What does one actually mean with S-matrix? Classical TGD forces to question even the basic ontology and strongly suggests the notion of zero energy ontology in which physical states possess vanishing net quantum numbers and are creatable from vacuum. U-matrix would characterize transition amplitudes between zero energy states and could have elements even between states belonging to different number fields. In particular, it could characterize transitions in which intention transforms to action. U-matrix would have M-matrices as its rows with M-matrices defining orthogonal basis of matrices expressible as product Hermitian square root of projection operator and unitary S-matrix. M-matrix would represent entanglement coefficients between positive and negative energy parts of the state. Given M-matrix in turn would define entanglement probabilities expressible as a weighted sum of projection operators. If the weights are identical, this would give rise to negentropic entanglement favored by Negentropy Maximization Principle (NMP) defining the variational principle behind state function reduction. Negentropic entanglement plays a key role in TGD inspired theory of consciousness and suggests a totally new interpretation for unitarity.

One can raise further questions. How does S-matrix relate to the usual S-matrix? What could S-matrix with a finite measurement resolution mean? What is the precise mathematical characterization of a physical state when the measurement resolution is finite? How does the fuzziness due to a finite measurement resolution affect the definition of transition probabilities defined by S-matrix?

The proper formulation of the notion of measurement resolution suggests a rather dramatic modification of the standard mathematical picture. S-matrix could be fractal and more or less the same for hyperfinite factor of type II_1 (HFF) $\mathcal{M}$ and its sub-factors. Transition probabilities would be defined by "quantum S-matrix" with non-commuting $\mathcal{N}$ valued elements in non-commutative fuzzy "quantum quantum state space" with $\mathcal{N}$ valued coefficients generated by $\mathcal{M}/\mathcal{N}$, where Jones inclusion $\mathcal{N} \subset \mathcal{M}$ defines the measurement resolution. Transition probabilities would be eigenvalues of the transition probability operators, which would be commuting Hermitian operators in $\mathcal{N}$.

At the more technical level the requirement of number theoretical universality leads to a rather concrete constraints on the general form of S-matrix based on the notion of number theoretic braid. This notion emerges also from the non-commutativity implied by the finite measurement resolution characterized in terms of Jones inclusions.

The improved understanding of super-conformal symmetries during last year provides powerful additional constraints and suggest a modification of stringy picture replacing the orbits of the ends of the number theoretic strings with number theoretic braids. Stringy variant of twistor Grassmann approach seems to define the most promising line of attack at this moment [K78].

7.2.1 About the general conceptual framework behind quantum TGD

Let us first list the basic conceptual framework in which I try to concretize the ideas about S-matrix.

$N = 4$ super-conformal invariance and light-like 3-surfaces as fundamental dynamical objects

Super-conformal symmetries generalized from string model context to TGD framework are symmetries of S-matrix and of its generalization to M-matrix. This is very powerful constraint but useless unless one has precisely defined ontology translated to a rigorous mathematical framework. The zero energy ontology of TGD is now rather well understood but differs dramatically from that of standard quantum field theories. Second deep difference is that path integral formalism is given up and the goal is to construct S-matrix as a generalization of braiding S-matrices with reaction vertices replaced with the replication of number theoretic braids associated with partonic 2-surfaces taking the role of vertices.

The path leading to the understanding of super-conformal invariance in TGD framework was long but the (one might hope) final outcome is briefly described. The are two kinds of super-conformal symmetries.

1. The first super-conformal invariance is associated with light-cone boundary and is due to its metric 2-dimensionality putting 4-D Minkowski space in a unique position. The symplectic transformations of $\delta H_{\pm} = \delta M^4_{\pm} \times CP_2$ are identified as isometries of WCW. The super-generators of super-symplectic algebra correspond to the gamma matrices of WCW.

2. Light-like partonic 3-surfaces X^3 are the basic dynamical objects and light-likeness is respected by the 3-D variant of Kac-Moody algebra of conformal transformations of imbedding space made local with respect to X^3. Ordinary 1-D Kac-Moody algebra with complex coordinate z replaced with a light-like radial coordinate r takes a special role and super Kac-Moody symmetry is associated with this. The conformal symmetries associated with X^2 are counterpart of stringy conformal symmetries but have a role analogous to the conformal symmetries of critical statistical systems.

3. WCW can be expressed as a union of coset spaces G/H, which are homogeneous or even symmetric spaces with isometries defined by the symplectic group G of $\delta M^{\pm} \times CP_2$. H corresponds to symplectic transformations leaving 3-surfaces at the end of CD invariant - that is inducing a mere diffeomorphism. G has symplectic isometries of $\delta M^{\pm} \times CP_2$ as a subgroup.

   WCW geometry has the Poisson algebra generated by super-generators of g identifiable as complexified gamma matrices of WCW and expressible in terms of modes of right-handed neutrino not mixing with left handed neutrino. Symplectic isometries correspond to the modes of the induced spinor field which are not elecro-weakly neutral.

   By the generalized coset construction the differences of super conformal generators assignable to g and sub-algebra h annihilate WCW spinor field at point of WCW which remains invariant under the action of H. Naturally this point corresponds to maximum of Kähler function and stationary point of the Morse function defined by the Kähler action associated with the Minkowskian space-time regions.

S-matrix in zero energy ontology

Zero energy ontology allows to construct unitary S-matrix in fermionic degrees of freedom as unitary entanglement coefficients between positive and negative energy parts of zero energy state. The basic properties of hyper-finite factor II_1 are absolutely crucial. The inclusion of bosonic degrees of freedom lead to a replacement of HFF of type II_1 with HFF of type $II_\infty = II_1 \otimes I_\infty$. However, normalizability of the states allows only a projection of S-matrix to a finite-dimensional subspace of incoming or outgoing states. Hence the S-matrix is effectively restricted to $II_1 \otimes I_n = II_1$ factor so that at the level of physical states HFF of type II_1 results. This is absolutely crucial for the unitary of the S-matrix since it makes possible to have $Tr(SS^\dagger) = Tr(Id) = 1$. If factor of type I is present as a tensor factor, thermal S-matrix is the only possibility and later arguments in favor of the idea that thermodynamics is unavoidable part of quantum theory in zero energy ontology will be developed.

One can worry whether unitarity condition is consistent with the idea that fermionic degrees of freedom should allow to represent Boolean functions in terms of time-like entanglement. That unitary time evolution is able to represent this kind of functions in the case of quantum computers suggests that unitarity is not too strong a restriction. The basic question is whether only a "cognitive" representation of physical S-matrix in terms of time like entanglement or a genuine physical S-matrix is in question. It seems that the latter option is the only possible one so that physical systems would represent the laws of physics.

U-matrix

Besides S-matrix there is also U-matrix defining the unitary process associated with the quantum jump. S- *resp.* U-matrix characterizes quantum state *resp.* quantum jump so that they cannot be one and same thing.

1. There are good arguments supporting the view that U-matrix is almost trivial, and the real importance of U-matrix seems to be related to the to the description of intentional action identified as a transition between p-adic and real zero energy states and to the possibility to perceive states rather than only changes as quantum jumps leaving the state almost unchanged.

2. State function reduction corresponds to a projection sub-factor in TGD inspired quantum measurement theory whereas U process in some sense corresponds its reversal. Therefore U matrix might correspond to unitary isomorphism mapping factor to a larger factor containing it.

3. State function reduction must be consistent with the unitarity of S-matrix defining time-like entanglement. Since state function reduction means essentially multiplication by a projector to a sub-space it seems that state function reduction for both incoming and outgoing states are possible and would naturally correspond to projections to sub-factors of corresponding HFFs of type II_1.

Unitarity of the counerpart of S-matrix is not necessary in zero energy ontology

U-matrix is necessarily unitary. There are good reasons to believe that this condition combined with Lorentz invariance makes it almost trivial. In the case of S-matrix unitarity is not absolutely necessary. M-matrix which is the counterpart of ordinary S-matrix need not be unitary and is most naturally the analog of thermal S-matrix in QFTs.

M-matrix need not be unitary. The density matrix associated with M-matrix is sum over projection operators corresponding to its eigenvalues. Degenerate eigenvalue with n-fold degeneracy corresponds to negentropic entanglement favoured by NMP. For unitary M-matrix resulting in state function reduction for zero energy state the entanglement would be negentropic since density matrix is proportional to unit matrix. NMP would make this kind of state stable against furthere state function reduction to either boundary.

The restriction of the time-like entanglement coefficients to a unitary M-matrix would conform with the idea that light-like partonic 2-surfaces represent a dynamical evolution at quantum level so that zero energy states must be orthogonal both with respect to positive and negative energy parts of the states. On the other hand, the light-like 3-surface can be chosen arbitrarily and its choice indeed affects S-matrix. Hence the theory cannot fully reduce to a 2-dimensional theory. The interpretation is that light-like 3-surfaces are in 1-1 correspondence with the ground states of super-conformal representations identifiable as light particles.

Imbedding space degrees of freedom naturally give rise to a factor of type I so that only thermal S-matrix defines a normalizable zero energy state. S-matrix as functor from the category of Feynman cobordisms to the category operators defining entanglement coefficients implies that S-matrix in fermionic degrees of freedom for a product of cobordisms is product of the S-matrices for cobordisms. This implies that in fermionic degrees of freedom S-matrix is thermal S-matrix with time parameter replaced with complex time parameter whose imaginary part corresponds to inverse temperature. Also an argument based on the existence of universal thermal S-matrix with a complex time parameter for hyper-finite factors of type III_1 supports the view that unitarity is not necessary. A further argument is based on the finding that in dimensions $D < 4$ unitary S-matrix exists only if cobordism is trivial so that topology change would not be possible. This raises the fascinating possibility that thermodynamics - in particular p-adic thermodynamics - is an unavoidable and inherent property of quantum TGD.

Quantum classical correspondence

Quantum classical correspondence states that there is a correspondence between quantum fluctuating degrees of freedom associated with partonic 2-surfaces and classical dynamics. The weakest form of this principle is that the ground states of partonic super-conformal representations (massless states which generate light masses observed in laboratory) correspond to the interior dynamics of space-time sheets containing the partonic 2-surfaces. At the space-time level there would be 1-1 correspondence with the maxima of Kähler function giving rise to the analog of spin glass energy landscape.

One could protest by saying that excited states of super-conformal representations have no space-time correlate in this picture. Quantum states are replaced with states in which the projection of S-matrix to a finite-dimensional space in bosonic degrees of freedom appears as time-like entanglement coefficients so that quantum classical correspondence is obtained in strict sense after all. These states states are formally analogous which raises the question whether an actual relationship exists. For HFFs of type III unitary time evolution and thermal equilibrium are indeed closely related aspects of states [A58] . $I_\infty \to I_n$ cutoff in the bosonic degrees of freedom would naturally have the discretization represented by number theoretic braids as a space-time correlate.

The effective elimination of the degrees of freedom associated with the space-time interior implied by the 1-1 correlation would allow to forget 4-D space-time degrees of freedom more or less completely as far as calculation of S-matrix is considered and everything would reduce to Fock space level as it does in quantum field theories. The functional integral around the maximum of Kähler function would select a set of preferred light-like partonic 3-surfaces. Quantum criticality suggests that the functional integral can be carried out exactly.

How TGD differs from string models

An important detail which deserves to be mentioned separately is one crucial deviation from string model picture: the stringy decays of partonic 2-surfaces or 3-surfaces are space-time correlates for the propagation of particle via several different routes rather than genuine particle decay. Note that partonic 2-surfaces can have arbitrarily large size and the outer boundary of any physical system represents the basic example of this kind of surface. Particle reactions correspond to branchings of light-like partonic 2-surfaces so that incoming and outgoing partons are glued together along their ends. This picture makes sense because quantum TGD reduces to almost topological conformal QFT at parton level (only light-likeness brings in the notion of metric).

Quantum classical correspondence allows to interpret light-like partonic 3-surface either as a time evolution of a highly non-deterministic 2-D system or as a 3-D system. This state-dynamics duality was discovered already in [K97] , where it was realized that topological quantum computation has interpretation either as a program (state) or running of program (dynamics). Complete reduction to 2-D dynamics is not possible since the light-like 3-surfaces associated with maxima of Kähler action define spin glass energy landscape such that each maximum corresponds to its own S-matrix.

In this picture particle reactions correspond classically to branchings of partonic 2-surfaces generalizing the branchings for lines in Feynman diagrams. The stringy vertices for decays of surfaces correspond in TGD framework to the classical space-time correlate for a particle travelling along different paths and the particle creation and annihilation is a generalization of what occurs in Feynman diagrams with vertices replaced with 2-dimensional partonic surfaces along which light-like partonic 3-surfaces meet.

The localization of the modes of induced spinor field with well-defined em charge to string world sheets or partonic 2-surfaces implies that string model like description appears as part of TGD.

One can indeed assign to elementary particles closed strings traversing from wormhole throat to another and returning back along second space-time sheet: together the sheets form 2-fold covering of some M^4 region. TGD is however more than string model: besides stringy conformal weight one has the super-conformal weight associated with the light-like radial coordinate of light-cone boundary and the integer characterizing the poly-locality of the generator of Yangian extending the super-conformal algebra. This makes three integers in accordance with 3-dimensionality of basic geometric objects.

Physics as a generalized number theory vision

TGD as a generalized number theory vision gives powerful constraints. New view about space-time involves p-adic space-time sheets as space-time correlates for cognitive representations in fermionic case and for intentions in the bosonic case. This leads to the notion of number theoretic braid belonging to the algebraic intersection of real and p-adic partonic surfaces obeying same algebraic equations.

The implication is that the data characterizing S-matrix elements should come from discrete algebraic points of number theoretic braids. The Galois groups for braids occupying regions of partonic 2-surface emerge as a new element and relate closely to the representations of braid groups in HFFs of type II_1. Number theoretic universality leads to the condition that S-matrix elements are algebraic numbers in the extension of rational defined by the extension of p-adic numbers involved.

The role of hyper-finite factors of type II_1

The Clifford algebra of WCW ("world of classical worlds") spinors is very naturally a hyper-finite factor of type II_1. During the last few years I have gradually learned something about the magnificent mathematical beauty of these objects.

1. TGD inspired quantum measurement theory with measurement resolution characterized in terms of Jones inclusion and based on HFFs of type II_1 brings in non-commutative quantum physics and leads to powerful general predictions [K99, K51] . The basic idea is that complex rays of the state space are replaced with $\mathcal{N}$ rays for Jones inclusion $\mathcal{N} \subset \mathcal{M}$. $\mathcal{N}$ defines the measurement resolution in the sense that the group G leaving elements of $\mathcal{N}$ invariant characterizes the measured quantum numbers.

2. Hyper-finite factors have the property that they are isomorphic with their tensor powers. This makes possible the construction of vertices as unitary isomorphisms between tensor products of HFFs of type II_1 associated with incoming and outgoing states. The core part of S-matrix boils down to a unitary isomorphism between tensor products of hyper-finite factors of type II_1 associated with incoming *resp.* outgoing partonic 3-surfaces whose ends meet at the partonic 2-surface representing reaction vertex.

3. The study of Jones inclusions leads to the idea that Planck constant is dynamical and quantized. The predicted hierarchy of Planck constants involving a generalization of imbedding space concept and an explanation of dark matter as macroscopic quantum phases [K27] . Here the special mathematical role of Jones inclusions with index $r \leq 4$ is crucial.

4. The properties of HFFs inspire also the idea that TGD based physics should able to mimic any imaginable quantum physical system defined by gauge theory or conformal field theory involving Kac-Moody symmetry. Thus the ultimate physics would be kind of analog for Turing machine. The prediction inspired by TGD based explanation of McKay correspondence [A17] is that TGD Universe is indeed able to simulate gauge and Kac-Moody dynamics of a very large subset of ADE type groups. In fact, also much more general prediction that simulation should be possible for any compact Lie group emerges.

5. HFFs of type II_1 lead also to deep connections with number theory [A17] and number theoretical braids can be interpreted in terms of representations of Galois groups assignable with partonic 2-surfaces in terms of HFFs of type II_1. Particle decay represents a replication of number theoretical braids and this together with p-adic fractality and hierarchy of Planck constants suggests strongly direct connections with genetic code and DNA.

Could TGD emerge from a local version of infinite-dimensional Clifford algebra?

A crucial step in the progress was the realization that TGD emerges from the mere idea that a local version of hyper-finite factor of type II_1 represented as an infinite-dimensional Clifford algebra must exist (as analog of say local gauge groups). This implies a connection with the classical number fields. Quantum version of complexified octonions defining the coordinate with respect to which one localizes is unique by its non-associativity allowing to uniquely separate the powers of octonionic coordinate from the associative infinite-dimensional Clifford algebra elements appearing as Taylor coefficients in the expansion of Clifford algebra valued field.

Associativity condition implies the classical and quantum dynamics of TGD. Space-time surfaces are hyper-quaternionic of co-hyper-quatenionic sub-manifolds of hyper-octonionic imbedding space HO. Also the interpretation as a four-surface in $H = M^4 \times CP_2$ emerges and implies $HO-H$ duality. What is also nice that Minkowski spaces correspond to the spectra for the eigenvalues of maximal set of commuting quantum coordinates of suitably defined quantum spaces. Thus Minkowski signature has quantal explanation.

Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.

2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.

3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [A85] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."* later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M} ...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of S-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation and automatic emerges of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

7.2.2 S-matrix as a functor in TQFTs

John Baez's [A42] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

Very briefl, almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. Feynman cobordism is the generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. This picture differs dramatically from that of string models. There is a functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function. Functor property generalizes the unitary condition and allows also thermal S-matrices which seem to be unavoidable since imbedding space degrees of freedom give rise to a factor of type I with $Tr(id) = \infty$.

The *-category of Hilbert spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T_Ψ^* mapping Hilbert space to C, inner products can be defined as morphisms $T_\Phi^* T_\Psi$.

The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

The monoidal *-category of Hilbert spaces and its counterpart at the level of nCob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

TQFT as a functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

1. Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail

to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

2. Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

3. What is the relevance of this result for quantum TGD?

7.2.3 S-matrix as a functor in quantum TGD

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [A104] only the exotic diffeo-structures modify the situation in 4-D case.

Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

Feynman cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of 2→2 reaction open string is pinched to a point at vertex. 1→2 vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

1. There is U-matrix between zero energy states. This is expected to be rather trivial but very important from the point of view of description of intentional actions as transitions transforming p-adic partonic 3-surfaces to their real counterparts.

2. The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics.

Time-like entanglement coefficients as a square root of density matrix?

All quantum states do not correspond to thermal states and one can wonder what might be the most general identification of the quantum state in zero energy ontology. Density matrix formalism defines a very general formulation of quantum theory. Since the quantum states in zero energy ontology are analogous to operators, the idea that time-like entanglement coefficients in some sense define a square root of density matrix is rather natural. This would give the defining conditions

$$\begin{aligned} \rho^{+} &= SS^{\dagger} \ ,\rho^{-} = S^{\dagger}S \ , \\ Tr(\rho^{\pm}) &= 1 \ . \end{aligned} \tag{7.2.1}$$

$\rho^{\pm}$ would define density matrix for positive/negative energy states. In the case HFFs of type II_1 one obtains unitary S-matrix and also the analogs of pure quantum states are possible for factors of type I. The numbers $p^{+}_{m,n} = |S^2_{m,n}|/\rho^{+}_{m,m}$ and $p^{-}_{m,n} = |S^2_{n,m}|/\rho^{-}_{m,m}$ give the counterparts of the usual scattering probabilities.

A physically well-motivated hypothesis would be that S has expression $S = \sqrt{\rho}S_0$ such that S_0 is a universal unitary S-matrix, and $\sqrt{\rho}$ is square root of a state dependent density matrix. Note that in general S is not diagonalizable in the algebraic extension involved so that it is not possible to reduce the scattering to a mere phase change by a suitable choice of state basis. Clearly, S-matrix can be seen as matrix valued generalization of Schrö;dinger amplitude. Note that the "indices" of the S-matrices correspond to WCW spinor s (fermions and their bound states giving rise to gauge bosons and gravitons) and to WCW degrees of freedom (world of classical worlds). For hyper-finite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [A9] . Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^{\dagger} = f\rho_{g,+}$ and $f^{\dagger}fg = \rho_{f,-}g$, and the conditions $ff^{\dagger} = \rho_{+}$ and $f^{\dagger}f = \rho_{-}$ are satisfied. Here $\rho_{\pm}$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^{\dagger}\rho^{-1}_{f,+}$ satisfies $ff_L^{-1} = Id_{+}$ and $f_R^{-1} = \rho^{-1}_{f,-}f^{\dagger}$ satisfies $f_R^{-1}f = Id_{-}$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

7.2.4 Number theoretic constraints on S-matrix

Number theoretical universality leads to the hypothesis that S-matrix elements must be algebraic numbers [K20] . This is achieved naturally if the definition of S-matrix elements involves only the data associated with the number theoretic braid. This leads naturally to a connection with braiding S-matrices also in the case of real-to-real transitions. Also the concept of number theoretic string emerges.

The partonic vertices appearing in S-matrix elements should be expressible in terms of N-point functions of almost topological $N = 4$ super-conformal field theory but with the p-adically questionable N-fold integrals over string replaced with sums over the strands of a braid: spin chain type string discretization could be in question [K20] . Propagators, that is correlations between partonic 2-surfaces, would be due to the interior dynamics of space-time sheets which means a deviation from super string theory. Another function of interior degrees of freedom is to provide zero modes of metric of WCW identifiable as classical degrees of freedom of quantum measurement theory entangling with quantal degrees of freedom at partonic 3-surfaces.

7.2.5 Stringy variant of twistor Grassmannian approach to scattering amplitudes

The stringy variant of twistor Grassmannian approach [K78] provides the most concrete and promising approach to the construction of the scattering amplitudes. I just summarize the essentials of this approach in following.

The thesis by Tim Adamo titled "Twistor actions for gauge theory and gravity" [B13] considers formulation of $N = 4$ SUSY gauge theory directly in twistor space instead of Minkowski space. The author is able to deduce MHV formalism, tree level amplitudes, and planar loop amplitudes from action in twistor space. Also local operators and null polygonal Wilson loops can be expressed twistorially. This approach is applied also to general relativity: one of the challenges is to deduce MHV amplitudes for Einstein gravity. The reading of the article inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD [K98, K101] giving a view about development of ideas).

Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1) \times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via superconformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for $M^4 \times CP_2$ serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of CP_2 twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor $1/p^2$ assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of $\mathcal{N} = 4$ SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.

2. This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators $G, G^\dagger$ and L bringing in

CP¡sub¿2¡/sub¿ scale appear in propagator lines: most importantly, the fact that G and $G^\dagger$ carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has $1/G$ and $1/G^\dagger$ at its ends. Twistorialization applies because all fermion lines are light-like.

3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to CP_2 scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting CP_2 scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

7.3 A vision about the role of HFFs in TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

7.3.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A35] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0,1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X,μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.
2. A positive linear functional is weight with $\omega(1)$ finite.
3. A state is a weight with $\omega(1) = 1$.
4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.
5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.
2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \qquad (7.3.1)$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.

3. The conjugation $x \rightarrow x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \rightarrow L^2(\mathcal{M})$ via $x \rightarrow x\Omega$. The map $S_0; x\Omega \rightarrow x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in zero energy ontology.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A59, A127] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \rightarrow \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_{∞} factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_{λ} this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a manner to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K99] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A3] and those of factors of type III by Alain Connes [A54] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A3] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a)\ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b)\ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{7.3.2}$$

   the numbers at right hand side are known as Beraha numbers [A108] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B39] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A85] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

Connes tensor product

The inclusions The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N} . \tag{7.3.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M} \otimes \mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

7.3.2 Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A92, A59, A127] . There are good arguments showing that in HFFS of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A127] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ \ , \ \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ \ .$$

An infinite hierarchy of inclusions of HFFS of type III_1s is induced by set theoretic inclusions.

7.3.3 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and zero energy ontology, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the modified Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the modified Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

Zero energy ontology and factors

The first question concerns the identification of the Hilbert space associated with the factors in zero energy ontology. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in zero energy ontology.

1. In zero energy ontology M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFS is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \rightarrow 0$ limit. In quantum TGD the exponent of modified Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Modified Dirac action can therefore be regarded as a "square root" of Kähler action.

3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in zero energy ontology based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the modified Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFS involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of modified Dirac action [K17] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

3. Quantum criticality means that modified Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of

quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two superconformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L^*_0$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing

of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K27] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

7.3.4 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

2. By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time

surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.
2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.
3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j\cdot A=0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j\cdot A=0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

2. If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence

C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{7.3.4}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Localization of the modes of Kähler-Dirac operator at string world sheets and definition of Dirac determinant

The condition that the modes of Kähler-Dirac operator have well defined electromagnetic charge eigenvalue implies that the modes are restricted to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces [K105]. In the generic case one would have a product of Dirac determinants associated with these 2-surfaces. This obviously simplifies dramatically the definition of Dirac determinant and suggests a reduction to stringy mathematics, where this kind of determinants appear routinely.

The construction of Dirac determinant could proceed in following manner.

1. The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

2. The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

3. The square of the Dirac determinant would be product of eigenvalues mass squared operator assignable to the eigenvalue equation $(p^k\gamma_k{+}\Gamma^n)^2\Psi = \Lambda_n)\Psi$. If the eigenvalues correspond up to multiplicative factor to integer valued conformal weights, the square of Dirac determinant would be the product of corresponding mass squared values equal to conformal weight with vacuum contribution. The square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + n$, where h is expressed using unit of mass determined by CP_2 radius.

4. One can of course ask whether it might be possible to define even the Dirac determinant itself. Here it seems that the only possible manner to proceed is number theoretic: the factors $p^k\gamma_k+\Gamma^n$ appearing in the formal Dirac determinant should be mapped to complexified octonions and the product of these factors should define Dirac determinant as complex quantity having interpretation as the product of exponents of Kähler for Euclidian and Minkowskian regions metting at wormhole throat. This would be a rather deep connection with the number theoretic approach.

5. Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

   The resulting determinant - if indeed well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

One must however be aware about possible objections against the hypothesis that the square of Dirac determinant gives the modulus squared for the vacuum functional.

1. It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

2. Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??**, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

Quantum TGD as square root of thermodynamics

Zero energy ontology (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A92] [K99]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A92], [K99] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the modified Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes. Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K27]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. The inclusions of these conformal algebras would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

Summarizing

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD and that different measurement interactions having interpretation in terms of different kind of quantum measurements causing wave function collapse in zero mode sector of WCW could correspond to various choices of ω. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

7.3.5 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in zero energy ontology

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

   The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

3. Zero energy ontology gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish zero energy ontology allows a symmetry breaking respecting a chosen Cartan algebra.

4. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFS as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \quad (7.3.5)$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \quad (7.3.6)$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^\dagger)|r_2\rangle \ . \quad (7.3.7)$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^\dagger) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \quad (7.3.8)$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In zero energy ontology the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A15] .

5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A15] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{7.3.9}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \quad (7.3.10)$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the modified Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain modified gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A41] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor

takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

7.3.6 Questions about quantum measurement theory in zero energy ontology

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+M^\dagger P_-M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

7.3.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [K20] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^nT_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^kT_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

7.3.8 Planar algebras and generalized Feynman diagrams

Planar algebras [A21] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A45] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K15] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

   [Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

7.3.9 Miscellaneous

The following considerations are somewhat out-of-date: hence the title 'Miscellaneous'.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A85] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A123] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A52] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_\mathcal{N} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K20] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A125] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic

size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K97] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K24] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A125] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

7.4 QFT limit of TGD

The understanding of the QFT limit of TGD has been one of the long-longstanding challenges in TGD. The considerations inspired by twistor approach to QFT led to the idea of bosonic emergence meaning that Dirac action coupled to gauge bosons and other particles could define YM part of the action as radiative corrections. This approach predicts the coupling constant evolution uniquely provided one finds a principle fixing the mass cutoff and hyperbolic cutoff. Zero energy ontology motivates the cutoffs and also leads to a set of conditions giving hopes of fixing hyperbolic cutoff uniquely as a function of the p-adic mass scale. The requirement is that bosonic $N > 2$-vertices defined by fermionic loops vanish for on mass-shell bosons by the defining property of vertex meaning that it does not represent scattering amplitude for on mass shell particles. These condition generalize also to the massive case and even to quantum TGD proper.

7.4.1 Bosonic emergence and QFT limit of TGD

In TGD framework S-matrix must be constructed without the help of path integral. In TGD only fermions appear as fundamental particles. This suggests a bootstrap program in which one starts from Dirac action for fermions with couplings to gauge potentials and generates the remaining n-point functions for bosons as radiative corrections for fermionic action with effective action. The success of twistorial unitary cut method in massless gauge theories suggests that its basic results such as recursive generation of tree diagrams might be given a status of axioms. Also massive particles should be treated in practical approach and this could be achieved by generalizing the twistors to 8-D twistors.

1. In [K17, K20] I have discussed how both field theoretic and stringy variants of the fermion propagator could arise via radiative self energy insertions described by a fundamental 2-vertex giving a contribution proportional to $p^k\gamma_k$ and leading a propagator containing the counterpart as a mass term expressed in terms of CP_2 gamma matrices so that massive particles can have fixed $M^4 \times CP_2$ chirality.

2. In TGD bosons are identified as bound states of fermion and anti-fermion at opposite wormhole throats so that bosonic n-vertex would correspond to the decay of bosons to fermion pairs in the loop. Purely bosonic gauge boson couplings would be generated radiatively from triangle and box diagrams involving only fermion-boson couplings. Also bosonic propagator would be generated as a self-energy loop: bosons would propagate by decaying to fermion-anti-fermion pair and then fusing back to the boson. Gauge theory dynamics would be emergent and bosonic couplings would have form factors with IR and UV behaviors allowing finiteness of the loops constructed from them.

3. The problem of this approach are UV divergences present unless one introduces cutoff in mass squared and hyperbolic angle. This kind of cutoffs are natural in zero energy ontology and would state that the radiative corrections for given causal diamond (CD) correspond to CDs in shorter scales and contained with the CD. p-Adic length scale hypothesis and the fractal structure of CDs suggest that mass scales come as half octaves fundamental scale. CP_2 mass scale defines a natural upper cutoff for mass scale and hyperbolic cutoff is expected to depend on the p-adic mass scale.

The considerations of [K64] lead to the conclusion that bosonic propagators could emerge from fermionic ones in the quantum field theory type description. This approach predicts all gauge couplings and assuming a geometrically very natural hyperbolic UV cutoff motivated by zero energy ontology one can understand the evolution of standard model gauge couplings and reproduce correctly the values of fine structure constant at electron and intermediate boson length scales. Also asymptotic freedom follows as a basic prediction. The UV cutoff for the hyperbolic angle as a function of p-adic length scale is the ad hoc element of the model in its recent form, and a quantitative model for how this function could be fixed by quantum criticality is formulated and studied.

These considerations and numerical calculations lead to a general vision about how real and p-adic variants of TGD relate to each other and how p-adic fractalization takes place. As in case of twistorialization Cutkosky rules allowing unitarization of the tree amplitudes in terms of $TT^\dagger$ contribution involving only light-like momenta seems to be the only working option and requires that $TT^\dagger$ makes sense p-adically. The vanishing of the fermionic loops defining bosonic vertices for the incoming massless momenta emerges as a consistency condition suggested also by quantum criticality and by the fact that only BFF vertex is fundamental vertex if bosonic emergence is accepted. The vanishing of on mass shell $N > 3$ bosonic vertices gives an infinite number of conditions on the hyperbolic cutoff as function of the integer k labeling p-adic length scale at the limit when bosons are massless and IR cutoff for the loop mass scale is taken to zero. It is not yet clear whether dynamical symmetries, in particular super-conformal symmetries, are involved with the realization of the vanishing conditions or whether hyperbolic cutoff is all that is needed.

It must be confessed that the original formulation of bosonic emergence was rather primitive and ad hoc looking. What was amazing that twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY led to an analogous picture: massless on mass shell bosons appearing in twistor diagrams however had complex momenta in this approach: for real on-mass-shell momenta all bosonic momenta would be parallel. Later TGD has led to a stringy variant of twistor approach in which fundamental fermions appearing in virtual lines are massless and on-mass shell but have non-physical polarizations.

7.4.2 Twistors and QFT limit of TGD

Twistors - a notion discovered by Penrose [B56] - have provided a fresh approach to the construction of perturbative scattering amplitudes in Yang-Mills theories and in $N = 4$ supersymmetric Yang-Mills theory. This approach was pioneered by Witten [B73] . The latest step in the progress was the proposal by Nima Arkani-Hamed and collaborators [B25] that super Yang Mills and super gravity amplitudes might be formulated in 8-D twistor space possessing real metric signature $(4, 4)$. The questions considered below are following.

1. Could twistor space could provide a natural realization of $N = 4$ super-conformal theory requiring critical dimension $D = 8$ and signature metric $(4, 4)$? Could string like objects in TGD sense be understood as strings in twistor space? More concretely, could one in some

sense lift quantum TGD from $M^4 \times CP_2$ to 8-D twistor space T so that one would have three equivalent descriptions of quantum TGD.

2. Could one construct the preferred extremals of Kähler action in terms of twistors -may be by mimicking the construction of hyper-quaternionic *resp.* co-hyper-quaternionic surfaces in M^8 as surfaces having hyper-quaternionic tangent space *resp.* normal space at each point with the additional property that one can assign to each point x a plane $M^2(x) \subset M^4$ as sub-space or as sub-space defined by light-like tangent vector in M^4. Could one mimic this construction by assigning to each point of X^4 regarded as a 4-surface in T a 4-D plane of twistor space satisfying some conditions making possible the interpretation as a tangent plane and guaranteeing the existence of a map of X^4 to a surface in $M^4 \times CP_2$. Could twistor formalism help to resolve the integrability conditions involved?

3. Could one modify the notion of Feynman diagram by allowing only massless loop momenta so that twistor formalism could be used in elegant manner to calculate loop integrals and whether the resulting amplitudes are finite in TGD framework where only fermions are elementary particles? Could one modify Feynman diagrams to twistor diagrams by replacing momentum eigenstates with light ray momentum eigenstates completely localized in transversal degrees of freedom?

The arguments of [K98] suggest some these questions might have affirmative answers.

Twistors and classical TGD

Consider first the twistorialization at the classical space-time level.

1. One can assign twistors to only 4-D Minkowski space (also to other than Lorentzian signature). One of the challenges of the twistor program is how to define twistors in the case of a general curved space-time. In TGD framework the structure of the imbedding space allows to circumvent this problem.

2. The lifting of classical TGD to twistor space level is a natural idea. Consider space-time surfaces representable as graphs of maps $M^4 \rightarrow CP_2$. At classical level the Hamilton-Jacobi structure [K9] required by the number theoretic compactification means dual slicings of the M^4 projection of the space-time surface X^4 by stringy word sheets and partonic two-surfaces. Stringy slicing allows to assign to each point of the projection of X^4 two light-like tangent vectors U and V parallel to light-like Hamilton-Jacobi coordinate curves. These vectors define components $\tilde{\mu}$ and λ of a projective twistor, and twistor equation assigns to this pair a point m of M^4. The conjecture is that for preferred extremals of Kähler action this point corresponds to the M^4 projection of the point in the natural M^4 coordinates associated with the upper or lower tip of causal diamond CD. If this conjecture is correct one can lift the M^4 projection of the space-time surface in $CD \times CP_2 \subset M^4 \times CP_2$ to a surface in $PT \times CP_2$, where CP_3 is projective twistor space $PT = CP_3$. Also induced spinor fields and induced gauge fields can be lifted to twistor space.

3. If one can fix the scales of the tangent vectors U and V and fix the phase of spinor λ one can consider also the lifting to 8-D twistor space T rather than 6-D projective twistor space PT. Kind of symmetry breaking would be in question. The proposal for how to achieve this relies on the notion of finite measurement resolution. The scale of V at partonic 2-surface $X^2 \subset \delta CD \times X^3_l$ would naturally correlate with the energy of the massless particle assignable to the light-like curve beginning from that point and thus fix the scale of V coordinate. Symplectic triangulation discussed in [K15] in turn allows to assign a phase factor to each strand of the number theoretic braid as the Kähler magnetic flux associated with the triangle having the point at its center. This allows to lift the stringy world sheets associated with number theoretic braids to their twistor variants but not the entire space-time surface. String model in twistor space is obtained in accordance with the fact that $N = 4$ super-conformal invariance is realized as a string model in a target space with $(4, 4)$ signature of metric. Note however that CP_2 defines additional degrees of freedom for the target space so that 12-D space is actually in question.

4. One can consider also a more general problem of identifying the counterparts for the preferred extremals of Kähler action with arbitrary dimensions of M^4 and CP_2 projections in 10-D space $PT \times CP2$. The key idea is the reduction of field equations to holomorphy as in Penrose's twistor representation of solutions of positive and negative frequency parts of free fields in M^4. A very helpful observation is that CP_2 as a sub-manifold of PT corresponds to the 2-D space of null rays of the complexified Minkowski space M^4_c. For the 5-D space $N \subset PT$ of null twistors this 2-D space contains 1-dimensional light ray in M^4 so that N parametrizes the light-rays of M^4. The idea is to consider holomorphic surfaces in $PT_\pm \times CP_2$ ($\pm$ correlates with positive and negative energy parts of zero energy state) having dimensions $D = 6, 8, 10$; restrict them to $N \times CP_2$, select a sub-manifold of light-rays from N, and select from each light-ray subset of points which can be discrete or portion of the light-ray in order to get a 4-D space-time surface. If integrability conditions for the resulting distribution of light-like vectors U and V can be satisfied (in other words they are gradients), a good candidate for a preferred extremal of Kähler action is obtained. Note that this construction raises light-rays to a role of fundamental geometric object.

Twistors and Feynman diagrams

The recent successes of twistor concept in the understanding of 4-D gauge theories and $N = 4$ SYM motivate the question of how twistorialization could help to understand construction of M-matrix in terms of Feynman diagrammatics or its generalization.

1. One of the basic problems of twistor program is how to treat massive particles. Massive four-momentum can be described in terms of two twistors but their choice is uniquely only modulo $SO(3)$ rotation. This is ugly and one can consider several cures to the situation.

   (a) Number theoretic compactification and hierarchy of Planck constants leading to a generalization of the notion of imbedding space assign to each sector of WCW defined by a particular CD a unique plane $M^2 \subset M^4$ defining quantization axes. The line connecting the tips of the CD selects also unique rest frame (time axis). The representation of a light-like four-momentum as a sum of four-momentum in this plane and second light-like momentum is unique and same is true for the spinors λ apart from the phase factors (the spinor associated with M^2 corresponds to spin up or spin down eigen state).

   (b) The tangent vectors of braid strands define light-like vectors in H and their M^4 projection is time-like vector allowing a representation as a combination of U and V. Could also massive momenta be represented as unique combinations of U and V?

   (c) One can consider also the possibility to represent massive particles as bound states of massless particles.

   It will be found that one can lift ordinary Feynman diagrams to spinor diagrams and integrations over loop momenta correspond to integrations over the spinors characterizing the momentum.

2. One assign to ordinary momentum eigen states spinor λ but it is not clear how to identify the spinor $\tilde{\mu}$ needed for a twistor.

   (a) Could one assign $\tilde{\mu}$ to spin polarization or perhaps to the spinor defined by the light-like M^2 part of the massive momentum? Or could λ and $\tilde{\mu}$ correspond to the vectors proportional to V and U needed to represent massive momentum?

   (b) Or is something more profound needed? The notion of light-ray is central for the proposed construction of preferred extremals. Should momentum eigen states be replaced with light ray momentum eigen states with a complete localization in degrees of freedom transversal to light-like momentum? This concept is favored both by the notion of number theoretic braid and by the massless extremals (MEs) representing "topological light rays" as analogs of laser beams and serving as space-time correlates for photons represented as wormhole contacts connecting two parallel MEs. The transversal position of the light ray would bring in $\tilde{\mu}$. This would require a modification of the perturbation theory and the introduction of the ray analog of Feynman propagator. This generalization would be M^4 counterpart for the highly successful twistor diagrammatics relying on twistor Fourier transform but making sense only for the (2,2) signature of Minkowski space.

3. In perturbation theory one can also consider the crazy idea of restricting the loop momenta to light-like momenta so that the auxiliary M^2 twistors would not be needed at all. This idea failed but led to a first precise proposal for how Feynman diagrammatics producing unitarity and UV finite S-matrix could emerge from TGD, where only fermions are elementary particles and all coupling constants are in principle predictions of the theory. Emergence would mean that the fundamental action is just the Dirac action with gauge boson couplings and containing no bosonic kinetic term, that the perturbative functional integral over the fermion fields in the construction of the effective action induces bosonic kinetic term radiatively, and that a further perturbative functional integral over the gauge boson fields gives an effective action in which all bosonic n-point functions have emerged from the fermionic dynamics. Physically this would mean that bosons interact only when the wormhole contact representing boson and carrying fermion and anti-fermion quantum numbers at the opposite light-like wormhole throats decays to a pair of fermion and anti-fermion represented by CP_2 type extremals with single wormhole throat only. Even fermionic propagators would emerge radiatively from the modified Dirac operator in more fundamental description [K20] . What is remarkable is that p-adic length scale hypothesis and the notion of finite measurement resolution lead to a precise proposal how UV divergences are tamed in a description taking into account the finite measurement resolution. The model of QFT limit based on these is discussed in separate chapter [K64] since the idea itself only marginally relates to twistors.

7.4.3 Stringy variant of twistor Grassmannian approach

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in elegant manner. One can imagin two manners to circumvent this problem. The first one is modification of the notion of massless to masslessness in 8-D sense. One can indeed imagine an 8-D generalization of the twistor approach of Penrose based on the notion of octonionic spinor [K117]. The status of octonionic spinors remains uncertain.

One can also consider a stringy variant of twistor Grassmannian approach [K78] in which fundamental fermions (as opposed to elementary fermions) are massless. Since this approach looks more promising it is briefly summarized below.

1. The approach is motivated by the stringy picture of elementary particles forced by the well-definedness of em charge for the modes of induced spinor field, and the assumption that elementary particles can be seen as bound states of massless fermions associated with the orbits of string ends at light-like orbits of partonic 2-surfaces. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

   This brings the desired purely physical IR cutoff expected to cancel IR divergences. The fermions are massless and on-shell, and one assigns the inverse of massless propagator to the line which corresponds to non-physical helicity. This picture follows from Feynman graph approach if one can perform residue integral over virtual fermion momenta.

2. In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that only the Chern-Simons terms associated with space-like ends of the space-time surface remain. These terms reduce to Chern-Simons terms only if one poses weak form of electric magnetic duality also here. This is not necessary.

3. The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse. Thermodynamics would naturally couple to the space-time geometry via the thermodynamical or quantum averages of the quantum numbers.

4. The basic vertex is essentially four-fermion vertex although two light-like momenta combine to form virtual bosonic wormhole contact with total four-momentum which can be also space-like. BCFW recursion formula is expected to hold for the diagrams when one interprets these lines as virtual bosons. This picture could be seen as reduction of bosonic lines fermion lines and replacement of point like elementary particles with stringy structures formed by pairs wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book).

5. Contrary to the original over-optimistic assumptions the logarithmic UV divergences do not cancel unless on assumes stringy picture. This means that one assigns to the ends of the fermion line the analog of super-conformal propagator and its Hermitian conjugate. The analog of super-conformal propagator is defined by the inverse G/L_0 of super-generator $G^\dagger$. This assignment allows to circumvent the problem due to the fact that G carries fermion number in TGD framework.

6. What is of special interest is that M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. For CP_2 the twistor space has interpretation as the space $SU(3)/U(1) \times U(1)$ for the choices of quantization axes for color quantum numbers. This kind of twistor space can be assigned even with WCW but it is not clear what the physical interpretation and mathematical role of this twistor space is.

7.4.4 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K18] . This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization

of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K20] .

3. The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

4. If Kähler action defines the modified Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

5. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n>0$ creates fermions whereas G_n, $n<0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{7.4.1}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n<0$ and $G_n^\dagger$, $n>0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K18] . Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K105].

   One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

3. The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M^4_\pm \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

4. An alternative identification for TGD parts of conformal fields is inspired by M^8-H duality. Conformal fields would be fields in WCW. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW

Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

7.5 How to define generalized Feynman diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in zero energy ontology (ZEO) [K74] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K98] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. Zero energy ontology (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K98] . Modified gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K17]. It took long time to realize that in zero energy ontology the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

   One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K28, K98] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B22] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of modified Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K40] in infinite-dimensional context already in the case of much simpler loop spaces [A71] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿ defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

7.5.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the modified Dirac action [K17] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F+n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F+n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

   In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. p^k is discretized by periodic boundary conditions at opposite boundaries of CD and has IR and UV cutoffs due to the finite size of CD and finite lower limit for the size of sub-CDs.

   One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

   By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that Kähler action reduces to the terms associated with space-like ends of the space-time surface. These terms reduce to Chern-Simons terms if one poses weak form of electric magnetic duality also here. The boundary condition for Kähler-Dirac equations states $\Gamma^n\Psi = 0$ so that incoming fundamental fermions are massless and there is a strong temptation to pose the additional condition $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$

   The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case

would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the modified Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub - CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for zero energy ontology.

4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

7.5.2 Generalized Feynman diagrams at fermionic and momentum space level

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. Zero energy ontology encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_{\pm}$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_{\pm}$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the modified Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^{\alpha}p_{\alpha} + \hat{\Gamma}^{\alpha}D_{\alpha} \ , \\ p_{\alpha} &= p_k\partial_{\alpha}h^k \ . \end{aligned} \tag{7.5.1}$$

   The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is modified gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D modified Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [K29] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N - 4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B7] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and modified Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [K29] .

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K24] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

7.5.3 Harmonic analysis in WCW as a manner to calculate WCW functional integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator

a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the modified Dirac operator D at propagator lines [K17] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \tag{7.5.2}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{7.5.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the modified Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \tag{7.5.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K18, K17]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \quad (7.5.5)$$

   works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

   The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [K20] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+,s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1,s^2)}{\partial(x^1_\pm,x^2_\pm)} d^2x_\pm \ . \quad (7.5.6)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+,s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J^-_{kl} \ , \\ J^{kl}_\pm &= (1+K_\pm)\partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_\pm \ . \end{aligned} \quad (7.5.7)$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A),Q(H_B)\} = Q(\{H_A,H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1,s^2)/\partial(x^1_\pm,x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

3. In zero energy ontology this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the modified Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

7.6 General vision about real and p-adic coupling constant evolution

The unification of super-symplectic and Super Kac-Moody symmetries allows new view about p-adic aspects of the theory forcing a considerable modification and refinement of the almost decade old first picture about color coupling constant evolution.

Perhaps the most important questions about coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

7.6.1 A general view about coupling constant evolution

Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events.

Einstein's equations, Equivalence Principle, and GRT and QFT limits of TGD

Coupling constant evolution makes sense in quantum field theory defined in fixed background space-time, say Minkowski space-time. In TGD framework imbedding space replaces this fixed space-time and in ZEO the hierarchy of causal diamonds replaces imbedding space. It is not at all clear whether at the level of basic TGD coupling constant evolution makes sense at all whereas it should make sense at QFT limit of TGD. This requires understanding of QFT and GRT limits of TGD including also Equivalence Principle.

At quantum level Equivalence Principle (EP) can be reduced to quantum classical correspondence: the conserved four-momentum associated with Kähler action equals to the eigenvalue of conserved quantal four-momentum assignable to Kähler-Dirac equation [K105]. This quantal four-momentum in turn can be associated with string world sheets which emerge naturally from Kähler-Doirac equation.

Einstein's equation give a purely local meaning for EP. How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book). .

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Similar description applies to induced electroweak gauge potentials and color gauge potentials: the sum of these gauge potentials over space-time sheets should define the classical gauge fields of QFT limit of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

What coupling constant evolution could mean in TGD framework? Kähler action and Kähler-Dirac action do not contain any fundamental couplings affecting to the dynamics. Kähler coupling strength does not affect classical dynamics and is analogous to critical temperature, and therefore invariant under renormalization group if defined in TGD framework. This suggests that the analog of renormalization group equations at space-time level does not look feasible. Continuous coupling constant evolution might be useful notion only at the QFT limit.

The natural length scale hierarchy associated with coupling constant evolution would be the hierarchy of length scales assignable to CDs. The minimal sizes of CDs assumed to be equal to secondary p-adic length scales in the case of elementary particles. More generally, number theoretical arguments suggest that the scales of CDs come as integer multiples of CP_2 radius. What is new that coupling constant evolution would be discretized, being labelled by integers. Primes and primes near powers of 2 could correspond to physically favored minimal size scales for CDs: kind of survivors in fight for survival. Discrete coupling constant evolution as evolution of various M-matrix elements as function of the size-scale of CD would look like a reasonable TGD counterpart of coupling constant evolution. For single CD one might say that system is quantum critical, and coupling constants do not evolve.

Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K20] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A63] known as hyperfinite factor of type II_1 (HFF) [K20, K99, K27] . HFF [A58, A84] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [A3] . The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A119] , anyons [D25] , quantum groups and conformal field theories [A85] , and knots and topological quantum field theories [A109, A125] .

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K20] . The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. The original hope was that Connes tensor product realizing mathematical the finite measurement resolution could fix M-matrix to high degree turned out be too optimistic.

How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale ($T_n = 2^nT_0$) implies in a natural manner coupling constant evolution. One must however emphasize that also the weaker condition $T_p = pT_0$, p prime, is possible, and would assign all p-adic time scales to the size scale hierarchy of CDs.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^nT_0$

(the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^kT_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have secondary Compton length Electron's secondary Compton time $T_e(127) = \sqrt{5}T_2(127) = .1$ seconds defines a fundamental biological rhythm. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

7.6.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

Symplectic invariance

Symplectic symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K21] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K62] . In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k \Phi_l = c_{kl}^m \Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s) d\mu_s \quad . \tag{7.6.1}$$

   Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1,s_2,s)) I d d\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1,s_2,s)) d\mu_s \quad . \tag{7.6.2}$$

   Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{7.6.3}$$

   of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle (\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c^r_{kl}\int f(\Delta A(s_1, s_2, s))\langle \Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \tag{7.6.4}$$
$$c^r_{kl}c_{rm}\int f(\Delta A(s_1, s_2, s))f(\Delta A(s, s_3, t))d\mu_s d\mu_t \ . \tag{7.6.5}$$

   Associativity requires that this expression equals to $\langle \Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1, s_2, s))d\mu_s \tag{7.6.6}$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

Generalization to quantum TGD

(Number theoretic) braids are identifiable as boundaries of string world sheets at which the modes of induced spinor fields are localized in the generic case in Minkowskian space-time regions. Fundamental fermions can be assigned to these lines. Braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2\times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli

space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [K20] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K88] .

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus

one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities could follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

UV finiteness is suggested also by the generalized Feynman rules providing a phenomenological view about what TGD predicts. According to these rules fundamental fermions propagate like massless particles. In twistor Grassmann approach residue integration is expected to reduce internal fermion lines to on mass shell propagation with non-physical helicity. The fundamental 4-fermion interaction is assignable to wormhole contact and corresponds to stringy exchange of four-momentum with propagator being defined by the inverse of super-conformal scaling generator $1/L_0$. Wormhole contacts carrying fermion and antifermion at their throats behave like fundamental bosons. Stringy propagators at wormhole contacts make TGD rules a hybrid of Feynmann and stringy rules. Stringy propagators are necessary in order to avoid logarithmic divergences. Higher mass excitations crucial for finiteness belong to the representations of super-conformal algebra and can be regarded as bound states of massless fermions. Massivation of external particles allows to avoid infrared divergences. Not only physical bosons but also physical fermions emerge from fundamental massless fermions.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-symplectic conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [K21] .

1. The state construction utilizes both super-symplectic and super Kac-Moody algebras. super-symplectic algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-symplectic algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-symplectic operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic

4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-symplectic super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the WCW super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to WCW gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since ordinary fermionic propagator and boson-emission vertices at the ends of the line containing WCW gamma matrix and its conjugate give compensating fermion numbers [K78].

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would correspond evolution as a function of the scale of CD. It might have interpretation also in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram as the size of CD is increased. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

7.7 The recent view about p-adic coupling constant evolution

One of the basic problems of quantum TGD is the understanding of p-adic coupling constant evolution.

1. Since neither classical field equations for Kähler action nor Kähler-Dirac action depend on coupling constants, one expects that at the level of TGD space-time the notion of coupling constant evolution is not well-defined or at least fails to be a fundamental notion. Coupling constant evolution would characterize GRT and QFT limits of TGD and since causal diamond (CD) is the basic unit, the scale of CD would serve as a fundamental scale. What would give rise to coupling constant evolution at long length scales, would be the replacement of many-sheeted space-time with GRT space-time containing gauge potentials which are sums of induced gauge potentials associated with various space-time sheets. The increase in the size of CD would correspond to the that in the size of the space-time sheet.

2. This evolution is discrete by p-adic length scale hypothesis justified by zero energy ontology, where CD sizes are assumed to come as integer multiples of CP_2 mass: the discretization is for number theoretical reasons and gives hopes of number theoretical universality. The most general option is that the CD sizes come as rational multiples of CP_2 size. Discreteness means that continuous mass scale is replaced by mass scales coming as half octaves of CP_2 mass. Kähler coupling strength α_K or gravitational coupling constant is assumed to remain invariant under p-adic coupling constant evolution. The basic problem is to understand the value of α_K and here p-adic mass calculations give strong constraints.

3. An attractive hypothesis is that Dirac determinant reduces to the vacuum functional identifiable as exponent of Kähler action S_K for a preferred extremal came first. The contribution

from Euclidian regions corresponds to Kähler function and that from Minkowskian regions serves as analog of Minkowskian action defining Morse function at the level of WCW.

There is no hope of reducing Kähler action to Dirac action since Kähler action and Kähler-Dirac action are in completely democratic position since Kähler-Dirac gamma matrices are defined in terms of the canonical momentumd densities for Kähler action. The value of Kähler coupling strength is however expected to follow from the condition that Dirac determinant equals to vacuum functional.

4. The realization that well-definedness of em charge requires the localization of the modes of induced spinor field to string world sheets or partonic 2-surfaces was an important step in process trying to make the notion of Dirac determinant more concrete [K105]. Dirac determinants reduce to those assignable to string world sheets and possibly also partonic 2-surfaces and would naturally correspond to square roots of determinants defined by the products of the eigenvalues of the mass squared operator for incoming on mass shell states and given by stringy mass formula. Zeta function regularization should allow to defined these determinants and one can hope that it reduces to the exponent of Kähler action for preferred extremal. Thus coupling constant evolution might allow a reduction to string model type description.

5. If weak form of electric magnetic duality and by $j \cdot A = 0$ condition for Kähler current and gauge potential in the interior of space-time sheets are satisfied, Kähler action reduces to Chern-Simons terms at light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface. Induced metric would apparently disappear from the action in accordance with the idea about TGD as almost topological QFT.

6. The boundary conditions for Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits can be expressed as generalized eigenvalue equation $D_{C-S})\Psi = p^k\gamma_k\Psi$ with p^k interpreted as virtual momentum of the fermion propagating along the boundary of string world sheets at which it is localized by the well-definedness of em charge. C-S-D term gives rise to massless fermion propagation. Also the external fermions are massless unless there are measurement interaction terms defined as Lagrangian multiplier terms forcing classical charges in Cartan algebra to be equal to their quantum counterparts for the space-time surfaces in the quantum superposition. This implies localization in WCW analogous to state function collapse and by quantum classical correspondence could accompany state function reduction. It would be very natural if this localization would happen in zero modes so that classical charges parametrize zero modes quantum charges correspond to wave function in quantum fluctuating modes.

The dream would be to have a formula for Kähler coupling strength in terms of a calculable and manifestly finite Dirac determinant without any need for zeta function regularization. In principle the formula would fix completely the number theoretic anatomy of Kähler coupling strength and of other gauge coupling strengths. When the formula for the gravitational constant involving Kähler coupling strength and the exponent of Kähler action for CP_2 type vacuum extremal - which remains still a conjecture - is combined with the number theoretical results and with the constraints from the predictions of p-adic mass calculations, one ends up to an identification of Kähler coupling strength as fine structure constant at electron length scale characterized by p-adic prime M_{127}. Also the number theoretic anatomy of the ratio $R^2/\hbar G$, where R is CP_2 size, can be deduce to high degree and a relationship between the p-adic evolutions of electromagnetic and color coupling strengths emerges.

In the following a general view about Dirac determinant is discussed assuming spinor modes are localizeed at string world sheets. Also some old speculations about p-adic coupling constant evolution are discussed. These formulas are guesses motivated by simple arguments: the reader can decide whether to take them seriously or not.

7.7.1 Dirac determinant assuming that spinor modes are localized to string world sheets

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

1. Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

2. Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

3. The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would

be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

4. The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

5. Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu(dx^\mu/ds)ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

6. The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

   The formula would be

$$K \quad \propto \quad \oint J(x,y) J_T dx^1 \wedge dx^2 \quad . \tag{7.7.1}$$

   This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

7.7.2 A revised view about coupling constant evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

1. The original and also recent hypothesis is that Kähler coupling strength is invariant under p-adic coupling constant evolution. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime M_{127}. Later I replaced fine structure constant with electro-weak U(1) coupling strength at this length scale.

2. The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [K17] . What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the modified Dirac operator. This formula dictates the number theoretical anatomy of g_K^2 and also of other coupling constants: the most general option is that α_K is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.

3. A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of $\sqrt{2}$. This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether $p \simeq 2^k$ should be replaced with 2^k in all formulas as the recent view about quantum TGD suggests.

4. The prediction is that Kähler coupling strength α_K is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared

and corresponds to the largest Mersenne prime (M_{127}), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter R^2/G p-adicization program allows to consider two options: either this constant is of form e^q or 2^q: in both cases q is rational number. $R^2/G = exp(q)$ allows only M_{127} gravitons if number theory is taken completely seriously. $R^2/G = 2^q$ allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.

5. A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(2S_K(CP_2))$ defining the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} \ . \tag{7.7.2}$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \tag{7.7.3}$$

$a < 1$ conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127}-1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

1. The formula for the gravitational constant

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r\hbar_0 G = L_p^2 \times exp(-2aS_K(CP_2)) \ , \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{7.7.4}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. What was noticed before is that this relationship allows even constant value of G if a has appropriate dependence on p.

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

1. I assumed that modulus squared for vacuum functional is in question: hence the factor $2a$ in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement $2a \rightarrow a$ is necessary.

2. Second wrong assumption was that graviton corresponds to CP_2 type vacuum extremal- that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by CP_2 vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor $2a$ in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to $exp(-aS_K(CP_2))$.

The basic constraint to the coupling constant evolution comes for the invariance of g_K^2 in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p,r)\pi^2}{log(pK)} \ , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r}\frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} \ . \end{aligned} \tag{7.7.5}$$

2. How to guarantee that g_K^2 is RG invariant and N:th root of rational?

Suppose that g_K^2 is N:th root of rational number and invariant under p-adic coupling constant evolution.

1. The most general manner to guarantee the expressibility of g_K^2 as N:th root of rational is guaranteed for both options by the condition

$$a(p,r) = \frac{g_K^2}{\pi^2}log(\frac{pK_0}{r}) \ . \tag{7.7.6}$$

   That a would depend logarithmically on p and $r = \hbar/\hbar_0$ looks rather natural. Even the invariance of G under p-adic coupling constant evolution can be considered.

2. The condition

$$\frac{r}{p} < K_0(p) \ . \tag{7.7.7}$$

   must hold true to guarantee the condition $a > 0$. Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition $a < 1$ is guaranteed by the condition

$$\frac{r}{p} > exp(-\frac{\pi^2}{g_K^2}) \times K_0(p) \ . \tag{7.7.8}$$

   The condition implies that for very large values of p the value of Planck constant must be larger than $\hbar_0$.

3. The two conditions are summarized by the formula

$$K_0(p) \times exp(-\frac{\pi^2}{g_K^2}) < \frac{r}{p} < K_0(p) \tag{7.7.9}$$

   characterizing the allowed interval for r/p. If G does not depend on p, the minimum value for r/p is constant. The factor $exp(-\frac{\pi^2}{g_K^2})$ equals to 1.8×10^{-47} for $\alpha_K = \alpha_{em}$ so that $r > 1$

is required for $p \geq 4.2\times 10^{-40}$. $M_{127}\sim 10^{38}$ is near the upper bound for p allowing $r=1$. The constraint on r would be roughly $r\geq 2^{k-131}$ and $p\simeq 2^{131}$ is the first p-adic prime for which $\hbar > 1$ is necessarily. The corresponding p-adic length scale is .1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for r behaves roughly as $r < 2.3\times 10^7 p$. This condition becomes relevant for gravitational Planck constant GM_1M_2/v_0 having gigantic values. For Earth-Sun system and for $v_0 = 2^{-11}$ the condition gives the rough estimate $p > 6\times 10^{63}$. The corresponding p-adic length scale would be of around $L(215)\sim 40$ meters.

4. p-Adic mass calculations predict the mass of electron as $m_e^2 = (5+Y_e)2^{-127}/R^2$ where $Y_e\in[0,1)$ parameterizes the not completely known second order contribution. Top quark mass favors a small value of Y_e (the original experimental estimates for m_t were above the range allowed by TGD but the recent estimates are consistent with small value Y_e [K57]). The range $[0,1)$ for Y_e restricts $K_0 = R^2/\hbar_0 G$ to the range $[2.3683, 2.5262]\times 10^7$.

5. The best value for the inverse of the fine structure constant is $1/\alpha_{em} = 137.035999070(98)$ and would correspond to $1/g_K^2 = 10.9050$ and to the range $(0.9757, 0.9763)$ for a for $\hbar=\hbar_0$ and $p = M_{127}$. Hence one can seriously consider the possibility that $\alpha_K = \alpha_{em}(M_{127}$ holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that α_K corresponds to electro-weak $U(1)$ coupling strength in this length scale. The fact that M_{127} defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, the recent view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that g_K^2 is root of rational number, possibly even rational, and can be assumed to be equal to e^2. Also $R^2/\hbar G$ could be rational. The new element is that G need not be proportional to p and can be even invariant under coupling constant evolution since the the parameter a can depend on both p and r. An unexpected constraint relating p and r for space-time sheets mediating gravitation emerges.

Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. One can consider two options.

1. The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings would approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

   Thus one would have

$$\frac{1}{\alpha_{U(1)}}+\frac{1}{\alpha_s} = \frac{1}{\alpha_K} \ . \tag{7.7.10}$$

   The relationship between $U(1)$ and em coupling strengths is

$$\begin{aligned}
\alpha_{U(1)} &= \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\
sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\
\alpha_{em}(M_{127}) &= 0.00729735253327 \ .
\end{aligned} \quad (7.7.11)$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [E35] is used. Note however that the previous argument implying $\alpha_K = \alpha_{em}(M_{127})$ excludes $\alpha = \alpha_{U(1)}(M_{127})$ option.

2. Second option is obtained by replacing $U(1)$ with electromagnetic gauge $U(1)_{em}$.

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \quad (7.7.12)$$

Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

1. In TGD framework coupling constant is discrete and comes as powers of $\sqrt{2}$ corresponding to p-adic primes $p \simeq 2^k$. Number theoretic considerations suggest that coupling constants g_i^2 are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have $g_i^2 = g_i^2(k)$. g_K^2 is predicted to be N:th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if $sin(\theta_W)$ and $cos(\theta_W)$ are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas $sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$ and $cos(\theta_W) = 2rs(r^2 + s^2)$.

2. A very strong prediction is that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.

3. $\alpha(M_{127}) = \alpha_K$ implies that M_{127} defines the confinement length scale in which the sign of α_s becomes negative. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [K84, L6] , [L6] . Hence one can argue that color coupling strength indeed diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha(M_{127})$. Therefore the precise knowledge of $\alpha(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron.

4. $\alpha_s(M_{89})$ is predicted to be $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$. $sin^2(\theta_W) = .23120$, $\alpha_{em}(M_{89}) \simeq 1/127$, and $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ give $1/\alpha_{U(1)}(M_{89}) = 97.6374$. $\alpha = \alpha_{em}$ option gives $1/\alpha_s(M_{89}) \simeq 10$, which is consistent with experimental facts. $\alpha = \alpha_{U(1)}$ option gives $\alpha_s(M_{89}) = 0.1572$, which is larger than QCD value. Hence $\alpha = \alpha_{em}$ option is favored.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

1. The natural guess for the general formula would be as

$$g^2(p,r) = kg_K^2 \times exp[-a_g(p,r) \times S_K(CP_2)] \ . \tag{7.7.13}$$

here k is a numerical constant.

2. The condition
$g_K^2 = e^2(M_{127})$ fixes the value of k if it's value does not depend on the character of gauge interaction:

$$k = exp[a_{gr}(M_{127}, r=1) \times S_K(CP_2)] \ . \tag{7.7.14}$$

Hence the general formula reads as

$$g^2(p,r) = g_K^2 \times exp[(-a_g(p,r) + a_{gr}(M_{127}), r=1)) \times S_K(CP_2)] \ . \tag{7.7.15}$$

The value of $a(M_{127}, r=1)$ is near to its maximum value so that the exponential factor tends to increase the value of g^2 from e^2. The formula can reproduce α_s and various electro-weak couplings although it is quite possibile that Weinberg angle corresponds to a group theoretic factor not representable in terms of $a_g(p,r)$. The volume of the CP_2 type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

3. α_{em} in very long length scales would correspond to

$$e^2(p \to \infty, r=1) = e^2 \times exp[(-1 + a(M_{127}), r=1)) \times S_K(CP_2)] = e^2 x \ , \tag{7.7.16}$$

where x is in the range $[0.6549, 0.6609]$.

Formula relating v_0 to α_K and R^2/G

The parameter $v_0 = 2^{-11}$ plays a key role in the formula for gravitational Planck constant and can be also seen as a fundamental constant in TGD framework. As a matter, factor v_0 has interpretation as velocity parameter and is dimensionless when $c=1$ is used.

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{\hbar G} \ . \end{aligned} \quad (7.7.17)$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 9.5.20 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi} \frac{2^{2m-1}}{K} \ . \quad (7.7.18)$$

For both $K = e^q$ with $q = 17$ and $K = 2^q$ option with $q = 24 + 1/2$ $m = 10$ is the smallest integer giving $b < 1$. $K = e^q$ option gives $b = .3302$ $(.0826)$ and $K = 2^q$ option gives $b = .3362$ $(.0841)$ for $m = 10$ $(m = 11)$.

$m = 10$ corresponds to one third of the action of free cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor rather near 1/12. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, suggests the identification of the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

Chapter 8

TGD and M-Theory

8.1 Introduction

In this chapter a critical comparison of M-theory [B48] and TGD (see [K96, K73, K60, K54, K74, K85, K82] and [K89, K12, K66, K10, K37, K46, K49, K81]) as two competing theories is carried out. Also some comments about the sociology of Big Science are made.

8.1.1 From hadronic string model to M-theory

The evolution of string theories began 1968 from Veneziano formula realizing duality symmetry of hadronic interactions. It took two years to realize that Veneziano amplitude could be interpreted in terms of interacting strings: Nambu, Susskind and Nielsen made the discovery simultaneously 1970. The need to describe also fermions led to the discovery of super-symmetry [B61] and Ramond and Neveu-Schwartz type superstrings in the beginning of seventies.

Gradually it became however clear that the strings do not describe hadrons: for instance, the critical dimensions for strings *resp.* superstrings where 26 *resp.* 10, and the breakthrough of QCD at 1973 meant an end for the era of hadronic string theory. 1974 Schwartz and Scherk proposed that strings might provide a quantum theory of gravitation [B62] if one accepts that space-time has compactified dimensions.

The first superstring revolution was initiated around 1984 by the paper by Green and Schwartz demonstrating the cancellation of anomalies in certain superstring theories [B33, B45] . The proposal was that superstrings might provide a divergence-free and anomaly-free quantum theory of gravitation. A crucial boost was given by Witten's interest on superstrings. Also the highly effective use of media played a key role in establishing superstring hegemony.

It became clear that superstrings come in five basic types [B52] . There are type I strings (both open and closed) with $N = 1$ super-symmetry and gauge group $SO(32)$, type IIA and IIB closed strings with $N = 2$ super-symmetry, and heterotic strings, which are closed and possess $N = 1$ super-symmetry with gauge groups $SO(32)$ and $E^8 \times E^8$. There is an entire landscape of solutions associated with each superstring theory defined by the compactifications whose dynamics is partially determined by the vanishing of conformal anomalies. For a moment it was believed that it would be an easy task to find which of the superstrings would allow the compactification which corresponds to the observed Universe but it became clear that this was too much to hope. In particular, the number 4 for non-compact space-time dimensions is by no means in a special position.

Around 1995 came the second superstring revolution with the idea that various superstring species could be unified in terms of an 11-dimensional M-theory with M meaning membrane in the lowest approximation [B48] . M-theory allowed to see various superstrings as limiting situations when 11-D theory reduces to 10-D one so that very special kind of membranes reduce to strings. This allowed to justify heuristically the claimed dualities between various superstrings [B52] . Matrix Theory as a proposal for a non-perturbative formulation of M-theory appeared 2 years later [B28] .

Now, almost a decade later, M-theory is in a deep crisis: the few predictions that the theory can make are definitely wrong and even anthropic principle is advocated as a means to save the theory [B68] . Despite this, very many people continue to work with M-theory and fill hep-th with highly speculative preprints proving that this is dual with that although the flow of papers dealing with strings and M-theory has reduced dramatically.

A reader interested in critical views about string theory can consult the article of Smolin [B66] criticizing anthropic principle, the web-lectures "Fantasy, Fashion, and Faith in Theoretical Physics" of Penrose [B57] as well as his article in New Scientist [B58] criticizing the notion of hidden space time dimensions, and the articles of Peter [C75] [B74] . Also the discussion group "Not Even Wrong" [B8] gives a critical perspective to the situation almost a decade after the birth of M-theory.

8.1.2 Evolution of TGD briefly

The first superstring revolution shattered the world at 1984, about two years after my own doctoral dissertation (1982), and four years after the Esalem conference in which the quantum consciousness movement started. Remarkably, David Finkelstein was one of the organizers of the conference besides being the chief editor of "International Journal of Theoretical Physics", in which I managed to publish first articles about TGD. The first and last contact with stars was Wheeler's review of my first article published in IJTP, and I cannot tell what my and TGD's fate had been without Wheeler's highly encouraging review.

During the 31 years after the discovery that space-times could be regarded as 4-surfaces as well as extended objects generalizing strings, I have devoted my time to the development of TGD. Without exaggeration I can say that life devoted to TGD has been much more successful project than I dared or even could dream and has led outside the very narrow realms of particle physics and quantum gravity. Indeed, without knowing anything about Finkelstein and Esalem at that time, I started to write a book about consciousness around 1995 when the second superstring revolution occurred. TGD inspired theory of consciousness has now materialized as 8 online books at my home page.

Altogether these 37 years boil down to eight online books [K96, K73, K60, K110, K111, K109, K85, K82] about TGD proper and eight online books about TGD inspired theory of consciousness and of quantum biology [K89, K12, K66, K10, K37, K46, K81, K104] plus printed book about TGD [K3] . This makes about 8000 pages of TGD spanning everything between elementary particle physics and cosmology. One might expect that the sheer waste amount of material at my web site might have stirred some interest in the physics community despite the fact that it became impossible to publish anything and to get anything into Los Alamos archives after the second super-string revolution. The only visible reaction has been from my Finnish colleagues and guarantees that I will remain unemployed in the foreseeable future. I will discuss some reasons for this state of affairs after comparing string models and TGD, and considering the reasons for the failure of the theory formerly known as superstring model.

Before continuing, I hasten to admit that I am not a string specialist and I do not handle the technicalities of M-theory. On the other hand, TGD has given quite a good perspective about the real problems of TOEs and provides also solutions to them. Hence it is relatively easy to identify the heuristic and usually slippery parts of various arguments from the formula jungle. Also I want to express my deep admiration for the people living in the theory world but from my own experience I know how easy it is to fall on wishful thinking and how necessary but painful it is to lose face now and then.

My humble suggestion is that M-theorists might gain a lot by asking what "What possibly went wrong?". This chapter suggests answers to this question. Perhaps M-theorists might also spend few hours in the web to check whether M-theory is indeed the only viable approach to quantum gravity: the material at my own home page might provide a surprise in this respect.

Ironically, TGD seems to be predict more stringy physics than string model. For instance, the well-definedness of em charge localizes the modes of induced spinor fields in generic case to 2-D surfaces so that strings become genuine part of TGD. Furthermore, string like objects defined by magnetic flux tubes appear in all scales, even in nuclear physics.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L22]. The topics relevant to this chapter are given by the following list.

- Comparison with other theories [L29]
- How TGD differs from standard model [L41]
- How quantum TGD differs from standard quantum physics [L40]
- Similarities between TGD and string models [L66]
- Differences between TGD and string models [L31]

8.2 A summary about the evolution of TGD

The basic idea about space-time as a 4-surface popped in my mind in autumn at 1978. The first implication was that I lost my job at Helsinki University. During the next 4 years this idea led to a thesis with the title "Topological GeometroDynamics" (TGD), which I think was suggested by David Finkelstein to distinguish TGD from Wheeler's GeometroDynamics.

8.2.1 Space-times as 4-surfaces

TGD (for a summary tgdevoI,tgdevoII) can be seen as as a solution to the energy problem of General Relativity via the unification of special and general relativities by assuming that space-times are representable as 4-surfaces in certain 8-dimensional space-time with the symmetries of empty Minkowski space. An alternative interpretation is as a generalization of string models by replacing strings with 3-dimensional surfaces: depending on their size they would represent elementary particles or the space we live in and anything between these extremes. From this point of view superstring theories are unique candidates for a Theory of Everything if space-time were 2- rather than 4-dimensional.

The first superstring revolution made me happy since I was convinced that it would be a matter of few years before TGD would replace superstring models as a natural generalization allowing to understand the four-dimensionality of the space-time. After all, only a half-page argument, a simple exercise in the realization of standard model symmetries, leads to a unique identification of the higher-dimensional imbedding space as a Cartesian product of Minkowski space and complex projective space CP_2 unifying electro-weak and color symmetries in terms of its holonomy and isometry groups. By the 4-dimensionality of the basic objects there was no need for the imbedding space geometry to be dynamical. Theory realized the dream about the geometrization of fundamental interactions and predicted the observed quantum numbers. In particular, the horrors of spontaneous compactification to be crystallized in the notion of M-theory landscape two decades later can be circumvented completely.

8.2.2 Uniqueness of the imbedding space from the requirement of infinite-dimensional Kähler geometric existence

Later I discovered heuristic mathematical arguments suggesting but not proving that the choice of the imbedding space is unique. The arguments relied on the uniqueness of the infinite-dimensional Kähler geometry of WCW of 3-surfaces. This uniqueness was discovered already in the context of loop spaces by Dan Freed [A71] .

CH, the "world of the classical worlds" serves as the arena of quantum dynamics [K18] , which reduces to the theory of classical spinor fields in CH and geometrizes fermionic anti-commutation relations and the notion of super-symmetry in terms of the gamma matrices of CH [K17] . Only quantum jump is the genuinely non-classical element of the theory in CH context. The heuristic argument states that CH geometry exists only for $H = M^4 \times CP_2$.

In particular, number theoretical arguments relating to quaternions and octonions fix the dimensions of space-time and imbedding space to four and 8 respectively. The fact that the space of quaternionic sub-spaces of octonion space containing preferred plane complex plane is CP_2 suggest an explanation for the special role of CP_2.

This stimulated a development, which led to notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [K88] . What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

The weakest form of number theoretical compactification states that light-like 3-surfaces $X_l^3 \subset HO$ are mapped to $X_l^3 \subset M^4 \times CP_2$ and requires only that one can assign preferred plane $M^2 \subset M^4$ to any connected component of X_l^3. This hyper-complex plane of hyper-quaternionic M^4 has interpretation as the plane of non-physical polarizations so that the gauge conditions of super string theories are obtained purely number theoretically. M^2 corresponds also to the degrees of freedom which do not contribute to the metric of WCW. The un-necessarily strong form would require that hyper-quaternionic 4-surfaces correspond to preferred extremals of Kähler action.

The requirement that M^2 belongs to the tangent space $T(X^4(X_l^3))$ at each point point of X_l^3 fixes also the boundary conditions for the preferred extremal of Kähler action. The construction of WCW spinor structure supports the conclusion that there must exist preferred coordinates of X^4 in which additional conditions $g_{ni} = 0$ and $J_{ni} = 0$ at X_l^3. The conditions state that induced metric and Kähler form are stationary at X_l^3. M^2 plays a key role also in many other constructions of quantum TGD, in particular the generalization of the imbedding space needed to realize the idea about hierarchy of Planck constant allowing to identify dark matter as matter with a non-standard value of Planck constant.

The realization of 4-D general coordinate invariance forces to assume that Kähler function assigns a unique space-time surface to a given 3-surface: by the breakdown of the strict classical determinism of Kähler action unions of 3-surfaces with time like separations must be however allowed as 3-D causal determinants and quantum classical correspondence allows to interpret them as representations of quantum jump sequences at space-time level. Space-time surface defined as absolute minimum or some more general preferred extremal [K88] of Kähler action is analogous to Bohr orbit so that classical physics becomes part of the definition of configuration space geometry rather than being a result of a stationary phase approximation.

8.2.3 TGD inspired theory of consciousness and other developments

During the last decade a lot has happened in TGD and it is sad that only those colleagues with mind open enough to make a visit my home page have had opportunity to be informed about this. Knowing the fact that a typical theoretical physicist reads only the articles published in respected journals about his own speciality, one can expect that the number of these physicists is not very high. Some examples of the work done during this decade are in order.

I have developed quantum TGD in a considerable detail with highly non-trivial number theoretical speculations relating to Riemann hypothesis and Riemann Zeta in riema . One outcome is a proposal for the proof of Riemann hypothesis [L1] .

During the same period I have constructed TGD inspired theory of consciousness [K89] . One outcome is a theory of quantum measurement and of observer having direct implications for the quantum TGD itself. The results of the modification of the double slit experiment carried out by Afshar [D11] , [J5] provides a difficult challenge for the existing interpretations of quantum theory and a support for the TGD view about quantum measurement in which space-time provides correlates for the non-deterministic process in question. The new views about energy and time have also profound technological implications.

TGD has forced the introduction of p-adic number fields besides real numbers and led to a generalization of number concept: p-adic number fields play a key role in the proposed physics of cognition and intentionality [K55, K31] . The notion of infinite primes [K86] leads to a visionc . Space-time point becomes infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

p-Adic number fields lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K55, K31] . One of the first applications was the calculation of elementary particle masses [K48, K48] . The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra is involved. Not only the fundamental mass scales reduce to number theory but also individual masses are predicted correctly under very mild assumptions. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K48] .

TGD inspired cosmology can be regarded as a fractal cosmology containing cosmologies within cosmologies [K80] . Sub-cosmology is defined in extremely general sense so that even the evolution of living organisms shares some crucial common aspects with cosmology in this sense. Initial singularities are absent. A period of flatness of 3-space following "big bang" is predicted by quantum criticality. The explanation of dark energy and dark matter are basically in terms of many-sheeted space-time although also new kinds of elementary particles are predicted (an entire hierarchy of asymptotically non-free standard model physics is possible). Dark matter and energy reside at larger space-time sheets, mainly magnetic flux quanta carrying magnetic and Z^0 magnetic fields. Solar corona represent a leakage of dark matter to our space-time sheets from magnetic flux tubes. Cosmological constant is predicted to have a spectrum given in terms of p-adic length scales characterizing the sizes of space-time sheets, and the deep puzzle produced by 10^{52}-fold discrepancy between experiment and theory disappears. Both the acceleration of cosmic expansion and the observed jerk [E15] is understood.

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extend that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges. The theory leads also to a prediction for the spectrum of Planck constants associated with M^4 and CP_2 degrees of freedom.

8.2.4 Von Neumann algebras and TGD

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

2. Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type

II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of WCW of 3-surfaces, etc....

Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

5. Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

8.2.5 Does dark matter at larger space-time sheets define super-quantal phase?

The last step in the rapid evolution of quantum TGD [K79] , [L4] was stimulated when I learned that D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.82 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The basic objection is that astrophysical systems are extremely classical whereas TGD predicts

macrotemporal quantum coherence in the scale of life time of gravitational bound states. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

The earlier work with topological quantum computation [K97] had already led to the idea that Planck constant could depend on the quantum phase $q = exp(i\pi/n)$. The first attempts to understand the large values of the Planck constant led to a badly wrong formula for this dependence. The improved understanding of Jones inclusions and their role in TGD [K99] allowed to deduce an extremely simple formula for the Planck constant, as a matter fact, for the two separate Planck constants assignable to with M^4 and CP_2 degrees of freedom appearing as scaling factors of the corresponding metrics. These Planck constants are given by the formulas $\hbar(M^4) = n(CP_2)\hbar_0$ and $\hbar(CP_2) = n(M^4)\hbar_0$ in terms of integers defining the corresponding quantum phases. The far reaching implication is that Planck constants can have arbitrarily large values. In this framework even imbedding space is a concept emerging from infinite-dimensional Clifford algebra but only the scaling factors of the metric can vary.

The general philosophy would be that when the quantum system becomes non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system. $q \neq 1$ quantum groups characterize non-perturbative phases.

The values of n for which the quantum phase is expressible using only iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. It should be noticed that this quantization does not depend at all on the parameter v_0 appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction. Also correct prediction for the ratio of densities of visible and dark matter emerges.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The value of v_0 has interpretation as velocity of distant stars around galaxies in the gravitational field of long cosmic string like objects traversing through galactic plane. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. Sub-harmonics would result when cosmic strings decay to magnetic flux tubes: magnetic energy density per unit length is quantized by the preferred extremal property and the simplest possibility is the reduction of the energy density by a factor $1/n^2$.

v_0 can be expressed in terms of Kähler coupling strength α_K and the parameter R^2/G characterizing CP_2 size. The value $v_0 = 2^{-11}$ favored both by the planetary Bohr orbitology and quantum model for living matter leads to new insights about coupling constant evolution. The surprising find was that α_K is very nearly equal to the electro-weak coupling $\alpha_{U(1)}$. This observation led to new insights about coupling constant evolution.

1. Contrary to the earlier beliefs, it is possible to assume that α_K is renormalization group invariant in strong sense if one assumes that gravitational interactions are mediated by space-time sheets labelled by M_{127}, the largest Mersenne prime which does not correspond to super-astronomical length scale.

2. Since classical color action reduces to Kähler action as does also electro-weak U(1) action, and since color holonomy is Abelian and induced spinors fields carry only anomalous color hyper charge as spinlike color quantum number identical with electroweak hypercharge, one can argue that the sum of color and U(1) actions equals to Kähler action implying $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K$ reducing the difficult-to-calculate evolution of color coupling strength to that of electroweak coupling constant evolution calculable perturbatively. The resulting

predictions are consistent with the empirical facts and electron mass and $\alpha_{U(1)}$ at electron length scale in principle fix the basic parameters of TGD completely.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with n=1 orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Needless to add, if the proposed general picture is correct, not much is left from the superstring/M-theory approach to quantum gravitation since perturbative quantum field theory as the fundamental corner stone must be given up and because the underlying physical picture about gravitational interaction is simply wrong.

8.3 Quantum TGD in nutshell

This section provides a summary about quantum TGD, which is essential for understanding the recent developments related to $M^8 - H$ duality. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits).

8.3.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of WCW forms what I have used to called super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum

TGD. The assumption that the commutator algebra of these super-symplectic and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of WCW Dirac equation.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. WCW spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

p-Adic physics as physics of cognition and intentionality

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also). The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K86] . It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

Hierarchy of Planck constants and dark matter hierarchy

The work with hyper-finite factors of type II_1 (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [K27] . The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by WCW spinor s. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞. The groups G should relate closely to finite groups defining inclusions of HFFs.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

3. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

8.3.2 The notions of imbedding space, 3-surface, and configuration space

The notions of imbedding space, 3-surface (and 4-surface), and WCW (world of classical worlds (WCW)) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [K87, K88, K86] .

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

2. With the discovery of zero energy ontology [K17, K21] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized in power-of-two multiples of CP_2 length, p-adic length scale hypothesis [K59] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [K27] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K65] .

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to equivalence believed to be implied by General Coordinate Invariance. There was a problem related to the realization of equivalence since it was not at all obvious why the preferred extremal (assumed to be absolute minimum) $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. Rather recently came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

The basic vision has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

1. The obvious guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice has some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections.

2. Much later number theoretical vision led to the conclusion that $X^4(X^3_{l,i})$, where $X^3_{l,i}$ denotes a connected component of the light-like 3-surfaces X^3_l, contain in their 4-D tangent space $T(X^4(X^3_{l,i}))$ a subspace $M^2_i \subset M^4$ having interpretation as the plane of non-physical polarizations. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

   In number theoretical framework M^2_i has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. A stronger condition would be that the condition holds true at all points of $X^4(X^3)$ for a global choice M^2 but this is un-necessary and leads to strong un-proven conjectures. The condition $M^2_i \subset T(X^4(X^3_{l,i}))$ in principle fixes the tangent space at $X^3_{l,i}$, and one has good hopes that the boundary value problem is well-defined and fixes $X^4(X^3)$ uniquely as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2_i \subset M^3$ plays also other important roles.

3. The next step [K17] was the realization that the construction of WCW geometry in terms of modified Dirac action strengthens the boundary conditions to the condition that there exists space-time coordinates in which the induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X^3_l. One could say that at X^3_l situation is static both metrically and for the Maxwell field defined by the induced Kähler form. There are reasons to hope that this is the final step in a long process.

4. The weakest form of number theoretic compactification states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred

extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$. A second interesting conjecture is that the hyper-quaternionic surfaces correspond to Kähler calibrations giving rise to absolute minima or maxima of Kähler action for M^8.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW (WCW) is a union of WCWs associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

8.4 Victories of M-theory from TGD view point

The basic victories of the M-theory relate to conformal symmetries and dualities and black hole physics and it is useful perform comparison with TGD.

8.4.1 Super-conformal symmetries

Space-time super-symmetries are regarded as one of the basic predictions of the super string model. Typically these super-symmetries appear at the level of effective quantum field theory limit derived from spontaneous compactification and predict that massless particles possess massless super partners, sparticles. The problem has been how to generalize Higgs mechanism to break the space-time super-symmetry. That sparticles have relatively low mass scale has been seen as one of the absolute predictions of M-theory and the ability to predict at least something has been counted as a success. Since sparticles have hitherto escaped the attempts to detect them, even this belief has been now challenged, and proposals has been made that perhaps M-theory might after all predict sparticles to be very massive.

Before continuing it must be emphasized that TGD and standard views about super-symmetry differ in many respects.

1. The standard view is inspired by the mathematically awkward and formal idea of assigning to the space-time coordinates anti-commuting super part. The belief is that string world sheet super-symmetries give rise to the space-time super symmetries of the low energy effective quantum field theory assigned to the string model.

2. In TGD the super-symmetry generators of the spectrum generating super-conformal algebra act as gamma matrices of WCW ("world of classical worlds"). The counterparts of the word sheet super-symmetries act as gauge super-symmetries at space-time level but do *not* give rise to global space-time super-symmetries at the level of imbedding space. Anti-commuting infinitesimals are encountered nowhere.

Super-symmetry at the space-time level

There have been a considerable progress in the understanding of super-conformal symmetries [K18, K105].

1. Super-symplectic algebra corresponds to the isometries of"world of classical worlds" (WCW) constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW.

2. One expects also Kac-Moody type gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

   The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X^3_l. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced WCW consisting of the light like 3-D CDs. The conformal symmetries in question suggests strongly a further degeneracy of the WCW metric and effective metric 2-dimensionality of 3-surfaces. These conformal symmetries could accompanied by super conformal symmetries defined by the solutions of the induced spinor fields.

   Contrary to the original beliefs, these conformal symmetries do not seem to be continuable to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate. The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional.

   A resolution of the interpretational problems came with the realization that it is hyper-quaternionic and -octonionic conformal symmetries, which are in question and that these symmetries are naturally associated with the description of the space-time surface as a 4-surface in hyper-quaternionic $HO = M^8$ rather than in H. These symmetries are realized also at the level of H. Note that hyper-quaternionic symmetries act trivially in the interior of X^4 but induce deformations of boundaries of X^4.

   The solutions of the modified Dirac equation $D\Psi = 0$, define the modes which do not contribute to the Dirac determinant of the modified Dirac operator in terms of which the vacuum functional assumed to correspond to the exponent of the Kähler action is defined.

Thus they define gauge super-symmetries. Usually D selects the physical helicities by the requirement that it annihilates physical states: now the situation is just the opposite. D^2 annihilates the generalized eigen states both at space-like and light like 3-surfaces. Hence the roles of the physical and non-physical helicities are switched. It is the generalized eigen modes of D with non-vanishing eigenvalues λ, which code for the physics whereas the solutions of the modified Dirac equation define super gauge symmetries.

At the space-like 3-surfaces associated with 7-D causal determinants the spinor harmonics of WCW satisfy the $M^4 \times CP_2$ counterpart of the massless Dirac equation so that non-physical helicities are eliminated in the standard sense at the imbedding space level. The right-handed neutrino does not generate an $N = 1$ space-time super-symmetry contrary to the long held belief.

Super-symmetry at the level of WCW

The gamma matrices of WCW are defined as matrix elements of properly chosen operators between right-handed neutrino and second quantized induced spinor field at space-like boundaries X^3 [K17] . These generators define the fermionic generators of what I call super-symplectic algebra. The right handed neutrino can be replaced with any spinor harmonic of the imbedding space to obtain an extended super-algebra, which can be used to construct the physical states.

The requirement that super-generators vanish for the vacuum extremals requires that the modified Dirac operator D_+ or the inverse of D_- appearing in the matrix element of the "Hermitian conjugate" $S^- = (S^+)^\dagger$ of the super charge S^+. Here $\pm$ refers to the negative and positive energy space-time sheets meeting at X^3 or to the two maximally deterministic space-time regions separated by the causal determinant. The operators D_+ and D_-^{-1} are restricted to the spinor modes not annihilated by $D_\pm$. The super-generator generated by the covariantly constant right handed neutrino vanishes identically: a more rigorous argument showing that $N = 1$ global super symmetry is indeed absent.

If WCW decomposes into a union of sectors labelled by unions of light cones having tips at arbitrary points of M^4, the spinor harmonics can be assumed to define plane waves in M^4 and even possess well-defined four-momenta and mass squared values. Same applies to the super-symplectic generators defined by their commutators. This means that the generators of the super-symplectic algebra generated in this manner would possess well defined four-momenta and thus their action would change the mass of the state. Space-time super-symmetries would be absent. Similar argument applies to the Kac Moody algebras associated with the light like 3-D causal determinants if super-symplectic Super Kac-Moody algebras provide dual representations of quantum states.

If the gist of these admittedly heuristic arguments is correct, they force to modify drastically the existing view about space-time super-symmetries. The problem how to break super-symmetry disappears since there is no space-time symmetry be broken down. Super-symmetries are realized as a spectrum generating algebra rather than symmetries in the standard sense.

I hasten to admit that I have myself believed that right handed neutrino defines a global super-symmetry and proposed that the topological condensation of sparticles and particles at space-time sheets with different p-adic primes would provide an elegant model for super-symmetry breaking using same general mass formulas but only a different mass scale. Giving up this assumption causes however only a sigh of relief. The predicted spectrum of massless states is reduced dramatically [K48] . p-Adic mass calculations based on p-adic thermodynamics and representations of super-conformal algebra are not affected since the global $N = 1$ super-symmetry implies only an additional vacuum degeneracy. Most predictions of TGD remain intact. The speculation that sneutrinos might be light and play a role in TGD based condensed matter physics is the only possible exception. One can however consider the possibility of light colored sneutrinos obtained by applying to a neutrino state a colored and thus non-vanishing super-symplectic generator defined by right handed antineutrino.

It deserves to be noticed that the notion super-symmetry in WCW sense was discovered with the advent of super string models and generalized to a space-time super-symmetry when gauge theories made their breakthrough. The notion of spontaneous compactification (we meet our friend again and again!) inspired then the hypothesis that this super-symmetry has a space-time counterpart and everyone believed. There is now an entire industry making similar purely formal out of context applications and generalizations of quantum groups, which originally emerged naturally in knot and braid theory and in the theory of von Neumann algebras [K86, K97] .

8.4.2 Dualities

The starting point of duality physics was the classical paper of Montonen and Olive about electric-magnetic Montonen which was generalized to what are known as S and T dualities in superstring context. The notion of duality is central also in TGD framework.

Dualities as victories of M-theory

Dualities [B52] allowing to unify various superstring models are regarded as basic victories of M-theory. The heuristic proofs for various dualities between various variants of superstring model that I have seen apply what might be called M-logic. Consider special examples defined by 11-dimensional super-gravity using a particular background and particular spontaneous compactification and demonstrate that these examples are consistent with the duality. Then generalize from special to general. For a non-specialist, it is difficult to decide, whether all this is just wishful thinking and clever choices of compactifications.

Mirror symmetry of Calabi-Yau manifolds

String theory has stimulated very general conjectures about the properties of Calabi-Yau manifolds, which have turned out to be correct. Calabi-Yau manifolds are 3-dimensional Kähler manifolds with $SU(3)$ (rather than $U(3)$) holonomy group and thus satisfy empty space Einstein equations implied by the requirement of the vanishing of conformal anomaly in closed super string models. The prediction of the mirror symmetry for Calabi-Yau manifolds [B53] emerged before the era of M-theory from the study of $N = 2$ super-conformal sigma models with Calabi-Yau manifold as a target space and closed string world sheet as the "space-time". In the 11-dimensional M-theory context Calabi-Yau manifolds are obtained only by a special compactification for which 11^{th} dimension corresponds to a circle. The argument taken from [B53] written in a physicist friendly manner runs as follows.

(a) In conformal field theories the so called marginal operators correspond to the deformations of the original conformal field theory respecting the property of being a conformal field theory, and thus the criticality of the physical system. In particular, the deformations of complex and Kähler structures of the target space, now Calabi-Yau space, induce this kind of deformations. The basic finding was that the operators inducing these two kinds of deformations differ only by the opposite sign of their U(1) charge associated with the $U(1)$ current of $N = 2$ super-symmetry algebra.

(b) The mere change of the sign of $U(1)$ charge would correspond to a permutation of the spaces of complex and Kähler moduli which means a rather drastic geometric and even a topological change. On the other hand, the physical change must be marginal since the system remains critical. Both signs of $U(1)$ charge seem highly plausible so that the hypothesis is that the Calabi-Yau manifolds appear a mirror pairs so that in a rough sense the moduli for Kähler and complex structures are permuted for the members of the mirror pair by performing a change of sign of $U(1)$ charge for the left moving modes of string. Actually a generalization of the notion of Kähler moduli is necessary. This is achieved by combining the Kähler form and antisymmetric field B defining a generalization of $U(1)$ gauge potential to form a imaginary and complex parts of a more general structure for which Kähler moduli space (Kähler cone) is complexified and by introducing so called extended Kähler cone combining the Kähler moduli associated with several Calabi-Yau spaces so that single Calabi-Yau manifold can have several mirrors [B53] .

There are two implications. First, two different Calabi-Yau geometries and even topologies give rise to the same conformally invariant physics: the physics $\leftrightarrow$ geometry identification of General Relativity is not strictly true anymore. Secondly, the continuous change of the complex moduli for the Calabi-Yau manifold corresponds to a topology change for the mirror manifold so that even topology change corresponds to a quite smooth change of physics, in fact a change respecting 2-dimensional criticality. Even the possibility that the change involves a temporary contraction of the Calabi-Yau to a point during the change cannot be excluded [B53] , which looks really weird. Also singular Calabi-Yau manifolds are possible and not mere limiting cases of non-singular ones [B53] .

These implications might be also seen as a failure of the theory basically due to the spontaneous compactification trick. In TGD imbedding space is fixed and similar phenomenon does not occur. The moduli space of conformal structures of the metrically 2-dimensional light like causal determinants effectively corresponding to closed string word sheets is however involved also now, and implies naturally the concept of elementary particle vacuum functional defined in the moduli space of complex structures characterizing the effectively 2-D induced metrics at causal determinants [K19] . The notion is essential for p-adic mass calculations and predicts correct ratios for electron, muon, and tau lepton masses [K48] .

To conclude, the discovery of the mirror symmetry is quite beautiful and impressive but as such does not provide support for the super string theory as a physical theory. The discovery could have been made by a conformal field theorist interested in two-dimensional critical statistical systems.

8.4.3 Dualities and conformal symmetries in TGD framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the modified Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of WCW geometry.

Electric-magnetic duality

Electric-magnetic duality for the induced Kähler induced field is present also in TGD (CP_2 Kähler form is self-dual). My original belief was that it corresponds to a self duality leaving Kähler coupling constant invariant as an analog of critical temperature: $\alpha_K \rightarrow \alpha_K$ in this transformation [K18] . This duality would allow to construct WCW Kähler metric in terms of Kähler electric or magnetic fluxes.

This duality relates in an interesting manner to the idea that space-time surfaces can be regarded either hyper-quaternionic sub-manifolds of M^8 endowed with hyper-octonionic tangent space or as 4-surfaces in $M^4 \times CP_2$ [K88] . The point is that one can consider also the dual definition for which the 4-D normal space defines 4-D subalgebra of 8-D algebra at each point of the space-time surface. One might speak of number theoretical spontaneous compactification. This duality corresponds to naturally to the decomposition of space-time surface to regions for which the signature of the induced metric is Minkowskian *resp.* Euclidian. Therefore there are reasons to expect that the dichotomies electric-magnetic, associative -co-assosociative, and Minkowskian-Euclidian correspond to one and same duality.

Quantum gravitational holography

The so called AdS/CFT duality of Maldacena Maldacena correspondence relates to quantum-gravitational holography states roughly that the gravitational theory in 10-dimensional $AdS_{10-n}\times S^n$ manifold is equivalent with the conformal field theory at the boundary of AdS_D factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD quantum gravitational holography is realized in terms of the Kähler-Dirac action at light like 3-D causal determinants [K17] , which by their metric 2-dimensionality allow superconformal invariance and are very much like world sheets of closed super string or the ends of an open string.

The condition that the value of electromagnetic charge is well-defined for the modes of the induced spinor field implies the localization of the modes to 2-D surfaces- string world sheets and possibly also partonic 2-surfaces- with the property that the induced W field and above weak scale also the induced Z^0 field vanish. This means that in fermionic sector one has a theory very similar to string model.

Weak form of electric magnetic duality and the proposed vanishing of the Coulombic contribution $j\cdot A$ to Kähler action density for preferred extremals would reduce Kähler action to mere Chern-Simons terms. If one adds compensating Chern-Simons term at partonic orbits, only the space-like ends of the space-time surface contribute to the Kähler action and Kähler function. This would reduce enormously difficult problem of identifying preferred extremals of Kähler action and calculating corresponding Kähler action to a local data at light-light-like 3-surfaces. A concrete realization of holography would be in question.

One can add also to Kähler-Dirac action Chern-Simons Dirac term at partonic orbits and the assumption that its action is equivalent with that of M^4 Dirac operator gives rise to the space-time analog of massless imbedding space Dirac equation. C-S-D action reduces to the analog of algebraic Dirac equation and one can assign to fermionic lines identified as boundaries of string world sheets massless Dirac propagator required by twistor Grassmannian approach.

Perhaps the most practical form of the quantum gravitational holography is implied by the generalized conformal invariance implying effective 2-dimensionality. This means that X^3_l represent generalized Feynman diagrams with lines representing by light-like 3-surfaces and vertices as 2-surfaces $X^2 \subset \delta CD\times CP_2$ at which these lines meet. Vertices can be expressed as N-point functions of super-conformal field theory at these 2-surfaces. Only effective two-dimensionality is in question since one has hierarchy of CDs within CDs and improvement of measurement resolution brings into consideration CDs with smaller size. Effective 2-dimensionality obvious means quantum holography in lower dimensional sense and this sequence of holographies continues down to the level of number theoretic braids with information about M-matrix coded by a set of discrete points at partonic 2-surfaces X^2.

Computationally TGD would reduce to almost string model type theory since light like 3-surfaces are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand. There is also an analogy with the Wess-Zumino-Witten model: light like causal determinants would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

8.4.4 Number theoretic compactification and M^8-H duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally M^8-H duality was introduced as a number theoretic explanation for $H=M^4\times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4\subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely.

One can question the feasibility of M^8-H duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of M^8-H duality.

The strong form M^8-H duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of H or as surfaces of M^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kḧler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4\times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering M^8-H duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick.

(a) The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

(b) Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the sub-space of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

(c) Ordinary quaternions Q are expressible as $q=q_0+q^kI_k$. Hyper-quaternions are expressible as $q=q_0+jq^kI_k$ and form a subspace of complexified quaternions $Q_c=Q\oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O\oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H=q_0+jq^kI_k+jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

(a) Kähler form for M^8 implies unique decomposition $M^8=M^4\times E^4$ needed to define M^8-H duality uniquely. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the

WCWs associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

(b) One can formulate associativity in M^8 by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

(c) Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

(d) What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8).

(a) All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the modified gamma matrices need not span the entire tangent space. The space spanned by the modified gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the modified gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

If the space spanned by modified gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

(b) For modified gamma matrices the notion of co-associativity can produce problems since modified gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by modified gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the modified gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the modified gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

(a) One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

(b) The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

(c) Quaternionic sub-algebras of M^8 are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

(d) The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

(a) The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

(b) Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

(c) One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

(d) Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

(e) The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

(f) Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

(a) This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

(b) The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K9]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

(c) Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K28] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of modified Dirac operator.

For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

(d) Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

(e) One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K98]).

(a) According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

(b) Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and modified gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to

Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

1. Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

2. Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

(a) By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

(b) One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

(c) Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

(d) The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

3. Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

(a) The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the intepretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

(b) One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

(c) One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

(d) The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1+ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

(e) In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

4. What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary

(O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H...$ iteration generating new solutions from existing ones.

1. Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1+io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $Re(Q_1) + iIm(Q_2)$ with signature (1,-1,-,1-,1) is non-vanishing. Co-associative surfaces would be surfaces for which the projections to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ so that only the projection to $iIm(O_2)$ with signature $(-1 - 1 - 1 - 1)$ is non-vanishing.

These sub-manifolds are excellent candidate for associative and co-associative 4-surfaces if one believes on the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

The alert reader has probably observed that the inverse image of the M^4 or E^4 as sub-space of O_c does not belong to $M^4 \times E^4$ sub-space of O_c. One can however assign to each point of this 4-surface a unique point of M^4 as projection and a unique point of CP_2 as characterization of the quaternionic tangent plane hence $O_c \to H$ correspondence holds true.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

2. Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

(a) Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

(b) If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

    One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

(c) Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

(d) Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$
\begin{aligned}
e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\
A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\
e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\
\Gamma_k &= e^A_k \gamma_A \ .
\end{aligned}
\tag{8.4.1}
$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly nontrivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$
F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{8.4.2}
$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associativity conditions.

(e) The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^By^Cz^D = 0$ of trilinear forms defined by the associators. The conditions say something only about the octonionic structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A43] expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "timelike" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

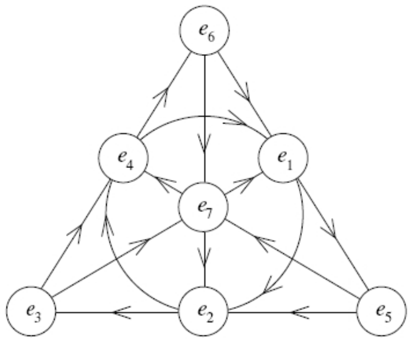

Figure 8.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A43] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaternionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

Questions

In following some questions related to $M^8 - H$ duality are represented.

1. Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

(a) For Kähler action the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

(b) The space spanned by the modified gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection modified gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence modified gamma matrices are flexible enough to realize associativity in H.

(c) Modified gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anticommutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

2. Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

3. Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

(a) If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

(b) $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

(c) $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

4. $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

(a) At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

(b) The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

(c) The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

(d) The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

(e) Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K57] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for

M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H$... is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

8.4.5 Configuration gamma matrices as hyper-octonionic conformal fields

The fact that the Clifford algebra generated by WCW gamma matrices forms a canonical representation for hyper-finite factor of type II_1 (HFFs) and led to a breakthrough in the understanding of quantum TGD. The inclusions of hyper-finite factors of type II_1 led to a realization of finite quantum measurement resolution as a basic principle govering dynamics and together with zero energy ontology this approach led to the generalization of S-matrix to M-matrix identified as time like entanglement coefficients between positive and negative energy parts of zero energy state and its identification as Connes tensor product. HFFs generated also ideas about how quantum TGD might be reducible to a generalization of HFFs to its local variant which is necessarily complex-octonionic as also to a construction of quantum variant of gamma matrix algebra leading to identification of quantum counterparts of hyper-octonions and hyper-quaternions as unique structures.

Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

The fantastic properties of hyperfinite factors of type II_1 (HFFs) inspire the idea that a localized hyper-octonionic version of Clifford algebra of WCW might allow to see space-time, embedding space, and WCW as structures emerging from a hyper-octonionic version of HFF. Surprisingly, commutativity and associativity imply most of the speculative "must-be-true's" of quantum TGD.

WCW gamma matrices act only in vibrational degrees of freedom of 3-surface. One must also include center of mass degrees of freedom which appear as zero modes. The natural idea is that the resulting local gamma matrices define a local version of HFF of type II_1 as a generalization of conformal field of gamma matrices appearing super string models obtained by replacing complex numbers with hyper-octonions identified as a subspace of complexified octonions.

As a matter fact, one can generalize octonions to quantum octonions for which quantum commutativity means restriction to a hyper-octonionic subspace of quantum octonions. Non-associativity is essential for obtaining something non-trivial: otherwise this algebra reduces to HFF of type II_1 since matrix algebra as a tensor factor would give an algebra isomorphic with the original one. The octonionic variant of conformal invariance fixes the dependence of local gamma matrix field on the coordinate of HO. The coefficients of Laurent expansion of this field must commute with octonions. !

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2, F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

Consider first hyper-quaternions and the emergence of M^4.

(a) The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \ , \tag{8.4.3}$$

read as

$$\begin{array}{ll} ab = qba \ , & ac = qac \ , \quad bd = qdb \ , cd = qdc \ , \\ [a,d] = (q-q^{-1})bc \ , & bc = cb \ . \end{array} \tag{8.4.4}$$

(b) These relations could be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a, a^\dagger] = [b, b^\dagger] = [c, c^\dagger] = [d, d^\dagger] = 1 \ . \tag{8.4.5}$$

This extension is not necessary for what comes.

(c) The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a-a^\dagger, d-d^\dagger, b-c^\dagger, c-b^\dagger\} \ . \end{array} \tag{8.4.6}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. These conditions need not be consistent with the commutation relations between a,b,c,d and their Hermitian conjugates. This is easy to see by noticing that the difference of $J_+ - J_-$ acts apart from imaginary unit like J_y and annihilates $j_y = 0$ state for every representation of rotation group diagonalized with respect to J_y.

(d) What is essential is that the operators of O are of form $A - A^\dagger$ and their commutators are also of the same form that the commutativity conditions reduce the condition that the Lie-algebra like structure generated by these operators annihilates the physical state. Hence it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

(e) $M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether also other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be not the case. For instance, the replacement $a - a^\dagger \rightarrow a + a^\dagger$ making also time variable Euclidian is impossible since $[a + a^\dagger, d - d^\dagger] = 2(q - q^{-1})(bc + b^\dagger c^\dagger$ is not proportional to a difference of operator and its hermitian conjugate and one does not obtain closed algebra.

What about M^8: does it have analogous description in terms of physical states annihilated by the Lie algebra generated by the differences $a_i - a_i^\dagger$, $i = 0,..7$?

(a) The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

(b) In quaternionic case basis for matrix algebra is formed by the sigma matrices and M^4 point is represented by a hermitian matrix expressible as linear combination of hermitian sigma matrices with coefficients which act on physical states like hermitian operators. In the hyper-octonionic case would expect that real octonion unit and octonionic imaginary units multiplied by commuting imaginary unit to define the counterparts of sigma matrices and that the physically representable sub-space of complex quantum octonions corresponds to operator valued coordinates which act like hermitian matrices. The restriction to complex quaternionic sub-space must give hyper-quaternions and M^4 so that the only sensible generalization is that M^8 holds quite generally. This is also required by SO^7 invariance allowing to choose the sub-space M^4 freely. Again the key point should be that the conditions giving rise to real eigenvalues give rise to a Lie-algebra which must annihilate the physical state. For other signatures one would not obtain Lie algebra.

(c) One can also make guess for the concrete realization of the algebra. Introduce the coefficients of E^4 gamma matrices having interpretation as quaternionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(ib+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

(d) The selections of complex and quaternionic sub-algebras of octonions are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of hyper-quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of hyper-octonions the selection of hyper-quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Hyper-quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned} [a_0, a_3] &= \frac{i}{2}(q - q^{-1})(a_1^2 - a_2^2) \ , \\ [a_i, a_j] &= 0 \ , \ i,j \neq 0,3 \ , \\ a_0 a_i &= q a_i a_0 \ , \ i \neq 0,3 \ , \\ a_3 a_i &= q a_i a_3 \ , \ i \neq 0,3 \ . \end{aligned} \tag{8.4.7}$$

Note that there is symmetry breaking in the sense that the commutation relations for sub-algebras relating to both M^4 and M^2 are in distinguished role.

Dimensions $D = 4$ and $D = 8$ are indeed unique if one takes this argument seriously.

(a) For dimensions other than $D = 4$ and $D = 8$ a representation of the point of M^D as element of Clifford algebra of M^D is needed. The coefficients should be real for the signatures and this requires that the elements of Clifford algebra are Hermitian.

Gamma matrices are the only natural candidates and when Majorana conditions can be satisfied one obtains quantum representation of M^D. 10-D Minkowski space of superstring models would represent one example of this kind of situation.

(b) For other dimensions $D \geq 8$ but now octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

WCW spinor fields as hyper-octonionic conformal fields

A further proposed application of this picture is to the construction of WCW spinor fields as generalizations of conformal fields. The basic problem is to treat center of mass degrees of freedom properly, and the idea that conformal invariance generalizes to hyper-octonionic - or at least hyper-quaternionic - conformal invariance is attractive. If so, the usual expansion in powers of complex coordinate z would be replaced in powers of hyper-octonionic coordinate h and the coefficients would be elements of Clifford algebra for sub-WCW consisting of light-like 3-surfaces with frozen center of mass degrees of freedom. This is possible if one can map the points of H to those of M^8 and $M^8 - H$ duality allows to achieve this.

The natural condition would be that N-point functions defined by WCW spinor fields for which M^8 coordinate labels the position of the tip of the causal diamond containing the zero energy state involve only those points which are mutually associative and would thus belong to a hyper-quaternionic sub-space $M^4 \subset M^8$ would be in question and the outcome would be the analog of M^4 quantum field theory.

Commutativity would restrict the points to $M^2 \subset M^4 \subset M^8$ and hyper-complex variant conformal field theory would result: this theory would be analogous with integrable models known as factorizing quantum field theories in M^2 in which particle scattering is almost trivial (interactions generate only phase lag).

8.4.6 Black hole physics

The hierarchy of Planck constants has forced to modify dramatically TGD based view about black holes. TGD black holes however have a lot of common with ordinary black holes.

M-theory and black holes

The reproduction of the formula for the black hole entropy [B67, B52] has been sold as a victory of M-theory. The first thing that has been forgotten is that GRT based formula has never been experimentally verified and could be even wrong.

One can also criticize the procedure leading to the formula.

(a) First M-theory is replaced by 11-D super gravity in order to calculate something. What this effectively means that, although the aim was to replace General Relativity with something more fundamental, one ends up with 11-D classical super-gravity after all.

(b) After this one finds black-hole type solutions and identifies them with M-branes. At this step one could protest by saying that the fundamental theory should replace black holes with something less singular.

(c) Next quantum gravitational holography is assumed and a conformal field theory on brane identified as a black hole horizon leads to an estimate for the entropy and estimates for what are known as greyness factors. The last step is nice in the 4-D situation and also TGD would suggest something very similar.

In Matrix Theory based estimate things look even less elegant. In [B28] a matrix theory based estimate for the entropy is made producing the correct order of magnitude for the entropy estimate using conformal field theory. An essential step is the estimate for the number N of 0-branes (ordinary particles) and is ad hoc (in particular one does not take the limit $N \to \infty$). I do not whether the arguments are more rigorous in other estimates but, to put it mildly, I do not find this argument is not too convincing.

Black holes in TGD framework

Black holes in the standard sense are possible in TGD framework but would be basically astrophysical objects and putting black holes and elementary particles in the same basket would be mixing apples with oranges. The vision about dark matter as a macroscopic quantum phases with large value of Planck constant (the value of gravitational Planck constant is enormous) forces to reconsider the identification of black holes. One can view TGD counterparts of black hole horizons as light-like 3-surfaces at which the signature of the induced metric changes. Black holes would be gigantic elementary particle (or rather parton-) like objects containing particles in anyonic phase with fractional charges guaranteeing confinement. Dark anyonic matter at light-like 3-surfaces of astrophysical size analogous to stringy black holes thought to be tightly tangled strings has several basic characteristics of black hole and would populate TGD Universe in all length scales.

In TGD Universe the role of black hole horizons is taken by light like 3-surfaces which are fundamental objects of the theory whereas the role of big bang is taken by the boundary δM^4_+ of causal diamond (CD). The basic difference to black hole horizons is that the signature of induced metric changes at the wormhole throat.

(a) The basic example is provided by elementary particle horizons surrounding the ends of the wormhole contacts having Euclidian signature of the induced metric and connecting with each other space-time sheets with Minkowskian signature of the induced metric. The light-like wormhole throats are carriers of fermion numbers. The interpretation of wormhole contacts is in terms of gauge bosons and Higgs bosons consisting of fermion and anti-fermion at the two wormhole throats. By its spin the only possible identification of graviton is as a pair of wormhole contacts connected by a flux tube carrying various gauge fluxes. Elementary fermions correspond to wormhole throats associated with CP_2 type vacuum extremals (note Euclidian signature of induced metric) glued to the background space-time with Minkowskian signature of metric.

(b) Second example is provided by light-like surfaces separating maximal deterministic regions of the space-time sheet. Light-like boundaries is a further example. By their metric 2-dimensionality various causal determinants indeed allow conformal field theory in an effectively 2-dimensional sense.

(c) The formula for the black hole entropy generalizes to elementary particle level and involves p-adic length scale hypothesis and p-adic mass calculations [K59] .

(d) The new element is the hierarchy of Planck constants [K79, K62, K27] inspired by the findings that gravitational Planck constant might have gigantic value [E27] . This leads to a vision about dark matter as phases of matter with large Planck constant and hence macroscopically quantum coherent since all quantum scales are scaled up. The space-time sheets mediating gravitational interaction would have gigantic value of Planck constant: $\hbar_{gr} = GM_1M_2/v_0$, $v_0 = 2^{-11}$ gives a good example about the situation. The implication is that black hole entropy proportional to $1/\hbar$ is of order unity if $\hbar_{gr} = GM^2/v_0$, $v_0 = 1/4$ holds true for black holes. This would change completely the view about black holes as highly entropic objects. In particular, Planck length scales as $\sqrt{\hbar}$ so that Schwartschild radius represents Planck length for this kind of black hole and defines naturally kind of minimum length scales below which the signature of induced metric becomes Euclidian in TGD Universe.

(e) The progress in the understanding of the realization of the hierarchy of Planck constants in terms of book like structure of imbedding space with the pages of book representing Cartesian products of singular coverings and factor spaces of causal diamond CD and CP_2 led to a detailed picture about identification of anyonic systems as macroscopic light-like 3-surfaces containing dark matter in anyonic form possessing fractional quantum numbers. Anyonicity means that the "partonic" 2-surface of macroscopic size system surrounds the tip of CD so that homologically non-trivial 2-surface is in question. Anyonic phase could be even responsible for the properties of living matter [K65, K23] . This also inspired the proposal that dark matter resides at light-like 3-surfaces of astrophysical and even cosmological size scale possessing very complex topology: typically spherical topologies glued together by flux tubes. Black holes in standard sense would result in gravitational collapse of this kind of systems. An open question is whether the topology actually transforms to simple spherical topology in this process or whether it is more or less conserved so that huge information about the topology of orbits of dark matter particles surrounding the object would be preserved.

More concrete ideas about black hole like structures emerged from the attempts to understand the strange events reported by RHIC (Relativistic Heavy Ion Collider) [C64, C57] during last years. This work led to a dramatic increase of understanding of TGD and allowed to fuse together separate threads of TGD [K80] .

(a) The scaled down TGD inspired cosmology involving (not so) big crunch followed by (not so) big bang serves as a model for the events, and predicts a new phase identifiable as color glass condensate identifiable as tightly tangled color magnetic flux tube modellable as a hadronic string in Hagedorn temperature.

 This state makes a phase transition to quark gluon plasma during a period of critical cosmology analogous to inflationary cosmology characterized completely by its duration and quark gluon plasma analogous to radiation dominated cosmology in turn hadronizes giving rise to the analog of matter dominated cosmology.

 The assumption that anyonicity is responsible for the formation of the gluonic Bose-Einstein condensate explains the liquid like character of color glass condensate. Anyonicity forces the system to behave like a single particle like unit since fractionally charged particles cannot leave the light-like 2-surface surrounding the tip of CD.

(b) RHIC events suggest processes analogous to the formation and evaporation of black hole. The TGD inspired description in terms of the formation of hadronic black hole and its evaporation and essentially identical with the description as a mini bang. The hadronic black hole is the same tightly tangled color magnetic flux tube that defines the initial state of the hadronic mini bang. The attribute 'hadronic' means that Planck length is replaced with hadronic length so that strong gravitation is in question. Black hole temperature is identifiable as Hagedorn temperature and predicted to be 195 MeV for bosonic strings in 4-D space-time and slightly higher than the hadronization temperature measured to be about 176 MeV [K80] .

(c) As also the small value of black hole entropy suggests, black holes and their scaled counterparts would not be merciless information destroyers in TGD Universe. The entanglement of particles possessing different conformal weights to give states with a vanishing net conformal weight and having particle like integrity would make black hole like states ideal candidates for quantum computer like systems [K97] . One could even imagine that the galactic black hole is a highly tangled cosmic string in Hagedorn temperature performing quantum computations the complexity of which is totally out of reach of human intellect! Indeed, TGD inspired consciousness predicts that evolution leads to the increase of information and intelligence, and the evolution of stars should not form exception to this. Also the interpretation of black hole as consisting of dark matter follows from this picture [K23] .

Concerning the mathematical description of dark matter - and of matter quite generally- TGD has led to amazingly simple mathematical framework, which might have something to with Matrix theory approach. The characteristic aspects of the classical dynamic determined by Kähler action is its vacuum degeneracy and this not only allows but even forces the notion of finite measurement resolution originally inspired by the inclusions of hyper-finite factors of type II_1 (HFFs) having WCW Clifford algebra as a canonical representative. The notion of finite measurement resolution leads to a discretization of physics in terms of number theoretic braids and finite number of fermionic oscillator operators characterizing any subsystem [K17] . Even the infinite-dimensional world of classical worlds can be described with arbitrary accuracy as a finite-dimensional space and these descriptions define a hierarchy of inclusions of HFFs associated with WCW Clifford algebra.

8.4.7 Zero energy ontology and Witten's approach to 3-D quantum gravitation

There is an interesting relationship of quantum TGD to the recent yet unpublished work of Witten related to 3-D quantum blackholes [B51] , which - despite that it does not directly relate to M-theory - provides additional perspective.

(a) The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

(b) This conformal theory would have the so called monster group [B51, B6] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [B42] . In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that gravitational radiation is not possible. HenceAdS_3 theory in Witten's sense might define this conformal field theory.

Witten's construction has obviously a strong structural similarity to TGD.

(a) Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory.

(b) Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.

(c) For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD, gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

(d) Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

There are also interesting differences.

(a) According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

(b) Also the fermionic counterpart of Chern-Simons action for the induced spinors whose form is dictated by the super-conformal symmetry is present. Furthermore, partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

(c) The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

(d) In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

(e) Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [K27] . Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [K27] . Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [K42] .

(f) To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   i. Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   ii. In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   iii. The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D light-like surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about M-matrix implied by the zero energy ontology.

(a) Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

(b) Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure (see discussion below). Without this generalization theory would reduce to a theory for 2-D fundamental objects.

(c) Theory becomes genuinely 4-D because M-matrix is not universal anymore but characterizes zero energy states.

(d) 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D light-like surfaces as sub-manifolds (analogs of blackhole horizons and light-like boundaries) [K18] . Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the modified Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [K17] .

8.5 What went wrong with string models?

As will be found, the few physical predictions of M-theory are wrong. It is instructive to try to understand what went wrong with M-theories and string models by comparing it with earlier successful theories and with TGD.

8.5.1 Problems of M-theory

At the physical side the situation in M-theory can be regarded as a catastrophe and without the association of the attribute "the only known candidate for the quantum theory of gravitation..." to the letter M bringing in mind Pavlov dogs, no-one could take it seriously. The various problems of M-theory have been discussed in the article of Smolin [B66] as also by Penrose in his lecture series "Fashion, Faith and Fantasy in Theoretical Physics" [B57] . The discussions of "Not Even Wrong" [B8] group provide a vivid critical view about the situation.

(a) M-theory has not been able to explain why the dimension of the space-time is four and has even failed to reproduce the standard model. Unless one assumes that the small dimensions form a singular manifold (something so ugly that it turns my stomach around), M-theory predicts chiral symmetry just like Kaluza-Klein theories: the symmetry is inconsistent with the standard model. Ironically, just this was the reason why superstrings replaced Kaluza-Klein theories in the first superstring revolution. This full π twist represents a good example of M-logic.

The predicted massless scalar fields have not been observed. The predicted low energy super-symmetry is experimentally absent, and now papers have begun to appear suggesting that M-theory after all might predict only high energy super-symmetry. One of the first findings after the second superstring revolution was that the prediction for the unification scale was wrong. I remember that Witten proposed at that time a suitable compactification of the 11^{th} dimension to a circle to circumvent this problem.

(b) Cosmological constant is now believed to be non-vanishing and positive [E18] whereas the cosmological constant predicted by M-theory is negative. M-theories provide no explanation for the accelerated expansion [E18] . There is a plethora of cosmological observations which M-theory cannot even address.

This sad state of affairs has led to the introduction of the anthropic principle [B68] but not in the sense that it would really predict something but as an M-logic proof that M-theory after all predicts among other things also the cosmological constant correctly. The premise is that M-theory is correct and the conclusion is that the observed universe must represent some distant corner of the M-landscape, and we must be ready to accept as a fact, that we will never be able to find our way to this distant part of the Theory Universe, and be happy with learning new dualities.

8.5.2 Mouse as a tailor

The history of string models differs dramatically from that for theories which has been successful as physical theories. As a rule, new theories have started from a precise problem which earlier theories have not been able to solve, and have led to a new ontology and inspired new mathematics.

String model was born as a model of hadrons. It however became gradually clear that the constraints on space-time dimensions make it unrealistic for this purpose. The conclusion of the mouse was not so humble as in the tale: admittedly string models fail for hadrons but who knows, they might describe everything.

After a decade of tailoring the cat was told that superstrings do not seem to make a TOE after all. The mouse said that he could tailor even something more grandiose just by sewing together all the previous failures. Now it has become clear that the result is an enormous bundle of solutions of the possibly existing M-theory, which at practical level is reduced after few heuristic arguments to compactifications of 11-D super gravity. There is still however a little problem: not a single one of these solutions seems to describe the Universe we live in. Now the mouse suggests that we should give up the dream about a theory of the observable universe as unrealistic, stop complaining and be happy with all these beautiful dualities.

Is the time ripe for the story to end as its original version did or shall the cat provide still another decade of financial support for the expensive tailor?

8.5.3 The dogma of reductionism

M-theory as an outcome of hard-nosed reductionism

The philosophical background of string models is hard-nosed reductionism taken down to Planck length: something taken to be so self-evident that it has not been even mentioned. Hence the theory cannot make any predictions about or utilize the rich experimental input coming from the known physics.

This means that string theorists do not pay any attention to the pressing problems of quantum measurement theory, to the problems related to the relationship between experienced and geometric time, and to the problems surrounding to the poor understanding of second law. Not to even mention the questions about the difference between animate and in-animate matter, and about what it means to be a conscious system.

The belief that the action defining functional integral summarizes the physics leads to an approach which is extremely pragmatic: start from the existing formulas of perturbative field theories and try to combine them in order to cook up a more general theory. The danger that theoreticians fall into a kind of mathematical insanity in this kind of situation is obvious, and the possible failure of reductionism means a tragic failure of the entire approach.

Giving up reductionism

TGD cannot be regarded as a success from the point of view of sociology of science but the success of TGD as a physical theory is undeniable and basically due to the facts that TGD emerged as a solution to a well-defined problem, and that the notion of many-sheeted space-time plus p-adic length scale hypothesis [K59] provide a precise quantitative formulation for how reductionism fails.

(a) I ended up with TGD by starting from a very real problem of general relativity and soon found that I could end up to TGD also from string models. From the beginning the contact of TGD with experimental physics was very intimate. Later the quantum classical correspondence has become a basic guide line in the construction of the theory.

(b) One cannot deny that string theories partially solved the divergence problem of perturbative quantum field theories. Unfortunately, is is highly implausible that the sum of the perturbation series would converge so that as such it is useless. This has in fact been seen as a victory of the theory since one can hope that a genuinely non-perturbative approach could lead to a unique theory.

    In TGD framework the absence of the basic divergences is highly plausible already from the basic construction involving new ontology of space-time. Vacuum functional identified as an exponent of Kähler function is not anymore a local functional of 3-surface so that basic perturbative divergences resulting from the micro-locality are absent. Also Gaussian and metric determinants cancel and the definition of Kähler function in terms of Dirac determinant is free of divergencies [K17] .

(c) The construction of quantum TGD was not possible without the theory of consciousness. Key element is the replacement of space-time micro-locality with classical locality in the "world of classical worlds" making possible to understand how macroscopic and macro-temporal quantum coherence are possible [K43, K11, K44] . Thanks to the notion of self [K77, K95, K16] , observer ceases to be an outsider and quantum measurement theory is becomes an essential part of the theory. Completely un-expected outcomes were the already mentioned generalizations of the number concept and the identification of the space-time correlates of cognition and intentionality.

(d) TGD generalizes in a dramatic manner the ontology of space-time in terms of the notion of the many-sheeted space-time involving also the new view about numbers. The identification of space-time sheets as space-time counterparts of physical objects resolves the question about the generation of structures. The ontology of quantum TGD is discussed in [K16] from the point of view of category theory. One important implication is that even quantum superposition and quantum logic can have space-time correlates at the level of many-sheeted space-time.

(e) TGD resolves the paradoxes due to the conflict between the non-determinism of quantum jump and determinism of Schrödinger equation and, by the classical non-determinism, quantum-classical correspondence can be realized at the space-time level even for quantum jump sequences. TGD leads to a new view about the relationship between geometric and subjectively experienced time rather than just identifying them [K95] .

(f) Zero energy ontology replaces positive energy ontology. Zero energy states are superpositions of pairs of positive and negative energy states with opposite energies and other conserved quantum numbers assignable to the boundaries of causal diamond (CD). In ordinary ontology they corresponds to events consisting of initial and final state.

Negative energies make possible what I call remote metabolism playing in key role in TGD inspired theory of consciousness and of quantum biology: the system can gain energy by sending negative energy to geometric past [K95, K43, K44] . Time mirror mechanism (see fig. `http://www.tgdtheory.fi/appfigures/timemirror.jpg` or fig. 24 in the appendix of this book) makes possible communications with geometric past and future and communications with an effectively super-luminal velocity become possible.

(g) The duality between theory and reality is resolved. TGD based ontology postulates only three levels of existence corresponding to existences in these sense of classical and quantum physics, and conscious existence which corresponds to the quantum jumps between the quantum states [K16] . The possibility that space-time points are infinitely structured in p-adic sense although this structure is not visible in real sense [K86] , would resolve the challenge posed by the question why all those structures that we can imagine mathematically, are not realized physically. Obviously, a reincarnation of the monad idea of Leibniz is in question.

8.5.4 The loosely defined M

In a sharp contrast with M-theory [B48] , Newton's mechanics and gravitational theory, Maxwell's electrodynamics, Special and General Relativities, and even Bohr's rules were from the beginning relatively precisely defined theories able to make testable predictions. The lack of a precise definition of what "M" means has led to a flood of speculations based on speculations based on...

"M" as "membrane" would be a rather precise definition but does not really make sense since the huge conformal invariance of string models is lost as objects become 2-dimensional. For this reason one prefers to replace "M" with Mystery, Mother, or perhaps Matrix, but still think in terms of membranes which behave like strings. It became however clear that also branes of various dimensions are needed as discovered by Polchinski [B59] and identified as non-perturbative objects at which string ends are attached to: this interpretation is the only possible one since otherwise momentum conservation would be lost for D-branes.

Needless to say, a theory using geometric structures consisting of parts possessing different dimensions does not satisfy the standards of the conventional mathematical aesthetics. An outsider could argue that the non-uniqueness of the boundary conditions (Neumann, Dirichlet and mixtures of them) is the fundamental failure of the string theory, and that a viable theory should predict the dynamics of boundaries. This is indeed the case in TGD where the criticality of the Kähler action guaranteeing general coordinate invariance in 4-D sense does this and implies that the space-time surface is a field theory counterpart of Bohr orbit.

A good example of brave new M-logic is provided by the construction of what is called Matrix Theory [B28] . One starts from M-theory "known" to have 11-D supergravity as a low energy limit, replaces it with a 11-D supergravity, restricts the consideration to N 0-branes (point particles) living in an effectively 10-D space, in an ad hoc manner replaces their position coordinates in 10-D space with non-commuting $N \times N$-matrix valued coordinates assuming that eigenvalues correspond to N space-time points, postulates a non-relativistic Schrödinger equation for this matrix, and by generalizing bravely the notion of holography, concludes that the original theory and even more follows from this very-very special theory at $N \to \infty$ limit. From Matrix Theory one then deduces all superstring dualities and and black hole physics using an argumentation with a comparable rigor.

It must be added that TGD predicts a rich variety of objects resulting as asymptotic self-organization patterns for which Kähler-Lorentz 4-force vanishes by quantum classical correspondence. The solutions are classified by the dimension of either their M^4 or CP_2 projection [K9] . This variety includes cosmic strings and magnetic flux tubes besides space-time sheets. Magnetic flux tubes and string like objects can indeed attach to the boundaries of space-time sheets and there are obvious correspondences with branes with dimensions of branes restricted to run from 0 to 4 ($p = -1, .., 3$) but only as objects obtained by idealizing 4-dimensional object with a lower-dimensional object.

Even the possibility of single space-time point or space-time curve to mimic the quantum dynamics of the quantum state of Universe is predicted but only at the level of cognition and relying on the new notion about what mathematical point is [K86] . I however do not think that this has much to do with Matrix Theory.

8.5.5 Los Alamos, M-theory, and TGD

String models have been seen not only as a kind of holy grail of modern physics but also as an ideology promising an Utopia. As a rule, ideologies have tried to establish the new world order using censorship. String model hegemony has followed the tradition.

For about decade ago it became impossible for me to get anything to hep-th and other physics related archives. Interestingly, for few years ago my article about Riemann hypothesis was accepted to the math archives of Los Alamos and is also published [L1] : it was however not possible to get it cross-listed to hep-th. For a few years American Mathematical Society has had a link to my homepage [A1] as one of the few examples about new mathematics related to quantum physics.

I have learned that I am not the only victim of the string revolution (see the comments in "Not Even Wrong" discussion group [B8]). Despite the official statement that anyone can contribute to LANL, an invisible peer system is acting. After 20 years of string revolutions it seems that physics itself has become the victim which has suffered the most severe injuries.

8.6 K-theory, branes, and TGD

K-theory has played important role in brane classification in super string models and M-theory. The excellent lectures by Harah Evslin with title *What doesn't K-theory classify?* [B30] make it possible to learn the basic motivations for the classification, what kind of classifications are possible, and what are the failures. Also the Wikipedia article [B5] gives a bird's eye of view about problems. As a by-product one learns something about the basic ideas of K-theory and about possible mathematical and physical problems of string theories and M-theory.

In the sequel I will discuss critically the basic assumptions of brane world scenario, sum up my understanding about the problems related to the topological classification of branes and also to the notion itself, ask what goes wrong with branes and demonstrate how the problems are avoided in TGD framework, and conclude with a proposal for a natural generalization of K-theory to include also the division of bundles inspired by the generalization of Feynman diagrammatics in quantum TGD, by zero energy ontology, and by the notion of finite measurement resolution.

8.6.1 Brane world scenario

The brane world scenario looks attractive from the mathematical point of view ine one is able to get accustomed with the idea that basic geometric objects have varying dimensions. Even accepting the varying dimensions, the basic physical assumptions behind this scenario are vulnerable to criticism.

(a) Branes are geometric objects of varying dimension in the 10-/11-dimensional space-time -call it M- of superstring theory/M-theory. In M-theory the fundamental strings are replaced with M-branes, which are 2-D membranes with 3-dimensional orbit having as its magnetic dual 6-D M5-brane. Branes are thought to emerge non-perturbatively from fundamental 2-branes but what this really means is not understood. One has D-p-branes with Dirichlet boundary conditions fixing a $p+1$-dimensional surface of M as brane orbit: one of the dimensions corresponds to time. Also S-branes localized in time have been proposed.

(b) In the description of the classical limit branes interact with the classical fields of the target space by the generalization of the minimal coupling of charged point-like particle to electromagnetic gauge potential. The coupling is simply the integral of the gauge potential over the world-line - the value of 1-form for the word-line. Point like particle represents 0-brane and in the case of p-brane the generalization is obtained by replacing the gauge potential represented by a 1-from with $p+1$-form. The exterior derivative of this $p+1$-form is $p+2$-form representing the analog of electromagnetic field. Complete dimensional democracy strongly suggests that string world sheets should be regarded as 1-branes.

(c) From TGD point of view the introduction of branes looks a rather ad hoc trick. By generalizing the coupling of electromagnetic gauge potential to the word line of point like particle one could introduce extended objects of various dimensions also in the ordinary 4-D Maxwell theory but they would be always interpreted as idealizations for the carriers of 4- currents. Therefore the crucial step leading to branes involves classical idealization in conflict with Uncertainty Principle and the genuine quantal description in terms of fields coupled to gauge potentials.

My view is that the most natural interpretation for what is behind branes is in terms of currents in D=10 or D= 11 space-time. In this scheme branes have role only as semi-classical idealizations making sense only above some scale. Both the reduction of string theories to quantum field theories by holography and the dynamical character of the metric of the target space conforms with super-gravity interpretation. Internal consistency requires also the identification of strings as branes so that superstring theories and M-theory would reduce to an idealization to 10-/11-dimensional quantum gravity.

In this framework the brave brane world episode would have been a very useful Odysseia. The possibility to interpret various geometric objects physically has proved to be an extremely powerful tool for building provable conjectures and has produced lots of immensely beautiful mathematics. As a fundamental theory this kind of approach does not look convincing to me.

8.6.2 The basic challenge: classify the conserved brane charges associated with branes

One can of course forget these critical arguments and look whether this general picture works. The first thing that one can do is to classify the branes topologically. I made the same question about 32 years ago in TGD framework: I thought that cobordism for 3-manifolds might give highly interesting topological conservation laws. I was disappointed. The results of Thom's classical article about manifold cobordism demonstrated that there is no hope for really interesting conservation laws. The assumption of Lorentz cobordism meaning the existence of global time-like vector field would make the situation more interesting but this condition looked too strong and I could not see a real justification for it. In generalized Feynman diagrammatics there is no need for this kind of condition.

There are many alternative approaches to the classification problem. One can use homotopy, homology, cohomology and their relative and other variants, topological or algebraic K-theory, twisted K-theory, and variants of K-theory not yet existing but to be proposed within next years. The list is probably endless unless something like motivic cohomology brings in enlightment.

(a) First of all one must decide whether one classifies p-dimensional time=constant sections of p-branes or their $p+1$-dimensional orbits. Both approaches have been applied although the first one is natural in the standard view about spontaneous compactification. For the first option topological invariants could be seen as conserved charges: homotopy invariants and homological and cohomological characteristics of branes provide this kind of invariants. For the latter option the invariants would be analogous to instanton number characterizing the change of magnetic charge.

(b) Purely topological invariants come first in mind. Homotopy groups of the brane are invariants inherent to the brane (the brane topology can however change). Homological and cohomological characteristics of branes in singular homology characterize the imbedding to the target space. There are also more delicate differential topological invariants such as de Rham cohomology defining invariants analogous to magnetic charges. Dolbeault cohomology emerges naturally for even-dimensional branes with complex structure.

(c) Gauge theories - both abelian and non-Abelian - define a standard approach to the construction of brane charges for the bundle structures assigned with branes. Chern-Simons classes are fundamental invariants of this kind. Also more delicate invariants associated with gauge potentials can be considered. Chern-Simons theory with vanishing field strengths for solutions of field equations provides a basic example about this. For instance, SU(2) Chern-Simons theory provides 3-D topological invariants and knot invariants.

(d) More refined approaches involve K-theory -closely related to motivic cohomology - and its twisted version. The idea is to reduce the classification of branes to the classification of the bundle structures associated with them. This approach has had remarkable successes but has also its short-comings.

The challenge is to find the mathematical classification which suits best the physical intuitions (, which might be fatally wrong as already proposed) but is universal at the same time. This challenge has turned out to be tough. The Ramond-Ramond (RR) p-form fields of type II superstring theory are rather delicate objects and a source of most of the problems. The difficulties emerge also by the presence of Neveu-Schwartz 3-form $H = dB$ defining classical background field.

K-theory has emerged as a good candidate for the classification of branes. It leaves the confines of homology and uses bundle structures associated with branes and classifies these. There are many K-theories. In topological K-theory bundles form an algebraic structure with sum, difference, and multiplication. Sum is simply the direct sum for the fibers of the bundle with common base space. Product reduces to a tensor product for the fibers. The difference of bundles represents a more abstract notion. It is obtained by replacing bundles with pairs in much the same way as rationals can be thought of as pairs of integers with equivalence $(m, n) = (km, kn)$, k integer. Pairs $(n, 1)$ representing integers and pairs $(1, n)$ their inverses. In the recent case one replaces multiplication with sum and regards bundle pairs and (E, F) and $(E+G, F+G)$ equivalent. Although the pair as such remains a formal notion, each pair must have also a real world representativs. Therefore the sign for the bundle must have meaning and corresponds to the sign of the charges assigned to the bundle. The charges are analogous to winding of the brane and one can call brane with negative winding antibrane. The interpretation in terms of orientation looks rather natural. Later a TGD inspired concrete interpretation for the bundle sum, difference, product and also division will be proposed.

8.6.3 Problems

The classification of brane structures has some problems and some of them could be argued to be not only technical but reflect the fact that the physical picture is wrong.

Problems related to the existence of spinor structure

Many problems in the classification of brane charges relate to the existence of spinor structure. The existence of spinor structure is a problem already in general general relativity since ordinary spinor structure exists only if the second Stiefel- Whitney class [A29] of the manifold is non-vanishing: if the third Stiefel-Whitney class vanishes one can introduce so called spin^c

structure. This kind of problems are encountered already in lattice QCD, where periodic boundary conditions imply non-uniqueness having interpretation in terms of 16 different spinor structures with no obvious physical interpretation. One the strengths of TGD is that the notion of induced spinor structure eliminates all problems of this kind completely. One can therefore find direct support for TGD based notion of spinor structure from the basic inconsistency of QCD lattice calculations!

(a) Freed-Witten anomaly [B32] appearing in type II string theories represents one of the problems. Freed and Witten show that in the case of 2-branes for which the generalized gauge potential is 3-form so called spin^c structure is needed and exists if the third Stiefel-Whitney class w_3 related to second Stiefel Whitney class whose vanishing guarantees the existence of ordinary spin structure (in TGD framework spin^c structure for CP_2 is absolutely essential for obtaining standard model symmetries).

    It can however happen that w_3 is non-vanishing. In this case it is possible to modify the spin^c structure if the condition $w_3 + [H] = 0$ holds true. It can however happen that there is an obstruction for having this structure - in other words $w_3 + [H]$ does not vanish - known as Freed-Witten anomaly. In this case K-theory classification fails. Witten and Freed argue that physically the wrapping of cycle with non-vanishing $w_3 + [H]$ by a Dp-brane requires the presence of $D(p-2)$ brane cancelling the anomaly. If $D(p-2)$ brane ends to anti-Dp in which case charge conservation is lost. If there is not place for it to end one has semi-infinite brane with infinite mass, which is also problematic physically. Witten calls these branes baryons: these physically very dubious objects are not classified by K-theory.

(b) The non-vanishing of $w_3 + [H] = 0$ forces to generalize K-theory to twisted K-theory [A34]. This means a modification of the exterior derivative to get twisted de Rham cohomology and twisted K-theory and the condition of closedness in this cohomology for certain form becomes the condition guaranteeing the existence of the modified spin^c structure. D-branes act as sources of these fields and the coupling is completely analogous to that in electrodynamics. In the presence of classical Neveu-Schwartz (NS-NS) 3-form field H associated with the back-ground geometry the field strength $G^{p+1} = dC_p$ is not gauge invariant anymore. One must replace the exterior derivative with its twisted version to get twisted de Rham cohomology:

$$d \rightarrow d + H \wedge \quad .$$

    There is a coupling between p- and p+2-forms together and gauge symmetries must be modified accordingly. The fluxes of twisted field strengths are not quantized but one can return to original p-forms which are quantized. The coupling to external sources also becomes more complicated and in the case of magnetic charges one obtains magnetically charged Dp-branes. Dp-brane serves as a source for $D(p-2)$- branes.

    This kind of twisted cohomology is known by mathematicians as Deligne cohomology. At the level of homology this means that if branes with dimension of p are presented then also branes with dimension $p + 2$ are there and serve as source of Dp-branes emanating from them or perhaps identifiable as their sub-manifolds. Ordinary homology fails in this kind of situation and the proposal is that so called twisted K-theory could allow to classify the brane charges.

(c) A Lagrangian formulation of brane dynamics based on the notion of p-brane democracy [B69] due to Peter Townsend has been developed by various authors.

Ashoke Sen has proposed a grand vision for understanding the brane classification in terms of tachyon condensation in absence of NS-NS field H [B65]. The basic observation is that stacks of space-filling D- and anti D-branes are unstable against process called tachyon condensation which however means fusion of $p+1$-D brane orbits rather than p-dimensional time slices of branes. These branes are however accompanied by lower-dimensional branes and the decay process cannot destroy these. Therefore the idea arises that suitable stacks of D9 branes and anti-D9-branes could code for all lower-dimensional brane configurations as the end products of the decay process.

This leads to a creation of lower-dimensional branes. All decay products of branes resulting in the decay cascade would be by definition equivalent. The basic step of the decay process is the fusion of D-branes in stack to single brane. In bundle theoretic language one can say that the D-branes and anti-D branes in the stack fuse together to single brane with bundle fiber which is direct sum of the fibers on the stack. This fusion process for the branes of stack would correspond in topological K-theory. The fusion of D-branes and anti-D branes would give rise to nothing since the fibers would have opposite sign. The classification would reduce to that for stacks of D9-branes and anti D9-branes.

Problems with Hodge duality and S-duality

The K-theory classification is plagued by problems all of which need not be only technical.

(a) R-R fields are self dual and since metric is involved with the mapping taking forms to their duals one encounters a problem. Chern characters appearing in K-theory are rational valued but the presence of metric implies that the Chern characters for the duals need not be rational valued. Hence K-theory must be replaced with something less demanding.

The geometric quantization inspired proposal of Diaconescu, Moore and Witten [B23] is based on the polarization using only one half of the forms to get rid of the proboem. This is like thinking the 10-D space-time as phase space and reducing it effectively to 5-D space: this brings strongly in mind the identification of space-time surfaces as hyper-quaternionic (associative) sub-manifolds of imbedding space with octonionic structure and one can ask whether the basic objects also in M-theory should be taken 5-dimensional if this line of thought is taken seriously. An alternative approach uses K-theory to classify the intersections of branes with 9-D space-time slice as has been porposed by Maldacena, Moore and Seiberg [B41].

(b) There another problem related to classification of the brane charges. Witten, Moore and Diaconescu [B23] have shown that there are also homology cycles which are unstable against decay and this means that twisted K-theory is inconsistent with the S-duality of type IIB string theory. Also these cycles should be eliminated in an improved classification if one takes charge conservation as the basic condition and an hitherto un-known modification of cohomology theory is needed.

(c) There is also the problem that K-theory for time slices classifies only the R-R field strengths. Also R-R gauge potentials carry information just as ordinary gauge potentials and this information is crucial in Chern-Simons type topological QFTs. K-theory for entire target space classifies D-branes as $p+1$-dimensional objects but in this case the classification of R-R field strengths is lost.

The existence of non-representable 7-D homology classes for targent space dimension $D > 9$

There is a further nasty problem which destroys the hopes that twisted K-theory could provide a satisfactory classification. Even worse, something might be wrong with the superstring theory itself. The problem is that not all homology classes allow a representation as non-singular manifolds. The first dimension in which this happens is $D = 10$, the dimension of super-string models! Situation is of course the same in M-theory. The existence of the non-representables was demonstrated by Thom - the creator of catastrophe theory and of cobordism theory for manifolds- for a long time ago.

What happens is that there can exist 7-D cycles which allow only singular imbeddings. A good example would be the imbedding of twistor space CP_3, whose orbit would have conical singularity for which CP_3 would contract to a point at the "moment of big bang". Therefore homological classification not only allows but demands branes which are orbifolds. Should orbifolds be excluded as unphysical? If so then homology gives too many branes and the singular branes must be excluded by replacing the homology with something else. Could twisted K-theory exclude non-representable branes as unstable ones by having non-vanishing $w_3 + [H]$? The answer to the question is negative: D6-branes with $w_3 + [H] = 0$ exist for which K-theory charges can be both vanishing or non-vanishing.

One can argue that non-representability is not a problem in superstring models (M-theory) since spontaneous compactification leads to $M \times X_6$ ($M \times X_7$). On the other hand, Cartesian product topology is an approximation which is expected to fail in high enough length scale resolution and near big bang so that one could encounter the problem. Most importantly, if M-theory is theory of everything it cannot contain this kind of beauty spots.

8.6.4 What could go wrong with super string theory and how TGD circumvents the problems?

As a proponent of TGD I cannot avoid the temptation to suggest that at least two things could go wrong in the fundamental physical assumptions of superstrings and M-theory.

(a) The basic failure would be the construction of quantum theory starting from semiclassical approximation assuming localization of currents of 10 - or 11-dimensional theory to lower-dimensional sub-manifolds. What should have been a generalization of QFT by replacing point-like particles with higher-dimensional objects would reduce to an approximation of 10- or 11-dimensional supergravity.

This argument does not bite in TGD. 4-D space-time surfaces are indeed fundamental objects in TGD as also partonic 2-surfaces and braids. This role emerges purely number theoretically inspiring the conjecture that space-time surfaces are associative sub-manifolds of octonionic imbedding spaces, from the requirement of extended conformal invariance, and from the non-dynamical character of the imbedding space.

(b) The condition that all homology equivalence classes are representable as manifolds excludes all dimensions $D > 9$ and thus super-strings and M-theory as a physical theory. This would be the case since branes are unavoidable in M-theory as is also the landscape of compactifications. In semiclassical supergravity interpretation this would not be catastrophe but if branes are fundamental objects this shortcoming is serious. If the condition of homological representability is accepted then target space must have dimension $D < 10$ and the arguments sequence leading to D=8 and TGD is rather short. The number theoretical vision provides the mathematical justification for TGD as the unique outcome.

(c) The existence of spin structure is clearly the source of many problems related to R-R form. In TGD framework the induction of spin^c structure of the imbedding space resolves all problems associated with sub-manifold spin structures. For some reason the notion of induced spinor structure has not gained attention in super string approach.

(d) Conservative experimental physicist might criticize the emergence of branes of various dimensions as something rather weird. In TGD framework electric-magnetic duality can be understood in terms of general coordinate invariance and holography and branes and their duals have dimension 2, 3, and 4 organize to sub-manifolds of space-time sheets. The TGD counterpart for the fundamental M-2-brane is light-like 3-surface. Its magnetic dual has dimension given by the general formula $p_{dual} = D - p - 4$, where D is the dimension of the target space [B27]. In TGD one has $D = 8$ giving $p_{dual} = 2$. The first interpretation is in terms of self-duality. A more plausible interpretation relies on the identification of the duals of light-like 3-surfaces as space-like 3-surfaces at the light-like boundaries of CD. General Coordinate Invariance in strong sense implies this duality. For partonic 2-surface one would have $p = 1$ and $p_{dual} = 3$. The identification of the dual would be as space-time surface. The crucial distinction to M-theory would be that branes of different dimension would be sub-manifolds of space-time surface.

(e) For $p = 0$ one would have $p_{dual} = 4$ assigning five-dimensional surface to orbits of point-like particles identifiable most naturally as braid strands. One cannot assign to it any direct physical meaning in TGD framework and gauge invariance for the analogs of brane gauge potentials indeed excludes even-dimensional branes in TGD since corresponding forms are proportional to Kähler gauge potential (so that they would be analogous to odd-dimensional branes allowed by type II_B superstrings).

4-branes could be however mathematically useful by allowing to define Morse theory for the critical points of the Minkowskian part of Kähler action. While writing this I learned that Witten has proposed a 4-D gauge theory approach with $\mathcal{N} = 4$ SUSY to the classification of knots. Witten also ends up with a Morse theory using 5-D space-times in the category-theoretical formulation of the theory [A72]. For some time ago I also proposed that TGD as almost topological QFT defines a theory of knots, knot braidings, and of 2-knots in terms of string world sheets [K41]. Maybe the 4-branes could be useful for understanding of the extrema of TGD of the Minkowskian part of Kähler action which would take take the same role as Hamiltonian in Floer homology: the extrema of 5-D brane action would connect these extrema.

(f) Light-like 3-surfaces could be seen as the analogs von Neuman branes for which the boundary conditions state that the ends of space-like 3-brane defined by the partonic 2-surfaces move with light-velocity. The interpretation of partonic 2-surfaces as space-like branes at the ends of CD would in turn make them D-branes so that one would have a duality between D-branes and N-brane interpretations. T-duality exchanges von Neumann and Dirichlet boundary conditions so that strong from of general coordinate invariance would correspond to both electric-magnetic and T-duality in TGD framework. Note that T-duality exchanges type II_A and type II_B super-strings with each other.

(g) What about causal diamonds and their 7-D light-like boundaries? Could one regard the light-like boundaries of CDs as analogs of 6-branes with light-like direction defining time-like direction so that space-time surfaces would be seen as 3-branes connecting them? This brane would not have magnetic dual since the formula for the dimensions of brane and its magnetic dual allows positive brane dimension p only in the range (1,3).

8.6.5 Can one identify the counterparts of R-R and NS-NS fields in TGD?

R-R and NS-NS 3-forms are clearly in fundamental role in M-theory. Since in TGD partonic 2-surfaces define the analogs of fundamental M-2-branes, one can wonder whether these 3-forms could have TGD counterparts.

(a) In TGD framework the 3-forms $G_{3,A} = dC_{2,A}$ defined as the exterior derivatives of the two-forms $C_{2,A}$ identified as products $C_{2,A} = H_A J$ of Hamiltonians H_A of $\delta M^4_\pm \times CP_2$ with Kähler forms of factors of $\delta M^4_\pm \times CP_2$ define an infinite family of closed 3-forms belonging to various irreducible representations of rotation group and color group. One can consider also the algebra generated by products $H_A A$, $H_A J$, $H_A A \wedge J$, $H_A J \wedge J$, where A *resp.* J denotes the Kähler gauge potential *resp.* Kähler form or either $\delta M^4_\pm$ or CP_2. A *resp.* Also the sum of Kähler potentials *resp.* forms of $\delta M^4_\pm$ and CP_2 can be considered.

(b) One can define the counterparts of the fluxes $\int Adx$ as fluxes of $H_A A$ over braid strands, $H_A J$ over partonic 2-surfaces and string world sheets, $H_A A \wedge J$ over 3-surfaces, and $H_A J \wedge J$ over space-time sheets.Gauge invariance however suggests that for non-constant Hamiltonians one must exclude the fluxes assigned to odd dimensional surfaces so that only odd-dimensional branes would be allowed. This would exclude 0-branes and the problematic 4-branes. These fluxes should be quantized for the critical values of the Minkowskian contributions and for the maxima with respect to zero modes for the Euclidian contributions to Kähler action. The interpretation would be in terms of Morse function and Kähler function if the proposed conjecture holds true. One could even hope that the charges in Cartan algebra are quantized for all preferred extremals and define charges in these irreducible representations for the isometry algebra of WCW. The quantization of electric fluxes for string world sheets would give rise to the familiar quantization of the rotation $\int E \cdot dl$ of electric field over a loop in time direction taking place in superconductivity.

(c) Should one interpret these fluxes as the analogs of NS-NS-fluxes or R-R fluxes? The exterior derivatives of the forms G_3 vanish which is the analog for the vanishing of magnetic charge densities (it is however possible to have the analogs of homological magnetic charge). The self-duality of Ramond p-forms could be posed formally ($G_p =^* G_{8-p}$) but does not have any implications for $p < 4$ since the space-time projections vanish in this case identically for $p > 3$. For $p = 4$ the dual of the instanton density $J \wedge J$ is proportional to volume form if M^4 and is not of topological interest. The approach of Witten eliminating one half of self dual R-R-fluxes would mean that only the above discussed series of fluxes need to be considered so that one would have no troubles with non-rational values of the fluxes nor with the lack of higher dimensional objects assignable to them. An interesting question is whether the fluxes could define some kind of K-theory invariants.

(d) In TGD imbedding space is non-dynamical and there seems to be no counterpart for the NS 3-form field $H = dB$. The only natural candidate would correspond to Hamiltonian $B = J$ giving $H = dB = 0$. At quantum level this might be understood in terms of bosonic emergence [K64] meaning that only Ramond representations for fermions are needed in the theory since bosons correspond to wormhole contacts with fermion and anti-fermions at opposite throats. Therefore twisted cohomology is not needed and there is no need to introduce the analogy of brane democracy and 4-D space-time surfaces containing the analogs of lower-dimensional brains as sub-manifolds are enough. The fluxes of these forms over partonic 2-surfaces and string world sheets defined non-abelian analogs of ordinary gauge fluxes reducing to rotations of vector potentials and suggested be crucial for understanding braidings of knots and 2-knots in TGD framework. [K41]. Note also that the unique dimension D=4 for space-time makes 4-D space-time surfaces homologically self-dual so that only they are needed.

8.6.6 What about counterparts of S and U dualities in TGD framework?

The natural question is what could be the TGD counterparts of $S-$, $T-$ and U-dualities. If one accepts the identification of U-duality as product $U = ST$ and the proposed counterpart of T duality as a strong form of general coordinate invariance, it remains to understand the TGD counterpart of S-duality - in other words electric-magnetic duality - relating the theories with gauge couplings g and $1/g$. Quantum criticality selects the preferred value of g_K: Kähler coupling strength is very near to fine structure constant at electron length scale and can be equal to it. Since there is no coupling constant evolution associated with α_K, it does not make sense to say that g_K becomes strong and is replaced with its inverse at some point. One should be able to formulate the counterpart of S-duality as an identity following from the weak form of electric-magnetic duality and the reduction of TGD to almost topological QFT. This seems to be the case.

(a) For preferred extremals the interior parts of Kähler action reduces to a boundary term because the term $j^\mu A_\mu$ vanishes. The weak form of electric-magnetic duality requires that Kähler electric charge is proportional to Kähler magnetic charge, which implies reduction to abelian Chern-Simons term: the Kähler coupling strength does not appear at all in Chern-Simons term. The proportionality constant beween the electric and magnetic parts J_E and J_B of Kähler form however enters into the dynamics through the boundary conditions stating the weak form of electric-magnetic duality. At the Minkowskian side the proportionality constant must be proportional to g_K^2 to guarantee a correct value for the unit of Kähler electric charge - equal to that for electric charge in electron length scale- from the assumption that electric charge is proportional to the topologically quantized magnetic charge. It has been assumed that

$$J_E = \alpha_K J_B$$

holds true at *both sides* of the wormhole throat but this is an un-necessarily strong assumption at the Euclidian side. In fact, the self-duality of CP_2 Kähler form stating

$$J_E = J_B$$

favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

(b) Minkowskian and Euclidian regions should correspond to a strongly/weakly interacting phase in which Kähler magnetic/electric charges provide the proper description. In Euclidian regions associated with CP_2 type extremals there is a natural interpretation of interactions between magnetic monopoles associated with the light-like throats: for CP_2 type vacuum extremal itself magnetic and electric charges are actually identical and cannot be distinguished from each other. Therefore the duality between strong and weak coupling phases seems to be trivially true in Euclidian regions if one has $J_B = J_E$ at Euclidian side of the wormhole throat. This is however an un-necessarily strong condition as the following argument shows.

(c) In Minkowskian regions the interaction is via Kähler electric charges and elementary particles have vanishing total Kähler magnetic charge consisting of pairs of Kähler magnetic monopoles so that one has confinement characteristic for strongly interacting phase. Therefore Minkowskian regions naturally correspond to a weakly interacting phase for Kähler electric charges. One can write the action density at the Minkowskian side of the wormhole throat as

$$\frac{(J_E^2 - J_B^2)}{\alpha_K} = \alpha_K J_B^2 - \frac{J_B^2}{\alpha_K} \ .$$

The exchange $J_E \leftrightarrow J_B$ accompanied by $\alpha_K \rightarrow -1/\alpha_K$ leaves the action density invariant. Since only the behavior of the vacuum functional infinitesimally near to the wormhole throat matters by almost topological QFT property, the duality is realized. Note that the argument goes through also in Euclidian regions so that it does not allow to decide which is the correct form of weak form of electric-magnetic duality.

(d) S-duality could correspond geometrically to the duality between partonic 2-surfaces responsible for magnetic fluxes and string worlds sheets responsible for electric fluxes as rotations of Kähler gauge potentials around them and would be very closely related with the counterpart of T-duality implied by the strong form of general coordinate invariance and saying that space-like 3-surfaces at the ends of space-time sheets are equivalent with light-like 3-surfaces connecting them.

The boundary condition $J_E = J_B$ at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole throat. Self-duality is indeed an un-necessarily strong condition.

Comparison with standard view about dualities

One can compare the proposed realization of T, S and U to the more general dualities defined by the modular group $SL(2, Z)$, which in QFT framework can hold true for the path integral over all possible gauge field configurations. In the resent case the dualities hold true for every preferred extremal separately and the functional integral is only over the space-time projections of fixed Kähler form of CP_2. Modular invariance for Maxwell action was discussed by E. Verlinde for Maxwell action with θ term for a general 4-D compact manifold with Euclidian signature of metric in [B71]. In this case one has path integral giving sum over infinite number of extrema characterized by the cohomological equivalence class of the Maxwell field the action exponential to a high degree. Modular invariance is broken for CP_2: one obtains invariance only for $\tau \rightarrow \tau + 2$ whereas S induces a phase factor to the path integral.

(a) In the recent case these homology equivalence classes would correspond to homology equivalence classes of holomorphic partonic 2-surfaces associated with the critical points of Kähler function with respect to zero modes.

(b) In the case that the Euclidian contribution to the Kähler action is expressible solely in terms of wormhole throat Chern-Simons terms, and one can neglect the measurement interaction terms fixing the values of some classical conserved quantities to be equal with their quantal counterparts for the space-time surfaces allowed in quantum superposition, the exponent of Kähler action can be expressed in terms of Chern-Simons action density as

$$\begin{aligned} L &= \tau L_{C-S} \ , \\ L_{C-S} &= J \wedge A \ , \\ \tau &= \frac{1}{g_K^2} + i\frac{k}{4\pi} \ , \quad k = 1 \ . \end{aligned} \tag{8.6.1}$$

Here the parameter τ transforms under full $SL(2,Z)$ group as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \ . \tag{8.6.2}$$

The generators of $SL(2,Z)$ transformations are $T : \tau \rightarrow \tau + 1$, $S : \tau \rightarrow -1/\tau$. The imaginary part in the exponents corresponds to Kac-Moody central extension $k = 1$.

This form corresponds also to the general form of Maxwell action with CP breaking θ term given by

$$L = \frac{1}{g_K^2} J \wedge^* J + i\frac{\theta}{8\pi^2} J \wedge J \ , \quad \theta = 2\pi \ . \tag{8.6.3}$$

Hence the Minkowskian part mimics the θ term but with a value of θ for which the term does not give rise to CP breaking in the case that the action is full action for CP_2 type vacuum extremal so that the phase equals to 2π and phase factor case is trivial. It would seem that the deviation from the full action for CP_2 due to the presence of wormhole throats reducing the value of the full Kähler action for CP_2 type vacuum extremal could give rise to CP breaking. One can visualize the excluded volume as homologically non-trivial geodesic spheres with some thickness in two transverse dimensions. At the limit of infinitely thin geodesic spheres CP breaking would vanish. The effect is exponentially sensitive to the volume deficit.

CP breaking and ground state degeneracy

Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

(a) In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

(b) A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

(c) There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and short-lived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral mesons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

Remark: The proportionality of Minkowskian and Euclidian contributions to the same Chern-Simons term implies that the critical points with respect to zero modes appear for both the phase and modulus of vacuum functional. The Kähler function property does not allow extrema for vacuum functional as a function of complex coordinates of WCW since this would mean Kähler metric with non-Euclidian signature. If this were not the case. the stationary values of phase factor and extrema of modulus of the vacuum functional would correspond to different configurations.

8.6.7 Could one divide bundles?

TGD differs from string models in one important aspects: stringy diagrams do not have interpretation as analogs of vertices of Feynman diagrams: the stringy decay of partonic 2-surface to two pieces does not represent particle decay but a propagation along different paths for incoming particle. Particle reactions in turn are described by the vertices of generalized Feynman diagrams in which the ends of incoming and outgoing particles meet along partonic 2-surface. This suggests a generalization of K-theory for bundles assignable to the partonic 2-surfaces. It is good to start with a guess for the concrete geometric realization of the sum and product of bundles in TGD framework.

(a) The analogs of string diagrams could represent the analog for direct sum. Difference between bundles could be defined geometrically in terms of trouser vertex $A + B \to C$. B would by definition represent $C - A$. Direct sum could make sense for single particle states and have as space-time correlate the conservation of braid strands.

(b) A possible concretization in TGD framework for the tensor product is in terms of the vertices of generalized Feynman diagrams at which incoming light-like 3-D orbits of partons meet along their ends. The tensor product of incoming state spaces defined by fermionic oscillator algebras is naturally formed. Tensor product would have also now as a space-time correlate conservation of braid strands. This does not mean that the number of braid strands is conserved in reactions if also particular exchanges can carry the braid strands of particles coming to the vertex.

Why not define also division of bundles in terms of the division for tensor product? In terms of the 3-vertex for generalized Feynman diagrams $A \otimes B = C$ representing tensor product B would be by definition C/A. Therefore TGD would extend the K-theory algebra by introducing also division as a natural operation necessitated by the presence of the join along ends vertices not present in string theory. I would be surprised if some mathematician would not have published the idea in some exotic journal. Below I represent an argument that this notion could be also applied in the mathematical description of finite measurement resolution in TGD framework using inclusions of hyper-finite factor. Division could make possible a rigorous definition for for non-commutative quantum spaces.

Tensor division could have also other natural applications in TGD framework.

(a) One could assign bundles M_+ and M_- to the upper and lower light-like boundaries of CD. The bundle M_+/M_- would be obtained by formally identifying the upper and lower light-like boundaries. More generally, one could assign to the boundaries of CD positive and negative energy parts of WCW spinor fields and corresponding bundle structures in "half WCW". Zero energy states could be seen as sections of the unit bundle just like infinite rationals reducing to real units as real numbers would represent zero energy states.

(b) Finite measurement resolution would encourage tensor division since finite measurement resolution means essentially the loss of information about everything below measurement resolution represented as a tensor product factor. The notion of coset space formed by hyper-finite factor and included factor could be understood in terms of tensor division and give rise to quantum group like space with fractional quantum dimension in the case of Jones inclusions [K99]. Finite measurement resolution would therefore define infinite hierarchy of finite dimensional non-commutative spaces characterized by fractional quantum dimension. In this case the notion of tensor product would be somewhat more delicate since complex numbers are effectively replaced by the included algebra whose action creates states not distinguishable from each other [K99]. The action of algebra elements to the state $|B\rangle$ in the inner product $\langle A|B\rangle$ must be equivalent with the action of its hermitian conjugate to the state $\langle A|$. Note that zero energy states are in question so that the included algebra generates always modifications of states which keep it as a zero energy state.

Part II

PHYSICS AS INFINITE-DIMENSIONAL SPINOR GEOMETRY AND GENERALIZED NUMBER THEORY: BASIC VISIONS

Chapter 9

The Geometry of the World of Classical Worlds

9.1 Introduction

In this chapter a summary about basic ideas related to the construction of the Kähler geometry of infinite-dimensional configuration of 3-surfaces (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits) or "world of classical worlds" (WCW).

9.1.1 The quantum states of Universe as modes of classical spinor field in the "world of classical worlds"

The vision behind the construction of WCW geometry is that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or $M^4 \times CP_2$, where M^4 and M^4_+ denote Minkowski space and its light cone respectively. This WCW might be called the "world of classical worlds".

Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. One of the basic features of the Kähler geometry is that it is solely determined by the so called , which defines both the J and the components of the g in complex coordinates via the general formulas [A117]

$$\begin{aligned} J &= i\partial_k\partial_{\bar{l}}K dz^k \wedge d\bar{z}^l \ , \\ ds^2 &= 2\partial_k\partial_{\bar{l}}K dz^k d\bar{z}^l \ . \end{aligned} \tag{9.1.1}$$

Kähler form is covariantly constant two-form and can be regarded as a representation of imaginary unit in the tangent space of the WCW

$$J_{mr}J^{rn} = -g_m^{\ n} \ . \tag{9.1.2}$$

As a consequence Kähler form defines also symplectic structure in WCW.

9.1.2 Definition of Kähler function

The task of finding Kähler geometry for the WCW reduces to that of finding Kähler function and identifying the complexification. The main constraints on the Kähler function result from the requirement of Diff^4 symmetry and degeneracy. requires that the definition of the Kähler function assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$, the generalized Bohr orbit defining the classical physics associated with X^3. The natural guess is that Kähler function is defined by what might be called Kähler action, which is essentially Maxwell action with Maxwell field expressible in terms of CP_2 coordinates. Absolute minimization is the first guess for how to fix $X^4(X^3)$ uniquely.

It has however become clear that this option might well imply that Kähler is negative and infinite for the entire Universe so that the vacuum functional would be identically vanishing. Quantum criticality suggests the correct principle to be the criticality, that is vanishing of the second variation of Kähler action. This principle now follows from the conservation of Nöether currents the modified Dirac action.

If Kähler action would define a strictly deterministic variational principle, Diff^4 degeneracy and general coordinate invariance would be achieved by restricting the consideration to 3-surfaces Y^3 at the boundary of M^4_+ and by defining Kähler function for 3-surfaces X^3 at $X^4(Y^3)$ and diffeo-related to Y^3 as $K(X^3) = K(Y^3)$. This reduction might be called . The classical non-determinism of the Kähler action however introduces complications which might be however overcome by generalizing the notion of quantum gravitational holography.

9.1.3 WCW metric from symmetries

A complementary approach to the problem of constructing configuration space geometry is based on symmetries. The work of Dan [A71] [A71] has demonstrated that the Kähler geometry of loop spaces is unique from the existence of Riemann connection and fixed completely by the Kac Moody symmetries of the space. In 3-dimensional context one has even better reasons to expect uniqueness. The guess is that WCW is a union of symmetric spaces labelled by zero modes not appearing in the line element as differentials. The generalized conformal invariance of metrically 2-dimensional light like 3-surfaces acting as causal determinants is the corner stone of the construction. The construction works only for 4-dimensional space-time and imbedding space which is a product of four-dimensional Minkowski space or its future light cone with CP_2.

9.1.4 What principle selects the preferred extremals?

In positive energy ontology space-time surfaces should be analogous to Bohr orbits in order to make possible possible realization of general coordinate invariance. The first guess was that absolute minimization of Kähler action might be the principle selecting preferred extremals. One can criticize the assumption that extremals correspond to the absolute minima of Kähler action, as too strong. In particular, the notion of absolute minimization does not make sense in p-adic context unless one manages to reduce it to purely algebraic conditions.

What is needed is the association of a unique space-time surface to given 3-surfaces and there are many manners to achieve this.Therefore it is better to talk just about preferred extremals of Kähler action and accept as the fact that there are several proposals for what this notion could mean. For instance, one can consider the identification of space-time surface as associative (co-associative) sub-manifold meaning that tangent space of space-time surface can be regarded as associative (co-associative) sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "associative sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred commmutative (co-commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K9] defining also this kind of slicing and the approaches could be equivalent.

In zero energy ontology (ZEO) 3-surfaces become pairs of space-like 3-surfaces at the boundaries of causal diamond (CD). Even the light-like partonic orbits could be included to give the analog of Wilson loop. In absence of non-determinism of Kähler action this would suggest that the attribute "preferred" is un-necessary. There are however excellent reasons to expect that there is an infinite gauge degeneracy assignable to quantum criticality and represented in terms of Kac-Moody type transformations of partonic orbits respecting their light-likeness and giving rise to the degeneracy behind hierarchy of Planck constants $h_{eff} = n \times h$. n would give the number of conformal equivalence classes of space-time surfaces with same ends. In given measurement resolution one might however hope that the "preferred" could be dropped away.

The construction of quantum TGD in terms of the modified Dirac action associated with Kähler action led to what looks like a final answer to the question about the principle selecting preferred extremals. The Noether currents associated with modified Dirac action are conserved if second variations of Kähler action vanish. This is nothing but space-time correlate for quantum criticality and it is amusing that I failed to realize this for so long time. A further very important result is that in generic case the modes of induced spinor field are localized at 2-D surfaces from the condition that em charge is well-defined quantum number (W fields must vanish and also Z^0 field above weak scale in order to avoid large parity breaking effects).

In this chapter I will first consider the basic properties of the WCW, discuss briefly the various approaches to the geometrization of the WCW, and introduce the two complementary strategies based on a direct guess of Kähler function and on the group theoretical approach assuming that WCW can be regarded as a union of symmetric spaces. After these preliminaries a definition of the Kähler function is proposed and various physical and mathematical motivations behind the proposed definition are discussed. The key feature of the Kähler action is classical non-determinism, and various implications of the classical non-determinism are discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L74]
- Geometry of WCW [L38]
- Structure of WCW [L68]
- Symmetries of WCW [L70]

9.2 How to generalize the construction of WCW geometry to take into account the classical non-determinism?

If the imbedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of WCW geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [B24, B49] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction.

9.2.1 Quantum holography in the sense of quantum gravity theories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture Maldacena which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [B24] , quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface X^3 at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of WCW consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between WCW spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of WCW geometry. One might however hope that the notion of quantum holography generalizes.

9.2.2 How the classical determinism fails in TGD?

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of preferred extremals $X^4(Y^3)$ of Kähler action so that one would get finite or at most enumerably infinite number of replicas of a given WCW region and the construction would still reduce to the light cone boundary.

(a) This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

(b) CP_2 type vacuum extremals are different since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

(c) The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial for the ordinary elementary particle physics. It seems that the conformal symmetries responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the WCW metric line element.

(d) The possibility of space-time sheets with a negative time orientation with ensuing negative sign of classical energy is a further blow against δM^4_+ reductionism. Space-time sheets can be created as pairs of positive and negative energy space-time sheet from vacuum and this forces to modify radically the ontology of physics. Crossing symmetry allows to interpret particle reactions as a creation of zero energy states from vacuum, and the identification of the gravitational energy as the difference between positive and negative energies of matter supports the view that the net inertial (conserved Poincare-) energy of the universe vanishes both in quantal and classical sense. This option resolves unpleasant questions about net conserved quantum numbers of Universe, and provides an elegant interpretation of the vacuum extremals as correlates for systems with vanishing Poincare energy. This option is the only possible alternative from the point of view of TGD inspired cosmology where Robertson-Walker metrics are vacuum extremals with respect to inertial energy. In particular, super-symplectic invariance transforms to a fundamental symmetry of elementary particle physics besides the conformal symmetry associated with 3-D light like causal determinants which means a dramatic departure from string models unless it is somehow equivalent with the super-symplectic symmetry.

The treatment of the non-determinism in a framework in which the prediction of time evolution is seen as initial value problem, seems to be difficult. Also the notion of WCW becomes a messy concept. Zero energy ontology changes the situation completely. Light-like 3-surfaces become representations of generalized Feynman diagrams and brings in the notion of finite time resolution. One obtains a direct connection with the concepts of quantum field theory with path integral with cutoff replaced with a sum over various preferred extremals with cutoff in time resolution.

9.2.3 The notions of imbedding space, 3-surface, and configuration space

The notions of imbedding space, 3-surface (and 4-surface), and configuration space ("world of classical worlds", WCW) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably. Therefore it seems better to begin directly from the recent picture.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [K87, K88, K86] .

(a) p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

(b) With the discovery of zero energy ontology [K17, K21] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [K59] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

(c) The realization of the hierarchy of Planck constants [K27] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

(d) The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components - present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K65] .

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial.

(a) The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to Equivalence implied by General Coordinate Invariance. There was a problem related to the realization of General Coordinate Invariance since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

(b) Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

(c) At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

(d) A further complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of General Coordinate Invariance has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The obvious but rather ad hoc guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice has some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections. This option however failed to have a direct analog in the p-adic sectors of the world of classical worlds (WCW). The reason is that minimization does not make sense for the p-adic valued counterpart of Kähler action since it is not even well-defined although the field equations make sense p-adically. Therefore, if absolute minimization makes sense it must have expression as purely algebraic conditions.

For this reason it is better to talk just about preferred extremals of Kähler action and accept as the fact that there are several proposals for what this notion could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K9] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K9].

The notion of number theoretical compactification led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

(a) The conclusion was that one can assign to the 4-D tangent space $T(X^4(X^3_l)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

(b) In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X^3_l)))$ in principle fixes the tangent space at X^3_l, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

(c) At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X^3_l)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X^3_l))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y_l^3 parallel to X_l^3 follows under certain conditions on the induced metric of $X^4(X_l^3)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level.

(d) The weakest form of number theoretic compactification [K88] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

The study of the modified Dirac equation meant further steps of progress and lead to a rather detailed view about what preferred extremals are.

(a) The detailed construction of the generalized eigen modes of the modified Dirac operator D_K associated with Kähler action [K17] relies on the vision that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action. The view about TGD as almost topological QFT is realized if the eigenmodes correspond to the solutions of D_K, which are effectively 3-dimensional. Otherwise almost topological QFT property would require Chern-Simons action alone and this choice is definitely un-physical. The first guess was that the eigenmodes are restricted to X_l^3 and therefore analogous to spinorial shock waves. As I realized that number theoretical compactification requires the slicing of $X^4(X_l^3)$ by light-like 3-surfaces Y_l^3 parallel to X_l^3, it became clear that super-conformal gauge invariance with respect to the coordinate labeling the slices is a more natural manner to realized effective 3-dimensionality and guarantees that Y_l^3 is gauge equivalent with X_l^3 (General Coordinate Invariance).

(b) The eigen modes of the modified Dirac operator D_K have the defining property that they are localized in regions of X_l^3, where the induced Kähler gauge field is non-vanishing. This guarantees that the number of generalized eigen modes is finite so that Dirac determinant is also finite and algebraic number if eigenvalues are algebraic numbers, and therefore makes sense also in p-adic context although Kähler action itself does not make sense p-adically.

(c) The construction of WCW geometry in terms of modified Dirac action strengthens also the boundary conditions to the condition that there exists space-time coordinates in which the induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X_l^3. One could say that at X_l^3 situation is static both metrically and for the Maxwell field defined by the induced Kähler form.

(d) The final step in the rapid evolution of ideas that too place during three months - at least I hope so since I do not want to continue this updating endlessly - was the realization that the introduction of imaginary CP breaking instanton part to the Kähler action is possible and also necessary if one wants a stringy variant of Feynman rules. Imaginary part does not contribute to the WCW metric. This enriches the spectrum of the modified Dirac operator with an infinite number of conformal excitations breaking the effective 2-dimensionality of 3-surfaces and exact holography. Conformal excitations make possible stringy Feynman diagrammatics [K20] . A breaking of effective 3-dimensionality of space-time surface comes through the non-determinism of Kähler action which indeed is the mechanism breaking the effective 2-dimensionality. Dirac determinant can be defined in terms of zeta function regularization using Riemann Zeta. Finite measurement resolution realized in terms of braids defined on basis of purely physical criteria however forces a cutoff in conformal weight and finiteness so that number theoretical universality is not lost.

(e) This picture relying crucially on the the slicing of $X^4(X^3)$ did not yet fix the definition of preferred extremals analytically at the level of field equations. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results [K40] . These conditions make sense also in p-adic context and have a number theoretical universal form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_+ \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_\pm \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

(a) For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

(b) With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

(c) This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW (WCW) is a union of WCWs associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

A further piece of understanding emerged from the following observations.

(a) The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta} J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the WCW metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at WCW level and TGD is a genuine theory of gravitation at this level.

(b) WCW can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

(c) This leads to the identification of the coset space structure of the sub-WCW associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). WCW in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about WCW which I gave up as too naive!

(d) Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

9.2.4 The treatment of non-determinism of Kähler action in zero energy ontology

The non-determinism of Kähler action means that the reduction of the construction of WCW geometry to the light cone boundary fails. Besides degeneracy of the preferred extrema of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary.

One can imagine two kinds of causal determinants.

(a) Elementary particle horizons and light-like boundaries $X^3_l \subset X^4$ of 4-surfaces representing wormhole throats act as causal determinants for the space-time dynamics defined by Kähler action. The boundary values of this dynamics have been already considered.

(b) At imbedding space level causal determinants correspond to light like CD forming a fractal hierarchy of CDs within CDs. These causal determinants determine the dynamics of zero energy states having interpretation as pairs of initial and final states in standard quantum theory.

The manner to treat the classical non-determinism would be roughly following.

(a) The replacement of space-like 3-surface X^3 with X^3_l transforms initial value problem for X^3 to a boundary value problem for X^3_l. In principle one can also use the surfaces $X^3 \subset \delta CD \times CP_2$ if X^3_l fixes $X^4(X^3_l)$ and thus X^3 uniquely. For years an important question was whether both X^3 and X^3_l contribute separately to WCW geometry or whether they provide descriptions, which are in some sense dual. This lead to the notion of 7-3 duality and I even considered the possibility that $\delta M^4_+ \times CP_2$ could be replaced with a more general surface $X^3_l \times CP_2$ allowing also generalized symplectic and conformal symmetries. 7-3 duality is not a good term since the actual duality actually relates descriptions based on space-like 3-surfaces X^3 and light-like 3-surfaces X^3_l. Hence it seems that the proper place for 7-3 duality is in paper basked.

(b) Only Super-Kac-Moody type conformal algebra makes sense in the interior of X^3_l. In the 2-D intersections of X^3_l with the boundary of causal diamond (CD) defined as intersection of future and past directed light-cones super-symplectic algebra makes sense.

This implies effective two-dimensionality which is broken by the non-determinism represented using the hierarchy of CDs meaning that the data from these 2-D surfaces and their normal spaces at boundaries of CDs in various scales determine the WCW metric.

(c) An important question has been whether Kac-Moody and super-symplectic algebras provide descriptions which are dual in some sense. At the level of Super-Virasoro algebras duality seems to be satisfied in the sense of generalized coset construction meaning that the differences of Super Virasoro generators of super-symplectic and super Kac-Moody algebras annihilate physical states. Among other things this means that four-momenta assignable to the two Super Virasoro representations are identical. T he interpretation is in terms of a generalization of Equivalence Principle [K17, K21] . This gives also a justification for p-adic thermodynamics applying only to Super Kac-Moody algebra.

(d) Light-like 3-surfaces can be regarded also as generalized Feynman diagrams. The finite length resolution mean means also a cutoff in the number of generalized Feynman diagrams and this number remains always finite if the light-like 3-surfaces identifiable as maxima of Kähler function correspond to the diagrams. The finiteness of this number is also essential for number theoretic universality since it guarantees that the elements of M-matrix are algebraic numbers if momenta and other quantum numbers have this property. The introduction of new sub-CDs means also introduction of zero energy states in corresponding time scale.

(e) The notion of finite measurement resolution expressed in terms of hierarchy of CDs within CDs is important for the treatment of classical non-determinism. In a given resolution the non-determinism of Kähler action remains invisible below the time scale assigned to the smallest CDs. One could also say that complete non-determinism characterized in terms path integral with cutoff is replaced in TGD framework with the partial failure of classical non-determinism leading to generalized Feynman diagrams. This gives rise to to discrete coupling constant evolution and avoids the mathematical ill-definedness and infinities plaguing path integral formalism since the functional integral over 3-surfaces is well defined.

(f) Dirac determinant defining vacuum functional is assumed to correspond to exponent of Kähler action for its preferred extremal. Dirac determinant is defined as a product of finite number of eigenvalues of the transverse part $D_K(X^2)$ of the modified Dirac operator D_K assumed to have decomposition $D_K = D_K(X^2) + D_K(Y^2)$ reflecting the dual slicings of X^4 to string world sheets Y^2 and partonic 2-surfaces X^2. The existence of the slicing is supported by the properties of known extremals of Kähler action and strongly suggested by number theoretical compactification, and it implies among other things dimensional reduction to Minkowskian string model like theory and its Euclidian equivalent allowing to understand how Equivalence Principle is realized at space-time level. Finite number for the eigenvalues raises even hope that in a given resolution the functional integral reduces to Gaussian integral over a finite-dimensional space of logarithms of eigenvalues.

(g) One can ask why Kähler action and playing with all these delicacies related to the failure of complete determinism. After all, one could formally replace Kähler action with 4-volume as in brane models. Space-time surfaces would be minimal surfaces and Dirac operator would be standard Dirac operator for the induced metric. Dirac determinant would however become infinite since the modes would not be anymore analogs of cyclotron states necessarily localized to a finite region of X_l^3. Recall that for Kähler action X_l^3 indeed decomposes into patches inside with induced Kähler form is non-vanishing and Dirac determinant defining the exponent of Kähler function is well-defined and finite without any regularization procedure. Hence Kähler action is completely unique.

9.2.5 Category theory and WCW geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of WCW is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of WCW geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions.

In zero energy ontology the effects of non-determinism are taken into account in terms of causal diamonds forming a hierarchical fractal structure. One must allow also the unions of CDs, CDs within CDs, and probably also overlapping of CDs, and there are good reasons to expert that CDs and corresponding algebraic structures could define categories. If one does not allow overlapping CDs then set theoretic inclusion map defines a natural arrow. If one allows both unions and intersections then CDs would form a structure analogous to the set of open sets used in set theoretic topology. One could indeed see CDs (or rather their Cartesian products with CP_2) as analogs of open sets in Minkowskian signature.

So called ribbon categories seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories. This discussion based on standard ontology. In [K15] rather detailed category theoretical constructions are discussed. Important role is played by the notion of operad operad,operads : this structure can be assigned with both generalized Feynman diagrams and with the hierarchy of symplectic fusion algebras realizing symplectic analogs of the fusion rules of conformal field theories.

9.3 Constraints on WCW geometry

The constraints on WCW geometry result both from the infinite dimension of WCW and from physically motivated symmetry requirements. There are three basic physical requirements on the WCW geometry: namely four-dimensional Diff invariance, Kähler property and the decomposition of WCW into a union $\cup_i G/H_i$ of symmetric spaces G/H_i, each coset space allowing G-invariant metric such that G is subgroup of some 'universal group' having natural action on 3-surfaces. Together with the infinite dimensionality of WCW these requirements pose extremely strong constraints on WCW geometry. In the following we shall consider these requirements in more detail.

9.3.1 WCW as "the world of classical worlds"

The first naive view about WCW of TGD was that it consists of all 3-surfaces of $M^4_+ \times CP_2$ containing sets of

(a) surfaces with all possible manifold topologies and arbitrary numbers of components (N-particle sectors)

(b) singular surfaces topologically intermediate between two manifold topologies (see Fig. 9.3.1)

The symbol $C(H)$ will be used to denote the set of 3-surfaces $X^3 \subset H$. It should be emphasized that surfaces related by $Diff^3$ transformations will be regarded as different surfaces in the sequel.

These surfaces form a connected(!) space since it is possible to glue various N-particle sectors to each other along their boundaries consisting of sets of singular surfaces topologically intermediate between corresponding manifold topologies. The connectedness of the WCW is a necessary prerequisite for the description of topology changing particle reactions as continuous paths in WCW (see Fig. 9.3.1).

9.3.2 Diff^4 invariance and Diff^4 degeneracy

Diff^4 plays fundamental role as the gauge group of General Relativity. In string models $Diff^2$ invariance ($Diff^2$ acts on the orbit of the string) plays central role in making possible the

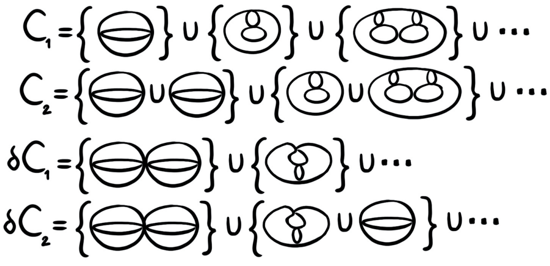

Figure 9.1: Structure of WCW: two-dimensional visualization

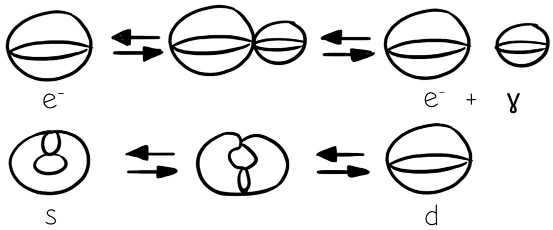

Figure 9.2: Two-dimensional visualization of topological description of particle reactions. a) Generalization of stringy diagram describing particle decay: 4-surface is smooth manifold and vertex a non-unique singular 3-manifold, b) Topological description of particle decay in terms of a singular 4-manifold but smooth and unique 3-manifold at vertex. c) Topological origin of Cabibbo mixing.

elimination of the time like and longitudinal vibrational degrees of freedom of string. Also in the present case the elimination of the tachyons (time like oscillatory modes of 3-surface) is a physical necessity and Diff^4 invariance provides an obvious manner to do the job.

In the standard functional integral formulation the realization of Diff^4 invariance is an easy task at the formal level. The problem is however that the path integral over four-surfaces is plagued by divergences and doesn't make sense. In the present case the WCW consists of 3-surfaces and only $Diff^3$ emerges automatically as the group of re-parameterizations of 3-surface. Obviously one should somehow define the action of Diff^4 in the space of 3-surfaces. Whatever the action of Diff^4 is it must leave the WCW metric invariant. Furthermore, the elimination of tachyons is expected to be possible only provided the time like deformations of the 3-surface correspond to zero norm vector fields of WCW so that 3-surface and its Diff^4 image have zero distance. The conclusion is that WCW metric should be both Diff^4 invariant and Diff^4 degenerate.

The problem is how to define the action of Diff^4 in $C(H)$. Obviously the only manner to achieve Diff^4 invariance is to require that the very definition of the WCW metric somehow associates a unique space-time surface to a given 3-surface for Diff^4 to act on! The obvious physical interpretation of this space time surface is as "classical space time" so that "Classical Physics" would be contained in WCW geometry. It is this requirement, which has turned out to be decisive concerning the understanding of the configuration space geometry. Amusingly enough, the historical development was not this: the definition of Diff^4 degenerate Kähler metric was found by a guess and only later it was realized that Diff^4 invariance and degeneracy could have been postulated from beginning!

9.3.3 Decomposition of WCW into a union of symmetric spaces G/H

The extremely beautiful theory of finite-dimensional symmetric spaces constructed by Elie Cartan suggests that WCW should possess a decomposition into a union of coset spaces $CH = \cup_i G/H_i$ such that the metric inside each coset space G/H_i is left invariant under the infinite dimensional isometry group G. The metric equivalence of surfaces inside each coset space G/H_i does not mean that 3-surfaces inside G/H_i are physically equivalent. The reason is that the vacuum functional is exponent of Kähler action which is not isometry invariant so that the 3-surfaces, which correspond to maxima of Kähler function for a given orbit, are in a preferred position physically. For instance, one can calculate functional integral around this maximum perturbatively. The sum of over i means actually integration over the zero modes of the metric (zero modes correspond to coordinates not appearing as coordinate differentials in the metric tensor).

The coset space G/H is a symmetric space only under very special Lie-algebraic conditions. Denoting the Cartan decomposition of the Lie-algebra g of G to the direct sum of H Lie-algebra h and its complement t by $g = h \oplus t$, one has

$$[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ .$$

This decomposition turn out to play crucial role in guaranteeing that G indeed acts as isometries and that the metric is Ricci flat.

The four-dimensional $Diff$ invariance indeed suggests to a beautiful solution of the problem of identifying G. The point is that any 3-surface X^3 is $Diff^4$ equivalent to the intersection of $X^4(X^3)$ with the light cone boundary. This in turn implies that 3-surfaces in the space $\delta H = \delta M^4_+ \times CP_2$ should be all what is needed to construct WCW geometry. The group G can be identified as some subgroup of diffeomorphisms of δH and H_i diffeomorphisms of the 3-surface X^3. Since G preserves topology, WCW must decompose into union $\cup_i G/H_i$, where i labels 3-topologies and various zero modes of the metric. For instance, the elements of the Lie-algebra of G invariant under WCW complexification correspond to zero modes.

The reduction to the light cone boundary, identifiable as the moment of big bang, looks perhaps odd at first. In fact, it turns out that the classical non-determinism of Kähler action forces does not allow the complete reduction to the light cone boundary: physically this is a highly desirable implication but means a considerable mathematical challenge.

Kähler property implies that the tangent space of the configuration space allows complexification and that there exists a covariantly constant two-form J_{kl}, which can be regarded as a representation of the imaginary unit in the tangent space of the WCW:

$$J_k^{\ r} J_{rl} = -G_{kl} \ . \tag{9.3.1}$$

There are several physical and mathematical reasons suggesting that WCW metric should possess Kähler property in some generalized sense.

(a) Kähler property turns out to be a necessary prerequisite for defining divergence free WCW integration. We will leave the demonstration of this fact later although the argument as such is completely general.

(b) Kähler property very probably implies an infinite-dimensional isometry Freed shows that loop group allows only single Kähler metric with well Riemann connection and this metric allows local G as its isometries!

To see this consider the construction of Riemannian connection for $Map(X^3, H)$. The defining formula for the connection is given by the expression

$$\begin{aligned} 2(\nabla_X Y, Z) &= X(Y,Z) + Y(Z,X) - Z(X,Y) \\ &+ ([X,Y],Z) + ([Z,X],Y) - ([Y,Z],X) \end{aligned} \tag{9.3.2}$$

X, Y, Z are smooth vector fields in $Map(X^3, G)$. This formula defines $\nabla_X Y$ uniquely provided the tangent space of Map is complete with respect to Riemann metric. In the finite-dimensional case completeness means that the inverse of the covariant metric tensor exists so that one can solve the components of connection from the conditions stating the covariant constancy of the metric. In the case of the loop spaces with Kähler metric this is however not the case.

Now the symmetry comes into the game: if X, Y, Z are left (local gauge) invariant vector fields defined by the Lie-algebra of local G then the first three terms drop away since the scalar products of left invariant vector fields are constants. The expression for the covariant derivative is given by

$$\nabla_X Y = (Ad_X Y - Ad_X^* Y - Ad_Y^* X)/2 \quad (9.3.3)$$

where Ad_X^* is the adjoint of Ad_X with respect to the metric of the loop space.

At this point it is important to realize that Freed's argument does not force the isometry group of WCW to be $Map(X^3, M^4 \times SU(3))$! Any symmetry group, whose Lie algebra is complete with respect to the WCW metric (in the sense that any tangent space vector is expressible as superposition of isometry generators modulo a zero norm tangent vector) is an acceptable alternative.

The Kähler property of the metric is quite essential in one-dimensional case in that it leads to the requirement of left invariance as a mathematical consistency condition and we expect that dimension three makes no exception in this respect. In 3-dimensional case the degeneracy of the metric turns out to be even larger than in 1-dimensional case due to the four-dimensional Diff degeneracy. So we expect that the metric ought to possess some infinite-dimensional isometry group and that the above formula generalizes also to the 3-dimensional case and to the case of local coset space. Note that in M^4 degrees of freedom $Map(X^3, M^4)$ invariance would imply the flatness of the metric in M^4 degrees of freedom.

The physical implications of the above purely mathematical conjecture should not be underestimated. For example, one natural looking manner to construct physical theory would be based on the idea that configuration space geometry is dynamical and this approach is followed in the attempts to construct string theories [B20] . Various physical considerations (in particular the need to obtain oscillator operator algebra) seem to imply that WCW geometry is necessarily Kähler. The above result however states that WCW Kähler geometry cannot be dynamical quantity and is dictated solely by the requirement of internal consistency. This result is extremely nice since it has been already found that the definition of the WCW metric must somehow associate a unique classical space time and "classical physics" to a given 3-surface: uniqueness of the geometry implies the uniqueness of the "classical physics".

(c) The choice of the imbedding space becomes highly unique. In fact, the requirement that WCW is not only symmetric space but also (contact) Kähler manifold inheriting its (degenerate) Kähler structure from the imbedding space suggests that spaces, which are products of four-dimensional Minkowski space with complex projective spaces CP_n, are perhaps the only possible candidates for H. The reason for the unique position of the four-dimensional Minkowski space turns out to be that the boundary of the light cone of D-dimensional Minkowski space is metrically a sphere S^{D-2} despite its topological dimension $D-1$: for $D = 4$ one obtains two-sphere allowing Kähler structure and infinite parameter group of conformal symmetries!

(d) It seems possible to understand the basic mathematical structures appearing in string model in terms of the Kähler geometry rather nicely.

   i. The projective representations of the infinite-dimensional isometry group (not necessarily Map!) correspond to the ordinary representations of the corresponding centrally extended group [A74]. The representations of Kac Moody group Schwartz,Green and WCW approach would explain their occurrence, not as a result of some quantization procedure, but as a consequence of symmetry of the underlying geometric structure.

ii. The bosonic oscillator operators of string models would correspond to centrally extended Lie-algebra generators of the isometry group acting on spinor fields of the WCW.

iii. The "fermionic" fields (Ramond fields, Schwartz,Green) should correspond to gamma matrices of the WCW. Fermionic oscillator operators would correspond simply to contractions of isometry generators j_A^k with complexified gamma matrices of WCW

$$\begin{aligned} \Gamma_A^{\pm} &= j_A^k \Gamma_k^{\pm} \\ \Gamma_k^{\pm} &= (\Gamma^k \pm J_l^k \Gamma^l)/\sqrt{2} \end{aligned} \tag{9.3.4}$$

(J_l^k is the Kähler form of WCW) and would create various spin excitations of WCW spinor field. $\Gamma_k^{\pm}$ are the complexified gamma matrices, complexification made possible by the Kähler structure of the WCW.

This suggests that some generalization of the so called Super Kac Moody algebra of string models [B64, B46] should be regarded as a spectrum generating algebra for the solutions of field equations in configuration space.

Although the Kähler structure seems to be physically well motivated there is a rather heavy counter argument against the whole idea. Kähler structure necessitates complex structure in the tangent space of WCW. In CP_2 degrees of freedom no obvious problems of principle are expected: WCW should inherit in some sense the complex structure of CP_2.

In Minkowski degrees of freedom the signature of the Minkowski metric seems to pose a serious obstacle for complexification: somehow one should get rid of two degrees of freedom so that only two Euclidian degrees of freedom remain. An analogous difficulty is encountered in quantum field theories: only two of the four possible polarizations of gauge boson correspond to physical degrees of freedom: mathematically the wrong polarizations correspond to zero norm states and transverse Hilbert space with Euclidian metric. Also in string model analogous situation occurs: in case of D-dimensional Minkowski space only $D-2$ transversal degrees of freedom are physical. The solution to the problem seems therefore obvious: WCW metric must be degenerate so that each vibrational mode spans effectively a 2-dimensional Euclidian plane allowing complexification.

It will be found that the definition of Kähler function to be proposed indeed provides a solution to this problem and also to the problems listed before.

(a) The definition of the metric doesn't differentiate between 1- and N-particle sectors, avoids spin statistics difficulty and has the physically appealing property that one can associate to each 3-surface a unique classical space time: classical physics is described by the geometry of WCW! And the geometry of WCW is determined uniquely by the requirement of mathematical consistency!

(b) Complexification is possible only provided the dimension of the Minkowski space equals to four(!).

(c) It is possible to identify a unique candidate for the necessary infinite-dimensional isometry group G. G is subgroup of the diffeomorphism group of $\delta M_+^4 \times CP_2$. Essential role is played by the fact that the boundary of the four-dimensional light cone, which, despite being topologically 3-dimensional, is metrically two-dimensional(!) Euclidian sphere, and therefore allows infinite-parameter groups of isometries as well as conformal and symplectic symmetries and also Kähler structure unlike the higher-dimensional light cone boundaries. Therefore WCW metric is Kähler only in the case of four-dimensional Minkowski space and allows symplectic $U(1)$ central extension without conflict with the no-go theorems about higher dimensional central extensions.

The study of the vacuum degeneracy of Kähler function defined by Kähler action forces to conclude that the isometry group must consist of the symplectic transformations of $\delta H = \delta M_+^4 \times CP_2$. The corresponding Lie algebra can be regarded as a loop algebra associated with the symplectic group of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light cone boundary. Thus the finite-dimensional group G defining loop group in case of string models extends to an infinite-dimensional group in TGD context. This group is a real monster! The radial Virasoro localized with respect to $S^2 \times CP_2$ defines naturally complexification for both G and H. The general form of the Kähler metric deduced on basis of this symmetry has same qualitative properties as that deduced from Kähler function identified as the absolute minimum of Kähler action. Also the zero modes, among them isometry invariants, can be identified.

(d) The construction of the WCW spinor structure is based on the identification of the WCW gamma matrices as linear superpositions of the oscillator operators associated with the induced spinor fields. The extension of the symplectic invariance to super symplectic invariance fixes the anti-commutation relations of the induced spinor fields, and WCW gamma matrices correspond directly to the super generators. Physics as number theory vision suggests strongly that WCW geometry exists for 8-dimensional imbedding space only and that the choice $M^4_+ \times CP_2$ for the imbedding space is the only possible one.

9.4 Identification of the Kähler function

There are two approaches to the construction of WCW geometry: a direct physics based guess of the Kähler function and a group theoretic approach based on the hypothesis that CH can be regarded as a union of symmetric spaces. The rest of this chapter is devoted to the first approach.

9.4.1 Definition of Kähler function

Quite generally, Kähler function K defines Kähler metric in complex coordinates via the following formula

$$J_{k\bar{l}} = ig_{k\bar{l}} = i\partial_k \partial_{\bar{l}} K \ . \qquad (9.4.1)$$

Kähler function is defined only modulo a real part of holomorphic function so that one has the gauge symmetry

$$K \rightarrow K + f + \overline{f} \ . \qquad (9.4.2)$$

Let X^3 be a given 3-surface and let X^4 be any four-surface containing X^3 as a sub-manifold: $X^4 \supset X^3$. The 4-surface X^4 possesses in general boundary. If the 3-surface X^3 has nonempty boundary δX^3 then the boundary of X^3 belongs to the boundary of X^4: $\delta X^3 \subset \delta X^4$.

The projection of CP_2 Kähler form J (induced Kähler form) defines Maxwell field on X^4. One can associate to Kähler form Maxwell action and also Chern-Simons anomaly term proportional to $\int_{X^4} J \wedge J$ in well known manner. Chern Simons term is purely topological term and well defined for orientable 4-manifolds, only. Since there is no deep reason for excluding non-orientable space-time surfaces it seems reasonable to drop Chern Simons term from consideration. Therefore Kähler action $S_K(X^4)$ can be defined as

$$S_K(X^4) = k_1 \int_{X^4; X^3 \subset X^4} J \wedge (*J) \ . \qquad (9.4.3)$$

The sign of the square root of the metric determinant, appearing implicitly in the formula, is defined in such a manner that the action density is negative for the Euclidian signature of the induced metric and such that for a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density.

The notational convention

$$k_1 \equiv \frac{1}{16\pi\alpha_K} , \tag{9.4.4}$$

where α_K will be referred as Kähler coupling strength will be used in the sequel. If the preferred extremals minimize/maximize [K88] the absolute value of the action in each region where action density has a definite sign, the value of α_K can depend on space-time sheet.

Induced Kähler form defines a Maxwell field and it is important to characterize precisely its relationship to the gauge fields as they are defined in gauge theories. Kähler form J is related to the corresponding Maxwell field F via the formula

$$J = \frac{g_K}{\hbar}F . \tag{9.4.5}$$

Similar relationship holds true also for the other induced gauge fields. The inverse proportionality of J to $\hbar$ does not matter in the ordinary gauge theory context where one routinely choses units by putting $\hbar = 1$ but becomes very important when one considers a hierarchy of Planck constants [K27] . By $\alpha_K = g_K^2/4\pi\hbar$ the large Planck constant means weaker interactions and convergence of the functional integral defined by the exponent of Kähler function and one can argue that the convergence of the functional integral is what forces the hierarchy of Planck constants. This is in accordance with the vision that Mother Nature likes theoreticians and takes care that the perturbation theory works by making a phase transition increasing the value of the Planck constant in the situation when perturbation theory fails. This leads to a replacement of the M^4 (or more precisely, causal diamond CD) and CP_2 factors of the imbedding space ($CD \times CP_2$) with its $r = \hbar/\hbar_0$-fold singular covering (one can consider also singular factor spaces). If the components of the space-time surfaces at the sheets of the covering are identical, one can interpret r-fold value of Kähler action as a sum of r identical contributions from the sheets of the covering with ordinary value of Planck constant and forget the presence of the covering. Physical states are however different even in the case that one assumes that sheets carry identical quantum states and anyonic phase could correspond to this kind of phase [K65] .

One can define the Kähler function in the following manner. Consider first the case $H = M^4_+ \times CP_2$ and neglect for a moment the non-determinism of Kähler action. Let X^3 be a 3-surface at the light-cone boundary $\delta M^4_+ \times CP_2$. Define the value $K(X^3)$ of Kähler function K as the value of the Kähler action for some preferred extremal in the set of four-surfaces containing X^3 as a sub-manifold:

$$K(X^3) = K(X^4_{pref}) , \quad X^4_{pref} \subset \{X^4|X^3 \subset X^4\} . \tag{9.4.6}$$

The original hypothesis was that the intersections of the four-surface with the boundary of the light cone ($\delta M^4_+ \times CP_2$) defined by the condition $a = \sqrt{(m^0)^2 - r_M^2} = 0$ and with the surface $a \to \infty$ are not subject to variational conditions since this would have meant that all universes have vanishing classical conserved quantities. Define the value $K(Y^3)$ of Kähler function for all Diff^4 related 3-surfaces at $X^4(X^3)$ as $K(X^3)$ so that the metric is $Diff^4$ degenerate.

Absolute minimization of Kähler action was the first identification for the principle selecting the preferred extremal. The worst that can happen for this option is that the value of Kähler action is negative and infinite for the entire Universe so that the vacuum functional defined by its exponent vanishes. A more plausible choice of the preferred extremal is based on the assumption that the absolute values of the contributions to Kähler action are separately minimized in regions of definite sign for Kähler action density. This implies the minimization of the absolute value of the net action and extremals are as near as possible to vacuum extremals, and minimize their energy: this gives hopes of constructing the extremals using only data at X^3. I ended up to this option from number theoretical vision, which also leads to an explicit proposal for how to construct these extremals of Kähler action [K88] .

This simple picture is too simple to be true and must be generalized even in case of M^4_+. It has however become clear that the gigantic symmetries associated with $\delta M^4_+ \times CP_2$ are also symmetries at the laboratory scale. Furthermore, M^4 is as a good option as M^4_+, and number theoretically even better since it allows interpretation as the space of hyperquaternions. Also exact Poincare invariance favors M^4 option.

M^4 option makes sense only if X^3 is selected uniquely by the internal geometry of X^4. The possibility of negative Poincare energies inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. By crossing symmetry this view is consistent with elementary particle physics.

Consistency with macroscopic physics can be achieved if gravitational energy is defined as the difference of Poincare energies of positive and negative energy matter. This definition indeed resolves the long lasting puzzle created by the fact that Robertson-Walker cosmologies correspond to vacuum extremals with respect to inertial energy and momentum. Space-time surfaces consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment to separate space-time sheets. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action. Also a natural fixing of Diff^4 gauge becomes possible. This view is also consistent with the non-determinism of Kähler action. This option works for both M^4_+ and M^4 and is very probably the correct one.

9.4.2 Minkowski space or its future light cone or something else?

The basic question is whether one should choose the imbedding space to be $M^4 \times CP_2$ or $M^4_+ \times CP_2$.

M^4_+ option has several nice features.

(a) Since future light cone corresponds to vacuum cosmology (cosmic time is Lorentz invariant distance) the latter choice seems to be more physical since it makes big bang cosmology a geometrical necessity and implies the arrow of time naturally. The loss of exact Poincare invariance could be seen as a problem. Even if one accepts light cone alternative as the correct one (as we shall cautiously do) there are two alternative definitions of the Kähler function.

(b) For M^4_+ option minimizing four-surfaces belong to the future light cone so that the presence of the light cone boundary reflects itself in the properties of minimizing four-surfaces: big bang cosmology is expected to manifest itself in the time development of four-surfaces. This alternative implies the loss of Poincare invariance in cosmological scales: in the laboratory scale Poincare invariance is of course practically exact since Poincare invariance is a symmetry of the extremals of Kähler action and broken only in the set of absolute minima.

(c) One could avoid the loss of Poincare invariance without totally giving up the light cone cosmology by defining the metric of $C(M^4_+ \times CP_2)$ as the restriction of the metric of $C(M^4 \times CP_2)$: minimizing four-surfaces would belong to M^4 although 3-surfaces belong to light cone. Poincare invariance becomes exact symmetry at the Lie algebra level broken only "kinematically". One can however heavily criticize this alternative: if one wants to interpret four-surface as an actual space-time then it is highly artificial to allow four-surfaces, which do not belong to the actual imbedding space. A second questionable feature is that the presence of the light cone boundary does not reflect itself in the properties of 4-surfaces as it should.

M^4 option makes many highly non-trivial and nice predictions which are allowed but not predicted by M^4_+ option. The mathematical elegance of M^4 option is definitely superior to that of M^4_+ alternative.

(a) Suppose that the classical non-determinism of Kähler action indeed implies that all light like 7-surfaces $X^3_l \times CP_2$, where X^3_l is light like surface of M^4_+, can act as causal determinants. As already noticed, this makes sense if pairs of space-time sheets having opposite time orientation and opposite energies can be created from vacuum at these 7-surfaces.

(b) For M^4 option the total energy of classical and by quantum-classical correspondence of also quantum universes must vanish and all matter would be created from vacuum. There would be no need to ponder the academic but very nasty question about total fermion numbers of the universe: all states of the universe would be vacua as far net quantum numbers are considered. Of course, also in the case of M^4_+ it is possible and natural to postulate that nothing flows out from the future light cone or into it and this would imply vanishing total quantum numbers.

(c) M^4 option allows both maximal space-time symmetries and forces the fractal hierarchy of cosmologies inside cosmologies defined by light cones inside light cones as does in fact also M^4_+ option. These cosmologies would be a result of dynamics rather than of the properties of the imbedding space. If the separation of positive and negative energy densities can be achieved in cosmological length scales, this option might work. The nice feature is that WCW becomes a union of WCWs associated with various light-like causal determinants $X^3_l \times CP_2$ with the most plausible identification of X^3_l being as a union of future and past directed light cone boundaries.

(d) Poincare transformations act as symmetries and one can assign to given space-time sheet unique value of geometric time as the moment of geometric time when it was created. This is of utmost importance concerning the understanding of the relationship between subjective and geometric time in TGD inspired theory of consciousness. It makes also possible to assign to S-matrix time parameter identifiable as interaction time without problems with energy conservation.

(e) For M^4 option the super conformal invariance associated with light like 3-surfaces $X^3_l \times CP_2$ and super-conformal invariance associated with 3-dimensional light-like boundaries and "elementary particle" horizons of space-time surfaces interact very naturally. The super conformal invariance associated with 3-dimensional light-like surfaces corresponds to the Super Kac Moody symmetries of string models with Poincare symmetry being exact, and determines mass squared formula. The super-symplectic invariance associated with $X^3_l \times CP_2$ is something new and it modifies that the stringy mass formula. The interaction of super Kac-Moody conformal algebra in super-symplectic algebra is of special significance in the construction of quantum theory.

(f) M^4 can be interpreted as the space of quaternions with Minkowski metric identifiable as the imaginary part of q^2. The imbedding space can be interpreted as a space having hyper-octonionic tangent space structure [K88] , and space-time surfaces as maximal associative sub-manifolds with hyper-quaternionic tangent space structure. Furthermore, the fact that CP_2 parameterizes hyper-quaternionic planes of hyper-octonion space, raises $M^4 \times CP_2$ in a completely unique position number theoretically.

Which of this alternatives is correct? At the practical laboratory level there are no testable differences between these options and it is very difficult to test whether the first moments of our cosmology are associated with a cosmology inside cosmology or M^4_+. One could however say that whereas M^4_+ option allows what seems to be the correct interpretation, M^4 option forces it, and its mathematical elegance is superior. For a long time I nearly-believed that M^4_+ alternative is the correct one but after a long period of certainty I began to feel more and more empathy towards M^4 option.

It actually turned out that both options are in a well-defined sense correct. The notion of zero energy ontology leads to the conclusion that WCW can be regarded as a union of sub-configuration spaces associated with spaces $CD \times CP_2$, where CD denotes what I have called causal diamond and defined as intersection of future and past directed light-cones of M^4. The position for the lower tip of CD varies in M^4 and defines the position of CD in M^4 since the temporal distance between lower and upper tips is assumed to be quantized as power of two multiple of CP_2 size (this predicts p-adic length scale hypothesis). At the level of single CD Poincare invariance is broken to Lorentz invariance but the union over sub-WCWs associated with CDs guarantees global Poincare invariance. These aspects are discussed in more detail in the next section.

9.4.3 The values of the Kähler coupling strength?

Since the vacuum functional of the theory turns out to be essentially the exponent $exp(K)$ of the Kähler function, the dynamics depends on the normalization of the Kähler function. Since the Theory of Everything should be unique it would be highly desirable to find arguments fixing the normalization or equivalently the possible values of the Kähler coupling strength α_K. Also a discrete spectrum of values is acceptable.

The quantization of Kähler form could result in the following manner. It will be found that Abelian extension of the isometry group results by coupling spinors of WCW to a multiple of Kähler potential. This means that Kähler potential plays role of gauge connection so that Kähler form must be integer valued by Dirac quantization condition for magnetic charge. So, if Kähler form is co-homologically nontrivial it is quantized.

Unfortunately, the exact definition of renormalization group concept is not at all obvious. There is however a much more general but more or less equivalent manner to formulate the condition fixing the value of α_K. Vacuum functional $exp(K)$ is analogous to the exponent $exp(-H/T)$ appearing in the definition of the partition function of a statistical system and S-matrix elements and other interesting physical quantities are integrals of type $\langle O\rangle = \int exp(K)O\sqrt{G}dV$ and therefore analogous to the thermal averages of various observables. α_K is completely analogous to temperature. The critical points of a statistical system correspond to critical temperatures T_c for which the partition function is non-analytic function of $T - T_c$ and according RGE hypothesis critical systems correspond to fixed points of renormalization group evolution. Therefore, a mathematically more precise manner to fix the value of α_K is to require that some integrals of type $\langle O\rangle$ (not necessary S-matrix elements) become non-analytic at $1/\alpha_K - 1/\alpha_K^c$.

This analogy suggests also a physical motivation for the unique value or value spectrum of α_K. Below the critical temperature critical systems suffer something analogous to spontaneous magnetization. At the critical point critical systems are characterized by long range correlations and arbitrarily large volumes of magnetized and non-magnetized phases are present. Spontaneous magnetization might correspond to the generation of Kähler magnetic fields: the most probable 3-surfaces are Kähler magnetized for subcritical values of α_K. At the critical values of α_K the most probable 3-surfaces contain regions dominated by either Kähler electric and or Kähler magnetic fields: by the compactness of CP_2 these regions have in general outer boundaries.

This suggests that 3-space has hierarchical, fractal like structure: 3-surfaces with all sizes (and with outer boundaries) are possible and they have suffered topological condensation on each other. Therefore the critical value of α_K allows the richest possible topological structure for the most probable 3-space. In fact, this hierarchical structure is in accordance with the basic ideas about renormalization group invariance. This hypothesis has highly nontrivial consequences even at the level of ordinary condensed matter physics.

The assumption about single critical value of α_K is probably too strong. p-Adic length scale hierarchy together with the immense vacuum degeneracy of the Kähler action leads to the hypothesis that different p-adic length scales correspond to different critical values of α_K, and that ordinary coupling constant evolution is replaced by a piecewise constant evolution induced by that for α_K.

Renormalization group invariance is closely related with criticality. The self duality of the Kähler form and Weyl tensor of CP_2 indeed suggest RG invariance. The point is that in

$N = 1$ super-symmetric field theories duality transformation relates the strong coupling limit for ordinary particles with the weak coupling limit for magnetic monopoles and vice versa. If the theory is self dual these limits must be identical so that action and coupling strength must be RG invariant quantities. The geometric realization of the duality transformation is easy to guess in the standard complex coordinates ξ_1, ξ_2 of CP_2 (see Appendix of the book). In these coordinates the metric and Kähler form are invariant under the permutation $\xi_1 \leftrightarrow \xi_2$ having Jacobian -1.

Consistency requires that particles of the theory are equivalent with magnetic monopoles: the so called CP_2 type extremals identified as elementary particles are isometric imbeddings of CP_2 and can be regarded as monopoles. The magnetic flux however flows in internal degrees of freedom (possible by nontrivial homology of CP_2) so that no long range $1/r^2$ magnetic field is created. The magnetic contribution to Kähler action is positive and this suggests that ordinary magnetic monopoles are not stable, since they do not minimize Kähler action: a cautious conclusion in accordance with the experimental evidence is that TGD does not predict magnetic monopoles. It must be emphasized that the prediction of monopoles of practically all gauge theories and string theories and follows from the existence of a conserved electromagnetic charge.

9.4.4 Is preferred extremal property needed at all in zero energy ontology?

A central conjecture has been that quaternionic 4-surfaces correspond to preferred extremals of Kähler action [K17] fixed by some principle.

The central question has been "How to identify preferred extremals?". Here I have made several proposals, which might be as such correct. It took surprisingly long time to make the trivial observation that in zero energy ontology (ZEO) the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is unique. Now the action principle is not-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and same Kähler action and same conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with 2-D criticality and is expected to be present also now.

A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

If one accepts this argument then the following sections relating to the question what preferred extremal property could mean, are obsolete. One can of course ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations might be present and correspond to the Bohr orbit property, space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

9.4.5 Absolute minimization or something else?

In this and following sections it is assumed that preferred extremals property is not obsolete notion. As notice, this is true in positive energy ontology but need not be true in ZEO unless one accepts additional correlations between 3-surfaces at the ends of CD suggested by Bohr orbitology and quantum classical correspondence (QCC).

The requirement that the 4-surface having given 3-surface as its sub-manifold is absolute minimum of the Kähler action is the most obvious guess for the principle selecting the preferred extremals and has been taken as a working hypothesis for about one and half decades.

The principle admittedly looks somewhat ad hoc, and in the beginning of 2005 I proposed that that absolute minimization principle should be perhaps relaxed in the sense that the absolute values of the contributions to the net Kähler action coming from regions where the action density has definite sign [K88] are separately minimized (or maximized in dual case). This would allow α_K to depend on space-time sheet and allow to understand p-adic evolution of α_K. The objection is that in p-adic context absolute minimization does not make sense. Absolute minimum and maximum can be defined algebraically (and thus p-adically) as extremal with non-degenerate Hessian but one cannot distinguish between minimum and maximum p-adically.

Later further number theoretical ideas and the proposal for the formulation of quantum TGD in terms of second quantized induced spinor fields at light-like 3-surfaces led to a mathematically beautiful and physically transparent vision about the choice of the preferred extremals $X^4(X^3)$ of Kähler action discussed in detail in [K17, K88] .

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

Further insights emerged through the realization that Noether currents modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

The vanishing of second variations of preferred extremals for deformations representing dynamical symmetries suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

(a) The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second and even third variations vanish identically (the "tip" of the multi-furcation set).

(b) The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

(c) The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

(d) Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

Is criticality consistent with absolute minimization?

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

(a) The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action need not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler cspin. The notion of p-adic manifold [K115] gives good hopes about defining p-adic variant of Kähler action as a kind of algebraic continuation of the real Kähler action.

(b) The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.

(c) These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

(a) Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).

(b) Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M_+^4 \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action.

9.4.6 How to identify the preferred extremals of Kähler action?

The identification of the preferred extremals turned out to be far from trivial and almost two decades was needed to realize that they are several possibly equivalent identifications and at this moment it is not possible to say anything final.

(a) The original physically motivated but otherwise ad hoc assumption was that preferred extremals correspond to absolute minima of Kähler action. This option failed to have a direct analog in the p-adic sectors of the world of classical worlds (WCW). The reason is that minimization does not make sense for the p-adic valued counterpart of Kähler action since it is not even well-defined although the field equations make sense p-adically. Therefore, if absolute minimization makes sense it must have expression as purely algebraic conditions.

(b) Much later number theoretic vision [K88] led to a more realistic proposal relying on the notion of number theoretic compactification stating that light-like 3-surfaces X_l^3 or even space-time surfaces $X^4(X_l^3)$ themselves can be regarded as surfaces in either M^8 or $M^4 \times CP_2$.

   i. Light-like 3-surfaces X_l^3 are the basic dynamical objects of quantum TGD and are defined by the throats of wormhole contacts and of CP_2 type vacuum extremals and have interpretation as elementary particles.

   ii. M^8 is regarded as a sub-space of complexified octonions with Minkowskian signature of natural metric (I have referred to M^8 as the space HO of hyperoctonions). The mapping of connected components of $X_l^3 \subset M^8$ to $X_l^3 \subset M^4 \times CP_2$ is possible if $X^4(X_l^3)$ has $M_i^2 \subset M^4$ as a subspace of its tangent space at each point of $X_{l,i}^3$. $X^4(X_l^3) \subset M^8$ would correspond to hyper-quaternionic 4-surface meaning that its tangent space is hyper-quaternionic at each point. $X^4(X_l^3) \subset M^4 \times CP_2$ would in turn be a preferred extremal of Kähler action. The condition that M_i^2 belongs to the tangent space of $X^4(X_l^3)$ at $X_{l,i}^3$ fixes at least partially the boundary conditions selecting preferred extremals of Kähler action in $M^4 \times CP_2$ and preferred hyper-quaternionic surfaces in M^8. M^2 has interpretation as the plane of non-physical polarizations.

   iii. The detailed construction of the generalized eigen modes of the Chern-Simons action [K17] relies on the requirement that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action. This is achieved if the eigenmodes correspond to singular shockwave type solutions of modified Dirac operator defined by Kähler action restricted to X_l^3. In the case of wormhole throats this leads to boundary conditions stating tha there exist coordinates in which $J_{ni} = 0$ and $g_{ni} = 0$ at X_l^3 [K17] . Therefore classical gravitational field is effectively static at X_l^3 and the Maxwell field defined by the induced Kähler form has only the magnetic part in these coordinates.

   iv. The basic conjecture motivating the construction is that the exponent of Kähler action defining vacuum functional equals to Dirac determinant for the eigenmodes having the defining property that they are localized in regions of X_l^3, where the induced Kähler gauge field is non-vanishing. This guarantees that the number of generalized eigen modes is finite so that Dirac determinant is finite and can be algebraic number, and therefore makes sense also in p-adic context although Kähler action does not make sense p-adically.

v. My basic sin during these years have been the strong tendency to make un-necessarily strong conjectures. Also now the original proposal stated that entire 4-surface $X^4(X^3_l)$ must contain M^2 in its tangent space in both M^8 and $M^4 \times CP_2$. This condition would force same plane of non-physical polarizations for all light-like 3-surfaces assignable to X^4. This condition is unnecessarily strong since light-like 3-surfaces are the basic physical objects. If the statement were true, it would allow to identify the preferred extremals of Kähler action as images of hyper-quaternionic surfaces of M^8 - an extremely powerful statement. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and also quite general class of known extremals of Kähler action however fail to satisfy this condition, which suggests that it is un-necessary strong. The weaker conjecture that $X^4(X^3)$ can be also regarded as a preferred extremal of Kähler action associated with $M^4 \times E^4$ might however make sense.

(c) A further step in progress was the emergence of zero energy ontology implying that causal diamonds CDs defined as intersections of future and past directed light-cones define the sectors of WCW as the set of light-like 3-surfaces in $CD \times CP_2$. The positions of the tips of CD in M^4 characterize the position of CD in M^4 and if the temporal distance between tips of CD is quantized in powers of two - as suggested by the geometry of CD - p-adic length scale hypothesis follows.

(d) The interpretation of light-like 3-surfaces as generalized Feynman diagrams - meaning that they are singular as 3-manifolds - is an important element of picture. The lines of diagrams represented by light-like 3-surfaces intersect at vertices, which are 2-D partonic surfaces at light-like boundaries of sub-CDs, and the fractal coupling constant evolution with improved measurement resolution described as addition of sub-CDs. The presence of sub-CDs also breaks effective 2-dimensionality implied by conformal invariants in light-like direction, and the outcome is 3-dimensionality in discretized sense.

(e) A further complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

It has gradually become clear that one can consider many identifications of preferred extremals. One approach relies on the identification of preferred extremal property in terms of criticality as space-time correlate of quantum criticality [K9]. One variant of this approach involves modified Dirac operator and Dirac determinant [K28]. Preferred extremals could be identified as quaternionic sub-manifolds meaning that the tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K9] defining also this kind of slicing and these two approaches could be equivalent.

In the sequel the considerations are restricted to 3-surfaces in $M^4_+ \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_+ \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs. It will be found that in the case of $M^4_+ \times CP_2$ Kähler geometry, or strictly speaking contact Kähler geometry, characterized by a degenerate Kähler form (Diff^4 degeneracy and plus possible other degeneracies) seems possible.

9.5 Construction of the WCW geometry from symmetry principles

Besides the direct guess of Kähler function one can also try to construct WCW geometry using symmetry principles. The mere existence of WCW geometry as a union of symmetric spaces requires maximal possible symmetries and means a reduction to single point of WCW with fixed values of zero modes. Therefore there are good hopes that the construction might work in practice.

9.5.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremal is absolute minimum of Kähler action.

This picture turned out to be too simple.

(a) Absolute minima had to be replaced by preferred extremals containing M^2 in the tangent space of X^4 at light-like 3-surfaces X^3_l. The reduction to the light-cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving light-like boundaries of causal diamonds CD already described.

(b) It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3.

This is indeed possible. The possibility of negative Poincare energies inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as a preferred extremal Kähler action in the set of 4-surfaces going through X^3.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X_l^3 of M^4 implies generalized conformal and symplectic invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

9.5.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X_l^3 of space-time surface appear as 3-D causal determinants. Examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M^4_\pm$ implies that the data at either X^3 or X_l^3 are enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 plus 4-D tangent space of X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory to string model like theory and does not occur even locally. Moreover, the reduction to effectively 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of causal diamonds (CDs) containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M^4_+ \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

The reduction of data to that associated with 2-D surfaces and their 4-D tangent space distributions conforms with the number theoretic vision about imbedding space as having hyper-octonionic structure [K88] : the commutative sub-manifolds of H have dimension not larger than two and for them tangent space is complex sub-space of complexified octonion tangent space. Number theoretic counterpart of quantum measurement theory forces the reduction of relevant data to 2-D commutative sub-manifolds of X^3. These points are discussed in more detail in the next chapter whereas in this chapter the consideration will be restricted to $X_l^3 = \delta M^4_+$ case which involves all essential aspects of the problem.

9.5.3 Magic properties of light-cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM^4_+, the boundary of four-dimensional light-cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light-cone boundary are parameterized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as absolute minimum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light-cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light-cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of WCW.

The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. This suggests that Kähler function is in a good approximation invariant under the symplectic transformations of CP_2 would mean that CP_2 symplectic transformations correspond to zero modes having zero norm in the Kähler metric of WCW.

The groups G and H, and thus WCW itself, should inherit the complex structure of the light-cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

9.5.4 Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

(a) The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

(b) The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

(c) The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that

CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

9.5.5 Symmetric space property reduces to conformal and symplectic invariance

The idea about symmetric space is extremely beautiful but it millenium had to change before time was ripe for identifying the precise form of the Cartan decomposition. The solution of the puzzle turned out to be amazingly simple.

The inspiration came from the finding that quantum TGD leads naturally to an extension of Super Algebras by combining Ramond and Neveu-Schwartz algebras into single algebra. This led to the introduction Virasoro generators and generators of symplectic algebra of CP_2 localized with respect to the light-cone boundary and carrying conformal weights with a half integer valued real part.

Soon came the realization that the conformal weights $h = -1/2 - i\sum_i y_i$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta, are excellent candidates for the super-symplectic ground state conformal weights. It took some time to answer affirmatively the question whether also the negatives of the trivial zeros $z = -2n$, $n > 0$ could be included. Thus the conjecture inspired by the work with Riemann hypothesis stating that the zeros of Riemann Zeta appear at the level of basic quantum TGD gets some support.

The main objection against this conjecture is that Riemann Zeta has no direct connection with basic quantum TGD. Rather, the zeta function $\zeta_D = \sum_n \lambda_n^{-s}$ - call it Dirac Zeta - defined by the eigenvaluesλ of the modified Dirac operator [K17] analogous to cyclotron energies looks physically better motivated than Riemann Zeta. The number of eigenvalues is finite and this has natural connection with the finite measurement resolution meaning that finite number of CDs contribute to the Dirac determinant. As a consequence the analytic continuation to all values of s exists automatically. The general vision about the spectrum of zeros for this zeta is lacking. In particular, the question under what conditions Riemann hypothesis holds true is lacking.

If the conjecture holds true, the generators whose commutators define the basis of the entire algebra have conformal weights given by the negatives of the zeros of Riemann Zeta or Dirac Zeta. The algebra is a direct sum $g = g_1 \oplus g_2$ such that g_1 has $h = n$ as conformal weights and g_2 $h = n - 1/2 + iy$, where y is sum over imaginary parts y_i of non-trivial zeros of Zeta. Only $h = 2n$, $n > 1$, and $h = -1/2 - iy + n$, such that n is even (odd) if y is sum of odd (even) number of y_i correspond to the weights labeling the generators of t in the Cartan decomposition $g = h + t$. The resulting super-symplectic algebra would quite well be christened as Riemann (or Dirac) algebra.

The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of WCW, which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories.

The light-cone conformal invariance differs in many respects from the conformal invariance of string theories. Finite-dimensional Kac-Moody group is replaced by an infinite-dimensional symplectic group. Conformal weights could correspond to zeros of Riemann zeta and suitable superpositions of them in case of trivial zeros, and physical states can have non-vanishing but real conformal weights just as the representations of color group in CP_2 can have non-vanishing color isospin and hyper charge. The conformal weights have also interpretation as quantum numbers associated with unitary representations of Lorentz group: thus there is no conflict between conformal invariance and Lorentz invariance in TGD framework. Complex conformal weights however correspond to complex values of mass squared and super-conformal invariance for physical plays fundamental role in string models. This suggest that 7-3-duality could in TGD framework translate to the statement that the sums of super-symplectic and Super Kac-Moody type super-conformal generators annihilate the physical states. This would generalize Goddard-Olive-Kent construction [A116] .

9.5.6 Attempts to identify WCW Hamiltonians

I have made several attempts to identify WCW Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate is based on the formulation of quantum TGD using 3-D light-like surfaces identified as orbits of partons. The proposal is out-of-date but the most recent proposal is obtained by a very straight-forward generalization from the proposal for magnetic Hamiltonians discussed below.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structures of WCW. There is almost unique identification for the symplectic structure. WCW counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$\begin{aligned}
Q_m(H_A, X^2) &= Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ , \\
Q^+_m(H_A, r_M) &= Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ , \\
J &\equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .
\end{aligned}$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q^+_m.

$$Q^{\alpha,\beta}_m(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q^+_m(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the WCW metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Generalization

The generalization for definition WCW super-Hamiltonians defining WCW gamma matrices is discussed in detail in [K116] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the modified Dirac action: in particular, about their localization at string worlds sheets (right handed neutrino could be an exception).

The basic formulas generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Right handed neutrino spinor is replaced with any mode of the modified Dirac operator localized at string world sheet in the case of Kac-Moody sub-algebra of super-symplectic algebra corresponding to symplectic isometries at light-cone boundary and CP_2. In the case of right- handed neutrino one obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

The resulting super Hamiltonians define WCW gamma matrices. They are labelled by two conformal weights. The first one is the conformal weight associated with the light-like coordinate of $\delta M^4_\pm \times CP_2$. Second conformal weight is associated with the spinor mode and the coordinate along stringy curve. One cannot exclude the possibility that the two conformal weights have same value. Effective 2-dimensionality and the fact that string coordinate cannot be always radial light-like coordinate would suggest that they are independent.

The presence of two conformal weights is in accordance with the idea that a generalization of conformal invariance to 4-D situation is in question. If Yangian extension of conformal symmetries is possible and would bring an additional integer n telling the degree of multilocality of Yangian generators defined as the number of partonic 2-surfaces at which the generator acts. For conformal algebra degree of multilocality equals to $n = 1$.

9.5.7 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of WCW. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the WCW. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) = 0 \ , \tag{9.5.1}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining WCW coordinates.

Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} = J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{9.5.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q_m^{\alpha,\beta}(H_{A,k})$ provide an explicit representation for the Hamiltonians at the level of WCW so that the components of the symplectic form of WCW are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light-cone boundary:

$$J(X(H_A), X(H_B)) = Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{9.5.3}$$

Recall that the superscript α, β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} = Q_e^{\alpha,\beta}(H_A) + Q_m^{\alpha,\beta}(H_A) = (1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{9.5.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A, H_B\}) \rightarrow (1+K)Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{9.5.5}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that WCW coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I, Q^J\} &= J^{IJ} = J_I \delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{9.5.6}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{9.5.7}$$

General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i \bar{Z}^j} = iG^{Z^i \bar{Z}^j} = \partial_{H^A} Z^i \partial_{H^B} \bar{Z}^j J^{AB} \ , \tag{9.5.8}$$

where J^{AB} is given by the classical Kähler charge for the light-cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \sum_I J(I)(\partial_{P^i}Z^i\partial_{Q^I}\bar{Z}^j - \partial_{Q^I}Z^i\partial_{P^I}\bar{Z}^j) \ . \quad (9.5.9)$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i}K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i}K \ . \end{aligned} \quad (9.5.10)$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K = \int_0^Z (A_{Z^i}dZ^i - A_{\bar{Z}^i}d\bar{Z}^i) \ . \quad (9.5.11)$$

$Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X^2_i.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X^2_i) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X^2_i-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_AQ(H_B|X^2_i) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r^n_M$ is given by $r^n_M dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X^2_i. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X^2_i induces a unique conformal structure and since the conformal transformations of X^2_i can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

(a) It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

(b) If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1_+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{9.5.12}$$

(c) If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{or} \ \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \ \text{or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{9.5.13}$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 9.5.15

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{9.5.14}$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of WCW provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of WCW (that is 3-surface) the proposed $g = t+h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of WCW Hamiltonians? Does the central extension of WCW reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

1. Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

(a) $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of WCW (unless it is flat) so that the task is to identify the point of WCW at which the proposed decomposition holds true.

(b) The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

(c) It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J_+^a = j^{ak}\partial_k$ and $j_-^a = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$\begin{aligned} \{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\ &= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j_+^a, j_-^b) \ . \end{aligned} \tag{9.5.15}$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\ (j^a, j^b) &= 2Re\left(i(j_+^a, j_-^b)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ . \end{aligned} \tag{9.5.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the WCW metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

(a) The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h, h\}_{-+} = 0 \ , \\ &Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\ &Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ . \end{aligned} \tag{9.5.17}$$

(b) The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h,h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

(c) The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t,t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t,t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

(d) The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

(e) Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the WCW the counterpart of the origin corresponds to the maximum of the Kähler function.

2. Cartan algebra decomposition at the level of configuration space

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of WCW. The use of the half bracket for WCW Hamiltonians in turn allows to calculate the matrix elements of the WCW metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X^3_l to $X^2 = X^3_l \cap \delta M^4_\pm \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [K9] . The construction of WCW spinor structure and metric in terms of the second quantized spinor fields [K17] relies to this picture as also the recent view about M-matrix [K20] .

In this framework the coset space decomposition becomes trivial.

(a) The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m,n,k) labeling the function basis of the light-cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

(b) Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [A71] , which served as the inspirer of the WCW geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k,A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k>0,A)\} \oplus \{X(k<0,A)\} \oplus \{X(k=0,A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1<0,A), X(k_2>0,B)) = k_2\delta(k_1+k_2)\delta(A,B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

(a) Central extension is most naturally induced by the natural central extension ($\{p,q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light-cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. light-cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light-cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$\begin{aligned} &g = h + t \ , \\ &[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{aligned} \tag{9.5.18}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the WCW metric.

The requirement $[t,t] \subset h$ and $[h,t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) = 0 = g([X,Y],Z) + g(Y,[X,Z]) \ . \tag{9.5.19}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (9.5.19) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) = 0 \ , \tag{9.5.20}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (9.5.20) as consistency conditions on the initial values of the time derivatives of imbedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

How to find Kähler function?

If one has found the expansion of WCW Kähler form in terms of electric fluxes one can solve also the Kähler function from the defining partial differential equations $J_{k\bar{l}} = \partial k \partial_{\bar{l}} K$. The solution is not unique since the equation allows the symmetry

$$K \rightarrow K + f(z^k) + \overline{f(z^k)} \ ,$$

where f is arbitrary holomorphic function of z^k. This non-uniqueness is probably eliminated by the requirement that Kähler function vanishes for vacuum extremals. This in turn makes in principle possible to find the maxima of Kähler function and to perform functional integration perturbatively around them.

Electric-magnetic duality implies that, apart from conformal factor depending on isometry invariants, one can solve Kähler metric without any knowledge on the initial values of the time derivatives of the imbedding space coordinates. Apart from conformal factor the resulting geometry is purely intrinsic to δCH. The role of Kähler action is only to to define $Diff^4$ invariance and give the rule how the metric is translated to metric on arbitrary point of CH. The degeneracy of the preferred extrema also implies that configuration space has multi-sheeted structure analogous to that encountered in case of Riemann surfaces.

The most promising concrete construction recipe for Kähler function is in terms of the modified Dirac operator [K17] . The recipe is described briefly in the introduction. If the conjecture that Dirac determinant coincides with the exponent of Kähler action for a preferred extremal is correct, the value of the Kähler coupling strength follows as a prediction of the theory. From the construction it is clear that Dirac determinant involves only a finite number of eigenvalues of the modified Dirac operator and can thus be an algebraic or even rational number if eigenvalues have this property. The most satisfactory property of the construction is that one can use the intuition from the behavior of 2-D magnetic systems.

9.6 Ricci flatness and divergence cancelation

Divergence cancelation in WCW integration requires Ricci flatness and in this section the arguments in favor of Ricci flatness are discussed in detail.

9.6.1 Inner product from divergence cancelation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of WCW over the reduced WCW containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('light-cone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \quad (9.6.1)$$

The degeneracy for the preferred extremals of Kähler action implies additional summation over the degenerate extremals associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by WCW integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of WCW integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the non-compact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [B72]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \ . \quad (9.6.2)$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancelation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancelation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the WCW into sectors D_P labeled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \to y,\beta) = \sum_{r,s} |S(r,\alpha \to s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \to s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

(a) Since WCW metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

(b) α_K is a natural small expansion parameter in WCW integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

(c) Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems (Duistermaat-Hecke theorem) stating that semiclassical approximation is exact for certain systems (for example for integrable systems [A64]). Symmetric space property suggests that Kähler function might possess the properties guaranteeing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving WCW spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that WCW integrals are continuable to p-adic number fields requires this kind of reduction.

9.6.2 Why Ricci flatness

It has been already found that the requirement of divergence cancelation poses extremely strong constraints on the metric of the WCW. The results obtained hitherto are the following.

(a) If the vacuum functional is the exponent of Kähler function one gets rid of the divergences resulting from the Gaussian determinants and metric determinants: determinants cancel each other.

(b) The non-locality of the Kähler action gives good hopes of obtaining divergence free perturbation theory.

The following arguments show that Ricci flatness of the metric is a highly desirable property.

(a) Dirac operator should be a well defined operator. In particular its square should be well defined. The problem is that the square of Dirac operator contains curvature scalar, which need not be finite since it is obtained via two infinite-dimensional trace operations from the curvature tensor. In case of loop spaces [A71] the Kähler property implies that even Ricci tensor is only conditionally convergent. In fact, loop spaces with Kähler metric are Einstein spaces (Ricci tensor is proportional to metric) and Ricci scalar is infinite.

   In 3-dimensional case situation is even worse since the trace operation involves 3 summation indices instead of one! The conclusion is that Ricci tensor had better to vanish in vibrational degrees of freedom.

(b) For Ricci flat metric the determinant of the metric is constant in geodesic complex coordinates as is seen from the expression for Ricci tensor [A117]

$$R_{k\bar{l}} = \partial_k \partial_{\bar{l}} ln(det(g)) \tag{9.6.3}$$

   in Kähler metric. This obviously simplifies considerably functional integration over WCW: one obtains just the standard perturbative field theory in the sense that metric determinant gives no contributions to the functional integration.

(c) The constancy of the metric determinant results not only in calculational simplifications: it also eliminates divergences. This is seen by expanding the determinant as a functional Taylor series with respect to the coordinates of WCW. In local complex coordinates the first term in the expansion of the metric determinant is determined by Ricci tensor

$$\delta\sqrt{g} \propto R_{k\bar{l}} z^k \bar{z}^l \ . \tag{9.6.4}$$

   In WCW integration using standard rules of Gaussian integration this term gives a contribution proportional to the contraction of the propagator with Ricci tensor. But since the propagator is just the contravariant metric one obtains Ricci scalar as result. So, in order to avoid divergences, Ricci scalar must be finite: this is certainly guaranteed if Ricci tensor vanishes.

(d) The following group theoretic argument suggests that Ricci tensor either vanishes or is divergent. The holonomy group of the WCW is a subgroup of $U(n = \infty)$ ($D = 2n$ is the dimension of the Kähler manifold) by Kähler property and Ricci flatness is guaranteed if the $U(1)$ factor is absent from the holonomy group. In fact Ricci tensor is proportional to the trace of the $U(1)$ generator and since this generator corresponds to an infinite dimensional unit matrix the trace diverges: therefore given element of the Ricci tensor is either infinite or vanishes. Therefore the vanishing of the Ricci tensor seems to be a mathematical necessity. This naive argument doesn't hold true in the case of loop spaces, for which Kähler metric with finite non-vanishing Ricci tensor exists [A71] . Note however that also in this case the sum defining Ricci tensor is only conditionally convergent.

There are indeed good hopes that Ricci tensor vanishes. By the previous argument the vanishing of the Ricci tensor is equivalent with the absence of divergences in WCW integration. That divergences are absent is suggested by the non-locality of the Kähler function as a functional of 3-surface: the divergences of local field theories result from the locality of interaction vertices. Ricci flatness in vibrational degrees of freedom is not only necessary mathematically. It is also appealing physically: one can regard Ricci flat WCW as a vacuum solution of Einstein's equations $G^{\alpha\beta} = 0$.

9.6.3 Ricci flatness and Hyper Kähler property

Ricci flatness property is guaranteed if WCW geometry is Hyper Kähler [A107, A40] (there exists 3 covariantly constant antisymmetric tensor fields, which can be regarded as representations of quaternionic imaginary units). Hyper Kähler property guarantees Ricci flatness because the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(n)$ generators instead of $U(n)$ generators so that the traces vanish. In the case of the loop spaces left invariance implies that Ricci tensor in the vibrational degrees is a multiple of the metric tensor so that Ricci scalar has an infinite value. This is basically due to the fact that Kac-Moody algebra has $U(1)$ central extension.

Consider now the arguments in favor of Ricci flatness of the WCW.

(a) The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

(b) The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

(c) In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property.

(a) The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of WCW in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of WCW. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

(b) S^2-fold degeneracy is indeed associated with the definition of the complex structure of WCW. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at δM^4_+ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of WCW and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

In the sequel we shall show that Ricci flatness is guaranteed provided that the holonomy group of WCW is isomorphic to some subgroup of $SU(n=\infty)$ instead of $U(n=\infty)$ (n is the complex dimension of WCW) implied by the Kähler property of the metric. We also derive an expression for the Ricci tensor in terms of the structure constants of the isometry algebra and WCW metric. The expression for the Ricci tensor is formally identical with that obtained by Freed for loop spaces: the only difference is that the structure constants of the finite-dimensional group are replaced with the group $Can(\delta H)$. Also the arguments in favor of Hyper Kähler property are discussed in more detail.

9.6.4 The conditions guaranteeing Ricci flatness

In the case of Kähler geometry Ricci flatness condition can be characterized purely Lie-algebraically: the holonomy group of the Riemann connection, which in general is subgroup of $U(n)$ for Kähler manifold of complex dimension n, must be subgroup of $SU(n)$ so that the Lie-algebra of this group consists of traceless matrices. This condition is easy to derive using complex coordinates. Ricci tensor is given by the following expression in complex vielbein basis

$$R^{A\bar{B}} = R^{A\bar{C}B}_{\ \ \ \ \bar{C}} \ , \tag{9.6.5}$$

where the latter summation is only over the antiholomorphic indices $\bar{C}$. Using the cyclic identities

$$\sum_{cycl\ \bar{C}B\bar{D}} R^{A\bar{C}B\bar{D}} = 0 \ , \tag{9.6.6}$$

the expression for Ricci tensor reduces to the form

$$R^{A\bar{B}} = R^{A\bar{B}C}_{\ \ \ \ C} \ , \tag{9.6.7}$$

where the summation is only over the holomorphic indices C. This expression can be regarded as a trace of the curvature tensor in the holonomy algebra of the Riemann connection. The trace is taken over holomorphic indices only: the traces over holomorphic and anti-holomorphic indices cancel each other by the antisymmetry of the curvature tensor. For

Kähler manifold holonomy algebra is subalgebra of $U(n)$, when the complex dimension of manifold is n and Ricci tensor vanishes if and only if the holonomy Lie-algebra consists of traceless matrices, or equivalently: holonomy group is subgroup of $SU(n)$. This condition is expected to generalize also to the infinite-dimensional case.

We shall now show that if WCW metric is Kähler and possesses infinite-dimensional isometry algebra with the property that its generators form a complete basis for the tangent space (every tangent vector is expressible as a superposition of the isometry generators plus zero norm vector) it is possible to derive a representation for the Ricci tensor in terms of the structure constants of the isometry algebra and of the components of the metric and its inverse in the basis formed by the isometry generators and that Ricci tensor vanishes identically for the proposed complexification of the WCW provided the generators $\{H_{A,m\neq 0}, H_{B,n\neq 0}\}$ correspond to zero norm vector fields of WCW.

The general definition of the curvature tensor as an operator acting on vector fields reads

$$R(X,Y)Z \;=\; [\nabla_X,\nabla_Y]Z - \nabla_{[X,Y]}Z \ . \tag{9.6.8}$$

If the vector fields considered are isometry generators the covariant derivative operator is given by the expression

$$\begin{aligned} \nabla_X Y &= (Ad_X Y - Ad^*_X Y - Ad^*_Y X)/2 \ , \\ (Ad^*_X Y, Z) &= (Y, Ad_X Z) \ , \end{aligned} \tag{9.6.9}$$

where $Ad_X Y = [X,Y]$ and Ad^*_X denotes the adjoint of Ad_X with respect to WCW metric.

In the sequel we shall assume that the vector fields in question belong to the basis formed by the isometry generators. The matrix representation of Ad_X in terms of the structure constants $C_{X,Y:Z}$ of the isometry algebra is given by the expression

$$\begin{aligned} Ad^m_{Xn} &= C_{X,Y:Z}\hat{Y}_n Z^m \ , \\ [X,Y] &= C_{X,Y:Z}Z \ , \\ \hat{Y} &= g^{-1}(Y,V)V \ , \end{aligned} \tag{9.6.10}$$

where the summation takes place over the repeated indices and $\hat{Y}$ denotes the dual vector field of Y with respect to the WCW metric. From its definition one obtains for Ad^*_X the matrix representation

$$\begin{aligned} Ad^{*m}_{Xn} &= C_{X,Y:Z}\hat{Y}^m Z_n \ , \\ Ad^*_X Y &= C_{X,U:V}g(Y,U)g^{-1}(V,W)W = g(Y,U)g^{-1}([X,U],W)W \ , \end{aligned} \tag{9.6.11}$$

where the summation takes place over the repeated indices.

Using the representations of ∇_X in terms of Ad_X and its adjoint and the representations of Ad_X and Ad^*_X in terms of the structure constants and some obvious identities (such as $C_{[X,Y],Z:V} = C_{X,Y:U}C_{U,Z:V}$) one can by a straightforward but tedious calculation derive a more detailed expression for the curvature tensor and Ricci tensor. Straightforward calculation of the Ricci tensor has however turned to be very tedious even in the case of the diagonal metric and in the following we shall use a more convenient representation [A71] of the curvature tensor applying in case of the Kähler geometry.

The expression of the curvature tensor is given in terms of the so called Toeplitz operators T_X defined as linear operators in the "positive energy part" G_+ of the isometry algebra spanned by the $(1,0)$ parts of the isometry generators. In present case the positive and negative energy parts and cm part of the algebra can be defined just as in the case of loop spaces:

$$\begin{aligned} G_+ &= \{H^{Ak}|k>0\} \ , \\ G_- &= \{H^{Ak}|k<0\} \ , \\ G_0 &= \{H^{Ak}|k=0\} \ . \end{aligned} \tag{9.6.12}$$

Here H^{Ak} denote the Hamiltonians generating the symplectic transformations of δH. The positive energy generators with non-vanishing norm have positive radial scaling dimension: $k \geq 0$, which corresponds to the imaginary part of the scaling momentum $K = k_1 + i\rho$ associated with the factors $(r_M/r_0)^K$. A priori the spectrum of ρ is continuous but it is quite possible that the spectrum of ρ is discrete and $\rho = 0$ does not appear at all in the spectrum in the sense that the flux Hamiltonians associated with $\rho = 0$ elements vanish for the maximum of Kähler function which can be taken to be the point where the calculations are done.

T_X differs from Ad_X in that the negative energy part of $Ad_X Y = [X,Y]$ is dropped away:

$$\begin{aligned} T_X : G_+ &\rightarrow G_+ \ , \\ Y &\rightarrow [X,Y]_+ \ . \end{aligned} \tag{9.6.13}$$

Here " + " denotes the projection to "positive energy" part of the algebra. Using Toeplitz operators one can associate to various isometry generators linear operators $\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ acting on G_+:

$$\begin{aligned} \Phi(X_0) &= T_{X_0} \ , \ X_0 \varepsilon G_0 \ , \\ \Phi(X_-) &= T_{X_-} \ , \ X_- \varepsilon G_- \ , \\ \Phi(X_+) &= -T^*_{X_-} \ , \ X_+ \varepsilon G_+ \ . \end{aligned} \tag{9.6.14}$$

Here "*" denotes hermitian conjugate in the diagonalized metric: the explicit representation $\Phi(X_+)$ is given by the expression [A71]

$$\begin{aligned} \Phi(X_+) &= D^{-1} T_{X_-} D \ , \\ DX_+ &= d(X) X_- \ , \\ d(X) &= g(X_-, X_+) \ . \end{aligned} \tag{9.6.15}$$

Here $d(X)$ is just the diagonal element of metric assumed to be diagonal in the basis used. denotes the conformal factor associated with the metric.

The representations for the action of ,$\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ in terms of metric and structure constants of the isometry algebra are in the case of the diagonal metric given by the expressions

$$\begin{aligned} \Phi(X_0)Y_+ &= C_{X_0,Y_+:U_+} U_+ \ , \\ \Phi(X_-)Y_+ &= C_{X_-,Y_+:U_+} U_+ \ , \\ \Phi(X_+)Y_+ &= \frac{d(Y)}{d(U)} C_{X_-,Y_-:U_-} U_+ \ . \end{aligned} \tag{9.6.16}$$

The expression for the action of the curvature tensor in positive energy part G_+ of the isometry algebra in terms of the these operators is given as [A71] :

$$R(X,Y)Z_+ \quad = \quad \{[\Phi(X),\Phi(Y)] - \Phi([X,Y])\}Z_+ \ . \tag{9.6.17}$$

The calculation of the Ricci tensor is based on the observation that for Kähler manifolds Ricci tensor is a tensor of type $(1,1)$, and therefore it is possible to calculate Ricci tensor as the trace of the curvature tensor with respect to indices associated with G_+.

$$Ricci(X_+,Y_-) \quad = \quad (\hat{Z}_+, R(X_+,Y_-)Z_+) \equiv Trace(R(X_+,Y_-)) \ , \tag{9.6.18}$$

where the summation over Z_+ generators is performed.

Using the explicit representations of the operators Φ one obtains the following explicit expression for the Ricci tensor

$$\begin{aligned} Ricci(X_+,Y_-) \quad &= \quad Trace\{[D^{-1}T_{X_+}D, T_{Y_-}] - T_{[X_+,Y_-]_{|G_0+G_-}} \\ &- \quad D^{-1}T_{[X_+,Y_-]_{|G_+}}D\} \ . \end{aligned} \tag{9.6.19}$$

This expression is identical to that encountered in case of loop spaces and the following arguments are repetition of those applying in the case of loop spaces.

The second term in the Ricci tensor is the only term present in the finite-dimensional case. This term vanishes if the Lie-algebra in question consists of traceless matrices. Since symplectic transformations are volume-preserving the traces of Lie-algebra generators vanish so that this term is absent. The last term gives a non-vanishing contribution to the trace for the same reason.

The first term is quadratic in structure constants and does not vanish in case of loop spaces. It can be written explicitly using the explicit representations of the various operators appearing in the formula:

$$\begin{aligned} Trace\{[D^{-1}T_{X_-}D, T_{Y_-}]\} \quad &= \quad \sum_{Z_+,U_+}[C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} \\ &- \quad C_{X_-,Z_-:U_-}C_{Y_-,U_+:Z_+}\frac{d(Z)}{d(U)}] \ . \end{aligned} \tag{9.6.20}$$

Each term is antisymmetric under the exchange of U and Z and one might fail to conclude that the sum vanishes identically. This is not the case. By the diagonality of the metric with respect to radial quantum number, one has $m(X_-) = m(Y_-)$ for the non-vanishing elements of the Ricci tensor. Furthermore, one has $m(U) = m(Z) - m(Y)$, which eliminates summation over $m(U)$ in the first term and summation over $m(Z)$ in the second term. Note however, that summation over other labels related to symplectic algebra are present.

By performing the change $U \rightarrow Z$ in the second term one can combine the sums together and as a result one has finite sum

$$\begin{aligned}
\sum_{0<m(Z)<m(X)} [C_{X_-,U_-:Z_-} C_{Y_-,Z_+:U_+} \frac{d(U)}{d(Z)} &= C \sum_{0<m(Z)<m(X)} \frac{m(X)}{m(Z)-m(X)} \ , \\
C &= \sum_{Z,U} C_{X,U:Z} C_{Y,Z:U} \frac{d_0(U)}{d_0(Z)} \ . \qquad (9.6.21)
\end{aligned}$$

Here the dependence of $d(X) = |m(X)|d_0(X)$ on $m(X)$ is factored out; $d_0(X)$ does not depend on k_X. The dependence on $m(X)$ in the resulting expression factorizes out, and one obtains just the purely group theoretic term C, which should vanish for the space to be Ricci flat.

The sum is quadratic in structure constants and can be visualized as a loop sum. It is instructive to write the sum in terms of the metric in the symplectic degrees of freedom to see the geometry behind the Ricci flatness:

$$C = \sum_{Z,U} g([Y,Z],U) g^{-1}([X,U],Z) \ . \qquad (9.6.22)$$

Each term of this sum involves a commutator of two generators with a non-vanishing norm. Since tangent space complexification is inherited from the local coset space, the non-vanishing commutators in complexified basis are always between generators in $Can_{\neq 0}$; that is they do not not belong to rigid $su(2)\times su(3)$.

The condition guaranteeing Ricci flatness at the maximum of Kähler function and thus everywhere is simple. All elements of type $[X_{\neq 0}, Y_{\neq 0}]$ vanish or have vanishing norm. In case of CP_2 Kähler geometry this would correspond to the vanishing of the $U(2)$ generators at the origin of CP_2 (note that the holonomy group is $U(2)$ in case of CP_2). At least formally stronger condition is that the algebra generated by elements of this type, the commutator algebra associated with $Can_{\neq 0}$, consist of elements of zero norm. Already the (possibly) weaker condition implies that adjoint map $Ad_{X\neq 0}$ and its hermitian adjoint $Ad^*_{X_{\neq 0}}$ create zero norm states. Since isometry conditions involve also adjoint action the condition also implies that $Can_{\neq 0}$ acts as isometries. More concrete form for the condition is that all flux factors involving double Poisson bracket and three generators in $Can_{\neq 0}$ vanish:

$$Q_e(\{H_A,\{H_B,H_C\}\}) = 0 \ , \ \text{for } H_A, H_B, H_C \text{ in } Can_{\neq 0} \ . \qquad (9.6.23)$$

The vanishing of fluxes involving two Poisson brackets and three Hamiltonians guarantees isometry invariance and Ricci flatness and, as found in [K18] , is implied by the $[t,t]\subset h$ property of the Lie-algebra of coset space G/H having symmetric space structure.

The conclusion is that the mere existence of the proposed isometry group (guaranteed by the symmetric space property) implies the vanishing of the Ricci tensor and vacuum Einstein equations. The existence of the infinite parameter isometry group in turn follows basically from the condition guaranteeing the existence of the Riemann connection. Therefore vacuum Einstein equations seem to arise, not only as a consequence of a physically motivated variational principle but as a mathematical consistency condition in infinite dimensional Kähler geometry. The flux representation seems to provide elegant manner to formulate and solve these conditions and isometry invariance implies Ricci flatness.

9.6.5 Is WCW metric Hyper Kähler?

The requirement that WCW integral integration is divergence free implies that WCW metric is Ricci flat. The so called Hyper-Kähler metrics [A107, A40] , [B12] are particularly nice representatives of Ricci flat metrics. In the following the basic properties of Hyper-Kähler metrics are briefly described and the problem whether Hyper Kähler property could realized in case of $M^4_+ \times CP_2$ is considered.

Hyper-Kähler property

Hyper-Kähler metric is a generalization of the Kähler metric. For Kähler metric metric tensor and Kähler form correspond to the complex numbers 1 and i and therefore define complex structure in the tangent space of the manifold. For Hyper Kähler metric tangent space allows three closed Kähler forms I, J, K, which with respect to the multiplication obey the algebra of quaternionic imaginary units and have square equal to - 1, which corresponds to the metric of Hyper Kähler space.

$$I^2 \;=\; J^2 = K^2 = -1 \;\; IJ = -JI = K, \text{ etc. } . \tag{9.6.24}$$

To define Kähler structure one must choose one of the Kähler forms or any linear combination of I, J and K with unit norm. The group $SO(3)$ rotates different Kähler structures to each other playing thus the role of quaternion automorphisms. This group acts also as coordinate transformations in Hyper Kähler manifold but in general fails to act as isometries.

If K is chosen to define complex structure then K is tensor of type $(1, 1)$ in complex coordinates, I and J being tensors of type $(2, 0) + (0, 2)$. The forms $I + iJ$ and $I - iJ$ are holomorphic and anti-holomorphic forms of type $(2, 0)$ and $(0, 2)$ respectively and defined standard step operators I_+ and I_- of $SU(2)$ algebra. The holonomy group of Hyper-Kähler metric is always $Sp(k)$, $k \leq dimM/4$, the group of $k \times k$ unitary matrices with quaternionic entries. This group is indeed subgroup of SU(2k), so that its generators are traceless and Hyper Kähler metric is therefore Ricci flat.

Hyper Kähler metrics have been encountered in the context of 3-dimensional super symmetric sigma models: a necessary prerequisite for obtaining $N = 4$ super-symmetric sigma model is that target space allows Hyper Kähler metric [B12, B14] . In particular, it has been found that Hyper Kähler property is decisive for the divergence cancelation.

Hyper-Kähler metrics arise also in monopole and instanton physics [A40] . The moduli spaces for monopoles have Hyper Kähler property. This suggests that Hyper Kähler property is characteristic for the configuration (or moduli) spaces of 4-dimensional Yang Mills types systems. Since YM action appears in the definition of WCW metric there are hopes that also in present case the metric possesses Hyper-Kähler property.

CP_2 allows what might be called almost Hyper-Kähler structure known as quaternionion structure. This means that the Weil tensor of CP_2 consists of three components in one-one correspondence with components of iso-spin and only one of them- the one corresponding to Kähler form- is covariantly constant. The physical interpretation is in terms of electroweak symmetry breaking selecting one isospin direction as a favored direction.

Does the 'almost' Hyper-Kähler structure of CP_2 lift to a genuine Hyper-Kähler structure in WCW?

The Hyper-Kähler property of WCW metric does not seem to be in conflict with the general structure of TGD.

(a) In string models the dimension of the "space-time" is two and Weyl invariance and complex structures play a decisive role in the theory. In present case the dimension of the space-time is four and one therefore might hope that quaternions play a similar role. Indeed, Weyl invariance implies YM action in dimension 4 and as already mentioned moduli spaces of instantons and monopoles enjoy the Hyper Kähler property.

(b) Also the dimension of the imbedding space is important. The dimension of Hyper Kähler manifold must be multiple of 4. The dimension of WCW is indeed infinite multiple of 8: each vibrational mode giving one "8".

(c) The complexification of the WCW in symplectic degrees of freedom is inherited from $S^2 \times CP_2$ and CP_2 Kähler form defines the symplectic form of WCW. The point is that CP_2 Weyl tensor has 3 covariantly constant components, having as their square metric apart from sign. One of them is Kähler form, which is closed whereas the other two are non-closed forms and therefore fail to define Kähler structure. The group $SU(2)$ of electro-weak isospin rotations rotate these forms to each other. It would not be too surprising if one could identify WCW counterparts of these forms as representations of quaternionic units at the level of WCW. The failure of the Hyper Kähler property at the level of CP_2 geometry is due to the electro-weak symmetry breaking and physical intuition (in particular, p-adic mass calculations [K54]) suggests that electro-weak symmetry might not be broken at the level of WCW geometry).

A possible topological obstruction for the Hyper Kähler property is related to the cohomology of WCW: the three Kähler forms must be co-homologically trivial as is clear from the following argument. If any of 3 quaternionic 2-form is cohomologically nontrivial then by $SO(3)$ symmetry rotating Kähler forms to each other all must be co-homologically nontrivial. On the other hand, electro-weak isospin rotation leads to a linear combination of 3 Kähler forms and the flux associated with this form is in general not integer valued. The point is however that Kähler form forms only the $(1,1)$ part of the symplectic form and must be co-homologically trivial whereas the zero mode part is same for all complexifications and can be co-homologically nontrivial. The co-homological non-triviality of the zero mode part of the symplectic form is indeed a nice feature since it fixes the normalization of the Kähler function apart from a multiplicative integer. On the other hand the hypothesis that Kähler coupling strength is analogous to critical temperature provides a dynamical (and perhaps equivalent) manner to fix the normalization of the Kähler function.

Since the properties of the WCW metric are inherited from $M^4_+ \times CP_2$ then also the Hyper Kähler property should be understandable in terms of the imbedding space geometry. In particular, the complex structure in CP_2 vibrational degrees of freedom is inherited from CP_2. Hyper Kähler property implies the existence of a continuum (sphere S^2) of complex structures: any linear superposition of 3 independent Kähler forms defines a respectable complex structure. Therefore also CP_2 should have this continuum of complex structures and this is certainly not the case.

Indeed, if we had instead of CP_2 Hyper Kähler manifold with 3 covariantly constant 2-forms then it would be easy to understand the Hyper Kähler structure of WCW. Given the Kähler structure of WCW would be obtained by replacing induced Kähler electric and magnetic fields in the definition of flux factors $Q(H_{A,m})$ with the appropriate component of the induced Weyl tensor. CP_2 indeed manages to be very nearly Hyper Kähler manifold!

How CP_2 fails to be Hyper Kähler manifold can be seen in the following manner. The Weyl tensor of CP_2 allows three independent components, which are self dual as 2-forms and rotated to each other by vielbein rotations.

$$\begin{aligned} W_{03} &= W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ W_{01} &= W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ W_{02} &= W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \end{aligned} \tag{9.6.25}$$

The component I_3 is just the Kähler form of CP_2. Remaining components are covariantly constant only with respect to spinor connection and not closed forms so that they cannot be interpreted as Maxwell fields. Their squares equal however apart from sign with the metric of CP_2, when appropriate normalization factor is used. If these forms were covariantly constant Kähler action defined by any linear superposition of these forms would indeed define Kähler structure in WCW and the group $SO(3)$ would rotate these forms to each other. The projections of the components of the Weyl tensor on 3-surface define 3 vector fields as their duals and only one of these vector fields (Kähler magnetic field) is divergenceless. One might regard these 3 vector fields as counter parts of quaternion units associated with the broken Hyper Kähler structure, that is quaternion structure. The interpretation in terms of electro-weak symmetry breaking is obvious.

One cannot exclude the possibility that the symplectic invariance of the induced Kähler electric field implies that the electric parts of the other two components of induced Weyl tensor are symplectic invariants. This is the minimum requirement. What is however obvious is that the magnetic parts cannot be closed forms for arbitrary 3-surfaces at light cone boundary. One counter example is enough and CP_2 type extremals seem to provide this counter example: the components of the induced Weyl tensor are just the same as they are for CP_2 and clearly not symplectically invariant.

Thus it seems that WCW could allow Hyper Kähler structure broken by electro-weak interactions but it cannot be inherited from CP_2. An open question is whether it allows genuine quaternionic structure. Good prospects for obtaining quaternionic structure are provided by the quaternionic counterpart QP_2 of CP_2, which is 8-dimensional and has coset space structure $QP_2 = Sp(3)/Sp(2) \times Sp(1)$. This choice does not seem to be consistent with the symmetries of the standard model. Note however that the over all symmetry group is obtained by replacing complex numbers with quaternions on the matrix representation of the standard model group.

Could different complexifications for M^4_+ and light like surfaces induce Hyper Kähler structure for WCW?

Quaternionic structure means also the existence of a family of complex structures parameterized by a sphere S^2. The complex structure of the WCW is inherited from the complex structure of some light like surface.

In the case of the light cone boundary δM^4_+ the complex structure corresponds to the choice of quantization axis of angular momentum for the sphere $r_M = constant$ so that the coordinates orthogonal to the quantization axis define a complex coordinate: the sphere S^2 parameterizes these choices. Thus there is a temptation to identify the choice of quantization axis with a particular imaginary unit and Hyper Kähler structure would directly relate to the properties rotation group. This would bring an additional item to the list of miraculous properties of light like surfaces of 4-dimensional space-times.

This might relate to the fact that WCW geometry is not determined by the symplectic algebra of CP_2 localized with respect to the light cone boundary as one might first expect but consists of $M^4_+ \times CP_2$ Hamiltonians so that infinitesimal symplectic transformation of CP_2 involves always also M^4_+-symplectic transformation. M^4_+ Hamiltonians are defined by a function basis generated as products of the Hamiltonians H_3 and $H_1 \pm iH_2$ generating rotations with respect to three orthogonal axes, and two of these Hamiltonians are complexified.

Also the light like 3-surfaces X^3_l associated with quaternion conformal invariance are determined by some 2-surface X^2 and the choice of complex coordinates and if X^2 is sphere the choices are labelled by S^2. In this case, the presence of quaternion conformal structure would be almost obvious since it is possible to choose some complex coordinate in several manners and the choices are labelled by S^2. The choice of the complex coordinate in turn fixes 2-surface X^2 as a surface for which the remaining coordinates are constant. X^2 need not however be located at the elementary particle horizon unless one poses additional constraint. One might hope that different choices of X^2 resulting in this manner correspond to all possible different selections of the complex structure and that this choice could fix uniquely the conformal equivalence class of X^2 appearing as argument in elementary particle vacuum functionals. If X^2 has a more complex topology the identification is not so clear but since conformal algebra SL(2,C) containing algebra of rotation group is involved, one might argue that the choice of quantization axis also now involves S^2 degeneracy. If these arguments are correct one could conclude that Hyper Kähler structure is implicitly involved and guarantees Ricci flatness of the WCW metric.

9.7 Does modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action. One cannot however get rid of Kähler action since the gamma matrices appearing in Kähler-Dirac action are defined in terms of canonical momentum densities of Kähler action. The most one can hope is that Dirac determinant reduces to the exponent of Kähler action for preferred extremals.

9.7.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

(a) Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. An infinite hierarchy of these currents is expected and they would define fermionic counterparts for zero modes. In number theoretic vision space-time surfaces are proposed to be identifiable as associative (co-associative) surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking [**?**]

(b) The purely quantal equations are associated with the representations of various superconformal algebras and with the modified Dirac (Kähler-Dirac) equation. The requirement that there are deformations of the space-time surface -actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

(c) The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [K28] , [L18] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [K28] , [L18] . If Kähler current defines Beltrami flow [B44] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

(d) The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

9.7.2 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a far reaching generalization of conformal symmetries. The development of the understanding of conservation laws has been slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

(a) The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J_k^\alpha \Psi \ , \\ J_k^\alpha &= \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^k + \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h^l}\delta h^l \ . \end{aligned} \quad (9.7.1)$$

Here h_β^k denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

(b) It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \quad (9.7.2)$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

(c) The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha \Psi \ . \quad (9.7.3)$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [A27, A30] . Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^{\alpha} = \overline{\Psi}\Gamma^k J_k^{\alpha}\Psi + \overline{\Psi}\hat{\Gamma}^{\alpha}\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^{\alpha}\Psi \ . \tag{9.7.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [A13] .

(d) Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

(e) The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

(a) Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

   i. Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [A116] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [A13] . Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

   ii. The understanding of the contributions to Kähler-Dirac action has been slow. It seems that what is needed is Chern-Simons Dirac action assigned to partonic orbits:

this was the original proposal. The condition that the action of C-S-D operator reduces to that of massless M^4 Dirac operator. $Gamma^n\Psi = p^k\gamma_k\Psi$ would be space-time counterpart for the massless Dirac equation at the level of imbedding space. I have called this condition earlier generalized eigenvalue condition.
The assumption that C-S-D is present strongly suggests that also Kähler action contains C-S term meaning that the C-S terms from Kähler action are cancelled at partonic orbits for preferred extremals. If C-S term is present also at space-like ends of space-time surface Kähler action and therefore also Kähler function vanishes identically. At the ends of space-time surface one would therefore have $\Gamma^n\Psi = 0$ if C-S-D term is not present. Hence this assumption seems unphysical. One would have massless Dirac propagator at the fermionic lines defined by the partonic boundaries of Kähler-Dirac equation and on-mass-shell condition at the space-like ends of the space-time surface.
If this is correct interpretation then the fermionic lines identified as boundaries of string world sheets correspond to massless fermion propagators and the stringy propagators $1/L_0$ could be associated with fermion fermion scattering at wormhole contacts (see fig. **??** in the appendix of this book) . The generalized Feynman diagrammatics would be a combination of stringy and Feynman diagrammatics. External fermion lines would carry massless on-shell momenta and wormhole contacts could be seen as massive bound states of massless fermions falling into representations of super-conformal algebras assignable to wormhole contacts. This would allow stringy variant of twistor approach.

(b) The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

(c) For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

(d) The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

(e) The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

(a) The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

(b) Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

(c) The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

(d) The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

(a) Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

(b) The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^\alpha_{\ k} + J_k^{\ \alpha})(J^\beta_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^\alpha_{\ k} = -J_k^{\ \alpha}$. The conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires additional conditions to be satisfied and the holomorphy of string world sheets (partonic 2-surfaces) and associated Kähler-Dirac gamma matrices makes this possible [K105].

(c) Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

(d) Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K27] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

(e) A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the understanding of the relationship between criticality and hierarchy of Planck constants [K40] , [L11] . Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

The vanishing of second variations of preferred extremals - at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

(a) The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

(b) The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that

WCW metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

(c) The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

(d) Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

(e) There is a possible connection with the notion of self-organized criticality [B10] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

9.7.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose modified Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

Why modified Dirac action?

1. Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K18, K88] .

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

2. Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{9.7.5}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{9.7.6}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{9.7.7}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R}\Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{9.7.8}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T^\alpha_l \Gamma^l \ . \end{aligned} \tag{9.7.9}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{9.7.10}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \tag{9.7.11}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

3. How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T^k_\mu T_{\nu k} \ . \tag{9.7.12}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

4. Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K88] . In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

Overall view about Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j \cdot A = 0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \quad (9.7.13)$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Some details about Chern-Simons Dirac equation

To avoid confusion some general comments are in order. Only the Chern-Simons Dirac operator will be considered. Modified gamma matrices contain also the contribution from the Lagrange multiplier term stating weak form of electric-magnetic duality. At space-like 3-surface one has also the contribution coming from the Lagrange multiplier terms identifying classical and quantal charges in Cartan algebra.

When C-S-D action at partonic orbits is included, one obtains what I have called generalized eigenvalue equation introduced in ad hoc manner in order to define Dirac determinant. Now Dirac determinant at least formally reduces to the same expression as in massless gauge theories. Dirac determinant could be also defined directly as the product of generalized eigenvalues $p^k\gamma_k$ defining virtual momenta propagating in fermion lines. Also the identification as hyperquaternions makes sense and the outcome is by symmetries real number or perhaps complex number.

One can of course wonder whether the Dirac determinant has anything to do with the exponent of Kähler action! Measurement interaction term states that the action of D_{C-S} modified by the contribution from em-duality constraint is identical with that of the Dirac operator of M^4 regarded as algebraic multiplication with $p^k\gamma_k$, where p^k is the four-momentum associated with the propagator line defined by the light-like orbit of parton. This simplifies the formalism enormously and gives a direct connection with similar condition posed independently in twistorial approach [K78].

One can require that the modes annihilated by Kähler-Dirac operator are eigenstates of C-S-D oeprator with generalized eigenvalues $p^k\gamma_k$ giving rise to fermion propagator Consider now the properties of eigenmodes of D_{C-S}.

(a) For $p^k = 0$ there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish.

(b) If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action.

$D(CP_2) \leq 2$ is inconsistent with the weak form of electric-magnetic duality. The extrema of Chern-Simons action have $D(CP_2) \leq 2$ and vanishing Chern-Simons density so that they would naturally represent on mass shell particles appearing as incoming and outgoing particles. This conforms with the interpretation of the basic extremals as free particles (massless extremals and cosmic strings with 2-D CP_2 projection). One could say that CP breaking is not present for free particles but unavoidably accompanies the propagator lines.

The explicit expression of D_{C-S} without constraint terms from the weak form of electric-magnetic duality is given by

$$\begin{aligned}
D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\
\hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k} \Gamma_k = \epsilon^{\mu\alpha\beta} \left[2 J_{kl} \partial_\alpha h^l A_\beta + J_{\alpha\beta} A_k \right] \Gamma^k D_\mu \ , \\
D_\mu \hat{\Gamma}^\mu &= B_K^\alpha (J_{k\alpha} + \partial_\alpha A_k) \ , \\
B_K^\alpha &= \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl} \partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma} \sqrt{g_3} \ .
\end{aligned} \tag{9.7.14}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma}$ does not depend on the induced metric.

The extremals of Chern-Simons action satisfy

$$B_K^\alpha (J_{kl} + \partial_l A_k) \partial_\alpha h^l = 0 \ , \ B_K^\alpha = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ . \tag{9.7.15}$$

For non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional and S^2 projection is 1-dimensional or vice versa. This implies a vanishing of Chern-Simons action for both options. Consider for the simplicity the case when S^2 projection is 1-dimensional.

(a) Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [K40] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

(b) With this assumption the Chern-Simons Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta} \left[2 J_{kl} \partial_\alpha h^l A_\beta + J_{\alpha\beta} A_k \right] \Gamma^k D_r \ . \tag{9.7.16}$$

(c) Consider first the general solutions of the modified Dirac equation when M^4 Dirac operator $p^k \gamma_k$ annhilates the spinor so that on mass shell massless fermion is in question. The spinor is covariantly constant with respect to the coordinate r:

$$D_r \Psi = 0 \ . \tag{9.7.17}$$

The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i \int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v} \Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X_l^3 is arbitrary.

(d) For internal lines $p^k\gamma_k$ does not annihilate the spinor although four-momentum can be still on mass shell if the spinor has unphysical helicity. In this case the equation is modified. Again the modes can be localized to 1-D curves.

(e) The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate.

The localization is of utmost importance since and is consistent with the localization of the modes (other than right-handed neutrino) of Kähler Dirac equation at string world sheets discussed in chapter [K105]. String ends would thus define braid strands. The absence of correlation between the behaviors with respect longitudinal coordinate and transversal coordinates looked very strange at first glance. System looked like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines.

A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

(a) The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

(b) One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

(c) What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

    i. The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

ii. It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^\alpha &= \overline{\Psi} O \hat{\Gamma}^\alpha \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \tag{9.7.18}$$

Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

iii. Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

iv. Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

v. There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

vi. The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

(d) Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

(e) Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

(f) The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

(g) One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z)+\overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

How to calculate Dirac determinant?

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If

Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

(a) Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

(b) Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

(c) The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

(d) The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

(e) Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu (dx^\mu/ds)ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

(f) The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

The formula would be

$$K \propto \oint J(x,y) J_T dx^1 \wedge dx^2 \quad . \tag{9.7.19}$$

This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

9.8 Representations for WCW gamma matrices in terms of super-symplectic charges at light cone boundary

During years I have considered several variants for the representation of WCW gamma matrices and each of these proposals has had some weakness.

(a) One question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians. Magnetic flux Hamiltonians do not refer to the space-time dynamics implying genuine 2-dimensionality, which is a catastrophe. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed effective 2-dimensionality is achieved. The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians.

(b) In the original proposal for WCW gamma matrices the covariantly constant right handed spinors played a key role. This led to interpretational problems with quarks. Are they needed at all or do leptons and quarks define somehow equivalent representations? I discovered only recently a brutally simple but deadly objection against this approach: the resulting WCW gamma matrices do not generate all WCW spinors from Fock vacuum. Therefore all modes of the induced spinor fields must be used.

The latter objection forced to realize that nothing is changed if one replaces the covariantly constant right handed neutrino with the collection of quark spinor modes q_n *resp.* leptonic spinor modes L_n multiplied by the contractions $J_{A+} = j^{Ak}\Gamma_k$ *resp.* its conjugate $J_{A-} = j^{A\overline{k}}\Gamma_{\overline{k}}$. It is essential that only of these contractions is used for a given H-chirality.

(a) If the anti-commutator of the spinor fields is or form $J = J_{\alpha\beta}\epsilon^{\alpha\beta}\delta^2(x,y)$ at X^2 for magnetic flux Hamiltonians and appropriate generalization of this fro the sum of magnetic and electric flux Hamiltonians, the "half-Poisson bracket" $\partial_k H_A J^{k\overline{l}}\partial_{\overline{l}} H_B$ from the quark spinor field and its conjugate as anti-commutator from the leptonic spinor field can combine to the full Poisson bracket if the remaining factors are identical.

(b) This happens if the quark modes and lepton-like modes are in 1-1 correspondence and the contractions of the eigenmodes resulting in the contraction satisfy $\overline{q}_m\gamma^0 q_n = \overline{L}_m\gamma^0 L_n = \Phi_{mn}$. The resulting Hamiltonians define an X^2-local algebra: that this extension is needed became obvious already earlier. A stronger condition is that the spinors can be expressed in terms of scalar function bases $\{\Phi_m\}$ so that one would have $q_{m,i} = \{\Phi_m\}q_i$ and $L_{m,i} = \{\Phi_m\}L_i$ so that one would assign to the super-currents the local Hamiltonians $\Phi_m H_A$.

(c) One could of course still argue that it is questionable to use sum of quark and lepton gamma matrices since this the resulting objects to not have a well defined fermion number and cannot be used to generate physical states from vacuum. How seriously this argument should be taken is not clear to me at this moment. One could of course consider also a scenario in which one divides leptonic (or quark) modes to two classes analogous to quark and lepton modes and uses J_{A_+} *resp.* J_{A_-} for these two classes.

In any case, the recent view is that all modes of the induced spinor fields must be used, that lepton-quark degeneracy is absolutely essential for the construction of WCW geometry, and that the original super-symmetrization of the flux Hamiltonians combined with weak electric-magnetic duality is the correct approach. There are also fermionic Noether charges and their super counterparts implied by the criticality but these can be assigned with zero modes.

This section represents both the earlier version of the construction of WCW gamma matrices and the construction introducing explicitly the notion of finite measurement resolution. The motivation for the latter option is that if the number the modes of modified Dirac operator is finite, strictly local anti-commutation relations fail unless one restricts the set of points included to that corresponding to number theoretic braid. In the following integral expressions for WCW Hamiltonians and their super-counterparts are derived first. After that the motivations for replacing integrals with sums are discussed and the expressions for Hamiltonians and super Hamiltonians are derived.

9.8.1 Magnetic flux representation of the super-symplectic algebra

In order to derive representation of WCW gamma matrices and super charges it is good to restate the basic facts about the magnetic flux representation of WCW gamma matrices using the original approach based on 2-dimensional integrals.

9.8.2 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

Generalized magnetic and electric fluxes

Isometry invariants are just a special case of fluxes defining natural coordinate variables for WCW. Canonical transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{s,n,k}$ defining the Lorentz covariant function basis H_A, $A \equiv (a,s,n,k)$ at the light cone boundary: $H_A = H_a \times f(s,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind magnetic or electric flux via the following formulas:

$$Q_{m/e}(H_A|X^2) = \int_{X^2} H_A J_{m/e} \ . \tag{9.8.1}$$

Here the magnetic (electric) flux J_m (J_e) denotes the flux associated with induced Kähler field and its dual which is well-defined since X^2 is part of 4-D space-time surface.

The flux Hamiltonians

$$Q_i(H_A|X^2) = Q_i(H_A|X^2) \ , \quad A \equiv (a,s,n,k) \tag{9.8.2}$$

provide a representation of WCW Hamiltonians as far as the "kinetic" part of Kähler form is considered.

Anti-commutation relations between oscillator operators associated with same partonic 2-surface

The construction of WCW gamma matrices leads to the anti-commutation relations given by

$$\begin{aligned} \{\overline{\Psi}(x)\gamma^0, \Psi(x)\} &= [J_e + J_m)\delta^2_{x,y} \ , \\ J_e &= \int J^{03}\sqrt{g_4} \ . \end{aligned} \tag{9.8.3}$$

Kähler magnetic flux $J_m = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ has no dependence on the induced metric.

If the weak- form of the electric-magnetic duality holds true, Kähler electric flux relates to it via the formula

$$J^{03}\sqrt{g_4} = KJ_{12} \ ,$$

where K is symplectic invariant and identifiable in terms of Kähler coupling strength from classical charge quantization condition for Kähler electric flux. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar = 4\pi\alpha_K$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant. The arguments leading to the identification $\epsilon \pm 1$ at the opposite boundaries of CD are discussed in [K40] , [L11] . An alternative identification is as $\epsilon = 0$ but predicts that WCW is trivial in M^4 degrees of freedom if Kähler function reduces to Chern-Simons terms.

The general form of the anti-commutation relations is therefore

$$\{\overline{\Psi}(x)\gamma^0, \Psi(x)\} \quad = \quad (1+K)J\delta^2_{x,y} \ . \tag{9.8.4}$$

What is nice that at the limit of vacuum extremals the right hand side vanishes when both J and J^1 vanish so that spinor fields become non-dynamical. One can criticize the non-vanishing of the anti-commutator for vacuum extremals of Kähler action.

For the latter option the fermionic counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned}
H_{A,\pm,n} &= \epsilon_q(A,\mp,n)H_{A,\pm,q,n} + \epsilon_L(A,\pm)H_{A,\mp,L,n} \quad , \\
H_{A,+,q,n} &= \oint \overline{\Psi} J^A_+ q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J^A_- \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi} J^A_+ L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J^A_- \Psi d^2x \ , \\
J^A_+ &= j^{Ak}\Gamma_k \ , \ J^A_- = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{9.8.5}$$

The commutative parameters $\epsilon_q(A,\pm,n)$ *resp.* $\epsilon_L(A,\pm,n)$ are assumed to carry quark *resp.* lepton number opposite to that of $H_{A,\mp,q,n}$ *resp.* $H_{A,\mp,L,n}$ and satisfy $\epsilon_i(A,+,n)\epsilon_i(A,-,n) = 1$. One encounters a hierarchy discrete algebras satisfying this condition in the construction of a symplectic analog of conformal quantum field theory required by the construction of quantum TGD [K74] . Associativity condition fixes uniquely the commutative multiplication of these units and analogs of plane waves with discrete momentum are in question.

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m,i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i\gamma^0 q_j &= \overline{L}_i\gamma^0 L_j = g_{ij} \ .
\end{aligned} \tag{9.8.6}$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij} \oint \overline{\Phi}_m \Phi_n H_A J d^2x \ . \quad (9.8.7)$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m \Phi_n = c_{mn}^{\ \ k} \Phi_k \ . \quad (9.8.8)$$

Note that the notion of symplectic QFT [K20] led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to WCW metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras associated with the regions in question. As far as coordinatization of the reduced WCW is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of WCW.

Generalization of WCW Hamiltonians and anti-commutation relations between flux Hamiltonians belonging to different ends of CD

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

(a) The proposed representation of WCW Hamiltonians as flux Hamiltonians [K18, K17] , [L12]

$$Q(H_A) = \int H_A J d^2x \ . \quad (9.8.9)$$

works for the kinetic terms only since J is not expectred to be the same at the ends of the line.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

(b) One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

(c) Perhaps the only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K20] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = (1+K)\int_{S^2_\pm} H_A X\delta^2(s_+,s_-)d^2s_\pm = (1+K)\int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1,s^2)}{\partial(x^1_\pm,x^2_\pm)} d^2x_\pm \quad (9.8.10)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+,s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J^+_{kl} + J^-_{kl} \ , \\ J^{kl}_\pm &= \partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_\pm \ . \end{aligned} \quad (9.8.11)$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

(d) One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A),Q(H_B)\} = Q(\{H_A,H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

(e) The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing J with $X\partial(s^1,s^2)/\partial(x^1_\pm,x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

9.8.3 Expressions for WCW super-symplectic generators in finite measurement resolution

The expressions of WCW Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

(a) In p-adic context integrals do not makes sense so that this representation fails in p-adic context (for pe-adic numbers see [A47]). Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

(b) The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

(c) Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_\pm \times CP_2$ contribute to WCW Hamiltonians and super Hamiltonians and therefore to the WCW metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of WCW as one can expect on basis of the fact that WCW Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that WCW can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and WCW spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} \ . \tag{9.8.12}$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should general coordinate invariant. The number theoretic vision leads to the notion of number theoretic braid defined as the set of points common to real and p-adic variant of X^2. The points of the number theoretic braid are excellent candidates for points x_n. The p-adic variant exists only if X^2 is defined by rational functions with coefficients which are possibly algebraic and thus make sense both in real and p-adic sense. These points belong to the algebraic extension of rational numbers appearing in the representation of X^2 as an algebraic surface but one can consider quite generally the possibility that the points of the number theoretic braid are rational or in a finite algebraic extension of rationals. What is important that if one restricts the consideration to rational points this criterion makes sense even if X^2 is not algebraic. In the generic case one can expect that the number of these points is finite.

9.8.4 WCW geometry and hierarchy of inclusions of hyper-finite factors of type II_1

The WCW metric defined as anti-commutators of the WCW gamma matrices is extremely degenerate since it effectively corresponds to a quadratic form in N-dimensional space, where N_m is the total number of the eigenmodes of D_K. Since two Hamiltonians whose values and corresponding Killing vector fields co-incide at the points of B are equivalent for given ray $M_\pm$, it is natural to pose a cutoff in the number of Hamiltonians used for the representation of reduced WCW in given region inside which induced Kähler form is non-vanishing. The natural manner to pose this cutoff is by ordering the representations with respect to dimension and eigenvalue of Casimir operator for the irreducible representations of $SO(3) \times SO(4)$ in case of M^8 and for the representations of $SO(3) \times SU(3)$ in case of H.

This boils down to a hierarchy of approximate representations of the WCW as Kähler manifold with spinor structure with a truncation of the Clifford algebra to a finite dimensional Clifford algebra. This is in spirit with the proposed interpretation of the inclusion sequence of hyper-finite factors of type II_1 and with the very notion of hyper-finiteness.

A rather concrete connection of WCW geometry with generalized eigenvalue spectrum of the Kähler-Dirac (K-D) operator and basic quantum physics suggests itself if the Dirac determinant can be identified as exponent of Kähler action. One must however be however aware of following points.

(a) It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

(b) Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

(c) The definition of the Dirac determinant is far from obvious. The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

The resulting determinant - if well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

If one accepts this questionable proposal, one can relate WCW geometry directly to elementary particle physics. For instance, from the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k \partial_{\bar{l}} K = \frac{\partial_k \partial_{\bar{l}} exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)} \frac{\partial_{\bar{l}} exp(K)}{exp(K)} \ , \tag{9.8.13}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of $D_K(X^3)$, the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k \partial_{\bar{l}} \lambda_i}{\lambda_i} - \frac{\partial_k \lambda_i}{\lambda_i} \frac{\partial_{\bar{l}} \ lambda_i}{\lambda_i} \tag{9.8.14}$$

for the WCW metric follows. Here complex coordinates refer to the complex coordinates of WCW.

A good candidate for these complex coordinates are the complex coordinates of $S^2 \times S$, $S = CP_2$ or E^4, for the points of B so that a close connection with the geometry of imbedding space is obtained. Once these coordinates have been specified G can be contracted with the Killing vector fields of WCW isometries defining the coordinates for the truncated WCW. By studying the behavior of eigenvalue spectrum under small deformations of X^3_l by symplectic transformations of $\delta CD \times S$ the components of G can be estimated.

9.9 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B7] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K18] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

(a) The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

(b) This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

(c) The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

(d) The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

(e) One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

9.9.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

(a) The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

(b) Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

(c) A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

(d) To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \tag{9.9.1}$$

A more general form of this duality is suggested by the considerations of [K40] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} . \tag{9.9.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

(e) Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} , \tag{9.9.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

(a) The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

(b) The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L5] , [L5] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (9.9.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (9.9.5)$$

(c) The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (9.9.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (9.9.7)$$

(d) There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

(a) The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

(b) The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K65] supports this interpretation.

(c) The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

(d) The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{9.9.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{9.9.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

(a) Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{9.9.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L5]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

(b) For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

(c) The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K70]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

(a) The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

(b) GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K93]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

(c) Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

(d) GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

9.9.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

(a) In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L \nu_R$. $\nu_L \overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

(b) One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D15] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K29] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

(a) Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

(b) The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

(c) How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K51] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

(d) What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K52] .

9.9.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

(a) Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

(b) If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

(a) For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K9]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

(b) The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \quad . \tag{9.9.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

(c) This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

(d) The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j^\alpha_K \partial_\alpha \phi = -j^\alpha A_\alpha \quad . \tag{9.9.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j^\alpha_K$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} \ = \ 0 \quad . \tag{9.9.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B44] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K9] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j^\alpha_I \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j^\alpha_K \phi$ and $j^\alpha_I \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

(e) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \ . \quad (9.9.14)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(f) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

(g) A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

(h) Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

9.9.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered. The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

(a) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^{\alpha}\partial_{\alpha}\phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_{\alpha}(j^{\alpha}\phi) \quad = \quad 0 \ . \tag{9.9.15}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_{\phi} = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_{\phi} = \sum\int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(b) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

(c) A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_{ϕ} and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

(a) The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

(b) If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^{\alpha}_{Q_i} D_{\alpha})\Psi = 0 \quad ,$$

where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

9.9.5 Kähler action for Euclidian regions as Kähler function and Kähler action for Minkowskian regions as Morse function?

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked between possibile connections between TGD and Floer homology [K102] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

(a) All arguments for this have been represented for Minkowskian regions [K28] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

(b) If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. I have earlier claimed that without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit and different coefficient. This statement is wrong since the space-like parts of the corresponding 3-surfaces are discjoint for Euclidian and Minkowskian regions.

(c) There is also an argument stating that Dirac determinant for Chern-Simons Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function which are definitely not proportional to each other.

CP breaking and ground state degeneracy

The Minkowskian contribution of Kähler action is imaginary due to the negativity of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

(a) In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

(b) A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

(c) There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

9.9.6 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

(a) The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

(b) Quite generally, one can write the field equations as conservation laws for I,J,I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A)-j^\alpha_K H^A+T^{\alpha\beta}j^l_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \qquad (9.9.16)$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha\left[j^\alpha_K H^A-T^{\alpha\beta}j^k_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \qquad (9.9.17)$$

For energy one has $H_A=1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

(c) One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j^\alpha_K J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j^\alpha_K D_\alpha H^A \quad = \quad j^\alpha_K J_\alpha{}^\beta j^A_\beta+T^{\alpha\beta}H^k_{\alpha\beta}j^A_k \ . \qquad (9.9.18)$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

(a) The most general assumption is that the conserved isometry currents

$$J_A^{\alpha} = j_K^{\alpha} H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \quad (9.9.19)$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

(b) The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \quad (9.9.20)$$

The conservation of J_A gives

$$d * (\Psi_A d\Phi_A) = 0 \ . \quad (9.9.21)$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

(c) The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d * d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \quad (9.9.22)$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

(a) The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{9.9.23}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{9.9.24}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

(b) Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A \Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{9.9.25}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K40] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

9.9.7 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

(a) Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

(b) The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

(c) This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

(d) A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional modified Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

(a) The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \quad (9.9.26)$$

The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \quad (9.9.27)$$

The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

(b) One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

(c) The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

(a) The modified gamma matrices are of form $T^\alpha_k\Gamma^k$, $T^\alpha_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^α_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^\alpha_k\Gamma^k = T^{\alpha A}\Gamma_A$.

(b) This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA} j_A D_t \Psi \quad = \quad 0 \ . \tag{9.9.28}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

Chapter 10

Classical TGD

10.1 Introduction

A brief summary of what might be called basic principles is in order to facilitate the reader to assimilate the basic tools and rules of intuitive thinking involved.

10.1.1 Quantum-classical correspondence

The fundamental meta level guiding principle is quantum-classical correspondence (classical physics is an exact part of quantum TGD). The principle states that all quantum aspects of the theory, which means also various aspects of consciousness such as volition, cognition, and intentionality, should have space-time correlates [K89] . Real space-time sheets provide kind of symbolic representations whereas p-adic space-time sheets provide correlates for cognition and intentions. All that we can symbolically communicate about conscious experience relies on quantal space-time engineering to build these representations.

The progress in the understanding of quantum TGD has demonstrated that quantum classical correspondence is more or less equivalent with holography, quantum criticality, and criticality as the principle selecting the preferred extremals of Kähler action. It also guarantees 1-1 correspondence between quantum states and classical states essential for quantum measurement theory.

10.1.2 Classical physics as exact part of quantum theory

Classical physics corresponds to the dynamics of space-time surfaces determined by the criticality in the sense that extremals allow an infinite number of deformations giving rise to a vanishing second variation of the Kähler action [K88] . This dynamics have several unconventional features basically due to the possibility to interpret the Kähler action as a Maxwell action expressible in terms of the induced metric defining classical gravitational field and induced Kähler form defining a non-linear Maxwell field not as such identifiable as electromagnetic field however.

Classical long ranged weak and color fields as signature for a fractal hierarchy of copies standard model physics

The geometrization of classical fields means that various classical fields are expressible in terms of imbedding space-coordinates and are thus not primary dynamical variables. This predicts the presence of long range weak and color (gluon) fields not possible in standard physics context. It took 26 years to end up with a convincing interpretation for this puzzling prediction.

What seems to be the correct interpretation is in terms of an infinite fractal hierarchy of copies of standard models physics with appropriately scaled down mass spectra for quarks, leptons, and gauge bosons. Both p-adic length scales and the values of Planck constant predicted by TGD [K99] label various physics in this hierarchy. Also other quantum numbers are predicted as labels. This means that universe would be analogous to an inverted Mandelbrot fractal with each bird's eye of view revealing new long length scale structures serving also as correlates for higher levels of self hierarchy.

Exotic dark weak forces and their dark variants are consistent with the experimental widths for ordinary weak gauge bosons since the particles belonging to different levels of the hierarchy do not have direct couplings at Feynman diagram level although they have indirect classical interactions and also the de-coherence reducing the value of $\hbar$ is possible. Classical long ranged weak fields play a key role in quantum control and communications in living matter [K30, K24] . Long ranged classical color force in turn is the backbone in the model of color vision [K34] : colors correspond to the increments of color quantum numbers in this model. The increments of weak isospin in turn could define the basic color like quale associated with hearing (black-white $\leftrightarrow$ to silence-sound [K34, K69, K71]).

Topological field quantization and the notion of many-sheeted space-time

The compactness of CP_2 implies the notions of many-sheeted space-time and topological field quantization. Topological field quantization means that various classical field configurations decompose into topological field quanta. One can see space-time as a gigantic Feynman diagram with lines thickened to 4-surfaces. Criticality of the preferred extremals implies that only selected field configurations analogous to Bohr's orbits are realized physically so that quantum-classical correspondence becomes very predictive. An interpretation as a 4-D quantum hologram is a further very useful picture [K43] but will not be discussed in this chapter in any detail.

Topological field quantization implies that the field patterns associated with material objects form extremely complex topological structures which can be said to belong to the material objects. The notion of field body , in particular magnetic body, typically much larger than the material system, differentiates between TGD and Maxwell's electrodynamics, and has turned out to be of fundamental importance in the TGD inspired theory of consciousness. One can say that field body provides an abstract representation of the material body.

One implication of many-sheetedness is the possibility of macroscopic quantum coherence. By quantum classical correspondence large space-time sheets as quantum coherence regions are macroscopic quantum systems and therefore ideal sites of the quantum control in living matter.

(a) The original argument was that each space-time sheet carrying matter has a temperature determined by its size and the mass of the particles residing at it via de Broglie wave length $\lambda_{dB} = \sqrt{2mE}$ assumed to define the p-adic length scale by the condition $L(k) < \lambda_{dB} < L(k_>)$. This would give very low temperatures when the size of the space-time sheet becomes large enough. The original belief indeed was that the large space-time sheets can be very cold because they are not in thermal equilibrium with the smaller space-time sheets at higher temperature.

(b) The assumption about thermal isolation is not needed if one accepts the possibility that Planck constant is dynamical and quantized and that dark matter corresponds to a hierarchy of phases characterized by increasing values of Planck constant [K99, K23] . From $E = hf$ relationship it is clear that arbitrarily low frequency dark photons (say EEG photons) can have energies above thermal energy which would explain the correlation of EEG with consciousness. This vision allows to formulate more precisely the basic notions of TGD inspired theory of consciousness and leads to a model of living matter giving precise quantitative predictions. Also the ability of this vision to generate new insights to quantum biology provides strong support for it [K24] .

Many-sheeted space-time predicts also fundamental mechanisms of metabolism based on the dropping of particles between space-time sheets with an ensuing liberation of the quantized zero point kinetic energy. Also the notion of many-sheeted laser follows naturally and population inverted many-sheeted lasers serve as storages of metabolic energy [K44] .

Space-time sheets topologically condense to larger space-time sheets by wormhole contacts which have Euclidian signature of metric. This implies causal horizon (or elementary particle horizon) at which the signature of the induced metric changes from Minkowskian to Euclidian. This forces to modify the notion of sub-system. What is new is that two systems represented by space-time sheets can be unentangled although their sub-systems bound state entangle with the mediation of the join along boundaries bonds connecting the boundaries of sub-system space-time sheets. This is not allowed by the notion of sub-system in ordinary quantum mechanics. This notion in turn implies the central concept of fusion and sharing of mental images by entanglement [K89] .

Zero energy ontology

The notion of zero energy ontology emerged implicitly in cosmological context from the observation that the imbeddings of Robertson-Walker metrics are always vacuum extremals. In fact, practically all solutions of Einstein's equations have this property very naturally. The explicit formulation emerged with the progress in the formulation of quantum TGD. In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T, and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T. One of the implications is a new view about fermions and bosons allowing to understand Higgs mechanism among other things.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach.

TGD Universe is quantum spin glass

Since Kähler action is Maxwell action with Maxwell field and induced metric expressed in terms of $M^4_+ \times CP_2$ coordinates, the gauge invariance of Maxwell action as a symmetry of the vacuum extremals (this implies is a gigantic vacuum degeneracy) but not of non-vacuum extremals. Gauge symmetry related space-time surfaces are not physically equivalent and gauge degeneracy transforms to a huge spin glass degeneracy. Spin glass degeneracy provides a universal mechanism of macro-temporal quantum coherence and predicts degrees of freedom called zero modes not possible in quantum field theories describing particles as point-like objects. Zero modes not contributing to the configuration space line element are identifiable as effectively classical variables characterizing the size and shape of the 3-surface as well as the induced Kähler field. Spin glass degeneracy as mechanism of macroscopic quantum coherence should be equivalent with dark matter hierarchy as a source of the coherence [K43] .

Classical and p-adic non-determinism

The vacuum degeneracy of Kähler action implies classical non-determinism, which means that space-like 3-surface is not enough to fix the space-time surface associated with it uniquely as an absolute minimum of action, and one must generalize the notion of 3-surface by allowing sequences of 3-surfaces with time like separations to achieve determinism in a generalized sense. These "association sequences" can be seen as symbolic representations for the sequences of quantum jumps defining selves and thus for contents of consciousness. Not only speech and written language define symbolic representations but all real space-time sheets of the space-time surfaces can be seen in a very general sense as symbolic representations of not only quantum states but also of quantum jump sequences. An important implication of the classical non-determinism is the possibility to have conscious experiences with contents localized with respect to geometric time. Without this non-determinism conscious experience would have no correlates localized at space-time surface, and there would be no psychological time.

p-Adic non-determinism follows from the inherent non-determinism of p-adic differential equations for any action principle and is due to the fact that integration constants, which by definition are functions with vanishing derivatives, are not constants but functions of the pinary cutoffs x_N defined as $x = \sum_k x_k p^k \rightarrow x_N = \sum_{k<N} x_k p^k$ of the arguments of the function. In p-adic topology one can therefore fix the behavior of the space-time surface at discrete set of space-time points *above* some length scale defined by p-adic concept of nearness by fixing the integration constants. In the real context this corresponds to the fixing the behavior *below* some time/length scales since points p-adically near to each other are in real sense faraway. This is a natural correlate for the possibility to plan the behavior and p-adic non-determinism is assumed to be a classical correlate for the non-determinism of intentionality, and perhaps also imagination and cognition.

These two non-determinisms allow to understand the

self-referentiality of consciousness at a very general level. In a given quantum jump a space-time surface can be created with the property that it represents symbolically or cognitively something about the contents of consciousness before the quantum jump. Thus it becomes possible to become conscious about being conscious of something. This is very much like mathematician expressing her thoughts as symbol sequences which provides feedback to go the next abstraction level.

Classical and p-adic non-determinisms force also the generalization of the notion of quantum entanglement. Time-like entanglement, crucial for understanding long term memory and precognition becomes possible. The notion of many-sheeted space-time forces also to modify the notion of sub-system, which implies that unentangled systems can have entangled sub-systems. One can partially understand this in terms of length scale dependent notion of entanglement (the entanglement of sub-systems is not seen in the length scale resolution defined by the size of unentangled systems) but only partially. The formation of join along boundaries bonds between sub-system space-time sheets and the fact that topologically condensed space-time sheets are separated by elementary particle horizons from larger space-time sheets, provide the deeper topological motivation for the generalization of sub-system concept.

Dark matter hierarchy and hierarchy of Planck constants

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant forces a further generalization of the notion of imbedding space and thus of space-time. One can say, that imbedding space is a book like structure obtained by gluing together infinite number of copies of the imbedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

The hierarchy of Planck constants can be reduced to the quantum criticality of Kähler action due to the non-determinism of Kähler action and the generalization of imbedding space is only a useful auxiliary tool to describe the situation mathematically.

All this is a work in progress and there are many uncertainties involved. Despite this it seems that it is good to sum up the recent view in order to make easier to refer to the new developments in the existing chapters.

p-Adic fractality of life and consciousness

p-Adic fractality of biology and consciousness has become an increasingly important guide line in the construction of the theory. This notion allows to relate phenomena occurring in the molecular level to phenomena like remote viewing and psychokinesis and it leads also to the view that topological field quanta of various fields of astrophysical size are crucial for the functioning of bio-systems. If one accepts p-adic fractality, the theory can be tested in unexpected manners, in particular in molecular and cellular length scales where the systems are much simpler. Sensory perception, long term memory, remote mental interactions, metabolism: all these phenomena rely on the same basic mechanisms. p-Adic length scale hypothesis allows to quantify the hypothesis with testable quantitative predictions.

Double slit experiment and classical non-determinism

Bohr's complementarity principle is the basic element of Copenhagen interpretation and at the same time one of the most poorly defined aspects of this interpretation. If the possibility of macroscopic quantum entanglement between measurement instrument and quantum system is accepted, complementary principle becomes un-necessary. This is however not all that is needed. If classical non-determinism makes it possible to represent quantum jump sequences at space-time level, a revision of space-time description of quantum measurement is necessary. This sounds very logical but to be honest, I write these lines only after having learned about the remarkable experiment done by Shahriar Afshar [J5] .

The variant of double slit experiment by Shahriar Afshar seems to contradict the Copenhagen interpretation which states that the particle and field aspects are complementarity and thus mutually exclusive. In the case of double slit experiment complementarity predicts that the measurement of whether the photon came to the detector through slit 1 or 2 should destroy the interference pattern of electromagnetic fields in the region behind the screen.

The experimental arrangement of Afshar differs from the standard double slit experiment in that a lens was added behind the screen. The lens transmitted the photons coming from slits 1 and 2 via mirrors to detectors A and B so that in particle picture a photon detected by A (B) could be regarded as coming from slit 1 (2). In the first step both slits were open and the detectors represented interference patterns representing diffraction through single slit. The other slit was then closed and metal wires at the positions of dark interference rings were added. These wires degraded somewhat the image in the second detector. After this the second slit was opened again. Surprisingly, the resulting interference pattern was the original one.

The measurement certainly measures the particle aspect of photons. On the other hand, the preservation of the detected patterns means that no photons did enter in the regions containing the wires so that also interference pattern is there. Hence wave and particle aspects seem to be mutually consistent.

This finding is difficult to understand in Copenhagen interpretation and also in the many-worlds interpretation of quantum mechanics. Afshar himself suggest that the very notion of photon must be questioned. It is however difficult to accept this view since the photon absorption quite concretely corresponds to a click in the detector and also because the mathematical formalism of second quantization works so fantastically.

The conclusion can be criticized. What is primarily measured is not basically through which slit the photons came but whether the direction of the momentum of the photon emerging from the lens was in the angle range characterizing the detector or not. One can however argue that in deterministic physics for fields the two measurements are equivalent so that the problem remains.

In TGD framework the classical physics is not completely deterministic and this has led to a generalization of the notion of quantum classical correspondence. Space-time surface provides a classical (unfaithful) representation not only for quantum states but for quantum jump sequences or equivalently, for sequences of quantum states. The most obvious identification for the quantum states is as the maximal non-deterministic regions of a given space-time sheet.

In the recent context this would mean that the fields in the region between the screen and lens represent the state before the state function reduction and thus the interference pattern, whereas the fields in the region between lens and detectors represent the situation after the state function reduction. The interaction with lens involves classical non-determinism.

This picture conforms also with the notion of topological field quantization. The space-time decomposes into space-time sheets interpreted, topological field quanta (topological light rays containing photons, flux quanta of magnetic field, etc..). Topological field quanta correspond to the coherence regions for classical fields with spinor fields included. De-coherence corresponds to the splitting of space-time sheet to smaller, possibly parallel space-time sheets. Topological field quantum carries classical fields inside it but behaves as a whole like particle. Hence particle and wave aspects are consistent in the sense that below the size scale L of the topological field quantum (say the thickness of a magnetic flux tube or topological light ray) the description as a wave applies and above L particle description makes sense. In the recent case the coherence is lost at the lens space-time sheet where the space-time sheet representing interference pattern decomposes to two sheets representing photon beams going to the two detectors.

10.1.3 Some basic ideas of TGD inspired theory of consciousness and quantum biology

The following ideas of TGD inspired theory of consciousness and of quantum biology are the most relevant ones for what will follow.

(a) "Everything is conscious and consciousness can be only lost" is the briefest manner to summarize TGD inspired theory of consciousness. Quantum jump as moment of consciousness and the notion of self are key concepts of the theory. Self is a system able to avoid bound state entanglement with environment and can be formally seen as an ensemble of quantum jumps. The contents of consciousness of self are defined by the averaged increments of quantum numbers and zero modes (sensory and geometric qualia). Moment of consciousness can be said to be the counterpart of elementary particle and self the counterpart of many-particle state, either bound and free. The selves formed by macro-temporal quantum coherence are in turn the counterparts of atoms, molecules and larger structures. Macro-temporal quantum coherence effectively binds a sequence of quantum jumps to a single quantum jump as far as conscious experience is considered. The idea that conscious experience is about changes amplified to macroscopic quantum phase transitions, is the key philosophical guideline in the construction of various models, such as the model of qualia, the capacitor model of sensory receptor, the model of cognitive representations, and declarative memories.

(b) Macro-temporal quantum coherence is a second consequence of the spin glass degeneracy [K43] . It is essentially due to the formation of bound states and has as a topological correlate the formation of join along boundaries bonds connecting the boundaries of the component systems. During macro-temporal coherence quantum jumps integrate effectively to single long-lasting quantum jump and one can say that system is in a state of oneness, eternal now, outside time. Macro-temporal quantum coherence makes possible stable non-entropic mental images. Negative energy MEs are one particular mechanism making possible macro-temporal quantum coherence via the formation of bound states, and remote metabolism and sharing of mental images are other facets of this mechanism. The real understanding of the origin of macroscopic quantum coherence requires the generalization of quantum theory allowing dynamical and quantized Planck constant [K23, K24] .

(c) p-Adic physics as physics of intentionality and of cognition is a further key idea of TGD inspired theory of consciousness. p-Adic space-time sheets as correlates for intentions and p-adic-to-real transformations of them as correlates for the transformation of intentions to actions allow deeper understanding of also psychological time as a front of p-adic-to-real transition propagating to the direction of the geometric future. Negative energy MEs are absolutely essential for the understanding of how precisely targeted intentionality is realized.

10.1.4 About preferred extremals

The understanding of preferred extremals of Kähler action is the basic challenge of classical TGD. The field equations are known locally but the key problem is to give a precise meaning to the "preferred". Various attempts in this direction are discussed in [K9, K117]. These options give different perspectives to the properties of preferred extremals but provide no magic formula.

Before continuing, it must be emphasized that the notion of preferred extremal originated in positive energy ontology. In ZEO 3-surfaces are pairs of space-like 3-surfaces at the boundaries of CD. Also the light-like partonic orbits at which the induced metric changes its signature could be included to get a closed 3-surface analogous to Wilson loop. In deterministic theory one would expect that the extremals are unique so that "preferred" would become obsolete. Kähler action is non-deterministic and quantum criticality suggests that the preferred extremals have Kac-Moody type symmetries are gauge symmetries deforming partonic orbits and preserving their light-likeness. The number of gauge equivalence classes would be finite and correspond to the integer n defining the value of effective Planck constant $h_{eff} = n \times h$. The conformal subalgebra with conformal weights coming as multiples of n would act as gauge symmetries. The most that one might expect that above measurement resolution the attribute "preferred" is un-necessary in ZEO.

The idea about Bohr orbitology would require that "preferred" is not an empty attribute even in ZEO. There would be strong correlations between the space-like 3-surfaces at the oppposite boundaries of CD: the pairs would be like point pairs at Bohr orbits connecting the boundaries of CD.

It is good to summarize the attempts to give meaning to "preferred". The properties assigned to preferred extremals characterize also the known extremals.

(a) The original proposal was that preferred extremals correspond to absolute minima of Kähler action. This makes sense only for Euclidian regions of space-time surfaces representing lines of generalized Feynman diagrams. This option is not number theoretically attractive since the very notion of minimum is p-adically poorly defined unless one can reduce absolute minimization to purely altebraic conditions making sense also p-adically.

(b) Later I ended up with the idea that preferred extremals are critical being analogous to saddle points of potential function: what this exactly means is not obvious [K9, K105]. This option is not consistent with absolute minimization. It took a long time to realize that Minkowskian and Euclidian regions give imaginary *resp.* real contributions to the exponent of the vacuum functional having interpretation as Morse function *resp.* Kähler function of WCW. One might ask whether absolute minimization works in Euclidian regions and criticality in Minkowskian regions.

(c) I have proposed the characterization of preferred extremal property in terms of Hamilton-Jacobi structure generalizing the notion of complex structure and being motivated by the huge super-conformal symmetries of "world of classical worlds" [K9]. This picture is consistent with the identification of Minkowskian regions consisting of massless and interaction topological field quanta since one can speak about local light direction and local polarization directions. This picture is very quantal since linear superposition is not possible and the counterpart of it is set theoretic union of space-time sheets representing quanta. Number theoretic vision based on classical number fields supports similar picture.

(d) Almost topological QFT property requires that Kähler action reduces to "boundary" terms transformable to Chern-Simons terms. This is guaranteed if the Kähler current is orthogonal to Kähler gauge potential ($j \cdot A = 0$) and the weak form of electric-magnetic duality holds true.

(e) It took decades to realize that GRT space-time is only an effective space-time obtained by replacing the sheets of many-sheeted space-time with single piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Equivalence Principle as it is expressed by Einstein's equations would follow from Poincare invariance in long length scales. Therefore it is not necessary that Einstein's equations are satisfy in TGD and the vanishing of the divergence of energy momentum tensor for Kähler action could be enough.

I have however considered also other alternatives before ending up this view.

i. Vacuum extremals of Kähler action seem to be excellent candidates for defining cosmological models. In light of what was said above this would mean that GRT space-times having imbedding as vacuum extremals are in favored position. This would not be surprising.

ii. An obvious question is whether Einstein equations could be true for preferred extremals [K9]. The condition that Kähler 4-force vanishes implies vanishing of the divergence of the energy momentum tensor. In general relativity this leads to Einstein's equations with cosmological constant term since the linear combination of Einstein tensor and metric tensor is automatically divergenceless. In TGD framework Einstein equations would have powerful consequences: for instance, curvature scalar would be constant so that mathematically highly interesting constant curvature spaces allowing to classify manifolds topologically, could emerge naturally. A weaker condition would be that energy momentum tensor is divergenceless only asymptotically when dissipation characterize by Lorentz force classically is absent.

iii. In TGD framework one can also consider a weaker form of Einstein equations with cosmological constant: several (at most two) cosmological "constants" would appear in the counterpart of Einstein equations [K114].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at http://www.tgdtheory.fi/cmaphtml.html [L21]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L22]. The topics relevant to this chapter are given by the following list.

- Classical TGD [L28]
- Topological field quantization [L80]
- Identification of preferred extremals of Kaehler action [L43]
- 4-D spin glass degeneracy [L23]
- TGD and GRT [L72]
- Equivalence Principle [L35]

10.2 Many-sheeted space-time, magnetic flux quanta, electrets and MEs

TGD inspired theory of consciousness and of living matter relies on space-time sheets carrying ordinary matter, topological light rays (massless extremals, MEs), and magnetic and electric flux quanta. There are some new results which motivate a separate discussion of them.

10.2.1 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also sub-harmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K79] .

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long ranged classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

Dark matter hierarchy and consciousness

The emergence of the vision about dark matter hierarchy has meant a revolution in TGD inspired theory of consciousness. Dark matter hierarchy means also a hierarchy of long term memories with the span of the memory identifiable as a typical geometric duration of moment of consciousness at the highest level of dark matter hierarchy associated with given self so that even human life cycle represents at this highest level single moment of consciousness.

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [K24] . The applications to living matter suggests that there is a hierarchy a hierarchy of Planck constants h_{eff} coming as integer multiples of ordinary Planck constant. The original - perhaps too limited - proposal was that in living matter there would be preferred values for the integr coming as power of 2^{11} $\hbar(k) = \lambda^k \hbar_0$, $\lambda = 2^{11}$ for $k = 0, 1, 2, ...$ [K24]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy [K27]. One can also ask whether number theoretically very simple integers could define the values of h_{eff}/h. One candidate for this class of integers characterize polygons constructible using only compass and ruler. The sine and cosine of the angle $2\pi/n$ characterizing the polygon reduces to expression involving only square root operations applied on rationals. These integers are products of power of two with a subset of distinct Fermat primes.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [K24] . Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K47, K24] . A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K24] .

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K23, K24] . The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to a single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

10.2.2 p-Adic length scale hypothesis and the connection between thermal de Broglie wave length and size of the space-time sheet

Also real space-time sheets are assumed to be characterized by p-adic prime p and assumed to have a size determined by primary p-adic length scale L_p or possibly n-ary p-adic length scale $L_p(n)$. Since multi-p-fractality is allowed [K87] , one cannot exclude even the possibility that each space-time dimension might correspond to its own p-adic length scale and even several p-adic primes could be associated with single dimension.

The possibility to assign a p-adic prime to the real space-time sheets is required by the success of the elementary particle mass calculations and various applications of the p-adic length scale hypothesis. Rational numbers are common to reals and all p-adic number fields. The p-adic-to-real transition transforming intentions to actions is made possible by a large number of common rational points between p-adic and real space-time surfaces, which supports the view that real space-time sheets obeys effective p-adic topology as an approximate topology in some resolution and below some length scale. p-Adic prime thus characterizes the classical non-determinism of the Kähler action.

Parallel space-time sheets with distance about 10^4 Planck lengths form a hierarchy. Each material object (...,atom, molecule, ..., cell,...) would correspond to this kind of space-time sheet. The p-adic primes $p \simeq 2^k$, k prime or power of prime, characterize the size scales of the space-time sheets in the hierarchy. The p-adic length scale $L(k)$ can be expressed in terms of cell membrane thickness as

$$L(k) = 2^{(k-151)/2} \times L(151) \ , \qquad (10.2.1)$$

$L(151) \simeq 10$ nm. These are so called primary p-adic length scales but there are also n-ary p-adic length scales related by a scaling of power of $\sqrt{p}$ to the primary p-adic length scale. Quite recent model for photosynthesis [K44] gives additional support for the importance of also n-ary p-adic length scales so that the relevant p-adic length scales would come as half-octaves in a good approximation but prime and power of prime values of k would be especially important.

10.2.3 Topological light rays (massless extremals, MEs)

I have described MEs, or "topological light rays", in detail in [L2] and in [K61] newphys, and describe here only very briefly the basic characteristics of MEs and concentrate on new idea about their possible role for consciousness and life.

What MEs are?

MEs (massless extremals, topological light rays) can be regarded as topological field quanta of classical radiation fields [K61, K7] . They are typically tubular space-time sheets inside which radiation fields propagate with light velocity in single direction without dispersion. The simplest case corresponds to a straight cylindrical ME but also curved MEs, kind of curved light rays, are possible. The initial values for a given moment of time are arbitrary by light likeness. Therefore MEs are ideal for precisely targeted communications. What distinguishes MEs from Maxwellian radiation fields in empty space is that light like vacuum 4-current is possible: ordinary Maxwell's equations would state that this current vanishes. Quite generally, purely geometric vacuum charge densities and 3-currents are purely TGD based prediction and could be seen as a classical correlate of the vacuum polarization predicted by quantum field theories.

MEs are fractal structures containing MEs within MEs. The so called scaling law of homeopathy predicts that the high frequency MEs inside low frequency MEs are in a ratio having discrete values [K39] . One can indeed justify this relationship. As ions drop from smaller space-time sheets to magnetic flux tubes, zero point kinetic energy is liberated as high frequency MEs, and the ions dropped to magnetic flux tubes generate cyclotron radiation, and the ratio of the fundamental frequencies is constant not depending on particle mass and being determined solely by p-adic length scale hypothesis. The model for the radio waves induced by the irradiation of DNA by laser light [I8] gives support for this picture [K43] .

Two basic types of MEs

MEs have 2-dimensional CP_2 projection which means that electro-weak holonomy group is Abelian (color holonomy is always Abelian which suggests that physical states in TGD Universe correspond to states of color multiplets with vanishing color hypercharge and isospin rather than color singlets). If CP_2 projection belongs to a homologically non-trivial geodesic sphere, only em and Z^0 fields and Abelian color gauge fields are present. In the homologically trivial case only classical W fields are non-vanishing.

(a) Neutral MEs can be assigned to various kinds of communications from biological body to the magnetic body and fractal hierarchy of EEGs and ZEGs represent the basic example in this respect [K24] .

(b) Dark W MEs serving as correlate for dark W exchanges induce an exotic ionization of atomic nuclei [K84, K25, K24] . This induces charge entanglement between magnetic body and biological body generating dark plasma oscillation patterns inducing nerve pulse patterns and ion waves at the space-time sheets occupied by the ordinary matter. The mechanism is based on many-sheeted Faraday law inducing electromagnetic fields at ordinary space-time sheet in turn giving rise to ohmic currents. State function reduction selects one of the exotically ionized configurations. This mechanism is the most plausible candidate for how magnetic body as an intentional agent controls biological body.

Negative energy MEs

MEs can have either positive or negative energy depending on the time orientation. The understanding of negative energy MEs has increased considerably. Phase conjugate laser beams [D27] are the most plausible standard physics counterparts of negative energy MEs since they can be interpreted as time reversed laser beams and do not possess direct Maxwellian analog. By quantum-classical correspondence one can interpret the frequencies associated with negative energy MEs as energies. One can also assume that the Bose-Einstein condensed photons associated with negative energy MEs and with the coherent light generated by the light like vacuum current have negative energies.

For frequencies for which energy is above the thermal energy there is no system which could interact with negative energy MEs or absorb negative energy photons. Therefore negative energy MEs and corresponding photons should propagate through matter practically without any interaction. Feinberg has demonstrated that phase conjugate laser beams behave similarly: for instance, one can see through chickens using these laser beams [D5] . This means that negative energy MEs do not respect Faraday cages and thus represent an attractive candidate for the hypothetical Psi field.

Negative energy MEs have many applications.

(a) Negative energy MEs ideal for generating time like entanglement. Since negative energies are involved, this entanglement can be seen as a correlate for the bound state entanglement leading to a macro-temporal quantum coherence. Negative energy MEs make thus possible telepathic sharing of mental images. Negative energy MEs are involved with both sensory perception, long term memory, and motor action. In the model for living matter [K24] The charge entanglement generated by W MEs inducing exotic weak charge and electromagnetic charge is assumed to be responsible for biocontrol whereas neutral MEs in general carrying both em and Z^0 fields are responsible for communications.

(b) Negative energy MEs are ideal for a precisely targeted realization of intentions. p-Adic ME having a large number of common rational points with negative energy ME is generated and transformed to a real ME in quantum jump. The system receives positive energy and momentum as a recoil effect and the transition is not masked by ordinary spontaneously occurring quantum transitions since the energy of the system increases. One can say that negative energy ME represents the desires communicated to the geometric past and inducing as a reaction the desired action realized as say neuronal activity and generation of positive energy MEs.

(c) The generation of negative energy MEs is also in a key role in remote metabolism and MEs serve as quantum credit cards implying an extreme flexibility of the metabolism. If the system receiving negative energy MEs is a population inverted laser or its many-sheeted counterpart, then quite a small field intensity associated with negative energy MEs (intensity of negative energy photons) can lead to the amplification of the time reflected positive energy signal. The reason is that the rate for the induced emission is proportional to the number of particles dropped to the ground state from the excited state. Therefore even negative energy bio-photons might serve as quantum controllers of metabolism and induce much more intense beams of positive energy photons, say when interacting with mitochondria.

10.2.4 Magnetic flux quanta and electrets

Magnetic flux tubes and electrets are extremals of Kähler action dual to each other. Also layer like magnetic flux quanta and their electric counterparts are possible. The magnetic/electric field is in a good approximation of constant magnitude but has varying direction.

Magnetic fields and life

The magnetic field associated with any material system is topologically quantized, and one can assign to any system a magnetic body. An attractive idea is that the relationship of the magnetic body to the material system is to some degree that of the manual to an electronic instrument. Quantitative arguments related to the dark matter hierarchy assuming that magnetic bodies are dark suggest that cognitions and emotions are regarded as somatosensory qualia of the magnetic body [K34, K24] . Magnetic body would in this case serve as a kind of computer screen at which the data items processes in say brain are communicated either classically (positive energy MEs) or by sharing of mental images (negative energy MEs).

Magnetic body is also an active intentional agent: motor actions are controlled from magnetic body and proceed as cascade like processes from long to short length and time scales as quantum communications of desires at various levels of hierarchy of magnetic bodies. Communication occurs backwards in geometric time by negative energy MEs. Motor action as a response to these desires occurs by classical communications by positive energy MEs and as neural activities. This explains the coherence and synchrony of motor actions difficult to understand in neuroscience framework. The sizes of flux quanta are astrophysical: for instance, EEG frequency of 7.8 Hz corresponds to a wave length defined by Earth's circumference. The non-locality in the length scale of magnetosphere, and even in length scales up to light life, is forced by Uncertainty Principle alone, if taken seriously in macroscopic length scales.

The leakage of supra currents of ions and their Cooper pairs from magnetic flux tubes of the Earth's magnetic field to smaller space-time sheets and their dropping back involving liberation of the zero point kinetic energy defines one particular metabolic "Karma's cycle".

In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

The dropping of protons from $k = 137$ atomic space-time sheet involved with the utilization of ATP molecules is only a special instance of the general mechanism involving an entire hierarchy of zero point kinetic energies defining universal metabolic currencies. This leads to the idea that the topologically quantized magnetic field of Earth defines the analog of central nervous system and blood circulation present already during the pre-biotic evolution and making possible primitive metabolism. This has far reaching implications for the understanding of how pre-biotic evolution led to living matter as we understand it [K30] .

For instance, it has recently become clear that the dropping of atoms and molecules from space-time sheets labelled by p-adic prime $p \simeq 2^k$, $k = 131$, liberates photons at visible and near infrared wave lengths. The hot $k = 131$ space-time sheets (with temperatures above 1000 K) could have served as a source of metabolic energy for life-forms at cool $k = 137$ sheets. Photosynthesis could have developed in the circumstances where solar radiation was replaced with these photons. The correct prediction is that chlorophylls should be especially sensitive to these wave lengths. In particular, it is predicted that also IR wave lengths 700-1000 nm should have been utilized. There indeed are bacteria using only this portion of solar radiation. This leads to a scenario making sense only in TGD universe. Pre-biotic life could have developed at the cool space-time sheets in the hot interior of Earth below crust, where $k = 131$ space-time sheets are possible and this life could still be there [K30] . Also the life as we know it, could involve hot spots generated by the cavitation of water inside cell. The classical repulsive Z^0 force causes a strong acceleration during final stages of bubble collapse creating high temperatures, and could explain also sono-luminescence [D13] , [D13] as suggested in [K25] .

Magnetic Mother Gaia could also form sensory and other representations receiving input from several brains via negative energy EEG MEs entangling magnetosphere with brains. The multi-brained magnetospheric selves could be responsible for the third person aspect of consciousness and for the evolution of social structures. For instance, the successful healing by prayer and meditation groups [J2] , and the experiments of Mark Germine [J9] provide support for the notion of multi-brained magnetospheric selves are involved. Magnetic flux tubes could function as wave guides for MEs and this aspect is crucial in the model of long term memory.

Electrets and bio-systems

Bio-systems are known to be full of electrets and liquid crystals [I10] . Perhaps the most fundamental electret structure is cell membrane. In particular, the water inside cells tends to be in gel phase which is liquid crystal phase. There are many good reasons for why water should be in ordered phase. One very fundamental reason is that bio-polymers are stable in liquid crystal/ordered water phase since there are no free water molecules available for the de-polymerization by hydration. In fact, only a couple of years ago it was experimentally discovered that bio-polymers can be stabilized around ice.

The capacitor model for sensory receptor is one very important application of the electret concept [K34] , [L3] . Sensory qualia result in the flow of particles with given quantum numbers from the plate to another one in quantum discharge. This kind of amplification of quantum number *resp.* zero mode increments would give rise to both geometric *resp.* non-geometric qualia [K34] .

Also micro-tubuli are electrets. Sol-gel transition, as any phase transition, is an good candidate for the representation of a conscious bit and controlled local sol-gel transitions between ordinary and liquid crystal water could be a basic control tool making possible cellular locomotion, changes of protein conformations, etc... The tubulin dimers of micro-tubuli could induce sol-gel transformations by generating negative energy MEs, and micro-tubular surface could provide bit maps of their environment somewhat like sensory areas of brain provide maps of body. If gel$\rightarrow$sol transition around tubulin inducing conformational change induces sol$\rightarrow$gel transformation in some point of environment as would be the case for the seesaw mechanism to be discussed below, a one-one correspondence would result. By this one-one correspondence micro-tubules would automatically generate kind of conscious log files about the control activities which could have evolved to micro-tubular declarative memory representations about what happens inside cell [K44] .

10.3 General view about field equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics.

10.3.1 Field equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k{}_l\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu{}^\beta - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \qquad (10.3.1)$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha{}^\beta h^k{}_\beta + J^k{}_l\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \qquad (10.3.2)$$

$H^k_{\alpha\beta}$ denotes the components of the

second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha{}^\beta\partial_\beta h^k + J^k{}_l)\partial_\alpha h^k) = 0 \ . \qquad (10.3.3)$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \qquad (10.3.4)$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K33] . Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K17] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{10.3.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{10.3.6}$$

The interpretation of this conditions is rather transparent.

(a) The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

(b) Third and fourth condition state that the induced Kähler field at X^3_l is purely magnetic and that the metric of x^3_l reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the modified Dirac operator is considered [K17] .

(c) The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X^3_l.

(d) The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X^3_l)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X^3_l)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y^3_l.

(e) The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

10.3.2 Topologization and light-likeness of the Kähler current as alternative manners to guarantee vanishing of Lorentz 4-force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \quad . \tag{10.3.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \quad . \end{aligned} \tag{10.3.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l} \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M_+^4 and CP_2 degrees of freedom to

$$T^{\alpha\beta} H^k_{\alpha\beta} = 0 \quad . \tag{10.3.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi \overline{B} \ , \quad \rho_I = \overline{B} \cdot \overline{A} \ . \tag{10.3.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla \times \overline{B} - \partial_t \overline{E} &= \overline{j} = \psi \overline{j}_I = \psi \left(\phi \overline{B} + \overline{E} \times \overline{A} \right) \ , \\ \nabla \cdot E &= \rho = \psi \rho_I \ . \end{aligned} \tag{10.3.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla \times \overline{B} = \alpha \overline{B} \ , \quad \alpha = \psi \phi \ . \tag{10.3.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B44].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I \overline{E} + \overline{j} \times \overline{B} = \psi \overline{B} \cdot \overline{AE} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \times \overline{B} = 0 \ . \tag{10.3.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j} \cdot \overline{E} = \psi \overline{B} \cdot \overline{E} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \cdot \overline{E} = 0 \ . \tag{10.3.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi \rho_I = \psi B \cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

(a) The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{10.3.15}$$

This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinates and and $\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

(b) The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta \overline{B} \tag{10.3.16}$$

The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla \times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the imbedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B18] .

In hydrodynamics the role of the magnetic field is taken by the velocity field. TGD based models for nuclei [K32] and condensed matter [K25] involve in an essential manner valence quarks having large $\hbar$ and exotic quarks giving nucleons anomalous color and weak charges creating long ranged color and weak forces. Weak forces have a range of order atomic radius and could be responsible for the repulsive core in van der Waals potential.

This raises the idea that the incompressible flow could occur along the field lines of the Z^0 magnetic field so that the velocity field would be proportional to the Z^0 magnetic field and the Beltrami condition for the velocity field would reduce to that for Z^0 magnetic field. Thus the flow lines of hydrodynamic flow would directly correspond to those of Z^0 magnetic field. The generalized Beltrami flow based on the topologization of the Z^0 current would allow to model also the non-stationary incompressible non-viscous hydrodynamical flows.

It would seem that one cannot describe viscous flows using flows satisfying generalized Beltrami conditions since the vanishing of the Lorentz 4-force says that there is no local dissipation of the classical field energy. One might claim that this is not a problem since in TGD framework viscous flow could be seen as a practical description of a quantum jump sequence by replacing the corresponding sequence of space-time surfaces with a single space-time surface.

One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Kähler fields, which are dissipative, and thus correspond to a non-vanishing Lorentz 4-force, represent one candidate for correlates of this kind. If this is the case, then the fields satisfying the generalized Beltrami condition provide space-time correlates only for the asymptotic self organization patterns for which the viscous effects are negligible, and also the solutions of field equations describing effects of viscosity should be possible.

One must however take this argument with a grain of salt. Dissipation, that is the transfer of conserved quantities to degrees of freedom corresponding to shorter scales, could correspond to a transfer of these quantities between different space-time sheets of the many-sheeted space-time. Here the opponent could however argue that larger space-time sheets mimic the dissipative dynamics in shorter scales and that classical currents represent "symbolically" averaged currents in shorter length scales, and that the local non-conservation of energy momentum tensor consistent with local conservation of isometry currents provides a unique manner to mimic the dissipative dynamics. This view will be developed in more detail below.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds [B29] . Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta} A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta} J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

(a) The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.

(b) This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B29] , and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B29] !

Beltrami flows associated with Euler equations are known to be unstable [B29] . Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

10.3.3 How to satisfy field equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l} \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \qquad (10.3.17)$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

(a) In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u,v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

(b) For Euclidian minimal surfaces (u,v) is replaced by complex coordinates $(w,\overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \qquad (10.3.18)$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla\times X\cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u=t-z$ and $v=t+z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

(a) Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t + z, S^- \equiv v = t - z, w = x + iy, \overline{w} = x - iy)$.

(b) In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \to v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \to S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t - z, t + z, x, y) \to S^-(t - z, t + z, x, y) = S^+(t + z, t - z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

(c) The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-} dS^+ dS^- + g_{w\overline{w}} dw d\overline{w} \\ &+ g_{+w} dS^+ dw + g_{+\overline{w}} dS^+ d\overline{w} \\ &+ g_{-w} dS^- dw + g_{-\overline{w}} dS^- d\overline{w} \quad . \end{aligned} \tag{10.3.19}$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \to f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \to f(S^+), S^- \to f(S^-)$ in longitudinal degrees of freedom preserve this structure.

(d) The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w \partial_{\overline{w}} K \ , \quad g_{+-} = \partial_{S^+} \partial_{S^-} K \ , \\ g_{w\pm} &= \partial_w \partial_{S^\pm} K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}} \partial_{S^\pm} K \ . \end{aligned} \tag{10.3.20}$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0 \overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{10.3.21}$$

The Christoffel symbols satisfy the conditions

$$\{^{\ k}_{w\ \overline{w}}\} = 0 \ , \quad \{^{\ k}_{+-}\} = 0 \ . \tag{10.3.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

(e) The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y^3_l labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y^3_l is implied. This is what number theoretic compactification and M^8-H duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K33, K88] .

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi,\overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w\partial_{\overline{w}}K \ , \quad s_{ws} = \partial_w\partial_s K \ , \quad s_{\overline{w}s} = \partial_{\overline{w}}\partial_s K \ . \tag{10.3.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{^{\ k}_{\xi\ \overline{\xi}}\}$.

$$\left\{ {k \atop \xi\ \overline{\xi}} \right\} = 0 \ . \qquad (10.3.24)$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta(J^{\alpha\beta}\sqrt{g}) &= 0 \text{ for } \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} &\neq 0 \ . \end{aligned} \qquad (10.3.25)$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

(a) The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \qquad (10.3.26)$$

Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

(b) Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

(c) By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi} s} = 0 \ . \qquad (10.3.27)$$

Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

(d) The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi,s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi,s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\bar{\xi}} &= m_{+w}\partial_{\bar{\xi}} w(\bar{\xi})\partial_s S^+(s) \ . \end{aligned} \quad (10.3.28)$$

Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

(e) The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\bar{\xi}}\sqrt{g} = \partial_\beta(J^{\bar{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \quad (10.3.29)$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \quad = \quad 0 \quad (10.3.30)$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

(a) The light-like character of the Kähler current brings in mind CP_2 extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\bar{\xi}}, J^{\xi -}, J^{\bar{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

(b) Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If absolute minima correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

(c) The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \bar{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

(d) Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y_l^3 of $X^4(X_l^3)$ "parallel" to X_l^3 requires only that gauge currents are parallel to Y_l^3 and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

(a) Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

(b) The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{10.3.31}$$

If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

(c) The components of the induced metric are

$$\begin{aligned} g_{ZZ} &= m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} &= m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} &= s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} &= s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} &= s_{s\xi} \ , & g_{s\overline{\xi}} &= s_{s\overline{\xi}} \ . \end{aligned} \tag{10.3.32}$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} = 0 \tag{10.3.33}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\ \partial_\xi(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) &= 0 \ . \end{aligned} \tag{10.3.34}$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} &\{^{\ k}_{T\ w}\} = 0 \ , \quad \{^{\ k}_{T\ \overline{w}}\} = 0 \ , \\ &\{^{\ k}_{Z\ w}\} = 0 \ , \quad \{^{\ k}_{Z\ \overline{w}}\} = 0 \end{aligned} \tag{10.3.35}$$

hold true. The conditions are equivalent with the conditions

$$\{^{\ k}_{\pm\ w}\} = 0 \ , \quad \{^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \tag{10.3.36}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s, Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

(a) Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.

(b) One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl} A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \tag{10.3.37}$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{10.3.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{ {}^{k}_{w+} \} = 0 \ , \quad \{ {}^{k}_{\overline{w}+} \} = 0 \ , \quad \{ {}^{k}_{++} \} = 0 \ . \tag{10.3.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \quad = \quad 0 \ . \tag{10.3.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \quad = \quad a_k \xi^k + \overline{a}_k \overline{\xi}^k \ . \tag{10.3.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j (J^{j\bar{i}} \sqrt{g}) = 0 \ , \quad \partial_{\bar{j}} (J^{\bar{j}i} \sqrt{g}) = 0 \ , \tag{10.3.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i} S^+ \partial_{\xi^j} S^- + m_{+-} \partial_{\xi^i} S^- \partial_{\xi^j} S^+)$ are non-vanishing.

(a) The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

(b) The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \tag{10.3.43}$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book). Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

(a) String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the imbedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

(b) Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi,\overline{\xi},u,v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s,r) = (s(u,v),r(u,v)) \quad , \quad \xi = constant \ , \tag{10.3.44}$$

and corresponds to a vanishing Kähler current.

(c) Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B}\cdot\overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s,r) = (s,r)\,(u,v,w,\overline{w}) \quad , \quad \xi = constant \ . \tag{10.3.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

(a) Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w,\overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

(b) The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \quad , \quad w = w(\xi) \quad , \quad S^- = s^- \quad , \quad S^+ = s^+ + f(\xi,\overline{\xi}) \ .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi\partial_+h^k$ and $\partial_{\overline{\xi}}\partial_+h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the imbeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K90] .

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta),\Phi)$ and let (t,ρ,ϕ,z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{array}{ll} U = U(\rho) \ , & \Phi = n\phi + kz \ , \\ J_{\rho\phi} = n\partial_\rho U \ , & J_{\rho z} = k\partial_\rho U \ . \end{array} \tag{10.3.46}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K90] . For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

10.3.4 $D_{CP_2} = 3$ phase allows infinite number of topological charges characterizing the linking of magnetic field lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B50] . Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A\cdot B$ as

$$\int dVA\cdot B = \int dVdV'B(r)\cdot\left(\frac{(r-r')}{|r-r'|^3}\times B(r')\right) \quad ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A\wedge dA$ is replaced by $Tr(A\wedge(dA + 2A\wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

10.3.5 Preferred extremal property and the topologization/light-likeness of Kähler current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

(a) Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

(b) One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X^3_l. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y^3_l varies. This conforms nicely with the notion of quantum gravitational holography.

(c) The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

(d) One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X^3_l)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

(a) $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized

Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

(b) One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X^3_l so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

(c) What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of zero energy ontology and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [L7] .

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

10.3.6 Generalized Beltrami fields and biological systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y^3_l associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

(a) Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

(b) Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B18] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ phase - if present at all- would correspond to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality. As already found, Maxwell phase is very probably not realized in this manner but is essentially outcome of many-sheeted space-time concept.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A} \cdot \overline{B} \neq 0$ and Kähler current $\overline{E} \times \overline{A} + \phi \overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [K30] . For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A57] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

(a) The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.

(b) If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

(a) The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.

(b) Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.

(c) If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing into a continuous Schrödinger time evolution.

(d) In TGD Schrödinger amplitudes are replaced by second quantized induced spinor fields. Hence one does not face the problem whether it makes sense to speak about Schrödinger time evolution of complex order parameter along the flow lines of a foliation or not. Also the fact that the "time evolution" for the modified Dirac operator corresponds to single position dependent generalized eigenvalue identified as Higgs expectation same for all transversal modes (essentially z^n labeled by conformal weight) is crucial since it saves from the problems caused by the possible non-existence of Schrödinger evolution.

4. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

(a) For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.

(b) For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.

(c) $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D imbedding space which defines one sector of the generalized imbedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

10.4 Basic extremals of Kähler action

The solutions of field equations can be divided to vacuum extremals and non-vacuum extremal. Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe.

10.4.1 CP_2 type vacuum extremals

These extremals correspond to various isometric imbeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the imbedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{10.4.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1, 2, 3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the symmetries of TGD. Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD.

The action for all extremals is same and given by the Kähler action for the imbedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} \ . \tag{10.4.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action. The the principle selecting preferred extremals of Kähler action suggests that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. Indeed, the nice generalization of the area law for the entropy of black hole [K59] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A73] . A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

10.4.2 Vacuum extremals with vanishing Kähler field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Lagrange manifold. The condition expressing Lagrange manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \quad (10.4.3)$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \quad (10.4.4)$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the WCW metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D = 3$ for CP_2 type extremals, $D = 2$ for string like objects, $D = 1$ for membranes and $D = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

The reason would be "Yin-Yang principle" discussed in [K9] .

(a) Consider first the option for which Kähler function corresponds to an absolute minimum of Kähler action. Vacuum functional as an exponent of Kähler function is expected to concentrate on those 3-surfaces for which the Kähler action is non-negative. On the other hand, the requirement that Kähler action is absolute minimum for the space-time associated with a given 3-surface, tends to make the action negative. Therefore the vacuum functional is expected to differ considerably from zero only for 3-surfaces with a vanishing Kähler action per volume. It could also occur that the degeneracy of 3-surfaces with same large negative action compensates the exponent of Kähler function.

(b) If preferred extrema correspond to Kähler calibrations or their duals [K88] , Yin-Yang principle is modified to a more local principle. For Kähler calibrations (their duals) the absolute value of action in given region is minimized (maximized). A given region with a positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically and implies that average Kähler action per volume vanishes. Positive and finite values of Kähler function are of course favored.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

10.4.3 Cosmic strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the principle selecting preferred extremals of the Kähler action is global rather than a local. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{10.4.5}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

10.4.4 Massless extremals

Massless extremals are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{aligned} & s^k = f^k(u,v) \ , \\ & u = p\cdot m \ , \qquad v = \varepsilon\cdot m \ , \\ & p\cdot\varepsilon = 0 \ , \qquad p^2 = 0 \ . \end{aligned}$$

CP_2 coordinates are arbitrary functions of $p{\cdot}m$ and $\varepsilon{\cdot}m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2+m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{aligned} & H(p\cdot m, \varepsilon\cdot m, \varepsilon_1\cdot m) = 0 \ , \\ & \varepsilon\cdot p = 0 \ , \qquad \varepsilon_1\cdot p = 0 \ , \quad \varepsilon\cdot\varepsilon_1 = 0 \ . \end{aligned} \tag{10.4.6}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many manners to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta}J^k_{\ l}\partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a manner to satisfy all boundary conditions but it is not clear whether there are any other manners to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamics this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are are bounded: this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

10.4.5 Generalization of the solution ansatz defining massless extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

(a) Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

(b) With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{10.4.7}$$

If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

(c) This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ \ , \ \ (\nabla_3 f)^2 = 1 \ \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many manners.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

(a) Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

(b) The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k \nabla S_+ \ \ ,$$

where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

(c) The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$

is satisfied: in this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

(d) The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$

$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \to -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

(a) Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

(b) Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

(c) The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then 'free' solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

(d) For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

(e) One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the imbedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{10.4.8}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{10.4.9}$$

The diagonalization of the metric seems to be a general aspect of absolute minima. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

(a) The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

(b) Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the imbedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

(c) These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the variational principle selecting preferred extremals of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of imbedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

Chapter 11

Physics as a Generalized Number Theory

11.1 Introduction

Physics as a generalized number theory program involves three threads: various p-adic physics and their fusion together with real number based physics to a larger structure [K87] , the attempt to understand basic physics in terms of classical number fields [K88] , and infinite primes [K86] whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. A common denominator of these approaches is a precise mathematical formulation for the notion of finite measurement resolution, which could be taken as one of the basic guiding principles of quantum TGD and is at quantum level realized in terms of inclusions of hyper-finite factors about which configuration space spinor fields provide an example [K99] . In the following these threads are described briefly. More detailed summaries will be given in separate articles.

11.1.1 p-Adic physics and unification of real and p-adic physics

p-Adic numbers [A75, A47, A50] became a part of TGD through the successes of p-adic thermodynamics in the description of elementary particle massivation [K54] . The p-adicization program attempts to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in an essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals.

Real and p-adic regions of the space-time as geometric correlates of matter and mind

One could end up with p-adic space-time sheets via field equations. The solutions of the equations determining space-time surfaces are restricted by the requirement that the coordinates are real. When this is not the case, one might apply instead of a real completion with some p-adic completion. It however seems that p-adicity is present at deeper level and automatically present via the generalization of the number concept obtained by fusing reals and p-adics along rationals and common algebraics.

p-Adic non-determinism due to the presence of non-constant functions with a vanishing derivative implies extreme flexibility and therefore suggests the identification of the p-adic regions as seats of cognitive representations. Unlike the completion of reals to complex numbers, the completions of p-adic numbers preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with 'mind like' regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of 'self' and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves!

The generalization of the notion of number

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

Zero energy ontology, cognition, and intentionality

One could argue that conservation laws forbid p-adic-real phase transitions in practice so that cognitions (intentions) realized as real-to-padic (p-adic-to-real) transitions would not be possible. The situation changes if one accepts zero energy ontology [K21, K20] .

1. Zero energy ontology classically

In TGD inspired cosmology [K80] the imbeddings of Robertson-Walker cosmologies are vacuum extremals. Same applies to the imbeddings of Reissner-Nordström solution [K93] and in practice to all solutions of Einstein's equations imbeddable as extremals of Kähler action. Since four-momentum currents define a collection of vector fields rather than a tensor in TGD, both positive and negative signs for energy corresponding to two possible assignments of the arrow of the geometric time to a given space-time surface are possible. This leads to the view that all physical states have vanishing net energy classically and that physically acceptable universes are creatable from vacuum.

The result is highly desirable since one can avoid unpleasant questions such as "What are the net values of conserved quantities like rest mass, baryon number, lepton number, and electric charge for the entire universe?", "What were the initial conditions in the big bang?", "If only single solution of field equations is selected, isn't the notion of physical theory meaningless since in principle it is not possible to compare solutions of the theory?". This picture fits also nicely with the view that entire universe understood as quantum counterpart 4-D space-time is recreated in each quantum jump and allows to understand evolution as a process of continual re-creation.

2. Zero energy ontology at quantum level

Also the construction of S-matrix [K20] leads to the conclusion that all physical states possess vanishing conserved quantum numbers. Furthermore, the entanglement coefficients between positive and negative energy components of the state have interpretation as M-matrix identifiable as a "complex square root" of density matrix expressible as a product of positive diagonal square root of the density matrix and of a unitary S-matrix. S-matrix thus becomes a property of the zero energy state and physical states code by their structure what is usually identified as quantum dynamics.

The collection of M-matrices defines an orthonormal state basis for zero energy states and together they define unitary U-matrix charactering transition amplitudes between zero energy states. This matrix would not be however the counterpart of the usual S-matrix. Rather the unitary matrix phase of a given M-matrix would define the S-matrix measured in laboratory. U-matrix would also characterize the transitions between different number fields possible in the intersection of rel and p-adic worlds and having interpretation in terms of intention and cognition.

At space-time level this would mean that positive energy component and negative energy component are at a temporal distance characterized by the time scale of the causal diamond (CD) and the rational (perhaps integer) characterizing the value of Planck constant for the state in question. The scale in question would also characterize the geometric duration of quantum jump and the size scale of space-time region contributing to the contents of conscious experience. The interpretation in terms of a mini bang followed by a mini crunch suggests itself also. CDs are indeed important also in TGD inspired cosmology [K80] .

3. Hyper-finite factors of type II_1 and new view about S-matrix

The representation of S-matrix as unitary entanglement coefficients would not make sense in ordinary quantum theory but in TGD the von Neumann algebra in question is not a type I factor as for quantum mechanics or a type III factor as for quantum field theories, but what is called hyper-finite factor of type II_1 [K99] . This algebra is an infinite-dimensional algebra with the almost defining, and at the first look very strange, property that the infinite-dimensional unit matrix has unit trace. The infinite dimensional Clifford algebra spanned by the configuration space gamma matrices (configuration space understood as the space of 3-surfaces, the "world of classical worlds") is indeed very naturally algebra of this kind since infinite-dimensional Clifford algebras provide a canonical representations for hyper-finite factors of type II_1.

4. The new view about quantum measurement theory

This mathematical framework leads to a new kind of quantum measurement theory. The basic assumption is that only a finite number of degrees of freedom can be quantum measured in a given measurement and the rest remain untouched. What is known as Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras allow to realize mathematically this idea [K99] . $\mathcal{N}$ characterizes measurement resolution and quantum measurement reduces the entanglement in the non-commutative quantum space $\mathcal{M}/\mathcal{N}$. The outcome of the quantum measurement is still represented by a unitary S-matrix but in the space characterized by $\mathcal{N}$. It is not possible to end up with a pure state with a finite sequence of quantum measurements.

The obvious objection is that the replacement of a universal S-matrix coding entire physics with a state dependent unitary entanglement matrix is too heavy a price to be paid for the resolution of the above mentioned paradoxes. Situation could be saved if the S-matrices have fractal structure. The quantum criticality of TGD Universe indeed implies fractality. The possibility of an infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras expresses this fractal character algebraically. Thus one can hope that the S-matrix appearing as entanglement coefficients is more or less universal in the same manner as Mandelbrot fractal looks more or less the same in all length scales and for all resolutions. Whether this kind of universality must be posed as an additional condition on entanglement coefficients or is an automatic consequence of unitarity in type II_1 sense is an open question.

5. The S-matrix for p-adic-real transitions makes sense

In zero energy ontology conservation laws do not forbid p-adic-real transitions and one can develop a relatively concrete vision about what happens in these kind of transitions. The starting point is the generalization of the number concept obtained by gluing p-adic number fields and real numbers along common rationals (expressing it very roughly). At the level of the imbedding space this means that p-adic and real space-time sheets intersect only along common rational points of the imbedding space and transcendental p-adic space-time points are infinite as real numbers so that they can be said to be infinite distant points so that intentionality and cognition become cosmic phenomena.

In this framework the long range correlations characterizing p-adic fractality can be interpreted as being due to a large number of common rational points of imbedding space for real space-time sheet and p-adic space-time sheet from which it resulted in the realization of intention in quantum jump. Thus real physics would carry direct signatures about the presence of intentionality. Intentional behavior is indeed characterized by short range randomness and long range correlations.

One can even develop a general vision about how to construct the S-matrix elements characterizing the process [K20] . The basic guideline is the vision that real and various p-adic physics as well as their hybrids are continuable from the rational physics. This means that these S-matrix elements must be characterizable using data at rational points of the imbedding space shared by p-adic and real space-time sheets so that more or less same formulas describe all these S-matrix elements. Note that also $p_1 \rightarrow p_2$ p-adic transitions are possible.

What number theoretical universality might mean?

Number theoretic universality has been one of the basic guide lines in the construction of quantum TGD. There are two forms of the principle.

(a) The strong form of number theoretical universality states that physics for any system should effectively reduce to a physics in algebraic extension of rational numbers at the level of M-matrix so that an interpretation in both real and p-adic sense (allowing a suitable algebraic extension of p-adics) is possible. One can however worry whether this principle only means that physics is algebraic so that there would be no need to talk about real and p-adic physics at the level of M-matrix elements. It is not possible to get rid of real and p-adic numbers at the level of classical physics since calculus is a prerequisite for the basic variational principles used to formulate the theory. For this option the possibility of completion is what poses conditions on M-matrix.

(b) The weak form of principle requires only that both real and p-adic variants of physics make sense and that the intersection of these physics consist of physics associated with various algebraic extensions of rational numbers. In this rational physics would be like rational numbers allowing infinite number of algebraic extensions and real numbers and p-adic number fields as its completions. Real and p-adic physics would be completions of rational physics. In this framework criticality with respect to phase transitions changing number field becomes a viable concept. This form of principle allows also purely p-adic phenomena such as p-adic pseudo non-determinism assigned to imagination and cognition. Genuinely p-adic physics does not however allow definition of notions like conserved quantities since the notion of definite integral is lacking and only the purely local form of real physics allows p-adic counterpart.

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough. It is however clear that number theoretical criticality could provide important insights to quantum TGD. p-Adic thermodynamics [K54] is an excellent example of this. In consciousness theory the transitions transforming intentions to actions and actions to cognitions would be key applications. Needless to say, zero energy ontology is absolutely essential: otherwise this kind of transitions would not make sense.

p-Adicization by algebraic continuation

The basic challenges of the p-adicization program are following.

(a) The first problem -the conceptual one- is the identification of preferred coordinates in which functions are algebraic and for which algebraic values of coordinates are in preferred position. This problem is encountered both at the level of space-time, imbedding space, and configuration space. Here the group theoretical considerations play decisive role and the selection of preferred coordinates relates closely to the selection of quantization axes. This selection has direct physical correlates at the level of imbedding space and the hierarchy of Planck constants has interpretation as a correlate for the selection of quantization axes [K27] .

Algebraization does not necessarily mean discretization at space-time level: for instance, the coordinates characterizing partonic 2-surface can be algebraic so that algebraic point of the configuration space results and surface is not discretized. If this kind of function spaces are finite-dimensional, it is possible to fix X^2 completely data for a finite number of points only.

(b) Local physics generalizes as such to p-adic context (field equations, etc...). The basic stumbling block of this program is integration already at space-time (Kähler action, flux Hamiltonians, etc..). The problem becomes really horrible looking at configuration space level (functional integral). Algebraic continuation could allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics. Also the existence of the Kähler geometry does this and the solution to the constraint is that WCW is a union of symmetric spaces.

In the case of symmetric spaces Fourier analysis generalizes to harmonics analysis and one can reduce integration to summation for functions allowing Fourier decomposition. In p-adic context the existence of plane waves requires an algebraic extension allowing roots of unity characterizing the measurement accuracy for angle like variables. This leads in the case of symmetric spaces to a general p-adicization recipe. One starts from a discrete variant of the symmetric space for which points correspond to roots of unity and replaces each discrete point with is p-adic completion representing the p-adic variant of the symmetric space so that kind of fractal variant of the symmetric space is obtained. There is an infinite hierarchy of p-adicizations corresponding to measurement resolutions and to the choice of preferred coordinates and the interpretation is in terms of cognitive representations. This requires a refined view about General Coordinate Invariance taking into account the fact that cognition is also part of the quantum state.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane.

(a) Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

(b) For instance, residue calculus essential in the construction of N-point functions of conformal field theory might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

(c) Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition and intentionality.

(d) For instance, the idea that number theoretically critical partonic 2-surfaces are expressible in terms of rational functions with rational or algebraic coefficients so that also p-adic variants of these surfaces make sense, is very attractive.

(e) Finite sums and products respect algebraic number property and the condition of finiteness is coded naturally by the notion of finite measurement resolution in terms of the notion of (number theoretic) braid. This simplifies dramatically the algebraic continuation since configuration space reduces to a finite-dimensional space and the space of configuration space spinor fields reduces to finite-dimensional function space.

The real configuration space can well contain sectors for which p-adicization does not make sense. For instance, if the exponent of Kähler function and Kähler are not expressible in terms of algebraic functions with rational or at most algebraic functions or more general functions making sense p-adically, the continuation is not possible. p-Adic non-determinism in p-adic sectors makes also impossible the continuation to real sector. All this is consistent with vision about rational and algebraic physics as as analog of rational and algebraic numbers allowing completion to various continuous number fields.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. For instance, most points p-adic space-time sheets reside at infinity in real sense and p-adically infinitesimal is infinite in real sense. Two types of cutoffs are predictedp-adic length scale cutoff and a cutoff due to phase resolution related to the hierarchy of Planck constants. Zero energy ontology provides natural realization for the p-adic length scale cutoff. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and quantum phases $exp(i2\pi/n)$, $n \geq 3$, coming as roots of unity and defining extensions of rationals and p-adics allowing to define p-adically sensible trigonometric functions These phases relate closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra.

11.1.2 TGD and classical number fields

This chapter is second one in a multi-chapter devoted to the vision about TGD as a generalized number theory. The basic theme is the role of classical number fields in quantum TGD. A central notion is M^8-H duality which might be also called number theoretic compactification. This duality allows to identify imbedding space equivalently either as M^8 or $M^4\times CP_2$ and explains the symmetries of standard model number theoretically. These number theoretical symmetries induce also the symmetries dictaging the geometry of the "world of classical worlds" (WCW) as a union of symmetric spaces. This infinite-dimensional Kähler geometry is expected to be highly unique from the mere requirement of its existence requiring infinite-dimensional symmetries provided by the generalized conformal symmetries of the light-cone boundary $\delta M^4_+\times S$ and of light-like 3-surfaces and the answer to the question what makes 8-D imbedding space and $S=CP_2$ so unique would be the reduction of these symmetries to number theory.

Zero energy ontology has become the corner stone of both quantum TGD and number theoretical vision. In zero energy ontology either light-like or space-like 3-surfaces can be identified as the fundamental dynamical objects, and the extension of general coordinate invariance leads to effective 2-dimensionality (strong form of holography) in the sense that the data associated with partonic 2-surfaces and the distribution of 4-D tangent spaces at them located at the light-like boundaries of causal diamonds (CDs) defined as intersections of future and past directed light-cones code for quantum physics and the geometry of WCW.

The basic number theoretical structures are complex numbers, quaternions and octonions, and their complexifications obtained by introducing additional commuting imaginary unit $\sqrt{-1}$. Hyper-octonionic (-quaternionic,-complex) sub-spaces for which octonionic imaginary units are multiplied by commuting $\sqrt{-1}$ have naturally Minkowskian signature of metric. The question is whether and how the hyper-structures could allow to understand quantum TGD in terms of classical number fields. The answer which looks the most convincing one relies on the existence of octonionic representation of 8-D gamma matrix algebra.

(a) The first guess is that associativity condition for the sub-algebras of the local Clifford algebra defined in this manner could select 4-D surfaces as associative (hyper-quaternionic) sub-spaces of this algebra and define WCW purely number theoretically. The associative sub-spaces in question would be spanned by the modified gamma matrices defined by the modified Dirac action fixed by the variational principle (Kähler action) selecting space-time surfaces as preferred extremals [K28] .

(b) This condition is quite not enough: one must strengthen it with the condition that a preferred commutative and thus hyper-complex sub-space is contained in the tangent space of the space-time surface. This condition actually generalizes somewhat since one can introduce a family of so called Hamilton-Jacobi coordinates for M^4 allowing an integrable distribution of decompositions of tangent space to the space of non-physical and physical polarizations [K9] . The physical interpretation is as a number theoretic realization of gauge invariance selecting a preferred local commutative plane of non-physical polarizations.

(c) Even this is not yet the whole story: one can define also the notions of co-associativity and co-commutativity applying in the regions of space-time surface with Euclidian signature of the induced metric. The basic unproven conjecture is that the decomposition of space-time surfaces to associative and co-associative regions containing preferred commutative *resp.* co-commutative 2-plane in the 4-D tangent plane is equivalent with the preferred extremal property of Kähler action and the hypothesis that space-time surface allows a slicing by string world sheets and by partonic 2-surfaces [K28] .

Hyper-octonions and hyper-quaternions

The discussions for years ago with Tony Smith [A113] stimulated very general ideas about space-time surface as an associative, quaternionic sub-manifold of octonionic 8-space (for octonions see [A19] . Also the observation that quaternionic and octonionic primes have norm squared equal to prime in complete accordance with p-adic length scale hypothesis, led to suspect that the notion of primeness for quaternions, and perhaps even for octonions, might be fundamental for the formulation of quantum TGD. The original idea was that space-time surfaces could be regarded as four-surfaces in 8-D imbedding space with the property that the tangent spaces of these spaces can be locally regarded as 4- *resp.* 8-dimensional quaternions and octonions.

It took some years to realize that the difficulties related to the realization of Lorentz invariance might be overcome by replacing quaternions and octonions with hyper-quaternions and hyper-octonions. Hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$ and can be regarded as a sub-space of complexified quaternions *resp.* octonions. The transition is the number theoretical counterpart of the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity. The loss of number field and even sub-algebra property is not fatal and has a clear physical meaning. The notion of primeness is inherited from that for complexified quaternions *resp.* octonions.

Note that hyper-variants of number fields make also sense p-adically unlike the notions of number fields themselves unless restricted to be algebraic extensions of rational variants of number fields. What deserves separate emphasis is that the basic structure of the standard model would reduce to number theory.

Number theoretical compactification and M^8-H duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H=M^4\times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4\subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of

M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naive and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

(a) The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X^3_l (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X^3_l is 1-dimensional. The surfaces $X^4(X^3_l) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.

(b) One can also consider a more local map of $X^4(X^3_l) \subset H$ to $X^4(X^3_l) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X^3_l) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X^3_l and represented by $X^4(X^3_l) \subset H$. Space-time surfaces $X^4(X^3_l) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of configuration space.

(c) Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X^3_l) \subset M^8$ and $X^4(X^3_l) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

(d) The map of $X^3_l \subset H \rightarrow X^3_l \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of imbedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

(e) The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

11.1.3 Infinite primes

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD. The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors. Many interpretations for infinite primes have been competing for survival but it seems that the recent state of TGD allows to exclude some of them from consideration.

The notion of infinite prime

Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense in the sequence of quantum jumps: the reason is simply that the size of primes is bounded below. This leads to a peculiar paradox: if the number of quantum jumps already occurred is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct infinite primes by repeating a procedure analogous to a quantization of a super symmetric arithmetic quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions with the corresponding decomposition of the infinite prime to primes at lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

This and other observations suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [A103] providing rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Somewhat surprisingly, infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively and this raises the question whether the tangent space for the configuration space of 3-surfaces could be regarded as the space of generalized 8-D hyper-octonionic numbers.

Infinite primes and physics in TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. Infinite primes, cognition, and intentionality

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

(a) The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.

(b) Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point -or more generally wave functions in the space of the units associated with the point- might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

(c) One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

2. Infinite primes and super-symmetric quantum field theory

Consider next the physical interpretation.

(a) The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that configuration space spinor fields or at least the ground states of associated super-conformal representations could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

(b) The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.

(c) The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. It turns out that this idea makes sense when one considers discrete wave functions in the space of infinite primes and that one can indeed represent standard model quantum numbers in this manner.

(d) The number theoretical supersymmetry suggests also space-time supersymmetry TGD framework. Space-time super-symmetry in its standard form is not possible in TGD Universe and this cheated me to believe that this supersymmetry is completely absent in TGD Universe. The progress in the understanding of the properties of the modified Dirac action however led to a generalization of the space-time super-symmetry as a dynamical and broken symmetry of quantum TGD [K29] .

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [K99] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on configuration space spinor fields representing physical states [K21] .

3. Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

G_2 acts as automorphisms of hyper-octonions and $SU(3)$ as its subgroup respecting the choice of a preferred imaginary unit. The discrete subgroups of $SU(3)$ permuting to each other hyper-octonionic primes are analogous to Galois group and turned out to play a crucial role in the understanding of the correspondence between infinite hyper-octonionic primes and quantum states predicted by quantum TGD.

4. The notion of finite measurement resolution as the key concept

TGD predicts several hierarchies: the hierarchy of space-time sheets, the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K99] , the dark matter hierarchy characterized by increasing values of $\hbar$ [K27] , the hierarchy of extensions of a given p-adic number field. TGD inspired theory of consciousness predicts the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. These hierarchies should be closely related.

The notion of finite measurement resolution turns out to be the key concept: the p-adic norm of the rational defined by the infinite prime characterizes the angle measurement resolution for given p-adic prime p. It is essential that one has what might be called a state function reduction selecting a fixed p-adic prime which could be also infinite. This gives direct connections with cognition and with the p-adicization program relying also on angle measurement resolution. Also the value the integers characterizing the singular coverings of CD and CP_2 defining as their product Planck constant characterize the measurement resolution for a given p-adic prime in CD and CP_2 degrees of freedom. This conforms with the fact that elementary particles are characterized by two infinite primes. Hence finite measurement resolution ties tightly together the three threads of the number theoretic vision. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory so that the characterization of the finite measurement resolution, which has been the ugly duckling of theoretical physics, transforms to a beautiful swan.

5. Space-time correlates of infinite primes

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. Note that this conjecture should be consistent with two other conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action and space-time surfaces as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8).

The representation of space-time surfaces as algebraic surfaces in M^8 is however too naive idea and the attempt to map hyper-octonionic infinite primes to algebraic surfaces has not led to any concrete progress.

The solution came from quantum classical correspondence, which requires the map of the quantum numbers of configuration space spinor fields to space-time geometry. The modified Dirac equation with measurement interaction term realizes this requirement. Therefore, if one wants to map infinite rationals to space-time geometry it is enough to map infinite primes to quantum numbers. This map can be indeed achieved thanks to the detailed picture about the interpretation of the symmetries of infinite primes in terms of standard model symmetries.

Generalization of ordinary number fields: infinite primes and cognition

Both fermions and p-adic space-time sheets are identified as correlates of cognition in TGD Universe. The attempt to relate these two identifications leads to a rather concrete model for how bosonic generators of super-algebras correspond to either real or p-adic space-time sheets (actions and intentions) and fermionic generators to pairs of real space-time sheets and their p-adic variants obtained by algebraic continuation (note the analogy with fermion hole pairs).

The introduction of infinite primes, integers, and rationals leads also to a generalization of classical number fields since an infinite algebra of real (complex, etc...) units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real and also more general units becomes infinitely degenerate.

Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point is even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units.

One might argue that for the real physics this structure is invisible and is relevant only for the physics of cognition. On the other hand, one can consider the possibility of mapping the configuration space and configuration space spinor fields to the number theoretical anatomies of a single point of imbedding space so that the structure of this point would code for the world of classical worlds and for the quantum states of the Universe. Quantum jumps would induce changes of configuration space spinor fields interpreted as wave functions in the set of number theoretical anatomies of single point of imbedding space in the ordinary sense of the word, and evolution would reduce to the evolution of the structure of a typical space-time point in the system. Physics would reduce to space-time level but in a generalized sense. Universe would be an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

Infinite rationals are in one-one correspondence with quantum states and in zero energy ontology hyper-octonionic units identified as ratios of the infinite integers associated with the positive and negative energy parts of the zero energy state define a representation of WCW spinor fields. The action of subgroups of SU(3) and rotation group SU(2) preserving hyper-octonionic and hyper-quaternionic primeness and identification of momentum and electro-weak charges in terms of components of hyper-octonionic primes makes this representation unique. Hence Brahman-Atman identity has a completely concrete realization and fixes completely the quantum number spectrum including particle masses and correlations between various quantum numbers.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Physics as generalized number theory [L58]
- Quantum physics as generalized number theory [L64]
- TGD and classical number fields [L71]
- $M^8 - H$ duality [L49]
- Basic notions behind $M^8 - H$ duality [L25]
- Quaternionic planes of octonions [L65]

11.2 p-Adic physics and the fusion of real and p-adic physics to a single coherent whole

In this section basic facts about p-adic numbers [A75, A47, A50] and the question about their relation to real numbers are discussed. Also the basic technicalities related to the notion of p-adic physics are discussed. Also included is a section about the physics in the intersection of real and p-adic worlds relevant to living systems in TGD Universe.

11.2.1 Background

It is good to start with a summary of the basic mathematical problems related to the p-adicization of physics and a rough formulation for how one might resolve these problems.

Problems

It is far from obvious what the p-adic counterpart of real physics could mean and how one could fuse together real and p-adic physics. Therefore it is good to list the basic problems and proposals for their solution.

The first problem concerns the correspondence between real and p-adic numbers.

(a) The success of p-adic mass calculations involves the notions of p-adic probability, thermodynamics, and the mapping of p-adic probabilities to the real ones by a continuous correspondence $x = \sum x_n p^n \rightarrow Id(x) = \sum x_n p^{-n}$ that I have christened canonical identification. The problem is that I n does not respect symmetries defined by isometries and also general coordinate invariance is possible only if one can identify preferred imbedding space coordinates. The reason is that I does not commute with the basic arithmetic operations. I allows several variants and it is possible to have correspondence which respects symmetries in arbitrary accuracy in preferred coordinates. Thus I can play a role at space-time level only if one defines symmetries modulo measurement resolution. I would make sense only in the interval defining the measurement resolution for a given coordinate variable and the p-adic effective topology would make sense just because the finite measurement resolution does not allow to well-order the points.

(b) The identification of real and p-adic numbers via rationals common to all number fields - or more generally along algebraic extension of rationals- respects symmetries and algebra but is not continuous. At the imbedding space level preferred coordinates are required also now. The maximal symmetries of the imbedding space allow identification of this kind of coordinates. They are not unique. For instance, M^4 linear coordinates look very natural but for CP_2 trigonometric functions of angle like coordinates look more suitablel and Fourier analysis suggests strongly the introduction of algebraic extensions involving roots of unity. Partly the non-uniqueness has an interpretation as an imbedding space correlate for the selection of the quantization axes. The symmetric space [A31] property of WCW gives hopes that general coordinate invariance in quantal sense can be realized. The existence of p-adic harmonic analysis suggests a discretization of the p-adic variant of imbedding space and WCW based on roots of unity.

(c) One can consider a compromise between the two correspondences. Discretization via common algebraic points can be completed to a p-adic continuum by assigning to each real discretization interval (say angle increment $2\pi/N$) p-adic numbers with norm smaller than one.

Second problem relates to integration and Fourier analysis. Both these procedures are fundamental for physics -be it classical or quantum. The p-adic variant of definite integral does not exist in the sense required by the action principles of physics although classical partial differential equations assigned to a particular variational principle make perfect sense. Fourier analysis is also possible only if one allows algebraic extension of p-adic numbers allowing a sufficient number of roots of unity correlating with the measurement resolution of angle. The finite number of them has interpretation in terms of finite angle resolution. Fourier analysis provides also an algebraic realization of definite integral when one integrates over the entire manifold as one indeed does in the case of WCW. If the space in question allows maximal symmetries as WCW and imbedding space do, there are excellent hopes of having p-adic variants of both integration and harmonic analysis and the above described procedure allows a precise completion of the discretized variant of real manifold to its continuous p-adic variant.

The third problem relates to the definitions of the p-adic variants of Riemannian, symplectic [A60, A33, A32], and Kähler [A14] geometries. It is possible to generalize formally the notion of Riemann metric although non-local quantities like areas and total curvatures do not make sense if defined in terms of integrals. If all relevant quantities assignable to the geometry (family of Hamiltonians defining isometries, Killing vector fields, components of metric and Kähler form, Kähler function, etc...) are expressible in terms of rational functions involving only rational numbers as coefficients of polynomials, they allow an algebraic continuation to the p-adic context and the p-adic variant of the geometry makes sense.

The fourth problem relates to the question what one means with p-adic quantum mechanics. In TGD framework p-adic quantum theory utilizes p-adic Hilbert space. The motivation is that the notions of p-adic probability and unitarity are well defined. From the beginning it was clear that the straightforward generalization of Schrödinger equation is not very interesting physically and gradually the conviction has developed that the most realistic approach must rely on the attempt to find the p-adic variant of the TGD inspired quantum physics in all its complexity. The recent approach starts from a rather concrete view about generalized Feynman diagrams defining the points of WCW and leads to a rather detailed view about what the p-adic variants of QM could be and how they could be fused with real QM to a larger structure. Even more, just the requirement that this p-adicization exists, gives very powerful constraints on the real variant of the quantum TGD.

The fifth problem relates to the notion of information in p-adic context. p-Adic thermodynamics leads naturally to the question what p-adic entropy might mean and this in turn leads to the realization that for rational or even algebraic probabilities p-adic variant of Shannon entropy can be negative and has minimum for a unique prime. One can say that the entanglement in the intersection of real and p-adic worlds is negentropic. This leads to rather fascinating vision about how negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) makes it possible for living systems to overcome the second law of thermodynamics. The formulation of quantum theory in the intersection of real and living worlds becomes the basic challenge.

The proposed solutions to the technical problems could be rephrased in terms of the notion of algebraic universality. Various p-adic physics are obtained as algebraic continuation of real physics through the common algebraic points of real and p-adic worlds and by performing completion in the sense that the interval corresponding to finite measurement resolution are replaced with their p-adic counterpart via canonical identification. This allows to have exact symmetries as their discrete variants and also a continuous correspondence if desired. Particular p-adicization is characterized by a choice fo preferred imbedding space coordinates, which has interpretation in terms of a particular cognitive representation. Hence one is forced to refine the view about general coordinate invariance. Different coordinate choices correspond to different cognitive representations having delicate effects on physics if it is assumed to include also the effects of cognition.

Program

These ideas lead to a reasonably well defined p-adicization program. Try to define precisely the concepts of the p-adic space-time and configuration space (WCW), formulate the finite-p p-adic versions of quantum TGD. Try to fuse together real and various p-adic quantum TGDs are to form a full theory of physics and cognition.

The construction of the p-adic TGD necessitates the generalization of the basic tools of standard physics such as differential and integral calculus, the concept of Hilbert space, Riemannian geometry, group theory, action principles, and the notions of probability and unitarity to the p-adic context. Also new physical thinking and philosophy is needed. The notions of zero energy ontology, hierarchy of Planck constants and the generalization of the notion of imbedding space required by it are essential but not discussed in detail in this article.

In the following I try to describe the most central problems and ideas of the p-adicization program. Page number of a readable article must be finite and this has forced to leave away a lot of topics. p-Adic mass calculations, which form the corner stone of the entire approach would require entire article series. The vision about how to define generalized Feynman diagrams and their p-adic variants by utilizing the assumption that WCW is symmetric space allowing algebraization of integration crucial for the entire approach is discussed in the May issue of this Journal [L14] . Negentropy Maximization Principle [K51] relevant for understanding the profound implications of the negentropic entanglement is not discussed. The applications of p-adic length scale hypothesis to the physics of living matter [K24] and the model of cognition and intentionality based on p-adic numbers [K58] have been also left out.

11.2.2 Summary of the basic physical ideas

In the following various manners to end up with p-adic physics and with the idea about p-adic topology as an effective topology of space-time surface are described.

p-Adic mass calculations briefly

p-Adic mass calculations based on p-adic thermodynamics with energy replaced with the generator $L_0 = zd/dz$ of infinitesimal scaling are described in the first part of [K54] .

(a) p-Adic thermodynamics is justified by the randomness of the motion of partonic 2-surfaces restricted only by the light-likeness of the orbit.

(b) It is essential that the conformal symmetries associated with the light-like coordinates of parton and light-cone boundary are not gauge symmetries but dynamical symmetries. The point is that there are two kinds of super-conformal symmetries [A27, A30] : the super-symplectic conformal symmetries assignable to the light-like boundaries of $CD \times CP_2$ and super Kac-Moody symmetries [A13] assignable to light-like 3-surfaces defining fundamental dynamical objects. In so called coset construction [A98] the differences of super-conformal generators of these algebras annihilate the physical states. This leads to a generalization of Equivalence Principle since one can assign four-momentum to the generators of both algebras identifiable as inertial *resp.* gravitational four-momentum. A second important consequence is that the generators of either algebra do not act like gauge transformations so that it makes sense to construct p-adic thermodynamics for them.

(c) In p-adic thermodynamics scaling generator L_0 having conformal weights as its eigen values replaces energy and Boltzmann weight $exp(H/T)$ is replaced by p^{L_0/T_p}. The quantization $T_p = 1/n$ of conformal temperature and thus quantization of mass squared scale is implied by number theoretical existence of Boltzmann weights. p-Adic length scale hypothesis states that primes $p \simeq 2^k$, k integer. A stronger hypothesis is that k is prime (in particular Mersenne prime or Gaussian Mersenne) makes the model very predictive and fine tuning is not possible.

Mersenne primes are very special number theoretically because bit as the unit of information unit corresponds to $log(2)$ and can be said to exists for M_n-adic topology. The reason is that $log(1+p)$ existing always p-adically corresponds for $M_n = 2^n - 1$ to $log(2^n) \equiv nlog(2)$ so that one has $log(2 \equiv log(1+M_n)/n$. Since the powers of 2 modulo p give all integers $n \in \{1, p-1\}$ by Fermat's theorem, one can say that the logarithms of all integers modulo M_n exist in this sense and therefore the logarithms of all p-adic integers not divisible by p exist. For other primes one must introduce a transcendental extension containing $log(a)$ where are is so called primitive root. One could criticize the identification since $log(1+M_n)$ corresponding in the real sense to n bits corresponds in p-adic sense to to a very small information content since the p-adic norm of the p-adic bit is $1/M_n$.

The basic mystery number of elementary particle physics defined by the ratio of Planck mass and proton mass follows thus from number theory once CP_2 radius is fixed to about 10^4 Planck lengths. Mass scale becomes additional discrete variable of particle physics so that there is not more need to force top quark and neutrinos with mass scales differing by 12 orders of magnitude to the same multiplet of gauge group. Electron, muon, and τ correspond to Mersenne prime $k = 127$ (the largest non-super-astrophysical Mersenne), and Mersenne primes $k = 113, 107$. Intermediate gauge bosons and photon correspond to Mersenne M_{89}, and graviton to M_{127}.

The value of k for quark can depend on hadronic environment [K57] and this would produce precise mass formulas for low energy hadrons. This kind of dependence conforms also with the indications that neutrino mass scale depends on environment [C21]. Amazingly, the biologically most relevant length scale range between 10 nm and 4 μm contains four Gaussian Mersennes $(1+i)^n - 1$, $n = 151, 157, 163, 167$ and scaled copies of standard model physics in cell length scale could be an essential aspect of macroscopic quantum coherence prevailing in cell length scale.

p-Adic mass thermodynamics is not quite enough: also Higgs boson is needed and wormhole contact carrying fermion and anti-fermion quantum numbers at the light-like wormhole throats is excellent candidate for Higgs [K48] . The coupling of Higgs to fermions can be small and induce only a small shift of fermion mass: this could explain why Higgs has not been observed. Also the Higgs contribution to mass squared can be understood thermodynamically if identified as absolute value for the thermal expectation value of the eigenvalues of the modified Dirac operator having interpretation as complex square root of conformal weight.

The original belief was that only Higgs corresponds to wormhole contact. The assumption that fermion fields are free in the conformal field theory applying at parton level forces to identify all gauge bosons as wormhole contacts connecting positive and negative energy space-time sheets [K48] . Fermions correspond to topologically condensed CP_2 type extremals with single light-like wormhole throat. Gravitons are identified as string like structures involving pair of fermions or gauge bosons connected by a flux tube. Partonic 2-surfaces are characterized by genus which explains family replication phenomenon and an explanation for why their number is three emerges [K19] . Gauge bosons are labeled by pairs (g_1, g_2) of handle numbers and can be arranged to octet and singlet representations of the resulting dynamical SU(3) symmetry. Ordinary gauge bosons are SU(3) singlets and the heaviness of octet bosons explains why higher boson families are effectively absent. The different character of bosons could also explain why the p-adic temperature for bosons is $T_p = 1/n < 1$ so that Higgs contribution to the mass dominates.

p-Adic length scale hypothesis, zero energy ontology, and hierarchy of Planck constants

Zero energy ontology and the hierarchy of Planck constants realized in terms of the generalization of the imbedding space lead to a deeper understanding of the origin of the p-adic length scale hypothesis.

1. Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the light-like boundaries of CD. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe?, Is theory building completely useless if only single solution of field equations is realized?). At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events.

2. Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K20] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra [A5] spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A63] known as hyperfinite factor of type II_1 (HFF) [K20, K99, K27] . HFF [A58, A84] is an algebraic fractal having infinite hierarchy of included sub-algebras isomorphic to the algebra itself [A3] . The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A119] , anyons [D25] , quantum groups and conformal field theories [A87] , and knots and topological quantum field theories [A109, A125] .

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

One can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of CD corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K20] . The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. Connes tensor product [A58] provides a mathematical description of the finite measurement resolution but does not fix the M-matrix as was the original hope. The remaining challenge is the calculation of M-matrix and the progress induced by zero energy ontology during last years has led to rather concrete proposal for the construction of M-matrix.

It turns out however that the mathematical representation for the notion of finite resolution for angle measurement serves as a common denominator for all basic approaches to quantum TGD: the Kähler geometry [A14] of WCW identified as a union of infinite-dimensional symmetric spaces, inclusions of hyper finite factors as representation of finite measurement resolution, p-adicization program, the role of classical number fields [A19, A7, A26] , and infinite primes so that it is fair to say that all approaches to TGD which originally seemed almost independent, converge to a coherent mathematical structure.

3. How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates CDs for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale ($T_n = 2^n T_0$) implies in a natural manner coupling constant evolution. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ (or $T_p = pT_0$) induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

4. Mersenne primes and Gaussian Mersennes

The generalization of the imbedding space required by the postulated hierarchy of Planck constants [K27] means a book like structure for which the pages are products of singular coverings or factor spaces of CD (causal diamond defined as intersection of future and past directed light-cones) and of CP_2 [K27] . This predicts that Planck constants are rationals and that a given value of Planck constant corresponds to an infinite number of different pages of the Big Book, which might be seen as a drawback. If only singular covering spaces are allowed the values of Planck constant are products of integers and given value of Planck constant corresponds to a finite number of pages given by the number of decompositions of the integer to two different integers. The definition of the book like structure assigns to a given CD preferred quantization axes and so that quantum measurement has direct correlate at the level of moduli space of CDs.

TGD inspired quantum biology and number theoretical considerations suggest preferred values for $r = \hbar/\hbar_0$. For the most general option the values of $\hbar$ are products and ratios of two integers n_a and n_b. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n_i)$, $i \in \{a, b\}$, in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of r.

One can however ask whether a more precise characterization of preferred Mersennes could exist and whether there could exists a stronger correlation between hierarchies of p-adic length scales and Planck constants. Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k-1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ are expected to be physically highly interesting and up to $k = 127$ indeed correspond to elementary particles. The number theoretical miracle is that all the four p-adic length scales with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm). The question has been whether these define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$. The proposal that this is the case and that these physics are in a well-defined sense induced by the dark scaled up variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$.

What induction means is that dark variant of exotic nuclear physics induces exotic physics with ordinary value of Planck constant in the new scale in a resonant manner: dark gauge bosons transform to their ordinary variants with the same Compton length. This transformation is natural since in length scales below the Compton length the gauge bosons behave as massless and free particles. As a consequence, lighter variants of weak bosons emerge and QCD confinement scale becomes longer.

This proposal will be referred to as Mersenne hypothesis. It leads to strong predictions about EEG [K24] since it predicts a spectrum of preferred Josephson frequencies for a given value of membrane potential and also assigns to a given value of $\hbar$ a fixed size scale having interpretation as the size scale of the body part or magnetic body. Also a vision about evolution of life emerges. Mersenne hypothesis is especially interesting as far as new physics in condensed matter length scales is considered: this includes exotic scaled up variants of the ordinary nuclear physics and their dark variants. Even dark nucleons are possible and this gives justification for the model of dark nucleons predicting the counterparts of DNA,RNA, tRNa, and amino-acids as well as realization of vertebrate genetic code [K94] .

These exotic nuclear physics with ordinary value of Planck constant could correspond to ground states that are almost vacuum extremals corresponding to homologically trivial geodesic sphere of CP_2 near criticality to a phase transition changing Planck constant. Ordinary nuclear physics would correspond to homologically non-trivial geodesic sphere and far from vacuum extremal property. For vacuum extremals of this kind classical Z^0 field proportional to electromagnetic field is present and this modifies dramatically the view about cell membrane as Josephson junction. The model for cell membrane as almost vacuum extremal indeed led to a quantitative breakthrough in TGD inspired model of EEG and is therefore something to be taken seriously. The safest option concerning empirical facts is that the copies of electro-weak and color physics with ordinary value of Planck constant are possible only for almost vacuum extremals - that is at criticality against phase transition changing Planck constant.

p-Adic physics and the notion of finite measurement resolution

Canonical identification mapping p-adic numbers to reals in a continuous manner plays a key role in some applications of TGD and together with the discretization necessary to define the p-adic variants of integration and harmonic analysis suggests that p-adic topology identified as an effective topology could provide an elegant manner to characterize finite measurement resolution.

(a) Finite measurement resolution can be characterized as an interval of minimum length. Below this length scale one cannot distinguish points from each other. A natural definition for this inability could be as an inability to well-order the points. The real topology is too strong in the modelling in kind of situation since it brings in large amount of processing of pseudo information whereas p-adic topology which lacks the notion of well-ordering could be more appropriate as effective topology and together with a pinary cutoff could allow to get rid of the irrelevant information.

(b) This suggest that canonical identification applies only inside the intervals defining finite measurement resolution in a given discretization of the space considered by say small cubes. The canonical identification is unique only modulo diffeomorphism applied on both real and p-adic side but this is not a problem since this would only reflect the absence of the well-ordering lost by finite measurement resolution. Also the fact that the map makes sense only at positive real axis would be natural if one accepts this identification.

This interpretation would suggest that there is an infinite hierarchy of measurement resolutions characterized by the value of the p-adic prime. This would mean quite interesting refinement of the notion of finite measurement resolution. At the level of quantum theory it could be interpreted as a maximization of p-adic entanglement negentropy as a function of the p-adic prime. Perhaps one might say that there is a unique p-adic effective topology allowing to maximize the information content of the theory relying on finite measurement resolution.

p-Adic numbers and the analogy of TGD with spin-glass

The vacuum degeneracy of the Kähler action leads to a precise spin glass analogy at the level of the WCW geometry and the generalization of the energy landscape concept to TGD context leads to the hypothesis about how p-adicity could be realized at the level of WCW. Also the concept of p-adic space-time surface emerges rather naturally.

1. Spin glass briefly

The basic characteristic of the spin glass phase [B21] is that the direction of the magnetization varies spatially, being constant inside a given spatial region, but does not depend on time. In the real context this usually leads to large surface energies on the surfaces at which the magnetization direction changes. Regions with different direction of magnetization clearly correspond non-vacuum regions separated by almost vacuum regions. Amusingly, if 3-space is effectively p-adic and if magnetization direction is p-adic pseudo constant, no surface energies are generated so that p-adics might be useful even in the context of the ordinary spin glasses.

Spin glass phase allows a great number of different ground states minimizing the free energy. For the ordinary spin glass, the partition function is the average over a probability distribution of the coupling constants for the partition function with Hamiltonian depending on the coupling constants. Free energy as a function of the coupling constants defines 'energy landscape' and the set of free energy minima can be endowed with an ultra-metric distance function using a standard construction [A106] .

2. Vacuum degeneracy of Kähler action

The Kähler action defining WCW geometry allows enormous vacuum degeneracy: any four-surface for which the induced Kähler form vanishes, is an extremal of the Kähler action. Induced Kähler form vanishes if the CP_2 projection of the space-time surface is Lagrangian manifold [A16] of CP_2: these manifolds are at most two-dimensional and any canonical transformation of CP_2 creates a new Lagrangian sub-manifold [A16] . An explicit representation for Lagrangian sub-manifolds is obtained using some canonical coordinates P_i, Q_i for CP_2: by assuming

$$P_i = \partial_i f(Q_1, Q_2) \ , \ i = 1, 2 \ ,$$

where f arbitrary function of its arguments. One obtains a 2-dimensional sub-manifold of CP_2 for which the induced Kähler form proportional to $dP_i \wedge dQ^i$ vanishes. The roles of P_i and Q_i can obviously be interchanged. A familiar example of Lagrange manifolds are $p_i = constant$ surfaces of the ordinary (p_i, q_i) phase space.

Since vacuum degeneracy is removed only by the classical gravitational interaction there are good reasons to expect large ground state degeneracy, when the system corresponds to a small deformation of a vacuum extremal. This degeneracy is very much analogous to the ground state degeneracy of spin glass but is 4-dimensional.

3. Vacuum degeneracy of the Kähler action and physical spin glass analogy

Quite generally, the dynamical reason for the physical spin glass degeneracy is the fact that Kähler action has a huge vacuum degeneracy. Any 4-surface with CP_2 projection, which is a Lagrangian sub-manifold (generically two-dimensional), is vacuum extremal. This implies that space-time decomposes into non-vacuum regions characterized by non-vanishing Kähler magnetic and electric fields such that the (presumably thin) regions between the the non-vacuum regions are vacuum extremals. Therefore no surface energies are generated. Also the fact that various charges and momentum and energy can flow to larger space-time sheets via wormholes is an important factor making possible strong field gradients without introducing large surfaces energies. From a given preferred extremal of Kähler action one obtains a new one by adding arbitrary space-time surfaces which is vacuum extremal and deforming them.

The symplectic invariance of the Kähler action for vacuum extremals allows a further understanding of the vacuum degeneracy. The presence of the classical gravitational interaction spoils the canonical group $Can(CP_2)$ as gauge symmetries of the action and transforms it to the isometry group of CH. As a consequence, the $U(1)$ gauge degeneracy is transformed to a spin glass type degeneracy and several, perhaps even infinite number of maxima of Kähler function become possible. Given sheet has naturally as its boundary the 3-surfaces for which two maxima of the Kähler function coalesce or are created from single maximum by a cusp catastrophe [A128] . In catastrophe regions there are several sheets and the value of the maximum Kähler function determines which give a measure for the importance of various sheets. The quantum jumps selecting one of these sheets can be regarded as phase transitions.

In TGD framework classical non-determinism forces to generalize the notion of the 3-surface by replacing it with a sequence of space like 3-surfaces having time like separations such that the sequence characterizes uniquely one branch of multi-furcation. This characterization works when non-determinism has discrete nature. For CP_2 type extremals which are bosonic vacua, basic objects are essentially four-dimensional since M^4_+ projection of CP_2 type extremal is random light like curve. This effective four-dimensionality of the basic objects makes it possible to topologize Feynman diagrammatics of quantum field theories by replacing the lines of Feynman diagrams with CP_2 type extremals.

In TGD framework spin glass analogy holds true also in the time direction, which reflects the fact that the vacuum extremals are non-deterministic. For instance, by gluing vacuum extremals with a finite space-time extension (also in time direction!) to a non-vacuum extremal and deforming slightly, one obtains good candidates for the degenerate preferred extremals. This non-determinism is expected to make the preferred extremals of the Kähler action highly degenerate. The construction of S-matrix at the high energy limit suggests that since a localization selecting one degenerate maximum occurs, one must accept as a fact that each choice of the parameters corresponds to a particular S-matrix and one must average over these choices to get scattering rates. This averaging for scattering rates corresponds to the averaging over the thermodynamical partition functions for spin glass. A more general is that one allows final state wave functions to depend on the zero modes which affect S-matrix elements: in the limit that wave functions are completely localized, one ends up with the simpler scenario.

4. p-Adic non-determinism and spin glass analogy

One must carefully distinguish between cognitive and physical spin-glass analogy. Cognitive spin-glass analogy is due to the p-adic non-determinism. p-Adic pseudo constants induce a non-determinism which essentially means that p-adic extrema depend on the p-adic pseudo constants which depend on a finite number of positive pinary digits of their arguments only. Thus p-adic extremals are glued from pieces for which the values of the integration constants are genuine constants. Obviously, an optimal cognitive representation is achieved if pseudo constants reduce to ordinary constants.

More precisely, any function

$$\begin{aligned} f(x) &= f(x_N) \ , \\ x_N &= \sum_{k\leq N} x_k p^k \ , \end{aligned} \tag{11.2.1}$$

which does not depend on the pinary digits x_n, $n > N$ has a vanishing p-adic derivative and is thus a pseudo constant. These functions are piecewise constant below some length scale, which in principle can be arbitrary small but finite. The result means that the constants appearing in the solutions the p-adic field equations are constants functions only below some length scale. For instance, for linear differential equations integration constants are arbitrary pseudo constants. In particular, the p-adic counterparts of the preferred extremals are highly degenerate because of the presence of the pseudo constants. This in turn means a characteristic randomness of the spin glass also in the time direction since the surfaces at which the pseudo constants change their values do not give rise to infinite surface energy densities as they would do in the real context.

The basic character of cognition would be spin glass like nature making possible 'engineering' at the level of thoughts (planning) whereas classical non-determinism of the Kähler action would make possible 'engineering' at the level of the real world.

Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

The possibility to define entropy differently for rational/algebraic entanglement and the fact that number theoretic entanglement entropy can be negative raises the question about which kind of systems can possess this kind of entanglement. I have considered several identifications but the most elegant interpretation is based on the idea that living matter resides in the intersection of real and p-adic worlds, somewhat like rational numbers live in the intersection of real and p-adic number fields.

The observation that Shannon entropy allows an infinite number of number theoretic variants for which the entropy can be negative in the case that probabilities are algebraic numbers leads to the idea that living matter in a well-defined sense corresponds to the intersection of real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

This picture has also a direct connection with consciousness.

(a) Algebraic entanglement is a prerequisite for the realization of intentions as transformations of p-adic space-time sheets to real space-time sheets representing actions. Essentially a leakage between p-adic and real worlds is in question and makes sense only in zero energy ontology. since various quantum numbers in real and p-adic sectors are not in general comparable in positive energy ontology so that conservation laws would be broken. Algebraic entanglement could be also called cognitive. The transformation can occur if the partonic 2-surfaces and their 4-D tangent space-distributions are representable using rational functions with rational coefficients in preferred coordinates for the imbedding space dictated by symmetry considerations. Intentional systems must live in the intersection of real and p-adic worlds. For the minimal option life would be also effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests.

(b) The generation of non-rational (non-algebraic) bound state entanglement between the system and external world means that the system loses consciousness during the state function reduction process following the U-process generating the entanglement. What happens that the Universe corresponding to given CD decomposes to two un-entangled subsystems, which in turn decompose, and the process continues until all subsystems have only entropic bound state entanglement or negentropic algebraic entanglement with the external world.

(c) If the sub-system generates entropic bound state entanglement in the the process, it loses consciousness. Note that the entanglement entropy of the sub-system is a sum over entanglement entropies over all subsystems involved. This hierarchy of subsystems corresponds to the hierarchy if sub-CDs so that the survival without a loss of consciousness depends on what happens at all levels below the highest level for a given self. In more concrete terms, ability to stay conscious depends on what happens at cellular level too. The stable evolution of systems having algebraic entanglement is expected to be a process proceeding from short to long length scales as the evolution of life indeed is.

(d) U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. This would suggest that the choice of the type of entanglement is a volitional selection. A possible interpretation is as a choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices.

(e) This formulation means a sharpening of the earlier statement "Everything is conscious and consciousness can be only lost" with the additional statement "This happens when non-algebraic bound state entanglement is generated and the system does not remain in the intersection of real and p-adic worlds anymore". Clearly, the quantum criticality of TGD Universe seems has very many aspects and life as a critical phenomenon in the number theoretical sense is only one of them besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua.

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

p-Adic physics as physics of cognition and intention

The vision about p-adic physics as physics of cognition has gradually established itself as one of the key idea of TGD inspired theory of consciousness. There are several motivations for this idea.

The strongest motivation is the vision about living matter as something residing in the intersection of real and p-adic worlds. One of the earliest motivations was p-adic non-determinism identified tentatively as a space-time correlqte for the non-determinism of imagination. p-Adic non-determinism follows from the fact that functions with vanishing derivatives are piecewise constant functions in the p-adic context. More precisely, p-adic pseudo constants depend on the pinary cutoff of their arguments and replace integration constants in p-adic differential equations. In the case of field equations this means roughly that the initial data are replaced with initial data given for a discrete set of time values chosen in such a manner that unique solution of field equations results. Solution can be fixed also in a discrete subset of rational points of the imbedding space. Presumably the uniqueness requirement implies some unique pinary cutoff. Thus the space-time surfaces representing solutions of p-adic field equations are analogous to space-time surfaces consisting of pieces of solutions of the real field equations. p-Adic reality is much like the dream reality consisting of rational fragments glued together in illogical manner or pieces of child's drawing of body containing body parts in more or less chaotic order.

The obvious looking interpretation for the solutions of the p-adic field equations is as a geometric correlate of imagination. Plans, intentions, expectations, dreams, and cognition in general are expected to have p-adic space-time sheets as their geometric correlates. This in the sense that p-adic space-time sheets somehow initiate the real neural processes providing symbolic counterparts for the cognitive representations provided by p-adic space-time sheets and p-adic fermions. A deep principle seems to be involved: incompleteness is characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

p-Adic space-time regions can suffer topological phase transitions to real topology and vice versa in quantum jumps replacing space-time surface with a new one. This process has interpretation as a topological correlate for the mind-matter interaction in the sense of transformation of intention to action and symbolic representation to cognitive representation. p-Adic cognitive representations could provide the physical correlates for the notions of memes [J4] and morphic fields [I16] . p-Adic real entanglement makes possible makes possible cognitive measurements and cognitive quantum computation like processes, and provides correlates for the experiences of understanding and confusion.

At the level of brain the fundamental sensory-motor loop could be seen as a loop in which real-to-p-adic phase transition occurs at the sensory step and its reverse at the motor step. Nerve pulse patterns would correspond to temporal sequences of quark like sub-CDs of duration

1 millisecond inside electronic sub-CD of duration .1 s with the states of sub-CDs allowing interpretation as a bit (this would give rise to memetic code). The real space-time sheets assignable to these sub-CDs are transformed to p-adic ones as sensory input transforms to thought. Intention in transforms to action in the reverse process in motor action. One can speak about creation of matter from vacuum in these time scales.

Although p-adic space-time sheets as such are not conscious, p-adic physics would provide beautiful mathematical realization for the intuitions of Descartes. The formidable challenge is to develop experimental tests for p-adic physics. The basic problem is that we can perceive p-adic reality only as 'thoughts' unlike the 'real' reality which represents itself to us as sensory experiences. Thus it would seem that we should be able generalize the physics of sensory experiences to physics of cognitive experiences.

11.2.3 p-Adic numbers

Basic properties of p-adic numbers

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A48] . p-Adic numbers are representable as power expansion of the prime number p of form:

$$x = \sum_{k\geq k_0} x(k)p^k ,\; x(k) = 0, ..., p-1 \; . \tag{11.2.2}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \; . \tag{11.2.3}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \; , \tag{11.2.4}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \; . \tag{11.2.5}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \; . \tag{11.2.6}$$

This division of the metric space into classes has following properties:

(a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

(b) Distances of points x and y inside single class are smaller than distances between different classes.

(c) Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B54] . The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Extensions of p-adic numbers

Algebraic democracy suggests that all possible real algebraic extensions of the p-adic numbers are possible. This conclusion is also suggested by various physical requirements, say the fact that the eigenvalues of a Hamiltonian representable as a rational or p-adic $N \times N$-matrix, being roots of N:th order polynomial equation, in general belong to an algebraic extension of rationals or p-adics. The dimension of the algebraic extension cannot be interpreted as physical dimension. Algebraic extensions are characteristic for cognitive physics and provide a new manner to code information. A possible interpretation for the algebraic dimension is as a dimension for a cognitive representation of space and might explain how it is possible to mathematically imagine spaces with all possible dimensions although physical space-time dimension is four. The idea of algebraic hologram and other ideas related to the physical interpretation of the algebraic extensions of p-adic numbers are discussed in [K87] .

It seems however that algebraic democracy must be extended to include also transcendentals in the sense that finite-dimensional extensions involving also transcendental numbers are possible: for instance, Neper number e defines a p-dimensional extension. It has become clear that these extensions fundamental for understanding how p-adic physics as physics of cognition is able to mimic real physics. The evolution of mathematical cognition can be seen as a process in which p-adic space-time sheets involving increasing value of p-adic prime p and increasing dimension of algebraic extension appear in quantum jumps.

1. Recipe for constructing algebraic extensions

Real numbers allow only complex numbers as an algebraic extension. For p-adic numbers algebraic extensions of arbitrary dimension are possible
[A48] . The simplest manner to construct (n+1)-dimensional extensions is to consider irreducible polynomials $P_n(t)$ in R_p assumed to have rational coefficients: irreducibility means that the polynomial does not possess roots in R_p so that one cannot decompose it into a product of lower order R_p valued polynomials. This condition is equivalent with the condition with irreducibility in the finite field $G(p,1)$, that is modulo p in R_p.

Denoting one of the roots of $P_n(t)$ by θ and defining $\theta^0 = 1$ the general form of the extension is given by

$$Z = \sum_{k=0,..,n-1} x_k \theta^k \ . \tag{11.2.7}$$

Since θ is root of the polynomial in R_p it follows that θ^n is expressible as a sum of lower powers of θ so that these numbers indeed form an n-dimensional linear space with respect to the p-adic topology.

Especially simple odd-dimensional extensions are cyclic extensions obtained by considering the roots of the polynomial

$$\begin{aligned} P_n(t) &= t^n + \epsilon d \ , \\ \epsilon &= \pm 1 \ . \end{aligned} \quad (11.2.8)$$

For $n = 2m+1$ and $(n = 2m, \epsilon = +1)$ the irreducibility of $P_n(t)$ is guaranteed if d does not possess n:th root in R_p. For $(n = 2m, \epsilon = -1)$ one must assume that $d^{1/2}$ does not exist p-adically. In this case θ is one of the roots of the equation

$$t^n = \pm d \ , \quad (11.2.9)$$

where d is a p-adic integer with a finite number of pinary digits. It is possible although not necessary to identify the roots as complex numbers. There exists n complex roots of d and θ can be chosen to be one of the real or complex roots satisfying the condition $\theta^n = \pm d$. The roots can be written in the general form

$$\begin{aligned} \theta &= d^{1/n} exp(i\phi(m)), \ m = 0, 1, ..., n-1 \ , \\ \phi(m) &= \frac{m2\pi}{n} \ \text{or} \ \frac{m\pi}{n} \ . \end{aligned} \quad (11.2.10)$$

Here $d^{1/n}$ denotes the real root of the equation $\theta^n = d$. Each of the phase factors $\phi(m)$ gives rise to an algebraically equivalent extension: only the representation is different. Physically these extensions need not be equivalent since the identification of the algebraically extended p-adic numbers with the complex numbers plays a fundamental role in the applications. The cases $\theta^n = \pm d$ are physically and mathematically quite different.

2. p-Adic valued norm for numbers in algebraic extension

The p-adic valued norm of an algebraically extended p-adic number x can be defined as some power of the ordinary p-adic norm of the determinant of the linear map $x :^e R_p^n \rightarrow^e R_p^n$ defined by the multiplication with x: $y \rightarrow xy$

$$N(x) = det(x)^{\alpha} \ , \ \alpha > 0 \ . \quad (11.2.11)$$

Real valued norm can be defined as the p-adic norm of $N(x)$. The requirement that the norm is homogenous function of degree one in the components of the algebraically extended 2-adic number (like also the standard norm of R^n) implies the condition $\alpha = 1/n$, where n is the dimension of the algebraic extension.

The canonical correspondence between the points of R_+ and R_p generalizes in obvious manner: the point $\sum_k x_k \theta^k$ of algebraic extension is identified as the point $(x_R^0, x_R^1, ..., x_R^k, ..,)$ of R^n using the pinary expansions of the components of p-adic number. The p-adic linear structure of the algebraic extension induces a linear structure in R_+^n and p-adic multiplication induces a multiplication for the vectors of R_+^n.

3. Algebraic extension allowing square root of ordinary p-adic numbers

The existence of a square root of an ordinary p-adic number is a common theme in various applications of the p-adic numbers and for long time I erratically believed that only this extension is involved with p-adic physics. Despite this square root allowing extension is of central importance and deserves a more detailed discussion.

(a) The p-adic generalization of the representation theory of the ordinary groups and Super Kac Moody and Super Virasoro algebras exists provided an extension of the p-adic numbers allowing square roots of the 'real' p-adic numbers is used. The reason is that the matrix elements of the raising and lowering operators in Lie-algebras as well as of oscillator operators typically involve square roots. The existence of square root might play a key role in various p-adic considerations.

(b) The existence of a square root of a real p-adic number is also a necessary ingredient in the definition of the p-adic unitarity and probability concepts since the solution of the requirement that $p_{mn} = S_{mn}\bar{S}_{mn}$ is ordinary p-adic number leads to expressions involving square roots.

(c) p-Adic length scales hypothesis states that the p-adic length scale is proportional to the square root of p-adic prime.

(d) Simple metric geometry of polygons involves square roots basically via the theorem of Pythagoras. p-Adic Riemannian geometry necessitates the existence of square root since the definition of the infinitesimal length ds involves square root. Note however that p-adic Riemannian geometry can be formulated as a mere differential geometry without any reference to global concepts like lengths, areas, or volumes.

The original belief that square root allowing extensions of p-adic numbers are exceptional seems to be wrong in light of TGD as a generalized number theory vision. All algebraic extensions of p-adic numbers a possible and the interpretation of algebraic dimension of the extension as a physical dimension is not the correct thing to do. Rather, the possibility of arbitrarily high algebraic dimension reflects the ability of mathematical cognition to imagine higher-dimensional spaces. Square root allowing extension of the p-adic numbers is the simplest one imaginable, and it is fascinating that it indeed is the dimension of space-time surface for $p > 2$ and dimension of imbedding space for $p = 2$. Thus the square root allowing extensions deserve to be discussed.

The results can be summarized as follows.

(a) In $p > 2$ case the general form of extension is

$$Z = (x + \theta y) + \sqrt{p}(u + \theta v) \ , \tag{11.2.12}$$

where the condition $\theta^2 = x$ for some p-adic number x not allowing square root as a p-adic number. For $p \ mod \ 4 = 3$ θ can be taken to be imaginary unit. This extension is natural for p-adication of space-time surface so that space-time can be regarded as a number field locally. Imbedding space can be regarded as a cartesian product of two 4-dimensional extensions locally.

(b) In $p = 2$ case 8-dimensional extension is needed to define square roots. The extension is defined by adding $\theta_1 = \sqrt{-1} \equiv i$, $\theta_2 = \sqrt{2}$, $\theta_3 = \sqrt{3}$ and the products of these so that the extension can be written in the form

$$Z = x_0 + \sum_k x_k\theta_k + \sum_{k<l} x_{kl}\theta_{kl} + x_{123}\theta_1\theta_2\theta_3 \ . \tag{11.2.13}$$

Clearly, $p = 2$ case is exceptional as far as the construction of the conformal field theory limit is considered since the structure of the representations of Virasoro algebra and groups in general changes drastically in $p = 2$ case. The result suggest that in $p = 2$ limit space-time surface and H are in same relation as real numbers and complex numbers: space-time surfaces defined as the absolute minima of 2-adiced Kähler action are perhaps identifiable as surfaces for which the imaginary part of 2-adically analytic function in H vanishes.

The physically interesting feature of p-adic group representations is that if one doesn't use $\sqrt{p}$ in the extension the number of allowed spins for representations of $SU(2)$ is finite: only spins $j < p$ are allowed. In $p = 3$ case just the spins $j \leq 2$ are possible. If 4-dimensional extension is used for $p = 2$ rather than 8-dimensional then one gets the same restriction for allowed spins.

4. Is e an exceptional transcendental?

One can consider also the possibility of transcental extensions of p-adic numbers and an open problem is whether the infinite-dimensional extensions involving powers of π and logarithms of primes make sense and whether they should be allowed. For instance, it is not clear whether the allowance of powers of π is consistent with the extensions based on roots of unity. This question is not academic since Feynman amplitudes in real context involve powers of π and algebraic universality forces the consider that also they p-adic variants might involve powers of π.

Neper number obviously defines the simplest transcendental extension since only the powers e^k, $k = 1, ..., p-1$ of e are needed to define p-adic counterpart of e^x for $x = n$ so that the extension is finite-dimensional. In the case of trigonometric functions deriving from e^{ix}, also e^i and its $p-1$ powers must belong to the extension.

An interesting question is whether e is a number theoretically exceptional transcendental or whether it could be easy to find also other transcendentals defining finite-dimensional extensions of p-adic numbers.

(a) Consider functions $f(x)$, which are analytic functions with rational Taylor coefficients, when expanded around origin for $x > 0$. The values of $f(n)$, $n = 1, ..., p-1$ should belong to an extension, which should be finite-dimensional.

(b) The expansion of these functions to Taylor series generalizes to the p-adic context if also the higher derivatives of f at $x = n$ belong to the extension. This is achieved if the higher derivatives are expressible in terms of the lower derivatives using rational coefficients and rational functions or functions, which are defined at integer points (such as exponential and logarithm) by construction. A differential equation of some finite order involving only rational functions with rational coefficients must therefore be satisfied (e^x satisfying the differential equation $df/dx = f$ is the optimal case in this sense). The higher derivatives could also reduce to rational functions at some step ($log(x)$ satisfying the differential equation $df/dx = 1/x$).

(c) The differential equation allows to develop $f(x)$ in power series, say in origin

$$f(x) = \sum f_n \frac{x^n}{n!}$$

such that f_{n+m} is expressible as a rational function of the m lower derivatives and is therefore a rational number.

The series converges when the p-adic norm of x satisfies $|x|_p \leq p^k$ for some k. For definiteness one can assume $k = 1$. For $x = 1, ..., p-1$ the series does not converge in this case, and one can introduce and extension containing the values $f(k)$ and hope that a finite-dimensional extension results.

Finite-dimensionality requires that the values are related to each other algebraically although they need not be algebraic numbers. This means symmetry. In the case of exponent function this relationship is exceptionally simple. The algebraic relationship reflects the fact that exponential map represents translation and exponent function is an eigen function of a translation operator. The necessary presence of symmetry might mean that the situation reduces always to either exponential action. Also the phase factors $exp(iq\pi)$ could be interpreted in terms of exponential symmetry. Hence the reason for the exceptional role of exponent function reduces to group theory.

Also other extensions than those defined by roots of e are possible. Any polynomial has n roots and for transcendental coefficients the roots define a finite-dimensional extension of rationals. It would seem that one could allow the coefficients of the polynomial to be functions in an extension of rationals by powers of a root of e and algebraic numbers so that one would obtain infinite hierarchy of transcendental extensions.

p-Adic Numbers and finite fields

Finite fields (Galois fields) consists of finite number of elements and allow sum, multiplication and division. A convenient representation for the elements of a finite field is as the roots of the polynomial equation $t^{p^m} - t = 0 \ mod \ p$, where p is prime, m an arbitrary integer and t is element of a field of characteristic p ($pt = 0$ for each t). The number of elements in a finite field is p^m, that is power of prime number and the multiplicative group of a finite field is group of order $p^m - 1$. $G(p, 1)$ is just cyclic group Z_p with respect to addition and $G(p, m)$ is in rough sense m:th Cartesian power of $G(p, 1)$.

The elements of the finite field $G(p, 1)$ can be identified as the p-adic numbers $0, ..., p - 1$ with p-adic arithmetics replaced with modulo p arithmetics. The finite fields $G(p, m)$ can be obtained from m-dimensional algebraic extensions of the p-adic numbers by replacing the p-adic arithmetics with the modulo p arithmetics. In TGD context only the finite fields $G(p > 2, 2)$, $p \ mod \ 4 = 3$ and $G(p = 2, 4)$ appear naturally. For $p > 2$, $p \ mod \ 4 = 3$ one has: $x + iy + \sqrt{p}(u + iv) \rightarrow x_0 + iy_0 \in G(p, 2)$.

An interesting observation is that the unitary representations of the p-adic scalings $x \rightarrow p^k x$ $k \in Z$ lead naturally to finite field structures. These representations reduce to representations of a finite cyclic group Z_m if $x \rightarrow p^m x$ acts trivially on representation functions for some value of m, $m = 1, 2, ...$ Representation functions, or equivalently the scaling momenta $k = 0, 1, ..., m - 1$ labelling them, have a structure of cyclic group. If $m \neq p$ is prime the scaling momenta form finite field $G(m, 1) = Z_m$ with respect to the summation and multiplication modulo m. Also the p-adic counterparts of the ordinary plane waves carrying p-adic momenta $k = 0, 1..., p - 1$ can be given the structure of Finite Field $G(p, 1)$: one can also define complexified plane waves as square roots of the real p-adic plane waves to obtain Finite Field $G(p, 2)$.

11.2.4 What is the correspondence between p-adic and real numbers?

There must be some kind of correspondence between reals and p-adic numbers. This correspondence can depend on context. In p-adic mass calculations one must map p-adic mass squared values to real numbers in a continuous manner and canonical identification $x = \sum x_n p^n \rightarrow Id(x) = \sum x_n p^{-n}$ is a natural first guess. Also p-adic probabilities could be mapped to their real counterparts by a suitable normalization. One can wonder whether p-adic valued S-matrices have any physical meaning and whether they could be obtained as algebraic continuation from a number theoretically universal S-matrix whose matrix elements are algebraic numbers allowing an interpretation as real or p-adic numbers in suitable algebraic extension: this would pose extremely strong constraints on S-matrix. If one wants to introduce p-adic physics at space-time level one must be able to relate p-adic and real space-time regions to each other and the identification along common rational points of real and various p-adic variants of the imbedding space suggests itself here.

Generalization of the number concept

The recent view about the unification of real and p-adic physics is based on the generalization of number concept obtained by fusing together real and p-adic number fields along common rationals (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also).

1. Rational numbers as numbers common to all number fields

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers appearing in the extension of p-adic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional algebraic and perhaps even transcendental extensions of p-adic numbers adds additional pages to this "Big Book".

This leads to a generalization of the notion of manifold as a collection of a real manifold and its p-adic variants glued together along common points. The outcome of experimentation is that this generalization makes sense under very high symmetries and that it is safest to lean strongly on the physical picture provided by quantum TGD.

(a) The most natural guess is that the coordinates of common points are rational or in some algebraic extension of rational numbers. General coordinate invariance and preservation of symmetries require preferred coordinates existing when the manifold has maximal number of isometries. This approach is especially natural in the case of linear spaces-in particular Minkowski space M^4. The natural coordinates are in this case linear Minkowski coordinates. The choice of coordinates is not completely unique and has interpretation as a geometric correlate for the choice of quantization axes for a given CD.

(b) As will be found, the need to have a well-defined integration based on Fourier analysis (or its generalization to harmonic analysis [A10] in symmetric spaces) poses very strong constraints and allows p-adicization only if the space has maximal symmetries. Fourier analysis requires the introduction of an algebraic extension of p-adic numbers containing sufficiently many roots of unity.

   i. This approach is especially natural in the case of compact symmetric spaces such as CP_2 [A6] .

   ii. Also symmetric spaces such the 3-D proper time $a = constant$ hyperboloid of M^4-call it $H(a)$ -allowing Lorentz group as isometries allows a p-adic variant utilizing the hyperbolic counterparts for the roots of unity. $M^4 \times H(a = 2^n a_0)$ appears as a part of the moduli space of CDs.

   iii. For light-cone boundaries associated with CDs $SO(3)$ invariant radial coordinate r_M defining the radius of sphere S^2 defines the hyperbolic coordinate and angle coordinates of S^2 would correspond to phase angles and $M^4_\pm$ projections for the common points of real and p-adic variants of partonic 2-surfaces would be this kind of points. Same applies to CP_2 projections. In the "intersection of real and p-adic worlds" real and p-adic partonic 2-surfaces would obey same algebraic equations and would be obtained by an algebraic continuation from the corresponding equations making sense in the discrete variant of $M^4_\pm \times CP_2$. This connection with discrete sub-manifolds geometries means very powerful constraints on the partonic 2-surfaces in the intersection.

(c) The common algebraic points of real and p-adic variant of the manifold form a discrete space but one could identify the p-adic counterpart of the real discretization intervals $(0, 2\pi/N)$ for angle like variables as p-adic numbers of norm smaller than 1 using canonical identification or some variant of it. Same applies to the the hyperbolic counterpart of this interval. The non-uniqueness of this map could be interpreted in terms of a finite measurement resolution. In particular, the condition that WCW allows Kähler geometry requires a decomposition to a union of symmetric spaces so that there are good hopes that p-adic counterpart is analogous to that assigned to CP_2.

2. How large p-adic space-time sheets can be?

Space-time region having finite size in the real sense can have arbitrarily large size in p-adic sense and vice versa. This raises a rather thought provoking questions. Could the p-adic space-time sheets have cosmological or even infinite size with respect to the real metric but have be p-adically finite? How large space-time surface is responsible for the p-adic representation of my body? Could the large or even infinite size of the cognitive space-time sheets explain why creatures of a finite physical size can invent the notion of infinity and construct cosmological theories? Could it be that pinary cutoff $O(p^n)$ defining the resolution of a p-adic cognitive representation would define the size of the space-time region needed to realize the cognitive representation?

In fact, the mere requirement that the neighborhood of a point of the p-adic space-time sheet contains points, which are p-adically infinitesimally near to it can mean that points infinitely distant from this point in the real sense are involved. A good example is provided by an integer valued point $x = n < p$ and the point $y = x + p^m$, $m > 0$: the p-adic distance of these points is p^{-m} whereas at the limit $m \rightarrow \infty$ the real distance goes as p^m and becomes infinite for infinitesimally near points. The points $n + y$, $y = \sum_{k>0} x_k p^k$, $0 < n < p$, form a p-adically continuous set around $x = n$. In the real topology this point set is discrete set with a minimum distance $\Delta x = p$ between neighboring points whereas in the p-adic topology every point has arbitrary nearby points. There are also rationals, which are arbitrarily near to each other both p-adically and in the real sense. Consider points $x = m/n$, m and n not divisible by p, and $y = (m/n) \times (1 + p^k r)/(1 + p^k s)$, $s = r + 1$ such that neither r or s is divisible by p and $k >> 1$ and $r >> p$. The p-adic and real distances are $|x - y|_p = p^{-k}$ and $|x - y| \simeq (m/n)/(r + 1)$ respectively. By choosing k and r large enough the points can be made arbitrarily close to each other both in the real and p-adic senses.

The idea about astrophysical size of the p-adic cognitive space-time sheets providing representation of body and brain is consistent with TGD inspired theory of consciousness, which forces to take very seriously the idea that even human consciousness involves astrophysical length scales.

3. Generalizing complex analysis by replacing complex numbers by generalized numbers

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions for which polynomials have rational coefficients are obviously functions satisfying this constraint. Algebraic functions for which polynomials have rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed.

For instance, one can ask whether residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the back of the book like structure (in very metaphorical sense) having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "Big Book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense. Contrary to the first expectations the algebraically continued residue calculus does not seem to have obvious applications in quantum TGD.

Canonical identification

Canonical There exists a natural continuous map $Id : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{11.2.14}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also desimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{11.2.15}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{11.2.16}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite number of pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite number of pinary digits. The finite number of pinary digits expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

1. *Canonical identification is a continuous map of non-negative reals to p-adics*

The topology induced by the inverse of the canonical identification map in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. 11.2.4) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. This allows two alternative interpretations. Either p-adic image of a physical systems provides a good representation of the system above some pinary cutoff or the physical system can be genuinely p-adic below certain length scale L_p and become in good approximation real, when a length scale resolution L_p is used in its description. The first interpretation is correct if canonical identification is interpreted as a cognitive map. p-Adic continuity implies ordinary continuity from right as is clear already

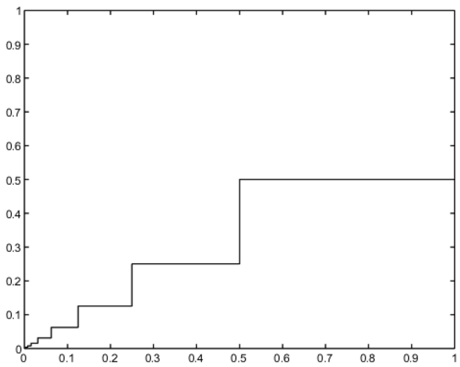

Figure 11.1: The real norm induced by canonical identification from 2-adic norm.

from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

If one considers seriously the application of canonical identification to basic quantum TGD one cannot avoid the question about the p-adic counterparts of the negative real numbers. There is no satisfactory manner to circumvent the fact that canonical images of p-adic numbers are naturally non-negative. This is not a problem if canonical identification applies only to the coordinate interval $(0, 2\pi/N)$ or its hyperbolic variant defining the finite measurement resolution. That p-adicization program works only for highly symmetric spaces is not a problem from the point of view of TGD.

2. Canonical identification maps the predictions of the p-adic probability calculus and statistical physics to real numbers

p-Adic mass calculations based on p-adic thermodynamics were the first and rather successful application of the p-adic physics (see the four chapters in [K54] . The essential element of the approach was the replacement of the Boltzmann weight $e^{-E/T}$ with its p-adic generalization p^{L_0/T_p}, where L_0 is the Virasoro generator corresponding to scaling and representing essentially mass squared operator instead of energy. T_p is inverse integer valued p-adic temperature. The predicted mass squared averages were mapped to real numbers by canonical identification.

One could also construct a real variant of this approach by considering instead of the ordinary Boltzmann weights the weights p^{-L_0/T_p}. The quantization of temperature to $T_p = log(p)/n$ would be a completely ad hoc assumption. In the case of real thermodynamics all particles are predicted to be light whereas in case of p-adic thermodynamics particle is light only if the ratio for the degeneracy of the lowest massive state to the degeneracy of the ground state is integer. Immense number of particles disappear from the spectrum of light particles by this criterion. For light particles the predictions are same as of p-adic thermodynamics in the lowest non-trivial order but in the next order deviations are possible.

Also p-adic probabilities and the p-adic entropy can be mapped to real numbers by canonical identification. The general idea is that a faithful enough cognitive representation of the real physics can by the number theoretical constraints involved make predictions, which would be extremely difficult to deduce from real physics.

3. The variant of canonical identification commuting with division of integers

The basic problems of canonical identification is that it does not respect unitarity. For this reason it is not well suited for relating p-adic and real scattering amplitudes. The problem of the correspondence via direct rationals or roots of unity is that it does not respect continuity.

The restriction of S-matrix to a discrete intersection of real and p-adic worlds is one manner to solve this difficulty.

One can also consider alternative approach to achieve a compromise between algebra and topology achieved by using a modification of canonical identification $I_{R_p\rightarrow R}$ defined as $I_1(r/s) = I(r)/I(s)$. If the conditions $r \ll p$ and $s \ll p$ hold true, the map respects algebraic operations and also unitarity and various symmetries. It seems that this option must be used to relate p-adic transition amplitudes to real ones and vice versa [K52] . In particular, real and p-adic coupling constants are related by this map. Also some problems related to p-adic mass calculations find a nice resolution when I_1 is used.

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q = r/s$ have no common factors. For integers $n < p$ it reduces to direct correspondence.

Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R\rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n/\sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n}/\sum s_n p^{-n}$ interpreted as a p-adic number:

$$q \ = \ \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} \ . \qquad (11.2.17)$$

R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R\rightarrow R_{p_2}} I_{R_{p_1}\rightarrow R}$.

This variant of canonical identification seems to be an excellent candidate for mapping the predictions of p-adic mass calculations to real numbers and also for relating p-adic and real scattering amplitudes to each other [K52] . The deviations of predictions from those for standard form of canonical identification are however small.

The cautious conclusion of this section is that symmetric space approach involving both the identification along common rationals of roots of unity in large and canonical identification below the measurement resolution provide the safest approach to the p-adicization of quantum TGD. The impossibility to well-order the points below measurement resolution explains why effective p-adic topology works in real context. The discussion of integration and Fourier analysis will provide further support for the conclusion.

11.2.5 p-Adic variants of the basic mathematical structures relevant to physics

The basic existential questions worrying a person planning to become a p-adic quantum physicist are rather obvious. How to define p-adic probabilities, p-adic thermodynamics, and p-adic unitarity and perhaps even p-adic Hilbert space? Is it possible to define the p-adic variant of the manifold concept? As already noticed for symmetric spaces p-adic variants might exist but what about space-time surfaces: could it be enough to consider only the p-adic variants of the partonic 2-surfaces in the manner already discussed? Can one somehow circumvent the difficulties related to the definition of the p-adic variant of definite integral? Perhaps by using Fourier analysis? How can one circumvent the fact that the basic variational principle involves integral over space-time surface which is p-adically notoriously difficult to define? Is all this just a waste of time or could it be that the enormous constraints from p-adicization could provide information about real physics not achievable otherwise (as in the case of p-adic mass calculations)?

p-Adic probabilities

p-Adic super conformal representations necessitate p-adic QM based on the p-adic unitarity and p-adic probability concepts. The concept of a p-adic probability indeed makes sense as shown by [A89] . p-Adic probabilities can be defined as relative frequencies N_i/N in a long series consisting of total number N of observations and N_i outcomes of type i. Probability conservation corresponds to

$$\sum_i N_i = N \ , \tag{11.2.18}$$

and the only difference as compared to the usual probability is that the frequencies are interpreted as p-adic numbers.

The interpretation as p-adic numbers means that the relative frequencies converge to probabilities in a p-adic rather than real sense in the limit of a large number N of observations. If one requires that probabilities are limiting values of the frequency ratios in p-adic sense one must pose restrictions on the possible numbers of the observations N if N is larger than p. For N smaller than p, the situation is similar to the real case. This means that for $p = M_{127} \simeq 10^{38}$, appropriate for the particle physics experiments, p-adic probability differs in no observable manner from the ordinary probability.

If the number of observations is larger than p, the situation changes. If N_1 and N_2 are two numbers of observations they are near to each other in the p-adic sense if they differ by a large power of p. A possible interpretation of this restriction is that the observer at the p:th level of the condensate cannot choose the number of the observations freely. The restrictions to this freedom come from the requirement that the sensible statistical questions in a p-adically conformally invariant world must respect p-adic conformal invariance [A27] .

The most important application of the p-adic probability is the description of the particle massivation based on p-adic thermodynamics. Instead of energy, Virasoro generator l is thermalized and in the low temperature phase temperature is quantized in the sense that the counterpart of the Boltzmann weight $exp(H/T)$ is $p^{L_0/T}$, where $T = 1/n$ from the requirement that Boltzmann weight exists (L_0 has integer spectrum). The surprising success of the mass calculations shows that p-adic probability theory is much more than a formal possibility.

In particle physics context coupling constant evolution is replaced with a discrete p-adic coupling constant evolution and the renormalization is related to the the change of the reduction of the p-adic length scale L_p in the length scale hierarchy rather than p-adic fractality for a fixed value of p. In zero energy ontology the evolution corresponds to the hierarchy of CDs with scales coming as powers of 2 in accordance with p-adic length scale hypothesis.

1. p-Adic probabilities and p-adic fractals

p-Adic probabilities are natural in the statistical description of the fractal structures, which can contain same structural detail with all possible sizes.

(a) The concept of a structural detail in a fractal seems to be reasonably well defined concept. The structural detail is clearly fixed by its topology and p-adic conformal invariants associated with it. Clearly, a finite resolution defined by some power of p of the p-adic cutoff scale must be present in the definition. For example, p-adic angles are conformal invariants in the p-adic case, too. The overall size of the detail doesn't matter. Let us therefore assume that it is possible to make a list, possibly infinite, of the structural details appearing in the p-adic fractal.

(b) What kind of questions related to the structural details of the p-adic fractal one can ask? The first thing one can ask is how many times i:th structural detail appears in a finite region of the fractal structure: although this number is infinite as a real number it might possess (and probably does so!) finite norm as a p-adic number and provides a useful p-adic invariant of the fractal. If a complete list about the structural details of the fractal is at use one can calculate also the total number of structural details defined as $N = \sum_i N_i$. This means that one can also define p-adic probability for the appearance of i:th structural detail as a relative frequency $p_i = N_i/N$.

(c) One can consider conditional probabilities, too. It is natural to ask what is the probability for the occurrence of the structural detail subject to the condition that part of the structural detail is fixed (apart from the p-adic conformal transformations). In order to evaluate these probabilities as relative frequencies one needs to look only for those structural details containing the substructure in question.

(d) The evaluation of the p-adic probabilities of occurrence can be done by evaluating the required numbers N_i and N in a given resolution. A better estimate is obtained by increasing the resolution and counting the numbers of the hitherto unobserved structural details. The increase in the resolution greatly increases the number of the observations in case of p-adic fractal and the fluctuations in the values of N_i and N increase with the resolution so that N_i/N has no well defined limit as a real number although one can define the probabilities of occurrence as a resolution dependent concept. In the p-adic sense the increase in the values of N_i and fluctuations are small and the procedure should converge rapidly so that reliable estimates should result with quite a reasonable resolution. Notice that the increase of the fluctuations in the real sense, when resolution is increased is in accordance with the criticality of the system.

(e) p-Adic frequencies and probabilities define via the canonical correspondence real valued invariants of the fractal structure.

p-Adic fractality in this sense could have practical applications only for small values of p. They could be important in the macroscopic length scales if the hierarchy of Planck constants meaning scaling up $L_p \rightarrow \sqrt{r}L_p$, $r = \hbar/\hbar_0$, of the p-adic length scales. In elementary particle physics L_p is of the order of the Compton length associated with the particle for $r = 1$ and already in the first downward step CP_2 length scale R is achieved whereas upward step gives astrophysical length scale in the case of electron ($p = M_{127} = 2^{127} - 1$) for instance. For large enough values of Planck constant and for small p-adic primes p the situation changes.

2. Relationship between p-adic and real probabilities

There are uniqueness problems related to the mapping of p-adic probabilities to real ones. These problems find a nice resolution from the requirement that the map respects probability conservation. The implied modification of the original mapping does not change measurably the predictions for the masses of light particles.

a) How unique the map of p-adic probabilities and mass squared values are mapped to real numbers is?

The mapping of p-adic thermodynamical probabilities and mass squared values to real numbers is not completely unique.

(a) The canonical identification $Id : \sum x_n p^n \rightarrow \sum x_n p^{-n}$ takes care of this mapping but does not respect the sum of probabilities so that the real images $I(p_n)$ of the probabilities must be normalized. This is a somewhat alarming feature.

(b) The modification of the canonical identification mapping rationals by the formula $I(r/s) = I(r)/I(s)$ has appeared naturally in various applications, in particular because it respects unitarity of unitary matrices with rational elements with $r < p, s < p$. In the case of p-adic thermodynamic the formula $I(g(n)p^n/Z) \rightarrow I(g(n)p^n)/I(Z)$ would be very natural although Z need not be rational anymore. For $g(n) < p$ the real counterparts of the p-adic probabilities would sum up to one automatically for this option. One cannot deny that this option is more convincing than the original one. The generalization of this formula to map p-adic mass squared to a real one is obvious.

(c) Options 1) and 2) differ dramatically when the $n=0$ massless ground state has ground state degeneracy $D>1$. For option 1) the real mass is predicted to be of order CP_2 mass whereas for option 2) it would be by a factor $1/D$ smaller than the minimum mass predicted by the option 1). Thus option 2) would predict a large number of additional exotic states. For those states which are light for option 1), the two options make identical predictions as far as the significant two lowest order terms are considered. Hence this interpretation would not change the predictions of the p-adic mass calculations in this respect. Option 2) is definitely more in accord with the real physics based intuitions and the main role of p-adic thermodynamics would be to guarantee the quantization of the temperature and fix practically uniquely the spectrum of the "Hamiltonian".

b) Under what conditions the mapping of p-adic ensemble probabilities to real probabilities respects probability conservation?

One can consider also a more general situation. Assume that one has an ensemble consisting of independent elementary events such that the number of events of type i is N_i. The probabilities are given by $p_i = N_i/N$ and $N = \sum N_i$ is the total number of elementary events. Even in the case that N is infinite as a real number it is natural to map the p-adic probabilities to their real counterparts using the rational canonical identification $I(p_i) = I(N_i)/I(N)$. Of course, N_i and N exist as well defined p-adic numbers under very stringent conditions only.

The question is under what conditions this map respects probability conservation. The answer becomes obvious by looking at the pinary expansions of N_i and N. If the integers N_i (possibly infinite as real integers) have pinary expansions having no common pinary digits, the sum of probabilities is conserved in the map. Note that this condition can assign also to a finite ensemble with finite number of a unique value of p.

This means that the selection of a basis for independent events corresponds to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory. What is physically highly non-trivial that this "orthogonalization" alone puts strong constraints on probabilities of the allowed elementary events. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product defined by point-wise multiplication in the space of pinary digits.

p-Adic thermodynamics for which Boltzmann weights $g(E)exp(-E/T)$ are replaced by $g(E)p^{E/T}$ such that one has $g(E) < p$ and E/T is integer valued, satisfies this constraint. The quantization of E/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance [A27, A30] in TGD applications, which is indeed a basic symmetry of the theory.

There are infinitely many ways to choose the elementary events and each choice corresponds to a decomposition of the infinite set of integers n labelling the powers of p to disjoint subsets. These subsets can be also infinite. One can assign to this kind of decomposition a resolution which is the poorer the larger the subsets involved are. p-Adic thermodynamics would represent the situation in which the resolution is maximal since each set contains only single pinary digit. Note the analogy with the basis of completely localized wave functions in a lattice.

c) How to map p-adic transition probabilities to real ones?

p-Adic variants of TGD, if they exist, give rise to S-matrices and transition probabilities P_{ij}, which are p-adic numbers.

(a) The p-adic probabilities defined by rows of S-matrix mapped to real numbers using canonical identification respecting the $q = r/s$ decomposition of rational number or its appropriate generalization should define real probabilities.

(b) The simplest example would simple renormalization for the real counterparts of the p-adic probabilities $(P_{ij})_R$ obtained by canonical identification (or more probably its appropriate modification).

$$\begin{aligned} P_{ij} &= \sum_{k\geq 0} P_{ij}^k p^k \ , \\ P_{ij} &\rightarrow \sum_{k\geq 0} P_{ij}^k p^{-k} \equiv (P_{ij})_R \ , \\ (P_{ij})_R &\rightarrow \frac{(P_{ij})_R}{\sum_j (P_{ij})_R} \equiv P_{ij}^R \ . \end{aligned} \tag{11.2.19}$$

The procedure converges rapidly in powers of p and resembles renormalization procedure of quantum field theories. The procedure automatically divides away one four-momentum delta function from the square of S-matrix element containing the square of delta function with no well defined mathematical meaning. Usually one gets rid of the delta function interpreting it as the inverse of the four-dimensional measurement volume so that transition rate instead of transition probability is obtained. Of course, also now same procedure should work either as a discrete or a continuous version.

(c) Probability interpretation would suggest that the real counterparts of p-adic probabilities sum up to unity. This condition is rather strong since it would hold separately for each row and column of the S-matrix.

(d) A further condition would be that the real counterparts of the p-adic probabilities for a given prime p are identical with the transition probabilities defined by the real S-matrix for real space-time sheets with effective p-adic topology characterized by p. This condition might allow to deduce all relevant phase information about real and corresponding p-adic S-matrices using as an input only the observable transition probabilities.

d) What it means that p-adically independent events are not independent in real sense?

A further condition would be that p-adic quantum transitions represent also in the real sense independent elementary events so that the real counterpart for a sum of the p-adic probabilities for a finite number of transitions equals to the sum of corresponding real probabilities. This condition is definitely too strong in the general case since only a single transition could correspond to a given p-adic norm of transition probability P_{ij} with i fixed. In p-adic thermodynamics it can be satisfied if the degeneracy for an energy eigenstate for a given eigen value $L_0 = n$ is not larger than p. This condition fails for large values of n for super Virasoro representations since the degeneracy grows exponentially. This has not practical implications for the large values of p considered.

The crucial question concerns the physical difference between the real counterpart for the sum of the p-adic transition probabilities and for the sum of the real counterparts of these probabilities, which are in general different:

$$(\sum_j P_{ij})_R \neq \sum_j (P_{ij})_R \ . \tag{11.2.20}$$

The suggestion is that p-adic sum of the transition probabilities corresponds to the experimental situation, when one does not monitor individual transitions but using some common experimental signature only looks whether the transition leads to this set of the final states or not. When one looks each transition separately or effectively performs different experiment by considering only one transition channel in each experiment one must use the sum of the real probabilities. More precisely, the choice of the experimental signatures divides the set U of the final states to a disjoint union $U = \cup_i U_i$ and one must define the real counterparts for the transition probabilities P_{iU_k} as

$$\begin{aligned} P_{iU_k} &= \sum_{j\in U_k} P_{ij} \ , \\ P_{iU_k} &\rightarrow (P_{iU_k})_R \ , \\ (P_{iU_k})_R &\rightarrow \frac{(P_{iU_k})_R}{\sum_l (P_{iU_l})_R} \equiv P^R_{iU_k} \ . \end{aligned} \tag{11.2.21}$$

The assumption means deep a departure from the ordinary probability theory. If p-adic physics is the physics of cognitive systems, there need not be anything mysterious in the dependence of the behavior of system on how it is monitored. At least half-jokingly one might argue that the behavior of an intelligent system indeed depends strongly on whether the boss is nearby or not. The precise definition for the monitoring could be based on the decomposition of the density matrix representing the entangled subsystem into a direct sum over the subspaces associated with the degenerate eigenvalues of the density matrix. This decomposition provides a natural definition for the notions of the monitoring and resolution.

The renormalization procedure is in fact familiar from standard physics. Assume that the labels j correspond to momenta. The division of momentum space to cells of a given size so that the individual momenta inside cells are not monitored separately means that momentum resolution is finite. Therefore one must perform p-adic summation over the cells and define the real probabilities in the proposed manner. p-Adic effects resulting from the difference between p-adic and real summations could be the counterpart of the renormalization effects in QFT. It should be added that similar resolution can be defined also for the initial states by decomposing them into a union of disjoint subsets.

An alternative interpretation for the degenerate eigenvalues has emerged years after writing this. The sub-spaces corresponding to given eigenvalue of density matrix represent entangled states resulting in state function reduction interpreted as measurement of density matrix. This entanglement would be negentropic and represent a rule/concept, whose instances the superposed state pairs are. The information measure would Shannon entropy based on the replacement of the probability appearing as argument of logarithm with its p-adic norm. This entropy would be negative and therefore measure the information associated with the entanglement. This number theoretic entropy characterizes two particle state rather than single particle state and has nothing to do with the ordinary Shannon entropy.

Maybe one could say that finite measurement resolution implies automatically conceptualization and rule building. Abstractions are indeed obtained by dropping out the details.

2. p-Adic thermodynamics

The p-adic field theory limit as such is not expected to give a realistic theory at elementary particle physics level. The point is that particles are expected to be either massless or possess mass of order 10^{-4} Planck mass. The p-adic description of particle massivation described in [K54] shows that p-adic thermodynamics provides the proper formulation of the problem. What is thermalized is Virasoro generator L_0 (mass squared contribution is not included to L_0 so that states do not have a fixed conformal weight). Temperature is quantized purely number theoretically in low temperature limit ($exp(H/kT) \rightarrow p^{L_0/T}$, $T = 1/n$): in fact, the partition function does not even exist in high temperature phase. The extremely small mixing of massless states with Planck mass states implies massivation and predictions of the p-adic thermodynamics for the fermionic masses are in excellent agreement with experimental masses. Thermodynamic approach also explains the emergence of the length scale L_p for a given p-adic condensation level and one can develop arguments explaining why primes near prime powers of two are favored.

It should be noticed that rational p-adic temperatures $1/T = k/n$ are possible, if one poses the restriction that thermal probabilities are non-vanishing only for some subalgebra of the Super Virasoro algebra isomorphic to the Super Virasoro algebra itself. The generators

L_{kn},G_{kn}, where k is a positive integer, indeed span this kind of a subalgebra by the fractality of the Super Virasoro algebra and $p^{L_0/T}$ is integer valued with this restriction.

One might apply thermodynamics approach should also in the calculation of S-matrix. What is is needed is thermodynamical expectation value for the transition amplitudes squared over incoming and outgoing states. In this expectation value 3-momenta are fixed and only mass squared varies.

3. Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = -\sum_n p_n log(p_n) \ . \qquad (11.2.22)$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

a) p-Adic entropies

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of x, where log_p denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$
\begin{aligned}
S_p &= \sum_n p_n k(p_n) \ , \\
k(p_n) &= -log_p(|p_n|) \ .
\end{aligned}
\tag{11.2.23}
$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$
S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n)log(p) + p^{k_n} log(p_n/p^{k_n}\right] \ . \tag{11.2.24}
$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$
\begin{aligned}
S_{p,R} &= (S_p)_R \times log(p)) \ , \\
(\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ .
\end{aligned}
\tag{11.2.25}
$$

The real counterpart of the p-adic entropy is non-negative.

b) Number theoretic entropies and metabolic energy

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|)log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$
I \equiv Max\{-S_p, \ p \text{ prime}\} \ . \tag{11.2.26}
$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP [K51] , and has a natural interpretation as a negentropic entanglement.

There is no need to interpret negentropic entanglement as bound state entanglement as was the original proposal. This together with the vision about life as something in the intersection of the real and p-adic worlds inspires the idea about a connection between information and metabolism in living matter. Metabolic energy could be carried by negentropic entanglement and the feed of metabolic energy would be also feed of negentropy. In particular, the poorly understood high energy phosphate bond could be identified as a bond involving negentropic entanglement [K26] . The prediction would be that the negentropic states of real systems form a number theoretical hierarchy according to the prime p and and dimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes in the intersection of real and p-adic worlds. p-Adic $\leftrightarrow$ real transitions make sense in the intersection with interpretation asa realization of intentional action and build-up of cognitive representations. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the U-matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

How to define integration and p-adic Fourier analysis, integral calculus, and p-adic counterparts of geometric objects?

p-Adic differential calculus exists and obeys essentially the same rules as ordinary differential calculus. The only difference from real context is the existence of p-adic pseudo constants: any function which depends on finite number of pinary digits has vanishing p-adic derivative. This implies non-determinism of p-adic differential equations. One can defined p-adic integral functions using the fact that indefinite integral is the inverse of differentiation. The basis problem with the definite integrals is that p-adic numbers are not well-ordered so that the crucial ordering of the points of real axis in definite integral is not unique. Also p-adic Fourier analysis is problematic since direct counterparts of ep(ix) and trigonometric functions are not periodic. Also exp(-x) fails to converse exponentially since it has p-adic norm equal to 1. Note also that these functions exists only when the p-adic norm of x is smaller than 1.

The following considerations support the view that the p-adic variant of a geometric objects, integration and p-adic Fourier analysis exists but only when one considers highly symmetric geometric objects such as symmetric spaces. This is wellcome news from the point of view of physics. At the level of space-time surfaces this is problematic. The field equations associated with Kähler action and modified Dirac equation make sense. Kähler action defined as integral over p-adic space-time surface fails to exist. If however the Kähler function identified as Kähler for a preferred extremal of Kähler action is rational or algebraic function of preferred complex coordinates of WCW with ratonal coefficients, its p-adic continuation is expected to exist.

1. Circle with rotational symmetries and its hyperbolic counterparts

Consider first circle with emphasis on symmetries and Fourier analysis.

(a) In this case angle coordinate ϕ is the natural coordinate. It however does not make sense as such p-adically and one must consider either trigonometric functions or the phase $exp(i\phi)$ instead. If one wants to do Fourier analysis on circle one must introduce roots $U_{n,N} = exp(in2\pi/N)$ of unity. This means discretization of the circle. Introducing all roots $U_{n,p} = exp(i2\pi n/p)$, such that p divides N, one can represent all $U_{k,n}$ up to $n = N$. Integration is naturally replaced with sum by using discrete Fourier analysis on circle. Note that the roots of unity can be expressed as products of powers of roots of unity $exp(in2\pi/p^k)$, where p^k divides N.

(b) There is a number theoretical delicacy involved. By Fermat's theorem $a^{p-1}\ mod\ p = 1$ for $a = 1, ...p - 1$ for a given p-adic prime so that for any integer M divisible by a factor of $p - 1$ the M:th roots of unity exist as ordinary p-adic numbers. The problem disappears if these values of M are excluded from the discretization for a given value of the p-adic prime. The manner to achieve this is to assume that N contains no divisors of $p-1$ and is consistent with the notion of finite measurement resolution. For instance, $N = p^n$ is an especially natural choice guaranteeing this.

(c) The p-adic integral defined as a Fourier sum does not reduce to a mere discretization of the real integral. In the real case the Fourier coefficients must approach to zero as the wave vector $k = n2\pi/N$ increases. In the p-adic case the condition consistent with the notion of finite measurement resolution for angles is that the p-adic valued Fourier coefficients approach to zero as n increases. This guarantees the p-adic convergence of the discrete approximation of the integral for large values of N as n increases. The map of p-adic Fourier coefficients to real ones by canonical identification could be used to relate p-adic and real variants of the function to each other.

This finding would suggests that p-adic geometries -in particular the p-adic counterpart of CP_2, are discrete. Variables which have the character of a radial coordinate are in natural manner p-adically continuous whereas phase angles are naturally discrete and described in terms of algebraic extensions. The conclusion is disappointing since one can quite well argue that the discrete structures can be regarded as real. Is there any manner to escape this conclusion?

(a) Exponential function $exp(ix)$ exists p-adically for $|x|_p \leq 1/p$ but is not periodic. It provides representation of p-adic variant of circle as group $U(1)$. One obtains actually a hierarchy of groups $U(1)_{p,n}$ corresponding to $|x|_p \leq 1/p^n$. One could consider a generalization of phases as products $Exp_p(N, n2\pi/N + x) = exp(in2\pi n/N)exp(ix)$ of roots of unity and exponent functions with an imaginary exponent. This would assign to each root of unity p-adic continuum interpreted as the analog of the interval between two subsequent roots of unity at circle. The hierarchies of measurement resolutions coming as $2\pi/p^n$ would be naturally accompanied by increasingly smaller p-adic groups $U(1)_{p,n}$.

(b) p-Adic integration would involve summation plus possibly also an integration over each p-adic variant of discretization interval. The summation over the roots of unity implies that the integral of $\int exp(inx)dx$ would appear for $n = 0$. Whatever the value of this integral is, it is compensated by a normalization factor guaranteeing orthonormality.

(c) If one interprets the p-adic coordinate as p-adic integer without the identification of points differing by a multiple of n as different points the question whether one should require p-adic continuity arises. Continuity is obtained if $U_n(x + mp^m) = U_n(x)$ for large values of m. This is obtained if one has $n = p^k$. In the spherical geometry this condition is not needed and would mean quantization of angular momentum as $L = p^k$, which does not look natural. If representations of translation group are considered the condition is natural and conforms with the spirit of the p-adic length scale hypothesis.

The hyperbolic counterpart of circle corresponds to the orbit of point under Lorentz group in two 2-D Minkowski space. Plane waves are replaced with exponentially decaying functions of the coordinate η replacing phase angle. Ordinary exponent function $exp(x)$ has unit p-adic norm when it exists so that it is not a suitable choice. The powers p^n existing for p-adic integers however approach to zero for large values of $x = n$. This forces discretization of η or rather the hyperbolic phase as powers of p^x, $x = n$. Also now one could introduce products of $Exp_p(nlog(p) + z) = p^n exp(x)$ to achieve a p-adic continuum. Also now the integral over the discretization interval is compensated by orthonormalization and can be forgotten. The integral of exponential function would reduce to a sum $\int Exp_p dx = \sum_k p^k = 1/(1-p)$. One can also introduce finite-dimensional but non-algebraic extensions of p-adic numbers allowing e and its roots $e^{1/n}$ since e^p exists p-adically.

2. Plane with translational and rotational symmetries

Consider first the situation by taking translational symmetries as a starting point. In this case Cartesian coordinates are natural and Fourier analysis based on plane waves is what one wants to define. As in the previous case, this can be done using roots of unity and one can also introduce p-adic continuum by using the p-adic variant of the exponent function. This would effectively reduce the plane to a box. As already noticed, in this case the quantization of wave vectors as multiples of $1/p^k$ is required by continuity.

One can take also rotational symmetries as a starting point. In this case cylindrical coordinates (ρ, ϕ) are natural.

(a) Radial coordinate can have arbitrary values. If one wants to keep the connection $\rho = \sqrt{x^2+y^2}$ with the Cartesian picture square root allowing extension is natural. Also the values of radial coordinate proportional to odd power of p are problematic since one should introduce $\sqrt{p}$: is this extension internally consistent? Does this mean that the points $\rho \propto p^{2n+1}$ are excluded so that the plane decomposes to annuli?

(b) As already found, angular momentum eigen states can be described in terms of roots of unity and one could obtain continuum by allowing also phases defined by p-adic exponent functions.

(c) In radial direction one should define the p-adic variants for the integrals of Bessel functions and they indeed might make sense by algebraic continuation if one consistently defines all functions as Fourier expansions. Delta-function renormalization causes technical problems for a continuum of radial wave vectors. One could avoid the problem by using exponentially decaying variants of Bessel function in the regions far from origin, and here the already proposed description of the hyperbolic counterparts of plane waves is suggestive.

(d) One could try to understand the situation also using Cartesian coordinates. In the case of sphere this is achieved by introducing two coordinate patches with Cartesian coordinates. Pythagorean phases are rational phases (orthogonal triangles for which all sides are integer valued) and form a dense set on circle. Complex rationals (orthogonal triangles with integer valued short sides) define a more general dense subset of circle. In both cases it is difficult to imagine a discretized version of integration over angles since discretization with constant angle incerement is not possible.

3. The case of sphere and more general symmetric space

In the case of sphere spherical coordinates are favored by symmetry considerations. For spherical coordinates $sin(\theta)$ is analogous to the radial coordinate of plane. Legendre polynomials expressible as polynomials of $sin(\theta)$ and $cos(\theta)$ are expressible in terms of phases and the integration measure $sin^2(\theta)d\theta d\phi$ reduces the integral of S^2 to summation. As before one can introduce also p-adic continuum. Algebraic cutoffs in both angular momentum l and m appear naturally. Similar cutoffs appear in the representations of quantum groups and there are good reasons to expect that these phenomena are correlated.

Exponent of Kähler function appears in the integration over WCW. From the expression of Kähler gauge potential given by $A_\alpha = J_\alpha{}^\theta \partial_\theta K$ one obtains using $A_\alpha = cos(\theta)\delta_{\alpha,\phi}$ and $J_{\theta\phi} = sin(\theta)$ the expression $exp(K) = sin(\theta)$. Hence the exponent of Kähler function is expressible in terms of spherical harmonics.

The completion of the discretized sphere to a p-adic continuum- and in fact any symmetric space- could be performed purely group theoretically.

(a) Exponential map maps the elements of the Lie-algebra to elements of Lie-group. This recipe generalizes to arbitrary symmetric space G/H by using the Cartan decomposition $g = t + h$, $[h,h] \subset h$,$[h,t] \subset t$,$[t,t] \subset h$. The exponentiation of t maps t to G/H in this case. The exponential map has a p-adic generalization obtained by considering Lie algebra with coefficients with p-adic norm smaller than one so that the p-adic exponent function exists. As a matter fact, one obtains a hierarchy of Lie-algebras corresponding to the upper bounds of the p-adic norm coming as p^{-k} and this hierarchy naturally corresponds to the hierarchy of angle resolutions coming as $2\pi/p^k$. By introducing finite-dimensional transcendental extensions containing roots of e one obtains also a hierarchy of p-adic Lie-algebras associated with transcendental extensions.

(b) In particular, one can exponentiate the complement of the $SO(2)$ sub-algebra of $SO(3)$ Lie-algebra in p-adic sense to obtain a p-adic completion of the discrete sphere. Each point of the discretized sphere would correspond to a p-adic continuous variant of sphere as a symmetric space. Similar construction applies in the case of CP_2. Quite generally, a kind of fractal or holographic symmetric space is obtained from a discrete variant of the symmetric space by replacing its points with the p-adic symmetric space.

(c) In the N-fold discretization of the coordinates of M-dimensional space t one $(N-1)^M$ discretization volumes which is the number of points with non-vanishing t-coordinates. It would be nice if one could map the p-adic discretization volumes with non-vanishing t-coordinates to their positive valued real counterparts by applying canonical identification. By group invariance it is enough to show that this works for a discretization volume assignable to the origin. Since the p-adic numbers with norm smaller than one are mapped to the real unit interval, the p-adic Lie algebra is mapped to the unit cell of the discretization lattice of the real variant of t. Hence by a proper normalization this mapping is possible.

The above considerations suggests that the hierarchies of measurement resolutions coming as $\Delta\phi = 2\pi/p^n$ are in a preferred role. One must be however cautious in order to avoid too strong assumptions. The above considerations suggest that the hierarchies of measurement resolutions coming as $\Delta\phi = 2\pi/p^n$ are in a preferred role. One must be however cautious in order to avoid too strong assumptions. The following arguments however support this identification.

(a) The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M unless one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. Otherwise the measurement resolution is just $\Delta\phi = 2\pi/p^n$. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.

(b) One can also interpret N as a p-adic integer and assume that state function reduction selects one particular prime (no superposition of quantum states with different p-adic topologies). For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M \mod p^n$. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$. One could assign to a given measurement resolution all the p-adic primes appearing as factors in N so that the notion of multi-p p-adicity would make sense. One can also consider the identification of the measurement resolution as $\Delta\phi = |N/M|_p = 2\pi/p^k$. This interpretation is supported by the approach based on infinite primes [K86] .

4. What about integrals over partonic 2-surfaces and space-time surface?

One can of course ask whether also the integrals over partonic 2-surfaces and space-time surface could be p-adicized by using the proposed method of discretization. Consider first the p-adic counterparts of the integrals over the partonic 2-surface X^2.

(a) WCW Hamiltonians and Kähler form are expressible using flux Hamiltonians defined in terms of X^2 integrals of JH_A, where H_A is $\delta CD \times CP_2$ Hamiltonian, which is a rational function of the preferred coordinates defined by the exponentials of the coordinates of the sub-space t in the appropriate Cartan algebra decomposition. The flux factor $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ is scalar and does not actually depend on the induced metric.

(b) The notion of finite measurement resolution would suggest that the discretization of X^2 is somehow induced by the discretization of $\delta CD \times CP_2$. The coordinates of X^2 could be taken to be the coordinates of the projection of X^2 to the sphere S^2 associated with $\delta M^4_\pm$ or to the homologically non-trivial geodesic sphere of CP_2 so that the discretization of the integral would reduce to that for S^2 and to a sum over points of S^2.

(c) To obtain an algebraic number as an outcome of the summation, one must pose additional conditions guaranteeing that both H_A and J are algebraic numbers at the points of discretization (recall that roots of unity are involved). Assume for definiteness that S^2 is $r_M = constant$ sphere. If the remaining preferred coordinates are functions of the preferred S^2 coordinates mapping phases to phases at discretization points, one obtains the desired outcome. These conditions are rather strong and mean that the various angles defining CP_2 coordinates -at least the two cyclic angle coordinates- are integer multiples of those assignable to S^2 at the points of discretization. This would be achieved if the preferred complex coordinates of CP_2 are powers of the preferred complex coordinate of S^2 at these points. One could say that X^2 is algebraically continued from a rational surface in the discretized variant of $\delta CD \times CP_2$. Furthermore, if the measurement resolutions come as $2\pi/p^n$ as p-adic continuity actually requires and if they correspond to the p-adic group $G_{p,n}$ for which group parameters satisfy $|t|_p \leq p^{-n}$, one can precisely characterize how a p-adic prime characterizes the real partonic 2-surface. This would be a fulfilment of one of the oldest dreams related to the p-adic vision.

A even more ambitious dream would be that even the integral of the Kähler action for preferred extremals could be defined using a similar procedure. The conjectured slicing of Minkowskian space-time sheets by string world sheets and partonic 2-surfaces encourages these hopes.

(a) One could introduce local coordinates of H at both ends of CD by introducing a continuous slicing of $M^4 \times CP_2$ by the translates of $\delta M^4_\pm \times CP_2$ in the direction of the time-like vector connecting the tips of CD. As space-time coordinates one could select four of the eight coordinates defining this slicing. For instance, for the regions of the space-time sheet representable as maps $M^4 \rightarrow CP_2$ one could use the preferred M^4 time coordinate, the radial coordinate of δM^4_+, and the angle coordinates of $r_M = constant$ sphere.

(b) Kähler action density should have algebraic values and this would require the strengthening of the proposed conditions for X^2 to apply to the entire slicing meaning that the discretized space-time surface is a rational surface in the discretized $CD \times CP_2$. If this condition applies to the entire space-time surface it would effectively mean the discretization of the classical physics to the level of finite geometries. This seems quite strong implication but is consistent with the preferred extremal property implying the generalized Bohr rules.

5. Tentative conclusions

These findings suggest following conclusions.

(a) Exponent functions play a key role in the proposed p-adicization. This is not an accident since exponent functions play a fundamental role in group theory and p-adic variants of real geometries exist only under symmetries- possibly maximal possible symmetries- since otherwise the notion of Fourier analysis making possible integration does not exist. The inner product defined in terms of integration reduce for functions representable in Fourier basis to sums and can be carried out by using orthogonality conditions. Convolution involving integration reduces to a product for Fourier components. In the case of imbedding space and WCW these conditions are satisfied but for space-time surfaces this is not possible.

(b) There are several manners to choose the Cartan algebra already in the case of sphere. In the case of plane one can consider either translations or rotations and this leads to different p-adic variants of plane. Also the realization of the hierarchy of Planck constants leads to the conclusion that the extended imbedding space and therefore also WCW contains sectors corresponding to different choices of quantization axes meaning that quantum measurement has a direct geometric correlate. One an imagine also other discretizations and choices of preferred coordinates and the interpretation is that they correspond to different cognitive representations and to to different p-adic physics. This means a refinement of General Coordinate Invariance taking into account cognition.

(c) The above described 2-D examples represent symplectic geometries for which one has natural decomposition of coordinates to canonical pairs of cyclic coordinate (phase angle) and corresponding canonical conjugate coordinate. p-Adicization depends on whether the conjugate corresponds to an angle or non-compact coordinate. In both cases it is however possible to define integration. For instance, in the case of CP_2 one would have two canonically conjugate pairs and one can define the p-adic counterparts of CP_2 partial waves by generalizing the procedure applied to spherical harmonics. Products of functions expressible using partial waves can be decomposed by tensor product decomposition to spherical harmonics and can be integrated. In particular inner products can be defined as integrals. The Hamiltonians generating isometries are rational functions of phases: this inspires the hope that also WCW Hamiltonians also rational functions of preferred WCW coordinates and thus allow p-adic variants.

(d) Discretization by introducing algebraic extensions seems unavoidable in the p-adicization of geometrical objects but one can have p-adic continuum as the analog of the discretization interval and in the function basis expressible in terms of phase factors and p-adic counterparts of exponent functions. As already described, the exponential map for Lie group provide an elegant manner to realize this. This would give a precise meaning for the p-adic counterparts of the imbedding space and WCW if the latter is a symmetric space allowing coordinatization in terms of phase angles and conjugate coordinates. The intersection of p-adic and real worlds in a given measurement resolution would be unique and correspond to the points defining the discretization.

p-Adic imbedding space

The construction of both quantum TGD and p-adic QFT limit requires p-adicization of the imbedding space geometry. Also the fact that p-adic Poincare invariance throws considerable light to the p-adic length scale hypothesis suggests that p-adic geometry is really needed. The construction of the p-adic version of the imbedding space geometry and spinor structure relies on the symmetry arguments and to the generalization of the analytic formulas of the real case almost. The essential element is the notion of finite measurement resolution leading to discretization in large and to p-adicization below the resolution scale. This approach leads to a highly nontrivial generalization of the symmetry concept and p-adic Poincare invariance throws light to the p-adic length scale hypothesis. An important delicacy is related to the identification of the fundamental p-adic length scale, which corresponds to the unit element of the p-adic number field and is mapped to the unit element of the real number field in the canonical identification mapping p-adic mass squared to its real counterpart.

1. p-Adic Riemannian geometry depends on cognitive representation

p-Adic Riemann geometry is a direct formal generalization of the ordinary Riemann geometry. In the minimal purely algebraic generalization one does not try to define concepts like arch length and volume involving definite integrals but simply defines the p-adic geometry via the metric identified as a quadratic form in the tangent space of the p-adic manifold. Canonical identification would make it possible to define p-adic variant of Riemann integral formally allowing to calculate arc lengths and similar quantities but looks like a trick. The realization that the p-adic variant of harmonic analysis makes it possible to define definite integrals in the case of symmetric space became possible only after a detailed vision about what quantum TGD is [K28] had emerged.

Symmetry considerations dictate the p-adic counterpart of the Riemann geometry for $M^4_+ \times CP_2$ to a high degree but not uniquely. This non-uniqueness might relate to the distinction between different cognitive representations. For instance, in the case of Euclidian plane one can introduce linear or cylindrical coordinates and the manifest symmetries dictating the preferred coordinates correspond to translational and rotational symmetries in these two cases and give rise to different p-adic variants of the plane. Both linear and cylindrical coordinates are fixed only modulo the action of group consisting of translations and rotations and the degeneracy of choices can be interpreted in terms of a choice of quantization axies of angular momentum and momenta.

The most natural looking manner to define the p-adic counterpart of M^4 is by using a p-adic completion for a subset of rational points in coordinates which are preferred on physical basis. In case of M^4 linear Minkowski coordinates are an obvious choice but also the counterparts of Robertson-Walker coordinates for M^4_+ defined as $[t,(z,x,y)] = a\times[cosh(\eta),sinh(\eta)(cos(\theta),sin(\theta)cos(\phi),sin(\theta)sin(\phi))]$ expressible in terms of phases and their hyperbolic counterparts and transforming nicely under the Cartan algebra of Lorentz group are possible. p-Adic variant is obtained by introducing finite measurement resolution for angle and replacing angle range by finite number of roots of unity. Same applies to hyperbolic angles.

Rational CP_2 could be defined as a coset space $SU(3,Q)/U(2,Q)$ associated with complex rational unitary 3×3-matrices. CP_2 could be defined as coset space of complex rational matrices by choosing one point in each coset $SU(3,Q)/U(2,Q)$ as a complex rational 3×3-matrix representable in terms of Pythagorean phases [A23] and performing a completion for the elements of this matrix by multiplying the elements with the p-adic exponentials $exp(iu)$, $|u|_p<1$ such that one obtains p-adically unitary matrix.

This option is not very natural as far as integration is considered. CP_2 however allows the analog of spherical coordinates for S^2 expressible in terms of angle variables alone and this suggests the introduction of the variant of CP_2 for which the coordinate values correspond to roots of unity. Completion would be performed in the same manner as for rational CP_2. This non-uniqueness need not be a drawback but could reflect the fact that the p-adic cognitive representation of real geometry are geometrically non-equivalent. This means a refinement of the principle of General Coordinate Invariance taking into account the fact that the cognitive representation of the real world affects the world with cognition included in a delicate manner.

2. The identification of the fundamental p-adic length scale

The fundamental p-adic length scale corresponds to the p-adic unit $e=1$ and is mapped to the unit of the real numbers in the canonical identification. The correct physical identification of the fundamental p-adic length scale is of crucial importance since the predictions of the theory for p-adic masses depend on the choice of this scale.

In TGD the 'radius' R of CP_2 is the fundamental length scale ($2\pi R$ is by definition the length of the CP_2 geodesics). In accordance with the idea that p-adic QFT limit makes sense only above length scales larger than the radius of CP_2 R is of same order of magnitude as the p-adic length scale defined as $l=\pi/m_0$, where m_0 is the fundamental mass scale and related to the 'cosmological constant' Λ ($R_{ij}=\Lambda s_{ij}$) of CP_2 by

$$m_0^2 = 2\Lambda \ . \tag{11.2.27}$$

The relationship between R and l is uniquely fixed:

$$R^2 = \frac{3}{m_0^3} = \frac{3}{2\Lambda} = \frac{3l^2}{\pi^2} \ . \tag{11.2.28}$$

Consider now the identification of the fundamental length scale.

(a) One must use R^2 or its integer multiple, rather than l^2, as the fundamental p-adic length scale squared in order to avoid the appearance of the p-adically ill defined π:s in various formulas of CP_2 geometry.

(b) The identification for the fundamental length scale as $1/m_0$ leads to difficulties.

i. The p-adic length for the CP_2 geodesic is proportional to $\sqrt{3}/m_0$. For the physically most interesting p-adic primes satisfying $p\ mod\ 4 = 3$ so that $\sqrt{-1}$ does not exist as an ordinary p-adic number, $\sqrt{3} = i\sqrt{-3}$ belongs to the complex extension of the p-adic numbers. Hence one has troubles in getting real length for the CP_2 geodesic.

ii. If m_0^2 is the fundamental mass squared scale then general quark states have mass squared, which is integer multiple of 1/3 rather than integer valued as in string models.

(c) These arguments suggest that the correct choice for the fundamental length scale is as $1/R$ so that $M^2 = 3/R^2$ appearing in the mass squared formulas is p-adically real and all values of the mass squared are integer multiples of $1/R^2$. This does not affect the real counterparts of the thermal expectation values of the mass squared in the lowest p-adic order but the effects, which are due to the modulo arithmetics, are seen in the higher order contributions to the mass squared. As a consequence, one must identify the p-adic length scale l as

$$l \equiv \pi R \ ,$$

rather than $l = \pi/m_0$. This is indeed a very natural identification. What is especially nice is that this identification also leads to a solution of some longstanding problems related to the p-adic mass calculations. It would be highly desirable to have the same p-adic temperature $T_p = 1$ for both the bosons and fermions rather than $T_p = 1/2$ for bosons and $T_p = 1$ for fermions. For instance, black hole elementary particle analogy as well as the need to get rid of light boson exotics suggests this strongly. It indeed turns out possible to achieve this with the proposed identification of the fundamental mass squared scale.

3. p-Adic counterpart of M^4_+

The construction of the p-adic counterpart of M^4_+ seems a relatively straightforward task and should reduce to the construction of the p-adic counterpart of the real axis with the standard metric. As already noticed, linear Minkowski coordinates are physically and mathematically preferred coordinates and it is natural to construct the metric in these coordinates.

There are some quite interesting delicacies related to the p-adic version of the Poincare invariance. Consider first translations. In order to have imaginary unit needed in the construction of the ordinary representations of the Poincare group one must have $p\ mod\ 4 = 3$ to guarantee that $\sqrt{-1}$ does not exist as an ordinary p-adic number. It however seems that the construction of the representations is at least formally possible by replacing imaginary unit with the square root of some other p-adic number not existing as a p-adic number.

It seems that only the discrete group of translations allows representations consisting of orthogonal plane waves. p-Adic plane waves can be defined in the lattice consisting of the multiples of $x_0 = m/n$ consisting of points with p-adic norm not larger that $|x_0|_p$ and the points $p^n x_0$ define fractally scaled-down versions of this set. In canonical identification these sets corresponds to volumes scaled by factors p^{-n}.

A physically interesting question is whether the Lorentz group should contain only the elements obtained by exponentiating the Lie-algebra generators of the Lorentz group or whether also large Lorentz transformations, containing as a subgroup the group of the rational Lorentz transformations, should be allowed. If the group contains only small Lorentz transformations, the quantization volume of M^4_+ (say the points with coordinates m^k having p-adic norm not larger than one) is also invariant under Lorentz transformations. This means that the quantization of the theory in the p-adic cube $|m^k| < p^n$ is a Poincare invariant procedure unlike in the real case.

The appearance of the square root of p, rather than the naively expected p, in the expression of the p-adic length scale can be undertood if the p-adic version of M^4 metric contains p as a scaling factor:

$$\begin{aligned} ds^2 &= pR^2 m_{kl} dm^k dm^l \ , \\ R &\leftrightarrow 1 \ , \end{aligned} \tag{11.2.29}$$

where m_{kl} is the standard M^4 metric $(1,-1,-1,-1)$. The p-adic distance function is obtained by integrating the line element using p-adic integral calculus and this gives for the distance along the k:th coordinate axis the expression

$$s = R\sqrt{p}m^k \ . \tag{11.2.30}$$

The map from p-adic M^4 to real M^4 is canonical identification plus a scaling determined from the requirement that the real counterpart of an infinitesimal p-adic geodesic segment is same as the length of the corresponding real geodesic segment:

$$m^k \rightarrow \pi(m^k)_R \ . \tag{11.2.31}$$

The p-adic distance along the k:th coordinate axis from the origin to the point $m^k = (p-1)(1+p+p^2+...) = -1$ on the boundary of the set of the p-adic numbers with norm not larger than one, corresponds to the fundamental p-adic length scale $L_p = \sqrt{p}l = \sqrt{p}\pi R$:

$$\sqrt{p}((p-1)(1+p+...))R \rightarrow \pi R\frac{(p-1)(1+p^{-1}+p^{-2}+...)}{\sqrt{p}} = L_p \ . \tag{11.2.32}$$

What is remarkable is that the shortest distance in the range $m^k = 1,..m-1$ is actually $L/\sqrt{p}$ rather than l so that p-adic numbers in range span the entire R_+ at the limit $p \rightarrow \infty$. Hence p-adic topology approaches real topology in the limit $p \rightarrow \infty$ in the sense that the length of the discretization step approaches to zero.

4. The two variants of CP_2

As noticed, CP_2 allows two variants based on rational discretization and on the discretization based on roots of unity. The root of unity option corresponds to the phases associated with $1/(1+r^2) = tan^2(u/2) = (1-cos(u))/(1+cos(u))$ and implies that integrals of spherical harmonics can be reduced to summations when angular resolution $\Delta u = 2\pi/N$ is introduced. In the p-adic context, one can replace distances with trigonometric functions of distances along zig zag curves connecting the points of the discretization. Physically this notion of distance is quite reasonable since distances are often measured using interferometer.

In the case of rtional variant of CP_2 one can proceed by defining the p-adic counterparts of $SU(3)$ and $U(2)$ and using the identification $CP_2 = SU(3)/U(2)$. The p-adic counterpart of $SU(3)$ consists of all 3×3 unitary matrices satisfying p-adic unitarity conditions (rows/colums are mutually orthogonal unit vectors) or its suitable subgroup: the minimal subgroup corresponds to the exponentials of the Lie-algebra generators. If one allows algebraic extensions of the p-adic numbers, one obtains several extensions of the group. The extension allowing the square root of a p-adically real number is the most interesting one in this respect since the general solution of the unitarity conditions involves square roots.

The subgroup of $SU(3)$ obtained by exponentiating the Lie-algebra generators of $SU(3)$ normalized so that their non-vanishing elements have unit p-adic norm, is of the form

$$SU(3)_0 = \{x = exp(\sum_k it_k X_k) \ ; \ |t_k|_p < 1\} = \{x = 1 + iy \ ; \ |y|_p < 1\} \ . \quad (11.2.33)$$

The diagonal elements of the matrices in this group are of the form $1 + O(p)$. In order $O(p)$ these matrices reduce to unit matrices.

Rational $SU(3)$ matrices do not in general allow a representation as an exponential. In the real case all $SU(3)$ matrices can be obtained from diagonalized matrices of the form

$$h = diag\{exp(i\phi_1), exp(i\phi_2), exp(exp(-i(\phi_1 + \phi_2)\} \ . \quad (11.2.34)$$

The exponentials are well defined provided that one has $|\phi_i|_p < 1$ and in this case the diagonal elements are of form $1+O(p)$. For $p \ mod \ 4 = 3$ one can however consider much more general diagonal matrices

$$h = diag\{z_1, z_2, z_3\} \ ,$$

for which the diagonal elements are rational complex numbers

$$z_i = \frac{(m_i + in_i)}{\sqrt{m_i^2 + n_i^2}} \ ,$$

satisfying $z_1 z_2 z_3 = 1$ such that the components of z_i are integers in the range $(0, p - 1)$ and the square roots appearing in the denominators exist as ordinary p-adic numbers. These matrices indeed form a group as is easy to see. By acting with $SU(3)_0$ to each element of this group and by applying all possible automorphisms $h \to ghg^{-1}$ using rational $SU(3)$ matrices one obtains entire $SU(3)$ as a union of an infinite number of disjoint components.

The simplest (unfortunately not physical) possibility is that the 'physical' $SU(3)$ corresponds to the connected component of $SU(3)$ represented by the matrices, which are unit matrices in order $O(p)$. In this case the construction of CP_2 is relatively straightforward and the real formalism should generalize as such. In particular, for $p \ mod \ 4 = 3$ it is possible to introduce complex coordinates ξ_1, ξ_2 using the complexification for the Lie-algebra complement of $su(2) \times u(1)$. The real counterparts of these coordinates vary in the range $[0, 1)$ and the end points correspond to the values of t_i equal to $t_i = 0$ and $t_i = -p$. The p-adic sphere S^2 appearing in the definition of the p-adic light cone is obtained as a geodesic sub-manifold of CP_2 ($\xi_1 = \xi_2$ is one possibility). From the requirement that real CP_2 can be mapped to its p-adic counterpart it is clear that one must allow all connected components of CP_2 obtained by applying discrete unitary matrices having no exponential representation to the basic connected component. In practice this corresponds to the allowance of all possible values of the p-adic norm for the components of the complex coordinates ξ_i of CP_2.

The simplest approach to the definition of the CP_2 metric is to replace the expression of the Kähler function in the real context with its p-adic counterpart. In standard complex coordinates for which the action of $U(2)$ subgroup is linear, the expression of the Kähler function reads as

$$K = log(1 + r^2) \ ,$$
$$r^2 = \sum_i \bar{\xi}_i \xi_i \ . \quad (11.2.35)$$

p-Adic logarithm exists provided r^2 is of order $O(p)$. This is the case when ξ_i is of order $O(p)$. The definition of the Kähler function in a more general case, when all possible values of the p-adic norm are allowed for r, is based on the introduction of a p-adic pseudo constant C to the argument of the Kähler function

$$K = log(\frac{1+r^2}{C}) \ .$$

C guarantees that the argument is of the form $\frac{1+r^2}{C} = 1+O(p)$ allowing a well-defined p-adic logarithm. This modification of the Kähler function leaves the definition of Kähler metric, Kähler form and spinor connection invariant.

A more elegant manner to avoid the difficulty is to use the exponent $\Omega = exp(K) = 1+r^2$ of the Kähler function instead of Kähler function, which indeed well defined for all coordinate values. In terms of Ω one can express the Kähler metric as

$$g_{k\bar{l}} = \frac{\partial_k \partial_{\bar{l}} \Omega}{\Omega} - \frac{\partial_k \Omega \partial_{\bar{l}} \Omega}{\Omega^2} \ . \quad (11.2.36)$$

The p-adic metric can be defined as

$$s_{i\bar{j}} \ = \ R^2 \partial_i \partial_{\bar{j}} K = R^2 \frac{(\delta_{i\bar{j}} r^2 - \bar{\xi}_i \xi_j)}{(1+r^2)^2} \ . \quad (11.2.37)$$

The expression for the Kähler form is the same as in the real case and the components of the Kähler form in the complex coordinates are numerically equal to those of the metric apart from the factor of i. The components in arbitrary coordinates can be deduced from these by the standard transformation formulas.

11.2.6 Quantum physics in the intersection of p-adic and real worlds

The p-adicization of quantum TGD means several challenges. One should define the notions of Riemann geometry and its variants such as Kähler geometry in the p-adic context. The notion of the p-adic space-time surface and its relationship to its real counterpart should be understood. Also the construction of Kähler geometry of "world of classical worlds" (WCW) in p-adic context should be carried out and the notion of WCW spinor fields should be defined in the p-adic context. The crucial technical problems relate to the notion of integral and Fourier analysis, which are the central elements of any physical theory. The basic challenge is to overcome the fact that although the field equations assignable to a given variational principle make sense p-adically, the action defined as an integral over arbitrary space-time surface has no natural p-adic counterpart as such in the generic case. What raises hopes that these challenges could be overcome is the symmetric space property of WCW and the idea of algebraic continuation. If WCW geometry is expressible in terms of rational functions with rational coefficients it allows a generalization to the p-adic context. Also integration can be reduced to Fourier analysis in the case of symmetric spaces. I have discussed the p-adicization and fusion of real and p-adic physics in earlier article [L14] and will not go to it here anymore. Suffice it to say that the notion of symmetric space allowing to algebraize the integration is central element of the approach.

The intersection of real and p-adic worlds is especially interesting as far as the physics of living system is considered in TGD framework and is discussed in this section.

What it means to be in the intersection of real and p-adic worlds?

The first question is what one really means when one speaks about a partonic 2-surface in the intersection of real and p-adic worlds or in the intersection of two p-adic worlds.

(a) Many algebraic numbers can be regarded also as ordinary p-adic numbers: square roots of roughly one half of integers provide a simple example about this. Should one assume that all algebraic numbers representable as ordinary p-adic numbers belong to the intersection of the real and p-adic variants of partonic 2-surface (or to the intersection of two different p-adic number fields)? Is there any hope that the listing of the points in the intersection is possible without a complete knowledge of the number theoretic anatomy of p-adic number fields in this kind of situation? And is the set of common algebraic points for real and p-adic variants of the partonic 2-surface X^2 quite too large- say a dense sub-set of X^2?

    This hopeless looking complexity is simplified considerably if one reduces the considerations to algebraic extensions of rationals since these induce the algebraic extensions of p-adic numbers. For instance, if the p-adic number field contains some n:th roots of integers in the range $(1, p-1)$ as ordinary p-adic numbers they are identified with their real counterparts. In principle one should be able to characterize the -probably infinite-dimensional- algebraic extension of rationals which is representable by a given p-adic number field as p-adic numbers of unit norm. This does not look very practical.

(b) At the level WCW one must direct the attention to the function spaces used to define partonic 2-surfaces. That is the spaces of rational functions or even algebraic functions with coefficients of polynomials in algebraic extensions of rational numbers making sense with arguments in all number fields so that algebraic extensions of rationals provide a neat hierarchy defining also the points of partonic 2-surfaces to be considered. If one considers only the algebraic points of X^2 belonging to the extension appearing in the definition the function space as common to various number fields one has good hopes that the number of common points is finite.

(c) Already the ratios of polynomials with rational coefficients lead to algebraic extensions of rationals via their roots. One can replace the coefficients of polynomials with numbers in algebraic extensions of rationals. Also algebraic functions involving roots of rational functions can be considered and force to introduce the algebraic extensions of p-adic numbers. For instance, an n:th root of a polynomial with rational coefficients is well defined if n:th roots of p-adic integers in the range $(1, p-1)$ are well well-defined. One clearly obtains an infinite hierarchy of function spaces. This would give rise to a natural hierarchy in which one introduces n:th roots for a minimum number of p-adic integers in the range $(1, p-1)$ in the range $1 \leq n \leq N$. Note that also the roots of unity would be introduced in a natural manner.

    The situation is made more complex because the partonic 2-surface is in general defined by the vanishing of six rational functions so that algebraic extensions are needed. An exception occurs when six preferred imbedding space coordinates are expressible as rational functions of the remaining two preferred coordinates. In this case the number of common rational points consists of all rational points associated with the remaining two coordinates. This situation is clearly non-generic. Usually the number of common points is much smaller (the set of rational points satisfying $x^n + y^n = z^n$ for $n > 2$ is a good example). This however suggests that these surfaces are of special importance since the naive expectation is that the amplitude for transformation of intention to action or its reversal is especially large in this case. This might also explain why these surfaces are easy to understand mathematically.

(d) These considerations suggest that the numbers common to reals and p-adics must be defined as rationals and algebraic numbers appearing explicitly in the algebraic extension or rationals associated with the function spaces used to define partonic 2-surfaces. This would make the deduction of the common points of partonic 2-surface a task possible at least in principle. Algebraic extensions of rationals rather than those of p-adic numbers would be in the fundamental role and induce the extensions of p-adic numbers.

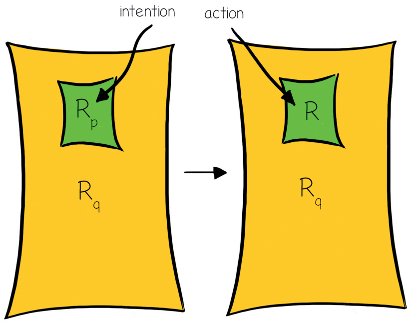

Figure 11.2: Transformation of intention to action as real-to-p-adc transformation and realization of intention as action as its reversal.

Braids and number theoretic braids

Braids -not necessary number theoretical- provide a realization discretization as a space-time correlate for the finite measurement resolution. The notion of braid was inspired by the idea about quantum TGD as almost topological quantum field theory. Although the original form of this idea has been buried, the notion of braid has survived: in the decomposition of space-time sheets to string world sheets, the ends of strings define representatives for braid strands at light-like 3-surfaces.

The notion of number theoretic universality inspired the much more restrictive notion of number theoretic braid requiring that the points in the intersection of the braid with the partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid or at least of the end points of the braid. The following consideration suggest that the number theoretic braids are not a useful notion in the generic case but make sense and are needed in the intersection of real and p-adic worlds which is in crucial role in TGD based vision about living matter [K51] .

It is only the braiding that matters in topological quantum field theories used to classify braids. Hence braid should require only the fixing of the end points of the braids at the intersection of the braid at the light-like boundaries of CDs and the braiding equivalence class of the braid itself. Therefore it is enough is to specify the topology of the braid and the end points of the braid in accordance with the attribute "number theoretic". Of course, the condition that all points of the strand of the number theoretic braid are algebraic is impossible to satisfy.

The situation in which the equations defining X^2 make sense both in real sense and p-adic sense using appropriate algebraic extension of p-adic number field is central in the TGD based vision about living matter [K51] . The reason is that in this case the notion of number theoretic entanglement entropy having negative values makes sense and entanglement becomes information carrying. This motivates the identification of life as something in the intersection of real and p-adic worlds. In this situation the identification of the ends of the number theoretic braid as points belonging to the intersection of real and p-adic worlds is natural. These points -call them briefly algebraic points- belong to the algebraic extension of rationals needed to define the algebraic extension of p-adic numbers. This definition however makes sense also when the equations defining the partonic 2-surfaces fail to make sense in both real and p-adic sense. In the generic case the set of points satisfying the conditions is discrete. For instance, according to Fermat's theorem the set of rational points satisfying $X^n + Y^n = Z^n$ reduces to the point $(0,0,0)$ for $n = 3,4,....$ Hence the constraint might be quite enough in the intersection of real and p-adic worlds where the choice of the algebraic extension is unique.

One can however criticize this proposal.

(a) One must fix the the number of points of the braid and outside the intersection and the non-uniqueness of the algebraic extension makes the situation problematic. Physical intuition suggests that the points of braid define carriers of quantum numbers assignable to second quantized induced spinor fields so that the total number of fermions anti-fermions would define the number of braids. In the intersection the highly non-trivial implication is that this number cannot exceed the number of algebraic points.

(b) In the generic case one expects that even the smallest deformation of the partonic 2-surface can change the number of algebraic points and also the character of the algebraic extension of rational numbers needed. The restriction to rational points is not expected to help in the generic case. If the notion of number theoretical braid is meant to be practical, must be able to decompose WCW to open sets inside which the numbers of algebraic points of braid at its ends are constant. For real topology this is expected to be impossible and it does not make sense to use p-adic topology for WCW whose points do not allow interpretation as p-adic partonic surfaces.

(c) In the intersection of real and p-adic worlds which corresponds to a discrete subset of WCW, the situation is different. Since the coefficients of polynomials involved with the definition of the partonic 2-surface must be rational or at most algebraic, continuous deformations are not possible so that one avoids the problem.

(d) This forces to ask the reason why for the number theoretic braids. In the generic case they seem to produce only troubles. In the intersection of real and p-adic worlds they could however allow the construction of the elements of M-matrix describing quantum transitions changing p-adic to real surfaces and vice versa as realizations of intentions and generation of cognitions. In this the case it is natural that only the data from the intersection of the two worlds are used. In [K51] I have sketched the idea about number theoretic quantum field theory as a description of intentional action and cognition.

There is also the the problem of fixing the interior points of the braid modulo deformations not affecting the topology of the braid.

(a) Infinite number of non-equivalent braidings are possible. Should one allow all possible braidings for a fixed light-like 3-surface and say that their existence is what makes the dynamics essentially three-dimensional even in the topological sense? In this case there would be no problems with the condition that the points at both ends of braid are algebraic.

(b) Or should one try to characterize the braiding uniquely for a given partonic 2-surfaces and corresponding 4-D tangent space distributions? The slicing of the space-time sheet by partonic 2-surfaces and string word sheets suggests that the ends of string world sheets could define the braid strands in the generic context when there is no algebraicity condition involved. This could be taken as a very natural manner to fix the topology of braid but leave the freedom to choose the representative for the braid. In the intersection of real and p-adic worlds there is no good reason for the end points of strands in this case to be algebraic at both ends of the string world sheet. One can however start from the braid defined by the end points of string world sheets, restrict the end points to be algebraic at the end with a smaller number of algebraic points and and then perform a topologically non-trivial deformation of the braid so that also the points at the other end are algebraic? Non-trivial deformations need not be possible for all possible choices of algebraic braid points at the other end of braid and different choices of the set of algebraic points would give rise to different braidings. A further constraint is that only the algebraic points at which one has assign fermion or anti-fermion are used so that the number of braid points is not always maximal.

(c) One can also ask whether one should perform the gauge fixing for the strands of the number theoretic braid using algebraic functions making sense both in real and p-adic context. This question does not seem terribly relevant since since it is only the topology of the braid that matters.

Number theoretical Quantum Mechanics

The vision about life as something in the intersection of the p-adic and real worlds requires a generalization of quantum theory to describe the U-process properly. One must answer several questions. What it means mathematically to be in this intersection? What the leakage between different sectors does mean? Is it really possible to formally extend quantum theory so that direct sums of Hilbert spaces in different number fields make sense? Or should one consider the possibility of using only complex, algebraic, or rational Hilbert spaces also in p-adic sectors so that p-adicization would take place only at the level of geometry?

1. What it means to be in the intersection of real and p-adic worlds?

The first question is what one really means when one speaks about a partonic 2-surface in the intersection of real and p-adic worlds or in the intersection of two p-adic worlds.

(a) Many algebraic numbers can be regarded also as ordinary p-adic numbers: square roots of roughly one half of integers provide a simple example about this. Should one assume that all algebraic numbers representable as ordinary p-adic numbers belong to the intersection of the real and p-adic variants of partonic 2-surface (or to the intersection of two different p-adic number fields)? Is there any hope that the listing of the points in the intersection is possible without a complete knowledge of the number theoretic anatomy of p-adic number fields in this kind of situation? And is the set of common algebraic points for real and p-adic variants of the partonic 2-surface X^2 quite too large- say a dense sub-set of X^2?

    This hopeless looking complexity is simplified considerably if one reduces the considerations to algebraic extensions of rationals since these induce the algebraic extensions of p-adic numbers. For instance, if the p-adic number field contains some n:th roots of integers in the range $(1, p-1)$ as ordinary p-adic numbers they are identified with their real counterparts. In principle one should be able to characterize the -probably infinite-dimensional- algebraic extension of rationals which is representable by a given p-adic number field as p-adic numbers of unit norm. This does not look very practical.

(b) At the level WCW one must direct the attention to the function spaces used to define partonic 2-surfaces. That is the spaces of rational functions or even algebraic functions with coefficients of polynomials in algebraic extensions of rational numbers making sense with arguments in all number fields so that algebraic extensions of rationals provide a neat hierarchy defining also the points of partonic 2-surfaces to be considered. If one considers only the algebraic points of X^2 belonging to the extension appearing in the definition the function space as common to various number fields one has good hopes that the number of common points is finite.

(c) Already the ratios of polynomials with rational coefficients lead to algebraic extensions of rationals via their roots. One can replace the coefficients of polynomials with numbers in algebraic extensions of rationals. Also algebraic functions involving roots of rational functions can be considered and force to introduce the algebraic extensions of p-adic numbers. For instance, an n:th root of a polynomial with rational coefficients is well defined if n:th roots of p-adic integers in the range $(1, p-1)$ are well well-defined. One clearly obtains an infinite hierarchy of function spaces. This would give rise to a natural hierarchy in which one introduces n:th roots for a minimum number of p-adic integers in the range $(1, p-1)$ in the range $1 \leq n \leq N$. Note that also the roots of unity would be introduced in a natural manner.

    The situation is made more complex because the partonic 2-surface is in general defined by the vanishing of six rational functions so that algebraic extensions are needed. An exception occurs when six preferred imbedding space coordinates are expressible as rational functions of the remaining two preferred coordinates. In this case the number of common rational points consists of all rational points associated with the remaining two coordinates. This situation is clearly non-generic. Usually the number of common points is much smaller (the set of rational points satisfying $x^n + y^n = z^n$ for $n > 2$ is a good example). This however suggests that these surfaces are of special importance since the naive expectation is that the amplitude for transformation of intention to action or its reversal is especially large in this case. This might also explain why these surfaces are easy to understand mathematically.

(d) These considerations suggest that the numbers common to reals and p-adics must be defined as rationals and algebraic numbers appearing explicitly in the algebraic extension or rationals associated with the function spaces used to define partonic 2-surfaces. This would make the deduction of the common points of partonic 2-surface a task possible at least in principle. Algebraic extensions of rationals rather than those of p-adic numbers would be in the fundamental role and induce the extensions of p-adic numbers.

Let us next try to summarize the geometrical picture at the level of WCW and WCW spinor fields.

(a) WCW decomposes into WCWs associated with CDs and there unions. For the unions one has Cartesian product of WCWs associated with CDs. At the level of WCW spinor fields one has tensor product.

(b) The WCW for a given CD decomposes into a union of sectors corresponding to various number fields and their algebraic extensions. The sub-WCW corresponding to the intersection consists of partonic 2-surfaces X^2 (plus distribution of 4-D tangent spaces $T(X^4)$ at X^2 - a complication which will not be considered in the sequel), whose mathematical representation makes sense in real number field and in some algebraic extensions of p-adic number fields. The extension of p-adic number fields needed for algebraic extension of rationals depends on p and is in general sub-extension of the extension of rationals. This sub-WCW is a sub-manifold of WCW itself. It has also a filtering by sub-manifolds of QCW. For instance, partonic 2-surfaces representable using ratios of polynomials with degree below fixed number N defines an inclusion hierarchy with levels labelled by N.

(c) The spaces of WCW spinors associated with these sectors are dictated by the second quantization of induced spinor fields with dynamics dictated by the modified Dirac action in more or less one-one correspondence. The dimension for the modes of induced spinor field (solutions of the modified Dirac equation at the space-time surface holographically assigned with X^2 plus the 4-D tangent space-space distribution) in general depends on the partonic 2-surface and the classical criticality of space-time surface suggests an inclusion hierarchy of super-conformal algebras corresponding to a hierarchy of criticalities. For instance, the partonic 2-surfaces X^2 having polynomial representations in referred coordinates could correspond to simplest possible surfaces nearest to the vacuum extremals and having in a well define sense smallest (but possibly infinite) dimension for the space of spinor modes.

(d) For each CD one can decompose the Hilbert space to a formal direct sum of orthogonal state spaces associated with various number fields

$$H = \oplus_F H_F \ . \tag{11.2.38}$$

Here F serves as a label for number fields. For the sake of simplicity and to get idea about what is involved, all complications due to algebraic extensions are neglected in the sequel so that only rational surfaces are regarded as being common to various sectors of WCW.

(e) The states in the direct sum make sense only formally since the formal inner product of these states would be a sum of numbers in different number fields unless one assigns complex Hilbert space with each sector or restricts the coefficients to be rational which is of course also possible. This problem is avoided if the state function reduction process induces inside each CD a choice of the number field. One could say that state function is a number theoretical necessity at least in this sense.

i. Should the state function reduction in this sense involve a reduction of entanglement between distinct CDs is not clear. One could indeed consider the possibility of a purely number theoretical reduction not induced by NMP and taking place in the absence of entanglement with reduction probabilities determined by the probabilities assignable to various number fields which should be rational or at most algebraic. Hard experience however suggests that one should not make exceptions from principles.

ii. The alternative is to allow the Hilbert spaces in question to have rational or at most algebraic coefficients in the intersection of real and various p-adic worlds. This means that the entanglement is algebraic and NMP need not lead to a pure state: the superposition of pairs of entangled states is however mathematically well defined since inner products give algebraic numbers. Cognitive entanglement stable under NMP would become possible. The experience of understanding could be a correlate for it. The pairs in the sum defining the entangled state defined the instances of a concept as a mapping of real world state to its symbol structurally analogous to a Boolean rule. The entangled states between different p-adic number fields would define maps between symbolic representations.

(f) Assume that each H_F allows a decomposition to a direct sum of two orthogonal parts corresponding to WCW spinor fields localized to the intersection of number fields and to the complements of the intersection:

$$\begin{aligned} H &= H_{nm} \oplus H_m \ , \\ H_{nm} &= \oplus_F H_{nm,F} \ , \ \ H_m = \oplus_F H_{m,F} \ . \end{aligned} \tag{11.2.39}$$

Here nm stands for 'no mixing' (no mixing between different number fields and localization to the complement of the intersection) and m for 'mixing' (mixing between different number fields in the intersection). F labels the number fields. Orthogonal direct sum might be mathematically rather singular and un-necessarily strong assumption but the notion of number theoretical criticality favors it.

2. The general structure of U-matrix neglecting the complexities due to algebraic extensions

M-matrix is diagonal with respect to the number field for obvious reasons. U-matrix can however induce a leakage between different number fields as well as entanglement between different number fields when unions of CDs are considered. The simplest assumption is that this entanglement is induced by the leakage between different number fields for single CD but not directly. For instance, the members of entangled pair of real states associated with two CDs leak to various p-adic sectors and induce in this manner entanglement beween different number fields. One must however notice that the part of U-matrix acting in the tensor product of Hilbert spaces assignable to separate CDs must be considered separately: it seems that the entanglement inducing part of U is diagonal with respect to number field except in the intersection.

To simplify the rather complex situation consider first the U matrix for a given CD by neglecting the possibility of algebraic extensions of the p-adic number fields. Restrict also the consideration to single CD.

(a) The unitarity conditions do not make sense in a completely general sense since one cannot add numbers belonging to different number fields. The problem can be circumvented if the U-matrix decomposes into a product of U-matrices, which both are such that unitarity conditions make sense for them. Here an essential assumption is that unit matrix and projection operators are number theoretically universal. In this spirit assume that for a given CD U decomposes to a product of two U-matrices U_{nm} inducing no mixing between different number fields and U_m inducing the mixing in the intersection:

$$U = U_{nm} U_m \ . \tag{11.2.40}$$

Here the subscript 'nm' (no mixing) having nothing to do with the induces of U as a matrix means that the action is restricted to a dispersion in a sector of WCW characterized by particular number field. The subscript 'm' (mixing) in turn means that the action corresponds to a leakage between different number fields possible in the intersection of worlds corresponding to different number fields and that U_m acts non-trivially in this intersection.

(b) Assume that U_{nm} decomposes into a formal direct sum of U-matrices associated with various number fields F:

$$U_{nm} = \oplus_F U_{nm,F} \ . \tag{11.2.41}$$

$U_{nm,F}$ acts inside H_F in both WCW and spin degrees of freedom, does not mix states belonging to different number fields, and creates a state which is always mathematically completely well defined in particular number field although the direct sum over number fields is only formally defined. Unitarity condition gives a direct sum of projection operators to Hilbert spaces associated with various number fields. One can assume that this object is number theoretically universal.

(c) U_m acts in the intersection of the real and p-adic worlds identified in the simplied picture in terms of surfaces representable using ratios of polynomials with rational coefficients. The resulting superposition of WCW spinor fields in different number fields is as such not mathematical sensible although the expression of U_m is mathematically well-defined. If the leakage takes place with same probability amplitude irrespective of the quantum state, U_m is a unitary operator, not affecting at all the spinor indices of WCW spinor fields characterizing quantum numbers of the state and whose action is analogous to unitary mixing of the identical copies of the state in various number fields.

The probability with which the intention is realized as action would not therefore depend at all on the quantum number fields, but only on the data at points common to the variants of the partonic 2-surface in various number fields. Intention would reduce completely to the algebraic geometry of partonic 2-surfaces. This assumption allows to write U in the form

$$U = U_{nm}U_m \ , \tag{11.2.42}$$

where U_m acts as an identity operator in H_{nm}.

3. The general structure of U-matrix when algebraic extensions of rationals are allowed

Consider now the generalization of the previous argument allowing also algebraic extensions.

(a) For each algebraic extension of rationals one can express WCW as a union of two parts. The first one corresponds to to 2-surfaces, which belong to the intersection of real and p-adic worlds. The second one corresponds to 2-surfaces in the algebraic extension of genuine p-adic numbers and having necessarily infinite size in real sense. Therefore the decomposition of U to a product $U = U_{nm}U_m$ makes sense also now.

(b) It is natural to assume that U_m decomposes to a product of two operators: $U_m = U_H U_Q$. The strictly horizontal operator U_H connects only same algebraic extensions of rationals assigned to different number fields. Here one must think that p-adic number fields represent a large number of algebraic extensions of rationals without need for an algebraic extension in the p-adic sense. The second unitary operator U_Q describes the leakage between different algebraic extensions of rationals. Number theoretical universality encourages the assumption that this unitary operator reduces to an operator U_Q acting on algebraic extensions of rationals regarded effectively as quantum states so that it would be same for all number fields. One can even consider the possibility that U_Q depends on the extensions of rationals only and not at all on partonic 2-surfaces. One cannot assume that U_Q corresponds just to an inclusion to a larger state space since this would give an infinite number of identical copies of same state and imply a non-normalizable state. Physically U_Q would define dispersion in the space of algebraic extension of rationals defining the rational function space giving rise to the sub-WCW. The simplest possibility is that U_Q between different algebraic extensions is just the projection operator to their intersection multiplied by a numerical constant determined number theoretical in terms of ratios of dimensions of the algebraic extensions so that the diffusion between extensions products unit norm states.

One must take into account the consistency conditions from the web of inclusions for the algebraic extensions of rationals inducing extensions of p-adic numbers.

(a) There is an infinite inverted pyramide-like web of natural inclusions of WCWs associated with algebraic extensions of rational numbers and one can assign a copy of this web to all number fields if a given p-adic number field is characterized by a web defined by algebraic extensions of rationals numbers, which it is able to represent without explicit introduction of the algebraic extension, so that the pyramide is same for all number fields. For instance, the WCW corresponding to p-adic numbers proper is included to the WCWs associated with any of its genuine algebraic extensions and defines the lower tip of the inverted pyramide. From this tip an arrow emerges connecting it to every algebraic extension defining a node of this web. Besides these arrows there are arrows from a given extension to all extensions containing it.

(b) These geometric inclusions induce inclusions of the corresponding Hilbert spaces defined by rational functions and possibly by algebraic functions in which case sub-web must be considered (all n:th roots of integers in the range $(1, p-1)$ must be introduced simultaneously). Leakage can occur between different extensions only through WCW spinor fields located in the common intersection of these spaces containing always the rational surfaces. The intersections of WCWs associated with various extensions of p-adic number fields correspond to WCWs assignable to rational functions with coefficients in various algebraic extensions of rationals using preferred coordinates of CD and CP_2.

Together with unitarity conditions this web poses strong constraints on the unitary matrices U_m and U_Q expressible conveniently in terms of commuting diagrams. There are two kinds of webs. The vertical webs are defined by the algebraic extensions of rationals. These form a larger web in which lines connect the nodes of identical webs associated with various p-adic number fields and represent algebraic extensions of rationals.

(a) One has the general product decomposition $U = U_{nm}U_QU_m$, where U_{nm} does not induce mixing between number fields, and U_m does it purely horizontally but without affecting quantum states in WCW spin degrees of freedom, and $P(H_{nm})$ projects to the complement of the intersection of number fields holds true also now.

(b) Each algebraic extension of rationals gives unitary conditions for the corresponding $U_{nm,F}$ for each p-adic number field with extensions included. These conditions are relatively simple and no commuting diagrams are needed.

(c) In the horizontal web U_m mixes the states in the intersections of two number fields but connects only same algebraic extensions so that the lines are strictly horizontal. U_Q acts strictly vertically in the web formed by algebraic extension of rationals and its action is unitary. One has infinite number of commuting diagrams involving U_m and U_Q since the actions along all routes connecting given points between p_1 and p_2 must be identical.

(d) If algebraic universality holds in the sense that U_m is expressible using only the data about the common points of 2-surfaces in the intersection defined by particular extensions using some universal functions, and U_Q is purely number theoretical unitary matrix having no dependence on partonic 2-surfaces, one can hope that the constraints due to commuting diagrams in the web of horizontal inclusions can be satisfied automatically and only the unitarity constraints remain. This web of inclusions brings strongly in mind the web of inclusions of hyper-finite factors.

11.3 TGD and classical number fields

This section is devoted to the vision about TGD as a generalized number theory. The basic theme is the role of classical number fields [A19, A7, A26] in quantum TGD. A central notion is M^8-H duality which might be also called number theoretic compactification. This duality allows to identify imbedding space equivalently either as M^8 or $M^4\times CP_2$ and explains the symmetries of standard model number theoretically. These number theoretical symmetries induce also the symmetries dictaying the geometry of the "world of classical worlds" (WCW) as a union of symmetric spaces [A31] . This infinite-dimensional Kähler geometry is expected to be highly unique from the mere requirement of its existence requiring infinite-dimensional symmetries provided by the generalized conformal symmetries of the light-cone boundary $\delta M^4_+\times S$ and of light-like 3-surfaces and the answer to the question what makes 8-D imbedding space and $S=CP_2$ so unique would be the reduction of these symmetries to number theory.

Zero energy ontology has become the corner stone of also number theoretical vision. In zero energy ontology either light-like or space-like 3-surfaces can be identified as the fundamental dynamical objects, and the extension of general coordinate invariance leads to effective 2-dimensionality (strong form of holography) in the sense that the data associated with partonic 2-surfaces and the distribution of 4-D tangent spaces at them located at the light-like boundaries of causal diamonds (CDs) defined as intersections of future and past directed light-cones code for quantum physics and the geometry of WCW. Also the hierarchy of Planck constants [K27] plays a role but not so important one.

The basic number theoretical structures are complex numbers, quaternions [A26] and octonions [A19] , and their complexifications obtained by introducing additional commuting imaginary unit $\sqrt{-1}$. Hyper-octonionic (-quaternionic,-complex) sub-spaces for which octonionic imaginary units are multiplied by commuting $\sqrt{-1}$ have naturally Minkowskian signature of metric. The question is whether and how the hyper-structures could allow to understand quantum TGD in terms of classical number fields. The answer which looks the most convincing one relies on the existence of octonionic representation of 8-D gamma matrix algebra.

(a) The first guess is that associativity condition for the sub-algebras of the local Clifford algebra defined in this manner could select 4-D surfaces as associative (hyper-quaternionic) sub-spaces of this algebra and define WCW purely number theoretically. The associative sub-spaces in question would be spanned by the modified gamma matrices defined by the modified Dirac action fixed by the variational principle (Kähler action) selecting space-time surfaces as preferred extremals [K28] .

(b) This condition is quite not enough: one must strengthen it with the condition that a preferred commutative and thus hyper-complex sub-algebra is contained in the tangent space of the space-time surface. This condition actually generalizes somewhat since one can introduce a family of so called Hamilton-Jacobi coordinates for M^4 allowing an integrable distribution of decompositions of tangent space to the space of non-physical and physical polarizations [K9] . The physical interpretation is as a number theoretic realization of gauge invariance selecting a preferred local commutative plane of non-physical polarizations.

(c) Even this is not yet the whole story: one can define also the notions of co-associativity and co-commutativity applying in the regions of space-time surface with Euclidian signature of the induced metric. The basic unproven conjecture is that the decomposition of space-time surfaces to associative and co-associative regions containing preferred commutative *resp.* co-commutative 2-plane in the 4-D tangent plane is equivalent with the preferred extremal property of Kähler action and the hypothesis that space-time surface allows a slicing by string world sheets and by partonic 2-surfaces [K28] .

11.3.1 Notations

Some notational conventions are in order before continuing. The fields of quaternions *resp.* octonions having dimension 4 *resp.* 8 and will be denoted by Q and O. Their complexified variants will be denoted by Q_C and O_C. The sub-spaces of hyper-quaternions HQ and hyper-octonions HO are obtained by multiplying the quaternionic and octonionic imaginary units by $\sqrt{-1}$. These sub-spaces are very intimately related with the corresponding algebras, and can be seen as Euclidian and Minkowkian variants of the same basic structure. Also the Abelianized versions of the hyper-quaternionic and -octonionic sub-spaces can be considered: these algebras have a representation in the space of spinors of imbedding space $H = M^4 \times CP_2$.

11.3.2 Quaternion and octonion structures and their hyper counterparts

In this introductory section the notions of quaternion and octonion structures and their hyper counterparts are introduced with strong emphasis on the physical interpretation. Literature contains several variants of these structures (Hyper-Kähler structure [A11] and quaternion Kähler structure possed also by CP_2 [A44]). The notion introduced here is inspired by the physical motivations coming from TGD. As usual the first proposal based on the notions of (hyper-)quaternion and (hyper-)octonion analyticity was not the correct one. Much later a local variant of the notion based on tangent space emerged.

Octonions and quaternions

In the following only the basic definitions relating to octonions and quaterions are given. There is an excellent article by John Baez [A19] describing octonions and their relations to the rest of mathematics and physics.

Octonions can be expressed as real linear combinations $\sum_k x^k I_k$ of the octonionic real unit $I_0 = 1$ (counterpart of the unit matrix) and imaginary units I_a, $a = 1, ..., 7$ satisfying

$$\begin{aligned} I_0^2 &= I_0 \equiv 1 \ , \\ I_a^2 &= -I_0 = -1 \ , \\ I_0 I_a &= I_a \ . \end{aligned} \quad (11.3.1)$$

Octonions are closed with respect to the ordinary sum of the 8-dimensional vector space and with respect to the octonionic multiplication, which is neither commutative ($ab \neq ba$ in general) nor associative ($a(bc) \neq (ab)c$ in general).

A concise manner to summarize octonionic multiplication is by using octonionic triangle. Each line (6 altogether) containing 3 octonionic imaginary units forms an associative triple which together with $I_0 = 1$ generate a division algebra of quaternions. Also the circle spanned by the 3 imaginary units at the middle of the sides of the triangle is associative triple. The multiplication rules for each associative triple are simple:

$$I_a I_b = \epsilon_{abc} I_c \ , \quad (11.3.2)$$

where ϵ_{abc} is 3-dimensional permutation symbol. $\epsilon_{abc} = 1$ for the clockwise sequence of vertices (the direction of the arrow along the circumference of the triangle and circle). As a special case this rule gives the multiplication table of quaternions. A crucial observation for what follows is that any pair of imaginary units belongs to one associative triple.

The non-vanishing structure constants $d_{ab}{}^c$ of the octonionic algebra can be read directly from the octonionic triangle. For a given pair I_a, I_b one has

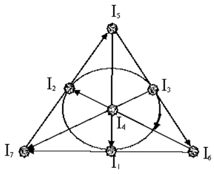

Figure 11.3: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

$$\begin{aligned}
I_aI_b &= d_{ab}{}^{c}I_c \ , \\
d_{ab} \quad c &= \epsilon_{ab}{}^{c} \ , \\
I_a^2 &= d_{aa}{}^{0}I_0 = -I_0 \ , \\
I_0^2 &= d_{00}{}^{0}I_0 \ , \\
I_0I_a &= d_{0a}{}^{a}I_a = I_a \ .
\end{aligned} \tag{11.3.3}$$

For ϵ_{abc} c belongs to the same associative triple as ab.

Non-associativity means that is not possible to represent octonions as matrices since matrix product is associative. Quaternions can be represented and the structure constants provide the defining representation as $I_a \rightarrow d_{abc}$, where b and c are regarded as matrix indices of 4×4 matrix. The algebra automorphisms of octonions form 14-dimensional group G_2, one of the so called exceptional Lie-groups. The isotropy group of imaginary octonion unit is the group $SU(3)$. The Euclidian inner product of the two octonions is defined as the real part of the product $\overline{x}y$

$$\begin{aligned}
(x,y) &= Re(\overline{x}y) = \sum_{k=0,..7} x_ky_k \ , \\
\overline{x} &= x^0I_0 - \sum_{i=1,..,7} x^kI_k \ ,
\end{aligned} \tag{11.3.4}$$

and is just the Euclidian norm of the 8-dimensional space.

Hyper-octonions and hyper-quaternions

The Euclidicity of the quaternion norm suggests that octonions are not a sensible concept in TGD context. One can imagine two manners to circumvent this conclusion.

(a) Minkowskian metric for octonions and quaternions is obtained by identifying Minkowski inner product xy as the real counterpart of the product

$$x \cdot y \equiv Re(xy) = x^0y^0 - \sum_k x^ky^k \quad . \tag{11.3.5}$$

$SO(1,7)$ ($SO(1,3)$ in quaternionic case) Lorentz invariance appears completely naturally as the symmetry of the real part of the octonion (quaternion) product and hence of octonions/quaternions and there is no need to perform the complexification of the octonion algebra. Furthermore, only the signature $(1,7)$ ($(1,3)$ in the quaternionic case) is possible and this would raise $M^4_+ \times CP_2$ in a preferred position.

This norm does not give rise to a number theoretic norm defining a homomorphism to real numbers. Indeed, the number theoretic norm defined by the determinant of the linear mapping defined by the multiplication with quaternion or octonion, is inherently Euclidian. This is in conflict with the idea that quaternionic and octonionic primes and their infinite variants should have key role in TGD [K86] .

(b) Hyper-octonions and hyper-quaternions provide a possible solution to these problems. These are obtained by multiplying imaginary units by commutative and associative $\sqrt{-1}$. These numbers form a sub-space of complexified octonions/quaternions and the cross product of imaginary parts leads out from this sub-space. In this case number theoretic norm induced from Q_C/O_C gives the fourth/eighth power of Minkowski length and Lorentz group acts as its symmetries. Light-like hyper-quaternions and -octonions causing the failure of the number field property have also a clear physical interpretation.

A criticism against the notion of hyper-quaternionic and octonionic primeness is that the tangent space as an algebra property is lost and the notion of primeness is inherited from Q_C/O_C. Also non-commutativity and non-associativity could cause difficulties.

Zero energy ontology leads to a possible physical interpretation of complexified octonions. The moduli space for causal diamonds corresponds to a Cartesian product of $M^4 \times CP_2$ whose points label the position of either tip of $CD \times CP_2$ and space I whose points label the relative positive of the second tip with respect to the first one. p-Adic length scale hypothesis results if one assumes that the proper time distance between the tips comes in powers of two so that one has union of hyperboloids $H_n \times CP_2$, $H_n = \{m \in M^4_+ | a = 2^n a_0)\}$. A further quantization of hyperboloids H_n is obtained by replacing it with a lattice like structure is highly suggestive and would correspond to an orbit of a point of H_n under a subgroup of $SL(2, Q_C)$ or $SL(2, Z_C)$ acting as Lorentz transformations in standard manner. Also algebraic extensions of Q_C and Z_C can be considered. Also in the case of CP_2 discretization is highly suggestive so that one would have an orbit of a point of CP_2 under a discrete subgroup of $SU(3, Q)$.

The outcome could be interpreted by saying that the moduli space in question is $H \times I$ such that H corresponds to hyper-octonions and I to a discretized version of $\sqrt{-1}H$ and thus a subspace of complexified octonions. An open question whether the quantization has some deeper mathematical meaning.

Basic constraints

Before going to details it is useful to make clear the constraints on the concept of the hyper-octonionic structure implied by TGD view about physics.

$M^4 \times CP_2$ cannot certainly be regarded as having any global octonionic structure (for instance in the sense that it could be regarded as a coset space associated with some exceptional group). There are however clear indications for the importance of the hyper-quaternionic and -octonionic structures.

(a) $SU(3)$ is the only simple 8-dimensional Lie-group and acts as the group of isometries of CP_2: if $SU(3)$ had some kind of octonionic structure, CP_2 would become unique candidate for the space S. The decomposition $SU(3) = h + t$ to $U(2)$ subalgebra and its complement corresponds rather closely to the decomposition of (hyper-)octonions to (hyper-)quaternionic sub-space and its complement. The electro-weak $U(2)$ algebra has a natural 1+3 decomposition and generators allow natural hyper-quaternionic structure. The components of the Weyl tensor of CP_2 behave with respect to multiplication like quaternionic imaginary units but only one of them is covariantly constant so that hyper Kähler structure [A11] with three covariantly constant quaternionic imaginary units represented by Kähler forms is not possible. These tensors and metric tensor however define quaternionic structure [A44] .

(b) M^4_+ has a natural 1+3 decomposition and a unique cosmic time coordinate defined as the light cone proper time. Hyper-quaternionic structure is consistent with the Minkowskian signature of the inner product and hyper quaternion units have a natural representation in terms of covariantly constant self-dual symplectic forms [A60, A33, A32] and their contractions with sigma matrices. It is not however clear whether this representation is physically interesting.

How to define hyper-quaternionic and hyper-octonionic structures?

I have considered several proposals for how to define quaternionic and octonionic structures and their hyper-counterparts.

(a) (Hyper-)octonionic manifolds would obtained by gluing together coordinate patches using (hyper-)octonion analytic functions with real Laurent coefficients (this guarantees associativity and commutativity). This definition does not yet involve metric or any other structures (such as Kähler structure). This approach does not seem to be physically realistic.

(b) Second option is based on the idea of representing quaternionic and octonionic imaginary units as antisymmetric tensors. This option makes sense for quaternionic manifolds [A25] and CP_2 indeed represents an example represents of this kind of manifold. The problem with the octonionic structure is that antisymmetric tensors cannot define non-associative product.

(c) If the manifold is endowed with metric, octonionic structure should be defined as a local tangent space structure analogous to eight-bein structure and local gauge algebra structures. This can be achieved by contracting octo-bein vectors with the standard octonionic basis to get octonion form I_k. Each vector field a^k defines naturally octonion field $A = a^k I_k$. The product of two vector fields can be defined by the octonionic multiplication and this leads to the introduction of a tensor field d_{klm} of these structure constants obtained as the contraction of the octobein vectors with the octonionic structure constants d_{abc}. Hyper-octonion structure can defined in a completely analogous manner.

It is possible to induce octonionic structure to any 4-dimensional space-time surface by forming the projection of I_k to the space-time surface and redefining the products of I_k:s by dropping away that part of the product, which is orthogonal to the space-time surface. This means that the structure constants of the new 4-dimensional algebra are the projections of d_{klm} to the space-time surface. One can also define similar induced algebra in the 4-dimensional normal space of the space-time surface. The hypothesis would be that the induced tangential is associative or hyper-quaternionic algebra. Also co-associativity defined as associativity of the normal space algebra is possible. This property would give for the 4-dimensionality of the space-time surface quite special algebraic meaning. The problem is now that there is no direct connection with quantum

TGD proper- in particular the connection with the classical dynamics defined by Kähler action is lacking.

(d) 8-dimensional gamma matrices allow a representation in terms of tensor products of octonions and 2×2 matrices. Genuine matrices are of course not in question since the product of the gamma matrices fails to be associative. An associative representation is obtained by restricting the matrices to a quaternionic plane of complex octonions. If the space-time surface is hyper-quaternionic in the sense that induced gamma matrices define a quaternionic plane of complexified octonions at each point of space-time surface the resulting local Clifford algebra is associative and structure constants define a matrix representation for the induced gamma matrices.

A more general definition allows gamma matrices to be modified gamma matrices defined by Kähler action appearing in the modified Dirac action and forced both by internal consistency and super-conformal symmetry [K17, K28] . The modified gamma matrices associated with Kähler action do not in general define tangent space of the space-time surface as the induced gamma matrices do. Also co-associativity can be considered if one can identify a preferred imaginary unit such that the multiplication of the modified gamma matrices with this unit gives a quaternionic basis. This condition makes sense only if the preferred extremals of the action are hyper-quaternionic surfaces in the sense defined by the action. That this is true for Kähler action at least is an is an unproven conjecture.

In the sequel only the fourh option will be considered.

How to end up to quantum TGD from number theory?

An interesting possibility is that quantum TGD could emerge from a condition that a local version of hyper-finite factor of type II_1 represented as a local version of infinite-dimensional Clifford algebra exists. The conditions are that "center or mass" degrees of freedom characterizing the position of CD separate uniquely from the "vibrational" degrees of freedom being represented in terms of octonions and that for physical states associativity holds true. The resulting local Clifford algebra would be identifiable as the local Clifford algebra of WCW (being an analog of local gauge groups and conformal fields [A27]).

The uniqueness of M^8 and $M^4 \times CP_2$ as well as the role of hyper-quaternionic space-time surfaces as fundamental dynamical objects indeed follow from rather weak conditions if one restricts the consideration to gamma matrices and spinors instead of assuming that M^8 coordinates are hyper-octonionic as was done in the first attempts.

(a) The unique feature of M^8 and any 8-dimensional space with Minkowski signature of metric is that it is possible to have an octonionic representation of the complexified gamma matrices [K17, K21] and of spinors. This does not require octonionic coordinates for M^8. The restriction to a quaternionic plane for both gamma matrices and spinors guarantees the associativity.

(b) One can also consider a local variant of the octonionic Clifford algebra in M^8. This algebra contains associative subalgebras for which one can assign to each point of M^8 a hyper-quaternionic plane. It is natural to assume that this plane is either a tangent plane of 4-D manifold defined naturally by the induced gamma matrices defining a basis of tangent space or more generally, by modified gamma matrices defined by a variational principle (these gamma matrices do not define tangent space in general). Kähler action defines a unique candidate for the variational principle in question. Associativity condition would automatically select sub-algebras associated with 4-D hyper-quaternionic space-time surfaces.

(c) This vision bears a very concrete connection to quantum TGD. In [K21] the octonionic formulation of the modified Dirac equation is studied and shown to lead to a highly unique general solution ansatz for the equation working also for the matrix representation of the Clifford algebra. An open question is whether the resulting solution as such defined also solutions of the modified Dirac equation for the matrix representation of gammas. Also a possible identification for 8-dimensional counterparts of twistors as octo-twistors follows: associativity implies that these twistors are very closely related to the ordinary twistors. In TGD framework octo-twistors provide an attractive manner to get rid of the difficulties posed by massive particles for the ordinary twistor formalism.

(d) Associativity implies hyperquaternionic space-time surfaces (in a more general sense as usual) and this leads naturally to the notion of WCW and local Clifford algebra in this space. Number theoretic arguments imply $M^8 - H$ duality. The resulting infinite-dimensional Clifford algebra would differ from von Neumann algebras in that the Clifford algebra and spinors assignable to the center of mass degrees of freedom of causal diamond CD would be expressed in terms of octonionic units although they are associative at space-time surfaces. One can therefore say that quantum TGD follows by assuming that the tangent space of the imbedding space corresponds to a classical number field with maximal dimension.

(e) The slicing of the Minkowskian space-time surface inside CD by stringy world sheets and by partonic 2-surfaces inspires the question whether the modified gamma matrices associated with the stringy world sheets *resp.* partonic 2-surfaces could be could commutative *resp.* co-commutative. Commutativity would also be seen as the justification for why the fundamental objects are effectively 2-dimensional.

This formulation is undeniably the most convincing one found hitherto since the notion of hyper-quaternionic structure is local and has elegant formulation in terms of modified gamma matrices.

11.3.3 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (or even its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely.

The basic ideas in nutshell

The vision about the physical role of the classical number fields relies on certain speculative questions and ideas.

(a) Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative) sub-space of octonions at each point of X^4 [K88]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative surfaces of H. Signature of M^8 could be a problem in M^8: M^8 can be regarded as linear sub-space of complexified octonions and the product of M^8 points does not belong to M^8. For tangent space this is not the case since one can complexify tangent space.

(b) Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K88]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

(c) $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely.

(d) G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

(e) One can also ask whether the notions of commutativity and co-commutavity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

One can go even further and ask whether one could somehow construct the preferred extremals of Kähler action using real-octonion analytic functions, call them generically f. For some time I believed to this idea but it seems I was wrong. The fact that octonion real-analytic functions in M^8 section of M^8_c have values in the space of complexified octonions makes the complexification of octonions necessary. The simplest guess would be that quaternionic 4-surfaces correspond to the loci at which the values of function f are real quaternionic. One clearly obtains quaternionic planes as trivial solutions but it is not clear whether their inverse images in general case are quaternionic surfaces and whether non-trivial surfaces with physical properties are obtained. In complex case Riemann zeta serves as a discouraging much simpler analogy since real sug-manifolds of complex plane are just pieces of real axis. Quaternionicity would be replaced with reality and the loci of zeros of the imaginary part of function should be pieces of real axes. Zeta is real at real axis and also at the line $Im(s) = 1/2$ but the inverse image of this line is not real line. Therefore this approach does not look promising.

Is Kähler action needed also at the level of M^8

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric.

Could they have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to (hyper-)quaterniocity at the level of tangent space and co-associativity to co(-hyper)-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kḧler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ making possible to identify M^4 point in $M^8 - H$ duality uniquely. It however turns out that M^4 point corresponds naturally to a projection of M^8 point to the quaternionic tangent space.

Definition of complexified octonions and quaternions

The Minkowskian signatures of M^8 and M^4 produce technical nuisance if one tries to define octonion-real- analyticity. One might try to overcome it by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides another approach giving hopes. Complexified tangent space must be introduced in any case so that its detailed definition deserves to discussed.

(a) The proper formulation for tangent space is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

(b) Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however define the notion of associativity for the sub-space of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

(c) Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$.

(d) One can consider two manners to identify the tangent space of H. Either as 8-D manifold for which tangent space is hyper-octonionic linear sub-space of complexified octonions O_c generated by sums and products of tangent vectors. Tangent space vectors of H could be also identified as hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units. This would imply an asymmetry between M^8 and H. The first option looks more elegant also because the composition of the duality maps can be iterated as maps of surfaces of H to those of H.

1. Are gamma matrices needed at all?

The recent definitions of associativity and M^8-H- duality has evolved slowly from in-accurate characterizations and there are still open questions.

(a) The standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H or even M^8 would mean doubling of the spinor structure: not an attractive idea.

It is however important to notice that the introduction of octonionic gamma matrices is not necessary. Simplest option is just the interpretation of tangent basis vectors are octonions: octonion basis is obtained as contractions of vielbein vectors with "flat space" octonions.

(b) The earlier formulation was in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest.

(c) The known extremals provide a test for the associativity (co-associativity) hypothesis. I have not demonstrated that the associativity works for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

(d) Could one define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8)? The basic problem is that the space spanned by the modified gamma matrices can have dimension smaller than that of 4 (so that co-basis would hve dimension larger than 4 if identified in terms of orthogonal complement). Second problem is that modified gammas are in general not in the tangent space of space-time surface as vectors of the imbedding space. Therefore the notions of associativity (co-associativity) defined in terms of tangent space (normal space) become problematic.

Basic formulation of $M^8 - H$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

Basic mathematical facts

The hard mathematical facts behind the notion of number theoretical compactification are following.

(a) One manner to define M^4 image of M^8 point uniquely would be to assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$ (it turns out that this is not the correct manner!). This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table. One can however do also without the introduction of this structure and use only the octonionic structure.

(b) The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

(c) Quaternionic sub-algebras of M^8 are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

(d) The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1} e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

1. Formulation of $M^8 - H$ duality

Consider now the formulation of $M^8 - H$ duality.

(a) The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \to S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

(b) One should be able to assign a unique point of M^4 to a given point of $X^4 \subset M^8$.

   i. The associative tangent space of space-time surface shifted to go through the origin of M^8 defines the preferred $M^4 \subset M^8$ uniquely, and one can projects the point of M^8 to this M^4 to get M^4 point. This identification implies that the dimension of tangent space projection to M^4 is maximum, and one avoids the situations in which the image surface of H has dimension smaller than 4.

   ii. One can imagine also second option which however fails. Since the Kähler structure of M^8 implies a unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \to M^4$ (this is modification to the original definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.
   One can however represent objection to this identification. The dimension of image in H is smaller than 4. For instance, hyperquaternionic plane M^4_1 which has M^2 the intersection with preferred M^4 corresponds to constant CP_2 point so that its H image is M^2.

2. Generalization to $H - H$ duality

As a matter fact, $M^8 - H$ duality might generalize to $H - H$ duality allowing to integrate space-time surfaces and thus WCW to a category.

(a) The map of space-time surfaces of M^8 to those of $H = M^4 \times CP_2$ need not imply that the image surfaces in H are quaternionic in H. If they are, then the construction can be iterated. It seems that one continue this series ad infinitum and could generate new solutions of field equations! If this is the case, one could iterate duality as a sequence $M^8 \to H \to H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. One would obtain basically a category of space-time surfaces with arrows defined by the duality. Same probably applies to co-associative surfaces. This certainly makes the heart of mathematician beat.

(b) It is not proven that associativity/co-associativity implies preferred extremal property for Kähler action. One thing to understand is why Kähler action. An argument in favor of preferred role of Kähler action is that only Kähler action allows localization of spinor modes to 2-D surfaces essential for the well-definedness of em charge [K105]. These surface would be string world sheets and possibly also partonic 2-surfaces and their could correspond to commutative and co-commutative 2-surfaces in number theoretic vision and be well-defined also for M^8. If so, Kähler action would provide a physical representation for the number theoretic notions like associativity and commutativity and their co-notions.

(c) If all goes as in dreams, the mere associativity or co-associativity in M^8 would code for the preferred extremal property of Kähler action in H and would imply this property in H. The surfaces with this property would form category with arrow defined by the duality.

(d) One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

3. Some comments

A couple of comments are in order.

(a) This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

(b) The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K9]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

(c) Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K28] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of modified Dirac operator.

For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

(d) Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. Not that the decomposition $M^8 = M^4 \times E^4$ is not necessary if M^4 projection is defined to the M^4 defined by hyper-quaternionic tangent place.

Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K98]).

(a) According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

(b) Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and modified gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

1. Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

2. Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

(a) By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

(b) One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

(c) Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

(d) The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

3. Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

(a) The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

(b) One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

(c) One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

(d) The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

(e) In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

4. What about the analog of Kähler-Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H...$ iteration generating new solutions from existing ones.

1.Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1+io_2) = N(o_1)-N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1+io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2)\oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1,-1,-,1-,1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1,-1,-1,-1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

2. Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

(a) Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a,b,c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

(b) If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

(c) Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a\times(b\times b)$ for different octonion units is involved.

(d) Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{11.3.6}$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{11.3.7}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

(e) The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^By^Cz^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A43] expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

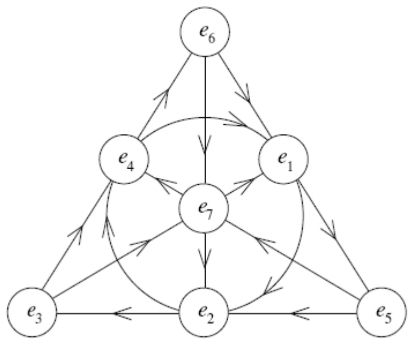

Figure 11.4: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A43] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

Questions

In following some questions related to $M^8 - H$ duality are represented.

1. Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

(a) For Kähler action the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k\gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.
The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

(b) The space spanned by the modified gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection modified gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence modified gamma matrices are flexible enough to realize associativity in H.

(c) Modified gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4\times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

2. Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

3. Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

(a) If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

(b) $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

(c) $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin. This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

4. $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

(a) At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

(b) The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

(c) The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

(d) The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

(e) Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K57] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

11.4 Infinite primes

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD.

The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also for their complexifications and one can speak about infinite primes in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors.

11.4.1 Basic ideas

The notion of infinite prime

The original motivation for the notion of infinite prime came from the first attempts to construct TGD inspired theory of consciousness (around 1995) [K89] . Suppose very naively that the 4-surfaces in a given sector of the "world of classical worlds" (WCW) are labelled by a fixed p-adic prime. The natural expectation is that evolution by quantum jumps means dispersion in the space of these sectors and leads to the increase of the p-adic prime characterizing the Universe. As one moves backwards in subjective time (sequence of quantum jumps) one ends up to the situation in which the prime characterizing the universe was $p = 2$. Should one assume that there was the first quantum jump when everything began? If not, then it would seem that the p-adic prime characterizing the Universe must be infinite. Second problem is that the p-adic length scales are finite and if the size scale of Universe is given by p-adic length scale the Universe has finite sized: this does not make sense in TGD framework. The only way out of the problems is the assumption that the p-adic prime characterizing the entire Universe is literally infinite and that p-adic primes characterizing space-time sheets are finite.

These argument, which are by no means central for the recent view about p-adic primes, motivated the attempt to construct a theory of infinite primes and to extend quantum TGD accordingly. This turns out to be possible. The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. It must be also emphasized that the notion of infinity is relativistic. With respect to the p-adic norm infinite primes have unit norm for all finite and infinite primes so that there is nothing to become scared of!

Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave functions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of imbedding space and space-time surface are subject to a number theoretic evolution. In philosophical mood one can of course also ask whether there exists a hierarchy of imbedding spaces in which the imbedding space at the lower level represents something with infinitesimal size in the sense of real topology and whether this hierarchy is accompanied also by a hierarchy of conscious entities.

This picture suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [A103] providing a rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively. Same generalization could make sense for all classical number fields [A19, A7, A26] .

Infinite primes and physics in TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. Infinite primes and super-symmetric quantum field theory

Consider next the physical interpretation.

(a) The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that configuration space spinor fields or at least the ground states of associated super-conformal representations [A30] (for super-conformal invariance see [A30] could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

(b) The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.

(c) The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. It turns out that this idea makes sense when one considers discrete wave functions in the space of infinite primes and that one can indeed represent standard model quantum numbers in this manner.

(d) The number theoretical supersymmetry suggests also space-time supersymmetry TGD framework. Space-time super-symmetry in its standard form is not possible in TGD Universe and this cheated me to believe that this supersymmetry is completely absent in TGD Universe. The progress in the understanding of the properties of the modified Dirac action however led to a generalization of the space-time super-symmetry as a dynamical and broken symmetry of quantum TGD [K29] .

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [K99] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on configuration space spinor fields representing physical states [K21] .

2. Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers [A75, A47, A50, A88] suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

G_2 acts as automorphisms of hyper-octonions and $SU(3)$ as its subgroup respecting the choice of a preferred imaginary unit. The discrete subgroups of $SU(3)$ permuting to each other hyper-octonionic primes are analogous to Galois group and turned out to play a crucial role in the understanding of the correspondence between infinite hyper-octonionic primes and quantum states predicted by quantum TGD.

3. The notion of finite measurement resolution as the key concept

TGD predicts several hierarchies: the hierarchy of space-time sheets, the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K99] , the dark matter hierarchy characterized by increasing values of $\hbar$ [K27] , the hierarchy of extensions of a given p-adic number field. TGD inspired theory of consciousness predictes the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. These hierarchies should be closely related.

The notion of finite measurement resolution turns out to be the key concept: the p-adic norm of the rational defined by the infinite prime characterizes the angle measurement resolution for given p-adic prime p. It is essential that one has what might be called a state function reduction selecting a fixed p-adic prime which could be also infinite. This gives direct connections with cognition and with the p-adicization program relying also on angle measurement resolution. Also the value the integers characterizing the singular coverings of CD and CP_2 defining as their product Planck constant characterize the measurement resolution for a given p-adic prime in CD and CP_2 degrees of freedom. This conforms with the fact that elementary particles are characterized by two infinite primes. Hence finite measurement resolution ties tightly together the three threads of the number theoretic vision. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory with finite measurement resolution.

4. Space-time correlates of infinite primes

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. This conjecture should be consistent with two other conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action and space-time surfaces as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8).

Quantum classical correspondence requires the map of the quantum numbers of WCW spinor fields to space-time geometry. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

Therefore, if one wants to map infinite rationals to space-time geometry it is enough to map infinite primes to quantum numbers. This map might be i achieved thanks to the detailed picture about the interpretation of the symmetries of infinite primes in terms of standard model symmetries. The notion of finite measurement resolution allows to deduce much more detailed about this correspondence. In particular, the rational defined by the infinite prime classifies the finite sub-manifold geometry defined by the discretization of the partonic 2-surface implied by the finite measurement resolution. Also a direct correlation between integers defining Planck constant and the "fermionic" part of the infinite prime emerges.

Infinite primes, cognition, and intentionality

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

(a) One can define the notion of prime also for the algebraic extensions of rationals. The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.

(b) The introduction of infinite primes, integers, and rationals leads also to a generalization of classical number fields since an infinite algebra of real (complex, etc...) units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum of infinitesimals (real zeros) is replaced by multiplication of real units meaning that the set of real and also more general units becomes infinitely degenerate.

(c) Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point -or more generally wave functions in the space of the units associated with the point- might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

(d) In zero energy ontology hyper-octonionic units identified as ratios of the infinite integers associated with the positive and negative energy parts of the zero energy state define a representation of WCW spinor fields. The action of subgroups of SU(3) and rotation group SU(2) preserving hyper-octonionic and hyper-quaternionic primeness and identification of momentum and electro-weak charges in terms of components of hyper-octonionic primes makes this representation unique. Hence Brahman-Atman identity has a completely concrete realization and fixes completely the quantum number spectrum including particle masses and correlations between various quantum numbers.

(e) One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

11.4.2 Infinite primes, integers, and rationals

The definition of the infinite integers and rationals is a straightforward procedure and structurally similar to a repeated second quantization of a super-symmetric quantum field theory but including also the number theoretic counterparts of bound states.

The first level of hierarchy

In the following the concept of infinite prime is developed gradually by stepwise procedure rather than giving directly the basic definitions. The hope is that the development of the concept in the same manner as it actually occurred would make it easier to understand it.

Step 1

One could try to define infinite primes P by starting from the basic idea in the proof of Euclid for the existence of infinite number of primes. Take the product of all finite primes and add 1 to get a new prime:

$$\begin{aligned} P &= 1 + X \ , \\ X &= \prod_p p \ . \end{aligned} \tag{11.4.1}$$

If P were divisible by finite prime then $P - X = 1$ would be divisible by finite prime and one would encounter contradiction. One could of course worry about the possible existence of infinite primes smaller than P and possibly dividing P. The numbers $N = P - k$, $k > 1$, are certainly not primes since k can be taken as a factor. The number $P' = P - 2 = -1 + X$ could however be prime. P is certainly not divisible by $P - 2$. It seems that one cannot express P and $P-2$ as product of infinite integer and finite integer. Neither it seems possible to express these numbers as products of more general numbers of form $\prod_{p\in U} p + q$, where U is infinite subset of finite primes and q is finite integer.

Step 2

P and $P - 2$ are not the only possible candidates for infinite primes. Numbers of form

$$\begin{aligned} P(\pm, n) &= \pm 1 + nX \ , \\ k(p) &= 0, 1, ... \ , \\ n &= \prod_p p^{k(p)} \ , \\ X &= \prod_p p \ , \end{aligned} \tag{11.4.2}$$

where $k(p) \neq 0$ holds true only in finite set of primes, are characterized by a integer n, and are also good prime candidates. The ratio of these primes to the prime candidate P is given by integer n. In general, the ratio of two prime candidates $P(m)$ and $P(n)$ is rational number m/n telling which of the prime candidates is larger. This number provides ordering of the prime candidates $P(n)$. The reason why these numbers are good candidates for infinite primes is the same as above. No finite prime p with $k(p) \neq 0$ appearing in the product can divide these numbers since, by the same arguments as appearing in Euclid's theorem, it would divide also 1. On the other hand it seems difficult to invent any decomposition of these numbers containing infinite numbers. Already at this stage one can notice the structural analogy with the construction of multiboson states in quantum field theory: the numbers $k(p)$ correspond to the occupation numbers of bosonic states of quantum field theory in one-dimensional box, which suggests that the basic structure of QFT might have number theoretic interpretation in some very general sense. It turns out that this analogy generalizes.

Step 3

All $P(n)$ satisfy $P(n) \geq P(1)$. One can however also the possibility that $P(1)$ is not the smallest infinite prime and consider even more general candidates for infinite primes, which are smaller than $P(1)$. The trick is to drop from the infinite product of primes $X = \prod_p p$ some primes away by dividing it by integer $s = \prod_{p_i} p_i$, multiply this number by an integer n not divisible by any prime dividing s and to add to/subtract from the resulting number

nX/s natural number ms such that m expressible as a product of powers of only those primes which appear in s to get

$$\begin{aligned} &P(\pm, m, n, s) = n\tfrac{X}{s} \pm ms \ , \\ &m = \prod_{p|s} p^{k(p)} \ , \\ &n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{11.4.3}$$

Here $x|y$ means 'x divides y'. To see that no prime p can divide this prime candidate it is enough to calculate $P(\pm, m, n, s)$ modulo p: depending on whether p divides s or not, the prime divides only the second term in the sum and the result is nonzero and finite (although its precise value is not known). The ratio of these prime candidates to $P(+, 1, 1, 1)$ is given by the rational number n/s: the ratio does not depend on the value of the integer m. One can however order the prime candidates with given values of n and s using the difference of two prime candidates as ordering criterion. Therefore these primes can be ordered.

One could ask whether also more general numbers of the form $n\frac{X}{s} \pm m$ are primes. In this case one cannot prove the indivisibility of the prime candidate by p not appearing in m. Furthermore, for $s \ mod \ 2 = 0$ and $m \ mod \ 2 \neq 0$, the resulting prime candidate would be even integer so that it looks improbable that one could obtain primes in more general case either.

Step 4

An even more general series of candidates for infinite primes is obtained by using the following ansatz which in principle is contained in the original ansatz allowing infinite values of n

$$\begin{aligned} &P(\pm, m, n, s|r) = nY^r \pm ms \ , \\ &Y = \tfrac{X}{s} \ , \\ &m = \prod_{p|s} p^{k(p)} \ , \\ &n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{11.4.4}$$

The proof that this number is not divisible by any finite prime is identical to that used in the previous case. It is not however clear whether the ansatz for given r is not divisible by infinite primes belonging to the lower level. A good example in $r = 2$ case is provided by the following unsuccessful ansatz

$$\begin{aligned} &N = (n_1Y + m_1s)(n_2Y + m_2s) = \tfrac{n_1n_2X^2}{s^2} - m_1m_2s^2 \ , \\ &Y = \tfrac{X}{s} \ , \\ &n_1m_2 - n_2m_1 = 0 \ . \end{aligned}$$

Note that the condition states that n_1/m_1 and $-n_2/m_2$ correspond to the same rational number or equivalently that (n_1, m_1) and (n_2, m_2) are linearly dependent as vectors. This encourages the guess that all other $r = 2$ prime candidates with finite values of n and m at least, are primes. For higher values of r one can deduce analogous conditions guaranteeing that the ansatz does not reduce to a product of infinite primes having smaller value of r. In fact, the conditions for primality state that the polynomial $P(n, m, r)(Y) = nY^r + m$ with integer valued coefficients ($n > 0$) defined by the prime candidate is irreducible in the field of integers, which means that it does not reduce to a product of lower order polynomials of same type.

Step 5

A further generalization of this ansatz is obtained by allowing infinite values for m, which leads to the following ansatz:

$$
\begin{array}{l}
P(\pm,m,n,s|r_1,r_2) = nY^{r_1} \pm ms \ , \\
m = P_{r_2}(Y)Y + m_0 \ , \\
Y = \frac{X}{s} \ , \\
m_0 = \prod_{p|s} p^{k(p)} \ , \\
n = \prod_{p|Y} p^{k(p)}, \ \ k(p) \geq 0 \ .
\end{array} \tag{11.4.5}
$$

Here the polynomial $P_{r_2}(Y)$ has order r_2 is divisible by the primes belonging to the complement of s so that only the finite part m_0 of m is relevant for the divisibility by finite primes. Note that the part proportional to s can be infinite as compared to the part proportional to Y^{r_1}: in this case one must however be careful with the signs to get the sign of the infinite prime correctly. By using same arguments as earlier one finds that these prime candidates are not divisible by finite primes. One must also require that the ansatz is not divisible by lower order infinite primes of the same type. These conditions are equivalent to the conditions guaranteeing the polynomial primeness for polynomials of form $P(Y) = nY^{r_1} \pm (P_{r_2}(Y)Y + m_0)s$ having integer-valued coefficients. The construction of these polynomials can be performed recursively by starting from the first order polynomials representing first level infinite primes: Y can be regarded as formal variable and one can forget that it is actually infinite number.

By finite-dimensional analogy, the infinite value of m means infinite occupation numbers for the modes represented by integer s in some sense. For finite values of m one can always write m as a product of powers of $p_i|s$. Introducing explicitly infinite powers of p_i is not in accordance with the idea that all exponents appearing in the formulas are finite and that the only infinite variables are X and possibly S (formulas are symmetric with respect to S and X/S). The proposed representation of m circumvents this difficulty in an elegant manner and allows to say that m is expressible as a product of infinite powers of p_i despite the fact that it is not possible to derive the infinite values of the exponents of p_i.

Summarizing, an infinite series of candidates for infinite primes has been found. The prime candidates $P(\pm,m,n,s)$ labeled by rational numbers n/s and integers m plus the primes $P(\pm,m,n,s|r_1,r_2)$ constructed as r_1:th or r_2:th order polynomials of $Y = X/s$: the latter ansatz reduces to the less general ansatz of infinite values of n are allowed.

One can ask whether the $p \ mod \ 4 = 3$ condition guaranteeing that the square root of -1 does not exist as a p-adic number, is satisfied for $P(\pm,m,n,s)$. $P(\pm,1,1,1) \ mod \ 4$ is either 3 or 1. The value of $P(\pm,m,n,s) \ mod \ 4$ for odd s on n only and is same for all states containing even/odd number of $p \ mod \ = 3$ excitations. For even s the value of $P(\pm,m,n,s) \ mod \ 4$ depends on m only and is same for all states containing even/odd number of $p \ mod \ = 3$ excitations. This condition resembles G-parity condition of Super Virasoro algebras. Note that either $P(+,m,n,s)$ or $P(-,m,n,s)$ but not both are physically interesting infinite primes ($2m \ mod \ 4 = 2$ for odd m) in the sense of allowing complex Hilbert space. Also the additional conditions satisfied by the states involving higher powers of X/s resemble to Virasoro conditions. An open problem is whether the analogy with the construction of the many-particle states in super-symmetric theory might be a hint about more deeper relationship with the representation of Super Virasoro algebras and related algebras.

It is not clear whether even more general prime candidates exist. An attractive hypothesis is that one could write explicit formulas for all infinite primes so that generalized theory of primes would reduce to the theory of finite primes.

Infinite primes form a hierarchy

By generalizing using general construction recipe, one can introduce the second level prime candidates as primes not divisible by any finite prime p or infinite prime candidate of type $P(\pm,m,n,s)$ (or more general prime at the first level: in the following we assume for simplicity that these are the only infinite primes at the first level). The general form of these prime candidates is exactly the same as at the first level. Particle-analogy makes it easy to express the construction receipe. In present case 'vacuum primes' at the lowest level are of the form

$$\begin{array}{l} \frac{X_1}{S} \pm S \ , \\ X_1 = X\prod_{P(\pm,m,n,s)} P(\pm,m,n,s) \ , \\ S = s\prod_{P_i} P_i \ , \\ s = \prod_{p_i} p_i \ . \end{array} \quad (11.4.6)$$

S is product or ordinary primes p and infinite primes $P_i(\pm,m,n,s)$. Primes correspond to physical states created by multiplying X_1/S (S) by integers not divisible by primes appearing S (X_1/S). The integer valued functions $k(p)$ and $K(p)$ of prime argument give the occupation numbers associated with X/s and s type 'bosons' respectively. The non-negative integer-valued function $K(P) = K(\pm,m,n,s)$ gives the occupation numbers associated with the infinite primes associated with X_1/S and S type 'bosons'. More general primes can be constructed by mimicking the previous procedure.

One can classify these primes by the value of the integer $K_{tot} = \sum_{P|X/S} K(P)$: for a given value of K_{tot} the ratio of these prime candidates is clearly finite and given by a rational number. At given level the ratio P_1/P_2 of two primes is given by the expression

$$\frac{P_1(\pm,m_1,n_1,s_1K_1,S_1}{P_2(\pm,m_2,n_2,s_2,K,S_2)} = \frac{n_1s_2}{n_2s_1}\prod_{\pm,m,n,s}(\frac{n}{s})^{K_1^+(\pm,n,m,s)-K_2^+(\pm,n,m,s)} \ . \quad (11.4.7)$$

Here K_i^+ denotes the restriction of $K_i(P)$ to the set of primes dividing X/S. This ratio must be smaller than 1 if it is to appear as the first order term $P_1P_2 \rightarrow P_1/P_2$ in the canonical identification and again it seems that it is not possible to get all rationals for a fixed value of P_2 unless one allows infinite values of N expressed neatly using the more general ansatz involving higher power of S.

Construction of infinite primes as a repeated quantization of a super-symmetric arithmetic quantum field theory

The procedure for constructing infinite primes is very much reminiscent of the second quantization of an super-symmetric arithmetic quantum field theory in which single particle fermion and boson states are labeled by primes. In particular, there is nothing especially frightening in the particle representation of infinite primes: theoretical physicists actually use these kind of representations quite routinely.

(a) The binary-valued function telling whether a given prime divides s can be interpreted as a fermion number associated with the fermion mode labeled by p. Therefore infinite prime is characterized by bosonic and fermionic occupation numbers as functions of the prime labeling various modes and situation is super-symmetric. X can be interpreted as the counterpart of Dirac sea in which every negative energy state state is occupied and $X/s \pm s$ corresponds to the state containing fermions understood as holes of Dirac sea associated with the modes labeled by primes dividing s.

(b) The multiplication of the 'vacuum' X/s with $n = \prod_{p|X/s} p^{k(p)}$ creates $k(p)$ 'p-bosons' in mode of type X/s and multiplication of the 'vacuum' s with $m = \prod_{p|s} p^{k(p)}$ creates $k(p)$ 'p-bosons'. in mode of type s (mode occupied by fermion). The vacuum states in which bosonic creation operators act, are tensor products of two vacuums with tensor product represented as sum

$$|vac(\pm)\rangle = |vac(\frac{X}{s})\rangle \otimes |vac(\pm s)\rangle \leftrightarrow \frac{X}{s} \pm s \quad (11.4.8)$$

obtained by shifting the prime powers dividing s from the vacuum $|vac(X)\rangle = X$ to the vacuum ± 1. One can also interpret various vacuums as many fermion states. Prime property follows directly from the fact that any prime of the previous level divides either the first or second factor in the decomposition $NX/S \pm MS$.

(c) This picture applies at each level of infinity. At a given level of hierarchy primes P correspond to all the Fock state basis of all possible many-particle states of second quantized super-symmetric theory. At the next level these many-particle states are regarded as single particle states and further second quantization is performed so that the primes become analogous to the momentum labels characterizing various single-particle states at the new level of hierarchy.

(d) There are two nonequivalent quantizations for each value of S due to the presence of $\pm$ sign factor. Two primes differing only by sign factor are like G-parity $+1$ and -1 states in the sense that these primes satisfy $P\ mod\ 4 = 3$ and $P\ mod\ 4 = 1$ respectively. The requirement that -1 does not have p-adic square root so that Hilbert space is complex, fixes G-parity to say $+1$. This observation suggests that there exists a close analogy with the theory of Super Virasoro algebras so that quantum TGD might have interpretation as number theory in infinite context. An alternative interpretation for the $\pm$ degeneracy is as counterpart for the possibility to choose the fermionic vacuum to be a state in which either all positive or all negative energy fermion states are occupied.

(e) One can also generalize the construction to include polynomials of $Y = X/S$ to get infinite hierarchy of primes labeled by the two integers r_1 and r_2 associated with the polynomials in question. An entire hierarchy of vacuums labeled by r_1 is obtained. A possible interpretation of these primes is as counterparts for the bound states of quantum field theory. The coefficient for the power $(X/s)^{r_1}$ appearing in the highest term of the general ansatz, codes the occupation numbers associated with vacuum $(X/s)^{r_1}$. All the remaining terms are proportional to s and combine to form, in general infinite, integer m characterizing various infinite occupation numbers for the subsystem characterized by s. The additional conditions guaranteeing prime number property are equivalent with the primality conditions for polynomials with integer valued coefficients and resemble Super Virasoro conditions. For $r_2 > 0$ bosonic occupation numbers associated with the modes with fermion number one are infinite and one cannot write explicit formula for the boson number.

(f) One could argue that the analogy with super-symmetry is not complete. The modes of Super Virasoro algebra are labeled by natural number whereas now modes are labeled by prime. This need not be a problem since one can label primes using natural number n. Also 8-valued spin index associated with fermionic and bosonic single particle states in TGD world is lacking (space-time is surface in 8-dimensional space). This index labels the spin states of 8-dimensional spinor with fixed chirality. One could perhaps get also spin index by considering infinite octonionic primes, which correspond to vectors of 8-dimensional integer lattice such that the length squared of the lattice vector is ordinary prime:

$$\sum_{k=1,\ldots,8} n_k^2 = \text{ prime } .$$

Thus one cannot exclude the possibility that TGD based physics might provide representation for octonions extended to include infinitely large octonions. The notion of prime octonion is well defined in the set of integer octonions and it is easy to show that the Euclidian norm squared for a prime octonion is prime. If this result generalizes then the construction of generalized prime octonions would generalize the construction of finite prime octonions. It would be interesting to know whether the results of finite-dimensional case might generalize to the infinite-dimensional context. One cannot exclude the possibility that prime octonions are in one-one correspondence with physical states in quantum TGD.

These observations suggest a close relationship between quantum TGD and the theory of infinite primes in some sense: even more, entire number theory and mathematics might be reducible to quantum physics understood properly or equivalently, physics might provide the representation of basic mathematics. Of course, already the uniqueness of the basic mathematical structure of quantum TGD points to this direction. Against this background the fact that 8-dimensionality of the imbedding space allows introduction of octonion structure (also p-adic algebraic extensions) acquires new meaning. Same is also suggested by the fact that the algebraic extensions of p-adic numbers allowing square root of real p-adic number are 4- and 8-dimensional.

What is especially interesting is that the core of number theory would be concentrated in finite primes since infinite primes are obtained by straightforward procedure providing explicit formulas for them. Repeated quantization provides also a model of abstraction process understood as construction of hierarchy of natural number valued functions about functions about ...... At the first level infinite primes are characterized by the integer valued function $k(p)$ giving occupation numbers plus subsystem-complement division (division to thinker and external world!). At the next level prime is characterized in a similar manner. One should also notice that infinite prime at given level is characterized by a pair $(R = MN, S)$ of integers at previous level. Equivalently, infinite prime at given level is characterized by fermionic and bosonic occupation numbers as functions in the set of primes at previous level.

Construction in the case of an arbitrary commutative number field

The basic construction recipe for infinite primes is simple and generalizes even to the case of algebraic extensions of rationals. Let $K = Q(\theta)$ be an algebraic number field (see the Appendix of [K87] for the basic definitions). In the general case the notion of prime must be replaced by the concept of irreducible defined as an algebraic integer with the property that all its decompositions to a product of two integers are such that second integer is always a unit (integer having unit algebraic norm, see Appendix of [K87]).

Assume that the irreducibles of $K = Q(\theta)$ are known. Define two irreducibles to be equivalent if they are related by a multiplication with a unit of K. Take one representative from each equivalence class of units. Define the irreducible to be positive if its first non-vanishing component in an ordered basis for the algebraic extension provided by the real unit and powers of θ, is positive. Form the counterpart of Fock vacuum as the product X of these representative irreducibles of K.

The unique factorization domain (UFD) property (see Appendix of [K87]) of infinite primes does not require the ring O_K of algebraic integers of K to be UFD although this property might be forced somehow. What is needed is to find the primes of K; to construct X as the product of all irreducibles of K but not counting units which are integers of K with unit norm; and to apply second quantization to get primes which are first order monomials. X is in general a product of powers of primes. Generating infinite primes at the first level correspond to generalized rationals for K having similar representation in terms of powers of primes as ordinary rational numbers using ordinary primes.

Mapping of infinite primes to polynomials and geometric objects

The mapping of the generating infinite primes to first order monomials labeled by their rational zeros is extremely simple at the first level of the hierarchy:

$$P_{\pm}(m,n,s) = \frac{mX}{s} \pm ns \rightarrow x_{\pm} \pm \frac{m}{sn} \quad . \tag{11.4.9}$$

Note that a monomial having zero as its root is not obtained. This mapping induces the mapping of all infinite primes to polynomials.

The simplest infinite primes are constructed using ordinary primes and second quantization of an arithmetic number theory corresponds in one-one manner to rationals. Indeed, the integer $s = \prod_i p_i^{k_i}$ defining the numbers k_i of bosons in modes k_i, where fermion number is one, and the integer r defining the numbers of bosons in modes where fermion number is zero, are co-prime. Moreover, the generating infinite primes can be written as $(n/s)X \pm ms$ corresponding to the two vacua $V = X \pm 1$ and the roots of corresponding monomials are positive *resp.* negative rationals.

More complex infinite primes correspond sums of powers of infinite primes with rational coefficients such that the corresponding polynomial has rational coefficients and roots which are not rational but belong to some algebraic extension of rationals. These infinite primes correspond simply to products of infinite primes associated with some algebraic extension of rationals. Obviously the construction of higher infinite primes gives rise to a hierarchy of higher algebraic extensions.

It is possible to continue the process indefinitely by constructing the Dirac vacuum at the n:th level as a product of primes of previous levels and applying the same procedure. At the second level Dirac vacuum $V = X \pm 1$ involves X which is the product of all primes at previous levels and in the polynomial correspondence X thus correspond to a new independent variable. At the n:th level one would have polynomials $P(q_1|q_2|...)$ of q_1 with coefficients which are rational functions of q_2 with coefficients which are.... The hierarchy of infinite primes would be thus mapped to the functional hierarchy in which polynomial coefficients depend on parameters depending on ....

At the second level one representation of infinite primes would be as algebraic curve resulting as a locus of $P(q_1|q_2) = 0$: this certainly makes sense if q_1 and q_2 commute. At higher levels the locus is a higher-dimensional surface.

How to order infinite primes?

One can order the infinite primes, integers and rationals. The ordering principle is simple: one can decompose infinite integers to two parts: the 'large' and the 'small' part such that the ratio of the small part with the large part vanishes. If the ratio of the large parts of two infinite integers is different from one or their sign is different, ordering is obvious. If the ratio of the large parts equals to one, one can perform same comparison for the small parts. This procedure can be continued indefinitely.

In case of infinite primes ordering procedure goes like follows. At given level the ratios are rational numbers. There exists infinite number of primes with ratio 1 at given level, namely the primes with same values of N and same S with MS infinitesimal as compared to NX/S. One can order these primes using either the relative sign or the ratio of $(M_1S_1)/(M_2S_2)$ of the small parts to decide which of the two is larger. If also this ratio equals to one, one can repeat the process for the small parts of M_iS_i. In principle one can repeat this process so many times that one can decide which of the two primes is larger. Same of course applies to infinite integers and also to infinite rationals build from primes with infinitesimal MS. If NS is not infinitesimal it is not obvious whether this procedure works. If $N_iX_i/M_iS_i = x_i$ is finite for both numbers (this need not be the case in general) then the ratio $\frac{M_1S_1}{M_2S_2}\frac{(1+x_2)}{(1+x_1)}$ provides the needed criterion. In case that this ratio equals one, one can consider use the ratio of the small parts multiplied by $\frac{(1+x_2)}{(1+x_1)}$ of M_iS_i as ordering criterion. Again the procedure can be repeated if needed.

What is the cardinality of infinite primes at given level?

The basic problem is to decide whether Nature allows also integers S , $R = MN$ represented as infinite product of primes or not. Infinite products correspond to subsystems of infinite size (S) and infinite total occupation number (R) in QFT analogy.

(a) One could argue that S should be a finite product of integers since it corresponds to the requirement of finite size for a physically acceptable subsystem. One could apply similar argument to R. In this case the set of primes at given level has the cardinality of integers ($alef_0$) and the cardinality of all infinite primes is that of integers. If also infinite integers R are assumed to involve only finite products of infinite primes the set of infinite integers is same as that for natural numbers.

(b) NMP is well defined in p-adic context also for infinite subsystems and this suggests that one should allow also infinite number of factors for both S and $R = MN$. Super symmetric analogy suggests the same: one can quite well consider the possibility that the total fermion number of the universe is infinite. It seems however natural to assume that the occupation numbers $K(P)$ associated with various primes P in the representations $R = \prod_P P^{K(P)}$ are finite but nonzero for infinite number of primes P. This requirement applied to the modes associated with S would require the integer m to be explicitly expressible in powers of $P_i|S$ ($P_{r_2} = 0$) whereas all values of r_1 are possible. If infinite number of prime factors is allowed in the definition of S, then the application of diagonal argument of Cantor shows that the number of infinite primes is larger than $alef_0$ already at the first level. The cardinality of the first level is $2^{alef_0} 2^{alef_0} == 2^{alef_0}$. The first factor is the cardinality of reals and comes from the fact that the sets S form the set of all possible subsets of primes, or equivalently the cardinality of all possible binary valued functions in the set of primes. The second factor comes from the fact that integers $R = NM$ (possibly infinite) correspond to all natural number-valued functions in the set of primes: if only finite powers $k(p)$ are allowed then one can map the space of these functions to the space of binary valued functions bijectively and the cardinality must be 2^{alef_0}. The general formula for the cardinality at given level is obvious: for instance, at the second level the cardinality is the cardinality of all possible subsets of reals. More generally, the cardinality for a given level is the cardinality for the subset of possible subsets of primes at the previous level.

How to generalize the concepts of infinite integer, rational and real?

The allowance of infinite primes forces to generalize also the concepts concepts of integer, rational and real number. It is not obvious how this could be achieved. The following arguments lead to a possible generalization which seems practical (yes!) and elegant.

1. Infinite integers form infinite-dimensional vector space with integer coefficients

The first guess is that infinite integers N could be defined as products of the powers of finite and infinite primes.

$$N = \prod_k p_k^{n_k} = nM \ , \ n_k \geq 0 \ , \qquad (11.4.10)$$

where n is finite integer and M is infinite integer containing only powers of infinite primes in its product expansion.

It is not however not clear whether the sums of infinite integers really allow similar decomposition. Even in the case that this decomposition exists, there seems to be no way of deriving it. This would suggest that one should regard sums

$$\sum_i n_i M_i$$

of infinite integers as infinite-dimensional linear space spanned by M_i so that the set of infinite integers would be analogous to an infinite-dimensional algebraic extension of say p-adic numbers such that each coordinate axes in the extension corresponds to single infinite integer of form $N = mM$. Thus the most general infinite integer N would have the form

$$N = m_0 + \sum m_i M_i \ . \qquad (11.4.11)$$

This representation of infinite integers indeed looks promising from the point of view of practical calculations. The representation looks also attractive physically. One can interpret the set of integers N as a linear space with integer coefficients m_0 and m_i:

$$N = m_0|1\rangle + \sum m_i|M_i\rangle \ . \tag{11.4.12}$$

$|M_i\rangle$ can be interpreted as a state basis representing many-particle states formed from bosons labeled by infinite primes p_k and $|1\rangle$ represents Fock vacuum. Therefore this representation is analogous to a quantum superposition of bosonic Fock states with integer, rather than complex valued, superposition coefficients. If one interprets M_i as orthogonal state basis and interprets m_i as p-adic integers, one can define inner product as

$$\langle N_a, N_b\rangle = m_0(a)m_0(b) + \sum_i m_i(a)m_i(b) \ . \tag{11.4.13}$$

This expression is well defined p-adic number if the sum contains only enumerable number of terms and is always bounded by p-adic ultra-metricity. It converges if the p-adic norm of of m_i approaches to zero when M_i increases.

2. Generalized rationals

Generalized rationals could be defined as ratios $R = M/N$ of the generalized integers. This works nicely when M and N are expressible as products of powers of finite or infinite primes but for more general integers the definition does not look attractive. This suggests that one should restrict the generalized rationals to be numbers having the expansion as a product of positive and negative primes, finite or infinite:

$$N = \prod_k p_k^{n_k} = \frac{n_1 M_1}{nM} \ . \tag{11.4.14}$$

3. Generalized reals form infinite-dimensional real vector space

One could consider the possibility of defining generalized reals as limiting values of the generalized rationals. A more practical definition of the generalized reals is based on the generalization of the pinary expansion of ordinary real number given by

$$\begin{aligned} x &= \sum_{n\geq n_0} x_n p^{-n} \ , \\ x_n &\in \{0,..,p-1\} \ . \end{aligned} \tag{11.4.15}$$

It is natural to try to generalize this expansion somehow. The natural requirement is that sums and products of the generalized reals and canonical identification map from the generalized reals to generalized p-adcs are readily calculable. Only in this manner the representation can have practical value.

These requirements suggest the following generalization

$$\begin{aligned} X &= x_0 + \sum_N x_N p^{-N} \ , \\ N &= \sum_i m_i M_i \ , \end{aligned} \tag{11.4.16}$$

where x_0 and x_N are ordinary reals. Note that N runs over infinite integers which has *vanishing finite part.* Note that generalized reals can be regarded as infinite-dimensional linear space such that each infinite integer N corresponds to one coordinate axis of this space. One could interpret generalized real as a superposition of bosonic Fock states formed from single single boson state labeled by prime p such that occupation number is either 0 or infinite integer N with a vanishing finite part:

$$X = x_0|0\rangle + \sum_N x_N|N> \quad . \tag{11.4.17}$$

The natural inner product is

$$\langle X, Y\rangle = x_0 y_0 + \sum_N x_N y_N \quad . \tag{11.4.18}$$

The inner product is well defined if the number of N:s in the sum is enumerable and x_N approaches zero sufficiently rapidly when N increases. Perhaps the most natural interpretation of the inner product is as R_p valued inner product.

The sum of two generalized reals can be readily calculated by using only sum for reals:

$$X + Y = x_0 + y_0 + \sum_N (x_N + y_N)p^{-N} \quad , \tag{11.4.19}$$

The product XY is expressible in the form

$$XY = x_0 y_0 + x_0 Y + X y_0 + \sum_{N_1,N_2} x_{N_1} y_{N_2} p^{-N_1-N_2} \quad , \tag{11.4.20}$$

If one assumes that infinite integers form infinite-dimensional vector space in the manner proposed, there are no problems and one can calculate the sums $N_1 + N_2$ by summing component wise manner the coefficients appearing in the sums defining N_1 and N_2 in terms of infinite integers M_i allowing expression as a product of infinite integers.

Canonical identification map from ordinary reals to p-adics

$$x = \sum_k x_k p^{-k} \quad \rightarrow \quad x_p = \sum_k x_k p^k \quad ,$$

generalizes to the form

$$x = x_0 + \sum_N x_N p^{-N} \quad \rightarrow \quad (x_0)_p + \sum_N (x_N)_p p^N \quad , \tag{11.4.21}$$

so that all the basic requirements making the concept of generalized real calculationally useful are satisfied.

There are several interesting questions related to generalized reals.

(a) Are the extensions of reals defined by various values of p-adic primes mathematically equivalent or not? One can map generalized reals associated with various choices of the base p to each other in one-one manner using the mapping

$$X = x_0 + \sum_N x_N p_1^{-N} \rightarrow x_0 + \sum_N x_N p_2^{-N} . \quad (11.4.22)$$

The ordinary real norms of *finite* (this is important!) generalized reals are identical since the representations associated with different values of base p differ from each other only infinitesimally. This would suggest that the extensions are physically equivalent. It these extensions are not mathematically equivalent then p-adic primes could have a deep role in the definition of the generalized reals.

(b) One can generalize previous formulas for the generalized reals by replacing the coefficients x_0 and x_i by complex numbers, quaternions or octonions so as to get generalized complex numbers, quaternions and octonions. Also inner product generalizes in an obvious manner. The 8-dimensionality of the imbedding space provokes the question whether it might be possible to regard the infinite-dimensional configuration space of 3-surfaces, or rather, its tangent space, as a Hilbert space realization of the generalized octonions. This kind of identification could perhaps reduce TGD based physics to generalized number theory.

Comparison with the approach of Cantor

The main difference between the approach of Cantor and the proposed approach is that Cantor uses only the basic arithmetic concepts such as sum and multiplication and the concept of successor defining ordering of both finite and infinite ordinals. Cantor's approach is also purely set theoretic. The problems of purely set theoretic approach are related to the question what the statement 'Set is Many allowing to regard itself as One' really means and to the fact that there is no obvious connection with physics.

The proposed approach is based on the introduction of the concept of prime as a basic concept whereas partial ordering is based on the use of ratios: using these one can recursively define partial ordering and get precise quantitative information based on finite reals. The ordering is only partial and there is infinite number of ratios of infinite integers giving rise to same real unit which in turn leads to the idea about number theoretic anatomy of real point.

The 'Set is Many allowing to regard itself as One' is defined as quantum physicist would define it: many particle states become single particle states in the second quantization describing the counterpart for the construction of the set of subsets of a given set. One could also say that integer as such corresponds to set as 'One' and its decomposition to a product of primes corresponds to the set as 'Many'. The concept of prime, the ultimate 'One', has as its physical counterpart the concept of elementary particle understood in very general sense. The new element is the physical interpretation: the sum of two numbers whose ratio is zero correspond to completely physical finite-subsystem-infinite complement division and the iterated construction of the set of subsets of a set at given level is basically p-adic evolution understood in the most general possible sense and realized as a repeated second quantization. What is attractive is that this repeated second quantization can be regarded also as a model of abstraction process and actually the process of abstraction itself.

The possibility to interpret the construction of infinite primes either as a repeated bosonic quantization involving subsystem-complement division or as a repeated super-symmetric quantization could have some deep meaning. A possible interpretation consistent with these two pictures is based on the hypothesis that fermions provide a reflective level of consciousness in the sense that the 2^N element Fock basis of many-fermion states formed from N single-fermion states can be regarded as a set of all possible statements about N basic statements. Statements about whether a given element of set X belongs to some subset S of X are certainly the fundamental statements from the point of view of mathematics. Hence one could argue that many-fermion states provide cognitive representation for the subsets of some set. Single fermion states represent the points of the set and many-fermion states represent possible subsets.

11.4.3 Can one generalize the notion of infinite prime to the non-commutative and non-associative context?

The notion of prime and more generally, that of irreducible, makes sense also in more general number fields and even algebras. The considerations of [K88] suggests that the notion of infinite prime should be generalized to the case of complex numbers, quaternions, and octonions as well as to their hyper counterparts which seem to be physically the most interesting ones [K88] . Also the hierarchy of infinite primes should generalize as also the representation of infinite primes as polynomials although associativity is expected to pose technical problems.

Quaternionic and octonionic primes and their hyper counterparts

The loss of commutativity and associativity implies that the definitions of quaternionic and octonionic primes are not completely straightforward.

1. Basic facts about quaternions and octonions

Both quaternions and octonions allow both Euclidian norm and the Minkowskian norm defined as a trace of the linear operator defined by the multiplication with octonion. Minkowskian norm has the metric signature of $H = M^4 \times CP_2$ or $M^4_+ \times CP_2$ so that H can be regarded locally as an octonionic space. Both norms are a multiplicative and the notions of both quaternionic and octonionic prime are well defined despite non-associativity. Quaternionic and octonionic primes have length squared equal to rational prime.

In the case of quaternions different basis of imaginary units I, J, K are related by 3-dimensional rotation group and different quaternionic basis span a 3-dimensional sphere. There is 2-sphere of complex structures since imaginary unit can be any unit vector of imaginary 3-space.

A basis for octonionic imaginary units J, K, L, M, N, O, P can be chosen in many manners and fourteen-dimensional subgroup G_2 of the group $SO(7)$ of rotations of imaginary units is the group labeling the octonionic structures related by octonionic automorphisms to each other. It deserves to be mentioned that G_2 is unique among the simple Lie-groups in that the ratio of the square roots of lengths for long and short roots of G_2 Lie-algebra are in ratio $3:1$ [H2] . For other Lie-groups this ratio is either 2:1 or all roots have same length. The set of equivalence classes of the octonion structures is $SO(7)/G_2 = S^7$. In the case of quaternions there is only one equivalence class.

The group of automorphisms for octonions with a fixed imaginary part is $SU(3)$. The coset space $S^6 = G_2/SU(3)$ labels possible complex structures of the octonion space specified by a selection of a preferred imaginary unit. $SU(3)/U(2) = CP_2$ could be thought of as the space of octonionic structures giving rise to a given quaternionic structure with complex structure fixed. This can be seen as follows. The units $1, I$ are $SU(3)$ singlets whereas J, J_1, J_2 and K, K_1, K_2 form $SU(3)$ triplet and antitriplet. Under $U(2)$ J and K transform like objects having vanishing $SU(3)$ isospin and suffer only a $U(1)$ phase transformation determined by multiplication with complex unit I and are mixed with each other in orthogonal mixture. Thus $1, I, J, K$ is transformed to itself under $U(2)$.

2. Quaternionic and octonionic primes

Quaternionic primes with $p \ mod \ 4 = 1$ can correspond to (n_1, n_2) with n_1 even and n_2 odd or vice versa. For $p \ mod \ 4 = 3$ (n_1, n_2, n_3) with n_i odd is the minimal option. In this case there is however large number of primes having only two components: in particular, Gaussian primes with $p \ mod \ 4 = 1$ define also quaternionic primes. Purely real Gaussian primes with

$p \ mod \ 4 = 3$ with norm $z\overline{z}$ equal to p^2 are not quaternionic primes, and are replaced with 3-component quaternionic primes allowing norm equal to p. Similar conclusions hold true for octonionic primes.

The reality condition for polynomials associated with Gaussian infinite primes requires that the products of generating prime and its conjugate are present so that the outcome is a real polynomial of second order.

3. Hyper primes

The notion of prime generalizes to hyper-quaternionic and octonionic case. The factorization $n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3)$ implies that any hyper-quaternionic and -octonionic prime has one particualr representative as $(n_0, n_3, 0, ...) = (n_3 + 1, n_3, 0, ...)$, $n_3 = (p-1)/2$ for $p > 2$. $p = 2$ is exceptional: a representation with minimal number of components is given by $(2, 1, 1, 0, ...)$.

Notice that the interpretation of hyper-quaternionic primes (or integers) as four-momenta implies that it is not possible to find rest system for them if one assumes the entire quaternionic prime as four-momentum: only a system where energy is minimum is possible. The introduction of a preferred hyper-complex plane necessary for several reasons- in particular for the possibility to identify standard model quantum numbers in terms of infinite primes- allows to identify the momentum of particle in the preferred plane as the first two components of the hyper prime in fixed coordinate frame. Note that this leads to a universal spectrum for mass squared.

For time like hyper-primes the momentum is always time like for hyper-primes. In this case it is possible to find a rest frame by applying a hyper-primeness preserving G_2 transformation so that the resulting momentum has no component in the preferred frame. As a matter fact, SU(3) rotation is enough for a suitable choice of SU(3). These transformations form a discrete subgroup of SU(3) since hyper-integer property must be preserved. Massless states correspond to a null norm for the corresponding hyper integer unless one allows also tachyonic hyper primes with minimal representatives $(n_3, n_3 - 1, 0, ...)$, $n_3 = (p-1)/2$. Note that Gaussian primes with $p \ mod 4 \ = 1$ are representable as space-like primes of form $(0, n_1, n_2, 0)$: $n_1^2 + n_2^2 = p$ and would correspond to genuine tachyons. Space-like primes with $p \ mod \ 4 = 3$ have at least 3 non-vanishing components which are odd integers.

The notion of "irreducible" (see Appendix of [K87]) is defined as the equivalence class of primes related by a multiplication with a unit and is more fundamental than that of prime. All Lorentz boosts of a hyper prime combine to form an irreducible. Note that the units cannot correspond to real particles in corresponding arithmetic quantum field theory.

If the situation for $p > 2$ is effectively 2-dimensional in the sense that it is always possible to transform the hyper prime to a 2-component form by multiplying it by a suitable unit representing Lorentz boost, the theory for time-like hyper primes effectively reduces to the 2-dimensional hyper-complex case when irreducibles are chosen to belong to H_2. The physical counterpart for the choice of H_2 would be the choice of the plane of longitudinal polarizations, or equivalently, of quantization axis for spin. This hypothesis is physically highly attractive since it would imply number theoretic universality and conform with the effective 2-dimensionality. Of course, the hyper-octonionic primes related by $SO(7,1)$ boosts need not represent physically equivalent states.

Also the rigorous notion of hyper primeness seems to require effective 2-dimensionality. If effective 2-dimensionality holds true, hyper integers have a decomposition to a product of hyper primes multiplied by a suitable unit. The representation is obtained by Lorentz boosting the hyper integer first to a 2-component form and then decomposing it to a product of hyper-complex primes.

Hyper-octonionic infinite primes

The infinite-primes associated with hyper-octonions are the most natural ones physically because of the underlying Lorentz invariance. It is however not possible to interpret them as as 8-momenta with mass squared equal to prime. The proper identification of standard model quantum numbers will be discussed later.

1. Should infinite primes be commutative and associative?

The basic objections against (hyper-)quaternionic and (hyper-)octonionic infinite primes relate to the non-commutativity and non-associativity.

In the case of quaternionic infinite primes non-commutativity, and in the case of octonionic infinite primes also non-associativity, might be expected to cause difficulties in the definition of X. Fortunately, the fact that all conjugates of a given finite prime appear in the product defining X, implies that the contribution from each irreducible with a given norm p is real and X is real. Therefore the multiplication and division of X with quaternionic or octonionic primes is a well-defined procedure, and generating infinite primes are well-defined apart from the degeneracy due to non-commutativity and non-associativity of the finite number of lower level primes. Also the products of infinite primes are well defined, since by the reality of X it is possible to tell how the products AB and BA differ. Of course, also infinite primes representing physical states containing infinite numbers of fermions and bosons are possible and infinite primes of this kind must be analogous to generators of a free algebra for which AB and BA are not related in any manner.

The original idea was that infinite hyper-octonionic primes could be mapped to polynomials and one could assign to these space-time surfaces in analogy with the identification of surfaces as zero locii of polynomals. Although this idea has been given up, it is good to make clear its problematic aspects.

(a) The sums of products of monomials of generating infinite primes define higher level infinite primes and also here non-commutativity and associativity cause potential technical difficulties. The assignment of a monomial to a quaternionic or octonionic infinite prime is not unique since the rational obtained by dividing the finite part mr with the integer n associated with infinite part can be defined either as $(1/n) \times mr$ or $mr \times (1/n)$ and the resulting non-commuting rationals are different.

(b) If the polynomial associated with infinite prime has real-rational coefficients, these difficulties do not appear. The problem is that the polynomials as such would not contain information about the number field in question.

(c) Commutativity requirement for infinite primes allows real-rationals or possibly algebraic extensions of them as the coefficients of the polynomials formed from hyper-octonionic infinite primes. If only infinite primes with complex rational coefficients are allowed and only the vacuum state $V_{\pm} = X \pm 1$ involving product over all primes of the number field, would reveal the number field. One could thus construct the generating infinite primes using the notion of hyper-octonionic prime for any algebraic extension of rationals.

The idea about mapping of infinite primes to polynomials in turn defining space-time surfaces is non-realistic. The recent view is more abstract and based on the mapping of wave functions in the space of hyper-octonion units assignable to single imbedding space point by its number-theoretic anatomy and a further mapping of quantum numbers to the geometry of space-time surface by the coupling of the modified Dirac action to the quantum numbers via measurement interaction. In this approach one cannot assume commutativity of hyper-octonionic primes at any level. The problems due to non-commutativity and non-associativity are however circumvented by assuming that permutations and associations of are represented as phase factors and therefore do not change the quantum state. This means the introduction of association statistics besides permutation statistics. Besides Fermi and Bose statistics one can consider braid statistics. Note that Fermi statistics makes sense only when the fermionic finite primes appearing in the state do not commute.

2. The construction recipe for hyper-octonionic infinite primes

The following argument represents the construction recipe for the first level hyper-octonionic primes without the restriction to rational infinite primes. If the reduction is possible always by a suitable G_2 rotation then the construction of the infinite primes analogous to bound states is obtained in trivial manner from that for rational variants of these primes. The recipe generalizes to the higher levels in trivial manner.

Each hyper-octonionic prime has a number of conjugates obtained by applying transformations of G_2 respecting the property of being hyper-octonionic integer.

(a) The number of conjugates of given finite prime depends on the number of non-vanishing components of the the prime with norm p in the minimal representation having minimal energy. Several primes with a given norm p not related by a multiplication with unit or by automorphism are in principle possible. The degeneracy is determined by the number of elements of a subgroup of Galois group acting non-trivially on the prime.

Galois group contains the permutations of 7 imaginary units and 7 conjugations of units consistent with the octonionic product. X is proportional to $p^{N(p)}$ where $N(p)$ in principle depends on p.

There could exist also G_2 transformations which change the number of components of the infinite prime. They satisfy tight number theoretical constraints since the quantity $\sum_{i=1}^{7} n_i^2$ must be preserved. For instance, for the transformation from standard form with two components to that with more than two components one has $n_1^2(i) = \sum_k n_k^2(f)$. For the transformation from 2-component prime to 3-component prime one has a condition characterizing Pythagorean triangle. One can however consider also a situation when no such G_2 transformation exist so that one has several G_2 orbits corresponding to the same rational prime.

The construction itself would be relatively straightforward. Consider first the construction of the "vacuum" primes.

(a) In the case of ordinary infinite primes there are two different vacuum primes $X \pm 1$. This is the case also now. I turns out that this degeneracy corresponds to the spin and orbital degrees of freedom for the spinor fields of WCW.

(b) The product X of all hyper-octonionic irreducibles can be regarded as the counterpart of Dirac vacuum in a rather concrete sense. Moreover, in the hyper-quaternionic and octonionic case the norm of X is analogous to the Dirac determinant of a fermionic field theory with prime valued mass spectrum and integer valued momentum components. The inclusion of only irreducible eliminates from the infinite product defining Dirac determinant product over various Lorentz boosts of $p^k\gamma_k - m$.

(c) Infinite prime property requires that X must be defined by taking one representative from each G_2 equivalence class representing irreducible and forming the product of all its G_2 conjugates. The standard representative for the hyper-octonionic primes can be taken to be time-like positive energy prime unless one allows also tachyonic primes in which case a natural representative has a vanishing real component. The conjugates of each irreducible appear in X so for a given norm p the net result is real for each rational prime p.

The construction of non-vacuum primes is equally straighforward.

(a) If the conjectured effective 2-dimensionality holds true, it is enough to construct hyper-complex primes first. To the finite hyper-complex primes appearing in these infinite primes one can apply transformations of G_2 mapping hyper-octonionic integers to hyper-octonionic integers. The infinite prime would have degeneracy defined by the product of G_2 orbits of finite primes involved. Every finite prime would be like particle possessing finite number of quantum states. If there are several G_2 orbits corresponding to the same finite prime exist they must be also included and the conjectured effective 2-dimensionality fails.

(b) An interesting question is what happens when the finite part of an infinite prime is multiplied by light like integer k. The first guess is that k describes the presence of a massless particle. If the resulting infinite integer is multiplied with conjugates $k_{c,i}$ of k

an integer of form $\prod_i k_{c,i} mX/n$ having formally zero norm results. It would thus seem that there is a kind of gauge invariance in the sense that infinite primes for which both finite and infinite part are multiplied with the same light-like primes, are divisors of zero and correspond to gauge degrees of freedom. This conclusion is supported by the interpretation of the projection of infinite prime to the preferred hyper-complex plane as momentum of particle in a preferred M^2 plane assigned by the hierarchy of Planck constants to each CD and also required by the p-adicization.

(c) More complex infinite hyper-octonionic primes can be constructed from rational hyper-complex and complex infinite primes using a representation in terms of polynomials and then acting on the finite primes appearing in their expression by elements of G_2 preserving integer property. This construction works at all levels of the hierarchy and one might hope that it is all that is needed. If there are several G_2 orbits for given finite prime p one encounters a problem since hyper-octonionic primes with more than 2 components do not allow associative and commutative polynomial representations. The interpretation as bound states is suggestive.

11.4.4 How to interpret the infinite hierarchy of infinite primes?

From the foregoing it should be clear that infinite primes might play key role in quantum physics. One can even consider the possibility that physics reduces to a generalized number theory, and that infinite primes are crucial for understanding mathematically consciousness and cognition. Of course, one must leave open the question whether infinite primes really provide really the mathematics of consciousness or whether they are only a beautiful but esoteric mathematical construct. In this spirit the following subsections give only different points of view to the problem with no attempt to a coherent overall view.

Infinite primes and hierarchy of super-symmetric arithmetic quantum field theories

Infinite primes are a generalization of the notion of prime. They turn out to provide number theoretic correlates of both free, interacting and bound states of a super-symmetric arithmetic quantum field theory. It turns also possible to assign to infinite prime space-time surface as a geometric correlate although the original proposal for how to achieve this failed. Hence infinite primes serve as a bridge between classical and quantum and realize quantum classical correspondence stating that quantum states have classical counterparts, and has served as a basic heuristic guideline of TGD. More precisely, the natural hypothesis is that infinite primes code for the ground states of super-symplectic representations (for instance, ordinary particles correspond to states of this kind).

1. Generating infinite primes as counterparts of Fock states of a super-symmetric arithmetic quantum field theory

The basic construction recipe for infinite primes is simple and generalizes to the quaternionic case.

(a) Form the product of all primes and call it X:

$$X = \prod_p p \quad .$$

(b) Form the vacuum states

$$V_{\pm} = X \pm 1 \quad .$$

(c) From these vacua construct all *generating* infinite primes by the following process. Kick out from the Dirac sea some negative energy fermions: they correspond to a product s of first powers of primes: $V \rightarrow X/s \pm s$ (s is thus square-free integer). This state represents a state with some fermions represented as holes in Dirac sea but no bosons. Add bosons by multiplying by integer r, which decomposes into parts as $r = mn$: m corresponding to bosons in X/s is product of powers of primes dividing X/s and n corresponds to bosons in s and is product of powers of primes dividing s. This step can be described as $X/s \pm s \rightarrow mX/s \pm ns$.

Generating infinite primes are thus in one-one correspondence with the Fock states of a super-symmetric arithmetic quantum field theory and can be written as

$$P_{\pm}(m,n,s) = \frac{mX}{s} \pm ns \ \ ,$$

where X is product of all primes at previous level. s is square free integer. m and n have no common factors, and neither m and s nor n and X/s have common factors.

The physical analog of the process is the creation of Fock states of a super-symmetric arithmetic quantum field theory. The factorization of s to a product of first powers of primes corresponds to many-fermion state and the decomposition of m and n to products of powers of prime correspond to bosonic Fock states since p^k corresponds to k-particle state in arithmetic quantum field theory.

2. More complex infinite primes as counterparts of bound states

Generating infinite primes are not all that are possible. One can construct also polynomials of the generating primes and under certain conditions these polynomials are non-divisible by both finite primes and infinite primes already constructed. As found, the conjectured effective 2-dimensionality for hyper-octonionic primes allows the reduction of polynomial representation of hyper-octonionic primes to that for hyper-complex primes. This would be in accordance with the effective 2-dimensionality of the basic objects of quantum TGD.

The physical counterpart of n:th order irreducible polynomial is as a bound state of n particles whereas infinite integers constructed as products of infinite primes correspond to non-bound but interacting states. This process can be repeated at the higher levels by defining the vacuum state to be the product of all primes at previous levels and repeating the process. A repeated second quantization of a super-symmetric arithmetic quantum field theory is in question.

The infinite primes represented by irreducible polynomials correspond to quantum states obtained by mapping the superposition of the products of the generating infinite primes to a superposition of the corresponding Fock states. If complex rationals are the coefficient field for infinite integers, this gives rise to states in a complex Hilbert space and irreducibility corresponds to a superposition of states with varying particle number and the presence of entanglement. For instance, the superpositions of several products of type $\prod_{i=1,..,n} P_i$ of n generating infinite primes are possible and in general give rise to irreducible infinite primes decomposing into a product of infinite primes in algebraic extension of rationals.

3. How infinite rationals correspond to quantum states and space-time surfaces?

The most promising answer to the question how infinite rationals correspond to space-time surfaces is discussed in detail in the next section. Here it is enough to give only the basic idea.

(a) In zero energy ontology hyper-octonionic units (in real sense) defined by ratios of infinite integers have an interpretation as representations for pairs of positive and negative energy states. Suppose that the quantum number combinations characterizing positive and negative energy quantum states are representable as superpositions of real units defined by ratios of infinite integers at each point of the space-time surface. If this is true, the quantum classical correspondence coded by the measurement interaction term of the modified Dirac action maps the quantum numbers also to space-time geometry and implies a correspondence between infinite rationals and space-time surfaces.

(b) The space-time surface associated with the infinite rational is in general not a union of the space-time surfaces associated with the primes composing the integers defining the rational. There the classical description of interactions emerges automatically. The description of classical states in terms of infinite integers would be analogous to the description of many particle states as finite integers in arithmetic quantum field theory. This mapping could in principle make sense both in real and p-adic sectors of WCW.

The finite primes which correspond to particles of an arithmetic quantum field theory present in Fock state, correspond to the space-time sheets of finite size serving as the building blocks of the space-time sheet characterized by infinite prime.

4. What is the interpretation of the higher level infinite primes?

Infinite hierarchy of infinite primes codes for a hierarchy of Fock states such that many-particle Fock states of a given level serve as elementary particles at next level. The unavoidable conclusion is that higher levels represent totally new physics not described by the standard quantization procedures. In particular, the assignment of fermion/boson property to arbitrarily large system would be in some sense exact. Topologically these higher level particles could correspond to space-time sheets containing many-particle states and behaving as higher level elementary particles.

This view suggests that the generating quantum numbers are present already at the lowest level and somehow coded by the hyper-octonionic primes taking the role of momentum quantum number they have in arithmetic quantum field theories. The task is to understand whether and how hyper-octonionic primes can code for quantum numbers predicted by quantum TGD.

The quantum numbers coding higher level states are collections of quantum numbers of lower level states. At geometric level the replacement of the coefficients of polynomials with rational functions is the equivalent of replacing single particle states with new single particle states consisting of many-particle states.

Infinite primes, the structure of many-sheeted space-time, and the notion of finite measurement resolution

The mapping of infinite primes to space-time surfaces codes the structure of infinite prime to the structure of space-time surface in a rather non-implicit manner, and the question arises about the concrete correspondence between the structure of infinite prime and topological structure of the space-time surface. It turns out that the notion of finite measurement resolution is the key concept: infinite prime characterizes angle measurement resolution. This gives a direct connection with the p-adicization program relying also on angle measurement resolution as well as a connection with the hierarchy of Planck constants. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory.

1. The first intuitions

The concrete prediction of the general vision is that the hierarchy of infinite primes should somehow correspond to the hierarchy of space-time sheets or partonic 2-surfaces if one accepts the effective 2-dimensionality. The challenge is to find space-time counterparts for infinite primes at the lowest level of the hierarchy.

One could hope that the Fock space structure of infinite prime would have a more concrete correspondence with the structure of the many-sheeted space-time. One might that the space-time sheets labeled by primes p would directly correspond to the primes appearing in the definition of infinite prime. This expectation seems to be too simplistic.

(a) What seems to be a safe guess is that the simplest infinite primes at the lowest level of the hierarchy should correspond to elementary particles. If inverses of infinite primes correspond to negative energy space-time sheets, this would explain why negative energy particles are not encountered in elementary particle physics.

(b) More complex infinite primes at the lowest level of the hierarchy could be interpreted in terms of structures formed by connecting these structures by join along boundaries bonds to get space-time correlates of bound states. Even simplest infinite primes must correspond to bound state structures if the condition that the corresponding polynomial has real-rational coefficients is taken seriously.

Infinite primes at the lowest level of hierarchy correspond to several finite primes rather than single finite prime. The number of finite primes is however finite.

(a) A possible interpretation for multi-p property is in terms of multi-p p-adic fractality prevailing in the interior of space-time surface. The effective p-adic topology of these space-time sheets would depend on length scale. In the longest scale the topology would correspond to p_n, in some shorter length scale there would be smaller structures with $p_{n-1} < p_n$-adic topology, and so on... . A good metaphor would be a wave containing ripples, which in turn would contain still smaller ripples. The multi-p p-adic fractality would be assigned with the 4-D space-time sheets associated with elementary particles. The concrete realization of multi-p p-adicity would be in terms of infinite integers coming as power series $\sum x_n N^n$ and having interpretation as p-adic numbers for any prime dividing N.

(b) Effective 2-dimensionality would suggest that the individual p-adic topologies could be assigned with the 2-dimensional partonic surfaces. Thus infinite prime would characterize at the lowest level space-time sheet and corresponding partonic 2-surfaces. There are however reasons to think that even single partonic 2-surface corresponds to a multi-p p-adic topology.

2. Do infinite primes code for the finite measurement resolution?

The above describe heuristic picture is not yet satisfactory. In order to proceed, it is good to ask what determines the finite prime or set of them associated with a given partonic 2-surface. It is good to recall first the recent view about the p-adicization program relying crucially on the notion of finite measurement resolution.

(a) The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M and measurement resolution does not depend on on the value of M. Situation is different if one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.

(b) One can also interpret N as a p-adic integer. For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M$ mod p^n. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$ if modular arithetics is used to define the the measurement resolution. This works at the first level of the hierarchy but not at higher levels. The alternative manner to assign a finite measurement resolution to M/N for given p is as $\Delta\phi = 2\pi|N/M|_p = 2\pi/p^n$. In this case the small fermionic part of the infinite prime would fix the measurement resolution. The argument below shows that only this option works also at the higher levels of hierarchy and is therefore more plausible.

(c) p-Adicization conditions in their strong form require that the notion of integration based on harmonic analysis [A10] in symmetric spaces [A31] makes sense even at the level of partonic 2-surfaces. These conditions are satisfied if the partonic 2-surfaces in a given measurement resolution can be regarded as algebraic continuations of discrete surfaces whose points belong to the discrete variant of the $\delta M^4_\pm \times CP_2$. This condition is extremely powerful since it effectively allows to code the geometry of partonic 2-surfaces by the geometry of finite sub-manifold geometries for a given measurement resolution. This condition assigns the integer N to a given partonic surface and all primes appearing as factors of N define possible effective p-adic topologies assignable to the partonic 2-surface.

How infinite primes could then code for the finite measurement resolution? Can one identify the measurement resolution for $M/N = M/(Rp^n)$ as $\Delta\phi = ((M/R)\ mod\ p^n) \times 2\pi/p^n$ or as $\Delta\phi = 2\pi/p^n$? The following argument allows only the latter option.

(a) Suppose that p-adic topology makes sense also for infinite primes and that state function reduction selects power of infinite prime P from the product of lower level infinite primes defining the integer N in M/N. Suppose that the rational defined by infinite integer defines measurement resolution also at the higher levels of the hierarchy.

(b) The infinite primes at the first level of hierarchy representing Fock states are in one-one correspondence with finite rationals M/N for which integers M and N can be chosen to characterize the infinite bosonic part and finite fermionic part of the infinite prime. This correspondence makes sense also at higher levels of the hierarchy but M and N are infinite integers. Also other option obtained by exchanging "bosonic" and "fermionic" but later it will be found that only the first identification makes sense.

(c) The first guess is that the rational M/N characterizing the infinite prime characterizes the measurement resolution for angles and therefore partially classifies also the finite sub-manifold geometry assignable to the partonic 2-surface. One should define what $M/N = ((M/R)\ mod\ P^n) \times P^{-n}$ is for infinite primes. This would require expression of M/R in modular arithmetics modulo P^n. This does not make sense.

(d) For the second option the measurement resolution defined as $\Delta\phi = 2\pi|N/M|_P = 2\pi/P^n$ makes sense. The Fourier basis obtained in this manner would be infinite but all states $exp(ik/P^n)$ would correspond in real sense to real unity unless one allows k to be infinite P-adic integer smaller than P^n and thus expressible as $k = \sum_{m<n} k_m P^m$, where k_m are infinite integers smaller than P. In real sense one obtains all roots $exp(iq2\pi)$ of unity with $q < 1$ rational. For instance, for $n = 1$ one can have $0 < k/P < 1$ for a suitably chosen infinite prime k. Thus one would have essentially continuum theory at higher levels of the hierarchy. The purely fermionic part N of the infinite prime would code for both the number of Fourier components in discretization for each power of prime involved and the ratio characterize the angle resolution.

The proposed relation between infinite prime and finite measurement resolution implies very strong number theoretic selection rules on the reaction vertices.

(a) The point is that the vertices of generalized Feyman diagrams correspond to partonic 2-surfaces at which the ends of light-like 3-surfaces describing the orbits of partonic 2-surfaces join together. Suppose that the partonic 2-surfaces appearing a both ends of the propagator lines correspond to same rational as finite sub-manifold geometries. If so, then for a given p-adic effective topology the integers assignable to all lines entering the vertex must contain this p-adic prime as a factor. Particles would correspond to integers and only the particles having common prime factors could appear in the same vertex.

(b) In fact, already the work with modelling dark matter [K27] led to ask whether particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It also seemed natural to assume that that only the space-time sheets containing common primes in this collection can interact. This inspired the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The possibility of multi-p p-adicity raises the question about how to fix the p-adic prime characterizing the mass of the particle. The mass scale of the contribution of a given throat to the mass squared is given by $p^{-n/2}$, where $T = 1/n$ corresponds to the p-adic temperature of throat. Hence the dominating contribution to the mass squared corresponds to the smallest prime power p^n associated with the throats of the particle. This works if the integers characterizing other particles than graviton are divisible by the gravitonic p-adic prime or a product of p-adic primes assignable to graviton. If the smallest power p^n assignable to the graviton is large enough, the mass of graviton is consistent with the empirical bounds on it. The same consideration applies in the case of photons. Recall that the number theoretically very natural condition that in zero energy ontology the number of generalized Feynman graphs contributing to a given process is finite is satisfied if all particles have a non-vanishing but arbitrarily small p-adic thermal mass [K28] .

3. Interpretational problem

The identification of infinite prime as a characterizer of finite measurement resolution looks nice but there is an interpretational problem.

(a) The model characterizing the quantum numbers of WCW spinor fields to be discussed in the next section involves a pair of infinite primes P_+ and P_- corresponding to the two vacuum primes $X \pm 1$. Do they correspond to two different measurement resolutions perhaps assignable to CD and CP_2 degrees of freedom?

(b) Different measurement resolutions in CD and CP_2 degrees of freedom need not be not a problem as long as one considers only the discrete variants of symmetric spaces involved. What might be a problem is that in the general case the p-adic primes associated with CD and CP_2 degrees of freedom would not be same unless the integers N_+ and N_- are assumed to have have same prime factors (they indeed do if $p^0 = 1$ is formally counted as prime power factors).

(c) The idea of assigning different p-adic effective topologies to CD and CP_2 does not look attractive. Both CD and CP_2 and thus also partonic 2-surface could however possess simultaneously both p-adic effective topologies. This kind of option might make sense since the integers representable as infinite powers series of integer N can be regarded as p-adic integers for all prime factors of N. As a matter fact, this kind of multi-p p-adicity could make sense also for the partonic 2-surfaces characterized by a measurement resolution $\Delta\phi = 2\pi M/N$. One would have what might be interpreted as N_+N_--adicity.

(d) It will be found that quantum measurement means also the measurement of the p-adic prime selecting same p-adic prime from N_+ and N_-. If $N_\pm$ is divisible only by $p^0 = 1$, the corresponding angle measurement resolution is trivial. From the point of view of consciousness state function reduction selects also the p-adic prime characterizing the cognitive representation which is very natural since quantum superpositions of different p-adic topologies are not natural physically.

How the hierarchy of Planck constants could relate to infinite primes and p-adic hierarchy?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K99] , the dark matter hierarchy characterized by increasing values of $\hbar$ [K27] , the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related. Number theoretical considerations give hopes about developing a more quantitative vision about the relationship between these hierarchies, in particular between the hierarchy of infinite primes, p-adic length scale hierarchy, and the hierarchy if Planck constants.

If infinite primes code for the hierarchy of measurement resolutions, the correlations between the p-adic hierarchy and the hierarchy of Planck constants indeed suggest themselves and allow also to select between two interpretations for the fact that two infinite primes N_+ and N_- are needed to characterize elementary particles (see the next section).

Recall that the hierarchy of Planck constants in the most general situation corresponds to a replacement M^4 and CP_2 factors of the imbedding space with singular coverings and factor spaces. The condition that Planck constant is integer valued allows only singular coverings characterized by two integers n_a *resp.* n_b assignable to CD *resp.* CP_2. This condition also guarantees that a given value of Planck constant corresponds to only a finite number of pages of the "Big Book" and therefore looks rather attractive mathematically. This option also forces evolution as a dispersion to the pages of the books characterized by increasing values of Planck constant.

Concerning the correspondence between the hierarchy of Planck constants and p-adic length scale hierarchy there seems to be only single working option. The following assumptions make precise the relationship between finite measurement resolution, infinite primes and hierarchy of Planck constants.

(a) Measurement resolution CD *resp.* CP_2 degrees of freedom is assumed to correspond to the rational M_+/N_+ *resp.* M_-/N_-. $N_\pm$ is identified as the integer assigned to the fermionic part of the infinite integer..

(b) One must always fix the consideration to a fixed p-adic prime. This process could be regarded as analogous to fixing the quantization axes and p would also characterize the p-adic cognitive space-time sheets involved. The p-adic prime is therefore same for CD and CP_2 degrees of freedom as required by internal consistency.

(c) The relationship to the hierarchy of Planck constants is fixed by the identifications $n_a = n_+(p)$ and $n_b = n_-(p)$ so that the number of sheets of the covering equals to the number of bosons in the fermionic mode p of the quantum state defined by infinite prime.

(d) A physically attractive hypothesis is that number theoretical bosons *resp.* fermions correspond to WCW orbital *resp.* spin degrees of freedom. The first ones correspond to the symplectic algebra [A60, A33, A32] of WCW and the latter one to purely fermionic degrees of freedom.

Consider now the basic consequences of these assumptions from the point of view of physics and cognition.

(a) Finite measurement resolution reduces for a given value of p to

$$\Delta\phi = \frac{2\pi}{p^{n_\pm(p)+1}} = \frac{2\pi}{p^{n_{a/b}}} \ ,$$

where $n_\pm(p) = n_{a/b} - 1$ is the number of bosons in the mode p in the fermionic part of the state. The number theoretical fermions and bosons and also their probably existing physical counterparts are necessary for a non-trivial angle measurement resolution. The value of Planck constant given by

$$\frac{\hbar}{\hbar_0} = n_a n_b = (n_+(p)+1) \times (n_-(p)+1)$$

tells the total number of bosons added to the fermionic mode p assigned to the infinite prime.

(b) The presence of $\hbar > \hbar_0$ partonic 2-surfaces is absolutely essential for a Universe able to measure its own state. This is in accordance with the interpretation of hierarchy of Planck constants in TGD inspired theory of consciousness. One can also say that $\hbar = 0$ sector does not allow cognition at all since $N_\pm = 1$ holds true. For given p $\hbar = n_a n_b = 0$ means that given fermionic prime corresponds to a fermion in the Dirac sea meaning $n_\pm(p) = -1$. Kicking out of fermions from Direac sea makes possible cognition. For purely bosonic vacuum primes one has $\hbar = 0$ meaning trivial measurement resolution so that the physics is purely classical and would correspond to the purely bosonic sector of the quantum TGD.

(c) For $\hbar = \hbar_0$ the number of bosons in the fermionic state vanishes and the general expression for the measurement resolution reduces to $\Delta\phi = 2\pi/p$. When one adds $n_\pm(p)$ bosons to the fermionic part of the infinite prime, the measurement resolution increases from $\Delta\phi = 2\pi/p$ to $\Delta\phi = 2\pi/p^{n_\pm(p)+1}$. Adding a sheet to the covering means addition of a number theoretic boson to the fermionic part of infinite prime. The presence of both number theoretic bosons and fermions with the values of p-adic prime $p_1 \neq p$ does not affect the measurement resolution $\Delta\phi = 2\pi/p^n$ for a given prime p.

(d) The resolutions in CD and CP_2 degrees of freedom correspond to the same value of the p-adic prime p so that one has dicretizations based on $\Delta\phi = 2\pi/p^{n_a}$ in CD degrees of freedom and $\Delta\phi = 2\pi/p^{n_b}$ in CP_2 degrees of freedom. The finite sub-manifold geometries make sense in this case and since the effective p-adic topology is same, the continuation to continuous p-adic partonic 2-surface is possible.

p-Adic thermodynamics involves the p-adic temperature $T = 1/n$ as basic parameter and the p-adic mass scale of the particle comes as $p^{-(n+1)/2}$. The natural question is whether one could assume the relation $T_\pm = 1/(n_\pm(p)+1)$ between p-adic temperature and infinite prime and thus the relations $T_a = 1/n_a(p)$ and $T_b = 1/n_b(p)$. This identification is not consistent with the recent physical interpretation of the p-adic thermodynamics nor with the view about dark matter hierarchy and must be given up.

(a) The minimal non-trivial measurement resolution with $n_i = 1$ and $\hbar = \hbar_0$ corresponds to the p-adic temperature $T_i = 1$. p-Adic mass calculations indeed predict $T = 1$ for fermions for $\hbar = \hbar_0$. In the case of gauge bosons $T \geq 2$ is favored so that gauge bosons would be dark. This would require that gauge bosons propagate along dark pages of the Big Book and become "visible" before entering to the interaction vertex.

(b) p-Adic thermodynamics also assumes same p-adic temperature in CD and CP_2 degrees of freedom but the proposed identification allows also different temperatures. In principle the separation of the super-conformal degrees of freedom of CD and CP_2 might allow different p-adic temperatures. This would assign to different p-adic mass scales to the particles and the larger mass scale should give the dominant contribution.

(c) For dark particles the p-adic mass scale would be by a factor $1/\sqrt{p}^{n_i(p)-1}$ lower than for ordinary particles. This is in conflict with the assumption that the mass of the particle does not depend on $\hbar$. This prediction would kill completely the recent vision about the dark matter.

11.4.5 How infinite primes could correspond to quantum states and space-time surfaces?

The hierarchy of infinite primes is in one-one correspondence with a hierarchy of second quantizations of an arithmetic quantum field theory. The additive quantum number in question is energy like quantity for ordinary primes and given by the logarithm of prime whereas p-adic length scale hypothesis suggests that the conserved quantity is proportional to the inverse of prime or its square root. For infinite primes at the first level of hierarchy these quantum numbers label single particles states having interpretation as ordinary elementary particles. For octonionic and hyper-octonionic primes the quantum number is analogous to a momentum with 8 components. The question is whether these number theoretic quantum numbers could have interpretation as genuine quantum numbers. Quantum classical correspondence raises another question. Is it possible to label space-time surfaces by infinite primes? Could this correspondence be even one-to-one?

I have considered these questions already more than decade ago. The discussion at that time was necessarily highly speculative and just a mathematical exercise. After that time however a lot of progress has taken place in quantum TGD and it is highly interaction to see what comes out from the interaction of the notion of infinite prime with the notions of zero energy ontology and generalized imbedding space, and with the recent vision about how measurement interaction term in the Kähler allows to code information about quantum numbers to the space-time geometry. The possibility of this coding allows to simplify the discussion dramatically. If one can map infinite hyper-octonionic primes to quantum numbers of the standard model naturally, then the their map of to the geometry of space-time surfaces realizes the coding of space-time surfaces by infinite primes (and more generally by integers and rationals). Also a detailed realization of number theoretic Brahman=Atman identity emerges as an outcome.

A brief summary about various moduli spaces and their symmetries

It is good to sum up the number theoretic symmetries before trying to construct an overall view about the situation. Several kinds of number theoretical symmetry groups are involved corresponding to symmetries in the moduli spaces of hyper-octonionic and hyper-quaternionic structures, symmetries mapping hyper-octonionic primes to hyper-octonionic primes, and translations acting in the space of causal diamonds (CDs) and shifting. The moduli space for CDs labeled by pairs of its tips that its pairs of points of $M^4 \times CP_2$ is also in important role.

(a) The basic idea is that color $SU(3) \subset G_2$ acts as automorphisms of hyper-octonion structure with a preferred imaginary unit. $SO(7,1)$ acts as symmetries in the moduli space of hyper-octonion structures. Associativity implies symmetry breaking so that only hyper-quaternionic structures are considered and $SO(3,1) \times SO(4)$ acts as symmetries of the moduli space for hyper-quaternionic structures.

(b) CP_2 parameterizes the moduli space of hyper-quaternionic structures induced from a given hyper-octonionic structure with preferred imaginary unit.

(c) Color group $SU(3)$ is the analog of Galois group for the extension of reals to octonions and has a natural action on the decompositions of rational infinite primes to hyper-octonionic infinite primes. For given hyper-octonionic prime one can identify a subgroup of SU(3) generating a finite set of hyper-octonionic primes for it at sphere S^7. This suggests wave function at the orbit of given hyper-octonionic prime in turn generalizing to wave functions in the space of infinite primes.

(d) Four-momenta correspond to translational degrees of freedom associated with the preferred points of M^4 coded by the infinite rational (tip of the light-cone). Color quantum numbers in cm degrees of freedom can be assigned to the CP_2 projection of the preferred point of H. As a matter fact, the definition of hyper-octonionic structure involves the choice of origin of M^8 giving rise to the preferred point of H.

These symmetries deserve a more detailed discussion.

(a) The choice of global hyper-octonionic coordinate is dictated only modulo a transformation of $SO(1,7)$ acting as isometries of hyper-octonionic norm and as transformations in moduli space of hyper-octonion structures. $SO(7)$ respects the choice of the real unit. $SO(1,3) \times SO(4)$ acts in the moduli space of global hyper-quaternionic structures identified as sub-structures of hyper-octonionic structure. The choice of global hyper-octonionic structures involves also a choice of origin implying preferred point of H. The M^4 projection of this point corresponds to the tip of CD. Since the integers representing physical states must be hyper-quaternionic by associativity conditions, the symmetry breaking ("number theoretic compactification") to $SO(1,3) \times SO(4)$ occurs very naturally. This group acts as spinor rotations in H picture and as isometries in M^8 picture. The choice of both tips of CD reduces $SO(1,3)$ to $SO(3)$.

(b) $SO(1,7)$ allows 3 different 8-dimensional representations (8_v, 8_s, and $\overline{8}_s$). All these representations must decompose under $SU(3)$ as $1+1+3+\overline{3}$ as little exercise with $SO(8)$ triality demonstrates. Under $SO(6) \cong SU(4)$ the decompositions are $1+1+6$ and $4+\overline{4}$ for 8_v and 8_s and its conjugate. Both hyper-octonion spinors and gamma matrices are identified as hyper-octonion units rather than as matrices. It would be natural to assign to bosonic M^8 primes 8_v and to fermionic M^8 primes 8_s and $\overline{8}_s$. One can distinguish between $8_v, 8_s$ and $\overline{8}_s$ for hyper-octonionic units only if one considers the full $SO(1,3) \times SO(4)$ action in the moduli space of hyper-octonionic structures.

(c) G_2 acts as automorphisms on octonionic imaginary units and $SU(3)$ respects the choice of preferred imaginary unit meaning a choice of preferred hyper-complex plane $M^4 \subset M^4$. Associativity requires a reduction to hyper-quaternionic primes and implies color confinement in number theoretical and as it turns also in physical sense. For hyper-quaternionic primes the automorphisms restrict to $SO(3)$ which has right/left action of fermionic hyper-quaternionic primes and adjoint action on bosonic hyper-quaternionc primes. The choice of hyper-quaternionic structure is global as opposed to the local choice of hyper-quaternionic tangent space of space-time surface assigning to a point of $HQ \subset HO$ a point of CP_2. $U(2) \subset SU(3)$ leaves invariant given hyper-quaternionic structure which are thus parameterized by CP_2. Color partial waves can be interpreted as partial waves in this moduli space.

Associativity and commutativity or only their quantum variants?

Associativity and commutativity conditions are absolutely essential notions in quantum TGD and also in the mapping of infinite primes to the space-time sheets. Hyper-quaternionicity formulated in terms of the modified gamma matrices defined by Kähler action fixes classical space-time dynamics and a very beautiful algebra formulation of quantum TGD in terms of the complexified local Clifford algebra of imbedding space emerges.

Associativity implies hyper-quaternionicity and commutativity requirement in turn leads to complex rational infinite primes. Since one can decompose complex rational primes to hyper-quaternionic and even hyper-octonionic primes, one might hope that this could allow to represent states which consist of colored constituents. This representations has however the flavor of a formal trick and the considerations related to concrete representations of infinite primes suggest that the rationality of infinite primes might be a too restrictive condition.

A more radical possibility is that physical states are only quantum associative and commutative. In case of associativity this means that they are obtained as quantum superpositions in the space of real units over all possible associations performed for a given product of hyper-octonion primes (for instance, $|A(BC)\rangle + |(AB)C\rangle$). These states would be associative in quantum sense but would not reduce to hyper-quaternionic primes. Also the notion of quantum commutativity makes sense. The fact that mesons are quantum superpositions of quark-antiquark pairs which each corresponds to different pair of hyper-quaternionic primes and are thus not representable classically, suggests that one can require only quantum associativity and quantum commutativity.

The correspondence between infinite primes and standard model quantum numbers

I have considered several candidates for the correspondence between infinite primes and standard model quantum numbers. The confusing aspect has been the dual nature of hyper-octonionic primes. One one hand they could be interpreted as components of 8-D momentum representing perhaps momentum and other quantum numbers. On the other hand, they transform like representations of $SU(3) \subset G_2$ and behave like color singlets and triplets so that the idea about quantum superpositions of infinite primes related by $SU(3)$ action is attractive. The second puzzling feature is that there are two kinds of infinite primes corresponding to two signs for the "small" part of the infinite prime. The following proposal leads to an interpretation for these aspects.

(a) The number of components of hyper-octonionic prime is 8 as is the dimension of the Cartan algebra of the product of Poincare group, color group SU(3) and electro-weak gauge group $SU(2)_L \times U(1)$ defining the quantum numbers of particles. One might therefore dream about a number theoretic interpretation of elementary particle quantum numbers by interpreting hyper-octonionic prime as 8-momentum. This form of the big idea fails. The point is that complexified basis for octonions consists of two color singlets and color triplet and its conjugate. For a given hyper-octonionic prime one can construct new primes by using a subgroup G of SU(3) by definition respecting the property that the values of the components of prime as integers and as a consequence also the modulus squared so that the primes are at sphere S^7. This group is analogous to Galois group. Identifying prime as an element of basis of quantum states, one can form wave functions at the discrete orbit of given prime transforming according to irreducible representations of color group. Triality $t \pm 1$ states correspond to color partial waves associated with quarks and antiquarks and triality $t = 0$ states to gluons and leptons and their color excitations. The states can be chosen to be eigenstates of the preferred hyper-octonionic imaginary unit ie_1. Additive four-momentum could be assigned the M^2 part of the hyper-octonion as will be found. Therefore the construction applies in special but natural coordinates assignable to the particle required also by zero energy ontology and hierarchy of Planck constants as well as by p-adicization program.

(b) This construction gives only the quantum numbers assignable to color partial waves in configuration space degrees of freedom. Also the quantum numbers assignable to imbedding space spinors are wanted. Luckily, there are two kinds of infinite primes, which might be denoted by $P_\pm$ because the sign of the "small" part of the infinite prime can be chosen freely. Super-conformal symmetry [A27] suggests that quantum numbers associated with spinorial and configuration space degrees freedom can be assigned to the infinite primes of these two types.

   i. In the case of spinor degrees of freedom one can restrict the multiplets to those generated by $SU(2)$ subgroup of SU(3) identified as rotation group. The interpretation is in terms of automorphism group of quaternions. Discrete subgroups of SU(2) generate the orbit of given hyper-octonionic prime and one obtains finite number of SU(2) multiplets having interpretation in terms of rotational degrees of freedom associated with the light-cone boundary. In the case of fermions (bosons) only half odd integer (integer) spins are allowed.

   ii. Remarkably, four of the hyper-octonionic units remain invariant under $SU(2)$. Also now only the hyper-complex projection in $M^2 \subset M^4$ can be interpreted as four-momentum in the preferred frame and the interpretation as a counterpart of Dirac equation eliminating four complex non-physical helicities of the imbedding spinor of given chirality. The states of same spin associated with the two spin doublets have interpretation as electro-weak doublets. As a representation of $SU(3)$ electro-weak doublets would correspond to quark and antiquark in color isospin doublet. This leaves two additional quantum numbers assignable to the color isospin singlets. The natural interpretation is in terms of electromagnetic charge and weak isospin. An analogous picture emerges also in the description of super-symmetric QFT limit of TGD [K29] replacing massless particles identified as light-like geodesics of M^4 with light like geodesics of $M^4 \times CP_2$ and assigning to them two quantum numbers in the Cartan algebra of $SU(3)$ and identified as electro-weak charges. Also conformal weight expressible in terms of stringy mass formula allows a description in terms of infinite primes. What is not achieved is the number theoretical description of genus of the partonic 2-surface and wave functions in the moduli space of the partonic 2-surfaces.

(c) In this picture leptons, gauge bosons, and gluons correspond to an infinite prime of type P_+ or P_- whereas quarks as well as color excitations of leptons correspond to a pair of primes of type P_+ and P_-. One can fix the notations by assigning color quantum numbers to P_+ and and spinorial quantum numbers to P_-. Both P_+ and P_- contribute to four-momentum. Each pair of infinite primes of this kind defines a finite-dimensional space of quantum states assignable to the subgroups of $SU(3)$ and $SU(2)$ respecting the prime property. Needless to say, this prediction is extremely powerful and fixes the spectrum of the quantum numbers almost completely!

(d) An interesting question is whether one can require number theoretical color confinement in the sense that the physical states resulting as tensor products of states assignable to a given infinite prime in P_+ are color singlets. This might be necessary to guarantee associativity. G_2 singletness would be even stronger condition but not possible for massless states. What is interesting is that spin and color in well-defined sense separate from each other. One can wonder whether this relates somehow to the spin puzzle of proton meaning that quarks do not seem to contribute to baryonic spin.

(e) The appearance of discrete subgroups of $SU(3)$ and $SU(2)$ strongly suggests a connection with the inclusions of the hyper-finite factors of type II_1 characterized by these subgroups, which are expected to play a fundamental role in quantum TGD. An interesting question is whether also infinite subgroups could be involved. For instance, one can consider the subgroups generated by discrete subgroup and infinite cyclic group and these might be involved with the inclusions for which the index is equal to four. The appearance of these groups suggests also a connection with the hierarchy of Planck constants and one can ask how the singular coverings defining the pages of the book like structure relate to the moduli space of causal diamonds.

The rather unexpected conclusion is that the wave functions in the discrete space defined by infinite primes are able to code for the quantum numbers of configuration space spinor fields and thus for configuration space spinor fields. A fascinating possibility is that even M-matrix- which is nothing but a characterization of zero energy state- could find an elegant formulation as entanglement coefficients associated with the pair of the integer and inverse integer characterizing the positive and negative energy states.

(a) The great vision is that associativity and commutativity conditions fix the number theoretical quantum dynamics completely. Quantum associativity states that the wave functions in the space of infinite primes, integers, and rationals are invariant under associations of finite hyper-octonionic primes ($A(BC)$ and $(AB)C$ are the basic associations), physics requires associativity only apart from a phase factor, in the simplest situation $+1/-1$ but in more general case phase factor. The condition of commutativity poses a more familiar condition implying that permutations induce only a phase factor which is +/- 1 for boson and fermion statistics and a more general phase for quantum group statistics for the anyonic phases, which correspond to nonstandard values of Planck constant in TGD framework. These symmetries induce time-like entanglement for zero energy stats and perhaps non-trivial enough M-matrix.

(b) One must also remember that besides the infinite primes defining the counterparts of free Fock states of supersymmetric QFT, also infinite primes analogous to bound states are predicted. The analogy with polynomial primes illustrates what is involved. In the space of polynomials with integer coefficients polynomials of degree one correspond free single particle states and one can form free many particle states as their products. Higher degree polynomials with algebraic roots correspond to bound states being not decomposable to a product of polynomials of first degree in the field of rationals. Could also positive and negative energy parts of zero energy states form a analog of bound state giving rise to highly non-trivial M-matrix?

How space-time geometry could be coded by infinite primes?

Second key question is whether space-time geometry could be characterized in terms of infinite primes (and integers and rationals in the most general case) and how this is achieved. The question is how the quantum states consisting of fundamental fermions serving as building bricks of elementary particles could be coded by infinite quaternionic integeres to which one can assign ordinary finite quaternionic primes.

The basic idea is roughly that at the first level of the hierarchy the finite primes appearing as building blocks of infinite prime correspond to structures formed by pairs or wormhole contacts assigned with elementary particles.

(a) The partonic orbits defined by wormhole throats could be characterized by finite primes specifying the preferred p-adic topology assignable to the p-adic "cognitive representation" of the throat.

(b) One could assign hyper-quaternionic integer to the real particle as its four-momentum. In this case the mass shell condition would fix the hyper-quaternionic integer to a high extent. All discrete Lorentz boosts of the particle state taking hyper-quaternionic integers to hyper-quaternionic integers would correspond to the same p-adic integer (prime) defined by the length of the Lorentz boosted hyper-quaternionic integer. The p-adic prime characterizing virtual particle would be one of the primes appearing in the factorization of this integer to a product of powers of prime, most naturally the one whose power is largest.

    Note that p-adic length scale hypothesis suggests that the p-adic primes near powers of two are favored for on mass shell particles and perhaps also for the virtual particles.

(c) For fundamental fermions associated with boundaries of string world sheets and appearing as building bricks of particles the masses would vanish on mass shell so that the hyper-quaternionic integer would in this case have vanishing norm.

    The virtual four-momentum assigned to a virtual fermion line as a generalized eigenvalue of Chern-Simons Dirac operator would correspond to hyper-quaternionic integer. In this case p-adic prime would be defined as for physical particles and would depend on the mass of the virtual particle. If the integration over virtual momenta by residue calculus effectively leads to an integral over on mass shell massless virtual momenta with non-physical spinor helicities then also virtual fundamental fermions would correspond to zero norm hyper-quaternionic integers.

(d) The correlation between particle's four-momentum and the p-adic prime characterizing corresponding cognitive representation would be in accordance with quantum classical correspondence.

(e) The hyperquaternionic primes appearing as largest factors in the factorizion of hyper-quaternionic integers assignable with physical particles could be interpreted as building bricks of an infinite hyperquaternionic prime characterizing the many-particle state and at least the boundaries of string world sheets. The idea that p-adic space-time surfaces defined "cognitive representations" as p-adic chart maps of real space-time surfaces and vice versa (as the TGD based definition of p-adic manifolds assumes) suggests that the p-adic primes in question characterize also space-time regions rather than only the boundaries of string world sheets.

A couple of comments about this speculation are in order.

(a) Zero energy ontology implies a hierarchy of CDs within CDs and this hierarchy as well as the hierarchy of space-time sheets corresponds naturally to the hierarchy of infinite primes. One can assign standard model quantum numbers to various partonic 2-surfaces with positive and negative energy parts of the quantum state assignable to the light-like boundaries of CD. Also infinite integers and rationals are possible and the inverses of infinite primes would naturally correspond to elementary particles with negative energy. The condition that zero energy state has vanishing net quantum numbers implies that the ratio of infinite integers assignable to zero energy state equals to real unit in real sense and has has vanishing total quantum numbers.

(b) Neither quantum numbers nor infinite primes coding them cannot characterize the partonic 2-surface itself completely since they say nothing about the deformation of the space-time surface but only about labels characterizing the WCW spinor field. Also the topology of partonic 2-surface fails to be coded. Quantum classical correspondence however suggests that this correspondence could be possible in a weaker sense. In the Gaussian approximation for functional integral over the world of classical worlds space-time surface and thus the collection of partonic 2-surfaces is effectively replaced with the one corresponding to the maximum of Kähler function, and in this sense one-one correspondence is possible unless the situation is non-perturbative. In this case the physics implied by the hierarchy of Planck constants could however guarantee uniqueness.

One of the basic ideas behind the identification of the dark matter as phases with non-standard value of Planck constant is that when perturbative description of the system fails, a phase transition increasing the value of Planck constant takes place and makes perturbative description possible. Geometrically this phase transition means a leakage to another sector of the imbedding space realized as a book like structure with pages partially labeled by the values of Planck constant. Anyonic phases and fractionization of quantum numbers is one possible outcome of this phase transition. An interesting question is what the fractionization of the quantum numbers means number theoretically.

How to achieve consistency with p-adic mass formula

The first argument against the proposal that infinite primes could code for four-momentum in preferred coordinates is that the logarithms of finite primes and even less those of hyper-octonionic primes are natural from the point of view of p-adic mass calculations predicting that the mass squared of particle behaves as $1/p$ for $T_p = 1$ (fermions) and $1/p^2$ for $T_p = 1/2$ (gauge bosons). This difficulty might be circumvented.

1. Ordinary primes

Consider first ordinary primes for which the inverse always exists.

(a) One can map finite primes p to phase factors $exp(i2\pi/p)$. The roots of unity play the role of primes in the decomposition of the roots of unity $exp(i2\pi/n)$, $n = \prod_i p_i^{n_i}$. $1/n$ is expressible as a sum of form

$$\begin{aligned} \frac{1}{n} &= \sum_i P_i \ , \\ P_i &= \frac{k_i}{p_i^{n_i}} \ . \end{aligned} \quad (11.4.23)$$

giving

$$exp(\frac{i2\pi}{n}) = exp(i2\pi \sum_i P_i) = exp(i2\pi \sum_i \frac{k_i}{p_i^{n_i}}) \ . \quad (11.4.24)$$

Apart from a common normalization factor one can interpret the coefficients P_i as energy like quantities assigned to the single particle states. The power $p_i^{n_i}$ would correspond to various p-adic inverse temperature $1/T_p = 2n_i$ in this expansion.

(b) The representation in terms of phase factors is not unique since P_i^k and $P_i^k + np_i^k$ define the same phase. This non-uniqueness is completely analogous to the non-uniqueness of momentum in the presence of a discrete translational symmetry and can be interpreted in terms of lattice momentum. Physically this corresponds to a finite measurement resolution. Also in the formulation of symplectic QFT defining one part of quantum TGD only phases defined by the roots of unity appear and similar non-uniqueness emerges and is due to the discretization serving as a space-time correlate for a finite measurement resolution implying UV cutoff.

(c) Mass squared is proportional to $1/p_i^2$ so that only the p-adic temperatures $T_p = 1/2n_i$ are possible for rational primes. For more general primes one can however have also a situation in which the modulus square of prime is ordinary prime. For instance, Gaussian (complex) primes $P = m + in$ satisfy $|P|^2 = p$ for $p \ mod \ 4 = 1$ and $|P|^2 = p^2$ for $p \ mod \ 4 = 3$ (for example, rational prime 5 decomposes as $5 = (2 + i)(2 - i)$). Therefore it is possible to have states satisfying $M^2 \propto 1/p$, p ordinary prime for hyper-octonionic primes. These primes correspond to the rational primes decomposing to the products of ordinary primes and also also higher roots of p might be possible. The finite prime assignable to the hyper-octonionic prime has a natural interpretation as the p-adic prime assignable to an elementary particle. In zero energy ontology this assignment makes sense also for virtual particles having interpretation as pairs of positive and negative energy on mass shell particles assignable to the light-like throats of wormhole contact.

2. Hyper-octonionic primes with inverse

Consider next the situation for hyper-octonionic primes when the integers in question have inverse. We are interested only in the longitudinal part of infinite prime in M^2. The phase factor makes sense also in the case of hyper-octonionic primes if the condition $|P| > 0$ holds true so that one has massive particles in 8-D sense possibly resulting via p-adic thermodynamics. If the imaginary unit appearing in the exponent is the imaginary unit i appearing in the complexification of octonions, the exponent has the character of a phase factor for hyper-octonionic primes. The reason is that $1/P = P^*/|P|^2$ is hyper-octonionic number of form $O_0 + iO_1$, where O_1 is a purely imaginary octonion. The exponent in the phase factor is therefore $2\pi(iO_0 - O_1)$ and involves only imaginary units, and one can write $exp(i2\pi(O_0 + iO_1)) = exp(iO_0) \times exp(-O_1)$. Both factors are phase factors. This condition analogous to unitarity is one further good reason for hyper-octonions and Minkowskian signature.

3. Light-like hyper-octonionic primes

The proposed representation as a phase factor fails for massless particles since light-like hyper-primes do not possess an inverse. One must therefore define the notion of primeness differently to see what might be the physical interpretation of these primes. Since the multiplication of hyper-octonionic integer by light-like prime yields zero norm prime, the natural interpretation would be as a gauge transformation and one might consider gauge transformations obtained by exponentiating Lie algebra with light-like coefficients.

One can consider two options depending on whether one requires that the relevant algebra has unit or not.

(a) For the first option hyper-octonionic light-like integers are of form $n(1 + e)$ and the product of two light-like integers $n_i(1 + e)$ is of form $2n_1n_2(1 + e)$. Here e could be arbitrary hyper-octonionic imaginary unit consistent with the prime property. This does not however allow unit light-like integer acting like unit since one has $(1+e)^2 = 2(1+e)$. All odd integers would be primes.

(b) The number $E = (1 + e)/2$ behaves as a unit. If one requires that unit is included in the algebra integers can be defined as numbers of form nE so that their product is n_1n_2E and equivalent with the ordinary product of integers so that primes correspond to ordinary primes.

One can construct the first level infinite primes from these primes just as in the case of ordinary primes. Now however $X = \prod p_i$ is replaced with $X = \prod_n [(2n + 1)(1 + e)]$ for the first option and equal to the $X = E \prod p_i$ for the second option.

The multiplicative phase factor could be defined for both options as $exp(i2\pi E/N)$ where N is a light-like hyper-octonionic integer. This definition would eliminate the singular $1/E$ factor and the situation reduces essentially to that for ordinary primes in the case of massless states. If the infinite prime $P_\pm$ is such that one can assign to it non-trivial multiplets in color or rotational degrees of freedom (half odd integer spin for fermions) it must have a part in the complement of M^2. For standard model elementary particles this is always the case. The energy spectrum is of form $1/2(2m+1)$ or $1/p$. For light-like hyper-octonions the projection to M^2 is in general time-like and quantized. If one does not allow the unit E in exponent the phase factor is ill-defined and one must identify the light-like hyper-octonionic primes as gauge degrees of freedom.

M^2 momentum is light-like only for states which are spinless color and electro-weak singlets having no counterpart in standard model counterpart nor in quantum TGD. Therefore light-like hyper-octonionic primes reducing to M^2 could correspond to gauge degrees of freedom. M^2 momentum is of form $P = (1,1)/2(2m+1)$ for the first option and of form $P = (1,1)/p$ for the second option. Even for graviton, photon, gluons, and right handed neutrino either hyper-octonionic prime is space-like if the state is massless. Light-like hyper-octonions can however characterize massive states but the proposed interpretation in terms of gauge degrees of freedom is highly suggestive.

If one interprets hyper-octonionic prime as 8-D momentum, which is of course not necessary in the recent framework, one could worry about conflict with TGD variant of twistor program. In accordance with associativity the role of 8-momentum in fermionic propagator is however taken by its projection to the hyper-quaternionic sub-space defined by the modified gamma matrices at given point of space-time sheet and masslessness holds for this projection so that 8-D tachyons are possible [K98] . This is highly analogous to the identification of the four-momentum as M^2 projection of hyperfinite prime.

4. The treatment of zero modes

There are also zero modes which are absolutely crucial for quantum measurement theory. They entangle with quantum fluctuating degrees of freedom in quantum measurement situation and thus map quantum numbers to positions of pointers. The interior degrees of freedom of space-time interior must correspond to zero modes and they represent space-time correlates for quantum states realized at light-like partonic 3-surfaces. Quantum measurement theory suggests 1-1 correspondence between zero modes and quantum fluctuating degrees of freedom so that also super-symmetry should have zero mode counterpart. The recent progress in understanding of the modified Dirac action [K28] leads to a concrete identification of the super-conformal algebra of zero modes as related to the deformations of the space-time surface defining vanishing second variations of Kähler action.

Complexification of octonions in zero energy ontology

The complexification of octonions plays a crucial role in the number theoretical vision and could be regarded as its weakest point. It has however a natural physical interpretation in zero energy ontology.

(a) CD has two tips, which correspond to the points of M^4. For M^4 the fixing of the quantization axes requires choosing a time-like direction fixing the rest system. This direction is naturally defined by the tips of CD. The moduli space for CDs is $M^4 \times M^4_+$. The realization of the hierarchy of Planck constants forces also a choice of a space-like direction fixing the quantization axes of spin.

(b) In the case of CP_2 the choice of the quantization axes requires fixing of a preferred point of CP_2 remaining invariant under $U(2)$ subgroup of $SU(3)$ acting linearly on complex coordinates having origin at this point and containing also the Cartan subgroup. This fixes the quantization axes of color hyper-charge. If the preferred CP_2 points associated with the light-like boundaries of CD are different they fix a unique geodesic circle of CP_2 fixing the quantization axes for color isospin. The moduli space is therefore $(CP_2)^2$.

(c) The full moduli space is $M^4 \times M^4_+ \times (CP_2)^2$. In M^8 description the moduli space would naturally correspond to pairs of points of M^4 and E^4 so that the moduli space for the choices CDs and quantization axes would be $M^4 \times M^4_+ \times (E^4)^2$. This space can be regarded locally as the space of complexified octonions.

(d) p-Adic length scale hypothesis follows if the time-like distance between the tips of CDs is quantized in powers of two so that a union of 3-D proper-time constant hyperboloids of M^4_+ results. Hierarchy of Planck constants implies rational multiples of these basic distances. Hyperboloids are coset spaces of Lorentz group and this suggests even more general quantization in which one replaces the hyperboloids with spaces obtained by identifying the points related by the action of a discrete subgroup of Lorentz group. This would give the analog of lattice cell obtained and one would obtain a lattice like structure consisting of unit cells labeled by the elements of the sub-group of Lorentz group. The interpretation of the moduli space of CDs as a discrete momentum space dual to the configuration space is suggestive. In the case of CP_2 similar quantization could correspond to the replacement of CP_2 with equivalence classes of points of CP_2 under action of a discrete subgroup of $SU(3)$.

(e) Could this discrete space be identified as the space of hyper-octonionic primes as looks natural? In other words, could the discrete points of the dual space $M^4_+ \times CP_2$ decompose to subsets in one-one corresponds with the orbits of G_+ and G_- appearing in the reductions $SO(7,1) \rightarrow SO(7) \rightarrow G_2 \rightarrow SU(3) \rightarrow G_+$ for primes in P_+ and $SO(7,1) \rightarrow SO(7) \rightarrow G_2 \rightarrow SU(3) \rightarrow SU(2) \rightarrow G_-$ in P_-? One can also consider the subgroups of G_2 respecting the hyperbolic prime property. This would allow to integrate $G_+ \times G_-$ multiplets to larger multiplets and get an over all view about multiplet structure. An interesting question is whether $SO(7,1)$ could contain non-compact discrete subgroups with infinite number of elements and respecting the property of being hyper-octonionic prime. If this idea is correct, the dual space $M^4_+ \times CP_2$ would play a role of heavenly sphere providing a representation for the quantum numbers labeling configuration space spinor fields.

The relation to number theoretic Brahman=Atman identity

Number theoretic Brahman=Atman identity -one might also use the term algebraic holography - states the number theoretic anatomy of single space-time point is enough to code for both WCW and and WCW spinors fields- the quantum states of entire Universe or at least the sub-Universe defined by CD. The entire quantum TGD could be represented in terms of 8-D imbedding space with the notion of number generalized to allow real units defined as ration of infinite integers and having number theoretical anatomy.

Before continuing it is perhaps good to represent the most obvious objection against the idea. The correspondence between WCW and WCW spinors with infinite rationals and their discreteness means that also WCW (world of classical worlds) and space of WCW spinors should be discrete. First this looks non-sensible but is indeed what one obtains if space-time surfaces correspond to light-like 3-surfaces expressible in terms of algebraic equations involving rational functions with rational coefficients.

By the above considerations it is indeed clear that zero energy states correspond to ratios of infinite integers boiling down to a hyper-octonionic unit with vanishing net four-momentum and electro-weak charges. Configuration space spinor fields can be mapped to wave functions in the space of these units and even the reduced configuration space consisting of the maxima of Kähler function could be coded by these wave functions. The wave functions in the space of hyper-octonion units would be induced by the discrete wave functions associated with the orbits of hyper-octonionic finite primes appearing in the decomposition of the infinite hyper-octonionic primes of type P_+ and P_-. The net color and quantum numbers and spin associated with the wave function in the space of hyper-octonionic units are vanishing. Clearly, a detailed realization of number theoretic Brahman=Atman identity emerges predicting reducing even the spectrum of possible quantum numbers to number theory.

In the original formulation of Brahman-Atman identity the description based on H was used. This leads to the conclusion that that the analog of a complex Schrödinger amplitude in the space of number-theoretic anatomies of a given imbedding space point represented by single point of H and represented as 8-tuples of real units should naturally represent the dependence of WCW spinors understood as ground states of super-conformal representations obtained as an 8-fold tensor power of a fundamental representation or product of representations perhaps differing somehow. The 8-tuples define a number theoretical analog of $U(1)^8$ group in terms of which all number theoretical symmetries are represented. This description should be equivalent with the use of single hyper-octonion unit.

Part III

HYPER-FINITE FACTORS OF TYPE II_1 AND HIERARCHY OF PLANCK CONSTANTS

Chapter 12

Was von Neumann Right After All?

12.1 Introduction

The work with TGD inspired model [K97] for topological quantum computation [K97] led to the realization that von Neumann algebras [A96, A121, A101, A63] , in particular so called hyper-finite factors of type II_1 [A84] , seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [A92] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD.

12.1.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A84] .

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

12.1.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [A62] based on the notion of delta function, plus the emergence of s [A70] , the possibility to formulate the notion of delta function rigorously in terms of distributions [A83, A112] , and the emergence of path integral approach [A102] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A109, A125] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A87] relate closely to type II_1 factors. In topological quantum computation [K97] based on braid groups [A126] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B34] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B63, B19] .

12.1.3 Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

(a) The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

(b) WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

(c) The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K80] . Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

(d) An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the modified gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with M^8-H duality [K17, K21] this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

12.1.4 Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

(a) The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

(b) Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

(c) The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

(d) There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to

Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

12.1.5 Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

(a) In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

(b) This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

(c) One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

12.1.6 Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

This chapter represents a summary about the development of the ideas with last sections representing the recent latest about the realization and role of HFFs in TGD. I have saved the reader from those speculations that have turned out to reflect my own ignorance or are inconsistent with what I regarded established parts of quantum TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Hyperfinite factors and TGD [L42]

12.2 Von Neumann algebras

In this section basic facts about von Neumann algebras are summarized using as a background material the concise summary given in the lecture notes of Longo [A92] .

12.2.1 Basic definitions

A formal definition of von Neumann algebra [A121, A101, A63] is as a *-subalgebra of the set of bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space $\mathcal{H}$ closed under weak operator topology, stable under the conjugation $J =^*$: $x \rightarrow x^*$, and containing identity operator Id. This definition allows also von Neumann algebras for which the trace of the unit operator is not finite.

Identity operator is the only operator commuting with a simple von Neumann algebra. A general von Neumann algebra allows a decomposition as a direct integral of simple algebras, which von Neumann called factors. Classification of von Neumann algebras reduces to that for factors.

$\mathcal{B}(\mathcal{H})$ has involution * and is thus a *-algebra. $\mathcal{B}(\mathcal{H})$ has order order structure $A \geq 0 : (Ax, x) \geq 0$. This is equivalent to $A = BB^*$ so that order structure is determined by algebraic structure. $\mathcal{B}(\mathcal{H})$ has metric structure in the sense that norm defined as supremum of $||Ax||$, $||x|| \leq 1$ defines the notion of continuity. $||A||^2 = inf\{\lambda > 0 : AA^* \leq \lambda I\}$ so that algebraic structure determines metric structure.

There are also other topologies for $\mathcal{B}(\mathcal{H})$ besides norm topology.

(a) $A_i \rightarrow A$ strongly if $||Ax - A_i x|| \rightarrow 0$ for all x. This topology defines the topology of C^* algebra. $\mathcal{B}(\mathcal{H})$ is a Banach algebra that is $||AB|| \leq ||A|| \times ||B||$ (inner product is not necessary) and also C^* algebra that is $||AA^*|| = ||A||^2$.

(b) $A_i \rightarrow A$ weakly if $(A_i x, y) \rightarrow (Ax, y)$ for all pairs (x, y) (inner product is necessary). This topology defines the topology of von Neumann algebra as a sub-algebra of $\mathcal{B}(\mathcal{H})$.

Denote by M' the commutant of $\mathcal{M}$ which is also algebra. Von Neumann's bicommutant theorem says that $\mathcal{M}$ equals to its own bi-commutant. Depending on whether the identity operator has a finite trace or not, one distinguishes between algebras of type II_1 and type II_∞. II_1 factor allow trace with properties $tr(Id) = 1$, $tr(xy) = tr(yx)$, and $tr(x^*x) > 0$, for all $x \neq 0$. Let $L^2(\mathcal{M})$ be the Hilbert space obtained by completing $\mathcal{M}$ respect to the inner product defined $\langle x|y\rangle = tr(x^*y)$ defines inner product in $\mathcal{M}$ interpreted as Hilbert space. The normalized trace induces a trace in M', natural trace $Tr_{M'}$, which is however not necessarily normalized. JxJ defines an element of M': if $\mathcal{H} = L^2(\mathcal{M})$, the natural trace is given by $Tr_{M'}(JxJ) = tr_M(x)$ for all $x \in M$ and bounded.

12.2.2 Basic classification of von Neumann algebras

Consider first some definitions. First of all, Hermitian operators with positive trace expressible as products xx^* are of special interest since their sums with positive coefficients are also positive.

In quantum mechanics Hermitian operators can be expressed in terms of projectors to the eigen states. There is a natural partial order in the set of isomorphism classes of projectors by inclusion: $E < F$ if the image of $\mathcal{H}$ by E is contained to the image of $\mathcal{H}$ by a suitable isomorph of F. Projectors are said to be metrically equivalent if there exist a partial isometry which maps the images $\mathcal{H}$ by them to each other. In the finite-dimensional case metric equivalence means that isomorphism classes are identical $E = F$.

The algebras possessing a minimal projection E_0 satisfying $E_0 \leq F$ for any F are called type I algebras. Bounded operators of n-dimensional Hilbert space define algebras I_n whereas the bounded operators of infinite-dimensional separable Hilbert space define the algebra I_∞. I_n and I_∞ correspond to the operator algebras of quantum mechanics. The states of harmonic oscillator correspond to a factor of type I.

The projection F is said to be finite if $F < E$ and $F \equiv E$ implies $F = E$. Hence metric equivalence means identity. Simple von Neumann algebras possessing finite projections but no minimal projections so that any projection E can be further decomposed as $E = F + G$, are called factors of type II.

Hyper-finiteness means that any finite set of elements can be approximated arbitrary well with the elements of a finite-dimensional sub-algebra. The hyper-finite II_∞ algebra can be regarded as a tensor product of hyper-finite II_1 and I_∞ algebras. Hyper-finite II_1 algebra can be regarded as a Clifford algebra of an infinite-dimensional separable Hilbert space subalgebra of I_∞.

Hyper-finite II_1 algebra can be constructed using Clifford algebras $C(2n)$ of $2n$-dimensional spaces and identifying the element x of $2^n \times 2^n$ dimensional $C(n)$ as the element $diag(x, x)/2$ of $2^{n+1} \times 2^{n+1}$-dimensional $C(n + 1)$. The union of algebras $C(n)$ is formed and completed in the weak operator topology to give a hyper-finite II_1 factor. This algebra defines the Clifford algebra of infinite-dimensional separable Hilbert space and is thus a sub-algebra of I_∞ so that hyper-finite II_1 algebra is more regular than I_∞.

von Neumann algebras possessing no finite projections (all traces are infinite or zero) are called algebras of type III. It was later shown by [A58] [A54] that these algebras are labeled by a parameter varying in the range $[0, 1]$, and referred to as algebras of type III_x. III_1 category contains a unique hyper-finite algebra. It has been found that the algebras of observables associated with bounded regions of 4-dimensional Minkowski space in quantum field theories correspond to hyper-finite factors of type III_1 [A92] . Also statistical systems at finite temperature correspond to factors of type III and temperature parameterizes one-parameter set of automorphisms of this algebra [B63] . Zero temperature limit correspond to I_∞ factor and infinite temperature limit to II_1 factor.

12.2.3 Non-commutative measure theory and non-commutative topologies and geometries

von Neumann algebras and C^* algebras give rise to non-commutative generalizations of ordinary measure theory (integration), topology, and geometry. It must be emphasized that these structures are completely natural aspects of quantum theory. In particular, for the hyper-finite type II_1 factors quantum groups and Kac Moody algebras [B39] emerge quite naturally without any need for ad hoc modifications such as making space-time coordinates non-commutative. The effective 2-dimensionality of quantum TGD (partonic or stringy 2-surfaces code for states) means that these structures appear completely naturally in TGD framework.

Non-commutative measure theory

von Neumann algebras define what might be a non-commutative generalization of measure theory and probability theory [A92] .

(a) Consider first the commutative case. Measure theory is something more general than topology since the existence of measure (integral) does not necessitate topology. Any measurable function f in the space $L^\infty(X,\mu)$ in measure space (X,μ) defines a bounded operator M_f in the space $\mathcal{B}(L^2(X,\mu))$ of bounded operators in the space $L^2(X,\mu)$ of square integrable functions with action of M_f defined as $M_f g = fg$.

(b) Integral over $\mathcal{M}$ is very much like trace of an operator $f_{x,y} = f(x)\delta(x,y)$. Thus trace is a natural non-commutative generalization of integral (measure) to the non-commutative case and defined for von Neumann algebras. In particular, generalization of probability measure results if the case $tr(Id) = 1$ and algebras of type I_n and II_1 are thus very natural from the point of view of non-commutative probability theory.

The trace can be expressed in terms of a cyclic vector Ω or vacuum/ground state in physicist's terminology. Ω is said to be cyclic if the completion $\overline{M\Omega} = H$ and separating if $x\Omega$ vanishes only for $x = 0$. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for M'. The expression for the trace given by

$$Tr(ab) = \left(\frac{(ab+ba)}{2},\Omega\right) \tag{12.2.1}$$

is symmetric and allows to defined also inner product as $(a,b) = Tr(a^*b)$ in $\mathcal{M}$. If Ω has unit norm $(\Omega,\Omega) = 1$, unit operator has unit norm and the algebra is of type II_1. Fermionic oscillator operator algebra with discrete index labeling the oscillators defines II_1 factor. Group algebra is second example of II_1 factor.

The notion of probability measure can be abstracted using the notion of state. State ω on a C^* algebra with unit is a positive linear functional on $\mathcal{U}$, $\omega(1) = 1$. By so called KMS construction [A92] any state ω in C^* algebra $\mathcal{U}$ can be expressed as $\omega(x) = (\pi(x)\Omega,\Omega)$ for some cyclic vector Ω and π is a homomorphism $\mathcal{U} \to \mathcal{B}(\mathcal{H})$.

Non-commutative topology and geometry

C^* algebras generalize in a well-defined sense ordinary topology to non-commutative topology.

(a) In the Abelian case Gelfand Naimark theorem [A92] states that there exists a contravariant functor F from the category of unital abelian C^* algebras and category of compact topological spaces. The inverse of this functor assigns to space X the continuous functions f on X with norm defined by the maximum of f. The functor assigns to these functions having interpretation as eigen states of mutually commuting observables defined by the function algebra. These eigen states are delta functions localized at single point of X. The points of X label the eigenfunctions and thus define the spectrum and obviously span X. The connection with topology comes from the fact that continuous map $Y \to X$ corresponds to homomorphism $C(X) \to C(Y)$.

(b) In non-commutative topology the function algebra $C(X)$ is replaced with a general C^* algebra. Spectrum is identified as labels of simultaneous eigen states of the Cartan algebra of C^* and defines what can be observed about non-commutative space X.

(c) Non-commutative geometry can be very roughly said to correspond to *-subalgebras of C^* algebras plus additional structure such as symmetries. The non-commutative geometry of Connes [A55] is a basic example here.

12.2.4 Modular automorphisms

von Neumann algebras allow a canonical unitary evolution associated with any state ω fixed by the selection of the vacuum state Ω [A92] . This unitary evolution is an automorphism fixed apart form unitary automorphisms $A \rightarrow UAU^*$ related with the choice of Ω.

Let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{12.2.2}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$.

12.2.5 Joint modular structure and sectors

Let $\mathcal{N} \subset \mathcal{M}$ be an inclusion. The unitary operator $\gamma = J_N J_M$ defines a canonical endomorphisms $M \rightarrow N$ in the sense that it depends only up to inner automorphism on $\mathcal{N}$, γ defines a sector of $\mathcal{M}$. The sectors of $\mathcal{M}$ are defined as $Sect(\mathcal{M}) = End(\mathcal{M})/Inn(\mathcal{M})$ and form a semi-ring with respected to direct sum and composition by the usual operator product. It allows also conjugation.

$L^2(\mathcal{M})$ is a normal bi-module in the sense that it allows commuting left and right multiplications. For $a, b \in M$ and $x \in L^2(\mathcal{M})$ these multiplications are defined as $axb = aJb^*Jx$ and it is easy to verify the commutativity using the factor $Jy^*J \in \mathcal{M}'$. [A58] [A55] has shown that all normal bi-modules arise in this way up to unitary equivalence so that representation concepts make sense. It is possible to assign to any endomorphism ρ index $Ind(\rho) \equiv M : \rho(\mathcal{M})$. This means that the sectors are in 1-1 correspondence with inclusions. For instance, in the case of hyper-finite II_1 they are labeled by Jones index. Furthermore, the objects with non-integral dimension $\sqrt{[\mathcal{M} : \rho(\mathcal{M})]}$ can be identified as quantum groups, loop groups, infinite-dimensional Lie algebras, etc...

12.2.6 Basic facts about hyper-finite factors of type III

Hyper-finite factors of type II_1, II_∞ and III_1, III_0, III_λ, $\lambda \in (0,1)$, allow by definition hierarchy of finite approximations and are unique as von Neumann algebras. Also hyper-finite factors of type II_∞ and type III could be relevant for the formulation of TGD. HFFs of type II_∞ and III could appear at the level operator algebra but that at the level of quantum states one would obtain HFFs of type II_1. These extended factors inspire highly non-trivial conjectures about quantum TGD. The book of Connes [A55] provides a detailed view about von Neumann algebras in general.

Basic definitions and facts

A highly non-trivial result is that HFFs of type II_∞ are expressible as tensor products $II_\infty = II_1 \otimes I_\infty$, where II_1 is hyper-finite [A55] .

1. The existence of one-parameter family of outer automorphisms

The unique feature of factors of type III is the existence of one-parameter unitary group of outer automorphisms. The automorphism group originates in the following manner.

(a) Introduce the notion of linear functional in the algebra as a map $\omega : \mathcal{M} \rightarrow C$. ω is said to be hermitian it respects conjugation in $\mathcal{M}$; positive if it is consistent with the notion of positivity for elements of $\mathcal{M}$ in which case it is called weight; state if it is positive and normalized meaning that $\omega(1) = 1$, faithful if $\omega(A) > 0$ for all positive A; a trace if $\omega(AB) = \omega(BA)$, a vector state if $\omega(A)$ is "vacuum expectation" $\omega_\Omega(A) = (\Omega, \omega(A)\Omega)$ for a non-degenerate representation $(\mathcal{H}, \pi)$ of $\mathcal{M}$ and some vector $\Omega \in \mathcal{H}$ with $||\Omega|| = 1$.

(b) The existence of trace is essential for hyper-finite factors of type II_1. Trace does not exist for factors of type III and is replaced with the weaker notion of state. State defines inner product via the formula $(x, y) = \phi(y^*x)$ and * is isometry of the inner product. *-operator has property known as pre-closedness implying polar decomposition $S = J\Delta^{1/2}$ of its closure. Δ is positive definite unbounded operator and J is isometry which restores the symmetry between $\mathcal{M}$ and its commutant $\mathcal{M}'$ in the Hilbert space $\mathcal{H}_\phi$, where $\mathcal{M}$ acts via left multiplication: $\mathcal{M}' = J\mathcal{M}J$.

(c) The basic result of Tomita-Takesaki theory is that Δ defines a one-parameter group $\sigma_\phi^t(x) = \Delta^{it}x\Delta^{-it}$ of automorphisms of $\mathcal{M}$ since one has $\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}$. This unitary evolution is an automorphism fixed apart from unitary automorphism $A \rightarrow UAU^*$ related with the choice of ϕ. For factors of type I and II this automorphism reduces to inner automorphism so that the group of outer automorphisms is trivial as is also the outer automorphism associated with ω. For factors of type III the group of these automorphisms divided by inner automorphisms gives a one-parameter group of $Out(\mathcal{M})$ of outer automorphisms, which does not depend at all on the choice of the state ϕ.

More precisely, let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{12.2.3}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$. What makes this result thought provoking is that it might mean a universal quantum dynamics apart from inner automorphisms and thus a realization of general coordinate invariance and gauge invariance at the level of Hilbert space.

2. Classification of HFFs of type III

Connes achieved an almost complete classification of hyper-finite factors of type III completed later by others. He demonstrated that they are labeled by single parameter $0 \leq \lambda \leq 1]$ and that factors of type III_λ, $0 \leq \lambda < 1$ are unique. Haagerup showed the uniqueness for $\lambda = 1$. The idea was that the the group has an invariant, the kernel $T(M)$ of the map from time like R to $Out(M)$, consisting of those values of the parameter t for which σ_ϕ^t reduces to an inner automorphism and to unity as outer automorphism. Connes also discovered also an invariant, which he called spectrum $S(\mathcal{M})$ of $\mathcal{M}$ identified as the intersection of spectra of $\Delta_\phi \backslash \{0\}$, which is closed multiplicative subgroup of R^+.

Connes showed that there are three cases according to whether $S(\mathcal{M})$ is

(a) R^+, type III_1

(b) $\{\lambda^n, n \in Z\}$, type III_λ.

(c) $\{1\}$, type III_0.

The value range of λ is this by convention. For the reversal of the automorphism it would be that associated with $1/\lambda$.

Connes constructed also an explicit representation of the factors $0 < \lambda < 1$ as crossed product II_∞ factor $\mathcal{N}$ and group Z represented as powers of automorphism of II_∞ factor inducing the scaling of trace by λ. The classification of HFFs of type III reduced thus to the classification of automorphisms of $\mathcal{N} \otimes \mathcal{B}(\mathcal{H}$. In this sense the theory of HFFs of type III was reduced to that for HFFs of type II_∞ or even II_1. The representation of Connes might be also physically interesting.

Probabilistic view about factors of type III

Second very concise representation of HFFs relies on thermodynamical thinking and realizes factors as infinite tensor product of finite-dimensional matrix algebras acting on state spaces of finite state systems with a varying and finite dimension n such that one assigns to each factor a density matrix characterized by its eigen values. Intuitively one can think the finite matrix factors as associated with n-state system characterized by its energies with density matrix ρ defining a thermodynamics. The logarithm of the ρ defines the single particle quantum Hamiltonian as $H = log(\rho)$ and $\Delta = \rho = exp(H)$ defines the automorphism σ_ϕ for each finite tensor factor as $exp(iHt)$. Obviously free field representation is in question.

Depending on the asymptotic behavior of the eigenvalue spectrum one obtains different factors [A55] .

(a) Factor of type I corresponds to ordinary thermodynamics for which the density matrix as a function of matrix factor approaches sufficiently fast that for a system for which only ground state has non-vanishing Boltzmann weight.

(b) Factor of type II_1 results if the density matrix approaches to identity matrix sufficiently fast. This means that the states are completely degenerate which for ordinary thermodynamics results only at the limit of infinite temperature. Spin glass could be a counterpart for this kind of situation.

(c) Factor of type III results if one of the eigenvalues is above some lower bound for all tensor factors in such a manner that neither factor of type I or II_1 results but thermodynamics for systems having infinite number of degrees of freedom could yield this kind of situation.

This construction demonstrates how varied representations factors can have, a fact which might look frustrating for a novice in the field. In particular, the infinite tensor power of $M(2, C)$ with state defined as an infinite tensor power of $M(2, C)$ state assigning to the matrix A the complex number $(\lambda^{1/2}A_{11} + \lambda^{-1/2}\ \phi(A) = A_{22})/(\lambda^{1/2} + \lambda^{-1/2})$ defines HFF III_λ [A55] , [C26] . Formally the same algebra which for $\lambda = 1$ gives ordinary trace and HFF of type II_1, gives III factor only by replacing trace with state. This simple model was discovered by Powers in 1967.

It is indeed the notion of state or thermodynamics is what distinguishes between factors. This looks somewhat weird unless one realizes that the Hilbert space inner product is defined by the "thermodynamical" state ϕ and thus probability distribution for operators and for their thermal expectation values. Inner product in turn defines the notion of norm and thus of continuity and it is this notion which differs dramatically for $\lambda = 1$ and $\lambda < 1$ so that the completions of the algebra differ dramatically.

In particular, there is no sign about I_∞ tensor factor or crossed product with Z represented as automorphisms inducing the scaling of trace by λ. By taking tensor product of I_∞ factor represented as tensor power with induces running from $-\infty$ to 0 and II_1 HFF with indices running from 1 to ∞ one can make explicit the representation of the automorphism of II_∞ factor inducing scaling of trace by λ and transforming matrix factors possessing trace given by square root of index $\mathcal{M} : \mathcal{N}$ to those with trace 2.

12.3 Braid group, von Neumann algebras, quantum TGD, and formation of bound states

The article of Vaughan Jones in [A126] discusses the relation between knot theory, statistical physics, and von Neumann algebras. The intriguing results represented stimulate concrete ideas about how to understand the formation of bound states quantitatively using the notion of join along boundaries bond. All mathematical results represented in the following discussion can be found in [A126] and in the references cited therein so that I will not bother to refer repeatedly to this article in the sequel.

12.3.1 Factors of von Neumann algebras

Von Neumann algebras M are algebras of bounded linear operators acting in Hilbert space. These algebras contain identity, are closed with respect to Hermitian conjugation, and are topologically complete. Finite-dimensional von Neuman algebras decompose into a direct sum of algebras M_n, which act essentially as matrix algebras in Hilbert spaces $\mathcal{H}_{nm}$, which are tensor products $C^n \otimes \mathcal{H}_m$. Here $\mathcal{H}_m$ is an m-dimensional Hilbert space in which M_n acts trivially. m is called the multiplicity of M_n.

A factor of von Neumann algebra is a von Neumann algebra whose center is just the scalar multiples of identity. The algebra of bounded operators in an infinite-dimensional Hilbert space is certainly a factor. This algebra decomposes into "atoms" represented by one-dimensional projection operators. This kind of von Neumann algebras are called type I factors.

The so called type II_1 factors and type III factors came as a surprise even for Murray and von Neumann. II_1 factors are infinite-dimensional and analogs of the matrix algebra factors M_n. They allow a trace making possible to define an inner product in the algebra. The trace defines a generalized dimension for any subspace as the trace of the corresponding projection operator. This dimension is however continuous and in the range $[0, 1]$: the finite-dimensional analog would be the dimension of the sub-space divided by the dimension of $\mathcal{H}_n$ and having values $(0, 1/n, 2/n, ..., 1)$. II_1 factors are isomorphic and there exists a minimal "hyper-finite" II_1 factor is contained by every other II_1 factor.

Just as in the finite-dimensional case, one can to assign a multiplicity to the Hilbert spaces where II_1 factors act on. This multiplicity, call it $dim_M(\mathcal{H})$ is analogous to the dimension of the Hilbert space tensor factor $\mathcal{H}_m$, in which II_1 factor acts trivially. This multiplicity can have all positive real values. Quite generally, von Neumann factors of type I and II_1 are in many respects analogous to the coefficient field of a vector space.

12.3.2 Sub-factors

Sub-factors $N \subset M$, where N and M are of type II_1 and have same identity, can be also defined. The observation that M is analogous to an algebraic extension of N motivates the introduction of index $|M : N|$, which is essentially the dimension of M with respect to N. This dimension is an analog for the complex dimension of CP_2 equal to 2 or for the algebraic dimension of the extension of p-adic numbers.

The following highly non-trivial results about the dimensions of the tensor factors hold true.

(a) If $N \subset M$ are II_1 factors and $|M : N| < 4$, there is an integer $n \geq 3$ such $|M : N| = r = 4cos^2(\pi/n)$, $n \geq 3$.

(b) For each number $r = 4cos^2(\pi/n)$ and for all $r \geq 4$ there is a sub-factor $R_r \subset R$ with $|R : R_r| = r$.

One can say that M effectively decomposes to a tensor product of N with a space, whose dimension is quantized to a certain algebraic number r. The values of r corresponding to $n = 3, 4, 5, 6...$ are $r = 1, 2, 1 + \Phi \simeq 2.61, 3, ...$ and approach to the limiting value $r = 4$. For $r \geq 4$ the dimension becomes continuous.

An even more intriguing result is that by starting from $N \subset M$ with a projection $e_N\colon M \to N$ one can extend M to a larger II_1 algebra $\langle M, e_N\rangle$ such that one has

$$
\begin{aligned}
|\langle M, e_N\rangle : M| &= |M : N| \ , \\
tr(xe_N) &= |M : N|^{-1} tr(x) \ , \ x \in M \ .
\end{aligned}
\tag{12.3.1}
$$

One can continue this process and the outcome is a tower of II_1 factors $M_i \subset M_{i+1}$ defined by $M_1 = N$, $M_2 = M$, $M_{i+1} = \langle M_i, e_{M_{i-1}}\rangle$. Furthermore, the projection operators $e_{M_i} \equiv e_i$ define a Temperley-Lieb representation of the braid algebra via the formulas

$$
\begin{aligned}
e_i^2 &= e_i \ , \\
e_i e_{i\pm 1} e_i &= \tau e_i \ , \ \tau = 1/|M : N| \\
e_i e_j &= e_j e_i \ , \ |i - j| \geq 2 \ .
\end{aligned}
\tag{12.3.2}
$$

Temperley Lieb algebra will be discussed in more detail later. Obviously the addition of a tensor factor of dimension r is analogous with the addition of a strand to a braid.

The hyper-finite algebra R is generated by the set of braid generators $\{e_1, e_2, ...\}$ in the braid representation corresponding to r. Sub-factor R_1 is obtained simply by dropping the lowest generator e_1, R_2 by dropping e_1 and e_2, etc..

12.3.3 II_1 factors and the spinor structure of WCW

The following observations serve as very suggestive guidelines for how one could interpret the above described results in TGD framework.

(a) The discrete spectrum of dimensions $1, 2, 1 + \Phi, 3, ..$ below $r < 4$ brings in mind the discrete energy spectrum for bound states whereas the for $r \geq 4$ the spectrum of dimensions is analogous to a continuum of unbound states. The fact that r is an algebraic number for $r < 4$ conforms with the vision that bound state entanglement corresponds to entanglement probabilities in an extension of rationals defining a finite-dimensional extension of p-adic numbers for every prime p.

(b) The discrete values of r correspond precisely to the angles ϕ allowed by the unitarity of Temperley-Lieb representations of the braid algebra with $d = -\sqrt{r}$. For $r \geq 4$ Temperley-Lieb representation is not unitary since $cos^2(\pi/n)$ becomes formally larger than one (n would become imaginary and continuous). This could mean that $r \geq 4$, which in the generic case is a transcendental number, represents unbound entanglement, which in TGD Universe is not stable against state preparation and state function reduction processes.

(c) The formula $tr(xe_N) = |M : N|^{-1} tr(x)$ is completely analogous to the formula characterizing the normalization of the link invariant induced by the second Markov move in which a new strand is added to a braid such that it braids only with the leftmost strand and therefore does not change the knot resulting as a link closure. Hence the addition of a single strand seems to correspond to an introduction of an r-dimensional sub-factor to II_1 factor.

In TGD framework the generation of bound state has the formation of (possibly braided join along boundaries bonds as a space-time correlate and this encourages a rather concrete interpretation of these findings. Also the I_1 factors themselves have a nice interpretation in terms of the WCW spinor structure.

1. The interpretation of II_1 factors in terms of Clifford algebra of WCW

The Clifford algebra of an infinite-dimensional Hilbert space defines a II_1 factor. The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus to operators of form $(1+\Sigma_{ij})/2$, defined by a subset of sigma matrices. The first guess is that the index pairs are $(i,j) = (1,2),(2,3),(3,4),.....$ The dimension of the Clifford algebra is 2^N for N-dimensional space so that $\Delta N = 1$ would correspond to $r = 2$ in the classical case and to one qubit. The problem with this interpretation is $r > 2$ has no physical interpretation: the formation of bound states is expected to reduce the value of r from its classical value rather than increase it.

One can however consider also the sequence $(i,j) = (1,1+k),(1+k,1+2k),(1+2k,1+3k),....$ For $k = 2$ the reduction of r from $r = 4$ would be due to the loss of degrees of freedom due to the formation of a bound state and $(r = 4, \Delta N = 2)$ would correspond to the classical limit resulting at the limit of weak binding. The effective elimination of the projection operators from the braid algebra would reflect this loss of degrees of freedom. This interpretation could at least be an appropriate starting point in TGD framework.

In TGD Universe physical states correspond to WCW spinor fields, whose gamma matrix algebra is constructed in terms of second quantized free induced spinor fields defined at space-time sheets. The original motivation was the idea that the quantum states of the Universe correspond to the modes of purely classical free spinor fields in the infinite-dimensional configuration space of 3-surfaces (the "world of classical worlds", WCW) possessing general coordinate invariant (in 4-dimensional sense!) Kähler geometry. Quantum information-theoretical motivation could have come from the requirement that these fields must be able to code information about the properties of the point (3-surface, and corresponding space-time sheet). Scalar fields would treat the 3-surfaces as points and are thus not enough. Induced spinor fields allow however an infinite number of modes: according to the naive Fourier analyst's intuition these modes are in one-one correspondence with the points of the 3-surface. Second quantization gives much more. Also non-local information about the induced geometry and topology must be coded, and here quantum entanglement for states generated by the fermionic oscillator operators coding information about the geometry of 3-surface provides enormous information storage capacity.

In algebraic geometry also the algebra of the imbedding space of algebraic variety divided by the ideal formed by functions vanishing on the surface codes information about the surface: for instance, the maximal ideals of this algebra code for the points of the surface (functions of imbedding space vanishing at a particular point). The function algebra of the imbedding space indeed plays a key role in the construction of WCW-geometry besides second quantized fermions.

The Clifford algebra generated by the WCW gamma matrices at a given point (3-surface) of WCW of 3-surfaces could be regarded as a II_1-factor associated with the local tangent space endowed with Hilbert space structure (WCW Kähler metric). The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus operators of form $(G_{\overline{A}B} \times 1 + \Sigma_{\overline{A}B})$ formed as linear combinations of components of the Kähler metric and of the sigma matrices defined by gamma matrices and contracted with the generators of the isometries of WCW. The addition of single complex degree of freedom corresponds to $\Delta N = 2$ and $r = 4$ and the classical limit and would correspond to the addition of single braid. $(r < 4, \Delta N < 2)$ would be due to the binding effects.

$r = 1$ corresponds to $\Delta N = 0$. The first interpretation is in terms of strong binding so that the addition of particle does not increase the number of degrees of freedom. In TGD framework $r = 1$ might also correspond to the addition of zero modes which do not contribute to the WCW metric and spinor structure but have a deep physical significance. $(r = 2, \Delta N = 1)$ would correspond to strong binding reducing the spinor and space-time degrees of freedom by a factor of half. $r = \Phi^2$ $(n = 5)$ *resp.* $r = 3$ $(n = 6)$ corresponds to $\Delta N_r \simeq 1.3885$ *resp.* $\Delta N_r = 1.585$. Using the terminology of quantum field theories, one might say that in the infinite-dimensional context a given complex bound state degree of freedom possesses anomalous real dimension $r < 2$. $r \geq 4$ would correspond to a unbound entanglement and increasingly classical behavior.

12.3.4 About possible space-time correlates for the hierarchy of II_1 sub-factors

By quantum classical correspondence the infinite-dimensional physics at WCW level should have definite space-time correlates. In particular, the dimension r should have some fractal dimension as a space-time correlate.

1. Quantum classical correspondence

Join along boundaries bonds serve as correlates for bound state formation. The presence of join along boundaries bonds would lead to a generation of bound states just by reducing the degrees of freedom to those of connected 3-surface. The bonds would constrain the two 3-surfaces to single space-like section of imbedding space.

This picture would allow to understand the difficulties related to Bethe-Salpeter equations for bound states based on the assumption that particles are points moving in M^4. The restriction of particles to time=constant section leads to a successful theory which is however non-relativistic. The basic binding energy would relate to the entanglement of the states associated with the bonded 3-surfaces. Since the classical energy associated with the bonds is positive, the binding energy tends to be reduced as r increases.

By spin glass degeneracy join along boundaries bonds have an infinite number of degrees of freedom in the ordinary sense. Since the system is infinite-dimensional and quantum critical, one expects that the number r of degrees freedom associated with a single join along boundaries bond is universal. Since join along boundaries bonds correspond to the strands of a braid and are correlates for the bound state formation, the natural guess is that $r = 4cos^2(\pi/n)$, $n = 3, 4, 5, ...$ holds true. $r < 4$ should characterize both binding energy and the dimension of the effective tensor factor introduced by a new join along boundaries bond.

The assignment of 2 "bare" and $\Delta N \leq 2$ renormalized real dimensions to single join along boundaries bond is consistent with the effective two-dimensionality of anyon systems and with the very notion of the braid group. The picture conforms also with the fact that the degrees of freedom in question are associated with metrically 2-dimensional light-like boundaries (of say magnetic flux tubes) acting as causal determinants. Also vibrational degrees of freedom described by Kac-Moody algebra are present and the effective 2-dimensionality means that these degrees of freedom are not excited and only topological degrees of freedom coded by the position of the puncture remain.

$(r \geq 4, \Delta N \geq 2)$, if possible at all, would mean that the tensor factor associated with the join along boundaries bond is effectively more than 4-dimensional due to the excitation of the vibrational Kac-Moody degrees of freedom. The finite value of r would mean that most of theme are eliminated also now but that their number is so large that bound state entanglement is not possible anymore.

The introduction of non-integer dimension could be seen as an effective description of an infinite-dimensional system as a finite-dimensional system in the spirit of renormalization group philosophy. The non-unitarity of $r \geq 4$ Temperley-Lieb representations could mean that they correspond to unbound entanglement unstable against state function reduction and preparation processes. Since this kind of entanglement does not survive in quantum jump it is not representable in terms of braid groups.

2. Does r define a fractal dimension of CP_2 projection of partonic 2-surface?

On basis of the quantum classical correspondence one expects that r should define some fractal dimension at the space-time level. Since r varies in the range $1, .., 4$ and corresponds to the fractal dimension of 2-D Clifford algebra the corresponding spinors would have dimension $d = \sqrt{r}$. There are two options.

(a) $D = r/2$ is suggested on basis of the construction of quantum version of M^d.

(b) $D = log_2(r)$ is natural on basis of the dimension $d = 2^{D/2}$ of spinors in D-dimensional space.

r can be assigned with CP_2 degrees of freedom in the model for the quantization of Planck constant based on the explicit identification of Josephson inclusions in terms of finite subgroups of $SU(2) \subset SU(3)$. Hence D should relate to the CP_2 projection of the partonic 2-surface and one could have $D = D(X^2)$, the latter being the average dimension of the CP_2 projection of the partonic 2-surface for the preferred extremals of Kähler action.

Since a strongly interacting non-perturbative phase should be in question, the dimension for the CP_2 projection of the space-time surface must be at least $D(X^4) = 2$ to guarantee that non-vacuum extremals are in question. This is true for $D(X^2) = r/2 \geq 1$. The logarithmic formula $D(X^2) = log_2(r) \geq 0$ gives $D(X^2) = 0$ for $n = 3$ meaning that partonic 2-surfaces are vacua: space-time surface can still be a non-vacuum extremal.

As n increases, the number of CP_2 points covering a given M^4 point and related by the finite subgroup of $G \subset SU(2) \subset SU(3)$ defining the inclusion increases so that the fractal dimension of the CP_2 projection is expected to increase also. $D(X^2) = 2$ would correspond to the space-time surfaces for which partons have topological magnetic charge forcing them to have a 2-dimensional CP_2 projection. There are reasons to believe that the projection must be homologically non-trivial geodesic sphere of CP_2.

12.3.5 Could binding energy spectra reflect the hierarchy of effective tensor factor dimensions?

If one takes completely seriously the idea that join along boundaries bonds are a correlate of binding then the spectrum of binding energies might reveal the hierarchy of the fractal dimensions $r(n)$. Hydrogen atom and harmonic oscillator have become symbols for bound state systems. Hence it is of interest to find whether the binding energy spectrum of these systems might be expressed in terms of the "binding dimension" $x(n) = 4 - r(n)$ characterizing the deviation of dimension from that at the limit of a vanishing binding energy. The binding energies of hydrogen atom are in a good approximation given by $E(n)/E(1) = 1/n^2$ whereas in the case of harmonic oscillator one has $E(n)/E_0 = 2n + 1$. The constraint $n \geq 3$ implies that the principal quantum number must correspond $n - 2$ in the case of hydrogen atom and to $n - 3$ in the case of harmonic oscillator.

Before continuing one must face an obvious objection. By previous arguments different values of r correspond to different values of $\hbar$. The value of $\hbar$ cannot however differ for the states of hydrogen atom. This is certainly true. The objection however leaves open the possibility that the states of the light-like boundaries of join along boundaries bonds correspond to reflective level and represent some aspects of the physics of, say, hydrogen atom.

In the general case the energy spectrum satisfies the condition

$$\frac{E_B(n)}{E_B(3)} = \frac{f(4 - r(n))}{f(3)} \ , \tag{12.3.3}$$

where f is some function. The simplest assumption is that the spectrum of binding energies $E_B(n) = E(n) - E(\infty)$ is a linear function of $r(n) - 4$:

$$\frac{E_B(n)}{E_B(3)} = \frac{4 - r(n)}{3} = \frac{4}{3} sin^2(\frac{\pi}{n}) \rightarrow \frac{4\pi^2}{3} \times \frac{1}{n^2} \ . \tag{12.3.4}$$

In the linear approximation the ratio $E(n+1)/E(n)$ approaches $(n/n+1)^2$ as in the case of hydrogen atom but for small values the linear approximation fails badly. An exact correspondence results for

$$\frac{E(n)}{E(1)} = \frac{1}{n^2} ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+2)/4}\right)} - 2 .$$

Also the ionized states with $r \geq 4$ would correspond to bound states in the sense that two particle would be constrained to move in the same space-like section of space-time surface and should be distinguished from genuinely free states when particles correspond to disjoint space-time sheets.

For the harmonic oscillator one express $E(n) - E(0)$ instead of $E(n) - E(\infty)$ as a function of $x = 4 - r$ and one would have

$$\frac{E(n)}{E(0)} = 2n + 1 ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+3)/4}\right)} - 3 .$$

In this case ionized states would not be possible due to the infinite depth of the harmonic oscillator potential well.

12.3.6 Four-color problem, II_1 factors, and anyons

The so called four-color problem can be phrased as a question whether it is possible to color the regions of a plane map using only four colors in such a manner that no adjacent regions have the same color (for an enjoyable discussion of the problem see [A43]). One might call this kind of coloring complete. There is no loss of generality in assuming that the map can be represented as a graph with regions represented as triangle shaped faces of the graph. For the dual graph the coloring of faces becomes coloring of vertices and the question becomes whether the coloring is possible in such a manner that no vertices at the ends of the same edge have same color. The problem can be generalized by replacing planar maps with maps defined on any two-dimensional surface with or without boundary and arbitrary topology. The four-color problem has been solved with an extensive use of computer [A39] but it would be nice to understand why the complete coloring with four colors is indeed possible.

There is a mysterious looking connection between four-color problem and the dimensions $r(n) = 4cos^2(\pi/n)$, which are in fact known as Beraha numbers in honor of the discoverer of this connection [A108] . Consider a more general problem of coloring two-dimensional map using m colors. One can construct a polynomial $P(m)$, so called chromatic polynomial, which tells the number of colorings satisfying the condition that no neighboring vertices have the same color. The vanishing of the chromatic polynomial for an integer value of m tells that the complete coloring using m colors is not possible.

$P(m)$ has also other than integer valued real roots. The strange discovery due to Beraha is that the numbers $B(n)$ appear as approximate roots of the chromatic polynomial in many situations. For instance, the four non-integral real roots of the chromatic polynomial of the truncated icosahedron are very close to $B(5)$, $B(7)$, $B(8)$ and $B(9)$. These findings led Beraha to formulate the following conjecture. Let P_i be a sequence of chromatic polynomials for a graph for which the number of vertices approaches infinity. If r_i is a root of the polynomial approaching a well-defined value at the limit $i \rightarrow \infty$, then the limiting value of $r(i)$ is Beraha number.

A physicist's proof for Beraha's conjecture based on quantum groups and conformal theory has been proposed [A108] . It is interesting to look for the a possible physical interpretation of 4-color problem and Beraha's conjecture in TGD framework.

(a) In TGD framework $B(n)$ corresponds to a renormalized dimension for a 2-spin system consisting of two qubits, which corresponds to 4 different colors. For $B(n) = 4$ two spin 1/2 fermions obeying Fermi statistics are in question. Since the system is 2-dimensional, the general case corresponds to two anyons with fractional spin $B(n)/4$ giving rise to $B(n) < 4$ colors and obeying fractional statistics instead of Fermi statistics. One can replace coloring problem with the problem whether an ideal antiferro-magnetic lattice using anyons with fractional spin $B(n)/4$ is possible energetically. In other words, does this system form a quantum mechanical bound state even at the limit when the lengths of the edges approach to zero.

(b) The failure of coloring means that there are at least two neighboring vertices in the lattice with the property that the spins at the ends of the same edge are in the same direction. Lattice defect would be in question. At the limit of an infinitesimally short edge length the failure of coloring is certainly not an energetically favored option for fermionic spins ($m = 4$) but is allowed by anyonic statistics for $m = B(n) < 4$. Thus one has reasons to expect that when anyonic spin is $B(n)/4$ the formation of a purely 2-anyon bound states becomes possible and they form at the limit of an infinitesimal edge length a kind of topological macroscopic quantum phase with a non-vanishing binding energy. That $B(n)$ are roots of the chromatic polynomial at the continuum limit would have a clear physical interpretation.

(c) Only $B(n) < 4$ defines a sub-factor of von Neumann algebra allowing unitary Temperley-Lieb representations. This is consistent with the fact that for $m = 4$ complete coloring must exists. The physical argument is that otherwise a macroscopic quantum phase with non-vanishing binding energy could result at the continuum limit and the upper bound for r from unitarity would be larger than 4. For $m = 4$ the completely anti-ferromagnetic state would represent the ground state and the absence of anyon-pair condensate would mean a vanishing binding energy.

12.4 Inclusions of II_1 and III_1 factors

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A3] and those of factors of type III by Alain Connes [A54] .

Sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

12.4.1 Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A3] .

(a) For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a) \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ , \quad h \geq 3 \ , \\ &b) \ \mathcal{M} : \mathcal{N} \geq 4 \ . \end{aligned} \quad (12.4.1)$$

the numbers at right hand side are known as Beraha numbers [A108] . The comments below give a rough idea about what finiteness of principal graph means.

(b) As explained in [B39] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A85] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

The interpretation of [A85] is that the subfactors correspond to inclusions $\mathcal{N} \subset \mathcal{M}$ defined in the following manner.

(a) Let G be a finite subgroup of SU(2). Denote by R the infinite-dimensional Clifford algebras resulting from infinite-dimensional tensor power of $M_2(C)$ and by R_0 its sub-algebra obtained by restricting $M_2(C)$ element of the first factor to be unit matrix. Let G act by automorphisms in each tensor factor. G leaves R_0 invariant. Denote by R_0^G and R^G the sub-algebras which remain element wise invariant under the action of G. The resulting Jones inclusions $R_0^G \subset R^G$ are consistent with the ADE correspondence.

(b) The argument suggests the existence of quantum versions of subgroups of SU(2) for which representations are truncations of those for ordinary subgroups. The results have been generalized to other Lie groups.

(c) Also $SL(2, C)$ acts as automorphisms of $M_2(C)$. An interesting question is what happens if one allows G to be any discrete subgroups of SL(2,C). Could this give inclusions with $\mathcal{M} : \mathcal{N} > 4$?. The strong analogy of the spectrum of indices with spectrum of energies with hydrogen atom would encourage this interpretation: the subgroup SL(2,C) not reducing to those of SU(2) would correspond to the possibility for the particle to move with respect to each other with constant velocity.

12.4.2 The fundamental construction and Temperley-Lieb algebras

It was shown by Jones [A68] that for a given Jones inclusion with $\beta = \mathcal{M} : \mathcal{N} < \infty$ there exists a tower of finite II_1 factors $\mathcal{M}_k$ for $k = 0, 1, 2, ...$ such that

(a) $\mathcal{M}_0 = \mathcal{N}$, $\mathcal{M}_1 = \mathcal{M}$,

(b) $\mathcal{M}_{k+1} = End_{\mathcal{M}_{k-1}} \mathcal{M}_k$ is the von Neumann algebra of operators on $L^2(\mathcal{M}_k)$ generated by $\mathcal{M}_k$ and an orthogonal projection $e_k : L^2(\mathcal{M}_k) \rightarrow L^2(\mathcal{M}_{k-1})$ for $k \geq 1$, where $\mathcal{M}_k$ is regarded as a subalgebra of $\mathcal{M}_{k+1}$ under right multiplication.

It can be shown that $\mathcal{M}_{k+1}$ is a finite factor. The sequence of projections on $\mathcal{M}_\infty = \cup_{k\geq 0} \mathcal{M}_k$ satisfies the relations

$$\begin{array}{lll} e_i^2 = e_i \ , & e_i^= e_i \ , & \\ e_i = \beta e_i e_j e_i & \text{for } \ |i-j| = 1 \ , & \\ e_i e_j = e_j e_i & \text{for } \ |i-j| \geq 2 \ . & \end{array} \tag{12.4.2}$$

The construction of hyper-finite II_1 factor using Clifford algebra $C(2)$ represented by 2×2 matrices allows to understand the theorem in $\beta = 4$ case in a straightforward manner. In particular, the second formula involving β follows from the identification of x at $(k-1)^{th}$ level with $(1/\beta) diag(x, x)$ at k^{th} level.

By replacing 2×2 matrices with $\sqrt{\beta}\times\sqrt{\beta}$ matrices one can understand heuristically what is involved in the more general case. $\mathcal{M}_k$ is $\mathcal{M}_{k-1}$ module with dimension $\sqrt{\beta}$ and $\mathcal{M}_{k+1}$ is the space of $\sqrt{\beta}\times\sqrt{\beta}$ matrices $\mathcal{M}_{k-1}$ valued entries acting in $\mathcal{M}_k$. The transition from $\mathcal{M}_k$ to $\mathcal{M}_{k-1}$ linear maps of $\mathcal{M}_k$ happens in the transition to the next level. x at $(k-1)^{th}$ level is identified as $(x/\beta)\times Id_{\sqrt{\beta}\times\sqrt{\beta}}$ at the next level. The projection e_k picks up the projection of the matrix with $\mathcal{M}_{k-1}$ valued entries in the direction of the $Id_{\sqrt{\beta}\times\sqrt{\beta}}$.

The union of algebras $A_{\beta,k}$ generated by $1, e_1, ..., e_k$ defines Temperley-Lieb algebra A_β [A119] . This algebra is naturally associated with braids. Addition of one strand to a braid adds one generator to this algebra and the representations of the Temperley Lieb algebra provide link, knot, and 3-manifold invariants [A126] . There is also a connection with systems of statistical physics and with Yang-Baxter algebras [A49] .

A further interesting fact about the inclusion hierarchy is that the elements in $\mathcal{M}_i$ belonging to the commutator $\mathcal{N}'$ of $\mathcal{N}$ form finite-dimensional spaces. Presumably the dimension approaches infinity for $n\rightarrow\infty$.

12.4.3 Connection with Dynkin diagrams

The possibility to assign Dynkin diagrams ($\beta < 4$) and extended Dynkin diagrams ($\beta = 4$ to Jones inclusions can be understood heuristically by considering a characterization of so called bipartite graphs [A86] , [B39] by the norm of the adjacency matrix of the graph.

Bipartite graphs Γ is a finite, connected graph with multiple edges and black and white vertices such that any edge connects white and black vertex and starts from a white one. Denote by $w(\Gamma)$ ($b(\Gamma)$) the number of white (black) vertices. Define the adjacency matrix $\Lambda = \Lambda(\Gamma)$ of size $b(\Gamma)\times w(\Gamma)$ by

$$w_{b,w} = \left\{ \begin{array}{ll} m(e) & \text{if there exists } e \text{ such that } \delta e = b - w \ , \\ 0 & \text{otherwise} \ . \end{array}\right. \quad (12.4.3)$$

Here $m(e)$ is the multiplicity of the edge e.

Define norm $||\Gamma||$ as

$$\begin{aligned} ||X|| &= max\{||X||;\ ||x||\leq 1\} \ , \\ ||\Gamma|| &= ||\Lambda(\Gamma)|| = \left|\left| \begin{array}{ll} 0 & \Lambda(\Gamma) \\ \Lambda(\Gamma)^t & 0 \end{array} \right|\right| \ . \end{aligned} \quad (12.4.4)$$

Note that the matrix appearing in the formula is $(m+n)\times(m+n)$ symmetric square matrix so that the norm is the eigenvalue with largest absolute value.

Suppose that Γ is a connected finite graph with multiple edges (sequences of edges are regarded as edges). Then

(a) If $||\Gamma||\leq 2$ and if Γ has a multiple edge, $||\Gamma|| = 2$ and $\Gamma = \tilde{A}_1$, the extended Dynkin diagram for $SU(2)$ Kac Moody algebra.

(b) $||\Gamma|| < 2$ if and only Γ is one of the Dynkin diagrams of A,D,E. In this case $||\Gamma|| = 2cos(\pi/h)$, where h is the Coxeter number of Γ.

(c) $||\Gamma|| = 2$ if and only if Γ is one of the extended Dynkin diagrams $\tilde{A}, \tilde{D}, \tilde{E}$.

This result suggests that one can indeed assign to the Jones inclusions Dynkin diagrams. To really understand how the inclusions can be characterized in terms bipartite diagrams would require a deeper understanding of von Neumann algebras. The following argument only demonstrates that bipartite graphs naturally describe inclusions of algebras.

(a) Consider a bipartite graph. Assign to each white vertex linear space $W(w)$ and to each edge of a linear space $W(b,w)$. Assign to a given black vertex the vector space $\oplus_{\delta e=b-w} W(b,w) \otimes W(w)$ where (b,w) corresponds to an edge ending to b.

(b) Define $\mathcal{N}$ as the direct sum of algebras $End(W(w))$ associated with white vertices and $\mathcal{M}$ as direct sum of algebras $\oplus_{\delta e=b-w} End(W(b,w)) \otimes End(W(w))$ associated with black vertices.

(c) There is homomorphism $N \to M$ defined by imbedding direct sum of white endomorphisms x to direct sum of tensor products x with the identity endomorphisms associated with the edges starting from x.

It is possible to show that Jones inclusions correspond to the Dynkin diagrams of A_n, D_{2n}, and E_6, E_8 and extended Dynkin diagrams of ADE type. In particular, the dual of the bipartite graph associated with $\mathcal{M}_{n-1} \subset \mathcal{M}_n$ obtained by exchanging the roles of white and black vertices describes the inclusion $\mathcal{M}_n \subset \mathcal{M}_{n+1}$ so that two subsequent Jones inclusions might define something fundamental (the corresponding space-time dimension is $2 \times log_2(\mathcal{M} : \mathcal{N}) \leq 4$.

12.4.4 Indices for the inclusions of type III_1 factors

Type III_1 factors appear in relativistic quantum field theory defined in 4-dimensional Minkowski space [B63] . An overall summary of basic results discovered in algebraic quantum field theory is described in the lectures of Longo [A92] . In this case the inclusions for algebras of observables are induced by the inclusions for bounded regions of M^4 in axiomatic quantum field theory. Tomita's theory of modular Hilbert algebras [A118] , [B19] forms the mathematical corner stone of the theory.

The basic notion is Haag-Kastler net [A100] consisting of bounded regions of M^4. Double cone serves as a representative example. The von Neumann algebra $\mathcal{A}(O)$ is generated by observables localized in bounded region O. The net satisfies the conditions implied by local causality:

(a) Isotony: $O_1 \subset O_2$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)$.

(b) Locality: $O_1 \subset O_2'$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ with O' defined as $\{x : \langle x,y\rangle < 0 \text{ for all } y \in O\}$.

(c) Haag duality $\mathcal{A}(O')' = \mathcal{A}(O)$.

Besides this Poincare covariance, positive energy condition, and the existence of vacuum state is assumed.

DHR (Doplicher-Haag-Roberts) [A105] theory allows to deduce the values of Jones index and they are squares of integers in dimensions $D > 2$ so that the situation is rather trivial. The 2-dimensional case is distinguished from higher dimensional situations in that braid group replaces permutation group since the paths representing the flows permuting identical particles can be linked in $X^2 \times T$ and anyonic statistics [D25, D24] becomes possible. In the case of 2-D Minkowski space M^2 Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ plus a set of discrete values of $\mathcal{M} : \mathcal{N}$ in the range $(4,6)$ are possible. In [A92] some values are given ($\mathcal{M} : \mathcal{N} = 5, 5.5049..., 5.236...., 5.828...$).

At least intersections of future and past light cones seem to appear naturally in TGD framework such that the boundaries of future/past directed light cones serve as seats for incoming/outgoing states defined as intersections of space-time surface with these light cones. III_1 sectors cannot thus be excluded as factors in TGD framework. On the other hand, the construction of S-matrix at space-time level is reduced to II_1 case by effective 2-dimensionality.

12.5 TGD and hyper-finite factors of type $\mathbf{II}_1$: ideas and questions

By effective 2-dimensionality of the construction of quantum states the hyper-finite factors of type II_1 fit naturally to TGD framework. In particular, infinite dimensional spinors define a canonical representations of this kind of factor. The basic question is whether only hyper-finite factors of type II_1 appear in TGD framework. Affirmative answer would allow to interpret physical M-matrix as time like entanglement coefficients.

12.5.1 What kind of hyper-finite factors one can imagine in TGD?

The working hypothesis has been that only hyper-finite factors of type II_1 appear in TGD. The basic motivation has been that they allow a new view about M-matrix as an operator representable as time-like entanglement coefficients of zero energy states so that physical states would represent laws of physics in their structure. They allow also the introduction of the notion of measurement resolution directly to the definition of reaction probabilities by using Jones inclusion and the replacement of state space with a finite-dimensional state space defined by quantum spinors. This hypothesis is of course just an attractive working hypothesis and deserves to be challenged.

WCW spinors

For WCW spinor s the HFF II_1 property is very natural because of the properties of infinite-dimensional Clifford algebra and the inner product defined by the WCW geometry does not allow other factors than this. A good guess is that the values of conformal weights label the factors appearing in the tensor power defining WCW spinor s. Because of the non-degeneracy and super-symplectic symmetries the density matrix representing metric must be essentially unit matrix for each conformal weight which would be the defining characteristic of hyper-finite factor of type II_1.

Bosonic degrees of freedom

The bosonic part of the super-symplectic algebra consists of Hamiltonians of CH in one-one correspondence with those of $\delta M^4_\pm \times CP_2$. Also the Kac-Moody algebra acting leaving the light-likeness of the partonic 3-surfaces intact contributes to the bosonic degrees of freedom. The commutator of these algebras annihilates physical states and there are also Virasoro conditions associated with ordinary conformal symmetries of partonic 2-surface [K21] . The labels of Hamiltonians of WCW and spin indices contribute to bosonic degrees of freedom.

Hyper-finite factors of type II_1 result naturally if the system is an infinite tensor product finite-dimensional matrix algebra associated with finite dimensional systems [A55] . Unfortunately, neither Virasoro, symplectic nor Kac-Moody algebras do have decomposition into this kind of infinite tensor product. If bosonic degrees for super-symplectic and super-Kac Moody algebra indeed give I_∞ factor one has HFF if type II_∞. This looks the most natural option but threatens to spoil the beautiful idea about M-matrix as time-like entanglement coefficients between positive and negative energy parts of zero energy state.

The resolution of the problem is surprisingly simple and trivial after one has discovered it. The requirement that state is normalizable forces to project M-matrix to a finite-dimensional sub-space in bosonic degrees of freedom so that the reduction $I_\infty \to I_n$ occurs and one has the reduction $II_\infty \to II_1 \times I_n = II_1$ to the desired HFF.

One can consider also the possibility of taking the limit $n \to \infty$. One could indeed say that since I_∞ factor can be mapped to an infinite tensor power of $M(2, C)$ characterized by a state which is not trace, it is possible to map this representation to HFF by replacing state with trace [A55] . The question is whether the forcing the bosonic foot to fermionic shoe is physically natural. One could also regard the II_1 type notion of probability as fundamental and also argue that it is required by full super-symmetry realized also at the level of many-particle states rather than mere single particle states.

How the bosonic cutoff is realized?

Normalizability of state requires that projection to a finite-dimensional bosonic sub-space is carried out for the bosonic part of the M-matrix. This requires a cutoff in quantum numbers of super-conformal algebras. The cutoff for the values of conformal weight could be formulated by replacing integers with Z_n or with some finite field $G(p,1)$. The cutoff for the labels associated with Hamiltonians defined as an upper bound for the dimension of the representation looks also natural.

Number theoretical braids which are discrete and finite structures would define space-time correlate for this cutoff. p-Adic length scale $p \simeq 2^k$ hypothesis could be interpreted as stating the fact that only powers of p up to p^k are significant in p-adic thermodynamics which would correspond to finite field $G(k,1)$ if k is prime. This has no consequences for p-adic mass calculations since already the first two terms give practically exact results for the large primes associated with elementary particles [K54] .

Finite number of strands for the theoretical braids would serve as a correlate for the reduction of the representation of Galois group S_∞ of rationals to an infinite produce of diagonal copies of finite-dimensional Galois group so that same braid would repeat itself like a unit cell of lattice i condensed matter [A17] .

HFF of type III for field operators and HFF of type II_1 for states?

One could also argue that the Hamiltonians with fixed conformal weight are included in fermionic II_1 factor and bosonic factor I_∞ factor, and that the inclusion of conformal weights leads to a factor of type III. Conformal weight could relate to the integer appearing in the crossed product representation $III = Z \times_{cr} II_\infty$ of HFF of type III [A55] .

The value of conformal weight is non-negative for physical states which suggests that Z reduces to semigroup N so that a factor of type III would reduce to a factor of type II_∞ since trace would become finite. If unitary process corresponds to an automorphism for II_∞ factor, the action of automorphisms affecting scaling must be uni-directional. Also thermodynamical irreversibility suggests the same. The assumption that state function reduction for positive energy part of state implies unitary process for negative energy state and vice versa would only mean that the shifts for positive and negative energy parts of state are opposite so that $Z \rightarrow N$ reduction would still hold true.

HFF of type II_1 for the maxima of Kähler function?

Probabilistic interpretation allows to gain heuristic insights about whether and how hyper-finite factors of type type II_1 might be associated with WCW degrees of freedom. They can appear both in quantum fluctuating degrees of freedom associated with a given maximum of Kähler function and in the discrete space of maxima of Kähler function.

Spin glass degeneracy is the basic prediction of classical TGD and means that instead of a single maximum of Kähler function analogous to single free energy minimum of a thermodynamical system there is a fractal spin glass energy landscape with valleys inside valleys. The discretization of WCW in terms of the maxima of Kähler function crucial for the p-adicization problem, leads to the analog of spin glass energy landscape and hyper-finite factor of type II_1 might be the appropriate description of the situation.

The presence of the tensor product structure is a powerful additional constraint and something analogous to this should emerge in WCW degrees of freedom. Fractality of the many-sheeted space-time is a natural candidate here since the decomposition of the original geometric structure to parts and replacing them with the scaled down variant of original structure is the geometric analog of forming a tensor power of the original structure.

12.5.2 Direct sum of HFFs of type II_1 as a minimal option

HFF II_1 property for the Clifford algebra of WCW means a definite distinction from the ordinary Clifford algebra defined by the fermionic oscillator operators since the trace of the unit matrix of the Clifford algebra is normalized to one. This does not affect the anti-commutation relations at the basic level and delta functions can appear in them at space-time level. At the level of momentum space I_∞ property requires discrete basis and anti-commutators involve only Kronecker deltas. This conforms with the fact that HFF of type II_1 can be identified as the Clifford algebra associated with a separable Hilbert space.

II_∞ factor or direct sum of HFFs of type II_1?

The expectation is that super-symplectic algebra is a direct sum over HFFs of type II_1 labeled by the radial conformal weight. In the same manner the algebra defined by fermionic anti-commutation relations at partonic 2-surface would decompose to a direct sum of algebras labeled by the conformal weight associated with the light-like coordinate of X^3_l. Super-conformal symmetry suggests that also the configuration space degrees of freedom correspond to a direct sum of HFFs of type II_1.

One can of course ask why not $II_\infty = I_\infty \times II_1$ structures so that one would have single factor rather than a direct sum of factors.

(a) The physical motivation is that the direct sum property allow to decompose M-matrix to direct summands associated with various sectors with weights whose moduli squared have an interpretation in terms of the density matrix. This is also consistent with p-adic thermodynamics where conformal weights take the place of energy eigen values.

(b) II_∞ property would predict automorphisms scaling the trace by an arbitrary positive real number $\lambda \in R_+$. These automorphisms would require the scaling of the trace of the projectors of Clifford algebra having values in the range $[0, 1]$ and it is difficult to imagine how these automorphisms could be realized geometrically.

How HFF property reflects itself in the construction of geometry of WCW?

The interesting question is what HFF property and finite measurement resolution realizing itself as the use of projection operators means concretely at the level of WCW geometry.

Super-Hamiltonians define the Clifford algebra of the configuration space. Super-conformal symmetry suggests that the unavoidable restriction to projection operators instead of complex rays is realized also WCW degrees of freedom. Of course, infinite precision in the determination of the shape of 3-surface would be physically a completely unrealistic idea.

In the fermionic situation the anti-commutators for the gamma matrices associated with WCW individual Hamiltonians in 3-D sense are replaced with anti-commutators where Hamiltonians are replaced with projectors to subspaces of the space spanned by Hamiltonians. This projection is realized by restricting the anti-commutator to partonic 2-surfaces so that the anti-commutator depends only the restriction of the Hamiltonian to those surfaces.

What is interesting that the measurement resolution has a concrete particle physical meaning since the parton content of the system characterizes the projection. The larger the number of partons, the better the resolution about WCW degrees of freedom is. The degeneracy of WCW metric would be interpreted in terms of finite measurement resolution inherent to HFFs of type II_1, which is not due to Jones inclusions but due to the fact that one can project only to infinite-D subspaces rather than complex rays.

Effective 2-dimensionality in the sense that WCW Hamiltonians reduce to functionals of the partonic 2-surfaces of X^3_l rather than functionals of X^3_l could be interpreted in this manner. For a wide class of Hamiltonians actually effective 1-dimensionality holds true in accordance with conformal invariance.

The generalization of WCW Hamiltonians and super-Hamiltonians by allowing integrals over the 2-D boundaries of the patches of X_l^3 would be natural and is suggested by the requirement of discretized 3-dimensionality at the level of WCW.

By quantum classical correspondence the inclusions of HFFs related to the measurement resolution should also have a geometric description. Measurement resolution corresponds to braids in given time scale and as already explained there is a hierarchy of braids in time scales coming as negative powers of two corresponding to the addition of zero energy components to positive/negative energy state. Note however that particle reactions understood as decays and fusions of braid strands could also lead to a notion of measurement resolution.

12.5.3 Bott periodicity, its generalization, and dimension $D = 8$ as an inherent property of the hyper-finite II_1 factor

Hyper-finite II_1 factor can be constructed as infinite-dimensional tensor power of the Clifford algebra $M_2(C) = C(2)$ in dimension $D = 2$. More precisely, one forms the union of the Clifford algebras $C(2n) = C(2)^{\otimes n}$ of $2n$-dimensional spaces by identifying the element $x \in C(2n)$ as block diagonal elements $diag(x, x)$ of $C(2(n + 1))$. The union of these algebras is completed in weak operator topology and can be regarded as a Clifford algebra of real infinite-dimensional separable Hilbert space and thus as sub-algebra of I_∞. Also generalizations obtained by replacing complex numbers by quaternions and octions are possible.

(a) The dimension 8 is an inherent property of the hyper-finite II_1 factor since Bott periodicity theorem states $C(n+8) = C_n(16)$. In other words, the Clifford algebra $C(n+8)$ is equivalent with the algebra of 16×16 matrices with entries in $C(n)$. Or articulating it still differently: $C(n + 8)$ can be regarded as 16×16 dimensional module with $C(n)$ valued coefficients. Hence the elements in the union defining the canonical representation of hyper-finite II_1 factor are $16^n \times 16^n$ matrices having $C(0)$, $C(2)$, $C(4)$ or $C(6)$ valued valued elements.

(b) The idea about a local variant of the infinite-dimensional Clifford algebra defined by power series of space-time coordinate with Taylor coefficients which are Clifford algebra elements fixes the interpretation. The representation as a linear combination of the generators of Clifford algebra of the finite-dimensional space allows quantum generalization only in the case of Minkowski spaces. However, if Clifford algebra generators are representable as gamma *matrices*, the powers of coordinate can be absorbed to the Clifford algebra and the local algebra is lost. Only if the generators are represented as quantum versions of octonions allowing no matrix representation because of their non-associativity, the local algebra makes sense. From this it is easy to deduce both quantum and classical TGD.

12.5.4 The interpretation of Jones inclusions in TGD framework

By the basic self-referential property of von Neumann algebras one can consider several interpretations of Jones inclusions consistent with sub-system-system relationship, and it is better to start by considering the options that one can imagine.

How Jones inclusions relate to the new view about sub-system?

Jones inclusion characterizes the imbedding of sub-system $\mathcal{N}$ to $\mathcal{M}$and $\mathcal{M}$ as a finite-dimensional $\mathcal{N}$-module is the counterpart for the tensor product in finite-dimensional context. The possibility to express $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ states fractality and can be regarded as a kind of self-referential "Brahman=Atman identity" at the level of infinite-dimensional systems.

Also the mysterious looking almost identity $CH^2 = CH$ for the WCW would fit nicely with the identity $M \oplus M = M$. $M \otimes M \subset M$ in WCW Clifford algebra degrees of freedom is also implied and the construction of $\mathcal{M}$ as a union of tensor powers of $C(2)$ suggests that $M \otimes M$ allows $\mathcal{M} : \mathcal{N} = 4$ inclusion to $\mathcal{M}$. This paradoxical result conforms with the strange self-referential property of factors of II_1.

The notion of many-sheeted space-time forces a considerable generalization of the notion of sub-system and simple tensor product description is not enough. Topological picture based on the length scale resolution suggests even the possibility of entanglement between sub-systems of un-entangled sub-systems. The possibility that hyper-finite II_1-factors describe the physics of TGD also in bosonic degrees of freedom is suggested by WCW super-symmetry. On the other hand, bosonic degrees could naturally correspond to I_∞ factor so that hyper-finite II_∞ would be the net result.

The most general view is that Jones inclusion describes all kinds of sub-system-system inclusions. The possibility to assign conformal field theory to the inclusion gives hopes of rather detailed view about dynamics of inclusion.

(a) The topological condensation of space-time sheet to a larger space-time sheet mediated by wormhole contacts could be regarded as Jones inclusion. $\mathcal{N}$ would correspond to the condensing space-time sheet, $\mathcal{M}$ to the system consisting of both space-time sheets, and $\sqrt{\mathcal{M} : \mathcal{N}}$ would characterize the number of quantum spinorial degrees of freedom associated with the interaction between space-time sheets. Note that by general results $\mathcal{M} : \mathcal{N}$ characterizes the fractal dimension of quantum group ($\mathcal{M} : \mathcal{N} < 4$) or Kac-Moody algebra ($\mathcal{M} : \mathcal{N} = 4$) [B39] .

(b) The branchings of space-time sheets (space-time surface is thus homologically like branching like of Feynman diagram) correspond naturally to n-particle vertices in TGD framework. What is nice is that vertices are nice 2-dimensional surfaces rather than surfaces having typically pinch singularities. Jones inclusion would naturally appear as inclusion of operator spaces $\mathcal{N}_i$ (essentially Fock spaces for fermionic oscillator operators) creating states at various lines as sub-spaces $N_i \subset M$ of operators creating states in common von Neumann factor $\mathcal{M}$. This would allow to construct vertices and vertices in natural manner using quantum groups or Kac-Moody algebras.

The fundamental $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}$ inclusion suggests a concrete representation based on the identification $N_i = M$, where M is the universal Clifford algebra associated with incoming line and $\mathcal{N}$ is defined by the condition that $\mathcal{M}/\mathcal{N}$ is the quantum variant of Clifford algebra of H. N-particle vertices could be defined as traces of Connes products of the operators creating incoming and outgoing states. It will be found that this leads to a master formula for S-matrix if the generalization of the old-fashioned string model duality implying that all generalized Feynman diagrams reduce to diagrams involving only single vertex is accepted.

(c) If 4-surfaces can branch as the construction of vertices requires, it is difficult to argue that 3-surfaces and partonic/stringy 2-surfaces could not do the same. As a matter fact, the master formula for S-matrix to be discussed later explains the branching of 4-surfaces as an apparent effect. Despite this one can consider the possibility that this kind of joins are possible so that a new kind of mechanism of topological condensation would become possible. 3-space-sheets and partonic 2-surfaces whose p-adic fractality is characterized by different p-adic primes could be connected by "joins" representing branchings of 2-surfaces. The structures formed by soap film foam provide a very concrete illustration about what would happen. In the TGD based model of hadrons [K57] it has been assumed that join along boundaries bonds (JABs) connect quark space-time space-time sheets to the hadronic space-time sheet. The problem is that, at least for identical primes, the formation of join along boundaries bond fuses two systems to single bound state. If JABs are replaced joins, this objection is circumvented.

(d) The space-time correlate for the formation of bound states is the formation of JABs. Standard intuition tells that the number of degrees of freedom associated with the bound state is smaller than the number of degrees of freedom associated with the pair of free systems. Hence the inclusion of the bound state to the tensor product could be regarded as Jones inclusion. On the other hand, one could argue that the JABs carry additional vibrational degrees of freedom so that the idea about reduction of degrees of freedom might be wrong: free system could be regarded as sub-system of bound state by Jones inclusion. The self-referential holographic properties of von Neumann algebras allow both interpretations: any system can be regarded as sub-system of any system in accordance with the bootstrap idea.

(e) Maximal deterministic regions inside given space-time sheet bounded by light-like causal determinants define also sub-systems in a natural manner and also their inclusions would naturally correspond to Jones inclusions.

(f) The TGD inspired model for topological quantum computation involves the magnetic flux tubes defined by join along boundaries bonds connecting space-time sheets having light-like boundaries. These tubes condensed to background 3-space can become linked and knotted and code for quantum computations in this manner. In this case the addition of new strand to the system corresponds to Jones inclusion in the hierarchy associated with inclusion $\mathcal{N} \subset \mathcal{M}$. The anyon states associated with strands would be represented by a finite tensor product of quantum spinors assignable to $\mathcal{M}/\mathcal{N}$ and representing quantum counterpart of H-spinors.

One can regard $\mathcal{M} : \mathcal{N}$ degrees of freedom correspond to quantum group or Kac-Moody degrees of freedom. Quantum group degrees of freedom relate closely to the conformal and topological degrees of freedom as the connection of II_1 factors with topological quantum field theories and braid matrices suggests itself. For the canonical inclusion this factorization would correspond to factorization of quantum H-spinor from WCW spinor .

A more detailed study of canonical inclusions to be carried out later demonstrates what this factorization corresponds at the space-time level to a formation of space-time sheets which can be regarded as multiple coverings of M^4 and CP_2 with invariance group $G = G_a \times G_b \subset SL(2,C) \times SU(2)$, $SU(2) \subset SU(3)$. The unexpected outcome is that Planck constants assignable to M^4 and CP_2 degrees of freedom depend on the canonical inclusions. The existence of macroscopic quantum phases with arbitrarily large Planck constants is predicted.

It would seem possible to assign the $\mathcal{M} : \mathcal{N}$ degrees quantum spinorial degrees of freedom to the interface between subsystems represented by $\mathcal{N}$ and $\mathcal{M}$. The interface could correspond to the wormhole contacts, joins, JABs, or light-like causal determinants serving as boundary between maximal deterministic regions, etc... In terms of the bipartite diagrams representing the inclusions, joins (say) would correspond to the edges connecting white vertices representing sub-system (the entire system without the joins) to black vertices (entire system).

About the interpretation of $\mathcal{M} : \mathcal{N}$ degrees of freedom

The Clifford algebra $\mathcal{N}$ associated with a system formed by two space-time sheet can be regarded as $1 \leq \mathcal{M} : \mathcal{N} \leq 4$-dimensional module having $\mathcal{N}$ as its coefficients. It is possible to imagine several interpretations the degrees of freedom labeled by β.

(a) The $\beta = \mathcal{M} : \mathcal{N}$ degrees of freedom could relate to the interaction of the space-time sheets. Beraha numbers appear in the construction of S-matrices of topological quantum field theories and an interpretation in terms of braids is possible. This would suggest that the interaction between space-time sheets can be described in terms of conformal quantum field theory and the S-matrices associated with braids describe this interaction. Jones inclusions would characterize the effective number of active conformal degrees of freedom. At $n = 3$ limit these degrees of freedom disappear completely since the conformal field theory defined by the Chern-Simons action describing this interaction would become trivial ($c = 0$ as will be found).

(b) The interpretation in terms of imbedding space Clifford algebra would suggest that β-dimensional Clifford algebra of $\sqrt{\beta}$-dimensional spinor space is in question. For $\beta = 4$ the algebra would be the Clifford algebra of 2-dimensional space. $\mathcal{M}/\mathcal{N}$ would have interpretation as complex quantum spinors with components satisfying $z_1 z_2 = q z_2 z_1$ and its conjugate and having fractal complex dimension $\sqrt{\beta}$. This would conform with the effective 2-dimensionality of TGD. For $\beta < 4$ the fractal dimension of partonic quantum spinors defining the basic conformal fields would be reduced and become $d = 1$ for $n = 3$: the interpretation is in terms of strong correlations caused by the non-commutativity of the components of quantum spinor. For number theoretical generalizations of infinite-dimensional Clifford algebras $Cl(C)$ obtained by replacing C with Abelian complexification of quaternions or octonions one would obtain higher-dimensional spinors.

12.5.5 WCW, space-time, and imbedding space and hyper-finite type II_1 factors

The preceding considerations have by-passed the question about the relationship of WCW tangent space to its Clifford algebra. Also the relationship between space-time and imbedding space and their quantum variants could be better. In particular, one should understand how effective 2-dimensionality can be consistent with the 4-dimensionality of space-time.

Super-conformal symmetry and WCW Poisson algebra as hyper-finite type II_1 factor

It would be highly desirable to achieve also a description of the WCW degrees of freedom using von Neumann algebras. Super-conformal symmetry relating fermionic degrees of freedom and WCW degrees of freedom suggests that this might be the case. Super-symplectic algebra has as its generators configuration space Hamiltonians and their super-counterparts identifiable as CH gamma matrices. Super-symmetry requires that the Clifford algebra of CH and the Hamiltonian vector fields of CH with symplectic central extension both define hyper-finite II_1 factors. By super-symmetry Poisson bracket corresponds to an anti-commutator for gamma matrices. The ordinary quantized version of Poisson bracket is obtained as $\{P_i, Q_j\} \rightarrow [P_i, Q_j] = J_{ij} Id$. Finite trace version results by assuming that Id corresponds to the projector CH Clifford algebra having unit norm. The presence of zero modes means direct integral over these factors.

WCW gamma matrices anti-commuting to identity operator with unit norm corresponds to the tangent space $T(CH)$ of CH. Thus it would be not be surprising if $T(CH)$ could be imbedded in the sigma matrix algebra as a sub-space of operators defined by the gamma matrices generating this algebra. At least for $\beta = 4$ construction of hyper-finite II_1 factor this definitely makes sense.

The dimension of WCW defined as the trace of the projection operator to the sub-space spanned by gamma matrices is obviously zero. Thus WCW has in this sense the dimensionality of single space-time point. This sounds perhaps absurd but the generalization of the number concept implied by infinite primes indeed leads to the view that single space-time point is infinitely structured in the number theoretical sense although in the real sense all states of the point are equivalen. The reason is that there is infinitely many numbers expressible as ratios of infinite integers having unit real norm in the real sense but having different p-adic norms.

How to understand the dimensions of space-time and imbedding space?

One should be able to understand the dimensions of 3-space, space-time and imbedding space in a convincing matter in the proposed framework. There is also the question whether space-time and imbedding space emerge uniquely from the mathematics of von Neumann algebras alone.

1. The dimensions of space-time and imbedding space

Two sub-sequent inclusions dual to each other define a special kind of inclusion giving rise to a quantum counterpart of $D = 4$ naturally. This would mean that space-time is something which emerges at the level of cognitive states.

The special role of classical division algebras in the construction of quantum TGD [K88] , $D = 8$ Bott periodicity generalized to quantum context, plus self-referential property of type II_1 factors might explain why 8-dimensional imbedding space is the only possibility.

State space has naturally quantum dimension $D \leq 8$ as the following simple argument shows. The space of quantum states has quark and lepton sectors which both are super-symmetric implying $D \leq 4$ for each. Since these sectors correspond to different Hamiltonian algebras (triality one for quarks and triality zero for leptonic sector), the state space has quantum dimension $D \leq 8$.

2. How the lacking two space-time dimensions emerge?

3-surface is the basic dynamical unit in TGD framework. This seems to be in conflict with the effective 2-dimensionality [K88] meaning that partonic 2-surface code for quantum states, and with the fact that hyper-finite II_1 factors have intrinsic quantum dimension 2.

A possible resolution of the problem is that the foliation of 3-surface by partonic two-surfaces defines a one-dimensional direct integral of isomorphic hyper-finite type II_1 factors, and the zero mode labeling the 2-surfaces in the foliation serves as the third spatial coordinate. For a given 3-surface the contribution to the WCW metric can come only from 2-D partonic surfaces defined as intersections of 3-D light-like CDs with $X^7_\pm$ [K18] . Hence the direct integral should somehow relate to the classical non-determinism of Kähler action.

(a) The one-parameter family of intersections of light-like CD with $X^7_\pm$ inside $X^4 \cap X^7_\pm$ could indeed be basically due to the classical non-determinism of Kähler action. The contribution to the metric from the normal light-like direction to $X^3 = X^4 \cap X^7_\pm$ can cause the vanishing of the metric determinant $\sqrt{g_4}$ of the space-time metric at $X^2 \subset X^3$ under some conditions on X^2. This would mean that the space-time surface $X^4(X^3)$ is not uniquely determined by the minimization principle defining the value of the Kähler action, and the complete dynamical specification of X^3 requires the specification of partonic 2-surfaces X^2_i with $\sqrt{g_4} = 0$.

(b) The known solutions of field equations [K9] define a double foliation of the space-time surface defined by Hamilton-Jacobi coordinates consisting of complex transversal coordinate and two light-like coordinates for M^4 (rather than space-time surface). Number theoretical considerations inspire the hypothesis that this foliation exists always [K88] . Hence a natural hypothesis is that the allowed partonic 2-surfaces correspond to the 2-surfaces in the restriction of the double foliation of the space-time surface by partonic 2-surfaces to X^3, and are thus locally parameterized by single parameter defining the third spatial coordinate.

(c) There is however also a second light-like coordinate involved and one might ask whether both light-like coordinates appear in the direct sum decomposition of II_1 factors defining $T(CH)$. The presence of two kinds of light-like CDs would provide the lacking two space-time coordinates and quantum dimension $D = 4$ would emerge at the limit of full non-determinism. Note that the duality of space-like partonic and light-like stringy 2-surfaces conforms with this interpretation since it corresponds to a selection of partonic/stringy 2-surface inside given 3-D CD whereas the dual pairs correspond to different CDs.

(d) That the quantum dimension would be $2D_q = \beta < 4$ above CP_2 length scale conforms with the fact that non-determinism is only partial and time direction is dynamically frozen to a high degree. For vacuum extremals there is strong non-determinism but in this case there is no real dynamics. For CP_2 type extremals, which are not vacuum extremals as far action and small perturbations are considered, and which correspond to $\beta = 4$ there is a complete non-determinism in time direction since the M^4 projection of the extremal is a light-like random curve and there is full 4-D dynamics. Light-likeness gives rise to conformal symmetry consistent with the emergence of Kac Moody algebra [K9] .

3. Time and cognition

In a completely deterministic physics time dimension is strictly speaking redundant since the information about physical states is coded by the initial values at 3-dimensional slice of space-time. Hence the notion of time should emerge at the level of cognitive representations possible by to the non-determinism of the classical dynamics of TGD.

Since Jones inclusion means the emergence of cognitive representation, the space-time view about physics should correspond to cognitive representations provided by Feynman diagram states with zero energy with entanglement defined by a two-sided projection of the lowest level S-matrix. These states would represent the "laws of quantum physics" cognitively. Also space-time surface serves as a classical correlate for the evolution by quantum jumps with maximal deterministic regions serving as correlates of quantum states. Thus the classical non-determinism making possible cognitive representations would bring in time. The fact that quantum dimension of space-time is smaller than $D = 4$ would reflect the fact that the loss of determinism is not complete.

4. Do space-time and imbedding space emerge from the theory of von Neumann algebras and number theory?

The considerations above force to ask whether the notions of space-time and imbedding space emerge from von Neumann algebras as predictions rather than input. The fact that it seems possible to formulate the S-matrix and its generalization in terms of inherent properties of infinite-dimensional Clifford algebras suggest that this might be the case.

Inner automorphisms as universal gauge symmetries?

The continuous outer automorphisms Δ^{it} of HFFs of type III are not completely unique and one can worry about the interpretation of the inner automorphisms. A possible resolution of the worries is that inner automorphisms act as universal gauge symmetries containing various super-conformal symmetries as a special case. For hyper-finite factors of type II_1 in the representation as an infinite tensor power of $M_2(C)$ this would mean that the transformations non-trivial in a finite number of tensor factors only act as analogs of local gauge symmetries. In the representation as a group algebra of S_∞ all unitary transformations acting on a finite number of braid strands act as gauge transformations whereas the infinite powers $P \times P \times ...$, $P \in S_n$, would act as counterparts of global gauge transformations. In particular, the Galois group of the closure of rationals would act as local gauge transformations but diagonally represented finite Galois groups would act like global gauge transformations and periodicity would make possible to have finite braids as space-time correlates without a loss of information.

Do unitary isomorphisms between tensor powers of II_1 define vertices?

What would be left would be the construction of unitary isomorphisms between the tensor products of the HFFs of type $II_1 \otimes I_n = II_1$ at the partonic 2-surfaces defining the vertices. This would be the only new element added to the construction of braiding M-matrices.

As a matter fact, this element is actually not completely new since it generalizes the fusion rules of conformal field theories, about which standard example is the fusion rule $\phi_i = c_i{}^{jk}\phi_j\phi_k$ for primary fields. These fusion rules would tell how a state of incoming HFF decomposes to the states of tensor product of two outgoing HFFs.

These rules indeed have interpretation in terms of Connes tensor products $\mathcal{M}\otimes_{\mathcal{N}}...\otimes_{\mathcal{N}}\mathcal{M}$ for which the sub-factor $\mathcal{N}$ takes the role of complex numbers [A67] so that one has $\mathcal{M}$ becomes $\mathcal{N}$ bimodule and "quantum quantum states" have $\mathcal{N}$ as coefficients instead of complex numbers. In TGD framework this has interpretation as quantum measurement resolution characterized by $\mathcal{N}$ (the group G characterizing leaving the elements of $\mathcal{N}$ invariant defines the measured quantum numbers).

12.5.6 Quaternions, octonions, and hyper-finite type II_1 factors

Quaternions and octonions as well as their hyper counterparts obtained by multiplying imaginary units by commuting $\sqrt{-1}$ and forming a sub-space of complexified division algebra, are in in a central role in the number theoretical vision about quantum TGD [K88] . Therefore the question arises whether complexified quaternions and perhaps even octonions could be somehow inherent properties of von Neumann algebras. One can also wonder whether the quantum counterparts of quaternions and octonions could emerge naturally from von Neumann algebras. The following considerations allow to get grasp of the problem.

Quantum quaternions and quantum octonions

Quantum quaternions have been constructed as deformation of quaternions [A97] . The key observation that the Glebsch Gordan coefficients for the tensor product $3 \otimes 3 = 5 \oplus \oplus 3 \oplus 1$ of spin 1 representation of $SU(2)$ with itself gives the anti-commutative part of quaternionic product as spin 1 part in the decomposition whereas the commutative part giving spin 0 representation is identifiable as the scalar product of the imaginary parts. By combining spin 0 and spin 1 representations, quaternionic product can be expressed in terms of Glebsh-Gordan coefficients. By replacing GGC:s by their quantum group versions for group $sl(2)_q$, one obtains quantum quaternions.

There are two different proposals for the construction of quantum octonions [A51, A2] . Also now the idea is to express quaternionic and octonionic multiplication in terms of Glebsch-Gordan coefficients and replace them with their quantum versions.

(a) The first proposal [A51] relies on the observation that for the tensor product of $j = 3$ representations of $SU(2)$ the Glebsch-Gordan coefficients for $7 \otimes 7 \rightarrow 7$ in $7 \otimes 7 = 9 \oplus 7 \oplus 5 \oplus 3 \oplus 1$ defines a product, which is equivalent with the antisymmetric part of the product of octonionic imaginary units. As a matter fact, the antisymmetry defines 7-dimensional Malcev algebra defined by the anti-commutator of octonion units and satisfying b definition the identity

$$[[x, y, z], x] = [x, y, [x, z]] \quad , \quad [x, y, z] \equiv [x, [y, z]] + [y, [z, x]] + [z, [x, y]] \quad . \tag{12.5.1}$$

7-element Malcev algebra defining derivations of octonionic algebra is the only complex Malcev algebra not reducing to a Lie algebra. The $j = 0$ part of the product corresponds also now to scalar product for imaginary units. Octonions are constructed as sums of $j = 0$ and $j = 3$ parts and quantum Glebsch-Gordan coefficients define the octonionic product.

(b) In the second proposal [A2] the quantum group associated with $SO(8)$ is used. This representation does not allow unit but produces a quantum version of octonionic triality assigning to three octonions a real number.

Quaternionic or octonionic quantum mechanics?

There have been numerous attempts to introduce quaternions and octonions to quantum theory. Quaternionic or octonionic quantum mechanics, which means the replacement of the complex numbers as coefficient field of Hilbert space with quaternions or octonions, is the most obvious approach (for example and references to the literature see for instance [A91] .

In both cases non-commutativity poses serious interpretational problems. In the octonionic case the non-associativity causes even more serious obstacles [B36, A91] , [B36] .

(a) Assuming that an orthonormalized state basis with respect to an octonion valued inner product has been found, the multiplication of any basis with octonion spoils the orthonormality. The proposal to circumvent this difficulty discussed in [B36] , [B36] eliminates non-associativity by assuming that octonions multiply states one by one (rather than multiplying each other before multiplying the state). Effectively this means that octonions are replaced with 8×8-matrices.

(b) The definition of the tensor product leads also to difficulties since associativity is lost (recall that Yang-Baxter equation codes for associativity in case of braid statistics [A87]).

(c) The notion of hermitian conjugation is problematic and forces a selection of a preferred imaginary unit, which does not look nice. Note however that the local selection of a preferred imaginary unit is in a key role in the proposed construction of space-time surfaces as

hyper-quaternionic or co-hyper-quaternionic surfaces and allows to interpret space-time surfaces either as surfaces in 8-D Minkowski space M^8 of hyper-octonions or in $M^4 \times CP_2$. This selection turns out to have quite different interpretation in the proposed framework.

Hyper-finite factor II_1 has a natural Hyper-Kähler structure

In the case of hyper-finite factors of type II_1 quaternions a more natural approach is based on the generalization of the Hyper-Kähler structure rather than quaternionic quantum mechanics. The reason is that also WCW tangent space should and is expected to have this structure [K18] . The Hilbert space remains a complex Hilbert space but the quaternionic units are represented as operators in Hilbert space. The selection of the preferred unit is necessary and natural. The identity operator representing quaternionic real unit has trace equal to one, is expected to give rise to the series of quantum quaternion algebras in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ having interpretation as N-modules.

The representation of the quaternion units is rather explicit in the structure of hyper-finite II_1 factor. The $\mathcal{M} : \mathcal{N} \equiv \beta = 4$ hierarchical construction can be regarded as Connes tensor product of infinite number of 4-D Clifford algebras of Euclidian plane with Euclidian signature of metric ($diag(-1,-1)$). This algebra is nothing but the quaternionic algebra in the representation of quaternionic imaginary units by Pauli spin matrices multiplied by i.

The imaginary unit of the underlying complex Hilbert space must be chosen and there is whole sphere S^2 of choices and in every point of WCW the choice can be made differently. The space-time correlate for this local choice of preferred hyper-octonionic unit [K88] . At the level of WCW geometry the quaternion structure of the tangent space means the existence of Hyper-Kähler structure guaranteeing that WCW has a vanishing Einstein tensor. It it would not vanish, curvature scalar would be infinite by symmetric space property (as in case of loop spaces) and induce a divergence in the functional integral over 3-surfaces from the expansion of $\sqrt{g}$ [K18] .

The quaternionic units for the II_1 factor, are simply limiting case for the direct sums of 2×2 units normalized to one. Generalizing from $\beta = 4$ to $\beta < 4$, the natural expectation is that the representation of the algebra as $\beta = \mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module gives rise to quantum quaternions with quaternion units defined as infinite sums of $\sqrt{\beta} \times \sqrt{\beta}$ matrices.

At Hilbert space level one has an infinite Connes tensor product of 2-component spinor spaces on which quaternionic matrices have a natural action. The tensor product of Clifford algebras gives the algebra of 2×2 quaternionic matrices acting on 2-component quaternionic spinors (complex 4-component spinors). Thus double inclusion could correspond to (hyper-)quaternionic structure at space-time level. Note however that the correspondence is not complete since hyper-quaternions appear at space-time level and quaternions at Hilbert space level.

Von Neumann algebras and octonions

The octonionic generalization of the Hyper-Kähler manifold does not make sense as such since octonionic units are not representable as linear operators. The allowance of anti-linear operators inherently present in von Neumann algebras could however save the situation.

Indeed, the Cayley-Dickson construction for the division algebras (for a nice explanation see [A43]), which allows to extend any * algebra, and thus also any von Neumann algebra, by adding an imaginary unit it and identified as *, comes in rescue.

The basic idea of the Cayley-Dickson construction is following. The * operator, call it J, representing a conjugation defines an *anti-linear* operator in the original algebra A. One can extend A by adding this operator as a new element to the algebra. The conditions satisfied by J are

$$a(Jb) = J(a^*b) \ , \quad (aJ)b = (ab^*)J \ , \quad (Ja)(bJ^{-1}) = (ab)^* \ . \tag{12.5.2}$$

In the associative case the conditions are equivalent to the first condition.

It is intuitively clear that this addition extends the hyper-Kähler structure to an octonionic structure at the level of the operator algebra. The quantum version of the octonionic algebra is fixed by the quantum quaternion algebra uniquely and is consistent with the Cayley-Dickson construction. It is not clear whether the construction is equivalent with either of the earlier proposals [A51, A2] . It would however seem that the proposal is simpler.

Physical interpretation of quantum octonion structure

Without further restrictions the extension by J would mean that vertices contain operators, which are superpositions of linear and anti-linear operators. This would give superpositions of states and their time-reversals and mean that state could be a superposition of states with opposite values of say fermion numbers. The problem disappears if either the linear operators A or anti-linear operators JA can be used to construct physical states from vacuum. The fact, that space-time surfaces are either hyper-quaternionic or co-hyper-quaternionic, is a space-time correlate for this restriction.

The $HQ-coHQ$ duality discussed in [K88] states that the descriptions based on hyper-quaternionic and co-hyper-quaternionic surfaces are dual to each other. The duality can have two meanings.

(a) The vacuum is invariant under J so that one can use either complexified quaternionic operators A or their co-counterparts of form JA to create physical states from vacuum.

(b) The vacuum is not invariant under J. This could relate to the breaking of CP and T invariance known to occur in meson-antimeson systems. In TGD framework two kinds of vacua are predicted corresponding intuitively to vacua in which either the product of all positive or negative energy fermionic oscillator operators defines the vacuum state, and these two vacua could correspond to a vacuum and its J conjugate, and thus to positive and negative energy states. In this case the two state spaces would not be equivalent although the physics associated with them would be equivalent.

The considerations of [K88] related to the detailed dynamics of $HQ-coHQ$ duality demonstrate that the variational principles defining the dynamics of hyper-quaternionic and co-hyper-quaternionic space-time surfaces are antagonistic and correspond to world as seen by a conscientous book-keeper on one hand and an imaginative artist on the other hand. HQ case is conservative: differences measured by the magnitude of Kähler action tend to be minimized, the dynamics is highly predictive, and minimizes the classical energy of the initial state. $coHQ$ case is radical: differences are maximized (this is what the construction of sensory representations would require). The interpretation proposed in [K88] was that the two space-time dynamics are just different predictions for what would happen (has happened) if no quantum jumps would occur (had occurred). A stronger assumption is that these two views are associated with systems related by time reversal symmetry.

What comes in mind first is that this antagonism follows from the assumption that these dynamics are actually time-reversals of each other with respect to M^4 time (the rapid elimination of differences in the first dynamics would correspond to their rapid enhancement in the second dynamics). This is not the case so that T and CP symmetries are predicted to be broken in accordance with the CP breaking in meson-antimeson systems [K52] and cosmological matter-antimatter asymmetry [K80] .

12.5.7 Does the hierarchy of infinite primes relate to the hierarchy of II_1 factors?

The hierarchy of Feynman diagrams accompanying the hierarchy defined by Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset ...$ gives a concrete representation for the hierarchy of cognitive dynamics providing a representation for the material world at the lowest level of the hierarchy. This hierarchy seems to relate directly to the hierarchy of space-time sheets.

Also the construction of infinite primes [K86] leads to an infinite hierarchy. Infinite primes at the lowest level correspond to polynomials of single variable x_1 with rational coefficients, next level to polynomials x_1 for which coefficients are rational functions of variable x_2, etc... so that a natural ordering of the variables is involved.

If the variables x_i are hyper-octonions (subs-space of complexified octonions for which elements are of form $x + \sqrt{-1}y$, where x is real number and y imaginary octonion and $\sqrt{-1}$ is commuting imaginary unit, this hierarchy of states could provide a realistic representation of physical states as far as quantum numbers related to imbedding space degrees of freedom are considered in M^8 picture dual to $M^4 \times CP_2$ picture [K88] . Infinite primes are mapped to space-time surfaces in a manner analogous to the mapping of polynomials to the loci of their zeros so that infinite primes, integers, and rationals become concrete geometrical objects.

Infinite primes are also obtained by a repeated second quantization of a super-symmetric arithmetic quantum field theory. Infinite rational numbers correspond in this description to pairs of positive energy and negative energy states of opposite energies having interpretation as pairs of initial and final states so that higher level states indeed represent transitions between the states. For these reasons this hierarchy has been interpreted as a correlate for a cognitive hierarchy coding information about quantum dynamics at lower levels. This hierarchy has also been assigned with the hierarchy of space-time sheets. Just as the hierarchy of generalized Feynman diagrams provides self representations of the lowest matter level and is coded by it, finite primes code the hierarchy of infinite primes.

Infinite primes, integers, and rationals have finite p-adic norms equal to 1, and one can wonder whether a Hilbert space like structure with dimension given by an infinite prime or integer makes sense, and whether it has anything to do with the Hilbert space for which dimension is infinite in the sense of the limiting value for a dimension of sub-space. The Hilbert spaces with dimension equal to infinite prime would define primes for the tensor product of these spaces. The dimension of this kind of space defined as any p-adic norm would be equal to one.

One cannot exclude the possibility that infinite primes could express the infinite dimensions of hyper-finite III_1 factors, which cannot be excluded and correspond to that part of quantum TGD which relates to the imbedding space rather than space-time surface. Indeed, infinite primes code naturally for the quantum numbers associated with the imbedding space. Secondly, the appearance of 7-D light-like causal determinants $X^7_\pm = M^4_\pm \times CP_2$ forming nested structures in the construction of S-matrix brings in mind similar nested structures of algebraic quantum field theory [B63] . If this is were the case, the hierarchy of Beraha numbers possibly associated with the phase resolution could correspond to hyper-finite factors of type II_1, and the decomposition of space-time surface to regions labeled by p-adic primes and characterized by infinite primes could correspond to hyper-finite factors of type III_1 and represent imbedding space degrees of freedom.

The state space would in this picture correspond to the tensor products of hyper-finite factors of type II_1 and III_1 (of course, also factors I_n and I_∞ are also possible). III_1 factors could be assigned to the sub-WCWs defined by 3-surfaces in regions of M^4 expressible in terms of unions and intersections of $X^7_\pm = M^4_\pm \times CP_2$. By conservation of four-momentum, bounded regions of this kind are possible only for the states of zero net energy appearing at the higher levels of hierarchy. These sub-WCWs would be characterized by the positions of the tips of light cones $M^4_\pm \subset M^4$ involved. This indeed brings in continuous spectrum of four-momenta forcing to introduce non-separable Hilbert spaces for momentum eigen states and necessitating III_1 factors. Infinities would be avoided since the dynamics proper would occur at the level of space-time surfaces and involve only II_1 factors.

12.6 Could HFFs of type III have application in TGD framework?

One can imagine several manners for how HFFs of type III could emerge in TGD although the proposed view about M-matrix in zero energy ontology suggests that HFFs of type III_1 should be only an auxiliary tool at best. Same is suggested with interpretational problems associated with them. Both TGD inspired quantum measurement theory, the idea about a variant of HFF of type II_1 analogous to a local gauge algebra, and some other arguments, suggest that HFFs of type III could be seen as a useful idealization allowing to make non-trivial conjectures both about quantum TGD and about HFFs of type III. Quantum fields would correspond to HFFs of type III and II_∞ whereas physical states (M-matrix) would correspond to HFF of type II_1. I have summarized first the problems of III_1 factors so that reader can decide whether the further reading is worth of it.

12.6.1 Problems associated with the physical interpretation of III_1 factors

Algebraic quantum field theory approach [B34, B63] has led to a considerable understanding of relativistic quantum field theories in terms of hyper-finite III_1 factors. There are however several reasons to suspect that the resulting picture is in conflict with physical intuition. Also the infinities of non-trivial relativistic QFTs suggest that something goes wrong.

Are the infinities of quantum field theories due the wrong type of von Neumann algebra?

The infinities of quantum field theories involve basically infinite traces and it is now known that the algebras of observables for relativistic quantum field theories for bounded regions of Minkowski space correspond to hyper-finite III_1 algebras, for which non-trivial traces are always infinite. This might be the basic cause of the divergence problems of relativistic quantum field theory.

On basis of this observations there is some temptation to think that the finite traces of hyper-finite II_1 algebras might provide a resolution to the problems but not necessarily in QFT context. One can play with the thought that the subtraction of infinities might be actually a process in which III_1 algebra is transformed to II_1 algebra. A more plausible idea suggested by dimensional regularization is that the elimination of infinities actually gives rise to II_1 inclusion at the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. It is indeed known that the dimensional regularization procedure of quantum field theories can be formulated in terms of bi-algebras assignable to Feynman diagrams and [A56] and the emergence of bi-algebras suggests that a connection with II_1 factors and critical role of dimension $D = 4$ might exist.

Continuum of inequivalent representations of commutation relations

There is also a second difficulty related to type III algebras. There is a continuum of inequivalent representations for canonical commutation relations [A76] . In thermodynamics this is blessing since temperature parameterizes these representations. In quantum field theory context situation is however different and this problem has been usually put under the rug.

Entanglement and von Neumann algebras

In quantum field theories where 4-D regions of space-time are assigned to observables. In this case hyper-finite type III_1 von Neumann factors appear. Also now inclusions make sense and has been studiedin fact, the parameters characterizing Jones inclusions appear also now and this due to the very general properties of the inclusions.

The algebras of type III_1 have rather counter-intuitive properties from the point of view of entanglement. For instance, product states between systems having space-like separation are not possible at all so that one can speak of intrinsic entanglement [A78] . What looks worse is that the decomposition of entangled state to product states is highly non-unique.

Mimicking the steps of von Neumann one could ask what the notion of observables could mean in TGD framework. Effective 2-dimensionality states that quantum states can be constructed using the data given at partonic or stringy 2-surfaces. This data includes also information about normal derivatives so that 3-dimensionality actually lurks in. In any case this would mean that observables are assignable to 2-D surfaces. This would suggest that hyper-finite II_1 factors appear in quantum TGD at least as the contribution of single space-time surface to S-matrix is considered. The contributions for WCW degrees of freedom meaning functional (not path-) integral over 3-surfaces could of course change the situation.

Also in case of II_1 factors, entanglement shows completely new features which need not however be in conflict with TGD inspired view about entanglement. The eigen values of density matrices are infinitely degenerate and quantum measurement can remove this degeneracy only partially. TGD inspired theory of consciousness has led to the identification of rational (more generally algebraic entanglement) as bound state entanglement stable in state function reduction. When an infinite number of states are entangled, the entanglement would correspond to rational (algebraic number) valued traces for the projections to the eigen states of the density matrix. The symplectic transformations of CP_2 are almost $U(1)$ gauge symmetries broken only by classical gravitation. They imply a gigantic spin glass degeneracy which could be behind the infinite degeneracies of eigen states of density matrices in case of II_1 factors.

12.6.2 Quantum measurement theory and HFFs of type III

The attempt to interpret the HFFs of type III in terms of quantum measurement theory based on Jones inclusions leads to highly non-trivial conjectures about these factors.

Could the scalings of trace relate to quantum measurements?

What should be understood is the physical meaning of the automorphism inducing the scaling of trace. In the representation based of factors based on infinite tensor powers the action of g should transform single $n \times n$ matrix factor with density matrix Id/n to a density matrix e_{11} of a pure state.

Obviously the number of degrees of freedom is affected and this can be interpreted in terms of appearance or disappearance of correlations. Quantization and emergence of non-commutativity indeed implies the emergence of correlations and effective reduction of degrees of freedom. In particular, the fundamental quantum Clifford algebra has reduced dimension $\mathcal{M} : \mathcal{N} = r \leq 4$ instead of $r = 4$ since the replacement of complex valued matrix elements with $\mathcal{N}$ valued ones implies non-commutativity and correlations.

The transformation would be induced by the shift of finite-dimensional state to right or left so that the number of matrix factors overlapping with I_∞ part increases or is reduced. Could it have interpretation in terms of quantum measurement for a quantum Clifford factor? Could quantum measurement for $\mathcal{M}/\mathcal{N}$ degrees of freedom reducing the state in these degrees of freedom to a pure state be interpreted as a transformation of single finite-dimensional matrix factor to a type I factor inducing the scaling of the trace and could the scalings associated with automorphisms of HFFs of type III also be interpreted in terms of quantum measurement?

This interpretation does not as such say anything about HFF factors of type III since only a decomposition of II_1 factor to I_2^k factor and II_1 factor with a reduced trace of projector to the latter. However, one can ask whether the scaling of trace for HFFs of type III could correspond to a situation in which infinite number of finite-dimensional factors have been quantum measured. This would correspond to the inclusion $\mathcal{N} \subset \mathcal{M}_\infty = \cup_n \mathcal{M}_n$ where $\mathcal{N} \subset \mathcal{M} \subset ...\mathcal{M}_n...$ defines the canonical inclusion sequence. Physicist can of course ask whether the presence of infinite number of I_2-, or more generally, I_n-factors is at all relevant to quantum measurement and it has already become clear that situation at the level of M-matrix reduces to I_n.

Could the theory of HHFs of type III relate to the theory of Jones inclusions?

The idea about a connection of HFFs of type III and quantum measurement theory seems to be consistent with the basic facts about inclusions and HFFs of type III_1.

(a) Quantum measurement would scale the trace by a factor $2^k/\sqrt{\mathcal{M}:\mathcal{N}}$ since the trace would become a product for the trace of the projector to the newly born $M(2,C)^{\otimes k}$ factor and the trace for the projection to $\mathcal{N}$ given by $1/\sqrt{\mathcal{M}:\mathcal{N}}$. The continuous range of values $\mathcal{M}:\mathcal{N} \geq 4$ gives good hopes that all values of λ are realized. The prediction would be that $2^k\sqrt{\mathcal{M}:\mathcal{N}} \geq 1$ holds always true.

(b) The values $\mathcal{M}:\mathcal{N} \in \{r_n = 4cos^2(\pi/n)\}$ for which the single $M(2,C)$ factor emerges in state function reduction would define preferred values of the inverse of $\lambda = \sqrt{\mathcal{M}:\mathcal{N}/4}$ parameterizing factors III_λ. These preferred values vary in the range $[1/2, 1]$.

(c) $\lambda = 1$ at the end of continuum would correspond to HFF III_1 and to Jones inclusions defined by infinite cyclic subgroups dense in $U(1) \subset SU(2)$ and this group combined with reflection. These groups correspond to the Dynkin diagrams A_∞ and D_∞. Also the classical values of $\mathcal{M}:\mathcal{N} = n^2$ characterizing the dimension of the quantum Clifford $\mathcal{M}:\mathcal{N}$ are possible. In this case the scaling of trace would be trivial since the factor n to the trace would be compensated by the factor $1/n$ due to the disappearance of $\mathcal{M}/\mathcal{N}$ factor III_1 factor.

(d) Inclusions with $\mathcal{M}:\mathcal{N} = \infty$ are also possible and they would correspond to $\lambda = 0$ so that also III_0 factor would also have a natural identification in this framework. These factors correspond to ergodic systems and one might perhaps argue that quantum measurement in this case would give infinite amount of information.

(e) This picture makes sense also physically. p-Adic thermodynamics for the representations of super-conformal algebra could be formulated in terms of factors of type I_∞ and in excellent approximation using factors I_n. The generation of arbitrary number of type II_1 factors in quantum measurement allow this possibility.

The end points of spectrum of preferred values of λ are physically special

The fact that the end points of the spectrum of preferred values of λ are physically special, supports the hopes that this picture might have something to do with reality.

(a) The Jones inclusion with $q = exp(i\pi/n)$, $n = 3$ (with principal diagram reducing to a Dynkin diagram of group SU(3)) corresponds to $\lambda = 1/2$, which corresponds to HFF III_1 differing in essential manner from factors III_λ, $\lambda < 1$. On the other hand, $SU(3)$ corresponds to color group which appears as an isometry group and important subgroup of automorphisms of octonions thus differs physically from the ADE gauge groups predicted to be realized dynamically by the TGD based view about McKay correspondence [A17] .

(b) For $r = 4$ SU(2) inclusion parameterized by extended ADE diagrams $M(2,C)^{\otimes 2}$ would be created in the state function reduction and also this would give $\lambda = 1/2$ and scaling by a factor of 2. Hence the end points of the range of discrete spectrum would correspond to the same scaling factor and same HFF of type III. SU(2) could be interpreted either as electro-weak gauge group, group of rotations of th geodesic sphere of $\delta M^4_\pm$, or a subgroup of SU(3). In TGD interpretation for McKay correspondence a phase transition replacing gauge symmetry with Kac-Moody symmetry.

(c) The scalings of trace by factor 2 seem to be preferred physically which should be contrasted with the fact that primes near prime powers of 2 and with the fact that quantum phases $q = exp(i\pi/n)$ with n equal to Fermat integer proportional to power of 2 and product of the Fermat primes (the known ones are 5, 17, 257, and $2^{16}+1$) are in a special role in TGD Universe.

12.6.3 What could one say about II_1 automorphism associated with the II_∞ automorphism defining factor of type III?

An interesting question relates to the interpretation of the automorphisms of II_∞ factor inducing the scaling of trace.

(a) If the automorphism for Jones inclusion involves the generator of cyclic automorphism sub-group Z_n of II_1 factor then it would seem that for other values of λ this group cannot be cyclic. SU(2) has discrete subgroups generated by arbitrary phase q and these are dense in $U(1) \subset SU(2)$ sub-group. If the interpretation in terms of Jones inclusion makes sense then the identification $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ makes sense.

(b) If HFF of type II_1 is realized as group algebra of infinite symmetric group [A17] , the outer automorphism induced by the diagonally imbedded finite Galois groups can induce only integer values of n and Z_n would correspond to cyclic subgroups. This interpretation conforms with the fact that the automorphisms in the completion of inner automorphisms of HFF of type II_1 induce trivial scalings. Therefore only automorphisms which do not belong to this completion can define HFFs of type III.

12.6.4 What could be the physical interpretation of two kinds of invariants associated with HFFs type III?

TGD predicts two kinds of counterparts for S-matrix: M-matrix and U-matrix. Both are expected to be more or less universal.

There are also *two* kinds of invariants and automorphisms associated with HFFs of type III.

(a) The first invariant corresponds to the scaling $\lambda \in]0, 1[$ of the trace associated with the automorphism of factor of II_∞. Also the end points of the interval make sense. The inverse of this scaling accompanies the inverse of this automorphism.

(b) Second invariant corresponds to the time scales $t = T_0$ for which the outer automorphism σ_t reduces to inner automorphism. It turns out that T_0 and λ are related by the formula $\lambda^{iT_0} = 1$, which gives the allowed values of T_0 as $T_0 = n2\pi/log(\lambda)$ [A55] . This formula can be understood intuitively by realizing that λ corresponds to the eigenvalue of the density matrix $\Delta = e^H$ in the simplest possible realization of the state ϕ.

The presence of two automorphisms and invariants brings in mind U matrix characterizing the unitary process occurring in quantum jump and M-matrix characterizing time like entanglement.

(a) If one accepts the vision based on quantum measurement theory then λ corresponds to the scaling of the trace resulting when quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ reduces to a tensor power of $M(2, C)$ factor in the state function reduction. The proposed interpretation for U process would be as the inverse of state function reduction transforming this factor back to $\mathcal{M}/\mathcal{N}$. Thus U process and state function reduction would correspond naturally to the scaling and its inverse. This picture might apply not only in single particle case but also for zero energy states which can be seen as states associated the a tensor power of HFFs of type II_1 associated with partons.

(b) The implication is that U process can occur only in the direction in which trace is reduced. This would suggest that the full III_1 factor is not a physical notion and that one must restrict the group Z in the crossed product $Z \times_{cr} II_\infty$ to the group N of non-negative integers. In this kind of situation the trace is well defined since the traces for the terms in the crossed product comes as powers λ^{-n} so that the net result is finite. This would mean a reduction to II_∞ factor.

(c) Since time t is a natural parameter in elementary particle physics experiment, one could argue that σ_t could define naturally M-matrix. Time parameter would most naturally correspond to a parameter of scaling affecting all $M^4_\pm$ coordinates rather than linear time. This conforms also with the fundamental role of conformal transformations and scalings in TGD framework.

The identification of the full M-matrix in terms of σ does not seem to make sense generally. It would however make sense for incoming and outgoing number theoretic braids so that σ could define universal braiding M-matrices. Inner automorphisms would bring in the dependence on experimental situation. The reduction of the braiding matrix to an inner automorphism for critical values of t which could be interpreted in terms of scaling by power of p. This trivialization would be a counterpart for the elimination of propagator legs from M-matrix element. Vertex itself could be interpreted as unitary isomorphism between tensor product of incoming and outgoing HFFs of type II_1 would code all what is relevant about the particle reaction.

12.6.5 Does the time parameter t represent time translation or scaling?

The connection $T_n = n2\pi/log(\lambda)$ would give a relationship between the scaling of trace and value of time parameter for which the outer automorphism represented by σ reduces to inner automorphism. It must be emphasized that the time parameter t appearing in σ need not have anything to do with time translation. The alternative interpretation is in terms of $M^4_\pm$ scaling (implying also time scaling) but one cannot exclude even preferred Lorentz boosts in the direction of quantization axis of angular momentum.

Could the time parameter correspond to scaling?

The central role of conformal invariance in quantum TGD suggests that t parameterizes scaling rather than translation. In this case scalings would correspond to powers of $(K\lambda)^n$. The numerical factor K which cannot be excluded a priori, seems to reduce to $K = 1$.

(a) The scalings by powers of p have a simple realization in terms of the representation of HFF of type II_∞ as infinite tensor power of $M(p, C)$ with suitably chosen densities matrices in factors to get product of I_∞ and II_1 factor. These matrix algebras have the remarkable property of defining prime tensor power factors of finite matrix algebras. Thus p-adic fractality would reflect directly basic properties of matrix algebras as suggested already earlier. That scalings by powers of p would correspond to automorphism reducing to inner automorphisms would conform with p-adic fractality.

(b) Also scalings by powers $[\sqrt{\mathcal{M} : \mathcal{N}}/2^k]^n$ would be physically preferred if one takes previous arguments about Jones inclusions seriously and if also in this case scalings are involved. For $q = exp(i\pi/n)$, $n = 5$ the minimal value of n allowing universal topological quantum computation would correspond to a scaling by Golden Mean and these fractal scalings indeed play a key role in living matter. In particular, Golden Mean makes it visible in the geometry of DNA.

Could the time parameter correspond to time translation?

One can consider also the interpretation of σ_t as time translation. TGD predicts a hierarchy of Planck constants parameterized by rational numbers such that integer multiples are favored.

In particular, integers defining ruler and compass polygons are predicted to be in a very special role physically. Since the geometric time span associated with zero energy state should scale as Planck constant one expects that preferred values of time t associated with σ are quantized as rational multiples of some fundamental time scales, say the basic time scale defined by CP_2 length or p-adic time scales.

(a) For $\lambda = 1/p$, p prime, the time scale would be $T_n = nT_1$, $T_1 = T_0 = 2\pi/log(p)$ which is not what p-adic length scale hypothesis would suggest.

(b) For Jones inclusions one would have $T_n/T_0 = n2\pi/log(2^{2k}/\mathcal{M} : \mathcal{N})$. In the limit when λ becomes very small (the number k of reduced $M(2, C)$ factors is large one obtains $T_n = (n/k)t_1$, $T_1 = T_0\pi/log(2)$. Approximate rational multiples of the basic length scale would be obtained as also predicted by the general quantization of Planck constant.

p-Adic thermodynamics from first principles

Quantum field theory at non-zero temperature can be formulated in the functional integral formalism by replacing the time parameter associated with the unitary time evolution operator $U(t)$ with a complexified time containing as imaginary part the inverse of the temperature: $t \to t + i\hbar/T$. In the framework of standard quantum field theory this is a mere computational trick but the time parameter associated with the automorphisms σ_t of HFF of type III is a temperature like parameter from the beginning, and its complexification would naturally lead to the analog of thermal QFT.

Thus thermal equilibrium state would be a genuine quantum state rather than fictive but useful auxiliary notion. Thermal equilibrium is defined separately for each incoming parton braid and perhaps even braid (partons can have arbitrarily large size). At elementary particle level p-adic thermodynamics could be in question so that particle massivation would have first principle description. p-Adic thermodynamics is under relatively mild conditions equivalent with its real counterpart obtained by the replacement of p^{L_0} interpreted as a p-adic number with p^{-L_0} interpreted as a real number.

12.6.6 Could HFFs of type III be associated with the dynamics in $M^4_\pm$ degrees of freedom?

HFFs of type III could be also assigned with the poorly understood dynamics in $M^4_\pm$ degrees of freedom which should have a lot of to do with four-dimensional quantum field theory. Hyper-finite factors of type III_1 might emerge when one extends II_1 to a local algebra by multiplying it with hyper-octonions replaced as analog of matrix factor and considers hyper-quaternionic subalgebra. The resulting algebra would be the analog of local gauge algebra and the elements of algebra would be analogous to conformal fields with complex argument replaced with hyper-octonionic, -quaternionic, or -complex one. Since quantum field theory in M^4 gives rise to hyper-finite III_1 factors one might guess that the hyper-quaternionic restriction indeed gives these factors.

The expansion of the local HFF II_∞ element as $O(m) = \sum_n m^n O_n$, where M^4 coordinate m is interpreted as hyper-quaternion, could have interpretation as expansion in which O_n belongs to $\mathcal{N}g^n$ in the crossed product $\mathcal{N} \times_{cr} \{g^n, n \in Z\}$. The analogy with conformal fields suggests that the power g^n inducing λ^n fold scaling of trace increases the conformal weight by n.

One can ask whether the scaling of trace by powers of λ defines an inclusion hierarchy of sub-algebras of conformal sub-algebras as suggested by previous arguments. One such hierarchy would be the hierarchy of sub-algebras containing only the generators O_m with conformal weight $m \geq n$, $n \in Z$.

It has been suggested that the automorphism Δ could correspond to scaling inside light-cone. This interpretation would fit nicely with Lorentz invariance and TGD in general. The factors III_λ with λ generating semi-subgroups of integers (in particular powers of primes) could be of special physical importance in TGD framework. The values of t for which automorphism reduces to inner automorphism should be of special physical importance in TGD framework. These automorphisms correspond to scalings identifiable in terms of powers of p-adic prime p so that p-adic fractality would find an explanation at the fundamental level.

If the above mentioned expansion in powers of m^n of $M^4_\pm$ coordinate makes sense then the action of σ^t representing a scaling by p^n would leave the elements O invariant or induce a mere inner automorphism. Conformal weight n corresponds naturally to n-ary p-adic length scale by uncertainty principle in p-adic mass calculations.

The basic question is the physical interpretation of the automorphism inducing the scaling of trace by λ and its detailed action in HFF. This scaling could relate to a scaling in M^4 and to the appearance in the trace of an integral over M^4 or subspace of it defining the trace. Fractal structures suggests itself strongly here. At the level of construction of physical states one always selects some minimum non-positive conformal weight defining the tachyonic ground state and physical states have non-negative conformal weights. The interpretation would be as a reduction to HHF of type II_∞ or even II_1.

12.6.7 Could the continuation of braidings to homotopies involve Δ^{it} automorphisms

The representation of braidings as special case of homotopies might lead from discrete automorphisms for HFFs type II_1 to continuous outer automorphisms for HFFs of type III_1. The question is whether the periodic automorphism of II_1 represented as a discrete sub-group of $U(1)$ would be continued to $U(1)$ in the transition.

The automorphism of II_∞ HFF associated with a given value of the scaling factor λ is unique. If Jones inclusions defined by the preferred values of λ as $\lambda = \sqrt{\mathcal{M}:\mathcal{N}}/2^k$ (see the previous considerations), then this automorphism could involve a periodic automorphism of II_1 factor defined by the generator of cyclic subgroup Z_n for $\mathcal{M}:\mathcal{N} < 4$ besides additional shift transforming II_1 factor to I_∞ factor and inducing the scaling.

12.6.8 HFFs of type III as super-structures providing additional uniqueness?

If the braiding M-matrices are as such highly unique. One could however consider the possibility that they are induced from the automorphisms σ_t for the HFFs of type III restricted to HFFs of type II_∞. If a reduction to inner automorphism in HFF of type III implies same with respect to HFF of type II_∞ and even II_1, they could be trivial for special values of time scaling t assignable to the partons and identifiable as a power of prime p characterizing the parton. This would allow to eliminate incoming and outgoing legs. This elimination would be the counterpart of the division of propagator legs in quantum field theories. Particle masses would however play no role in this process now although the power of padic prime would fix the mass scale of the particle.

12.7 A vision about the role of HFFs in TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

12.7.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi,\xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A35] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

(a) Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

(b) A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

(c) Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

(d) For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

(a) Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

(b) For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0, 1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

(c) Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

(d) Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X, μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

(a) A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

(b) A positive linear functional is weight with $\omega(1)$ finite.

(c) A state is a weight with $\omega(1) = 1$.

(d) A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

(e) A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

(a) Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

(b) Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{12.7.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*^{\ *} = \mathcal{M}$.

(c) The conjugation $x \rightarrow x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \rightarrow L^2(\mathcal{M})$ via $x \rightarrow x\Omega$. The map $S_0; x\Omega \rightarrow x^*\Omega$ is however non-isometric.

(d) Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

(e) What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in zero energy ontology.

The basic results of Tomita-Takesaki theory are following.

(a) The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

(b) The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A59, A127] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

(c) $\omega \rightarrow \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

(a) One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

(b) The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

(c) Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a manner to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K99] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A3] and those of factors of type III by Alain Connes [A54] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M}:\mathcal{N}$ for the inclusion $\mathcal{N}\subset\mathcal{M}$ can be defined as $\mathcal{M}:\mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M}:\mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M}:\mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A3] .

(a) For pairs $\mathcal{N}\subset\mathcal{M}$ with a finite principal graph the values of $\mathcal{M}:\mathcal{N}$ are given by

$$\begin{aligned} a)\ \ \mathcal{M}:\mathcal{N} &= 4cos^2(\pi/h)\ \ ,\quad h\geq 3\ \ ,\\ b)\ \ \mathcal{M}:\mathcal{N} &\geq 4\ \ . \end{aligned} \tag{12.7.2}$$

the numbers at right hand side are known as Beraha numbers [A108] . The comments below give a rough idea about what finiteness of principal graph means.

(b) As explained in [B39] , for $\mathcal{M}:\mathcal{N}<4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g)-r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M}:\mathcal{N}=4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A85] is following. The ADE diagrams are associated with the $n=\infty$ case having $\mathcal{M}:\mathcal{N}\geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M}:\mathcal{N}<4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

Connes tensor product

The inclusions The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} = \mathcal{M}/\mathcal{N}\otimes\mathcal{N}\ \ . \tag{12.7.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n\times n$ matrices acts on V from right, V can be regarded as a space formed by $m\times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n\times r$ matrices.

(a) In the first representation the Connes tensor product of spaces V and W consists of $m\times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

(b) An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn}\otimes e^{nr}$$

in the tensor product $V\otimes W$.

(c) One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger\otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m=r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

(d) Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

12.7.2 Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A92, A59, A127] . There are good arguments showing that in HFFS of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A127] deserve to be listed since they are suggestive also from the point of view of TGD.

(a) Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E\in\mathcal{M}(\mathcal{O})$ can be written as WW^* with $W\in\mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W=1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

(b) Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

(c) The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFS of type III_1s is induced by set theoretic inclusions.

12.7.3 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

(a) Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

(b) Is the identification $M = \Delta^{it}$ sensible is quantum TGD and zero energy ontology, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

(c) TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

(d) What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

(a) What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

(b) What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

(c) Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the modified Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the modified Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

(a) Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

(b) Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

Zero energy ontology and factors

The first question concerns the identification of the Hilbert space associated with the factors in zero energy ontology. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

(a) The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

(b) The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in zero energy ontology.

(a) In zero energy ontology M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFS is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

(b) Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \rightarrow 0$ limit. In quantum TGD the exponent of modified Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Modified Dirac action can therefore be regarded as a "square root" of Kähler action.

(c) The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in zero energy ontology based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

(d) In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the modified Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFS involves further conceptual problems.

(a) The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of modified Dirac action [K17] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

(b) Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

(c) Quantum criticality means that modified Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

(d) The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

(e) Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

(a) The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

(b) The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

(c) The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

(a) The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

(b) The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

(c) Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

(d) One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

(a) In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K27] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

(b) Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

(c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

12.7.4 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j\cdot A = 0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j\cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{12.7.4}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Localization of the modes of Kähler-Dirac operator at string world sheets and definition of Dirac determinant

The condition that the modes of Kähler-Dirac operator have well defined electromagnetic charge eigenvalue implies that the modes are restricted to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces [K105]. In the generic case one would have a product of Dirac determinants associated with these 2-surfaces. This obviously simplifies dramatically the definition of Dirac determinant and suggests a reduction to stringy mathematics, where this kind of determinants appear routinely.

The construction of Dirac determinant could proceed in following manner.

(a) The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

(b) The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

(c) The square of the Dirac determinant would be product of eigenvalues mass squared operator assignable to the eigenvalue equation $(p^k\gamma_k+\Gamma^n)^2\Psi = \Lambda_n)\Psi$. If the eigenvalues correspond up to multiplicative factor to integer valued conformal weights, the square of Dirac determinant would be the product of corresponding mass squared values equal to conformal weight with vacuum contribution. The square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + n$, where h is expressed using unit of mass determined by CP_2 radius.

(d) One can of course ask whether it might be possible to define even the Dirac determinant itself. Here it seems that the only possible manner to proceed is number theoretic: the factors $p^k\gamma_k + \Gamma^n$ appearing in the formal Dirac determinant should be mapped to complexified octonions and the product of these factors should define Dirac determinant as complex quantity having interpretation as the product of exponents of Kähler for Euclidian and Minkowskian regions metting at wormhole throat. This would be a rather deep connection with the number theoretic approach.

(e) Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

The resulting determinant - if indeed well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

One must however be aware about possible objections against the hypothesis that the square of Dirac determinant gives the modulus squared for the vacuum functional.

(a) It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

(b) Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

(a) At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

(b) In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

(c) Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

(d) Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

(e) The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

(f) Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??**, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

Quantum TGD as square root of thermodynamics

Zero energy ontology (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A92] [K99]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A92], [K99] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the modified Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes. Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K27]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. The inclusions of these conformal algebras would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

Summarizing

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD and that different measurement interactions having interpretation in terms of different kind of quantum measurements causing wave function collapse in zero mode sector of WCW could correspond to various choices of ω. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

12.7.5 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in zero energy ontology

Some clarifications concerning the notion of observable in zero energy ontology are in order.

(a) As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

(b) Also the conjugation $A \to JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

(c) Zero energy ontology gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish zero energy ontology allows a symmetry breaking respecting a chosen Cartan algebra.

(d) In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFS as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

(a) Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

(b) The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

(c) The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{12.7.5}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{12.7.6}$$

(d) Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{12.7.7}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M}:\mathcal{N} \times Tr_N(SS^\dagger) = \mathcal{M}:\mathcal{N} \times Tr(P_N) = 1 \ . \qquad (12.7.8)$$

(e) Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

(f) This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

(a) Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

(b) The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

(c) For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

(a) In zero energy ontology the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

(b) The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

(c) The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

(d) In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A15] .

(e) Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A15] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

(f) If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

(a) Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \quad (12.7.9)$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

(b) $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

(c) Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \quad (12.7.10)$$

for any physically reasonable choice $\mathcal{N}$.

(d) In TGD framework the condition would say that the M-matrix defined by the modified Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain modified gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A41] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

(a) The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

(b) One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

(a) Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

(b) The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

(c) The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

12.7.6 Questions about quantum measurement theory in zero energy ontology

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

12.7.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [K20] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

(d) The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

12.7.8 Planar algebras and generalized Feynman diagrams

Planar algebras [A21] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A45] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K15] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

(a) One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

(b) One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

(c) One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

(d) It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

(a) Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

(b) Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

(c) The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

(d) The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

(e) One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

(a) The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

(b) The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

[Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

(c) Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

(d) Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

(e) There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

12.7.9 Miscellaneous

The following considerations are somewhat out-of-date: hence the title 'Miscellaneous'.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A85] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A123] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

(a) For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

(b) The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

(c) A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A52] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k+2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_\mathcal{N} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K20] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern-Simons action [A125] .

(a) The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

(b) Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K97] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K24] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A125] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

12.8 Fresh view about hyper-finite factors in TGD framework

In the following I will discuss the basic ideas about the role of hyper-finite factors in TGD with the background given by a work of more than half decade. First I summarize the input ideas which I combine with the TGD inspired intuitive wisdom about HFFs of type II_1 and their inclusions allowing to represent finite measurement resolution and leading to notion of quantum spaces with algebraic number valued dimension defined by the index of the inclusion.

Also an argument suggesting that the inclusions define "skewed" inclusions of lattices to larger lattices giving rise to quasicrystals is proposed. The core of the argument is that the included HFF of type II_1 algebra is a projection of the including algebra to a subspace with dimension $D \leq 1$. The projection operator defines the analog of a projection of a bigger lattice to the included lattice. Also the fact that the dimension of the tensor product is product of dimensions of factors just like the number of elements in finite group is product of numbers of elements of coset space and subgroup, supports this interpretation.

One also ends up with a detailed identification of the hyper-finite factors in orbital degrees of freedom in terms of symplectic group associated with $\delta M^4_\pm \times CP_2$ and the group algebras of their discrete subgroups define what could be called "orbital degrees of freedom" for WCW spinor fields. By very general argument this group algebra is HFF of type II, maybe even II_1.

12.8.1 Crystals, quasicrystals, non-commutativity and inclusions of hyperfinite factors of type II_1

I list first the basic ideas about non-commutative geometries and give simple argument suggesting that inclusions of HFFs correspond to "skewed" inclusions of lattices as quasicrystals.

(a) Quasicrystals (say Penrose tilings) [A24] can be regarded as subsets of real crystals and one can speak about "skewed" inclusion of real lattice to larger lattice as quasicrystal. What this means that included lattice is obtained by projecting the larger lattice to some lower-dimensional subspace of lattice.

(b) The argument of Connes concerning definition of non-commutative geometry can be found in the book of Michel Lapidus at page 200. Quantum space is identified as a space of equivalence classes. One assigns to pairs of elements inside equivalence class matrix elements having the element pair as indices (one assumes numerable equivalence class). One considers irreducible representations of the algebra defined by the matrices and identifies the equivalent irreducible representations. If I have understood correctly, the equivalence classes of irreps define a discrete point set representing the equivalence class and it can often happen that there is just single point as one might expect. This I do not quite understand since it requires that all irreps are equivalent.

(c) It seems that in the case of linear spaces - von Neumann algebras and accompanying Hilbert spaces - one obtains a connection with the inclusions of HFFs and corresponding quantum factor spaces that should exist as analogs of quantum plane. One replaces matrices with elements labelled by element pairs with linear operators in HFF of type II_1. Index pairs correspond to pairs in linear basis for the HFF or corresponding Hilbert space.

(d) Discrete infinite enumerable basis for these operators as a linear space generates a lattice in summation. Inclusion $N \subset M$ defines inclusion of the lattice/crystal for N to the corresponding lattice of M. Physical intuition suggests that if this inclusion is "skewed" one obtains quasicrystal. The fact the index of the inclusion is algebraic number suggests that the coset space M/N is indeed analogous to quasicrystal.

More precisely, the index of inclusion is defined for hyper-finite factors of type II_1 using the fact that quantum trace of unit matrix equals to unity $Tr(Id(M)) = 1$, and from the tensor product composition $M = (M/N) \times N$ given $Tr(Id(M)) = 1 = Ind(M/N)Tr(P(M \rightarrow N))$, where $P(M \rightarrow N$ is projection operator from M to N. Clearly, $Ind(M/N) = 1/Tr(P(M \rightarrow N))$ defines index as a dimension of quantum space M/N.

For Jones inclusions characterized by quantum phases $q = exp(i2\pi/n)$, $n = 3, 4, ...$ the values of index are given by $Ind(M/N) = 4cos^2(\pi/n)$, $n = 3, 4, ...$ There is also another range inclusions $Ind(M/N) \geq 4$: note that $Tr(P(M \rightarrow N))$ defining the dimension of N as included sub-space is never larger than one for HFFs of type II_1. The projection operator $P(M \rightarrow N)$ is obviously the counterpart of the projector projecting lattice to some lower-dimensional sub-space of the lattice.

(e) Jones inclusions are between linear spaces but there is a strong analogy with non-linear coset spaces since for the tensor product the dimension is product of dimensions and for discrete coset spaces G/H one has also the product formula $n(G) = n(H) \times n(G/H)$ for the numbers of elements. Noticing that space of quantum amplitudes in discrete space has dimension equal to the number of elements of the space, one could say that Jones inclusion represents quantized variant for classical inclusion raised from the level of discrete space to the level of space of quantum states with the number of elements of set replaced by dimension. In fact, group algebras of infinite and enumerable groups defined HFFs of type II under rather general conditions (see below).

Could one generalize Jones inclusions so that they would apply to non-linear coset spaces analogs of the linear spaces involved ? For instance, could one think of infinite-dimensional groups G and H for which Lie-algebras defining their tangent spaces can be regarded as HFFs of type II_1? The dimension of the tangent space is dimension of the non-linear manifold: could this mean that the non-linear infinite-dimensional inclusions reduce to tangent space level and thus to the inclusions for Lie-algebras regarded hyper-finite factors of type II_1 or more generally, type II? This would would rise to quantum spaces which have finite but algebraic valued quantum dimension and in TGD framework take into account the finite measurement resolution.

(f) To concretize this analogy one can check what is the number of points map from 5-D space containing aperiodic lattice as a projection to a 2-D irrational plane containing only origin as common point with the 5-D lattice. It is easy to get convinced that the projection is 1-to-1 so that the number of points projected to a given point is 1. By the analogy with Jones inclusions this would mean that the included space has same von Neumann dimension 1 - just like the including one. In this case quantum phase equals $q = exp(i2\pi/n)$, $n = 3$ - the lowest possible value of n. Could one imagine the analogs of $n > 3$ inclusions for which the number of points projected to a given point would be larger than 1? In 1-D case the rational lines $y = (k/l)x$ define 1-D rational analogs of quasi crystals. The points $(x, y) = (m, n)$, $m \ mod \ l = 0$ are projected to the same point. The number of points is now infinite and the ratio of points of 2-D lattice and 1-D crystal like structure equals to l and serves as the analog for the quantum dimension $d_q = 4cos^2(\pi/n)$.

To sum up, this this is just physicist's intuition: it could be wrong or something totally trivial from the point of view of mathematician. The main message is that the inclusions of HFFs might define also inclusions of lattices as quasicrystals.

12.8.2 HFFs and their inclusions in TGD framework

In TGD framework the inclusions of HFFs have interpretation in terms of finite measurement resolution. If the inclusions define quasicrystals then finite measurement resolution would lead to quasicrystals.

(a) The automorphic action of N in $M \supset N$ and in associated Hilbert space H_M where N acts generates physical operators and accompanying stas (operator rays and rays) not distinguishable from the original one. States in finite measurement resolution correspond to N-rays rather than complex rays. It might be natural to restrict to unitary elements of N.

    This leads to the need to construct the counterpart of coset space M/N and corresponding linear space H_M/H_N. Physical intuition tells that the indices of inclusions defining the "dimension" of M/N are algebraic numbers given by Jones index formula.

(b) Here the above argument would assign to the inclusions also inclusions of lattices as quasicrystals.

Degrees of freedom for WCW spinor field

Consider first the identification of various kinds of degrees of freedom in TGD Universe.

(a) Very roughly, WCW ("world of classical worlds") spinor is a state generated by fermionic creation operators from vacuum at given 3-surface. WCW spinor field assigns this kind of spinor to each 3-surface. WCW spinor fields decompose to tensor product of spin part (Fock state) and orbital part ("wave" in WCW) just as ordinary spinor fields.

(b) The conjecture motivated by super-symmetry has been that both WCW spinors and their orbital parts (analogs of scalar field) define HFFs of type II_1 in quantum fluctuating degrees of freedom.

(c) Besides these there are zero modes, which by definition do not contribute to WCW Kähler metric.

    i. If the zero zero modes are symplectic invariants, they appear only in conformal factor of WCW metric. Symplectically invariant zero modes represent purely classical degrees of freedom - direction of a pointer of measurement apparatus in quantum measurement - and in given experimental arrangement they entangle with quantum fluctuating degrees of freedom in one-one manner so that state function reduction assigns to the outcome of state function reduction position of pointer. I forget symplectically invariant zero modes and other analogous variables in the following and concentrate to the degrees of freedom contributing WCW line-element.

    ii. There are also zero modes which are not symplectic invariants and are analogous to degrees of freedom generated by the generators of Kac-Moody algebra having vanishing conformal weight. They represent "center of mass degrees of freedom" and this part of symmetric algebra creates the representations representing the ground states of Kac-Moody representations. Restriction to these degrees of freedom gives QFT limit in string theory. In the following I will speak about "cm degrees of freedom".

The general vision about symplectic degrees of freedom (the analog of "orbital degrees of freedom" for ordinary spinor field) is following.

(a) WCW (assignable to given CD) is a union over the sub-WCWs labeled by zero modes and each sub-WCW representing quantum fluctuating degrees of freedom and "cm degrees of freedom" is infinite-D symmetric space. If symplectic group assignable to $\delta M^4_+ \times CP_2$ acts as as isometries of WCW then "orbital degrees of freedom" are parametrized by the symplectic group or its coset space (note that light-cone boundary is 3-D but radial dimension is light-like so that symplectic - or rather contact structure - exists).

Let S^2 be $r_M = constant$ sphere at light-cone boundary (r_M is the radial light-like coordinate fixed apart from Lorentz transformation). The full symplectic group would act as isometries of WCW but does not - nor cannot do so - act as symmetries of Kähler action except in the huge vacuum sector of the theory correspond to vacuum extremals.

(b) WCW Hamiltonians can be deduced as "fluxes" of the Hamiltonians of $\delta M^4_+ \times CP_2$ taken over partonic 2-surfaces. These Hamiltonians expressed as products of Hamiltonians of S^2 and CP_2 multiplied by powers r_M^n. Note that r_M plays the role of the complex coordinate z for Kac-Moody algebras and the group G defining KM is replaced with symplectic group of $S^2 \times CP_2$. Hamiltonians can be assumed to have well-defined spin (SO(3)) and color (SU(3)) quantum numbers.

(c) The generators with vanishing radial conformal weight ($n = 0$) correspond to the symplectic group of $S^2 \times CP_2$. They are not symplectic invariants but are zero modes. They would correspond to "cm degrees of freedom" characterizing the ground states of representations of the full symplectic group.

Discretization at the level of WCW

The general vision about finite measurement resolution implies discretization at the level of WCW.

(a) Finite measurement resolution at the level of WCW means discretization. Therefore the symplectic groups of $\delta M^4_+ \times CP_2$ *resp.* $S^2 \times CP_2$ are replaced by an enumerable discrete subgroup. WCW is discretized in both quantum fluctuating degrees of freedom and "center of mass" degrees of freedom.

(b) The elements of the group algebras of these discrete groups define the "orbitals parts" of WCW spinor fields in discretization. I will later develop an argument stating that they are HFFs of type II - maybe even II_1. Note that also function spaces associated with the coset spaces of these discrete subgroups could be considered.

(c) Discretization applies also in the spin degrees of freedom. Since fermionic Fock basis generates quantum counterpart of Boolean algebra the interpretation in terms of the physical correlates of Boolean cognition is motivated (fermion number 1/0 and various spins in decomposition to a tensor product of lower-dimensional spinors represent bits). Note that in ZEO fermion number conservation does not pose problems and zero states actually define what might be regarded as quantum counterparts of Boolean rules $A \rightarrow B$.

(d) Note that 3-surfaces correspond by the strong form of GCI/holography to collections of partonic 2-surfaces and string world sheets of space-time surface intersecting at discrete set of points carrying fermionic quantum numbers. WCW spinors are constructed from second quantized induced spinor fields and fermionic Fock algebra generates HFF of type II_1.

Does WCW spinor field decompose to a tensor product of two HFFs of type II_1?

The group algebras associated with infinite discrete subgroups of the symplectic group define the discretized analogs of waves in WCW having quantum fluctuating part and cm part.

The proposal is that these group algebras are HFFs of type II_1. The spinorial degrees of freedom correspond to fermionic Fock space and this is known to be HFF. Therefore WCW spinor fields would defined tensor product of HFFs of type II_1. The interpretation would be in terms of supersymmetry at the level of WCW. Super-conformal symmetry is indeed the basic symmetry of TGD so that this result is a physical "must". The argument goes as follows.

(a) In non-zero modes WCW is symplectic group of $\delta M^4_+ \times CP_2$ (call this group just *Sympl*) reduces to the analog of Kac-Moody group associated with $S^2 \times CP_2$, where S^2 is $r_M =$ constant sphere of light-cone boundary and z is replaced with radial coordinate. The Hamiltonians, which do not depend on r_M would correspond to zero modes and one could not assign metric to them although symplectic structure is possible. In "cm degrees of freedom" one has symplectic group associated with $S^2 \times CP_2$.

(b) Finite measurement resolution, which seems to be coded already in the structure of the preferred extremals and of the solutions of the modified Dirac equation, suggests strongly that this symplectic group is replaced by its discrete subgroup or symmetric coset space. What this group is, depends on measurement resolution defined by the cutoffs inherent to the solutions. These subgroups and coset spaces would define the analogs of Platonic solids in WCW!

(c) Why the discrete infinite subgroups of *Sympl* would lead naturally to HFFs of type II? There is a very general result stating that group algebra of an enumerable discrete group, which has infinite conjugacy classes, and is amenable so that its regular representation in group algebra decomposes to all unitary irreducibles is HFF of type II. See for examples about HFFs of type II listed in Wikipedia article [A12].

(d) Suppose that the group algebras associated the discrete subgroups *Sympl* are indeed HFFs of type II or even type II_1. Their inclusions would define finite measurement resolution the orbital degrees of freedom for WCW spinor fields. Included algebra would create rays of state space not distinguishable experimentally. The inclusion would be characterized by the inclusion of the lattice defined by the generators of included algebra by linearity. One would have inclusion of this lattice to a lattice associated with a larger discrete group. Inclusions of lattices are however known to give rise to quasicrystals (Penrose tilings are basic example), which define basic non-commutative structures. This is indeed what one expects since the dimension of the coset space defined by inclusion is algebraic number rather than integer.

(e) Also in fermionic degrees of freedom finite measurement resolution would be realized in terms of inclusions of HFFs- now certainly of type II_1. Therefore one could obtain hierarchies of lattices included as quasicrystals.

What about zero modes which are symplectic invariants and define classical variables? They are certainly discretized too. One might hope that one-one correlation between zero modes (classical variables) and quantum fluctuating degrees of freedom suggested by quantum measurement theory allows to effectively eliminate them. Besides zero modes there are also modular degrees of freedom associated with partonic 2-surfaces defining together with their 4-D tangent space data basis objects by strong form of holography. Also these degrees of freedom are automatically discretized. But could one consider finite measurement resolution also in these degrees of freedom. If the symplectic group of $S^2 \times CP_2$ defines zero modes then one could apply similar argument also in these degrees of freedom to discrete subgroups of $S^2 \times CP_2$.

12.8.3 Little Appendix: Comparison of WCW spinor fields with ordinary second quantized spinor fields

In TGD one identifies states of Hilbert space as WCW spinor fields. The analogy with ordinary spinor field helps to understand what they are. I try to explain by comparison with QFT.

Ordinary second quantized spinor fields

Consider first ordinary fermionic QFT in fixed space-time. Ordinary spinor is attached to an space-time point and there is $2^{D/2}$ dimensional space of spin degrees of freedom. Spinor field attaches spinor to every point of space-time in a continuous/smooth manner. Spinor fields satisfying Dirac equation define in Euclidian metric a Hilbert space with a unitary inner product. In Minkowskian case this does not work and one must introduce second quantization and Fock space to get a unitary inner product. This brings in what is essentially a basic realization of HFF of type II_1 as allowed operators acting in this Fock space. It is operator algebra rather than state space which is HFF of type II_1 but they are of course closely related.

Classical WCW spinor fields as quantum states

What happens TGD where one has quantum superpositions of 4-surface/3-surfaces by GCI/partonic 2-surfaces with 4-D tangent space data by strong form of GCI.

(a) First guess: space-time point is replaced with 3-surface. Point like particle becomes 3-surface representing particle. WCW spinors are fermionic Fock states at this surface. WCW spinor fields are Fock state as a functional of 3-surface. Inner product decomposes to Fock space inner product plus functional integral over 3-surfaces (no path integral!). One could speak of quantum multiverse. Not single space-time but quantum superposition of them. This quantum multiverse character is something new as compared to QFT.

(b) Second guess: forced by ZEO, by geometrization of Feynman diagrams, etc.

   i. 3-surfaces are actually not connected 3-surfaces. They are collections of components at both ends of CD and connected to single connected structure by 4-surface. Components of 3-surface are like incoming and outgoing particles in connected Feynman diagrams. Lines are identified as regions of Euclidian signature or equivalently as the 3-D light-like boundaries between Minkowskian and Euclidian signature of the induced metric.

   ii. Spinors(!!) are defined now by the fermionic Fock space of second quantized induced spinor fields at these 3-surfaced and by holography at 4-surface. This fermionic Fock space is assigned to all multicomponent 3-surfaces defined in this manner and WCW spinor fields are defined as in the first guess. This brings integration over WCW to the inner product.

(c) Third, even more improved guess: motivated by the solution ansatz for preferred extremals and for modified Dirac equation [K105] giving a connection with string models.

   The general solution ansatz restricts all spinor components but right-handed neutrino to string world sheets and partonic 2-surfaces: this means effective 2-dimensionality. String world sheets and partonic 2-surfaces intersect at the common ends of light-like and space-like braids at ends of CD and at along wormhole throat orbits so that effectively discretization occurs. This fermionic Fock space replaces the Fock space of ordinary second quantization.

12.9 Jones inclusions and cognitive consciousness

WCW spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of WCWs spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

12.9.1 Does one have a hierarchy of U- and M-matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question. The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2,...$ TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K86] and Jones inclusions are suggestive.

12.9.2 Feynman diagrams as higher level particles and their scattering as dynamics of self consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [K20] .

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K97] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

(a) These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

(b) States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^\dagger) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

(a) Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a manner that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

(b) The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

(c) After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

(d) One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

(e) This picture could also relate to the suggested duality between string and parton pictures [K88] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

(a) Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.

(b) The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.

(c) In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

12.9.3 Logic, beliefs, and spinor fields in the world of classical worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

WCW spinors as logic statements

In TGD framework the infinite-dimensional WCW (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [K17] . CH spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [K86] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

(a) Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.

(b) One can wonder what is the difference between real and p-adic variants of CH spinor fields and whether they could represent reality and beliefs about reality. CH spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real CH spinors as different objects. Real/ p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real CH spinors.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic CH spinor fields could serve as representations of beliefs and real CH spinor fields as representations of reality looks very nice but the fact that the outcomes of p-adic-to-real phase transition and its reversal are highly non-predictable does not support it as such.

Quantum statistical determinism could however come into rescue. Belief could be represented as an ensemble of p-adic mental images resulting in transitions of real mental images representing reality to p-adic states. p-Adic ensemble average would represent the belief.

It is not at all clear whether real-to-padic transitions can occur at high enough rate since p-adic-to-real transition are expected to be highly irreversible. The real initial states much have nearly vanishing quantum numbers emitted in the transition to p-adic state to guarantee conservation laws (p-adic conservation laws hold true only piecewise since conserved quantities are pseudo constants). The system defined by an ensemble of real Boolean mental images representing reality would automatically generate a p-adic variant representing a belief about reality.

p-Adic CH spinors can also represent the cognitive aspects of intention whereas p-adic space-time sheets would represent its geometric aspects reflected in sensory experience.p-Adic space-time sheet could also serve only as a space-time correlate for the fundamental representation of intention in terms of p-adic CH spinor field. This view is consistent with the proposed identification of beliefs since the transitions associated with intentions *resp.* beliefs would be p-adic-to-real *resp.* real-to-padic.

12.9.4 Jones inclusions for hyperfinite factors of type II_1 as a model for symbolic and cognitive representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with CH spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ $(n = \infty)$ the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a manner that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since CH spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d < 2$. The maximal reduction occurs to $d = 1$ for $n = 3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n = 3$ corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d < 2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy

Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d < 2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \rightarrow p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

(a) Since the Hermitian operators $X_1 = (z^1\overline{z^1}+\overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2}+\overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

(b) By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as $(1/2 + n_1 q^{n_2})r$ and $(1/2 + n_2 q^{n_1})r$. The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \rightarrow \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \rightarrow \infty$.

(c) The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2 + N_1 n, 1/2 + N_2 n)/[1 + (N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \rightarrow \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

(a) Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.

(b) Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
Knowledge and misbelief would involve both the measurement of real and p-adic probabilities .

(c) Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.

Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.

(d) Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.

(e) Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \rightarrow \infty$, which according to the proposal of [K79] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation. A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

12.9.5 Intentional comparison of beliefs by topological quantum computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [K97] . The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

12.9.6 The stability of fuzzy qbits and quantum computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [K97] .

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [K50] . For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

12.9.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [J7] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J10] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00}=P_{11}=cos^2(\alpha-\beta)/2$ and $P_{01}=P_{10}=sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

(a) One has four identities $P_{i,i}+P_{i,i+1}=P_{ii}+P_{i+1,i}=1/2$ having interpretation in terms of probability conservation. Experimental data of [J7] are not consistent with this prediction [J1] and this is identified as the anomaly.

(b) The QM prediction $E(\alpha,\beta)=\sum_i(P_{i,i}-P_{i,i+1})=cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J1] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} &\rightarrow P^2P_{i,j} + (1-P)^2P_{i+1,j+1} \\ &+ P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \ . \end{aligned} \tag{12.9.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned} P_{0,0} = P_{1,1} &\rightarrow A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ P_{0,1} = P_{1,0} &\rightarrow A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ A &= P^2 + (1-P)^2 \ , \ \ B = 2P(1-P) \ . \end{aligned} \tag{12.9.2}$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha-\beta$ are scaled by a factor $1-4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i(P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \rightarrow \left[1-4P(1-P)\right]E(\alpha,\beta) \ . \tag{12.9.3}$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha-\beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E = \pm 1$ to $E = \pm(1-4P(1-P))$.

From the figure 1b) of [J1] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P = 1-\epsilon$ one has $A-B \simeq 1-4\epsilon$ and $B \simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1-4\epsilon$. The graph would give $1-P \simeq \epsilon \simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [J10, J1] .

A further prediction is that the identities $P(i,i)+P(i+1,i) = 1/2$ should still hold true since one has $P_{i,i} + P_{i,i+1} = (A-B)/2 + B = 1$. This is implied also by probability conservation. The four curves corresponding to these identities do not however co-incide as the figure 6 of [J1] demonstrates. This is regarded as the basic anomaly in [J10, J1] . From the same figure it is also clear that below $\alpha-\beta < 10$ degrees $P_{++} = P_{--}$ $\Delta P_{+-} = -\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying

$\Delta P_{++} = -\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij} P_{ij} = 1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii} + P_{ii+1} = 1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i} + P_{ii+1} = 1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-} + P_{--}$ and $P_{+-} + P_{++}$. The constraint $\sum_{ij} P_{ij} = 1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha - \beta > 10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

(a) The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative "mundane" explanation.

(b) The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.

(c) One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P = P(i,j)$ so that one has $(P(-,+), P(+,-)) = (P + \Delta, P - \Delta)$ and $(P(-,-), P(+,+)) = (P + \Delta_1, P - \Delta_1)$. $\Delta \simeq .025$ and $\Delta_1 \simeq \Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i) + P(i,i+1) \geq 1/2$. Only if the procedure extracting the correlations uses the constraint $\sum_{i,j} P_{ij} = 1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1 \simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of $N_i n$ one would have $(N_1 - N_2)/(N_1 + N_2) \simeq .4$ so that one cannot say anything about n in this case. $(N_1, N_2) = (3, 1)$ satisfies the condition exactly. For $n = 3$, the smallest possible value of n, this would give $p_1 \simeq .88$ and for $n = 4$ $p_1 = .41$. With high enough precision it might be possible to select between $n = 3$ and $n = 4$ options if small values of N_i are accepted.

12.9.8 Category theoretic formulation for quantum measurement theory with finite measurement resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at http://kea-monad.blogspot.com/) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [A8] . In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [K16]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [A22] . These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [A20] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [A28] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition and intentionality with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

(a) Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?

(b) How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum sub-spaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space $\{0,1\}$ would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [K74] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized imbedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \rightarrow M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

12.10 Appendix: Inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A53] . It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

(a) According to [A53] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

(b) Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A53] . For any amenable group G the the inclusion is also unique apart from outer automorphism [A67] .

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [A53] . The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M} .... \otimes_{\mathcal{N}} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A53] .

12.10.1 Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n) | n \geq 3\} \cap [4, \infty)$ [A53] . They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i - j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A53] . It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is

$\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis [K27] that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$ define orbifold coverings of $H_\pm = M^4_\pm \times CP_2 \rightarrow H_\pm/G_a \times G_b$.

12.10.2 Wassermann's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [A53] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence ... $\subset \mathcal{N} \subset \mathcal{M} \subset$ ... since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \rightarrow H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction [A17] . The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

12.10.3 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A124] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M}:\mathcal{N} \quad = \prod_{1\leq r<s\leq k} \frac{sin^2\left((\lambda_r-\lambda_s+s-r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{12.10.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l-k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \rightarrow M^4/G_a$ or $CP_2 \rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of SU(2) the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1)\times SU(3)$ and $SL(2,C)\times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4\times CP_2$.

(a) $n>2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

(b) $n>3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Chapter 13

Does TGD Predict a Spectrum of Planck Constants?

13.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [E27] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence. This approach led to the the formula $h_{eff} = n \times h$. Quite recently (2014) it became clear that for microscopic systems the identification $h_{eff} = h_{gr}$ makes sense and predicts universal energy spectrum for cyclotron energies of dark photons identifiable as energy spectrum of bio-photons in TGD inspired quantum biology.

As almost all chapters of the books, also this chapter should be seen as a story about evolution of ideas rather than final summary. I have moved some purely mathematical speculations to second chapter to keep emphasis on TGD inspired physics.

13.1.1 The evolution of mathematical ideas

From the beginning the basic challenge -besides the need to deduce a general formula for the quantized Planck constant- was to understand how the quantization of Planck constant is mathematically possible. From the beginning it was clear that since particles with different values of Planck constant cannot appear in the same vertex, a generalization of space-time concept is needed to achieve this.

During last five years or so many deep ideas -both physical and mathematical- related to the construction of quantum TGD have emerged and this has led to a profound change of perspective in this and also other chapters. The overall view about TGD is described briefly in [L9] .

(a) For more than five years ago I realized that von Neumann algebras known as hyperfinite factors of type II_1 (HFFs) are highly relevant for quantum TGD since the Clifford algebra of configuration space ("world of classical worlds", WCW) is direct sum over HFFs. Jones inclusions are particular class of inclusions of HFFs and quantum groups are closely related to them. This led to the conviction that Jones inclusions can provide a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom (later I replaced M^4 by its light cone $M^4_\pm$ and finally with the causal diamond CD defined as intersection of future and past light-cones of M^4). The idea about connection with Jones inclusion can be however questioned and is left another chapter.

(b) The notion of zero energy ontology (ZEO) replaces physical states with zero energy states consisting of pairs of positive and negative energy states at the light-like boundaries $\delta M^4_\pm \times CP_2$ of CDs forming a fractal hierarchy containing CDs within CDs. In standard ontology zero energy state corresponds to a physical event, say particle reaction. This led to the generalization of S-matrix to M-matrix - possibly identified as Connes tensor product - characterizing time like entanglement between positive and negative energy states. M-matrix is product of square root of density matrix and unitary S-matrix just like Schrödinger amplitude is product of modulus and phase, which means that thermodynamics becomes part of quantum theory and thermodynamical ensembles are realized as single particle quantum states. This led also to a solution of long standing problem of understanding how geometric time of the physicist is related to the experienced time identified as a sequence of quantum jumps interpreted as moments of consciousness [L7] in TGD inspired theory of consciousness which can be also seen as a generalization of quantum measurement theory [L8] .

(c) Another closely related idea was the emergence of measurement resolution as the basic element of quantum theory. Measurement resolution is characterized by inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ characterizing the measurement resolution in the sense that the action of $\mathcal{M}$ creates states which cannot be distinguished from each other within measurement resolution used. Hence complex rays of state space are replaced with $\mathcal{M}$ rays. One of the basic challenges is to define the nebulous factor space $\mathcal{N}/\mathcal{M}$ having finite fractional dimension $\mathcal{N} : \mathcal{M}$ given by the index of inclusion. It was clear that this space should correspond to quantum counterpart of Clifford algebra of world of classical worlds reduced to a finite-quantum dimensional algebra by the finite measurement resolution [K17] .

(d) The realization that light-like 3-surfaces at which the signature of induced metric of space-time surface changes from Minkowskian to Euclidian are ideal candidates for basic dynamical objects besides light-like boundaries of space-time surface was a further decisive step or progress. This led to vision that quantum TGD is almost topological quantum field theory ("almost" because light-likeness brings in induced metric) characterized by Chern-Simons action for induced Kähler gauge potential of CP_2. Together with zero energy ontology this led to the generalization of the notion of Feynman diagram to a light-like 3-surface for which lines correspond to light-like 3-surfaces and vertices to 2-D partonic surface at which these 3-D surface meet. This means a strong departure from string model picture. The interaction vertices should be given by N-point functions of a conformal field theory with second quantized induced spinor fields defining the basic fields in terms of which also the gamma matrices of world of classical worlds could be constructed as super generators of super conformal symmetries [K17] .

(e) By quantum classical correspondence finite measurement resolution should have a space-time correlate. The obvious guess was that this correlate is discretization at the level of construction of M-matrix. In almost-TQFT context the effective replacement of light-like 3-surface with braids defining basic objects of TQFTs is the obvious guess. Also number theoretic universality necessary for the p-adicization of quantum TGD by a process analogous to the completion of rationals to reals and various p-adic number fields requires discretization since only rational and possibly some algebraic points of the imbedding space (in suitable preferred coordinates) allow interpretation both as real and p-adic points. It was clear that the construction of M-matrix boils to the precise understanding of number theoretic braids [K17] .

(f) The interaction with M-theory dualities [K83] led to a handful of speculations about dualities possible in TGD framework, and one of these dualities- $M^8 - M^4 \times CP_2$ duality - eventually suggests a highly unique identification of number theoretic braids. The dimensions of partonic 2-surface, space-time, and imbedding space strongly suggest that classical number fields, or more precisely their complexifications might help to understand quantum TGD. If the choice of imbedding space is unique because of uniqueness of infinite-dimensional Kähler geometric existence of world of classical worlds then standard model symmetries coded by $M^4 \times CP_2$ should have some deeper meaning and the most obvious guess is that $M^4 \times CP_2$ can be understood geometrically. $SU(3)$ belongs to the automorphism group of octonions as well as hyper-octonions M^8 identified by subspace of complexified octonions with Minkowskian signature of induced metric. This led to the discovery that hyper-quaternionic 4-surfaces in M^8 can be mapped to $M^4 \times CP_2$ provided their tangent space contains preferred $M^2 \subset M^4 \subset M^4 \times E^4$. Years later I realized that the map generalizes so that M^2 can depend on the point of X^4. The interpretation of $M^2(x)$ is both as a preferred hyper-complex (commutative) sub-space of M^8 and as a local plane of non-physical polarizations so that a purely number theoretic interpretation of gauge conditions emerges in TGD framework. This led to a rapid progress in the construction of the quantum TGD. In particular, the challenge of identifying the preferred extremal of Kähler action associated with a given light-like 3-surface X_l^3 could be solved and the precise relation between M^8 and $M^4 \times CP_2$ descriptions was understood [K17] .

(g) Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $h_{eff} = n \times h$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. This reduces the understanding of the effective hierarchy of Planck constants to quantum variant of multi-furcations for the dynamics of preferred extremals of Kähler action. The number of branches of multi-furcation defines the integer n in $\hbar_{eff} = n\hbar$.

13.1.2 The evolution of physical ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

(a) The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.

(b) The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K65] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E27] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

(c) Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K91] , [I13] .

(d) Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L6, K91] , [L6] .

13.1.3 Brief summary about the generalization of the imbedding space concept

A brief summary of the basic vision in order might help reader to assimilate the more detailed representation about the generalization of imbedding space, which has turned to be only a useful auxiliary tool of the theory rather than basic postulate.

(a) The original belief was that the hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time since particles with different values of Planck constant cannot appear in the same interaction vertex. This suggests some kind of book like structure for both M^4 and CP_2 factors of the generalized imbedding space is suggestive. It has turned out that the view about hierarchy of Planck constants as effective hierarchy allows to regard the singular coverings of imbedding space as the natural *auxiliary* tool to describe the quantum view about multi-furcations of preferred extremals.

(b) Schrödinger equation suggests that Planck constant corresponds to a scaling factor of M^4 metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in M^4 factor is possible so that the scaling of $\hbar$ corresponds to the scaling of the size of causal diamond CD defined as the intersection of future and past directed light-cones. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states. The infinite-D spaces formed by these 3-surfaces define the fundamental sectors of the configuration space (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces, the coding of radiative corrections to the geometry of space-time sheets becomes possible and Kähler action can be seen as expansion in powers of $\hbar/\hbar_0$.

(c) Quantum criticality of the TGD Universe is one of the key postulates of quantum TGD. The most important implication is that Kähler coupling strength is analogous to critical temperature. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality would mean that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes into regions corresponding to different values of Planck constant much like matter decomposes to several phases at thermodynamical criticality.

13.1.4 Basic physical picture as it is now

The basic phenomenological rules are simple and remained roughly the same during years.

(a) The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K92].

(b) Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K65] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

In astrophysics and cosmology the implications are even more dramatic. It was Nottale [E27] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

(c) Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

13.1.5 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K105] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis. coverings of H are just an auxiliary tool.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Quantum criticality can be now understood as a direct consequence of the non-determinism of Kähler action and relates directly to h_{eff} hierarchy. There is also a direct relation to generalized conformal symmetries. Critical deformations correspond to Kac-Moody type algebra deforming light-like orbits of partons and respecting their light-likeness and leaving the partonic 2-surfaces at their ends invariant. There are n conformal equivalence classes of these deformations and n defines the value of $h_{eff} = n \times h$.

In this chapter I try to summarize the evolution of the ideas related to Planck constant without systematic attempt to achieve complete internal consistency. I have left the summary about the recent views to the end of the chapter and the reader might find it a good idea to begin from this section.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L39]
- Geometrization of fields [L37]
- Magnetic body [L51]

13.2 Experimental input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

13.2.1 Hints for the existence of large $\hbar$ phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [K79] . This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

(a) With inspiration coming from the finding of Nottale [E27] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [K79] . Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.

(b) Living matter could represent a basic example of large $\hbar$ phase [K23, K7] . Even ordinary condensed matter could be "partially dark" in many-sheeted space-time [K25] . In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.

(c) There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [C57]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [K80] . The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [K57, K52] - something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.

(d) I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [K25, K23] .

13.2.2 Quantum coherent dark matter and $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED$e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the modified Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in WCW degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

(a) The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.

(b) The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of WCW geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.

(c) In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to WCW functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about imbedding space and forces to generalize the notion of imbedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of imbedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the imbedding space concept.

13.2.3 The phase transition changing the value of Planck constant as a transition to non-perturbative phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulomb contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength αQ_iQ_j, where α is the appropriate coupling strength and Q_iQ_j represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_iQ_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

13.3 A generalization of the notion of imbedding space as a realization of the hierarchy of Planck constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the imbedding space is given. In [K65] the important delicacies associated with the Kähler structure of generalized imbedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactification leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of modified Dirac action fix to a high degree the vision about generalized imbedding space.

13.3.1 Basic ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of imbedding space geometry provided it is such that it is possible to compare the regions of generalized imbedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

(a) The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a manner that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.

(b) The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

(c) Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$ in CD degrees of freedom and its analog in CP_2 degrees of freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, modified Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the modified Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define WCW spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over WCW defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant WCW metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semiclassical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that WCW Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [K99] . A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} = B_n$. $\sqrt{b_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{b_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized imbedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action- finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anti-commutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized imbedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tetrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2,C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two manners for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

13.3.2 The vision

A brief summary of the basic vision behind the generalization of the imbedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

(a) The hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized imbedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.

(b) The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.

(c) The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of WCW (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.

(d) The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

(a) The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions.

(b) The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

(c) G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [K65] .

(d) Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$ appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.

(e) Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.

(f) The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

13.3.3 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

(a) The starting point was the proposal of Nottale [E27] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K79] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

(b) Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K80] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

(c) The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

(d) Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K91] .

(e) The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K65] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E27] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

(f) Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L6, K91] , [L6] .

The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

(a) The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

(b) CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is unacceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

i. The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

ii. The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

(c) For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP_2}\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$, $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP_2}/G_b$.

(d) The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

(a) How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

(b) One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

(c) A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

(a) One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

(b) If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

(c) The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K24].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1 + i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K24] . This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

13.4 Updated view about the hierarchy of Planck constants

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis was that the hierarchy is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization has remained somewhat fuzzy [K65]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to N branches is not general enough: the N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

13.4.1 Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

(a) The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K92].

(b) Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K65] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

(c) In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E27] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation.

(d) Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

13.4.2 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K105] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

13.4.3 The relationship to the original view about the hierarchy of Planck constants

Originally the hierarchy of Planck constant was assumed to correspond to a book like structure having as pages the n-fold coverings of the imbedding space for various values of n serving therefore as a page number. The pages are glued together along a 4-D "back" at which the branches of n-furcations are degenerate. This leads to a very elegant picture about how the particles belonging to the different pages of the book interact. All vertices are local and involve only particles with the same value of Planck constant: this is enough for darkness in the sense of particle physics. The interactions between particles belonging to different pages involve exchange of a particle travelling from page to another through the back of the book and thus experiencing a phase transition changing the value of Planck constant.

Is this picture consistent with the picture based on n-furcations? This seems to be the case. The conservation of energy in n-furcation in which several sheets are realized simultaneously is consistent with the conservation of classical conserved quantities only if the space-time sheet before n-furcation involves n identical copies of the original space-time sheet or if the Planck constant is $h_{eff} = nh$. This kind of degenerate many-sheetedness is encountered also in the case of branes. The first option means an n-fold covering of imbedding space and h_{eff} is indeed effective Planck constant. Second option means a genuine quantization of Planck constant due to the fact the value of Kähler coupling strength $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is scaled down by $1/n$ factor. The scaling of Planck constant consistent with classical field equations since they involve α_K as an overall multiplicative factor only.

13.4.4 Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

(a) The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

(b) The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

(c) In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

(d) For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

(a) The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n + 1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

(b) It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

(a) The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) automatically.

(b) If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

(c) Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

13.4.5 Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

(a) In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in

WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

(b) The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionization and fractionization of spin.

(c) The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N+1$:th branch corresponds to the original one. This suggests that angular momentum fractionization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionization however leads to problems with fractionization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

13.4.6 Negentropic entanglement between branches of multi-furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism [K44] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy based on the p-adic norm for the probability appearing as argument of logarithm [K51], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs $a_i \otimes b_i$ in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large $\hbar$ photons.

How the large $\hbar$ photons could carry negentropic entanglement? There are several options to consider and at this stage it is not possible to pinpoint anyone of them as the only possible one. Several of them could also be realized.

(a) In zero energy ontology large $\hbar$ photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.

(b) The negentropic entanglement of large $\hbar$ photon could be also associated with its positive or energy part or both. Large $\hbar_{eff} = n\hbar$ photon with n-fold energy $E = n \times hf$ is n-sheeted structure consisting of n-photons with energy $E = hf$ de-localized in the discrete space formed by the N space-time sheets. The n single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for N-fold branching the superposition of all $N!/(N-n)!n!$ states obtained by selecting n branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would the quintessence of life.

(c) A further very attractive possibility discovered quite recently is that large $h_{eff} = nh$ is closely related to the negentropic entanglement between the states of *two* n-furcated - that is dark - space-time sheets. In the most recent formulation negentropic entanglement corresponds to a state characterized by $n \times n$ identity matrix resulting from the measurement of density matrix. The number theoretic entanglement negentropy is positive for primes dividing p and there is unique prime for which it is maximal.

The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option.

(a) One proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.

The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large $\hbar$ variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW. If each of the n branches carries $1/n$:th part of electron one would have an anyonic state in WCW.

(b) One can also make a really crazy question. Could it be that ATP and various bio-molecules form n-particle states at the n-sheet of n-furcation and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [K6] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given CD this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

13.4.7 Dark variants of nuclear and atomic physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [K94].

Before the real understanding what charge fractionization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form n-particle states associated with n-branches of N-furcation with $n = 1, ..., N$. The fractionization for a single particle state de-localized completely to the discrete space of N branches as the analog of plane wave means that single branch carriers charge $1/N$.

The new element is the possibility of n-particle states populating n branches of the N-furcation: note that there is superposition over the states corresponding to different selections of these n branches. $N-k$ and k-nuclei/atoms are in sense conjugates of each other and they can fuse to form N-nuclei/N-atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that N-atoms and even N-molecules could make possible the emergence of symbolic representations with $n \leq N$ serving as a name of atom/molecule and that k- and $N-k$ atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form N-atom/-molecule. For instance, in bio-catalysis k- and $N-k$-atoms/molecules would be paired. The recent picture about n and $N-k$ atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their n-multiples. Synchronized quantum jumps would allow to avoid the disastrous effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

13.4.8 What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

(a) Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

(b) At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \ . \tag{13.4.1}$$

Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \ .$$

$\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

(c) Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{13.4.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

(d) Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

(e) One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta}L_\alpha L_\beta = g_{eff}^{kl}L_k L_l = l(l+1)\hbar^2 \quad . \tag{13.4.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{13.4.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4-momentum squared, then Equivalence Principle implies that h_{gr} can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying h_{gr} can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

13.4.9 Hierarchy of Planck constants and non-determinism of Kähler action

Originally the hierarchy of Planck constant was inspired by empirical inputs from neuroscience, biology, and from Nottale's observations. That it is possible to understand the hierarchy in terms of non-determinism of Kähler action - the fundamental difference between TGD and quantum field theories and string models - is a victory for TGD approach (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Recall that non-determinism means that all space-time surfaces with CP_2 projection, which is Lagrangian sub-manifold (at most 2-D) of CP_2, carries a vanishing induced Kähler form and is vacuum extremal. The first guess would be that there is a finite number n of space-time sheets connecting given pair of 3-surfaces at the ends of space-time surface at the light-like boundaries of causal diamond (CD). Planck constant would be given as $h_{eff} = n \times h$ in accordance with the earlier interpretation. The degenerate extremals would have same Kähler action and conserved quantities as assumed also in the earlier approach. That the degenerate extremals co-incide at the ends of space-time surface was motivated by mathemtical aesthetics in the earlier approach but finds an interpretation in terms of non-uniqueness of the preferred extremals.

It is essential that these n degrees of freedom are regarded as genuine physical degrees of freedom, which are however discrete. Negentropic entanglement and dark matter would be associated with them naturally. The effective description would be in terms of n-fold singular covering of imbedding space becoming singular at the ends of the space-time surface.

I have also assigned hierarchy of Planck constants with the quantum criticality. Quantum criticality means the existence of an entire continuous family of deformations of space-time sheet with same Kähler action and conserved quantities. The deformations would by definition vanish at the ends of space-time surface. The critical deformations would act as gauge transformations identifiable as conformal symmetries indeed expected to be presents since WCW isometries form a conformal algebra and there is also Kac-Moody type algebra present. The proposal has been that the sub-algebras of conformal algebra for which conformal weights are integer multiples of integer $n = 1, 2, ..$ defined a hierarchy of gauge algebras so that the dynamical algebra reduces to n-dimensional one.

These two identifications seem to be mutually inconsistent. The resolution of the conflict comes from the gauge invariance. For a given Kähler action and conserved quantities there would be n conformal equivalence classes of these 4-surfaces rather than n surfaces, and one would have n-fold degeneracy but for conformal equivalence classes of 4-surfaces rather than 4-surfaces. In Minkowskian regions the degenerate preferred extremals are sheets (graphs of a map from M^4 to CP_2).

13.4.10 Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E17, E31] discussed in [K112] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

(a) This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

(b) That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

(c) The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K108, K107] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   i. If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K24] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   ii. For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

   iii. Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies. Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

   It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

   iv. The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

(d) An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

(e) The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

(f) There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

13.4.11 How the effective hierarchy of Planck constants could reveal itself in condensed matter physics

Anderson - one of the gurus of condensed matter physics - has stated that there exists no theory of condensed matter: experiments produce repeatedly surprises and theoreticians do their best to explain them in the framework of existing quantum theory.

This suggests that condensed matter physics might allow room even for new physics. Indeed, the model for fractional quantum Hall effect (FQHE) [K65] strengthened the feeling that the many-sheeted physics of TGD could play a key role in condensed matter physics often thought to be a closed chapter in physics. One implication would be that space-time regions with Euclidian signature of the induced metric would represent the space-time sheet assignable to condensed matter object as a whole as analog of a line of a generalized Feynman diagram. Also the hierarchy of effective Planck constants $\hbar_{eff} = n\hbar$ appears in the model of FQHE.

The recent discussion of possibility of quantum description of psychokinesis [L20] boils down to a model for intentional action based on the notion of magnetic flux tube carrying dark matter and dark photons and inducing macroscopic quantum superpositions of magnetic bubbles of ferromagnet with opposite magnetization. As a by-product the model leads to the proposal that the conduction electrons responsible for ferromagnetism are actually dark (in the sense of having large value of effective Planck constant) and assignable to a multi-sheeted singular covering of space-time sheet assignable to second quantization multi-furcation of the preferred extremal of Kähler action made possible by its huge vacuum degeneracy.

What might be the signatures for $\hbar_{eff} = n\hbar$ states in condensed matter physics and could one interpret some exotic phenomena of condensed matter physics in terms of these states for electrons?

(a) The basic signature for the many-electron states associated with multi-sheeted covering is a sharp peak in the density of states due to the presence of new degrees of freedom. In ferromagnets this kind of sharp peak is indeed observed at Fermi energy [D7].

(b) In the theory of super-conductivity Cooper pairs are identified as bosons. In TGD framework all bosons - also photons - emerge as wormhole contacts with throats carrying fermion and anti-fermion. I have always felt uneasy with the assumption that two-fermion states obey exact Bose-Einstein statistics at the level of oscillator operators: they are after all two-fermion states. The sheets of multi-sheeted covering resulting in a multi-furcation could however carry both photons identified as fermion-anti-fermion pairs and Cooper pairs and this could naturally give rise to Bose-Einstein statistics in strong sense and also be involved with Bose-Einstein condensates. The maximum number of photons/Cooper pairs in the Bose-Einstein condensate would be given by the number of sheets. Note that in zero energy ontology also the counterparts of coherent states of Cooper pairs are possible: in positive energy ontology they have ill-defined fermion number and also this has made me feel uneasy.

(c) Majorana fermions [D3] have become one of the hot topics of condensed matter physics recently.

   i. Majorana particles are actually quasiparticles which can be said to be half-electrons and half-holes. In the language of anyons would would have charge fractionization $e \rightarrow e/2$. The oscillator operator $a^{\dagger}(E)$ creating the hole with energy E defined as the difference of real energy and Fermi energy equals to the annihilation operator $a(-E)$ creating a hole: $a^{\dagger}(E) = a(-E)$. If the energy of excitation is $E = 0$ one obtains $a^{\dagger}(0) = a(-0)$.
   Since oscillator operators generate a Clifford algebra just like gamma matrices do, one can argue that one has Majorana fermions at the level of Fock space rather than at the level of spinors. Note that one cannot define Fock vacuum as a state annihilated by $a(0)$. Since the creation of particle generates a hole equal to particle for $E = 0$, Majorana particles come always in pairs. A fusion of two Majorana particles produces an ordinary fermion.

   ii. Purely mathematically Majorana fermion as a quasiparticle would be highly analogous to covariantly constant right-handed neutrino spinor in TGD with vanishing four-momentum. Note that right-handed neutrino allows 4-dimensional modes as a solution of the modified Dirac equation whereas other spinor modes localized to partonic 2-surfaces and string world sheets. The recent view is however that covariantly constant right-handed neutrino cannot not give rise to the TGD counterpart of standard space-time SUSY.

   iii. In TGD framework the description that suggests itself is in terms of bifurcation of space-time sheet. Charge -e/2 states would be electrons de-localized to two sheets. Charge fractionization would occur in the sense that both sheets would carry charge -e/2. Bifurcation could also carry two electrons giving charge -e at both sheets. Two-sheeted analog of Cooper pair would be in question. Ordinary Cooper pair would in turn be localized in single sheet of a multi-furcation. The two-sheeted analog of Cooper pair could be regarded as a pair of Majorana particles if the measured charge of electron corresponds to its charge at single sheet of bifurcation (this assumption made also in the case of FQHE is crucial!). Whether this is the case, remains unclear to me.

iv. Fractional Josephson effect in which the current carriers of Josephson current become electrons or quasiparticles with the quantum numbers of electron has been suggested to serve as a signature of Majorana quasiparticles [D4]. An explanation consistent with above assumption is as a two-sheeted analog of Cooper pair associated with a bifurcated space-time sheets.
If the measurements of Josephson current measure the current associated with single branch of bifurcation the unit of Josephson current is indeed halved from -2e to -e. These 2-sheeted Cooper pairs behave like dark matter with respect to ordinary matter so that dissipation free current flow would become possible.
Note that ordinary Cooper pair Bose-Einstein condensate would correspond to N-furcation with N identified as the number of Cooper pairs in the condensate if the above speculation is correct. Fractional Josephson effect generated in external field would correspond to a formation of mini Bose-Einstein condensates in this framework and also smaller fractional charges are expected. In this case the interpretation as Majorana fermion does not seem to make sense.

13.4.12 Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and it is somewhat a matter of taste whether one regards the hierarchy as only effective or real. The non-determinism of Kähler action implies the hierarchy. Anyonic physics and effective charge fractionization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K105]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K101] so that conformal dynamics represents conformal evolution) [K88].

13.5 Vision about dark matter as phases with non-standard value of Planck constant

13.5.1 Dark rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

(a) Generalized imbedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.

(b) Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [I13] . This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.

(c) The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [K65] a detailed model for the Kähler structure of the generalized imbedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full imbedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

13.5.2 Phase transitions changing Planck constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized imbedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

(a) The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.

(b) Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a manner that regions in which induced Kähler form is non-vanishing are contained within single page of imbedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD \times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for appropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

13.5.3 Coupling constant evolution and hierarchy of Planck constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K58] .

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tetrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5 and and thus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

(a) In [K5] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.

(b) The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0, 1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [K5] .

13.6 Some applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

13.6.1 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{13.6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2] .

The model of Laughlin [D24] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D22] . Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

(a) The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

(b) With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

(c) Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [D2, D16] . The fractionized charge is $e/4$ in the latter case [D16, D26] . Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D22] . $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

(d) A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

(e) Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

(f) The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather ad hoc. Therefore the model can be taken as a warm-up exercise only. In [K65] , where the delicacies of Kähler structure of generalized imbedding space are discussed, also a more detailed of QHE is discussed.

13.6.2 Gravitational Bohr orbitology

The basic question concerns justification for gravitational Bohr orbitology [K79] . The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K79] .

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Schrödinger equation holds true only inside flux quanta for the dark matter.

(a) During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).

(b) In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.

(c) The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n\ mod\ 5 = 0$ for some reason.

(d) A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

(a) The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

(b) Nottale [E27] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [K79]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [K79] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}]*X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet)n_{F,b}(Earth)/n_{F,a}(Earth)n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^{4}\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 1. The table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k\prod_k F_{n_k}$, where $F_i = 2^{2^i}+1$, $i = 0,1,2,3,4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254}\times 5\times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K24] . For $v_0 = 4.6\times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1+\epsilon)$ of the total mass for Earth gives

$$1+\epsilon = \frac{17}{16} \quad (13.6.2)$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1-\epsilon) \simeq 4.6\times 10^{-4}$ equals with $v_0(eff) = 1/(2^7\times F_2) = 4.5956\times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

(a) $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v\cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

(b) $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

(a) The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

(b) One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface) . Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of imbedding space indeed involve quantum groups central in the modelling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [K65] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized imbedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized imbedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [K93] .

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

13.6.3 Accelerating periods of cosmic expansion as phase transitions increasing the value of Planck constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E10, E5] . Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. *Supernovas of type Ia*

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be .5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the imbeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D imbedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D imbedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D imbedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to imbedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

13.6.4 Phase transition changing Planck constant and expanding Earth theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

(a) These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.

(b) The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.

(c) This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3,4, and 5 are present.

(d) Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [F8] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [F1] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

(a) The radius of Earth has increased during last 185 million years (dinosaurs [I2] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of

4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.

(b) Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.

(c) Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.

(d) The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.

(e) I am not sure whether Adams mentions the following objections [F2] . Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.

(f) As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [F5] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [F3] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [F4] (orogeny), earth quake zones, and associated zones of volcanic activity [F6] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

(a) There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.

(b) There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.

(c) One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [F2] , whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

(a) 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.

(b) Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.

(c) The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

(a) The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.

(b) Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.

(c) The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.

(d) The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.

(e) One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...

(f) From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.

(g) If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

(a) Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I9] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

    Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.

(b) TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs. The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

(c) A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

(d) Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I15] , which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I12] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [K30] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

(a) Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

(b) Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

(c) What applies to Earth should apply also to other similar planets and Mars [E6] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is .131 times that for Earth so that surface gravity would be .532 of that for Earth now and would be reduced to .131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

13.6.5 Allais effect as evidence for large values of gravitational Planck constant?

Allais effect [E1, E11] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E11] .

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5 \times 10^{-4}$ [E1, E20] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E25] .

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K8] . If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M,and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

13.6.6 Applications to elementary particle physics, nuclear physics, and condensed matter physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water- might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron [K92] . Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [C76] . Towards the end of 2008 CDF anomaly [C15] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [K92] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [D23] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [L6] , [L6] . In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the cathode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

(a) The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E13]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.

(b) The so called $H_{1.5}O$ anomaly of water [D9, D8, D12, D20] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.

(c) The mysterious behavior burning salt water [D1] can be also understood in the same framework.

(d) The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [C54, C69] . Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

13.6.7 Applications to biology and neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

(a) There is evidence for an icosahedral clustering in [D21] [D10] . Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.

(b) It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

(c) What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, amino-acids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [K13] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc... [D29, D28]
.

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [D29] .

The TGD based model for high T_c super-conductivity [K13] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

(a) Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.

(b) The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.

(c) The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [K13] . p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

(a) An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1-5$ μm, the cell nucleus length scale. Hence type I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

(b) At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

(c) The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of .05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [K24] .

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [K24] .

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [K13, K14] .

Dark cyclotron radiation with photon energy above thermal energy could be used for coordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [I11] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [K30] .

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

(a) The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

(b) The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

(c) The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.

(d) The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.

(e) Each physical system corresponds to an onion-like hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.

(f) At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of TQC in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

(a) Sharing of labor means conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [I8] . Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to TQC must be weakened appropriately.

(b) The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [C65] generalizes. The ends of the space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical TQC program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that TQC program is automatically written to memory as the braiding of the threads during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

(a) Darwinian selection for which standard theory of self-organization [B9] provides a model, should apply also to TQC programs. TQC programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.

(b) Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.

(c) The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions,...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQC's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.

(d) Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.

(e) The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.

(f) There are several problems related to the details of the realization. How nucleotides A,T,C,G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

(a) In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

(b) The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

(c) Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

(d) The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

(a) The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of TQC. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

(b) During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

13.7 Appendix

13.7.1 About inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A53] . It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

(a) According to [A53] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

(b) Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A53] . For any amenable group G the the inclusion is also unique apart from outer automorphism [A67] .

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [A53] . The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M} .... \otimes_{\mathcal{N}} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A53] .

1. Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n)|n \geq 3\} \cap [4, \infty)$ [A53] . They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i - j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A53] . It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$ define orbifold coverings of $H_\pm = CD \times CP_2 \rightarrow H_\pm / G_a \times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [A53] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2, C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2, C)$ is effectively replaced with the orbifold $M(2, C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \to H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

13.7.2 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A124] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} . \tag{13.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \to M^4/G_a$ or $CP_2 \to CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

(a) $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

(b) $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Part IV

APPLICATIONS

Chapter 14

Cosmology and Astrophysics in Many-Sheeted Space-Time

14.1 Introduction

This chapter is devoted to the applications of TGD to astrophysics and cosmology are discussed. It must be admitted that the development of the proper interpretation of the theory has been rather slow and involved rather weird twists motivated by conformist attitudes. Typically these attempts have brought into theory ad hoc identifications of say gravitational four-momentum although theory itself has from very beginning provided completely general formulas.

Perhaps the real problem has been that radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology (ZEO) states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

14.1.1 Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state.

Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K33, K21] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

Zero energy states decompose into positive and negative energy parts having identification as initial and final states of particle reaction in time scales of perception longer than the geometro-temporal separation T of positive and negative energy parts of the state. If the time scale of perception is smaller than T, the usual positive energy ontology applies.

In zero energy ontology inertial four-momentum is a quantity depending on the temporal time scale T used and in time scales longer than T the contribution of zero energy states with parameter $T_1 < T$ to four-momentum vanishes. This scale dependence alone implies that it does not make sense to speak about conservation of inertial four-momentum in cosmological scales. Hence it would be in principle possible to identify inertial and gravitational four-momenta and achieve strong form of Equivalence Principle. It however seems that this is not the correct approach to follow.

The the relationship between TGD and GRT was understood quite recently (2014). GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical form of Equivalence Principle (EP) for the GRT limit in long length scales at least expressed in terms of Einstein's equations in given resolution scale with space-time sheets with size smaller than resolution scale represented as external currents.

One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

The vacuum extremals are absolutely essential for the TGD based view about long length scale limit about gravitation. Effective GRT space time would be imbeddable as a vacuum extremal to H. This is just assumption albeit coming first in mind - especially so when one has not yet understood how GRT space-time emerges from TGD!

Already the Kähler action defined by CP_2 Kähler form J allows enormous vacuum degeneracy: any four-surface having Lagrangian sub-manifold of CP_2 as its CP_2 projection is a vacuum extremal. The dimension of these sub-manifolds is at most two. Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K33, K21] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

In TGD framework topological field quantization leads to the hypothesis that quantum concepts should have geometric counterparts and also potential energy should have precise correlate at the level of description based on topological field quanta. This indeed seems to be the case. As already explained, TGD allows space-time sheets to have both positive and negative time orientations. This in turn implies that also the sign of energy can be also negative. This suggests that the generation of negative energy space-time sheets representing virtual gravitons together with energy conservation makes possible the generation of huge gravitationally induced kinetic energies and gravitational collapse. In this process inertial energy would be conserved since instead, of positive energy gravitons, the inertial energy would go to the energy of matter.

This picture has a direct correlate in quantum field theory where the exchange negative energy virtual bosons gives rise to the interaction potential. The gravitational red-shift of microwave background photons is the strongest support for the non-conservation of energy in General Relativity. In TGD it could have concrete explanation in terms of absorption of negative energy virtual gravitons by photons leading to gradual reduction of their energies. This explanation is consistent with the classical geometry based explanation of the red-shift based on the stretching of electromagnetic wave lengths. This explanation is also consistent with the intuition based on Feynman diagram description of gravitational acceleration in terms of graviton exchanges.

14.1.2 Dark matter hierarchy and hierarchy of Planck constants

The idea about hierarchy of Planck constants relying on generalization of the imbedding space was inspired both by empirical input (Bohr quantization of planetary orbits and anomalies of biology) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence. Consider first the mathematical structure in question.

(a) The Clifford algebra of World of Classical Worlds (WCW) creating many fermion states is a standard example of an algebra expressible as a direct integral of copies of von Neumann algebras known as hyper-finite factor of type II_1 (HFFs). The inclusions of HFFs relate very intimately to the notion of finite measurement resolution. There is a canonical hierarchy of Jones inclusions [A3] labeled by finite subgroups of SU(2) [A85] . Quantum classical correspondence suggests that these inclusions have space-time correlates [K99, K27] and the generalization of imbedding space would provide these correlates.

(b) The space $CD \times CP_2$, where $CD \subset M^4$ is so called causal diamond identified as the intersection of future and past directed light-cones defines the basic geometric structure in zero energy ontology. The positive (negative) energy part of the zero energy state is located to the lower (upper) light-like boundaries of $CD \times CP_2$ and has interpretation as the initial (final) state of the physical event in standard positive energy ontology. p-Adic length scale hypothesis follows if one assumes that the temporal distance between the tips of CD comes as an octave of fundamental time scale defined by the size of CP_2. The "world of classical worlds" (WCW) is union of sub-WCWs associated with spaces $CD \times CP_2$ with different locations in $M^4 \times CP_2$.

(c) One can say that causal diamond CD and the space CP_2 appearing as factors in $CD \times CP_2$ forms the basic geometric structure in zero energy ontology, is replaced with a book like structure obtained by gluing together infinite number of singular coverings and factor spaces of CD *resp.* CP_2 together. The copies are glued together along a common "back" $M^2 \subset M^2$ of the book in the case of CD. In the case of CP_2 the most general option allows two backs corresponding to the two non-isometric geodesic spheres S^2_i, $i = I, II$, represented as sub-manifolds $\xi^1 = \overline{\xi}^2$ and $\xi^1 = \xi^2$ in complex coordinates transforming linearly under $U(2) \subset SU(3)$. Color rotations in CP_2 produce different choices of this pair.

(d) The selection of geodesic spheres S^2 and M^2 is an imbedding space correlate for the fixing of quantization axes and means symmetry breaking at the level of imbedding space geometry. WCW is union over all possible choices of CD and pairs of geodesic spheres so that at the level no symmetry breaking takes place. The points of M^2 and S^2 have a physical interpretation in terms of quantum criticality with respect to the phase transition changing Planck constant (leakage to another page of the book through the back of the book).

(e) The pages of the singular coverings are characterized by finite subgroups G_a and G_b of $SU(2)$ and these groups act in covering or leave the points of factor space invariant. The pages are labeled by Planck constants $\hbar(CD) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$, where n_a and n_b are integers characterizing the orders of maximal cyclic subgroups of G_a and G_b. For singular factor spaces one has $\hbar(CD) = \hbar_0/n_a$ and $\hbar(CP_2) = \hbar_0/n_b$. The observed Planck constant corresponds to $\hbar = (\hbar(CD)/\hbar(CP_2)) \times \hbar_0$. What is also important is that $(\hbar/\hbar_0)^2$ appears as a scaling factor of M^4 covariant metric so that

Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

The interpretation in terms of dark matter comes as follows.

(a) Large values of $\hbar$ make possible macroscopic quantum phase since all quantum scales are scaled upwards by $\hbar/\hbar_0$. Anyonic and charge fractionization effects allow to "measure" $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio. $\hbar(CD) = \hbar(CP_2) = \hbar_0$ corresponds to what might be called standard physics without any anyonic effects and visible matter is identified as this phase.

(b) Particle states belonging to different pages of the book can interact via classical fields and by exchanging particles, such as photons, which leak between the pages of the book. This leakage means a scaling of frequency and wavelength in such a manner that energy and momentum of photon are conserved. Direct interactions in which particles from different pages appear in the same vertex of generalized Feynman diagram are impossible. This seems to be enough to explain what is known about dark matter. This picture differs in many respects from more conventional models of dark matter making much stronger assumptions and has far reaching implications for quantum biology, which also provides support for this view about dark matter.

This is the basic picture. One can imagine large number of speculative applications.

(a) The number theoretically simple ruler-and-compass integers n having as factors only first powers of Fermat primes and power of 2 would define a physically preferred values of n_a and n_b and thus a sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5- multiples should be important.

(b) G_a could correspond directly to the observed symmetries of visible matter induced by the underlying dark matter if singular factor space is in question [K27] . For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K27, K13] and recently reported strange properties of graphene [D17] . Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K25] , [D21] .

(c) A further fascinating possibility is that the evidence for Bohr orbit quantization of planetary orbits [E27] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K79] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

(d) Since the gravitational Planck constant $\hbar_{gr} = GM_1m/v_0$, $v_0 = 2^{-11}$ for the inner planets, is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

14.1.3 Many-sheeted cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

The most important differences are following.

(a) Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.

(b) TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.

(c) The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.

(d) Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.

(e) Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

14.1.4 Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically.

TGD predicts two basic types of strings.

(a) The analogs of hadronic strings correspond to deformations of vacuum extremals carrying non-vanishing induced Kähler fields. p-Adic thermodynamics for super-symplectic quanta condensed on them with additivity of mass squared yields without further assumptions stringy mass formula. These strings are associated with various fractally scaled up variants of hadron physics.

(b) Cosmic strings correspond to homologically non-trivial geodesic sphere of CP_2 (more generally to complex sub-manifolds of CP_2) and have a huge string tension. These strings are expected to have deformations with smaller string tension which look like magnetic flux tubes with finite thickness in M^4 degrees of freedom. The signature of these strings would be the homological non-triviality of the CP_2 projection of the transverse section of the string.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Classical TGD [L28]
- TGD inspired cosmology [L76]
- Astrophysics and TGD [L24]
- TGD and GRT [L72]
- Cosmic strings [L30]

14.2 Basic principles of General Relativity from TGD point of view

General Coordinate Invariance, Equivalence Principle are corner stones of general relativity and one expects that they hold true also in TGD some sense. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

Objections against TGD have turned out to be the best route to the correct interpretation of the theory. A very general objection against TGD relies on the notion of induced gauge fields and metric implying extremely strong constraints between classical gauge fields for preferred extremals. These constraints cannot hold true for gauge fields in the usual sense. Also linear superposition is lost. The solution of the problem comes from simple observation: it is not fields which superpose but their effects on test particle topologically condensed to space-time sheets carrying the classical fields. Superposition is replaced with set theoretic union. This leads also naturally to explicit identification of the effective metric and gauge potentials defined in M^4 and defining GRT limit of TGD.

Finite length scale resolution is central notion in TGD and implies that the topological inhomogenities (space-time sheets and other topological inhomogenities) are treated as point-like objects and described in terms of energy momentum tensor of matter and various currents coupling to effective YM fields and effective metric important in length scales above the resolution scale. Einstein's equations with coupling to gauge fields and matter relate these currents to the Einstein tensor and metric tensor of the effective metric of M^4. The topological inhomogenities below cutoff scale serve determine the curvature of the effective metric.

The original proposal, which I called smoothed out space-time, took into account the topological inhomogenities but neglected many-sheetedness in length scales above resolution scale. I also identified the effective metric can be identified as induced metric: this is very strong assumption although the properties of vacuum extremals support this identification at least in some important special cases.

The attempts to understand Kähler-Dirac (or modified Dirac-) action has provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions: for instance, if is far from clear how em charge can be well-defined for the modes of the induced spinor field and how the effective absence of weak bosons above weak scale is realized at classical level for Kähler-Dirac action.

14.2.1 General Coordinate Invariance

General Coordinate Invariance plays in the formulation of quantum TGD even deeper role than in that of GRT. Since the fundamental objects are 3-D surfaces, the construction of the geometry of the configuration space of 3-surfaces (the world of classical worlds, WCW) requires that the definition of the geometry assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$. This space-time surface is completely analogous to Bohr orbit, which means a completely unexpected connection with quantum theory.

General Coordinate Invariance is analogous to gauge symmetry and requires gauge fixing. The definition assigning $X^4(X^3)$ to given X^3 must be such that the outcome is same for all 4-diffeomorphs of X^3. This condition is highly non-trivial since $X^4(X^3) = X^4(Y^3)$ must hold true if X^3 and Y^3 are 4-diffeomorphs. One manner to satisfy this condition is by assuming quantum holography and weakened form of General Coordinate Invariance: there exists physically preferred 3-surfaces X^3 defining $X^4(X^3)$, and the 4-diffeomorphs Y^3 of X^3 at $X^4(X^3)$ provide classical holograms of X^3: $X^4(Y^3) = X^4(X^3)$ is trivially true. Zero energy ontology allows to realize this form of General Coordinate Invariance.

(a) In zero energy ontology WCW decomposes into a union of sub-WCWs associated with causal diamonds $CD \times CP_2$ (CD denotes the intersection of future and past directed light-cones of M^4), and the intersections of space-time surface with the light-light boundaries of $CD \times CP_2$ are excellent candidates for preferred space-like 3-surfaces X^3. The 3-surfaces at $\delta CD \times CP_2$ are indeed physically special since they carry the quantum numbers of positive and negative energy parts of the zero energy state.

(b) Preferred 3-surfaces could be also identified as light-like 3-surfaces X_l^3 at which the Euclidian signature of the induced space-time metric changes to Minkowskian. Also light-like boundaries of X^4 can be considered. These 3-surfaces are assumed to carry elementary particle quantum numbers and their intersections with the space-like 3-surfaces X^3 are 2-dimensional partonic surfaces so that effective 2-dimensionality consistent with the conformal symmetries of X_l^3 results if the identifications of 3-surfaces are physically equivalent. Light-like 3-surfaces are identified as generalized Feynman diagrams and due to the presence of 2-D partonic 2-surfaces representing vertices fail to be 3-manifolds. Generalized Feynman diagrams could be also identified as Euclidian regions of space-time surface.

(c) General Coordinate Invariance in minimal form requires that the slicing of $X^4(X_l^3)$ by light light 3-surfaces Y_l^3 "parallel" to X_l^3 predicted by number theoretic compactification gives rise to quantum holography in the sense that the data associated with any Y_l^3 allows an equivalent formulation of quantum TGD. This poses a strong condition on the spectra of the modified Dirac operator at Y_l^3 and thus to the preferred extremals of Kähler action since the WCW Kähler functions defined by various choices of Y_l^3 can differ only by a sum of a holomorphic function and its conjugate [K17, K21] .

14.2.2 The basic objection against TGD

The basic objection against TGD is that induced metrics for space-time surfaces in $M^4 \times CP_2$ form an extremely limited set in the space of all space-time metrics appearing in the path integral formulation of General Relativity. Even special metrics like the metric of a rotating black hole fail to be imbeddable as an induced metric. For instance, one can argue that TGD cannot reproduce the post-Newtonian approximation to General Relativity since it involves linear superposition of gravitational fields of massive objects. As a matter fact, Holger B. Nielsen- one of the very few colleagues who has shown interest in my work - made this objection for at least two decades ago in some conference and I remember vividly the discussion in which I tried to defend TGD with my poor English.

The objection generalizes also to induced gauge fields expressible solely in terms of CP_2 coordinates and their gradients. This argument is not so strong as one might think first since in standard model only classical electromagnetic field plays an important role.

(a) Any electromagnetic gauge potential has in principle a local imbedding in some region. Preferred extremal property poses strong additional constraints and the linear superposition of massless modes possible in Maxwell's electrodynamics is not possible.

(b) There are also global constraints leading to topological quantization playing a central role in the interpretation of TGD and leads to the notions of field body and magnetic body having non-trivial application even in non-perturbative hadron physics. For a very large class of preferred extremals space-time sheets decompose into regions having interpretation as geometric counterparts for massless quanta characterized by local polarization and momentum directions. Therefore it seems that TGD space-time is very quantal. Is it possible to obtain from TGD what we have used to call classical physics at all?

The imbeddability constraint has actually highly desirable implications in cosmology. The enormously tight constraints from imbeddability imply that imbeddable Robertson-Walker cosmologies with infinite duration are sub-critical so that the most pressing problem of General Relativity disappears. Critical and over-critical cosmologies are unique apart from a parameter characterizing their duration and critical cosmology replaces both inflationary cosmology and cosmology characterized by accelerating expansion. In inflationary theories the situation is just the opposite of this: one ends up with fine tuning of inflaton potential in order to obtain recent day cosmology.

Despite these and many other nice implications of the induced field concept and of sub-manifold gravity the basic question remains. Is the imbeddability condition too strong physically? What about linear superposition of fields which is exact for Maxwell's electrodynamics in vacuum and a good approximation central also in gauge theories. Can one obtain linear superposition in some sense?

(a) Linear superposition for small deformations of gauge fields makes sense also in TGD but for space-time sheets the field variables would be the deformations of CP_2 coordinates which are scalar fields. One could use preferred complex coordinates determined about SU(3) rotation to do perturbation theory but the idea about perturbations of metric and gauge fields would be lost. This does not look promising. Could linear superposition for fields be replaced with something more general but physically equivalent?

(b) This is indeed possible. The basic observation is utterly simple: what we know is that the *effects* of gauge fields superpose. The assumption that fields superpose is un-necessary! This is a highly non-trivial lesson in what operationalism means for theoreticians tending to take these kind of considerations as mere "philosophy".

(c) The hypothesis is that the superposition of effects of gauge fields occurs when the M^4 projections of space-time sheets carrying gauge and gravitational fields intersect so that the sheets are extremely near to each other and can touch each other (CP_2 size is the relevant scale).

A more detailed formulation goes as follows.

(a) One can introduce common M^4 coordinates for the space-time sheets. A test particle (or real particle) is identifiable as a wormhole contact and is therefore point-like in excellent approximation. In the intersection region for M^4 projections of space-time sheets the particle forms topological sum contacts with all the space-time sheets for which M^4 projections intersect.

(b) The test particle experiences the sum of various gauge potentials of space-time sheets involved. For Maxwellian gauge fields linear superposition is obtained. For non-Abelian gauge fields gauge fields contain interaction terms between gauge potentials associated with different space-time sheets. Also the quantum generalization is obvious. The sum of the fields induces quantum transitions for states of individual space time sheets in some sense stationary in their internal gauge potentials.

(c) The linear superposition applies also in the case of gravitation. The induced metric for each space-time sheet can be expressed as a sum of Minkowski metric and CP_2 part having interpretation as gravitational field. The natural hypothesis that in the above kind of situation the effective metric is sum of Minkowski metric with the sum of the CP_2 contributions from various sheets. The effective metric for the system is well-defined and one can calculate a curvature tensor for it among other things and it contains naturally the interaction terms between different space-time sheets. At the Newtonian limit one obtains linear superposition of gravitational potentials. One can also postulate that test particles moving along geodesics in the effective metric. These geodesics are not geodesics in the metrics of the space-time sheets.

(d) This picture makes it possible to interpret classical physics as the physics based on effective gauge and gravitational fields and applying in the regions where there are many space-time sheets which M^4 intersections are non-empty. The loss of quantum coherence would be due to the effective superposition of very many modes having random phases.

The effective superposition of the CP_2 parts of the induced metrics gives rise to an effective metric which is not in general imbeddable to $M^4 \times CP_2$. Therefore many-sheeted space-time makes possible a rather wide repertoire of 4-metrics realized as effective metrics as one might have expected and the basic objection can be circumvented In asymptotic regions where one can expect single sheetedness, only a rather narrow repertoire of "archetypal" field patterns of gauge fields and gravitational fields defined by topological field quanta is possible.

The skeptic can argue that this still need not make possible the imbedding of a rotating black hole metric as induced metric in any physically natural manner. This might be the case but need of course not be a catastrophe. We do not really know whether rotating blackhole metric is realized in Nature. I have indeed proposed that TGD predicts new physics new physics in rotating systems. Unfortunately, gravity probe B could not check whether this new physics is there since it was located at equator where the new effects vanish.

14.2.3 How GRT and Equivalence Principle emerge from TGD?

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) . This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?", "How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?" [K68].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

TGD and GRT

Concerning GRT limit the basic questions are the following ones.

(a) Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 imbedding space coordinates (actually only 4 by general coordinate invariance?

(b) What happens to Einstein equations?

(c) What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?

(d) What about Equivalence Principle (EP)

(e) Can one predict the value of gravitational constant?

(f) What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.

(c) This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.

(d) One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.

(e) Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.

(f) The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

(a) EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.

(b) The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

(a) Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to imbedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.

(b) TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [K93]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.

(c) Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.

(d) The notion of gravitational Planck constant $h_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar et al [E17, E31] [K68].

For few ago entropic gravity [B70, B37] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [K93].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

(a) In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave functions having interpretation as square roots of thermal distributions in ZEO.

The quantum model for cell membrane [K24] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.

(b) A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

Equivalence Principle

Equivalence Principle has several interpretations.

(a) The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.

(b) The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

(a) In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.

(b) Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.

(c) In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

(a) Inertial momentum is defined as Noether charge for Kähler action.

(b) One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [K28, K105]. Its conservation is however not at all all trivial since imbedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [K105]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.

(c) Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.

(d) QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the imbedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

(a) Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.

(b) Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.

(c) By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

(a) In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?

(b) Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other manners. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

EP as quantum classical correspondence

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

(a) The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

(b) Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This view might be equivalent with coset space view. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K93].

    A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the modified Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

(c) A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

14.2.4 The recent view about Kähler-Dirac action

The understanding of Kähler-Dirac action and equation have provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of howe EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

Kähler-Dirac action

14.2.5 Kähler-Dirac action

Kähler-Dirac equation

14.2.6 Kähler-Dirac equation in the interior of space-time surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

(a) Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.

(b) Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.

(c) What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of imbedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that imbedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the imbedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the imbedding space isometries?

(a) Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.

(b) Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to imbedding space coordinates would not vanish in general.

(c) The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n\times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

14.2.7 Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.

The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

(b) If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \quad (14.2.1)$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

14.2.8 About the notion of four-momentum in TGD framework

The starting point of TGD was the energy problem of General Relativity [K93]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K93, K114]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K110] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K101, K78]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B15]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K80], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K6, K106], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K106]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K105].

Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

(a) Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

(b) QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K114]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes.

The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

The objection against the identification h in the decomposition $g = t + h$ of the symplectic algebra as Kac-Moody algebra is that this does not make sense mathematically. The strong form of holography implied by strong form of General Coordinate Invariance however implies that the action of Kac-Moody algebra for the maxima of Kähler function induces unique action of sub-algebra of symplectic algebra so that the identification makes sense after all [K18].

(c) Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K110] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors would be extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep, and it seems that the coset option is definitely wrong: the reason is that for H in G/H decomposition the four-momentum vanishes.

TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces.

Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

1. Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - -> T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2. The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B15]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K101], where also references to the work of pioneers can be found.

1. Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K101] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B43] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2. How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A13] and Virasoro algebras [A30] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

14.3 TGD inspired cosmology

TGD Universe consists of quantum counterparts of a statistical system at critical temperature. As a consequence, topological condensate is expected to possess hierarchical, fractal like structure containing topologically condensed 3-surfaces with all possible sizes. Both Kähler magnetized and Kähler electric 3-surfaces ought to be important and string like objects indeed provide a good example of Kähler magnetic structures important in TGD inspired cosmology. In particular space-time is expected to be many-sheeted even at cosmological scales and ordinary cosmology must be replaced with many-sheeted cosmology. The presence of vapor phase consisting of free cosmic strings and possibly also elementary particles is second crucial aspects of TGD inspired cosmology.

It should be made clear from beginning that many-sheeted cosmology involves a vulnerable asumption. It is assumed that single-sheeted space-time surface is enough to model the cosmology. This need not to be the case. GRT limit of TGD is obtained by lumping together the sheets of many-sheeted space-time to a piece of Minkowski space and endowing it with an effective metric, which is sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Hence the proposed models make sense only if GRT limits allowing imbedding as a vacuum extremal of Kähler action have special physical role.

Quantum criticality of TGD Universe (Kähler coupling strength is analogous to critical temperature) supports the view that many-sheeted cosmology is in some sense critical. Criticality in turn suggests fractality. Phase transitions, in particular the topological phase transitions giving rise to new space-time sheets, are (quantum) critical phenomena involving no scales. If the curvature of the 3-space does not vanish, it defines scale: hence the flatness of the cosmic time=constant section of the cosmology implied by the criticality is consistent with the scale invariance of the critical phenomena. This motivates the assumption that the new space-time sheets created in topological phase transitions are in good approximation modellable as critical Robertson-Walker cosmologies for some period of time at least.

Any one-dimensional sub-manifold allows global imbeddings of subcritical cosmologies whereas for a given 2-dimensional Lagrange manifold of CP_2 critical and overcritical cosmologies allow only one-parameter family of partial imbeddings. The infinite size of the horizon for the imbeddable critical cosmologies is in accordance with the presence of arbitrarily long range quantum fluctuations at criticality and guarantees the average *isotropy* of the cosmology. Imbedding is possible for some critical duration of time. The parameter labelling these cosmologies is a scale factor characterizing the duration of the critical period. These cosmologies have the same optical properties as inflationary cosmologies but exponential expansion is replaced with logarithmic one. Critical cosmology can be regarded as a 'Silent Whisper amplified to Bang' rather than 'Big Bang' and transformed to hyperbolic cosmology before its imbedding fails. Split strings decay to elementary particles in this transition and give rise to seeds of galaxies. In some later stage the hyperbolic cosmology can decompose to disjoint 3-surfaces. Thus each sub-cosmology is analogous to biological growth process leading eventually to death.

The critical cosmologies can be used as a building blocks of a fractal cosmology containing cosmologies containing ... cosmologies. p-Adic length scale hypothesis allows a quantitative formulation of the fractality [K79] . Fractal cosmology predicts cosmos to have essentially same optical properties as inflationary scenario. Fractal cosmology explains the paradoxical result that the observed density of the matter is much lower than the critical density associated with the largest space-time sheet of the fractal cosmology. Also the observation that some astrophysical objects seem to be older than the Universe, finds a nice explanation.

Absolutely essential element of the considerations (and longstanding puzzle of TGD inspired cosmology) is the conservation of energy implied by Poincare invariance which seems to be in conflict with the non-conservation of gravitational energy. It took long time to discover the natural resolution of the paradox. In TGD Universe matter and antimatter have opposite energies and gravitational four-momentum is identified as difference of the four momenta of matter and antimatter (or vice versa, so that gravitational energy is positive). The assumption that the net inertial energy density vanishes in cosmological length scales is the proper interpretation for the fact that Robertson-Walker cosmologies correspond to vacuum extremals of Kähler action.

Tightly bound, possibly coiled pairs of cosmic strings are the basic building block of TGD inspired cosmology and all al structures including large voids, galaxies, stars, and even planets can be seen as pearls in a cosmic fractal necklace consisting of cosmic strings containing smaller cosmic strings linked around them containing... During cosmological evolution the cosmic strings are transformed to magnetic flux tubes and these structures are also key players in TGD inspired quantum biology.

Negative energy virtual gravitons represented by topological quanta having negative time orientation and hence also negative energy. The absorption of negative energy gravitons by photons could explain gradual red-shifting of the microwave background radiation at particle level. Negative energy virtual gravitons give also rise to a negative gravitational potential energy. Quite generally, negative energy virtual bosons build up the negative interaction potential energy. An important constraint to TGD inspired cosmology is the requirement that Hagedorn temperature $T_H \sim 1/R$, where R is CP_2 size, is the limiting temperature of radiation dominated phase.

14.3.1 Robertson-Walker cosmologies

Robertson-Walker cosmologies are the basic building block of standard cosmologies and sub-critical R-W cosmologies have a very natural place in TGD framework as Lorentz invariant cosmologies. Inflationary cosmologies are replaced with critical cosmologies being parameterized by a single parameter telling the duration of the critical cosmology. Over-critical cosmologies are not possible at all.

Why Robertson-Walker cosmologies?

One can hope Robertson Walker cosmology represented as a vacuum extremal of the Kähler action to be a reasonable idealization only in the length scales, where the density of the Kähler charge vanishes. Since (visible) matter and antimatter carry Kähler charges of opposite sign this means that Kähler charge density vanishes in length scales, where matter-antimatter asymmetry disappears on the average. This length scale is certainly very large in present day cosmology: in the proposed model for cosmology its present value is of the order of 10^8 light years: the size of the observed regions containing visible matter predominantly on their boundaries [E37] . That only matter is observed can be understood from the fact that fermions reside dominantly at future oriented space-time sheets and anti-fermions on past-oriented space-time sheets.

Robertson Walker cosmology is expected to apply in the description of the condensate locally at each condensate level and it is assumed that the GRT based criteria for the formation of "structures" apply. In particular, the Jeans criterion stating that density fluctuations with size between Jeans length and horizon size can lead to the development of the "structures" will be applied.

Imbeddability requirement for RW cosmologies

Standard Robertson-Walker cosmology is characterized by the line element [E35]

$$ds^2 = f(a)da^2 - a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ , \tag{14.3.1}$$

where the values $k = 0, \pm 1$ of k are possible.

The line element of the light cone is given by the expression

$$ds^2 = da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \ . \tag{14.3.2}$$

Here the variables a and r are defined in terms of standard Minkowksi coordinates as

$$\begin{aligned} a &= \sqrt{(m^0)^2 - r_M^2} \ , \\ r_M &= ar \ . \end{aligned} \tag{14.3.3}$$

Light cone clearly corresponds to mass density zero cosmology with $k = -1$ and this makes the case $k = -1$ is rather special as far imbeddings are considered since any Lorentz invariant map $M^4_+ \rightarrow CP_2$ defines imbedding

$$s^k = f^k(a) \ . \tag{14.3.4}$$

Here f^k are arbitrary functions of a.

$k = -1$ requirement guarantees imbeddability if the matter density is positive as is easy to see. The matter density is given by the expression

$$\rho = \frac{3}{8\pi G a^2}(\frac{1}{g_{aa}} + k) \ . \tag{14.3.5}$$

A typical imbedding of $k = -1$ cosmology is given by

$$\begin{aligned} \phi &= f(a) \ , \\ g_{aa} &= 1 - \frac{R^2}{4}(\partial_a f)^2 \ . \end{aligned} \tag{14.3.6}$$

where ϕ can be chosen to be the angular coordinate associated with a geodesic sphere of CP_2 (any one-dimensional sub-manifold of CP_2 works equally well). The square root term is always positive by the positivity of the mass density and the imbedding is indeed well defined. Since g_{aa} is smaller than one, the matter density is necessarily positive.

Critical and over-critical cosmologies

TGD allows vacuum extremal imbeddings of a one-parameter family of critical over-critical cosmologies. Critical cosmologies are however not inflationary in the sense that they would involve the presence of scalar fields. Exponential expansion is replaced with a logarithmic one so that the cosmologies are in this sense exact opposites of each other. Critical cosmology has been used hitherto as a possible model for the very early cosmology. What is remarkable that this cosmology becomes vacuum at the moment of 'Big Bang' since mass density behaves as $1/a^2$ as function of the light cone proper time. Instead of 'Big Bang' one could talk about 'Small Whisper' amplified to bang gradually. This is consistent with the idea that space-time sheet begins as a vacuum space-time sheet for some moment of cosmic time. As an imbedded 4-surface this cosmology would correspond to a deformed future light cone having its tip inside the future light cone. The interpretation of the tip as a seed of a phase transition is possible. The imbedding makes sense up to some moment of cosmic time after which the cosmology becomes necessarily hyperbolic. At later time hyperbolic cosmology stops expanding and decomposes to disjoint 3-surfaces behaving as particle like objects co-moving at larger cosmological space-time sheet. These 3-surfaces topologically condense on larger space-time sheets representing new critical cosmologies.

Consider now in more detail the imbeddings of the critical and overcritical cosmologies. For $k = 0, 1$ the imbeddability requirement fixes the cosmology almost uniquely. To see this, consider as an example of $k = 0/1$ imbedding the map from the light cone to S^2, where S^2 is a geodesic sphere of CP_2 with a vanishing Kähler form (any Lagrage manifold of CP_2 would do instead of S^2). In the standard coordinates (Θ, Φ) for S^2 and Robertson-Walker coordinates (a, r, θ, ϕ) for future light cone (, which can be regarded as empty hyperbolic cosmology), the imbedding is given as

$$\begin{aligned} sin(\Theta) &= \frac{a}{a_1} \ , \\ (\partial_r \Phi)^2 &= \frac{1}{K_0}\left[\frac{1}{1-kr^2} - \frac{1}{1+r^2}\right] \ , \\ K_0 &= \frac{R^2}{4a_1^2} \ , \ k = 0, 1 \ , \end{aligned} \tag{14.3.7}$$

when Robertson-Walker coordinates are used for both the future light cone and space-time surface.

The differential equation for Φ can be written as

$$\partial_r \Phi = \pm\sqrt{\frac{1}{K_0}\left[\frac{1}{1-kr^2} - \frac{1}{1+r^2}\right]} \ . \tag{14.3.8}$$

For $k = 0$ case the solution exists for all values of r. For $k = 1$ the solution extends only to $r = 1$, which corresponds to a 4-surface $r_M = m^0/\sqrt{2}$ identifiable as a ball expanding with the velocity $v = c/\sqrt{2}$. For $r \rightarrow 1$ Φ approaches constant Φ_0 as $\Phi - \Phi_0 \propto \sqrt{1-r}$. The space-time sheets corresponding to the two signs in the previous equation can be glued together at $r = 1$ to obtain sphere S^3.

The expression of the induced metric follows from the line element of future light cone

$$ds^2 = da^2 - a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ . \tag{14.3.9}$$

The imbeddability requirement fixes almost uniquely the dependence of the S^2 coordinates a and r and the g_{aa} component of the metric is given by the same expression for both $k = 0$ and $k = 1$.

$$\begin{aligned} g_{aa} &= 1 - K \ , \\ K &\equiv K_0 \frac{1}{(1-u^2)} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{14.3.10}$$

The imbedding fails for $a \geq a_1$. For $a_1 \gg R$ the cosmology is essentially flat up to immediate vicinity of $a = a_1$. Energy density and "pressure" follow from the general equation of Einstein tensor and are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi G a^2}(\frac{1}{g_{aa}} + k) \ , \ k = 0, 1 \ , \\ \frac{1}{g_{aa}} &= \frac{1}{1-K} \ , \\ p &= -(\rho + \frac{a\partial_a \rho}{3}) = -\frac{\rho}{3} + \frac{2}{3}K_0 u^2 \frac{1}{(1-K)(1-u^2)^2}\rho_{cr} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{14.3.11}$$

Here the subscript 'cr' refers to $k = 0$ case. Since the time component g_{aa} of the metric approaches constant for very small values of the cosmic time, there are no horizons associated with this metric. This is clear from the formula

$$r(a) = \int_0^a \sqrt{g_{aa}}\frac{da}{a}$$

for the horizon radius.

The mass density associated with these cosmologies behaves as $\rho \propto 1/a^2$ for very small values of the M^4_+ proper time. The mass in a co-moving volume is proportional to $a/(1-K)$ and goes to zero at the limit $a \to 0$. Thus, instead of Big Bang one has 'Silent Whisper' gradually amplifying to Big Bang. The imbedding fails at the limit $a \to a_1$. At this limit energy density becomes infinite. This cosmology can be regarded as a cosmology for which co-moving strings ($\rho \propto 1/a^2$) dominate the mass density as is clear also from the fact that the "pressure" becomes negative at big bang ($p \to -\rho/3$) reflecting the presence of the string tension. The natural interpretation is that cosmic strings condense on the space-time sheet which is originally empty.

The facts that the imbedding fails and gravitational energy density diverges for $a = a_1$ necessitates a transition to a hyperbolic cosmology. For instance, a transition to radiation or matter dominated hyperbolic cosmology can occur at the limit $\theta \to \pi/2$. At this limit $\phi(r)$ must transform to a function $\phi(a)$. The fact, that vacuum extremals of Kähler action are in question, allows large flexibility for the modelling of what happens in this transition. Quantum criticality and p-adic fractality suggest the presence of an entire fractal hierarchy of space-time sheets representing critical cosmologies created at certain values of cosmic time and having as their light cone projection sub-light cone with its tip at some a=constant hyperboloid.

More general imbeddings of critical and over-critical cosmologies as vacuum extremals

In order to obtain imbeddings as more general vacuum extremals, one must pose the condition guaranteeing the vanishing of corresponding the induced Kähler form (see the Appendix of this book). Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2 the surfaces in question can be expressed as

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D|k+u| \ , \\ u &\equiv cos(\Theta) \ , \ D = \frac{r_0^2}{1+r_0^2}\times\frac{1}{C} \ , \quad C = |k+cos(\Theta_0)| \ . \end{aligned} \tag{14.3.12}$$

Here C and D are integration constants.

These imbeddings generalize to imbeddings to $M^4 \times Y^2$, where Y^2 belongs to a family of Lagrange manifolds described in the Appendix of this book with induced metric

$$\begin{aligned} ds^2_{eff} &= \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X\times\left[\frac{(1-u^2)}{(k+u)^2}\times\frac{1}{1-X}+1-X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X\times\left[(1-X)(k+u)^2+1-u^2\right] \ . \end{aligned} \tag{14.3.13}$$

For $k \neq 1$ $u = \pm 1$ corresponds in general to circle rather than single point as is clear from the fact that $s^{eff}_{\Phi\Phi}$ is non-vanishing at $u = \pm 1$ so that u and Φ parameterize a piece of cylinder. The generalization of the previous imbedding is as

$$sin(\Theta) = ka \quad \rightarrow \quad \sqrt{s^{eff}_{\Phi\Phi}} = ka \ . \tag{14.3.14}$$

For Φ the expression is as in the previous case and determined by the requirement that g_{rr} corresponds to $k = 0, 1$.

The time component of the metric can be expressed as

$$g_{aa} = 1 - \frac{R^2k^2}{4}\frac{s^{eff}_{\Theta\Theta}}{\frac{d\sqrt{s^{eff}_{\Phi\Phi}}}{d\Theta}} \tag{14.3.15}$$

In this case the $1/(1-k^2a^2)$ singularity of the density of gravitational mass at $\Theta = \pi/2$ is shifted to the maximum of $s^{eff}_{\Phi\Phi}$ as function of Θ defining the maximal value a_{max} of a for which the imbedding exists at all. Already for $a_0 < a_{max}$ the vanishing of g_{aa} implies the non-physicality of the imbedding since gravitational mass density becomes infinite.

The geometric properties of critical cosmology change radically in the transition to the radiation dominated cosmology: before the transition the CP_2 projection of the critical cosmology is two-dimensional. After the transition it is one-dimensional. Also the isometry group of the cosmology changes from $SO(3)\times E^3$ to $SO(3,1)$ in the transition. One could say that critical cosmology represents Galilean Universe whereas hyperbolic cosmology represents Lorentzian Universe.

String dominated cosmology

A particularly interesting cosmology is string dominated cosmology with very nearly critical mass density. Assuming that strings are co-moving the mass density of this cosmology is proportional to $1/a^2$ instead of the $1/a^3$ behavior characteristic to the standard matter dominated cosmology. The line element of this metric is very simple: the time component of the metric is simply constant smaller than 1:

$$g_{aa} = K < 1 \ . \tag{14.3.16}$$

The Hubble constant for this cosmology is given by

$$H = \frac{1}{\sqrt{K}a} \ , \tag{14.3.17}$$

and the so called acceleration parameter [E35] k_0 proportional to the second derivative $\ddot{a}$ therefore vanishes. Mass density and pressure are given by the expression

$$\rho = \frac{3}{8\pi GKa^2}(1-K) = -3p \ . \tag{14.3.18}$$

What makes this cosmology so interesting is the absence of the horizons. The comparison with the critical cosmology shows that these two cosmologies resemble each other very closely and both could be used as a model for the very early cosmology.

Stationary cosmology

An interesting candidate for the asymptotic cosmology is stationary cosmology for which gravitational four-momentum currents (and also gravitational color currents) are conserved. This cosmology extremizes the Einstein-Hilbert action with cosmological term given by $\int(kR+\lambda)\sqrt{g}d^4x+\lambda$ and is obtained as a sub-manifold $X^4 \subset M^4_+ \times S^1$, where S^1 is the geodesic circle of CP_2 (note that imbedding is now unique apart from isometries by variational principle).

For a vanishing cosmological constant, field equations reduce to the conservation law for the isometry associated with S^1 and read

$$\partial_a(G^{aa}\partial_a\phi\sqrt{g}) = 0 , \quad (14.3.19)$$

where ϕ denotes the angle coordinate associated with S^1. From this one finds for the relevant component of the metric the expression

$$\begin{aligned} g_{aa} &= \frac{(1-2x)}{(1-x)} , \\ x &= (\frac{C}{a})^{2/3} . \end{aligned} \quad (14.3.20)$$

The mass density and "pressure" of this cosmology are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi Ga^2}\frac{x}{(1-2x)} , \\ p &= -(\rho+\frac{a\partial_a\rho}{3}) = -\frac{\rho}{9}\left[3-\frac{2}{(1-2x)}\right] . \end{aligned} \quad (14.3.21)$$

The asymptotic behavior of the energy density is $\rho \propto a^{-8/3}$. "Pressure" becomes negative indicating that this cosmology is dominated by the string like objects, whose string tension gives negative contribution to to the "pressure". Also this cosmology is horizon free as are all string dominated cosmologies: this is of crucial importance in TGD inspired cosmology.

It should be noticed that energy density for this cosmology becomes infinite for $x=(C/a)^{2/3}=1/2$ implying that this cosmology doesn't make sense at very early times so that the non-conservation of gravitational energy is necessary during the early stages of the cosmology.

Non-conservation of gravitational energy in RW cosmologies

In RW cosmology the gravitational energy in a given co-moving sphere of radius r in local light cone coordinates (a,r,θ,ϕ) is given by

$$E = \int \rho g^{aa}\partial_a m^0\sqrt{g}dV . \quad (14.3.22)$$

The rate characterizing the non-conservation of gravitational energy is determined by the parameter X defined as

$$X \equiv \frac{(dE/da)_{vap}}{E} = \frac{(dE/da+\int|g^{rr}|p\partial_r m^0\sqrt{g}d\Omega)}{E} , \quad (14.3.23)$$

where p denotes the pressure and $d\Omega$ denotes angular integration over a sphere with radius r. The latter term subtracts the energy flow through the boundary of the sphere.

The generation of the pairs of positive and negative (inertial) energy space-time sheets leads to non-conservation of gravitational energy. The generation of pairs of positive and negative energy cosmic strings would be involved with the generation of a critical sub-cosmology.

For RW cosmology with subcritical mass density the calculation gives

$$X = \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} + \frac{3pg_{aa}}{\rho a} . \tag{14.3.24}$$

This formula applies to any infinitesimal volume. The rate doesn't depend on the details of the imbedding (recall that practically any one-dimensional sub-manifold of CP_2 defines a huge family of subcritical cosmologies). Apart from the numerical factors, the rate behaves as $1/a$ in the most physically interesting RW cosmologies. In the radiation dominated and matter dominated cosmologies one has $X = -1/a$ and $X = -1/2a$ respectively so that gravitational energy decreases in radiation and matter dominated cosmologies. For the string dominated cosmology with $k = -1$ having $g_{aa} = K$ one has $X = 2/a$ so that gravitational energy increases: this might be due to the generation of dark matter due to pairs of cosmic strings with vanishing net inertial energy.

For the cosmology with exactly critical mass density Lorentz invariance is broken and the contribution of the rate from 3-volume depends on the position of the co-moving volume. Taking the limit of infinitesimal volume one obtains for the parameter X the expression

$$\begin{aligned} X &= X_1 + X_2 , \\ X_1 &= \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} , \\ X_2 &= \frac{pg_{aa}}{\rho a} \times \frac{3+2r^2}{(1+r^2)^{3/2}} . \end{aligned} \tag{14.3.25}$$

Here r refers to the position of the infinitesimal volume. Simple calculation gives

$$\begin{aligned} &X = X_1 + X_2 , \\ &X_1 = \frac{1}{a}\left[1 + 3K_0u^2\frac{1}{1-K}\right] , \\ &X_2 = -\frac{1}{3a}\left[1 - K - \frac{2K_0u^2}{(1-u^2)^2}\right] \times \frac{3+2r^2}{(1+r^2)^{3/2}} , \\ &\quad K = \frac{K_0}{1-u^2} , \quad u = \frac{a}{a_0} , \quad K_0 = \frac{R^2}{4a_0^2} . \end{aligned} \tag{14.3.26}$$

The positive density term X_1 corresponds to increase of gravitational energy which is gradually amplified whereas pressure term ($p < 0$) corresponds to a decrease of gravitational energy changing however its sign at the limit $a \to a_0$.

The interpretation is in terms of creation of pairs of positive and negative energy particles contributing nothing to the inertial energy. Also pairs of positive energy gravitons and negative anti-gravitons are involved. The contributions of all particle species are determined by thermal arguments so that gravitons should not play any special role as thought originally.

Pressure term is negligible at the limit $r \to \infty$ so that topological condensation occurs all the time at this limit. For $a \to 0, r \to 0$ one has $X > 0 \to 0$ so that condensation starts from zero at $r = 0$. For $a \to 0, r \to \infty$ one has $X = 1/a$ which means that topological condensation is present already at the limit $a \to 0$.

Both the existence of the finite limiting temperature and of the critical mass density imply separately finite energy per co-moving volume for the condensate at the very early stages of the cosmic evolution. In fact, the mere requirement that the energy per co-moving volume in the vapor phase remains finite and non-vanishing at the limit $a \to 0$ implies string dominance as the following argument shows.

Assuming that the mass density of the condensate behaves as $\rho \propto 1/a^{2(1+\alpha)}$ one finds from the expression

$$\rho \propto \frac{(\frac{1}{g_{aa}} - 1)}{a^2} ,$$

that the time component of the metric behaves as $g_{aa} \propto a^{\alpha}$. Unless the condition $\alpha < 1/3$ is satisfied or equivalently the condition

$$\rho < \frac{k}{a^{2+2/3}} \quad (14.3.27)$$

is satisfied, gravitational energy density is reduced. In fact, the limiting behavior corresponds to the stationary cosmology, which is not imbeddable for the small values of the cosmic time. For stationary cosmology gravitational energy density is conserved which suggests that the reduction of the density of cosmic strings is solely due to the cosmic expansion.

14.3.2 Free cosmic strings

The free cosmic strings correspond to four-surfaces of type $X^2 \times S^2$, where S^2 is the homologically nontrivial geodesic sphere of CP_2 [L5] , [L5] and X^2 is minimal surface in M^4_+. As a matter fact, any complex manifold $Y^2 \subset CP_2$ is possible. In this section, a co-moving cosmic string solution inside the light cone $M^4_+(m)$ associated with a given m point of M^4_+ will be constructed.

Recall that the line element of the light cone in co-moving coordinates inside the light cone is given by

$$ds^2 = da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) . \quad (14.3.28)$$

Outside the light cone the line element is given

$$ds^2 = -da^2 - a^2(-\frac{dr^2}{1-r^2} + r^2 d\Omega^2) , \quad (14.3.29)$$

and is obtained from the line element inside the light cone by replacements $a \to ia$ and $r \to -ir$.

Simplest solutions

Using the coordinates ($a=\sqrt{(m^0)^2-r_M^2}, ar=r_M$) for X^2 the orbit of the cosmic string is given by

$$\begin{aligned} \theta &= \frac{\pi}{2} \ , \\ \phi &= f(r) \ . \end{aligned} \tag{14.3.30}$$

Inside the light cone the line element of the induced metric of X^2 is given by

$$ds^2 = da^2 - a^2\left(\frac{1}{1+r^2} + r^2 f_{,r}^2\right)dr^2 \ . \tag{14.3.31}$$

The equations stating the minimal surface property of X^2 can be expressed as a differential conservation law for energy or equivalently for the component of the angular momentum in the direction orthogonal to the plane of the string. The conservation of the energy current T^α gives

$$\begin{aligned} T^\alpha_{,\alpha} &= 0 \ , \\ T^\alpha &= Tg^{\alpha\beta}m^0_{,\beta}\sqrt{g} \ , \\ T &= \frac{1}{8\alpha_K R^2} \simeq .52\times 10^{-6}\frac{1}{G} \ . \end{aligned} \tag{14.3.32}$$

The numerical estimate $TG \simeq .52\times 10^{-6}$ for the string tension is upper bound and corresponds to a situation in which the entire area of S^2 contributes to the tension. It has been obtained using $\alpha_K/104$ and $R^2/G = 2.5\times 10^7 G$ given by the most recent version of p-adic mass calculations (the earlier estimate was roughly by a factor 1/2 too small due to error in the calculation [K33, K5]). The string tension belongs to the range $TG \in [10^{-6}-10^{-7}]$ predicted for GUT strings [E23] . WMAP data give the upper bound $TG \in [10^{-6}-10^{-7}]$, which does not however hold true in the recent case since criticality predicts adiabatic spectrum of perturbations as in the inflationary scenarios.

The non-vanishing components of energy current are given by

$$\begin{aligned} T^a &= TUa \ , \\ T^r &= -T\frac{r}{U} \ , \\ U &= \sqrt{1+r^2(1+r^2)f_{,r}^2} \ . \end{aligned} \tag{14.3.33}$$

The equations of motion give

$$U = \frac{r}{\sqrt{r^2-r_0^2}} \ , \tag{14.3.34}$$

or equivalently

$$\phi_{,r} = \frac{r_0}{r\sqrt{(r^2-r_0^2)(1+r^2)}} \ , \tag{14.3.35}$$

where r_0 is an integration constant to be determined later. Outside the light cone the solution has the form

$$\phi_{,r} = \frac{r_0}{\sqrt{r^2 + r_0^2 r \sqrt{1-r^2}}} \quad . \tag{14.3.36}$$

In the region inside the light cone, where the conditions

$$r_0 << r << 1 \tag{14.3.37}$$

hold, the solution has the form

$$\begin{aligned} \phi(r) &\simeq \phi_0 + \frac{v}{r} \ , \\ v &= \frac{r_0}{\sqrt{1+r_0^2}} \ , \end{aligned} \tag{14.3.38}$$

corresponding to the linearized equations of motion

$$f_{,rr} + \frac{2f_{,r}}{r} = 0 \ , \tag{14.3.39}$$

obtained most nicely from the angular momentum conservation condition.

Cosmic string is stationary in comoving coordinates

In co-moving coordinates (in general the co-moving coordinates of sub-light-cone M^4_+!) the string is stationary. In Minkowski coordinates string rotates with an angular velocity inversely proportional to the distance from the origin

$$\omega \simeq \frac{v}{r_M} \tag{14.3.40}$$

so that the orbital velocity of the string becomes essentially constant in this region. For very large values of r the orbital velocity of the string vanishes as $1/r$. Outside the light cone the variable r is in the role of time and for a given value of the time variable r strings are straight and one can regard the string as a rigidly rotating straight string in this region.

Inside the light cone, the solution becomes ill defined for the values of r smaller than the critical value r_0. Although the derivative $\phi_{,r}$ becomes infinite at this limit, the limiting value of ϕ is finite so that strings winds through a finite angle. The normal component T^r of the energy momentum current vanishes at $r = r_0$ identically, which means that no energy flows out at the end of the string. The coordinate variable r becomes however bad at $r = r_0$ (string resembles a circle at r_0) and this conclusion must be checked using ϕ as coordinate instead of r. The result is that the normal component of the energy current indeed vanishes.

Field equations are not however satisfied at the end of the string since the normal component of the angular momentum current (in z- direction) is non-vanishing at the boundary and given by

$$J^r = Tr^2 a \ . \qquad (14.3.41)$$

This means that the string loses angular momentum through its ends although the angular momentum density of the string is vanishing. The angular momentum lost at moment a is given by

$$J = \frac{Tr^2a^2}{2} = \frac{Tr_M^2}{2} \ . \qquad (14.3.42)$$

This angular momentum is of the same order of magnitude as the angular momentum of a typical galaxy [E29] .

In M^4 coordinates singularity corresponds to a disk in the plane of string growing with a constant velocity, when the coordinate m^0 is positive

$$\begin{aligned} r_M &= vm^0 \ , \\ v &= \frac{r_0}{\sqrt{1+r_0^2}} \ . \end{aligned} \qquad (14.3.43)$$

From the expression of the energy density of the string

$$\begin{aligned} T^a &= T\frac{ar}{\sqrt{r^2-r_0^2}} \ , \\ T &= \frac{1}{8\alpha_K R^2} \ , \end{aligned} \qquad (14.3.44)$$

it is clear that energy density diverges at the singularity.

Energy of the cosmic string

As already noticed, the string tension is by a factor of order 10^{-6} smaller than the critical string tension $T_{cr} = 1/4G$ implying angle deficit of 2π in GRT so that there seems to be no conflict with General Relativity (unlike in the original scenario, in which the CP_2 radius was of order Planck length).

The energy of the string portion ranging from r_0 to r_1 is given by

$$E = T\sqrt{(r_1^2-r_0^2)a} = T\sqrt{\delta r_M^2} \ . \qquad (14.3.45)$$

It should be noticed that M^4 time development of the string can be regarded as a scaling: each point of the string moves to radial direction with a constant velocity v.

One can calculate the total change of the angle ϕ from the integral

$$\Delta\phi = \sqrt{\frac{r_0^2}{1+r_0^2}}\int_{r_0}^{\infty} dr\frac{1}{r\sqrt{(r^2-r_0^2)(1+r^2)}} \ . \qquad (14.3.46)$$

The upper bound of this quantity is obtained at the limit $r_0 \rightarrow 0$ and equals to $\Delta\phi = \pi/2$.

14.3.3 Cosmic strings and cosmology

The model for cosmic strings has forced to question all cherished assumptions including positive energy ontology, Equivalence Principle, and positivity of gravitational mass. The final outcome turned out to be rather conservative. Zero energy ontology is unavoidable, Equivalence Principle holds true universally but its general relativistic formulation makes sense only in long length scales, and gravitational mass has definite sign for positive/negative energy states. As a matter fact, all problems were created by the failure to realize that the expression of gravitational energy in terms of Einstein's tensor does not hold true in short length scales and must be replaced with the stringy expression resulting naturally by dimensional reduction of quantum TGD to string model like theory [K17, K33, K5].

The realization that GRT is only an effective description of many-sheeted space-time as Minkowski space M^4 endowed with effective metric whose deviation from flat metric is the sum of the corresponding deviations for space-time sheets in the region of M^4 considered resolved finally the problems and allowed to reduced Equivalence Principle to its form in GRT. Similar description applies also to gauge interactions.

TGD is therefore a microscopic theory and the physics for single space-time sheet is expected to be extremely simpler, much simpler than in gauge theory and general relativity already due to the fact that only four bosonic variables (4 imbedding space coordinates) defined the dynamics at this level.

Zero energy ontology and cosmic strings

There are two kinds of cosmic strings: free and topological condensed ones and both are important in TGD inspired cosmology.

(a) Free cosmic strings are not absolute minima of the Kähler action (the action has wrong sign). In the original identification of preferred extremals as absolute minima of Kähler action this was a problem. In the new formulation preferred extremals correspond to quantum criticality identified as the vanishing of the second variation of Kähler action at least for the deformations defining symmetries of Kähler action [K17, K33]. The symmetries very probably correspond to conformal symmetries acting as or almost as gauge symmetries. The number of conformal equivalence classes of space-time sheets with same Kähler action and conserved charges is expected to be finite and correspond to n in $h_{eff} = n \times h$ defining the hierarchy of Planck constants labelling phases of dark matter (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Criticality guarantees the conservation of the Noether charges assignable to the modified Dirac action. Ideal cosmic strings are excluded because they fail to satisfy the conditions characterizing the preferred extremal as a space-time surface containing regions with both Euclidian and Minkowskian signature of the induced metric with light-like 3-surface separating them identified as orbits of partonic 2-surfaces carrying elementary particle quantum numbers. The topological condensation of CP_2 type vacuum extremals representing fermons generates negative contribution to the action and reduces the string tension and leaves cosmic strings still free.

(b) If the topologically condensate of fermions has net Kähler charges as the model for matter antimatter asymmetry suggests, the repulsive interaction of the particles tends to thicken the cosmic string by increasing the thickness of its infinitely thin M^4 projection so that Kähler magnetic flux tubes result. These flux tubes are ideal candidates for the carriers of dark matter with a large value of Planck constant. The criterion for the phase transition increasing $\hbar$ is indeed the presence of a sufficiently dense plasma implying that perturbation theory in terms of $Z^2\alpha_{em}$ (Z is the effective number of charges with interacting with each other without screening effects) fails for the standard value of Planck constant. The phase transition $\hbar_0 \rightarrow \hbar$ reduces the value of $\alpha_{em} = e^2/4\pi\hbar$ so that perturbation theory works. This phase transition scales up also the transversal size of the cosmic string. Similar criterion works also for other charges. The resulting phase is anyonic if the resulting 2-surfaces containing almost spherical portions connected by flux tubes to each other encloses the tip of the causal diamond (CD). The proposal is that dark matter resides on complex anyonic 2-surfaces surrounding the tips of CDs.

(c) The topological condensation of cosmic strings generates wormhole contacts represented as pieces of CP_2 type vacuum extremals identified as bosons composed of fermion-anti-fermion pairs. Also this generates negative action and can make cosmic string a preferred extremal of Kähler action. The earliest picture was based on dynamical cancelation mechanism involving generation of strong Kähler electric fields in the condensation whose action compensated for Kähler magnetic action [K2] . Also this mechanism might be at work. Cosmic strings could also form bound states by the formation graviton like flux tubes connecting them and having wormhole contacts at their ends so that again action is reduced.

(d) One can argue that in long enough length and time scales Kähler action per volume must vanish so that the idealization of cosmology as a vacuum extremal becomes possible and there must be some mechanism compensating the positive action of the free cosmic strings. The general mechanism could be topological condensation of fermions and creation of bosons by topological condensation of cosmic strings to space-time sheets.

In this framework zero energy states correspond to cosmologies leading from big bang to big crunch separated by some time interval T of geometric time. Quantum jumps can gradually increase the value T and TGD inspired theory of consciousness suggests that the increase of T might relate to the shift for the contents of conscious experience towards geometric future. In particular, what is usually regarded as cosmology could have started from zero energy state with a small value of T.

Topological condensation of cosmic strings

In the original vision about topological condensation of cosmic strings I assumed that large voids represented by space-time sheets contain "big" cosmic string in their interior and galactic strings near their boundaries. The recent much simpler view is that there are just galactic strings which carry net fermion numbers (matter antimatter asymmetry). If they have also net em charge they have a repulsive interaction and tend to end up to the boundaries of the large void. Since this slows down the expansive motion of strings, the repulsive interaction energy increases and a phase transition increasing Planck constant and scaling up the size of the void occurs after which cosmic strings are again driven towards the boundary of the resulting larger void.

One cannot assume that the exterior metric of the galactic strings is the one predicted by assuming General Relativity in the exterior region. This would mean that metric decomposes as $g = g_2(X^2) + g_2(Y^2)$. $g(X^2)$ would be flat as also $g_2(Y^2)$ expect at the position of string. The resulting angle defect due to the replacement of plane Y^2 with cone would be large and give rise to lense effect of same magnitude as in the case of GUT cosmic strings. Lense effect has not been observed.

This suggests that General Relativity fails in the length scale of large void as far as the description of topologically condensed cosmic strings is considered. The constant velocity spectrum for distant stars of galaxies and the fact that galaxies are organized along strings suggests that these string generate in a good approximation Newtonian potential. This potential predicts constant velocity spectrum with a correct value velocity.

In the stationary situation one expects that the exterior metric of galactic string corresponds to a small deformation of vacuum extremal of Kähler action which is also extremal of the curvature scalar in the induced metric. This allows a solution ansatz which conforms with Newtonian intuitions and for which metric decomposes as $g = g_1 + g_3$, where g_1 corresponds to axis in the direction of string and g_3 remaining $1 + 2$ directions.

Dark energy is replaced with dark matter in TGD framework

The observed accelerating expansion of the Universe has forced to introduce the notion of cosmological constant in the GRT based cosmology. In TGD framework the situation is different.

(a) The gigantic value of gravitational Planck constant implies that dark matter makes TGD Universe a macroscopic quantum system even in cosmological length scales. Astrophysical systems become stationary quantum systems which participate in cosmic expansion only via quantum phase transitions increasing the value of gravitational Planck constant.

(b) Critical cosmologies, which are determined apart from a single parameter in TGD Universe, are natural during all quantum phase transitions, in particular the phase transition periods increasing the size of large voids and having interpretation in terms of an increase of gravitational Planck constant. Cosmic expansion is predicted to be accelerating during these periods. The mere criticality requires that besides ordinary matter there is a contribution $\Omega_\Lambda \simeq .74$ to the mass density besides visible matter and dark matter. In fact, also for the over-critical cosmologies expansion is accelerating.

(c) In GRT framework the essential characteristic of dark energy is its negative pressure. In TGD framework critical and over-critical cosmologies have automatically effective negative pressure. This is essentially due to the constraint that Lorentz invariant vacuum extremal of Kähler action is in question. The mysterious negative pressure would be thus a signal about the representability of space-time as 4-surface in H and there is no need for any microscopic description in terms of exotic thermodynamics.

The values for the TGD counterpart of cosmological constant

One can introduce a parameter characterizing the contribution of dark mass to the mass density during critical periods and call it cosmological constant recalling however that the contribution does not correspond to dark energy. The value of this parameter is same as in the standard cosmology from mere criticality assumption.

What is new that p-adic fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing Λ. The order of magnitude for the recent value of the cosmological constant comes out correctly. The gravitational energy density assignable to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

The naive expectation would be the density of cosmic strings would behave as $1/a^2$ as function of M^4_+ proper time. The vision about dark matter as a phase characterized by gigantic Planck constant however implies that large voids do not expand in continuous manner during cosmic evolution but in discrete quantum jumps increasing the value of the gravitational Planck constant and thus increasing the size of the large void as a quantum state. Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids.

TGD cosmic strings are consistent with the fluctuations of CMB

GUT cosmic strings were excluded by the fluctuation spectrum of the CMB background [E2] . In GRT framework these fluctuations can be classified to adiabatic density perturbations and isocurvature density perturbations. Adiabatic density perturbations correspond to overall scaling of various densities and do not affect the vanishing curvature scalar. For isocurvature density fluctuations the net energy density remains invariant. GUT cosmic strings predict isocurvature density perturbations while inflationary scenario predicts adiabatic density fluctuations.

In TGD framework inflation is replaced with quantum criticality of the phase transition period leading from the cosmic string dominated phase to matter dominated phase. Since curvature scalar vanishes during this period, the density perturbations are indeed adiabatic.

Matter-antimatter asymmetry and cosmic strings

Despite huge amount of work done during last decades (during the GUT era the problem was regarded as being solved!) matter-antimatter asymmetry remains still an unresolved problem of cosmology. A possible resolution of the problem is matter-antimatter asymmetry in the sense that cosmic strings contain antimatter and their exteriors matter. The challenge would be to understand the mechanism generating this asymmetry. The vanishing of the net gauge charges of cosmic string allows this symmetry since electro-weak charges of quarks and leptons can cancel each other.

The challenge is to identify the mechanism inducing the CP breaking necessary for the matter-antimatter asymmetry. Quite a small CP breaking inside cosmic strings would be enough.

(a) The key observation is that vacuum extremals as such are not physically acceptable: small deformations of vacuum extremals to non-vacua are required. This applies also to cosmic strings since as such they do note present preferred extremals. The reason is that the preferred extremals involve necessary regions with Euclidian signature providing four-dimensional representations of generalized Feynman diagrams with particle quantum numbers at the light-like 3-surfaces at which the induced metric is degenerate.

(b) The simplest deformation of vacuum extremals and cosmic strings would be induced by the topological condensation of CP_2 type vacuum extremals representing fermions. The topological condensation at larger space-time surface in turn creates bosons as wormhole contacts.

(c) This process induces a Kähler electric fields and could induce a small Kähler electric charge inside cosmic string. This in turn would induce CP breaking inside cosmic string inducing matter antimatter asymmetry by the minimization of the ground state energy. Conservation of Kähler charge in turn would induce asymmetry outside cosmic string and the annihilation of matter and antimatter would then lead to a situation in which there is only matter.

(d) Either galactic cosmic strings or big cosmic strings (in the sense of having large string tension) at the centers of galactic voids or both could generate the asymmetry and in the recent scenario big strings are not necessary. One might argue that the photon to baryon ratio $r \sim 10^{-9}$ characterizing matter asymmetry quantitatively must be expressible in terms of some fundamental constant possibly characterizing cosmic strings. The ratio $\epsilon = G/\hbar R^2 \simeq 4 \times 10^{-8}$ is certainly a fundamental constant in TGD Universe. By replacing R with $2\pi R$ would give $\epsilon = G/(2\pi R)^2 \simeq 1.0 \times 10^{-9}$. It would not be surprising if this parameter would determine the value of r.

The model can be criticized.

(a) The model suggest only a mechanism and one can argue that the Kähler electric fields created by topological condensates could be random and would not generate any Kähler electric charge. Also the sign of the asymmetry could depend on cosmic string. A CP breaking at the fundamental level might be necessary to fix the sign of the breaking locally.

(b) The model is not the only one that one can imagine. It is only required that antimatter is somewhere else. Antimatter could reside also at other p-adic space-time sheets and at the dark space-time sheets with different values of Planck constant.

The needed CP breaking is indeed predicted by the fundamental formulation of quantum TGD in terms of the modified Dirac action associated with Kähler action and its generalization allowing include instanton term as imaginary part of Kähler action inducing CP breaking [K17, K65] .

(a) The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal, whose proper identification becomes a challenge. In zero energy ontology (ZEO) 3-surfaces are pairs of space-like 3-surfaces assignable to the boundaries of causal diamond (CD) and for deterministic action principle this suggests that the extremals are unique. In presence of non-determinism the situation changes.

(b) The huge vacuum degeneracy of Kähler action suggests that for given pair of 3-surfaces at the boundaries of CD there is a continuum of extremals with the same Kähler action and conserved charges obtained from each other by conformal transformations acting as gauge symmetries and respecting the light-likeness of wormhole throats (as well as the vanishing of the determinant of space-time metric at them). The interpretation is in terms of quantum criticality with the hierarchy of symmetries defining a hierarchy of criticalities analogous to the hierarchy defined by the rank of the matrix defined by the second derivatives of potential function in Thom's catastrophe theory.

(c) The number of gauge equivalence classes is expected to be finite integer n and the proposal is that it corresponds to the value of the effective Planck constant $h_{eff} = n \times h$ so that a connection with dark matter hierarchy labelled by values of n emerges [K27].

(d) This representation generalizes - at least formally. One could add an imaginary instanton term to the Kähler function and corresponding modified Dirac operator D_K so that the generalized eigenvalues assignable to D_K become complex. The generalized eigenvalues correspond to the square roots of the eigenvalues of the operator $DD^\dagger = (p^k\gamma_k + \Gamma^n)(p^k\gamma_k + \Gamma^n)^\dagger$ acting at the boundaries of string world sheets carrying fermion modes and it seems that only space-like 3-surfaces contribute. Γ^n is the normal component of the vector defined by Kähler-Dirac gamma matrices. One can define Dirac determinant formally as the product of the eigenvalues of $DD^\dagger$.

The conjecture is that the resulting Dirac determinant equals to the exponent of Kähler action and imaginary instanton term for the preferred extremal. The instanton term does not contribute to the WCW metric but could provide a first principle description for CP breaking and anyonic effects. It also predicts the dependence of these effects on the page of the book like structure defined by the generalized imbedding space realizing the dark matter hierarchy with levels labeled by the value of Planck constant.

(e) In the case of cosmic strings CP breaking could be especially significant and force the generation of Kähler electric charge. Instanton term is proportional to $1/\hbar$ so that CP breaking would be small for the gigantic values of $\hbar$ characterizing dark matter. For small values of $\hbar$ the breaking is large provided that the topological condensation is able to make the CP_2 projection of cosmic string four-dimensional so that the instanton contribution to the complexified Kähler action is non-vanishing and large enough. Since instanton contribution as a local divergence reduces to the contributions assignable to the light-like 3-surfaces X_l^3 representing topologically condensed particles, CP breaking is large if the density of topologically condensed fermions and wormhole contacts generated by the condensation of cosmic strings is high enough.

CP breaking at the level of CKM matrix

The CKM matrix for quarks contains CP breaking phase factors and this could lead to different evaporation rates for baryons and anti-baryons are different (quark cannot appear as vapor phase particle since vapor phase particle must have vanishing color gauge charges and in the recent vision about quantum TGD CP_2 type vacuum extremal which has not suffered topological condensation represents vacuum). The CP breaking at the level of CKM matrix would be implied by the instanton term present in the complexified Kähler action and modified Dirac operator. The mechanism might rely on hadronic Kähler electric fields which are accompanied by color electric gauge fields proportional to induced Kähler form.

The topological condensation of quarks on hadronic strings containing weak color electric fields proportional to Kähler electric fields should be responsible for its string tension and this should in turn generate CP breaking. At the parton level the presence of CP breaking phase factor $exp(ikS_{CS})$, where $S_{CS} = \int_{X^4} J \wedge J + boundary\ term$ is purely topological Chern Simons term and naturally associated with the boundaries of space-time sheets with at most $D = 3$-dimensional CP_2 projection, could have something to do with the matter antimatter asymmetry. Note however that TGD predicts no strong CP breaking as QCD does [K5] .

Development of strings in the string dominated cosmology

The development of the string perturbations in the Robertson Walker cosmology has been studied [E33] and the general conclusion seems to be that that all the details smaller than horizon are rapidly smoothed out. One must of course take very cautiously the application of these result in TGD framework.

In present case, the horizon has an infinite size so that details in all scales should die away. To see what actually happens consider small perturbations of a static string along z-axis. Restrict the consideration to a perturbation in the y-direction. Using instead of the proper time coordinate t the "conformal time coordinate" τ defined by $d\tau = dt/a$ the equations of motion read [E33]

$$\begin{aligned}(\partial_\tau + \frac{2\dot{a}}{a})(\dot{y}U) &= \partial_z(y'U) \ , \\ U &= \frac{1}{\sqrt{1+(y')^2-\dot{y}^2}} \ . \end{aligned} \quad (14.3.47)$$

Restrict the consideration to small perturbations for which the condition $U \simeq 1$ holds. For the string dominated cosmology the quantity $\dot{a}/a = 1/\sqrt{K}$ is constant and the equations of motion reduce to a very simple approximate form

$$\ddot{y} + \frac{2}{\sqrt{K}}\dot{y} - y'' = 0 \ . \quad (14.3.48)$$

The separable solutions of this equation are of type

$$\begin{aligned} y &= g(a)(C\sin(kz) + D\cos(kz)) \ , \\ g(a) &= (\frac{a}{a_0})^r \ . \end{aligned} \quad (14.3.49)$$

where r is a solution of the characteristic equation $r^2 + 2r/\sqrt{K} + k^2 = 0$:

$$r = -\frac{1}{\sqrt{K}}(1 \pm \sqrt{1-k^2K}) \ . \quad (14.3.50)$$

For perturbations of small wavelength $k > 1/\sqrt{K}$, an extremely rapid attenuation occurs; $1/\sqrt{K} \simeq 10^{27}$! For the long wavelength perturbations with $k << 1/\sqrt{K}$ (physical wavelength is larger than t) the attenuation is milder for the second root of above equation: attenuation takes place as $(a/a_0)^{\sqrt{K}k^2/2}$. The conclusion is that irregularities in all scales are smoothed away but that attenuation is much slower for the long wave length perturbations.

The absence of horizons in the string dominated phase has a rather interesting consequence. According to the well known Jeans criterion the size L of density fluctuations leading to the formation of structures [E33] must satisfy the following conditions

$$l_J < L < l_H , \tag{14.3.51}$$

where l_H denotes the size of horizon and l_J denotes the Jeans length related to the sound velocity v_s and cosmic proper time as [E33]

$$l_J \simeq 10 v_s t . \tag{14.3.52}$$

For a string dominated cosmology the size of the horizon is infinite so that no upper bound for the size of the possible structures results. These structures of course, correspond to string like objects of various sizes in the microscopic description. This suggests that primordial fluctuations create structures of arbitrary large size, which become visible at much later time, when cosmology becomes string dominated again.

Limiting temperature

Since particles are extended objects in TGD, one expects the existence of the limiting temperature T_H (Hagedorn temperature as it is called in string models) so that the primordial cosmology is in Hagedorn temperature. A special consequence is that the contribution of the light particles to the energy density becomes negligible: this is in accordance with the string dominance of the critical mass cosmology. The value of T_H is of order $T_H \sim \hbar/R$, where R is CP_2 radius of order $R \sim 10^{3.5}\sqrt{G}$ and thus considerably smaller than Planck temperature. Note that T_H increases with Planck constant and one can wonder whether this increase continues only up to $T_H = \hbar_{cr}/R = \sqrt{\hbar_{cr}/G}$, which corresponds to the critical value $\hbar_{cr} = R^2/G$. The value $R^2/G = 3 \times 20^{23}\hbar_0$ is consistent with p-adic mass calculations and is favored by by number theoretical arguments [K33, K5] .

The existence of limiting temperature gives strong constraint to the value of the light cone proper time a_F when radiation dominance must have established itself in the critical cosmology which gave rise to our sub-cosmology. Before the moment of transition to hyperbolic cosmology critical cosmology is string dominated and the generation of negative energy virtual gravitons builds up gradually the huge energy density density, which can lead to gravitational collapse, splitting of the strings and establishment of thermal equilibrium with gradually rising temperature. This temperature cannot however become higher than Hagedorn temperature T_H, which serves thus as the highest possible temperature of the effectively radiation dominated cosmology following the critical period. The decay of the split strings generates elementary particles providing the seeds of galaxies.

If most strings decay to light particles then energy density is certainly of the form $1/a^4$ of radiation dominated cosmology. This is not the only manner to obtain effective radiation dominance. Part of the thermal energy goes to the kinetic energy of the vibrational motion of strings and energy density $\rho \propto 1/a^2$ cannot hold anymore. The strings of the condensate is expected to obey the scaling law $\rho \propto 1/a^4$, $p = \rho/3$ [E33] . The simulations with string networks suggest that the energy density of the string network behaves as $\rho \propto 1/a^{2(1+v^2)}$, where v^2 is the mean square velocity of the point of the string [E12] . Therefore, if the value of the mean square velocity approaches light velocity, effective radiation dominance results even when strings dominate [E22] . In radiation dominated cosmology the velocity of sound is $v = 1/\sqrt{3}$. When v lowers to sound velocity one obtains stationary cosmology which is string dominated.

An estimate for a_F is obtained from the requirement that the temperature of the radiation dominated cosmology, when extrapolated from its value $T_R \simeq .3\text{eV}$ at the time about $a_R \sim 3 \times 10^7$ years for the decoupling of radiation and matter to $a = a_F$ using the scaling law $T \propto 1/a$, corresponds to Hagedorn temperature. This gives

$$a_F = a_R \frac{T_R}{T_H} \ , \qquad T_H = \frac{n}{R} \ , \qquad a_R \sim 3\times 10^7 \ y \ , \quad T_R = .27 \ eV \ . \tag{14.3.53}$$

This gives a rough estimate $a_F \sim 3\times 10^{-10}$ seconds, which corresponds to length scale of order 7.7×10^{-2} meters. The value of a_F is quite large.

The result does not mean that radiation dominated sub-cosmologies might have not developed before $a = a_F$. In fact, entire series of critical sub-cosmologies could have developed to radiation dominated phase before the final one leading to our sub-cosmology is actually possible. The contribution of sub-cosmology i to the total energy density of recent cosmology is in the first approximation equal to the fraction $(a_F(i)/a_F)^4$. This ratio is multiplied by a ratio of numerical factors telling the number of effectively massless particle species present in the condensate if elementary particles dominate the mass density. If strings dominate the mass density (as expected) the numerical factor is absent.

For some reason the later critical cosmologies have not evolved to the radiation dominated phase. This might be due to the reduced density of cosmic strings in the vapor phase caused by the formation of the earlier cosmologies which does not allow sufficiently strong gravitational collapse to develop and implies that critical cosmology transforms directly to stationary cosmology without the intervening effectively radiation dominated phase. Indeed, condensed cosmic strings develop Kähler electric field compensating the huge positive Kähler action of free string and can survive the decay to light particles if they are not split. The density of split strings yielding light particles is presumably the proper parameter in this respect.

p-Adic length scale hypothesis allows rather predictive quantitative model for the series of sub-cosmologies [K79] predicting the number of them and allowing to estimate the moments of their birth, the durations of the critical periods and also the durations of radiation dominated phases. p-Adic length scale hypothesis allows also to estimate the maximum temperature achieved during the critical period: this temperature depends on the duration of the critical period a_1 as $T \sim n/a_1$, where n turns out to be of order 10^{30}. This means that if the duration of the critical period is long enough, transition to string dominated asymptotic cosmology occurs with the intervening decay of cosmic strings leading to the radiation dominated phase.

The existence of the limiting temperature has radical consequences concerning the properties of the very early cosmology. The contribution of a given massless particle to the energy density becomes constant. So, unless the number of the effectively massless particle families $N(a)$ increases too fast the contribution of the effectively massless particles to the energy density becomes negligible. The massive excitations of large size (string like objects) are indeed expected to become dominant in the mass density.

What about thermodynamical implications of dark matter hierarchy?

The previous discussion has not mentioned dark matter hierarchy labeled by increasing values of Planck constants and predicted macroscopic quantum coherence in arbitrarily long scales. In TGD Universe dark matter hierarchy means also a hierarchy of conscious entities with increasingly long span of memory and higher intelligence [K89, K24] .

This forces to ask whether the second law is really a fundamental law and whether it could reflect a wrong view about existence resulting resulting when all these dark matter levels and information associated with conscious experiences at these levels is neglected. For instance, biological evolution difficult to understand in a universe obeying second law relies crucially on evolution as gradual progress in which sudden leaps occur as new dark matter levels emerge.

TGD inspired consciousness suggests that Second Law holds true only for the mental images of a given self (a system able to avoid bound state entanglement with environment [K89]) rather than being a universal physical law. Besides these mental images there is irreducible basic awareness of self and second law does not apply to it. Also the hierarchy of higher level conscious entities is there. In this framework second law would basically reflect the exclusion of conscious observers from the physical model of the Universe.

14.3.4 Mechanism of accelerated expansion in TGD Universe

In TGD framework the most plausible identification for the accelerated periods of cosmic expansion is in terms of phase transitions increasing gravitational Planck constant. These phase transitions would in average sense provide quantum counterpart for smooth cosmic expansion. These phase transitions might be initiated by the repulsive Coulomb interaction between cosmic strings driven to the boundaries of the large voids. It is interesting to see how this view relates with the assumption of positive cosmological constant.

How accelerated expansion results in standard cosmology?

The accelerated of cosmic expansion means that the deceleration parameter

$$q = -(ad^2a/ds^2)/(da/ds)^2$$

is negative. For Robertson-Walker cosmologies one has

$$\begin{aligned} H^2 &\equiv (\frac{da/ds}{a})^2 = \frac{8\pi G\rho + \Lambda}{3} - K/a^2, \quad K = 0, \pm 1 \ , \\ 3\frac{d^2a/ds^2}{a} &= \Lambda - 4\pi G(\rho + 3p) \equiv -4\pi G(1+3w)\rho \ . \end{aligned} \tag{14.3.54}$$

It is clear that the accelerated expansion requires positive value of Λ.

The deceleration parameter can be expressed as $q = \frac{1}{2}(1+3w)(1+K/(aH)^2)$. $K =, 0, 1, -1$ tells whether the cosmology is flat, hyper-spherical, or hyperbolic. The rate for the change of Hubble constant can be expressed as $(dH/ds)/H^2 = (1+q)$ and the acceleration of cosmic expansion means $q < -1$. All particle models predict $q \geq -1$.

On basis of modified Einstein's equations written for the recent metric convention (+,-,-.-) (note that opposite signature changes the sign of the left hand side)

$$-G^{\alpha\beta} - \Lambda g^{\alpha\beta} = 8\pi G T^{\alpha\beta} \tag{14.3.55}$$

it is clear that the introduction of a positive cosmological constant could be interpreted by saying that for gravitational vacuum carries energy density equal to $\Lambda/8\pi$ and negative pressure. The negative gravitational pressure would induce the acceleration.

Cosmological term at the level of field equations could be also interpreted by saying that Einstein's equations hold true in the original sense but that energy momentum tensor contains besides the density of inertial mass also a positive density of purely gravitational mass: $T \rightarrow T + \Lambda g$ so that Equivalence Principle fails. Since cosmological constant means effectively negative pressure $p = -\Lambda/8\pi$ the introduction of the cosmological constant means the effective replacement $\rho + 3p \rightarrow \rho + 3p - 2\Lambda/8\pi)$. In the so called $\Lambda - CDM$ model [E5] the densities of dark energy, ordinary matter, and dark matter are assumed to sum up to critical mass density $\rho_{cr} = 3/(8\pi g_{aa} G a^2)$. The fraction of dark matter density is deduced to be $\Omega_\Lambda = .74$ from mere criticality.

Critical cosmology predicts accelerated expansion

In order to get clue about the mechanism of accelerated cosmic expansion in TGD framework it is useful to study the deceleration parameter for various cosmologies in TGD framework.

In standard Friedmann cosmology with non-vanishing cosmological constant one has

$$3\frac{d^2a/ds^2}{a} = \Lambda - 4\pi G(\rho + 3p) \ . \qquad (14.3.56)$$

From this form it is obvious why $\Lambda > 0$ is required in order to obtain accelerating expansion.

Deceleration parameter is a purely geometric property of cosmology and defined as

$$q \equiv -a\frac{d^2a/ds^2}{(da/ds)^2} \ . \qquad (14.3.57)$$

During radiation and matter dominated phases the value of q is positive. In TGD framework there are several metrics which are independent of details of dynamics.

1. String dominated cosmology

String dominated cosmology is hyperbolic cosmology and might serve as a model for very early cosmology corresponds to the metric

$$g_{aa} \equiv (ds/da)^2 = 1 - K_0 \ . \qquad (14.3.58)$$

In this case one has $q = 0$.

2. Critical cosmology

Critical cosmology with flat 3-space corresponds to

$$\begin{aligned} g_{aa} &= 1 - K \ , \\ K &\equiv \frac{K_0}{1-u^2} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \qquad (14.3.59)$$

g_{aa} has the same form also for over-critical cosmologies. Both cosmologies have finite duration. In this case q is given by

$$q = -K_0\frac{K_0u^2}{1-u^2-K_0} < 0 \ , \qquad (14.3.60)$$

and is negative. The rate of change for Hubble constant is

$$\frac{dH/ds}{H^2} = -(1+q) \ , \qquad (14.3.61)$$

so that one must have $q < -1$ in order to have acceleration. This holds true for $a > \sqrt{(1-K_0)/(1+K_0)}a_1$.

Quantum critical cosmology could be seen as a universal characteristic of quantum critical phases associated with phase transition like phenomena. No assumptions about the mechanism behind the transition are made. There is great temptation to assign this cosmology to the phase transitions increasing the size of large voids occurring during late cosmology. The observed jerk assumed to lead from de-accelerated to accelerated expansion for about 13 billion years ago might have interpretation as a transition of this kind.

3. Stationary cosmology

TGD predicts a one-parameter family of stationary cosmologies from the requirement that the density of gravitational 4-momentum is conserved. This is guaranteed if curvature scalar is extremized. These cosmologies are expected to define asymptotic cosmologies or at least characterize the stationary phases between quantum phase transitions. The metric is given by

$$\begin{aligned} g_{aa} &= \frac{1-2x}{1-x} \ , \\ x &= (\frac{a_0}{a})^{2/3} \ . \end{aligned} \tag{14.3.62}$$

The deceleration parameter

$$q = \frac{1}{3}\frac{x}{(1-2x)(1-x)} \ . \tag{14.3.63}$$

is positive so that it seems that TGD does not lead to a continual acceleration which might be regarded as tearing galaxies into pieces.

If quantum critical phases correspond to the expansion of large voids induced by the accelerated radial motion of galactic strings as they reach the boundaries of the voids, one can consider a series of phase transitions between stationary cosmologies in which the value of gravitational Planck constant and the parameter a_0 characterizing the stationary cosmology increase by some even power of two as the ruler-and-compass integer hypothesis [K33, K27] and p-adic length scale hypothesis suggests.

4. Summary

One can safely conclude that TGD predict accelerated cosmic expansion during critical periods and that dark energy is replaced with dark matter in TGD framework. There is also a rather clear view about detailed mechanism leading to the accelerated expansion at "microscopic" level. Some summarizing remarks are in order.

(a) Accelerated expansion is predicted only during periods of over-critical and critical cosmologies parameterized essentially by their duration. The microscopic description would be in terms of phase transitions increasing the size scale of large void. This phase transition is basically a quantum jump increasing gravitational Planck constant and thus the size of the large void. p-Adic length scales are favored sizes of the large voids. A large piece of 4-D cosmological history would be replaced by a new one in this transition so that quite a dramatic event would be in question.

(b) p-Adic fractality forces to ask whether there is a fractal hierarchy of time scales in which Equivalence Principle in the formulation provided by General Relativity sense fails locally (no failure in stringy sense). This would predict a fractal hierarchy of large voids and phase transitions during which accelerated expansion occurs.

(c) Cosmological constant can be said to be vanishing in TGD framework and the description of accelerated expansion in terms of a positive cosmological constant is not equivalent with TGD description since only effective pressure is negative. TGD description has some resemblance to the description in terms of quintessence [E7] , a hypothetical form of matter for which equation of state is of form $p = -w\rho$, $w < -1/3$, so that one has $\rho + 3p = 1 - w < 0$ and deceleration parameter can be negative. The energy density of quintessence is however positive. TGD does not predict endlessly accelerated acceleration tearing galaxies into pieces if the total purely gravitational energy of large voids is assumed to vanish so that Equivalence Principle holds above this length scale.

TGD counterpart of Λ as a density of dark matter rather than dark energy

The value of Λ is expressed usually as a fraction of vacuum energy density from the critical mass density. Combining the data about acceleration of cosmic expansion with the data about cosmic microwave background gives $\Omega_\Lambda \simeq .74$.

(a) Critical mass density requires also in TGD framework the presence of dark contribution since visible matter contribute only a few percent of the total mass density and $\Omega_\Lambda \simeq .-74$ characterizes this contribution. Since the acceleration mechanism has nothing to do with dark energy, dark energy can be replaced with dark matter in TGD framework.

(b) The dark matter hierarchy labeled by the values of Planck constant suggests itself. The $1/a^2$ behavior of dark matter density suggests an interpretation as dark matter topologically condensed on cosmic strings. Besides ordinary particles also super-symplectic bosons and their super partners playing a key role in the model of hadrons and black holes suggest themselves.

(c) Stationary cosmology predicts that the density of stringy matter and thus dark matter decreases like $1/a^2$ as a function of M^4_+ proper time. This behavior is very natural in cosmic string dominated cosmology and one expects that the TGD counterpart of cosmological constant should behave as $\Lambda \propto 1/a^2$ in average sense. At primordial period cosmological constant would be gigantic but its recent value would be extremely small and naturally of correct order of magnitude if the fraction of positive gravitational energy is few per cent about negative gravitational energy. Hence the basic problem of the standard cosmology would find an elegant solution.

Piecewise constancy of TGD counterpart of Λ and p-adic length scale hypothesis

There are good reasons to believe that TGD counterpart of Λ is piecewise constant. Classical picture suggests that the sizes of large voids increase in discrete jumps. The transitions increasing the size of the void would occur when the galactic strings end up to the boundary of the large void and large repulsive Coulomb energy forces the phase transition increasing Planck constant.

Also the quantum astrophysics based on the notion of gravitational Planck constant strongly suggests that astrophysical systems are analogous to stationary states of atoms so that the sizes of astrophysical systems remain constant during the cosmological expansion, and can change only in quantum jumps increasing the value of Planck constant and therefore increasing the radius of the large void regarded as dark matter bound state.

Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids with phase transitions increasing k by even integer. What values of k are realized depends on the time scale of the dynamics driving the galactic strings to the boundaries of expanded large void. Even if all values of k are realized the transitions becomes very rare for large values of a.

p-Adic fractality predicts that the effective cosmological constant Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing the value of effective cosmological constant Λ. As noticed, the allowed values of k would be of form $k = k_0+2n$, where however all integer value need not be realized. By p-adic length scale hypothesis primes are candidates for k. The recent value of the effective cosmological constant can be understood. The gravitational energy density usually assigned to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic prediction is consistent with the recent study [E34] according to which cosmological constant has not changed during the last 8 billion years: the conclusion comes from the reshifts of supernovae of type I_a. If p-adic length scales $L_e(k) = p \simeq 2^k$, k any positive integer, are allowed, the finding gives the lower bound $T_N > \sqrt(2)/(\sqrt{2}-1)) \times 8 = 27.3$ billion years for the recent age of the universe.

Brad Shaefer from Lousiana University has studied the red shifts of gamma ray bursters up to a red shift $z = 6.3$, which corresponds to a distance of 13 billion light years [E30] , and claims that the fit to the data is not consistent with the time independence of the cosmological constant. In TGD framework this would mean that a phase transition changing the value of the cosmological constant must have occurred during last 13 billion years. In principle the phase transitions increasing the size of large voids could be observed as sudden changes of sign for the deceleration parameter.

The reported cosmic jerk as an accelerated period of cosmic expansion

There is an objection against the hypothesis that cosmological constant has been gradually decreasing during the cosmic evolution. Type Ia supernovae at red shift $z \sim .45$ are fainter than expected, and the interpretation is in terms of an accelerated cosmic expansion [E26] . If a period of an accelerated expansion has been preceded by a decelerated one, one would naively expect that for older supernovae from the period of decelerating expansion, say at redshifts about $z > 1$, the effect should be opposite. The team led by Adam Riess [E16] has identified 16 type Ia supernovae at redshifts $z > 1.25$ and concluded that these supernovae are indeed brighter. The conclusion is that about about 5 billion years ago corresponding to $z \simeq .48$, the expansion of the Universe has suffered a cosmic jerk and transformed from a decelerated to an accelerated expansion.

The apparent dimming/brightening of supernovae at the period of accelerated/decelerated expansion the follows from the luminosity distance relation

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} , \quad (14.3.64)$$

where $\mathcal{L}$ is actual luminosity and $\mathcal{F}$ measured luminosity, and from the expression for the distance d_L in flat cosmology in terms of red shift z in a flat Universe

$$\begin{aligned} d_L &= (1+z)\int_0^z \frac{du}{H(u)} \\ &= (1+z)H_0^{-1}\int_0^z exp\left[-\int_0^u du\,[1+q(u)]\,d(ln(1+u)\right] du , \end{aligned} \quad (14.3.65)$$

where one has

$$\begin{aligned} H(z) &= \frac{dln(a)}{ds} , \\ q &\equiv -\frac{d^2a/ds^2}{aH^2} = \frac{dH^{-1}}{ds} - 1 . \end{aligned} \quad (14.3.66)$$

In TGD framework a corresponds to the light-cone proper time and s to the proper time of Robertson-Walker cosmology. Depending on the sign of the deceleration parameter q, the distance d_L is larger or smaller and accordingly the object looks dimmer or brighter.

The natural interpretation for the jerk would be as a period of accelerated cosmic expansion due to a phase transition increasing the value of gravitational Planck constant.

14.4 Microscopic description of black-holes in TGD Universe

In TGD framework the imbedding of the metric for the interior of Schwartshild black-hole fails below some critical radius. This strongly suggests that only the exterior metric of black-hole makes sense in TGD framework and that TGD must provide a microscopic description of black-holes. Somewhat unexpectedly, I ended up with this description from a model of hadrons.

Super-symplectic algebra is a generalization of Kac-Moody algebra obtained by replacing the finite-dimensional group G with the group of symplectic transformations of $\delta M^4_\pm \times CP_2$. This algebra defines the group of isometries for the "world of classical worlds" and together with the Kac-Moody algebra assignable to the deformations of light-like 3-surfaces representing orbits of 2-D partonic surfaces it defines the mathematical backbone of quantum TGD as almost topological QFT.

From the point of view of experimentalist the basic question is how these super-symplectic degrees of freedom reflect themselves in existing physics and the pleasant surprise was that super-symplectic bosons explain what might be called the missing hadronic mass and spin. The point is that quarks explain only about 170 MeV of proton mass. Also the spin puzzle of proton is known for years. Also precise mass formulas for hadrons emerge.

Super-symplectic degrees of freedom represent dark matter in electro-weak sense and highly entangled hadronic strings in Hagedorn temperature are very much analogous to black-holes. This indeed generalizes to a microscopic model for black-holes created when hadronic strings fuse together in high density.

14.4.1 Super-symplectic bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K18, K17] , whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5,6,58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K57] and here only a brief summary is given.

As explained in [K57] , the assignment of these bosons to hadronic space-time sheet is an attractive idea.

(a) Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.

(b) Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.

(c) Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

(d) Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C28] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C37]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

14.4.2 Are ordinary black-holes replaced with super-symplectic black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-hole like states associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

(a) Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \quad (14.4.1)$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

(b) If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \quad (14.4.2)$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of $\hbar_0$ for all values of $\hbar$.

(c) Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \quad (14.4.3)$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

(d) The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K79] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \quad v_0 = 1/2 \ . \quad (14.4.4)$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K79] . Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

(e) The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

(f) Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.

(g) In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

14.4.3 Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy - being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic. $\hbar = GM^2/v_0$, $v_0 = 1/4$, would hold true for an ideal black hole with Planck length $(\hbar G)^{1/2}$ equal to Schwartshild radius $2GM$. Since black hole entropy is inversely proportional to $\hbar$, this would predict black hole entropy to be of order single bit. This of course looks totally non-sensible if one believes in standard thermodynamics. For the star with mass equal to 10^{40} Planck masses the entropy associated with the initial state of the star would be roughly the number of atoms in star equal to about 10^{60}. Black hole entropy proportional to $GM^2/\hbar$ would be of order 10^{80} provided the standard value of $\hbar$ is used as unit. This stimulates some questions.

(a) Does second law pose an upper bound on the value of $\hbar$ of dark black hole from the requirement that black hole has at least the entropy of the initial state. The maximum value of $\hbar$ would be given by the ratio of black hole entropy to the entropy of the initial state and about 10^{20} in the example consider to be compared with $GM^2/v_0 \sim 10^{80}$.

(b) Or should one generalize thermodynamics in a manner suggested by zero energy ontology by making explicit distinction between subjective time (sequence of quantum jumps) and geometric time? The arrow of geometric time would correlate with that of subjective time. One can argue that the geometric time has opposite direction for the positive and negative energy parts of the zero energy state interpreted in standard ontology as initial and final states of quantum event. If second law would hold true with respect to subjective time, the formation of ideal dark black hole would destroy entropy only from the point of view of observer with standard arrow of geometric time. The behavior of phase conjugate laser light would be a more mundane example. Do self assembly processes serve as example of non-standard arrow of geometric time in biological systems? In fact, zero energy state is geometrically analogous to a big bang followed by big crunch. One can however criticize the basic assumption as ad hoc guess. One should really understand the the arrow of geometric time. This is discussed in detail in [L7] .

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

14.5 A quantum model for the formation of astrophysical structures and dark matter?

D. Da Rocha and Laurent Nottale, the developer of Scale Relativity, have ended up with an highly interesting quantum theory like model for the evolution of astrophysical systems [E27] (I am grateful for Victor Christianito for informing me about the article). In particular, this model applies to planetary orbits. I learned later that also A. Rubric and J. Rubric have proposed a Bohr model for planetary orbits [E28] already 1998.

The model is simply Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant

$$\hbar \rightarrow \hbar_{gr} = \frac{GmM}{v_0} \ . \tag{14.5.1}$$

Here I have used units $\hbar = c = 1$. v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. The peak orbital velocity of stars in galactic halos is 142 ± 2 km/s whereas the average velocity is 156 ± 2 km/s. Also sub-harmonics and harmonics of v_0 seem to appear.

The model makes fascinating predictions which hold true. For instance, the radii of planetary orbits fit nicely with the prediction of the hydrogen atom like model. The inner solar system (planets up to Mars) corresponds to v_0 and outer solar system to $v_0/5$.

The predictions for the distribution of major axis and eccentricities have been tested successfully also for exoplanets. Also the periods of 3 planets around pulsar PSR B1257+12 fit with the predictions with a relative accuracy of few hours/per several months. Also predictions for the distribution of stars in the regions where morphogenesis occurs follow from the gravitational Schödinger equation.

What is important is that there are no free parameters besides v_0. In [E27] a wide variety of astrophysical data is discussed and it seem that the model works and has already now made predictions which have been later verified. In the following I shall discuss Nottale's model from the point of view of TGD.

14.5.1 TGD prediction for the parameter v_0

One of the basic questions is the origin of the parameter v_0, which according to a rich amount of experimental data discussed in [E27] seems to play a role of a constant of Nature. One of the first applications of cosmic strings in TGD sense was an explanation of the velocity spectrum of stars in the galactic halo in terms of dark matter which could consists of cosmic strings. Cosmic strings could be orthogonal to the galactic plane going through the nucleus (jets) or they could be in galactic plane in which case the strings and their decay products would explain dark matter assuming that the length of cosmic string inside a sphere of radius R is or has been roughly R [K22] . The predicted value of the string tension is determined by the CP_2 radius whose ratio to Planck length is fixed by electron mass via p-adic mass calculations. The resulting prediction for the v_0 is correct and provides a working model for the constant orbital velocity of stars in the galactic halo.

The parameter $v_0 \simeq 2^{-11}$, which has actually the dimension of velocity unless on puts $c = 1$, and also its harmonics and sub-harmonics appear in the scaling of $\hbar$. v_0 corresponds to the velocity of distant stars in the model of galactic dark matter. TGD allows to identify this parameter as the parameter

$$\begin{aligned} v_0 &= 2\sqrt{TG} = \sqrt{\frac{1}{2\alpha_K}}\sqrt{\frac{G}{R^2}} \ , \\ T &= \frac{1}{8\alpha_K}\frac{\hbar_0}{R^2} \ . \end{aligned} \tag{14.5.2}$$

Here T is the string tension of cosmic strings, R denotes the "radius" of CP_2 ($2R$ is the radius of geodesic sphere of CP_2). α_K is Kähler coupling strength, the basic coupling constant strength of TGD, whose evolution as a function of p-adic length scale is fixed by quantum criticality. The condition that G is invariant in the p-adic coupling constant evolution and number theoretical arguments predict

$$\begin{aligned} \alpha_K(p) &= k\frac{1}{log(p) + log(K)} \ , \\ K &= \frac{R^2}{\hbar_0 G} = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23 \ , \ \ k\simeq \pi/4 \ . \end{aligned} \tag{14.5.3}$$

The predicted value of v_0 depends logarithmically on the p-adic length scale and for $p \simeq 2^{127}-1$ (electron's p-adic length scale) one has $v_0 \simeq 2^{-11}$.

14.5.2 Model for planetary orbits without $v_0 \rightarrow v_0/5$ scaling

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. For instance, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas inner planets correspond to v_0. Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E27] is that the different values of n relate to fractality. This scaling is not necessary for the planetary orbits in TGD based model.

Effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question in the case of outer planets. If one accepts the interpretation that visible matter has concentrated around dark matter, which is in macroscopic quantum phase around Bohr orbits, this allows to consider also the possibility that $\hbar_{gr}$ has the same value for all planets.

(a) Some gravitational perturbation has kicked dark matter from the region of the asteroid belt to $n \simeq 5k$, $k = 2,..,6$, orbits. The best fit is obtained by using values of n deviating somewhat from multiples of 5 which suggests that the scaling of v_0 is not needed. Gravitational perturbations might have caused the same for the visible matter. The fact that the tilt angles of Earth and outer planets other than Pluto are nearly the same suggests that the orbits of these planets might be an outcome of some violent quantum process for dark matter preserving the orbital plane in a good approximation. Pluto might in turn have experienced some violent collision changing its orbital plane.

(b) There could exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter.

	Exp.	T-B	Bohr$_1$	Bohr$_2$
Planet	R/R_M	R/R_M	$[n, R/R_M]$	$[n, R/R_M]$
Mercury	1	1	[3, 1]	
Venus	1.89	1.75	[4, 1.8]	
Earth	2.6	2.5	[5, 2.8]	
Mars	3.9	4	[6, 4]	
Asteroids	6.1-8.7	7	[(7, 8, 9), (5.4, 7.1, 9)]	
Jupiter	13.7	13	[11, 13.4]	[2×5,11.1]
Saturn	25.0	25	[3×5, 25]	[3×5, 25]
Uranus	51.5	49	[22, 53.8]	[4×5,44.4]
Neptune	78.9	97	[27 , 81]	[5×5, 69.4]
Pluto	105.2	97	[31, 106.7]	[6×5,100]

Table 1. The table represents the experimental average orbital radii of planets, the predictions of Titius-Bode law (note the failure for Neptune), and the predictions of Bohr orbit model assuming a) that the principal quantum number n corresponds to best possible fit, b) the scaling $v_0 \rightarrow v_0/5$ for outer planets. Option a) gives the best fit with errors being considerably smaller than the maximal error $|\Delta R|/R \simeq 1/n$ except for Uranus. R_M denotes the orbital radius of Mercury. T-B refers to Titius-Bode law.

How to understand the harmonics and sub-harmonics of v_0 in TGD framework?

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. In particular, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas inner planets correspond to v_0 in this model. As already found, TGD allows also an alternative explanation.

Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E27] is that the different values of n relate to fractality. This quantization is a challenge for TGD since v_0 certainly defines a fundamental constant in TGD Universe.

(a) Consider first the harmonics of v_0. Besides cosmic strings of type $X^2 \times S^2 \subset M^4 \times CP_2$ one can consider also deformations of these strings defining their multiple coverings so that the deformation is n-valued as a function of S^2-coordinates (Θ, Φ) and the projection to S^2 is thus an $n \rightarrow 1$ map. The solutions are higher dimensional analogs of originally closed orbits which after perturbation close only after n turns. This kind of surfaces emerge in the TGD inspired model of quantum Hall effect naturally [K97] and $n \to \infty$ limit has an interpretation as an approach to chaos [K90] .

Using the coordinates (x, y, θ, ϕ) of $X^2 \times S^2$ and coordinates m^k for M^4 of the unperturbed solution the space-time surface the deformation can be expressed as

$$\begin{aligned} m^k &= m^k(x, y, \theta, \phi) \ , \\ (\Theta, \Phi) &= (\theta, n\phi) \ . \end{aligned} \tag{14.5.4}$$

The value of the string tension would be indeed n^2-fold in the first approximation since the induced Kähler form defining the Kähler magnetic field would be $J_{\theta\phi} = nsin(\Theta)$ and one would have $v_n = nv_0$. At the limit $m^k = m^k(x, y)$ different branches for these solutions collapse together.

(b) Consider next how sub-harmonics appear in TGD framework. Suppose that cosmic strings decay to magnetic flux tube structures. This could the counterpart for cosmic expansion. The Kähler magnetic flux $\Phi = BS$ is conserved in the process but the thickness of the M^4 projection of the cosmic string increases field strength is reduced. This means that string tension, which is proportional to B^2S, is reduced (so that also Kähler action is reduced). The fact that space-time surface is Bohr orbit in generalized sense means that the reduced string tension (magnetic energy per unit length) is quantized.

The task is to guess how the quantization occurs. There are two options.

(a) The simplest explanation for the reduction of v_0 is based on the decay of a flux tube resembling a disk with a hole to n identical flux tubes so that $v_0 \to v_0/n$ results for the resulting flux tubes. It turns out that this mechanism is favored and explains elegantly the value of $\hbar_{gr}$ for outer planetary system. One can also consider small-p p-adicity so that n would be prime.

(b) Second explanation is more intricate. Consider a magnetic flux tube. Since magnetic flux is quantized, the magnetic field strengths are quantized in integer multiples of basic strength: $B = nB_0$ and would rather naturally correspond to the multiple coverings of the original magnetic flux tube with magnetic energy quantized in multiples of n^2. The idea is to require internal consistency in the sense that the allowed reduced field strengths are such that the spectrum associated with B_0 is contained to the spectrum associated with the quantized field strengths $B_1 > B_0$. This would allow only field strengths $B = B_S/n^2$, where B_S denotes the field strength of the fundamental cosmic string and one would have $v_n = v_0/n$. Flux conservation requires that the area of the flux tube scales as n^2.

Sub-harmonics might appear in the outer planetary system and there are indications for the higher harmonics below the inner planetary system [E27] : for instance, solar radius corresponds to $n = 1$ orbital for $v_3 = 3v_0$. This would suggest that Sun and also planets have an onion like structure with highest harmonics of v_0 and strongest string tensions appearing in the solar core and highest sub-harmonics appearing in the outer regions. If the matter results as decay remnants of cosmic strings this means that the mass density inside Sun should correlate strongly with the local value of n characterizing the multiple covering of cosmic strings.

One can ask whether the very process of the formation of the structures could have excited the higher values of n just like closed orbits in a perturbed system become closed only after n turns. The energy density of the cosmic string is about one Planck mass per $\sim 10^7$ Planck lengths so that $n > 1$ excitation increasing this density by a factor of n^2 is obviously impossible except under the primordial cosmic string dominated period of cosmology during which the net inertial energy density must have vanished. The structure of the future solar system would have been dictated already during the primordial phase of cosmology when negative energy cosmic string suffered a time reflection to positive energy cosmic strings.

Nottale equation is consistent with the TGD based model for dark matter

TGD allows two models of dark matter. The first one is spherically symmetric and the second one cylindrically symmetric. The first thing to do is to check whether these models are consistent with the gravitational Schrödinger equation/Bohr quantization.

1. Spherically symmetric model for the dark matter

The following argument based on Bohr orbit quantization demonstrates that this is indeed the case for the spherically symmetric model for dark matter. The argument generalizes in a trivial manner to the cylindrically symmetric case.

(a) The gravitational potential energy $V(r)$ for a mass distribution $M(r) = xTr$ (T denotes string tension) is given by

$$V(r) = Gm\int_r^{R_0} \frac{M(r)}{r^2}dr = GmxTlog(\frac{r}{R_0}) \ . \tag{14.5.5}$$

Here R_0 corresponds to a large radius so that the potential is negative as it should in the region where binding energy is negative.

(b) The Newton equation $\frac{mv^2}{r} = \frac{GmxT}{r}$ for circular orbits gives

$$v = xGT \ . \tag{14.5.6}$$

(c) Bohr quantization condition for angular momentum by replacing $\hbar$ with $\hbar_{gr}$ reads as $mvr = n\hbar_{gr}$ and gives

$$\begin{aligned} r_n &= \frac{n\hbar_{gr}}{mv} = nr_1 \ , \\ r_1 &= \frac{GM}{vv_0} \ . \end{aligned} \tag{14.5.7}$$

Here v is rather near to v_0.

(d) Bound state energies are given by

$$E_n = \frac{mv^2}{2} - xTlog(\frac{r_1}{R_0}) + xTlog(n) \ . \tag{14.5.8}$$

The energies depend only weakly on the radius of the orbit.

(e) The centrifugal potential $l(l+1)/r^2$ in the Schrödinger equation is negligible as compared to the potential term at large distances so that one expects that degeneracies of orbits with small values of l do not depend on the radius. This would mean that each orbit is occupied with same probability irrespective of value of its radius. If the mass distribution for the starts does not depend on r, the number of stars rotating around galactic nucleus is simply the number of orbits inside sphere of radius R and thus given by $N(R) \propto R/r_0$ so that one has $M(R) \propto R$. Hence the model is self consistent in the sense that one can regard the orbiting stars as remnants of cosmic strings and thus obeying same mass distribution.

2. Cylindrically symmetric model for the galactic dark matter

TGD allows also a model of the dark matter based on cylindrical symmetry. In this case the dark matter would correspond to the mass of a cosmic string orthogonal to the galactic plane and traversing through the galactic nucleus. The string tension would the one predicted by TGD. In the directions orthogonal to the plane of galaxy the motion would be free motion so that the orbits would be helical, and this should make it possible to test the model. The quantization of radii of the orbits would be exactly the same as in the spherically symmetric model. Also the quantization of inclinations predicted by the spherically symmetric model could serve as a sensitive test. In this kind of situation general theory of relativity would predict only an angle deficit giving rise to a lens effect. TGD predicts a Newtonian $1/\rho$ potential in a good approximation.

Spiral galaxies are accompanied by jets orthogonal to the galactic plane and a good guess is that they are associated with the cosmic strings. The two models need not exclude each other. The vision about astrophysical structures as pearls of a fractal necklace would suggest that the visible matter has resulted in the decay of cosmic strings originally linked around the cosmic string going through the galactic plane and creating $M(R) \propto R$ for the density of the visible matter in the galactic bulge. The finding that galaxies are organized along linear structures [E37] fits nicely with this picture.

MOND and TGD

TGD based model explains also the MOND (Modified Newton Dynamics) model of Milgrom [E24] for the dark matter. Instead of dark matter the model assumes a modification of Newton's laws. The model is based on the observation that the transition to a constant velocity spectrum seems in the galactic halos seems to occur at a constant value of the stellar acceleration equal to $a_0 \simeq 10^{-11}g$, where g is the gravitational acceleration at the Earth. MOND theory assumes that Newtonian laws are modified below a_0.

The explanation relies on Bohr quantization. Since the stellar radii in the halo are quantized in integer multiples of a basic radius and since also rotation velocity v_0 is constant, the values of the acceleration are quantized as $a(n) = v_0^2/r(n)$ and a_0 correspond to the radius $r(n)$ of the smallest Bohr orbit for which the velocity is still constant. For larger orbital radii the acceleration would indeed be below a_0. a_0 would correspond to the distance above which the density of the visible matter does not appreciably perturb the gravitational potential of the straight string. This of course requires that gravitational potential is that given by Newton's theory and is indeed allowed by TGD.

The MOND theory [E24] and its variants predict that there is a critical acceleration below which Newtonian gravity fails. This would mean that Newtonian gravitation is modified at large distances. String models and also TGD predict just the opposite since in this regime General Relativity should be a good approximation.

(a) The $1/r^2$ force would transform to $1/r$ force at some critical acceleration of about $a = 10^{-10}$ m/s^2: this is a fraction of 10^{-11} about the gravitational acceleration at the Earth's surface.

(b) The recent empirical study [E36] giving support for this kind of transition in the dynamics of stars at large distances and therefore breakdown of Newtonian gravity in MOND like theories.

In TGD framework critical acceleration is predicted but the recent experiment does not force to modify Newton's laws. Since Big Science is like market economy in the sense that funding is more important than truth, the attempts to communicate TGD based view about dark matter [K27, K79, K62, K80, K22] have turned out to be hopeless. Serious Scientist does not read anything not written on silk paper.

(a) One manner to produce this spectrum is to assume density of dark matter such that the mass inside sphere of radius R is proportional to R at last distances [K22]. Decay products of and ideal cosmic strings would predict this. The value of the string tension predicted correctly by TGD using the constraint that p-adic mass calculations give electron mass correctly [K48].

(b) One could also assume that galaxies are distributed along cosmic string like pearls in necklace. The mass of the cosmic string would predict correct value for the velocity of distant stars. In the ideal case there would be no dark matter outside these cosmic strings.

i. The difference with respect to the first mechanism is that this case gravitational acceleration would vanish along the direction of string and motion would be free motion. The prediction is that this kind of motions take place along observed linear structures formed by galaxies and also along larger structures.
ii. An attractive assumption is that dark matter corresponds to phases with large value of Planck constant is concentrated on magnetic flux tubes. Holography would suggest that the density of the magnetic energy is just the density of the matter condensed at wormhole throats associated with the topologically condensed cosmic string.
iii. Cosmic evolution modifies the ideal cosmic strings and their Minkowski space projection gets gradually thicker and thicker and their energy density - magnetic energy - characterized by string tension could be affected

TGD option differs from MOND in some respects and it is possible to test empirically which option is nearer to the truth.

(a) The transition at same critical acceleration is predicted universally by this option for all systems-now stars- with given mass scale if they are distributed along cosmic strings like like pearls in necklace. The gravitational acceleration due the necklace simply wins the gravitational acceleration due to the pearl. Fractality encourages to think like this.
(b) The critical acceleration predicted by TGDr depends on the mass scale as $a \propto GT^2/M$, where M is the mass of the object- now star. Since the recent study considers only stars with solar mass it does not allow to choose between MOND and TGD and Newton can continue to rest in peace in TGD Universe. Only a study using stars with different masses would allow to compare the predictions of MOND and TGD and kill either option or both. Second test distinguishing between MOND and TGD is the prediction of large scale free motions by TGD option.

TGD option explains also other strange findings of cosmology.

(a) The basic prediction is the large scale motions of dark matter along cosmic strings. The characteristic length and time scale of dynamics is scaled up by the scaling factor of $\hbar$. This could explain the observed large scale motion of galaxy clusters -dark flow [E3]- assigned with dark matter in conflict with the expectations of standard cosmology.
(b) Cosmic strings could also relate to the strange relativistic jet like structures [E8] meaning correlations between very distant objects. Universe would be a spaghetti of cosmic strings around which matter is concentrated.
(c) The TGD based model for the final state of star [K93] actually predicts the presence of string like object defining preferred rotation axis. The beams of light emerging from supernovae would be preferentially directed along this lines- actually magnetic flux tubes. Same would apply to the gamma ray bursts [E4] from quasars, which would not be distributed evenly in all directions but would be like laser beams along cosmic strings.

14.5.3 The interpretation of $\hbar_{gr}$ and pre-planetary period

$\hbar_{gr}$ could corresponds to a unit of angular momentum for quantum coherent states at magnetic flux tubes or walls containing macroscopic quantum states. Quantitative estimate demonstrates that $\hbar_{gr}$ for astrophysical objects cannot correspond to spin angular momentum. For Sun-Earth system one would have $\hbar_{gr} \simeq 10^{77}\hbar$. This amount of angular momentum realized as a mere spin would require 10^{77} particles! Hence the only possible interpretation is as a unit of orbital angular momentum. The linear dependence of $\hbar_{gr}$ on m is consistent with the additivity of angular momenta in the fusion of magnetic flux tubes to larger units if the angular momentum associated with the tubes is proportional to both m and M.

Just as the gravitational acceleration is a more natural concept than gravitational force, also $\hbar_{gr}/m = GM/v_0$ could be more natural unit than $\hbar_{gr}$. It would define a universal unit for the circulation $\oint v \cdot dl$, which is apart from $1/m$-factor equal to the phase integral $\oint p_\phi d\phi$ appearing in Bohr rules for angular momentum. The circulation could be associated with the flow associated with outer boundaries of magnetic flux tubes surrounding the orbit of mass m around the central mass $M \gg m$ and defining light like 3-D CDs analogous to black hole horizons.

The expression of $\hbar_{gr}$ depends on masses M and m and can apply only in space-time regions carrying information about the space-time sheets of M and and the orbit of m. Quantum gravitational holography suggests that the formula applies at 3-D light like causal determinant (CD) X^3_l defined by the wormhole contacts gluing the space-time sheet X^3_l of the planet to that of Sun. More generally, X^3_l could be the space-time sheet containing the planet, most naturally the magnetic flux tube surrounding the orbit of the planet and possibly containing dark matter in super-conducting state. This would give a precise meaning for $\hbar_{gr}$ and explain why $\hbar_{gr}$ does not depend on the masses of other planets.

The simplest option consistent with the quantization rules and with the explanatory role of magnetic flux structures is perhaps the following one.

(a) X^3_l is a torus like surface around the orbit of the planet containing de-localized dark matter. The key role of magnetic flux quantization in understanding the values of v_0 suggests the interpretation of the torus as a magnetic or Z^0 magnetic flux tube. At pre-planetary period the dark matter formed a torus like quantum object. The conditions defining the radii of Bohr orbits follow from the requirement that the torus-like object is in an eigen state of angular momentum in the center of mass rotational degrees of freedom. The requirement that rotations do not leave the torus-like object invariant is obviously satisfied. Newton's law required by the quantum-classical correspondence stating that the orbit corresponds to a geodesic line in general relativistic framework gives the additional condition implying Bohr quantization.

(b) A simple mechanism leading to the localization of the matter would have been the pinching of the torus causing kind of a traffic jam leading to the formation of the planet. This process could quite well have involved a flow of matter to a smaller planet space-time sheet Y^3_l topologically condensed at X^3_l. Most of the angular momentum associated with torus like object would have transformed to that of planet and situation would have become effectively classical.

(c) The conservation of magnetic flux means that the splitting of the orbital torus would generate a pair of Kähler magnetic charges. It is not clear whether this is possible dynamically and hence the torus could still be there. In fact, TGD explanation for the tritium beta decay anomaly citeTroitsk,Mainz in terms of classical Z^0 force [K84] requires the existence of this kind of torus containing neutrino cloud whose density varies along the torus. This picture suggests that the lacking $n = 1$ and $n = 2$ orbits in the region between Sun and Mercury are still in magnetic flux tube state containing mostly dark matter.

(d) The fact that $\hbar_{gr}$ is proportional to m means that it could have varied continuously during the accumulation of the planetary mass without any effect in the planetary motion: this is of course nothing but a manifestation of Equivalence Principle.

(e) It is interesting to look for the scaled up versions of Planck mass $m_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = \sqrt{M_1 M_2/v_0}$ and Planck length $L_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = G\sqrt{M_1 M_2/v_0}$. For $M_1 = M_2 = M$ this gives $m_{Pl} = M/\sqrt{v_0} \simeq 45.6 \times M$ and $L_{Pl} = r_S/2\sqrt{v_0} \simeq 22.8 \times r_S$, where r_S is Schwartshild radius. For Sun r_S is about 2.9 km so that one has $L_{Pl} \simeq 66$ km. For a few years ago it was found that Sun contains "inner-inner" core of radius about $R = 300$ km [F7] which is about $4.5 \times L_{Pl}$.

14.5.4 Inclinations for the planetary orbits and the quantum evolution of the planetary system

The inclinations of planetary orbits provide a test bed for the theory. The semiclassical quantization of angular momentum gives the directions of angular momentum from the formula

$$cos(\theta) = \frac{m}{\sqrt{j(j+1)}} \quad , \quad |m| \leq j \quad . \tag{14.5.9}$$

where θ is the angle between angular momentum and quantization axis and thus also that between orbital plane and (x,y)-plane. This angle defines the angle of tilt between the orbital plane and (x,y)-plane.

$m = j = n$ gives minimal value of angle of tilt for a given value of n of the principal quantum number as

$$cos(\theta) = \frac{n}{\sqrt{n(n+1)}} \quad . \tag{14.5.10}$$

For $n = 3, 4, 5$ (Mercury, Venus, Earth) this gives $\theta = 30.0$, 26.6, and 24.0 degrees respectively.

Only the relative tilt angles can be compared with the experimental data. Taking as usual the Earth's orbital plane as the reference the relative tilt angles give what are known as inclinations. The predicted inclinations are 6 degrees for Mercury and 2.6 degrees for Venus. The observed values [E9] are 7.0 and 3.4 degrees so that the agreement is satisfactory. If one allows half-odd integer spin the fit is improved. For $j = m = n - 1/2$ the predictions are 7.1 and 2.9 degrees for Mercury and Venus respectively. For Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto the inclinations are 1.9, 1.3, 2.5, 0.8, 1.8, 17.1 degrees. For Mars and outer planets the tilt angles are predicted to have wrong sign for $m = j$. In a good approximation the inclinations vanish for outer planets except Pluto and this would allow to determine m as $m \simeq \sqrt{5n(n+1)/6}$: the fit is not good.

The assumption that matter has condensed from a matter rotating in (x,y)-plane orthogonal to the quantization axis suggests that the directions of the planetary rotation axes are more or less the same and by angular momentum conservation have not changed appreciably. The prediction for the tilt of the rotation axis of the Earth is 24 degrees of freedom in the limit that the Earth's spin can be treated completely classically, that is for $m = j >> 1$ in the units used for the quantization of the Earth's angular momentum. What is the value of $\hbar_{gr}$ for Earth is not obvious (using the unit $\hbar_{gr} = GM^2/v_0$ the Earth's angular momentum would be much smaller than one). The tilt of the rotation axis of Earth with respect to the orbit plane is 23.5 degrees so that the agreement is again satisfactory. This prediction is essentially quantal: in purely classical theory the most natural guess for the tilt angle for planetary spins is 0 degrees.

The observation that the inner planets Mercury, Venus, and Earth have in a reasonable approximation the predicted inclinations suggest that they originate from a primordial period during which they formed spherical cells of dark matter and had thus full rotational degrees of freedom and were in eigen states of angular momentum corresponding to a full rotational symmetry. The subsequent $SO(3) \rightarrow SO(2)$ symmetry breaking leading to the formation of torus like configurations did not destroy the information about this period since the information about the value of j and m was coded by the inclination of the planetary orbit.

In contrast to this, the dark matter associated with Earth and outer planets up to Neptune formed a flattened magnetic or Z^0 magnetic flux tube resembling a disk with a hole and the subsequent symmetry breaking broke it to separate flux tubes. Earth's spherical disk was joined to the disk formed by the outer planets. The spherical disk could be still present and contain super-conducting dark matter. The presence of this "heavenly sphere" might closely relate to the fact that Earth is a living planet. The time scale $T = 2\pi R/c$ is very nearly equal to 5 minutes and defines a candidate for a bio-rhythm.

If this flux tube carried the same magnetic flux as the flux tubes associated with the inner planets, the decomposition of the disk with a hole to 5 flux tubes corresponding to Earth and to the outer planets Mars, Jupiter, Saturn and Neptune, would explain the value of v_0 correctly and also the small inclinations of outer planets. That Pluto would not originate from this structure, is consistent with its anomalously large values of inclination $i = 17.1$ degrees, small value of eccentricity $e = .248$, and anomalously large value of inclination of equator to orbit about 122 degrees as compared to 23.5 degrees in the case of Earth [E9] .

14.5.5 Eccentricities and comets

Bohr-Sommerfeld quantization allows also to deduce the eccentricities of the planetary and comet orbits. One can write the quantization of energy as

$$\begin{aligned} \frac{p_r^2}{2m_1} + \frac{p_\theta^2}{2m_1r^2} + \frac{p_\phi^2}{2m_1r^2sin^2(\theta)} - \frac{k}{r} &= -\frac{E_1}{n^2} \ , \\ E_1 = \frac{k^2}{2\hbar_{gr}^2} \times m_1 &= \frac{v_0^2}{2} \times m_1 \ . \end{aligned} \tag{14.5.11}$$

Here one has $k = GMm_1$. E_1 is the binding energy of $n = 1$ state. In the orbital plane $(\theta = \pi/2, p_\theta = 0)$ the conditions are simplified. Bohr quantization gives $p_\phi = m\hbar_{gr}$ implying

$$\frac{p_r^2}{2m_1} + \frac{k^2\hbar_{gr}^2}{2m_1r^2} - \frac{k}{r} = -\frac{E_1}{n^2} \ . \tag{14.5.12}$$

For $p_r = 0$ the formula gives maximum and minimum radii $r_\pm$ and eccentricity is given by

$$e^2 = \frac{r_+ - r_-}{r_+} = \frac{2\sqrt{1-\frac{m^2}{n^2}}}{1+\sqrt{1-\frac{m^2}{n^2}}} \ . \tag{14.5.13}$$

For small values of n the eccentricities are very large except for $m = n$. For instance, for $(m = n - 1, n)$ for $n = 3, 4, 5$ gives $e = (.93, .89, .86)$ to be compared with the experimental values (.206, .007, .0167). Thus the planetary eccentricities with Pluto included ($e = .248$) must vanish in the lowest order approximation and must result as a perturbation of the magnetic flux tube.

The large eccentricities of comet orbits might however have an interpretation in terms of $m < n$ states. The prediction is that comets with small eccentricities have very large orbital radius. Oort's cloud is a system weakly bound to a solar system extending up to 3 light years. This gives the upper bound $n \leq 700$ if the comets of the cloud belong to the same family as Mercury, otherwise the bound is smaller. This gives a lower bound to the eccentricity of not nearly circular orbits in the Oort cloud as $e > .32$.

14.5.6 Why the quantum coherent dark matter is not visible?

The obvious objection against quantal astrophysics is that astrophysical systems look extremely classical. Quantal dark matter in many-sheeted space-time resolves this counter argument. As already explained, the sequence of symmetry breakings of the rotational symmetry would explain nicely why astral Bohr rules work. The prediction is however that de-localized quantal dark matter is probably still present at (the boundaries of) magnetic flux tubes and spherical shells. It is however the entire structure defined by the orbit which behaves like a single extended particle so that the localization in quantum measurement does not mean a localization to a point of the orbit. Planet itself corresponds to a smaller localized space-time sheet condensed at the flux tube.

One should however understand why this dark matter with a gigantic Planck constant is not visible. The simplest explanation is that there cannot be any direct quantum interactions between ordinary and dark matter in the sense that particles with different values of Planck constant could appear in the same particle vertex. This would allow also a fractal hierarchy copies of standard model physics to exist with different p-adic mass scales.

There is also second argument. The inability to observe dark matter could mean inability to perform state function reduction localizing the dark matter. The probability for this should be proportional to the strength of the measurement interaction. For photons the strength of the interaction is characterized by the fine structure constant. In the case of dark matter the fine structure constant is replaced with

$$\alpha_{em,gr} = \alpha_{em} \times \frac{\hbar}{\hbar_{gr}} = \alpha_{em} \times \frac{v_0}{GMm} . \tag{14.5.14}$$

For $M = m = m_{Pl} \simeq 10^{-8}$ kg the value of the fine structure constant is smaller than $\alpha_{em} v_0$ and completely negligible for astrophysical masses. However, for processes for which the lowest order classical rates are non-vanishing, rates are not affected in the lowest order since the increase of the Compton length compensates the reduction of α. Higher order corrections become however small. What makes dark matter invisible is not the smallness of α_{em} but the fact that the binding energies of say hydrogen atom proportional to $\alpha^2 m_e$ are scaled as $1/\hbar^2$ so that the spectrum is scaled down.

14.5.7 Quantum interpretation of gravitational Schrödinger equation

Schrödinger equation in astrophysical length scales with a gigantic value of Planck constant looks sheer madness idea from the standard physics point of view. In TGD Universe situation might be different.

(a) In TGD inertial four-momentum (or conserved four-momentum) is not positive definite and the net four-momentum of the Universe vanishes. Already in cosmological length scales the density of inertial mass vanishes. Gravitational masses and inertial masses can be identified only at the limit when one can neglect the interaction between positive and negative energy matter. The masses appearing in the gravitational Schrödinger equation are gravitational masses and one can ask whether inertial and gravitational Planck constants are different.

(b) The fractality of the many-sheeted space-time predicts that quantum effects appear in all length and time scales. In particular, dark matter is at larger space-time sheets and hence almost invisible.

(c) An even more weirder looks the idea that Planck constant could have a gigantic value in astrophysical length scales being of order of magnitude of product of masses using Planck mass as a unit for $\hbar = c = 1$. This would mean that gravitation at space-time sheets of astrophysical size would have super quantal character! But even the gigantic value of Planck constant might be understood in TGD framework.

Jones inclusions and quantization of Planck constants

Quantum TGD emerges from infinite-dimensional Clifford algebra defined as infinite power of 8-dimensional Clifford algebra $C(8)$ generalized to a local algebra by constructing power series of quantum octonionic variable having the elements of this Clifford algebra as coefficients. The eigenstates for the commuting hermitian coordinates assignable to this octonionic variable have M^8 as spectrum and extremely general arguments imply both classical and quantum TGD. The construction works only for $D = 8$ (by non-associativity of the octonionic units) since for other dimensions the local field defined by algebra could not be distinguished from algebra itself.

Perhaps the most important outcome is a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products and new view theory of quantum measurement. Further outcomes are prediction the spectra of the quantized values of M^4 and CP_2 Planck constants as characterizers of Jones inclusions associated with quantum phases $q = exp(i\pi/n)$.

1. Some background

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

2. How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

(a) Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

(b) Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. One would get tensor product of two algebra.

(c) There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! One can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now one cannot anymore absorb the tensor factor to the Clifford algebra and one gets a genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

(d) Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of WCW of 3-surfaces, etc....

3. Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

(a) A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. One can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

(b) At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

(c) For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

(d) The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

4. The quantization of Planck constant and ADE hierarchies

The quantization of Planck constant has been the basic them of TGD for more than one and half years and leads also the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

(a) The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: no landscape.

(b) In ordinary phase Planck constants $\hbar(M^4)$ and $\hbar(CP_2)$ are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL_e(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups. Mass spectrum is invariant under these scalings. The values of Planck constants are $\hbar(M^4) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$ and scaling factor of M^4 covariant metric is n_b and that of CP_2 metric n_a. In Kähler action only the ratio n_a/n_b occurs and the Planck constant $\hbar_{eff}$ occurring in Schrödinger equation is by quantum classical correspondence $\hbar_{eff}/\hbar_0 = n_a/n_b$.

(c) This predicts automatically arbitrarily large and also small values of Planck constant depending in the value of the ratio n_a/n_b and assigns the preferred values of Planck constant to quantum phases $q = exp(i\pi/n_i)$, $i = a, b$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter correspond to these special values of Planck constant. This model reproduces also the other aspects of the general vision. The subgroups of $SL_e(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL_e(2,C) \times SU(2)$.

(d) These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_2n, E_6, E_8 are possible so that TGD seems to be able to mimic these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

Bohr quantization of planetary orbits and prediction for Planck constant

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. Generalization of the p-adic length scale hypothesis

The evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only square roots of rationals are number theoretically very special since they correspond to algebraic extensions of p-adic numbers involving only square roots which should emerge first and therefore systems involving these values of q should be especially abundant in Nature.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K58] .

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constants assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

2. Do the values of gravitational Planck constant correspond to polygons obtained by ruler and compass construction?

Since the macroscopic quantum phases with minimum dimension of algebraic extension should be especially abundant in the universe, the natural guess is that the values of the gravitational Planck constant correspond to n_F-multiples of ordinary Planck constant.

(a) The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

(b) Nottale [E27] has suggested that also the harmonics and subharmonics of λ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}]*X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet)n_{F,b}(Earth)/n_{F,a}(Earth)n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^4\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 1. The table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k \prod_k F_{n_k}$, where $F_i = 2^{2^i} + 1$, $i = 0, 1, 2, 3, 4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254} \times 5 \times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K24] . For $v_0 = 4.6 \times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1 + \epsilon)$ of the total mass for Earth gives

$$1 + \epsilon = \frac{17}{16} \quad (14.5.15)$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1 - \epsilon) \simeq 4.6 \times 10^{-4}$ equals with $v_0(eff) = 1/(2^7 \times F_2) = 4.5956 \times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

3. Can one really identify gravitational and inertial Planck constants?

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

(a) $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This suggests that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

(b) $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of M^4 seems to resolve these problems.

(a) The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

(b) One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of M^4, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantization as a means of avoiding gravitational collapse

Schrödinger equation provided a solution to the infrared catastrophe of the classical model of atom: the classical prediction was that electron would radiate its energy as brehmstrahlung and would be captured by the nucleus. The gravitational variant of this process would be the capture of the planet by a black hole, and more generally, a collapse of the star to a black hole. Gravitational Schrödinger equation could obviously prevent the catastrophe.

For $1/r$ gravitation potential the Bohr radius is given by $a_{gr} = GM/v_0^2 = r_S/2v_0^2$, where $r_S = 2GM$ is the Schwartchild radius of the mass creating the gravitational potential: obviously Bohr radius is much larger than the Schwartschild radius. That the gravitational Bohr radius does not depend on m conforms with Equivalence Principle, and the proportionality $\hbar_{gr} \propto Mm$ can be deduced from it. Gravitational Bohr radius is by a factor $1/2v_0^2$ larger than black hole radius so that black hole can swallow the piece of matter with a considerable rate only if it is in the ground state and also in this state the rate is proportional to the black hole volume to the volume defined by the black hole radius given by $2^3 v_0^6 \sim 10^{-20}$.

The $\hbar_{gr} \to \infty$ limit for $1/r$ gravitational potential means that the exponential factor $exp(-r/a_0)$ of the wave function becomes constant: on the other hand, also Schwartshild and Bohr radii become infinite at this limit. The gravitational Compton length associated with mass m does not depend on m and is given by GM/v_0 and the time $T = E_{gr}/\hbar_{gr}$ defined by the gravitational binding energy is twice the time taken to travel a distance defined by the radius of the orbit with velocity v_0 which suggests that signals travelling with a maximal velocity v_0 are involved with the quantum dynamics.

In the case of planetary system the proportionality $\hbar_{gr} \propto mM$ creates problems of principle since the influence of the other planets is not taken account. One might argue that the generalization of the formula should be such that M is determined by the gravitational field experienced by mass m and thus contains also the effect of other planets. The problem is that this field depends on the position of m which would mean that $\hbar_{gr}$ itself would become kind of field quantity.

Does the transition to non-perturbative phase correspond to a change in the value of $\hbar$?

Nature is populated by systems for which perturbative quantum theory does not work. Examples are atoms with $Z_1Z_2e^2/4\pi\hbar > 1$ for which the binding energy becomes larger than rest mass, non-perturbative QCD resulting for $Q_{s,1}Q_{s,2}g_s^2/4\pi\hbar > 1$, and gravitational systems satisfying $GM_1M_2/4\pi\hbar > 1$. Quite generally, the condition guaranteeing troubles is of the form $Q_1Q_2g^2/4\pi\hbar > 1$. There is no general mathematical approach for solving the quantum physics of these systems but it is believed that a phase transition to a new phase of some kind occurs.

The gravitational Schrödinger equation forces to ask whether Nature herself takes care of the problem so that this phase transition would involve a change of the value of the Planck constant to guarantee that the perturbative approach works. The values of $\hbar$ would vary in a stepwise manner from $\hbar(\infty)$ to $\hbar(3) = \hbar(\infty)/4$. The non-perturbative phase transition would correspond to transition to the value of

$$\frac{\hbar}{\hbar_0} \rightarrow \left[\frac{Q_1Q_2g^2}{v}\right] \tag{14.5.16}$$

where $[x]$ is the integer nearest to x, inducing

$$\frac{Q_1Q_2g^2}{4\pi\hbar} \rightarrow \frac{v}{4\pi} \ . \tag{14.5.17}$$

The simplest (and of course ad hoc) assumption making sense in TGD Universe is that v is a harmonic or subharmonic of v_0 appearing in the gravitational Schrödinger equation. For instance, for the Kepler problem the spectrum of binding energies would be universal (independent of the values of charges) and given by $E_n = v^2m/2n^2$ with v playing the role of small coupling. Bohr radius would be g^2Q_2/v^2 for $Q_2 \gg Q_1$.

This provides a new insight to the problems encountered in quantizing gravity. QED started from the model of atom solving the infrared catastrophe. In quantum gravity theories one has started directly from the quantum field theory level and the recent decline of the M-theory shows that we are still practically where we started. If the gravitational Schrödinger equation indeed allows quantum interpretation, one could be more modest and start from the solution of the gravitational IR catastrophe by assuming a dynamical spectrum of $\hbar$ comes as integer multiples of ordinary Planck constant. The implications would be profound: the whole program of quantum gravity would have been misled as far as the quantization of systems with $GM_1M_2/\hbar > 1$ is considered. In practice, these systems are the most interesting ones and the prejudice that their quantization is a mere academic exercise would have been completely wrong.

An alternative formulation for the occurrence of a transition increasing the value of $\hbar$ could rely on the requirement that classical bound states have reasonable quantum counterparts. In the gravitational case one would have $r_n = n^2\hbar_{gr}^2/GM_1^2M$, for $M_1 \ll M$, which is extremely small distance for $\hbar_{gr} = \hbar$ and reasonable values of n. Hence, either n is so large that the system is classical or $\hbar_{gr}/\hbar$ is very large. Equivalence Principle requires the independence of r_n on M_1, which gives $\hbar = kGM_1M_2$ giving $r_n = n^2kGM$. The requirement that the radius is above Schwartshild radius gives $k \geq 2$. In the case of Dirac equation the solutions cease to exist for $Z \geq 137$ and which suggests that $\hbar$ is large for hypothetical atoms having $Z \geq 137$.

14.5.8 How do the magnetic flux tube structures and quantum gravitational bound states relate?

In the case of stars in galactic halo the appearance of the parameter v_0 characterizing cosmic strings as orbital rotation velocity can be understood classically. That v_0 appears also in the gravitational dynamics of planetary orbits could relate to the dark matter at magnetic flux tubes. The argument explaining the harmonics and sub-harmonics of v_0 in terms of properties of cosmic strings and magnetic flux tubes identifiable as their descendants strengthens this expectation.

The notion of magnetic body

In TGD inspired theory of consciousness the notion of magnetic body plays a key role: magnetic body is the ultimate intentional agent, experiencer, and performer of bio-control and can have astrophysical size: this does not sound so counter-intuitive if one takes seriously the idea that cognition has p-adic space-time sheets as space-time correlates and that rational points are common to real and p-adic number fields. The point is that infinitesimal in p-adic topology corresponds to infinite in real sense so that cognitive and intentional structures would have literally infinite size.

The magnetic flux tubes carrying various supra phases can be interpreted as special instance of dark energy and dark matter. This suggests a correlation between gravitational self-organization and quantum phases at the magnetic flux tubes and that the gravitational Schrödinger equation somehow relates to the ordinary Schrödinger equation satisfied by the macroscopic quantum phases at magnetic flux tubes. Interestingly, the transition to large Planck constant phase should occur when the masses of interacting is above Planck mass since gravitational self-interaction energy is $V \sim GM^2/R$. For the density of water about 10^3 kg/m^3 the volume carrying a Planck mass correspond to a cube with side 2.8×10^{-4} meters. This corresponds to a volume of a large neuron, which suggests that this phase transition might play an important role in neuronal dynamics.

Could gravitational Schrödinger equation relate to a quantum control at magnetic flux tubes?

An infinite self hierarchy is the basic prediction of TGD inspired theory of consciousness ("everything is conscious and consciousness can be only lost"). Topological quantization allows to assign to any material system a field body as the topologically quantized field pattern created by the system [K95, K30] . This field body can have an astrophysical size and would utilize the material body as a sensory receptor and motor instrument.

Magnetic flux tube and flux wall structures are natural candidates for the field bodies. Various empirical inputs have led to the hypothesis that the magnetic flux tube structures define a hierarchy of magnetic bodies, and that even Earth and larger astrophysical systems possess magnetic body which makes them conscious self-organizing living systems. In particular, life at Earth would have developed first as a self-organization of the super-conducting dark matter at magnetic flux tubes [K30] .

For instance, EEG frequencies corresponds to wavelengths of order Earth size scale and the strange findings of Libet about time delays of conscious experience [J12, J6] find an elegant explanation in terms of time taken for signals propagate from brain to the magnetic body [K95] . Cyclotron frequencies, various cavity frequencies, and the frequencies associated with various p-adic frequency scales are in a key role in the model of bio-control performed by the magnetic body. The cyclotron frequency scale is given by $f = eB/m$ and rather low as are also cavity frequencies such as Schumann frequencies: the lowest Schumann frequency is in a good approximation given by $f = 1/2\pi R$ for Earth and equals to 7.8 Hz.

1. Quantum time scales as "bio-rhythms" in solar system?

To get some idea about the possible connection of the quantum control possibly performed by the dark matter with gravitational Schrödinger equation, it is useful to look for the values of the periods defined by the gravitational binding energies of test particles in the fields of Sun and Earth and look whether they correspond to some natural time scales. For instance, the period $T = 2GM_S n^2/v_0^3$ defined by the energy of n^{th} planetary orbit depends only on the mass of Sun and defines thus an ideal candidate for a universal "bio-rhythm".

For Sun black hole radius is about 2.9 km. The period defined by the binding energy of lowest state in the gravitational field of Sun is given $T_S = 2GM_S/v_0^3$ and equals to 23.979 hours for $v_0/c = 4.8233 \times 10^{-4}$. Within experimental limits for v_0/c the prediction is consistent with 24 hours! The value of v_0 corresponding to exactly 24 hours would be $v_0 = 144.6578$ km/s (as a matter fact, the rotational period of Earth is 23.9345 hours). As if as the frequency defined by the lowest energy state would define a "biological" clock at Earth! Mars is now a strong candidate for a seat of life and the day in Mars lasts 24hr 37m 23s! $n = 1$ and $n = 2$ are orbitals are not realized in solar system as planets but there is evidence for the $n = 1$ orbital as being realized as a peak in the density of IR-dust [E27] . One can of course consider the possibility that these levels are populated by small dark matter planets with matter at larger space-time sheets. Bet as it may, the result supports the notion of quantum gravitational entrainment in the solar system.

The slower rhythms would become as n^2 sub-harmonics of this time scale. Earth itself corresponds to $n = 5$ state and to a rhythm of .96 hours: perhaps the choice of 1 hour to serve as a fundamental time unit is not merely accidental. The magnetic field with a typical ionic cyclotron frequency around 24 hours would be very weak: for 10 Hz cyclotron frequency in Earth's magnetic field the field strength would about 10^{-11} T. However, $T = 24$ hours corresponds with 6 per cent accuracy to the p-adic time scale $T(k = 280) = 2^{13}T(2, 127)$, where $T(2, 127)$ corresponds to the secondary p-adic time scale of .1 s associated with the Mersenne prime $M_{127} = 2^{127} - 1$ characterizing electron and defining a fundamental bio-rhythm and the duration of memetic codon [K37] .

Comorosan effect [K100] , [I4, I14] demonstrates rather peculiar looking facts about the interaction of organic molecules with visible laser light at wavelength $\lambda = 546\ nm$. As a result of irradiation molecules seem to undergo a transition $S \rightarrow S^*$. S^* state has anomalously long lifetime and stability in solution. $S \rightarrow S^*$ transition has been detected through the interaction of S^* molecules with different biological macromolecules, like enzymes and cellular receptors. Later Comorosan found that the effect occurs also in non-living matter. The basic time scale is $\tau = 5$ seconds. p-Adic length scale hypothesis does not explain τ, and it does not correspond to any obvious astrophysical time scale and has remained a mystery.

The idea about astro-quantal dark matter as a fundamental bio-controller inspires the guess that τ could correspond to some Bohr radius R for a solar system via the correspondence $\tau = R/c$. As observed by Nottale, $n = 1$ orbit for $v_0 \rightarrow 3v_0$ corresponds in a good approximation to the solar radius and to $\tau = 2.18$ seconds. For $v_0 \rightarrow 2v_0$ $n = 1$ orbit corresponds to $\tau = AU/(4 \times 25) = 4.992$ seconds: here $R = AU$ is the astronomical unit equal to the average distance of Earth from Sun. The deviation from τ_C is only one per cent and of the same order of magnitude as the variation of the radius for the orbit due to orbital eccentricity $(a - b)/a = .0167$ [E9] .

2. Earth-Moon system

For Earth serving as the central mass the Bohr radius is about 18.7 km, much smaller than Earth radius so that Moon would correspond to $n = 147.47$ for v_0 and $n = 1.02$ for the sub-harmonic $v_0/12$ of v_0. For an afficionado of cosmic jokes or a numerologist the presence of the number of months in this formula might be of some interest. Those knowing that the Mayan calendar had 11 months and that Moon is receding from Earth might rush to check whether a transition from $v/11$ to $v/12$ state has occurred after the Mayan culture ceased to exist: the increase of the orbital radius by about 3 per cent would be required! Returning to a more serious mode, an interesting question is whether light satellites of Earth consisting of dark matter at larger space-time sheets could be present. For instance, in [K30] I have discussed the possibility that the larger space-time sheets of Earth could carry some kind of intelligent life crucial for the bio-control in the Earth's length scale.

The period corresponding to the lowest energy state is from the ratio of the masses of Earth and Sun given by $M_E/M_S = (5.974/1.989) \times 10^{-6}$ given by $T_E = (M_E/M_S) \times T_S = .2595$ s. The corresponding frequency $f_E = 3.8535$ Hz frequency is at the lower end of the theta band in EEG and is by 10 per cent higher than the p-adic frequency $f(251) = 3.5355$ Hz associated with the p-adic prime $p \simeq 2^k$, $k = 251$. The corresponding wavelength is 2.02

times Earth's circumference. Note that the cyclotron frequencies of Nn, Fe, Co, Ni, and Cu are $5.5, 5.0, 5.2, 4.8$ Hz in the magnetic field of $.5 \times 10^{-4}$ Tesla, which is the nominal value of the Earth's magnetic field. In [K71] I have proposed that the cyclotron frequencies of Fe and Co could define biological rhythms important for brain functioning. For $v_0/12$ associated with Moon orbit the period would be 7.47 s: I do not know whether this corresponds to some bio-rhythm.

It is better to leave for the reader to decide whether these findings support the idea that the super conducting cold dark matter at the magnetic flux tubes could perform bio-control and whether the gravitational quantum states and ordinary quantum states associated with the magnetic flux tubes couple to each other and are synchronized.

14.5.9 p-Adic length scale hypothesis and $v_0 \rightarrow v_0/5$ transition at inner-outer border for planetary system

$v_0 \rightarrow v_0/5$ transition would allow to interpret the orbits of outer planets as $n \geq 1$ orbits. The obvious question is whether inner to outer zone as $v_0 \rightarrow v_0/5$ transition could be interpreted in terms of the p-adic length scale hierarchy.

(a) The most important p-adic length scale are given by primary p-adic length scales $L_e(k) = 2^{(k-151)/2} \times 10$ nm and secondary p-adic length scales $L_e(2,k) = 2^{k-151} \times 10$ nm, k prime.

(b) The p-adic scale $L_e(2,139) = 114$ Mkm is slightly above the orbital radius 109.4 Mkm of Venus. The p-adic length scale $L_e(2,137) \simeq 28.5$ Mkm is roughly one half of Mercury's orbital radius 57.9 Mkm. Thus strong form of p-adic length scale hypothesis could explain why the transition $v_0 \rightarrow v_0/5$ occurs in the region between Venus and Earth ($n = 5$ orbit for v_0 layer and $n = 1$ orbit for $v_0/5$ layer).

(c) Interestingly, the *primary* p-adic length scales $L_e(137)$ and $L_e(139)$ correspond to fundamental atomic length scales which suggests that solar system be seen as a fractally scaled up "secondary" version of atomic system.

(d) Planetary radii have been fitted also using Titius-Bode law predicting $r(n) = r_0 + r_1 \times 2^n$. Hence on can ask whether planets are in one-one correspondence with primary and secondary p-adic length scales $L_e(k)$. For the orbital radii $58, 110, 150, 228$ Mkm of Mercury, Venus, Earth, and Mars indeed correspond approximately to k= $276, 278, 279, 281$: note the special position of Earth with respect to its predecessor. For Jupiter, Saturn, Uranus, Neptune, and Pluto the radii are 52,95,191,301,395 Mkm and would correspond to p-adic length scales $L_e(280 + 2n))$, $n = 0, ..., 3$. Obviously the transition $v_0 \rightarrow v_0/5$ could occur in order to make the planet–p-adic length scale one-one correspondence possible.

(e) It is interesting to look whether the p-adic length scale hierarchy applies also to the solar structure. In a good approximation solar radius .696 Mkm corresponds to $L_e(270)$, the lower radius .496 Mkm of the convective zone corresponds to $L_e(269)$, and the lower radius .174 Mkm of the radiative zone (radius of the solar core) corresponds to $L_e(266)$. This encourages the hypothesis that solar core has an onion like sub-structure corresponding to various p-adic length scales. In particular, $L_e(2,127)$ ($L_e(127)$ corresponds to electron) would correspond to 28 Mm. The core is believed to contain a structure with radius of about 10 km: this would correspond to $L_e(231)$. This picture would suggest universality of star structure in the sense that stars would differ basically by the number of the onion like shells having standard sizes.

Quite generally, in TGD Universe the formation of join along boundaries bonds is the space-time correlate for the formation of bound states. This encourages to think that (Z^0) magnetic flux tubes are involved with the formation of gravitational bound states and that for $v_0 \rightarrow v_0/k$ corresponds either to a splitting of a flux tube resembling a disk with a whole to k pieces, or to the scaling down $B \rightarrow B/k^2$ so that the magnetic energy for the flux tube thickened and stretched by the same factor k^2 would not change.

14.5.10 About the interpretation of the parameter v_0

The formula for the gravitational Planck constant contains the parameter $v_0/c = 2^{-11}$. This velocity defines the rotation velocities of distant stars around galaxies. The presence of a parameter with dimensions of velocity should carry some important information about the geometry of dark matter space-time sheets.

Velocity like parameters appear also in other contexts. There is evidence for the Tifft's quantization of cosmic redshifts in multiples of $v_0/c = 2.68 \times 10^{-5}/3$: also other units of quantization have been proposed but they are multiples of v_0 [E32] .

The strange behavior of graphene includes high conductivity with conduction electrons behaving like massless particles with light velocity replaced with $v_0/c = 1/300$. The TGD inspired model [K13] explains the high conductivity as being due to the Planck constant $\hbar(M^4) = 6\hbar_0$ increasing the de-localization length scale of electron pairs associated with hexagonal rings of mono-atomic graphene layer by a factor 6 and thus making possible overlap of electron orbitals. This explains also the anomalous conductivity of DNA containing 5- and 6-cycles [K13] .

Is dark matter warped?

The reduced light velocity could be due to the warping of the space-time sheet associated with dark electrons. TGD predicts besides gravitational red-shift a non-gravitational red-shift due to the warping of space-time sheets possible because space-time is 4-surface rather than abstract 4-manifold. A simple example of everyday life is the warping of a paper sheet: it bends but is not stretched, which means that the induced metric remains flat although one of its component scales (distance becomes longer along direction of bending). For instance, empty Minkowski space represented canonically as a surface of $M^4 \times CP_2$ with constant CP_2 coordinates can become periodically warped in time direction because of the bending in CP_2 direction. As a consequence, the distance in time direction shortens and effective light-velocity decreases when determined from the comparison of the time taken for signal to propagate from A to B along warped space-time sheet with propagation time along a non-warped space-time sheet.

The simplest warped imbedding defined by the map $M^4 \rightarrow S^1$, S^1 a geodesic circle of CP_2. Let the angle coordinate of S^1 depend linearly on time: $\Phi = \omega t$. g_{tt} component of metric becomes $1-R^2\omega^2$ so that the light velocity is reduced to $v_0/c = \sqrt{1-R^2\omega^2}$. No gravitational field is present.

The fact that M^4 Planck constant $n_a\hbar_0$ defines the scaling factor n_a^2 of CP_2 metric could explain why dark matter resides around strongly warped imbeddings of M^4. The quantization of the scaling factor of CP_2 by $R^2 \rightarrow n_a^2 R^2$ implies that the initial small warping in the time direction given by $g_{tt} = 1-\epsilon$, $\epsilon = R^2\omega^2$, will be amplified to $g_{tt} = 1 - n_a^2\epsilon$ if ω is not affected in the transition to dark matter phase. $n_a = 6$ in the case of graphene would give $1 - x \simeq 1 - 1/36$ so that only a one per cent reduction of light velocity is enough to explain the strong reduction of light velocity for dark matter.

Is c/v_0 quantized in terms of ruler and compass rationals?

The known cases suggests that c/v_0 is always a rational number expressible as a ratio of integers associated with n-polygons constructible using only ruler and compass.

(a) $c/v_0 = 300$ would explain graphene. The nearest rational satisfying the ruler and compass constraint would be $q = 5 \times 2^{10}/17 \simeq 301.18$.

(b) If dark matter space-time sheets are warped with $c_0/v = 2^{11}$ one can understand Nottale's quantization for the radii of the inner planets. For dark matter space-time sheets associated with outer planets one would have $c/v_0 = 5 \times 2^{11}$.

(c) If Tifft's red-shifts relate to the warping of dark matter space-time sheets, warping would correspond to $v_0/c = 2.68 \times 10^{-5}/3$. $c/v_0 = 2^5 \times 17 \times 257/5$ holds true with an error smaller than .1 per cent.

Tifft's quantization and cosmic quantum coherence

An explanation for Tifft's quantization in terms of Jones inclusions could be that the subgroup G of Lorentz group defining the inclusion consists of boosts defined by multiples $\eta = n\eta_0$ of the hyperbolic angle $\eta_0 \simeq v_0/c$. This would give $v/c = sinh(n\eta_0) \simeq nv_0/c$. Thus the dark matter systems around which visible matter is condensed would be exact copies of each other in cosmic length scales since G would be an exact symmetry. The property of being an exact copy applies of course only in single level in the dark matter hierarchy. This would mean a de-localization of elementary particles in cosmological length scales made possible by the huge values of Planck constant. A precise cosmic analog for the de-localization of electron pairs in benzene ring would be in question.

Why then η_0 should be quantized as ruler and compass rationals? In the case of Planck constants the quantum phases $q = exp(im\pi/n_F)$ are number theoretically simple for n_F a ruler and compass integer. If the boost $exp(\eta)$ is represented as a unitary phase $exp(im\eta)$ at the level of discretely de-localized dark matter wave functions, the quantization $\eta_0 = n/n_F$ would give rise to number theoretically simple phases. Note that this quantization is more general than $\eta_0 = n_{F,1}/n_{F,2}$.

Chapter 15

Overall View About TGD from Particle Physics Perspective

15.1 Introduction

Topological Geometrodynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

During these 32 years TGD has become quite an extensive theory involving also applications to quantum biology and quantum consciousness theory. Therefore it is difficult to decide in which order to proceed. Should one represent first the purely mathematical theory as done in the articles in Prespace-time Journal [L11, L12, L17, L18, L15, L10, L16, L19]? Or should one start from the TGD inspired heuristic view about space-time and particle physics and represent the vision about construction of quantum TGD briefly after that? In this and other two chapters I have chosen the latter approach since the emphasis is on the applications on particle physics.

Second problem is to decide about how much material one should cover. If the representation is too brief no-one understands and if it is too detailed no-one bothers to read. I do not know whether the outcome was a success or whether there is any way to success but in any case I have been sweating a lot in trying to decide what would be the optimum dose of details.

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the imbedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces associated with light-like boundaries of so called causal diamonds defined as intersections of future and past directed light-cones (CDs) and with light-like 3-surfaces. Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become

part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The understanding of the relationship between TGD and GRT and quantum and classical variants of Equivalence Principle (EP) in TGD have develope rather slowly but the recent picture is rather feasible.

  (a) The recent view is that EP at quantum level reduces to Quantum Classical Correpondence (QCC) in the sense that Cartan algebra Noether charges assignable to 3-surface in case of Kähler action (inertial charges) are identical with eigenvalues of the quantal variants of Noether charges for Kähler-Dirac action (gravitational charges). The well-definedness of the latter charges is due to the conformal invariance assignable to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) at which the spinor modes are localized in generic case. This localization follows from the condition that em charge has well defined value for the spinor modes. The localization is possibly only for the Kähler-Dirac action and key role is played by the modification of gamma matrices to Kähler-Dirac gamma matrices. The gravitational four-momentum is thus completely analogous to stringy four-momentum.

  (b) At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. Similar procedure applies to induced gauge fields.

- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electroweak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.

- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves three Dirac operators. The Kähler Dirac equation holds true in the interior of space-time surface and its solutions have a natural interpretation in terms of description of matter, in particular condensed matter. Chern-Simons Dirac action is associated with the light-like 3-surfaces and space-like 3-surfaces at ends of space-time surface at light-like boundaries of CD. One can assign to it a generalized eigenvalue equation and the matrix valued eigenvalues correspond to the the action of Dirac operator on momentum eigenstates. Momenta are however not usual momenta but pseudo-momenta very much analogous to region momenta of twistor approach. The third Dirac operator is associated with super Virasoro generators and super Virasoro conditions define Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S-matrix to a collection of what I call M-matrices defining the rows of unitary U-matrix defining unitary process.

- Twistor approach has inspired several ideas in quantum TGD during the last years and it seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness.

The discussion of this chapter is rather sketchy and the reader interesting in details can consult the books about TGD [K96, K73, K60, K54, K74, K85, K90] .

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- Overall view about TGD [L55]
- What TGD is [L84]
- TGD as unified theory of fundamental interactions [L75]
- Key notions and ideas of TGD [L47]
- Basic TGD [L27]
- Space-time as 4-surface in $M^4 \times CP_2$ [L67]
- Classical TGD [L28]
- Manysheeted space-time [L52]
- Geometrization of fields [L37]
- TGD and GRT [L72]
- Zero Energy Ontology (ZEO) [L85]
- Vacuum functional in TGD [L82]
- Quantum Classical Correspondence [L59]
- Quantum criticality [L61]
- Symmetries of WCW [L70]
- TGD as ATQFT [L73]
- KD equation [L46]
- Kaehler-Dirac action [L45]
- The unique role of twistors in TGD [L79]
- Twistors and TGD [L81]

15.2 Some aspects of quantum TGD

In the following I summarize very briefly those basic notions of TGD which are especially relevant for the applications to particle physics. The representation will be practically formula free. The article series published in Prespace-time Journal [L11, L12, L17, L18, L15, L10, L16, L19] describes the mathematical theory behind TGD. The seven books about TGD [K96, K73, K60, K111, K85, K110, K109, K82] provide a detailed summary about the recent state of TGD.

15.2.1 New space-time concept

The physical motivation for TGD was what I have christened the energy problem of General Relativity. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The way out is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [L19] , is the unique choice.

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the space-time in this framework is much more richer than in general relativity.

(a) Space-time decomposes into space-time sheets (see fig. 2.3.2 in the appendix of this book) with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

(b) Elementary particles are identified as topological inhomogenuities glued to these space-time sheets (see figs. `http://www.tgdtheory.fi/appfigures/particletgd.jpg`, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg`, which are also in the appendix of this book). In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in general relativity.

(c) Also the view about classical fields changes. One can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K61] . One can speak about field body or magnetic body of the system.

Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C3] and the color magnetic body of u quark whose size is somethat larger than the Bohr radius could explain the anomaly [K52] .

15.2.2 Zero energy ontology

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events. By quantum classical correspondences zero energy states must have space-time and imbedding space correlates.

(a) Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone (see ig. 2.3.5 in the appendix of this book)). The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.

(b) p-Adic length scale hypothesis [K55] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves $T = 2^n T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K7] . This means a direct coupling between microscopic and macroscopic scales.

Zero energy ontology conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

For thermodynamical states this is indeed the case and this leads to the idea that quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K51] as the fundamental variational principle. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump. From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal.

15.2.3 The hierarchy of Planck constants

The motivations for the hierarchy of Planck constants come from both astrophysics and biology [K70, K24] . In astrophysics the observation of Nottale [E27] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K79, K62] .

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the imbedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite (see fig. **??** in the appendix of this book).

A convenient technical manner to treat the situation is to replace imbedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $\hbar = n\hbar_0$. This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/\hbar$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [D2] is almost obvious. At the more detailed level one finds that the spectrum of Planck constants would be given by $\hbar = n_a n_b \hbar_0$ [K27] .

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoome up version of quantum world at elementary particle length scales.

(a) Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.

(b) Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J3] . Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.

(c) The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.

(d) The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and $\hbar$ changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [K30] (see fig. **??** in the appendix of this book). The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K26] .

(e) p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like $\hbar$. This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C11] . The classical Z^0 and possibly also W fields responsible for parity breaking effects must be experienced by fundamental fermions in cellular length scale. This is not possible for ordinary value of Planck constant above weak scale since the induced spinor modes are restricted on string world sheets at which W and Z^0 fields vanish: this follows from the well-definedness of em charge. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I5] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K70] .

15.2.4 p-Adic physics and number theoretic universality

p-Adic physics [K54, K88] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition and intentionality. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

p-Adic number fields

p-Adic number fields Q_p [A50] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x,y) = d(x-y)$ depends on the lowest pinary digit of $x-y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x-y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultra-metricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x-y) \leq Min\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics -not for energy but infinitesimal scaling generator of so called super-conformal algebra [A30] acting as symmetries of quantum TGD [K73] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K54] . The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1+i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5 μm contains as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ assignable to Gaussian Mersennes $M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$, [K70] . This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition and intention [K58] . Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes. One can also consider p-adic space-time sheets as correlates of intentions. The quantum jump in which p-adic space-time sheet is replaced with a real one could serve as a quantum correlate of intentional action (see fig. 3c in the appendix of this book). This process is forbidden by conservation laws in standard ontology: one cannot even compare real and p-adic variants of the conserved quantities like energy in the general case. In zero energy ontology the net values of conserved quantities for zero energy states vanish so that conservation laws allow these transitions.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

(a) The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.

(b) The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $log(p_n)$ with $-log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Negentropic entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely (see fig. **??** in the appendix of this book).

Negentropy Maximization Principle [K51] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I3] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [K30] .

15.3 Symmetries of quantum TGD

15.4 Symmetries of TGD

Symmetry principles play key role in the construction of WCW geometry have become and deserve a separate explicit treatment even at the risk of repetitions. Symmetries of course manifest themselves also at space-time level and space-time supersymmetry - possibly present also in TGD - is the most non-trivial example of this.

15.4.1 General Coordinate Invariance

General coordinate invariance is certainly of the most important guidelines and is much more powerful in TGD framework thanin GRT context.

(a) General coordinate transformations as a gauge symmetry so that the diffeomorphic slices of space-time surface equivalent physically. 3-D light-like 3-surfaces defined by wormhole throats define preferred slices and allows to fix the gauge partially apart from the remaining 3-D variant of general coordinate invariance and possible gauge degeneracy related to the choice of the light-like 3-surface due to the Kac-Moody invariance. This would mean that the random light-likeness represents gauge degree of freedom except at the ends of the light-like 3-surfaces.

(b) GCI can be strengthened so that the pairs of space-like ends of space-like 3-surfaces at CDs are equivalent with light-like 3-surfaces connecting them. The outcome is effective 2-dimensionality because their intersections at the boundaries of CDs must carry the physically relevant information.

15.4.2 Generalized conformal symmetries

One can assign Kac-Moody type conformal symmetries to light-like 3-surfaces as isometries of H localized with respect to light-like 3-surfaces. Kac Moody algebra essentially the Lie algebra of gauge group with central extension meaning that projective representation in which representation matrices are defined only modulo a phase factor. Kac-Moody symmetry is not quite a pure gauge symmetry.

One can assign a generalization of Kac-Moody symmetries to the boundaries of CD by replacing Lie-group of Kac-Moody algebra with the group of symplectic (contact-) transformations [A60, A33, A32] of H_+ provided with a degenerate Kähler structure made possible by the effective 2-dimensionality of δM^4_+. The light-like radial coordinate of δM^4_+ plays the role of the complex coordinate of conformal transformations or their hyper-complex analogs. The basic hypothesis is that these transformations define the isometry algebra of WCW.

p-Adic mass calculations require also second super-conformal symmetry. It is defined by Kac-Moody algebra assignable to the isometries of the imbedding space or possibly those of δCD. This algebra must appear together with symplectic algebra as a direct sum. The original guess was that Kac-Moody algebra is associated with light-like 3-surfaces as a local algebra localized by hand with respect to the internal coordinates. A more elegant identification emerged in light of the wisdom gained from the solutions of the modified Dirac equation. Neutrino modes and symplectic Hamiltonians generate symplectic algebra and the remaining fermion modes and Hamiltonians of symplectic isometries generate the Kac-Moody algebra and the direct sum of these algebras acts naturally on physical states.

A further physically well-motivated hypothesis inspired by holography and extended GCI is that these symmetries extend so that they apply at the entire space-time sheet and also at the level of imbedding space.

(a) The extension to the entire space-time surface requires the slicing of space-time surface by partonic 2- surfaces and by stringy world sheets such that each point of stringy world sheet defines a partonic 2-surface and vice versa. This slicing has deep physical motivations since it realizes geometrically standard facts about gauge invariance (partonic 2-surface defines the space of physical polarizations and stringy space-time sheet corresponds to non-physical polarizations) and its existence is a hypothesis about the properties of the preferred extremals of Kähler action.

    There is a similar decomposition also at the level of CD and so called Hamilton-Jacobi coordinates for M^4_+ [K9] define this kind of slicings. This slicing can induced the slicing of the space-time sheet. The number theoretic vision gives a further justification for this hypothesis and also strengthens it by postulating the presence of the preferred time direction having interpretation in terms of real unit of octonions. In ZEO this time direction corresponds to the time-like vector connecting the tips of CD.

(b) The simplest extension of the symplectic algebra at the level of imbedding space is by parallel translating the light-cone boundary. This would imply duality of the formulations using light-like and space-like 3-surfaces and Equivalence Principle (EP) might correspond to this duality in turn implied by strong form of general coordinate invariance (GCI).

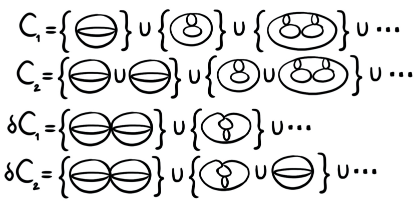

Figure 15.1: Conformal symmetry preserves angles in complex plane

Conformal symmetries would provide the realization of WCW as a union of symmetric spaces. Symmetric spaces are coset spaces of form G/H. The natural identification of G and H is as groups of symplectic transformations and its subroup leaving preferred 3-surface invariant (acting as diffeomorphisms for it). Quantum fluctuating (metrically non-trivial) degrees of freedom would correspond to symplectic transformations of H_+ and fluxes of the induced Kähler form would define a local representation for zero modes: not necessarily all of them.

15.4.3 Equivalence Principle and super-conformal symmetries

Equivalence Principle (EP) is a second corner stone of General Relativity and together with GCI leads to Einstein's equations. What EP states is that inertial and gravitational masses are identical. In this form it is not well-defined even in GRT since the definition of gravitational and inertial four-momenta is highly problematic because Noether theorem is not available. Therefore the realization is in terms of local equations identifying energy momentum tensor with Einstein tensor.

Thinking EP in terms of scattering amplitudes for graviton exchange, it seems obvious that EP is true in TGD since all particles are string like objects. How EP is realized in TGD has been a longstanding open question [K93]. The problem has been that at the classical level EP in its GRT form can hold true only in long enough length scales and it took long to time to realize that only the stringy form of this principle is required. The first question is how to identify the gravitational and inertial four-momenta. I have considered very many proposals in this regard!

The first idea was that one could associate to the two types super-conformal algebras g and h assigned with light-like 3-surfaces and space-like 3-surfaces four-momenta to both. EP would state that these four-momenta are identical and is equivalent with the generalization of GCI and effective 2-dimensionality. The condition generalizes so that it applies to the generators of super-conformal algebras associated with the two super-conformal symmetries. Ironically, this idea is rather compelling if the two super-conformal algebras correspond to symplectic symmetries acting at space-like *resp.* light-like 3-surfaces and both decompose to direct sums of representations of super-symplectic and super Kac-Moody algebras. Here I however made mis-identification and was led to a wrong track. The rediscovery of the correct (really?) interpretation took almost twenty years!

The imbeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals [K93] . Gravitational mass density does not however vanish for vacuum extremals. This forces to ask whether Equivalence Principle (EP) fails in TGD Universe. General arguments at the level of representations of super-conformal algebras however suggest that EP holds in generalised sense. Also GRT identified as limiting theory lumping many-sheeted space-time to M^4 endowed with an effective metric with Einstein's equations reflecting underlying Poincare invariance supports this intuition.

One could argue that Equivalence Principle (EP) reduces to a mere tautology in TGD framework since stringy picture implies stringy scattering amplitudes for graviton exchanges. This might be the case at quantum level. There are however problems: how the exact Poincare invariance can be consistent with the non-conservation of four-momentum in GRT based cosmologies? What EP could mean at quantum level? Does EP reduce at classical level to Einstein's equations in some sense. How to take into account the many-sheetedness of TGD space-time? The following represents the latest vision about EP in TGD.

1. ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

2. Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

(a) At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

(b) The most recent view (2014) about understanding how EP emerges in TGD is described in [K93] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

(c) The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the modified Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

3. Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K114].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

15.4.4 Extension of super-conformal symmetries

The original idea behind the extension of conformal symmetries to super-conformal symmetries was the observation that isometry currents defining infinitesimal isometries of WCW have natural super-counterparts obtained by contracting the Killing vector fields with the complexified gamma matrices of the imbedding space.

This vision has generalized considerably as the construction of WCW spinor structure in terms of modified Dirac action has developed. The basic philosophy behind this idea is that WCW spinor structure must relate directly to the fermionic sector of quantum physics. In particular, modified gamma matrices should be expressible in terms of the fermionic oscillator operators associated with the second quantized induced spinor fields. The explicit realization of this program leads to an identification of rich spectrum of super-conformal symmetries and generalization of the ordinary notion of space-time supersymmetry. What happens that all fermionic oscillator operator generate broken super-symmetries whereas in SUSYs there is only finite number of them. One can however identify sub-algebra of super-conformal symmetries associated with right handed neutrino and this gives $\mathcal{N} = 1$ super-symmetry [B11] of SUSYs [K29] .

15.4.5 Does TGD allow the counterpart of space-time super-symmetry

It has been clear from the beginning that the notion of super-conformal symmetry crucial for the successes of super-string models generalizes in TGD framework. The answer to the question whether space-time SUSY makes sense in TGD framework has not been obvious at all but it seems now that the answer is affirmative. The evolution of the ideas relevant for the formulation of SUSY in TGD framework is summarized in the chapters of [K74] . The chapters devoted to the notion of bosonic emergence [K64] , to the SUSY QFT limit of TGD [K29] , to twistor approach to TGD [K98] , and to the generalization of Yangian symmetry of $\mathcal{N} = 4$ SYM manifest in the Grassmannian twistor approach [B26] to a multi-local variant of super-conformal symmetries [K101] represent a gradual development of the ideas about how super-symmetric M-matrix could be constructed in TGD framework. A warning to the reader is in order. In their recent form these chapters do not represent the final outcome but just an evolution of ideas proceeding by trial and error. There are however good reasons to believe that the chapter about Yangian symmetry is nearest to the correct physical interpretation and mathematical formulation.

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the modified Dirac action [K17, K28, K21] . It is possible to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation $\mathcal{N} = 2N$ SUSY algebra (an inherent cutoff on the number of fermionic modes at light-like wormhole throat) or fermionic part of super-conformal algebra with infinite number of oscillator operators results. The addition of fermion in particular mode would define particular super-symmetry. This super-symmetry is badly broken due to the dynamics of the modified Dirac operator which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

Zero energy ontology combined with the analog of the twistor approach to $\mathcal{N} = 4$ SYMs and weak form of electric-magnetic duality has actually led to this kind of formulation [K101] . What is new that also virtual particles have massless fermions as their building blocks. This implies manifest finiteness of loop integrals so that the situation simplifies dramatically. What is also new element that physical particles and also string like objects correspond to bound states of massless fermions.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

(a) In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

(b) The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the super-charges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D = 10$ and $D = 11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel. It indeed turns out that TGD view about space-time SUSY is internally consistent. Even more, the separate conservation of quark and lepton number is essential for the internal consistency of this view [K29] .

(c) The massivation of particles is the basic problem of both SUSYs and twistor approach. I have discussed several solutions to this problem [K98, K101] . The simplest and most convincing solution of the problem is following and inspired by twistor Grassmannian approach to $\mathcal{N} = 4$ SYM and the generalization of the Yangian symmetry of this theory. In zero energy ontology one can construct physical particles as bound states of massless particles associated with the opposite wormhole throats. If the particles have opposite 3-momenta the resulting state is automatically massive. In fact, this forces massivation of also spin one bosons since the fermion and anti-fermion must move in opposite directions for their spins to be parallel so that the net mass is non-vanishing: note that this means that even photon, gluons, and graviton have small mass. This mechanism makes topologically condensed fermions massive and padic thermodynamics allows to describe the massivation in terms of zero energy states and M-matrix. Bosons receive to their mass besides the small mass coming from thermodynamics also a contribution which is counterpart of the contribution coming from Higgs vacuum expectation value and Higgs gives rise to longitudinal polarizations. No Higgs potential is however needed. The cancellation of infrared divergences necessary for exact Yangian symmetry and the observation that even photon receives small mass suggest that scalar Higgs would disappear completely from the spectrum.

Basic data bits

Let us first summarize the data bits about possible relevance of super-symmetry for TGD before the addition of the 3-D measurement interaction term to the modified Dirac action [K17, K28] .

(a) Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A117, A80, A99, A73] .

(b) In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, covariantly constant right-handed neutrino spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K48, K56] . The negative conformal weight of the vacuum also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.

(c) Right-handed covariantly constant neutrino allows to construct the gamma matrices of the world of classical worlds (WCW) as fermionic counterparts of Hamiltonians of WCW. This gives rise super-symplectic symmetry algebra having interpretation also as a conformal algebra. Also more general super-conformal symmetries exist.

(d) Space-time (in the sense of Minkowski space M^4) super-symmetry in the conventional sense of the word is impossible in TGD framework since it would require require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break separate conservation of lepton and baryon numbers in TGD framework.

Could one generalize super-symmetry?

Could one then consider a more general space-time super-symmetry with "space-time" identified as space-time surface rather than Minkowski space?

(a) The TGD variant of the super-symmetry could correspond quite concretely to the addition to fermion and boson states right-handed neutrinos. Since right-handed neutrinos do not have electro-weak interactions, the addition might not appreciably affect the mass formula although it could affect the p-adic prime defining the mass scale.

(b) The problem is to understand what this addition of the right-handed neutrino means. To begin with, notice that in TGD Universe fermions reside at light-like 3-surfaces at which the signature of induced metric changes. Bosons correspond to pairs of light-like wormhole throats with wormhole contact having Euclidian signature of the induced metric. The long standing problem has been that for bosons with parallel light-like four-momenta with same sign of energy the spins of fermion and anti-fermion are opposite so that one would obtain only scalar bosons!

I have considered several solutions to the problem but the final solution came from the basic problem of twistor approach to $\mathcal{N} = 4$ SUSY. This theory is believed to be UV finite but has IR divergences spoiling the Yangian SUSY. These infinities cancel if the physical particles are bound states of pairs of wormhole throats with light-like momenta. Just the requirement that spin is equal to one forces massivation. This is true for all spin 1 particles, also those regarded as massless. Massivation of the photon is not a problem if the mass corresponds to the IR cutoff determined by the largest causal diamond (CD) defining the measurement resolution. For electron the size of CD corresponds to the size scale of Earth. The basic prediction is that Higgs disappears completely from the spectrum so that this mechanism is testable at LHC.

The first proposal to the solution of problem was that either fermion or anti-fermion in the boson state carries what might be called un-physical polarization in the standard conceptual framework. This means that it has negative energy but three-momentum parallel to that of the second wormhole throat. The assumption that the bosonic wormhole throats correspond to positive and negative energy space-time sheets realizes this constraint in the framework of zero energy ontology. It however turned out that for light-like momenta these states have more natural interpretation in terms of virtual bosons able to have space-like momenta. This means that one can realize virtual particles as pairs of on mass shell wormhole throats with either sign of energy and 3-momentum so that the basic condition of twistorial approach is satisfied. The conservation of 4-momentum at vertices gives extremely powerful kinematical constraints so that there are excellent hopes about cancellation of UV divergences of loop integrals.

(c) The super-symmetry as an addition to the fermion state a second wormhole throats carrying right handed neutrino quantum numbers does not make sense since the resulting state cannot be distinguished from gauge boson or Higgs type particle. The light-like 3-surfaces can however carry fermion numbers up to the number of modes of the induced spinor field, which is expected to be infinite inside string like objects having wormhole throats at ends and finite when one has space time sheets containing the throats [K28] . In very general sense one could say that each mode defines a very large broken N-super-symmetry with the value of N depending on state and light-like 3-surface. The breaking of this super-symmetry would come from electro-weak - , color - , and gravitational interactions. Right-handed neutrino would by its electro-weak and color inertness define a minimally broken super-symmetry.

(d) What this addition of the right handed neutrinos or more general fermion modes could precisely mean? One cannot assign fermionic oscillator operators to right handed neutrinos which are covariantly constant in both M^4 and CP_2 degrees of freedom since the modes with vanishing energy (frequency) cannot correspond to fermionic oscillator operator creating a physical state since one would have $a = a^\dagger$. The intuitive view is that all the spinor modes move in an exactly collinear manner -somewhat like quarks inside hadron do approximately.

Modified Dirac equation briefly

The answer to the question what "collinear motion" means mathematically emerged from the recent progress in the understanding of the modified Dirac equation.

(a) The modified Dirac action involves two terms. Besides the original 4-D modified Dirac action there is measurement interaction which can be localized to wormhole throat or to any light-like 3-surfaces "parallel" to it in the slicing of space-time sheet by light-like 3-surfaces. This term correlates space-time geometry with quantum numbers assignable to super-conformal representations and is also necessary to obtain almost- stringy propagator.

(b) The modified Dirac equation with measurement action added reads as

$$\begin{aligned} D_K\Psi &= 0 \ , \\ D_3\Psi &= (D_{C-S} + Q\times O)\Psi = 0 \ , \\ [D_3, D_K]\,\Psi &= 0 \ . \end{aligned} \tag{15.4.1}$$

i. D_K corresponds formally to 4-D massless Dirac equation in X^4. D_3 realizes measurement interaction. D_{C-S} is the 3-D modified Dirac action defined by Chern-Simons action.

ii. Q is linear in Cartan algebra generators of the isometry algebra of imbedding space (color isospin and hypercharge plus four-momentum or two components of four momentum and spin and boost in direction of 3-momentum). Q is expressible as

$$Q = Q_A\partial_\alpha h^k g^{AB} j_{Bk}\hat{\Gamma}^{\alpha}_{CS} \ . \tag{15.4.2}$$

Here Q_A is Cartan algebra generator acting on physical states. Physical states must be eigen states of Q_A since otherwise the equations do not make sense. g^{AB} is the inverse of the matrix defined by the imbedding space inner product of Killing vector fields j_A^k and j_B^l: its existence allows only Cartan algebra charges. $\hat{\Gamma}^\alpha_{CS}$ is the modified gamma matrix associated with the Chern-Simons action.

iii. In general case the modified gamma matrices are defined in terms of action density L as

$$\hat{\Gamma}^\alpha = \frac{\partial L}{\partial_\alpha h^k}\gamma^k \quad . \tag{15.4.3}$$

γ^k denotes imbedding space gamma matrices.

iv. The operator O characterizes the conserved fermionic current to which Cartan algebra generators of isometries couple. The simplest conserved currents correspond to quark or lepton currents and corresponding vectorial isospin- and spin currents [K28]. Besides this there is an infinite hierarchy of conserved currents relating to quantum criticality and in one-one correspondence with vanishing second variations of Kähler action for preferred extremal. These couplings allow to represent measurement interaction for any observable.

(c) The equation $D_3\nu_R = 0$ would reduce for vanishing color charges and covariantly constant spinor to the analog of algebraic fermionic on mass shell condition $p_A\gamma^A\nu_R = 0$ since Q is obtained by projecting the total four-momentum of the parton state interpreted as a vector-field of H to the space-time surface and by replacing ordinary gamma matrices with the modified ones. This equation cannot be exact since Q depends on the point of the light-like 3-surface so that covariant constancy fails and D_{C-S} cannot annihilate the state. This is the space-time correlate for the breaking of super-symmetry. The action of the Cartan algebra generators is purely algebraic and on the state of super-conformal representations rather than that of a differential operator on spinor field. The modified equation implies that all spinor modes represent fermions moving collinearly in the sense an equation with the same total four-momentum and total color quantum numbers is satisfied by all of them. Note that p_A represents the total four-momentum of the state rather than individual four-momenta of fermions.

TGD counterpart of space-time super-symmetry

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

(a) One can in principle construct many-fermion states containing both fermions and anti-fermions at given light-like 3-surface. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the modified gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

(b) For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry.

(c) The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

(a) For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.

(b) The quantum field theoretic description should be based on QFT limit of TGD formulated in terms of bosonic emergence [K64] . This formulation should allow to calculate the propagators of the super-partners in terms of fermionic loops.

(c) This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true. The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.

(d) It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0, 1, 2, 3/2, 2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

15.4.6 What could be the generalization of Yangian symmetry of $\mathcal{N} = 4$ SUSY in TGD framework?

There has been impressive steps in the understanding of $\mathcal{N} = 4$ maximally sypersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

The notion of Yangian symmetry would have a generalization in TGD framework obtained by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2. The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of $CP_3 \times CP_3$ mapped to 6-surfaces dual $CP_3 \times CP_3$, which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

Background

I am outsider as far as concrete calculations in $\mathcal{N}=4$ SUSY are considered and the following discussion of the background probably makes this obvious. My hope is that the reader had patience to not care about this and try to see the big pattern.

The developments began from the observation of Parke and Taylor [B55] that n-gluon tree amplitudes with less than two negative helicities vanish and those with two negative helicities have unexpectedly simple form when expressed in terms of spinor variables used to represent light-like momentum. In fact, in the formalism based on Grassmanian integrals the reduced tree amplitude for two negative helicities is just "1" and defines Yangian invariant. The article *Perturbative Gauge Theory As a String Theory In Twistor Space* [B73] by Witten led to so called Britto-Cachazo-Feng-Witten (BCFW) recursion relations for tree level amplitudes [B60, B31, B60] allowing to construct tree amplitudes using the analogs of Feynman rules in which vertices correspond to maximally helicity violating tree amplitudes (2 negative helicity gluons) and propagator is massless Feynman propagator for boson. The progress inspired the idea that the theory might be completely integrable meaning the existence of infinite-dimensional un-usual symmetry. This symmetry would be so called Yangian symmetry [K101] assigned to the super counterpart of the conformal group of 4-D Minkowski space.

Drumond, Henn, and Plefka represent in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [B40] an argument suggesting that the Yangian invariance of the scattering amplitudes ins an intrinsic property of planar $\mathcal{N}=4$ super Yang Mills at least at tree level.

The latest step in the progress was taken by Arkani-Hamed, Bourjaily, Cachazo, Carot-Huot, and Trnka and represented in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [B26] . At the same day there was also the article of Rutger Boels entitled *On BCFW shifts of integrands and integrals* [B17] in the archive. Arkani-Hamed *et al* argue that a full Yangian symmetry of the theory allows to generalize the BCFW recursion relation for tree amplitudes to all loop orders at planar limit (planar means that Feynman diagram allows imbedding to plane without intersecting lines). On mass shell scattering amplitudes are in question.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K101] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B43] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N}=4$ SUSY). One of the conditions conditions is that the tensor product $R\otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, I have nothing to say. I am just perplexed. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A13] and Virasoro algebras [A30] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context;-)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras ontains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N}=4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of partonic 2-surface. A coset construction is applied to these two Virasoro algebras so that the differences of the corresponding Super-Virasoro generators and Kac-Moody generators annihilate physical states. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Is there any hope about description in terms of Grassmannians?

At technical level the successes of the twistor approach rely on the observation that the amplitudes can be expressed in terms of very simple integrals over sub-manifolds of the space consisting of k-dimensional planes of n-dimensional space defined by delta function appearing in the integrand. These integrals define super-conformal Yangian invariants appearing in twistorial amplitudes and the belief is that by a proper choice of the surfaces of the twistor space one can construct all invariants. One can construct also the counterparts of loop corrections by starting from tree diagrams and annihilating pair of particles by connecting the lines and quantum entangling the states at the ends in the manner dictated by the integration over loop momentum. These operations can be defined as operations for Grassmannian integrals in general changing the values of n and k. This description looks extremely powerful and elegant and nosta importantly involves only the external momenta.

The obvious question is whether one could use similar invariants in TGD framework to construct the momentum dependence of amplitudes.

(a) The first thing to notice is that the super algebras in question act on infinite-dimensional representations and basically in the world of classical worlds assigned to the partonic 2-surfaces correlated by the fact that they are associated with the same space-time surface. This does not promise anything very practical. On the other hand, one can hope that everything related to other than M^4 degrees of freedom could be treated like color degrees of freedom in $\mathcal{N}=4$ SYM and would boil down to indices labeling the quantum states. The Yangian conditions coming from isometry quantum numbers, color quantum numbers, and electroweak quantum numbers are of course expected to be highly non-trivial and could fix the coefficients of various singlets resulting in the tensor product of incoming and outgoing states.

(b) The fact that incoming particles can be also massive seems to exclude the use of the twistor space. The following observation however raises hopes. The Dirac propagator for wormhole throat is massless propagator but for what I call pseudo momentum. It is still unclear how this momentum relates to the actual four-momentum. Could it be actually equal to it? The recent view about pseudo-momentum does not support this view but it is better to keep mind open. In any case this finding suggests that twistorial approach could work in in more or less standard form. What would be needed is a representation for massive incoming particles as bound states of massless partons. In particular, the massive states of super-conformal representations should allow this kind of description.

Could zero energy ontology allow to achieve this dream?

(a) As far as divergence cancellation is considered, zero energy ontology suggests a totally new approach producing the basic nice aspects of QFT approach, in particular unitarity and coupling constant evolution. The big idea related to zero energy ontology is that all virtual particle particles correspond to wormhole throats, which are pairs of on mass shell particles. If their momentum directions are different, one obtains time-like continuum of virtual momenta and if the signs of energy are opposite one obtains also space-like virtual momenta. The on mass shell property for virtual partons (massive in general) implies extremely strong constraints on loops and one expect that only very few loops remain and that they are finite since loop integration reduces to integration over much lower-dimensional space than in the QFT approach. There are also excellent hopes about Cutkoski rules.

(b) Could zero energy ontology make also possible to construct massive incoming particles from massless ones? Could one construct the representations of the super conformal algebras using only massless states so that at the fundamental level incoming particles would be massless and one could apply twistor formalism and build the momentum dependence of amplitudes using Grassmannian integrals.

One could indeed construct on mass shell massive states from massless states with momenta along the same line but with three-momenta at opposite directions. Mass squared is given by $M^2 = 4E^2$ in the coordinate frame, where the momenta are opposite and of same magnitude. One could also argue that partonic 2-surfaces carrying quantum numbers of fermions and their superpartners serve as the analogs of point like massless particles and that topologically condensed fermions and gauge bosons plus their superpartners correspond to pairs of wormhole throats. Stringy objects would correspond to pairs of wormhole throats at the same space-time sheet in accordance with the fact that space-time sheet allows a slicing by string worlds sheets with ends at different wormhole throats and defining time like braiding.

The weak form of electric magnetic duality indeed supports this picture. To understand how, one must explain a little bit what the weak form of electric magnetic duality means.

(a) Elementary particles correspond to light-like orbits of partonic 2-surfaces identified as 3-D surfaces at which the signature of the induced metric of space-time surface changes from Euclidian to Minkowskian and 4-D metric is therefore degenerate. The analogy with black hole horizon is obvious but only partial. Weak form of electric-magnetic duality states that the Kähler electric field at the wormhole throat and also at space-like 3-surfaces defining the ends of the space-time surface at the upper and lower light-like boundaries of the causal diamond is proportonial to Kähler magnetic field so that Kähler electric flux is proportional Kähler magnetic flux. This implies classical quantization of Kähler electric charge and fixes the value of the proportionality constant.

(b) There are also much more profound implications. The vision about TGD as almost topological QFT suggests that Kähler function defining the Kähler geometry of the "world of classical worlds" (WCW) and identified as Kähler action for its preferred extremal reduces to the 3-D Chern-Simons action evaluated at wormhole throats and possible boundary components. Chern-Simons action would be subject to constraints.

Wormhole throats and space-like 3-surfaces would represent extremals of Chern-Simons action restricted by the constraint force stating electric-magnetic duality (and realized in terms of Lagrange multipliers as usual).

If one assumes that Kähler current and other conserved currents are proportional to current defining Beltrami flow whose flow lines by definition define coordinate curves of a globally defined coordinate, the Coulombic term of Kähler action vanishes and it reduces to Chern-Simons action if the weak form of electric-magnetic duality holds true. One obtains almost topological QFT. The absolutely essential attribute "almost" comes from the fact that Chern-Simons action is subject to constraints. As a consequence, one obtains non-vanishing four-momenta and WCW geometry is non-trivial in M^4 degrees of freedom. Otherwise one would have only topological QFT not terribly interesting physically.

Consider now the question how one could understand stringy objects as bound states of massless particles.

(a) The observed elementary particles are not Kähler monopoles and there much exist a mechanism neutralizing the monopole charge. The only possibility seems to be that there is opposite Kähler magnetic charge at second wormhole throat. The assumption is that in the case of color neutral particles this throat is at a distance of order intermediate gauge boson Compton length. This throat would carry weak isospin neutralizing that of the fermion and only electromagnetic charge would be visible at longer length scales. One could speak of electro-weak confinement. Also color confinement could be realized in analogous manner by requiring the cancellation of monopole charge for many-parton states only. What comes out are string like objects defined by Kähler magnetic fluxes and having magnetic monopoles at ends. Also more general objects with three strings branching from the vertex appear in the case of baryons. The natural guess is that the partons at the ends of strings and more general objects are massless for incoming particles but that the 3-momenta are in opposite directions so that stringy mass spectrum and representations of relevant super-conformal algebras are obtained. This description brings in mind the description of hadrons in terms of partons moving in parallel apart from transversal momentum about which only momentum squared is taken as observable.

(b) Quite generally, one expects for the preferred extremals of Kähler action the slicing of space-time surface with string world sheets with stringy curves connecting wormhole throats. The ends of the stringy curves can be identified as light-like braid strands. Note that the strings themselves define a space-like braiding and the two braidings are in some sense dual. This has a concrete application in TGD inspired quantum biology, where time-like braiding defines topological quantum computer programs and the space-like braidings induced by it its storage into memory. Stringlike objects defining representations of super-conformal algebras must correspond to states involving at least two wormhole throats. Magnetic flux tubes connecting the ends of magnetically charged throats provide a particular realization of stringy on mass shell states. This would give rise to massless propagation at the parton level. The stringy quantization condition for mass squared would read as $4E^2 = n$ in suitable units for the representations of super-conformal algebra associated with the isometries. For pairs of throats of the same wormhole contact stringy spectrum does not seem plausible since the wormhole contact is in the direction of CP_2. One can however expect generation of small mass as deviation of vacuum conformal weight from half integer in the case of gauge bosons.

If this picture is correct, one might be able to determine the momentum dependence of the scattering amplitudes by replacing free fermions with pairs of monopoles at the ends of string and topologically condensed fermions gauge bosons with pairs of this kind of objects with wormhole throat replaced by a pair of wormhole throats. This would mean suitable number of doublings of the Grassmannian integrations with additional constraints on the incoming momenta posed by the mass shell conditions for massive states.

Could zero energy ontology make possible full Yangian symmetry?

The partons in the loops are on mass shell particles have a discrete mass spectrum but both signs of energy are possible for opposite wormhole throats. This implies that in the rules for constructing loop amplitudes from tree amplitudes, propagator entanglement is restricted to that corresponding to pairs of partonic on mass shell states with both signs of energy. As emphasized in [B26], it is the Grassmannian integrands and leading order singularities of $\mathcal{N} = 4$ SYM, which possess the full Yangian symmetry. The full integral over the loop momenta breaks the Yangian symmetry and brings in IR singularities. Zero energy ontologist finds it natural to ask whether QFT approach shows its inadequacy both via the UV divergences and via the loss of full Yangian symmetry. The restriction of virtual partons to discrete mass shells with positive or negative sign of energy imposes extremely powerful restrictions on loop integrals and resembles the restriction to leading order singularities. Could this restriction guarantee full Yangian symmetry and remove also IR singularities?

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

15.5 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B7] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K18] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

(a) The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

(b) This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

(c) The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

(d) The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

(e) One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

    The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

15.5.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

(a) The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2

looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

(b) Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

(c) A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

(d) To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{15.5.1}$$

A more general form of this duality is suggested by the considerations of [K40] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{15.5.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

(e) Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{15.5.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

(a) The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

(b) The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L5] , [L5] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (15.5.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (15.5.5)$$

(c) The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (15.5.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (15.5.7)$$

(d) There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

(a) The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

(b) The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K65] supports this interpretation.

(c) The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

(d) The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z . \quad (15.5.8)$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} . \quad (15.5.9)$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \to 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

(a) Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{15.5.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L5]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

(b) For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

(c) The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K70]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

(a) The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

(b) GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K93]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

(c) Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

(d) GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

15.5.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

(a) In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

(b) One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of

M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D15] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K29] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

(a) Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

(b) The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

(c) How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K51] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

(d) What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K52] .

15.6 Quantum TGD very briefly

15.6.1 Two approaches to quantum TGD

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry [A14] for the "world of classical worlds" (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question. The second vision is the identification of physics as a generalized number theory involving p-adic number fields and the fusion of real numbers and p-adic numbers to a larger structure, classical number fields, and the notion of infinite prime.

With a better resolution one can distinguish also other visions crucial for quantum TGD. Indeed, the notion of finite measurement resolution realized in terms of hyper-finite factors, TGD as almost topological quantum field theory, twistor approach, zero energy ontology, and weak form of electric-magnetic duality play a decisive role in the actual construction and interpretation of the theory. One can however argue that these visions are not so fundamental for the formulation of the theory than the first two.

Physics as infinite-dimensional geometry

It is good to start with an attempt to give overall view about what the dream about physics as infinite-dimensional geometry is. The basic vision is generalization of the Einstein's program for the geometrization of classical physics so that entire quantum physics would be geometrized. Finite-dimensional geometry is certainly not enough for this purposed but physics as infinite-dimensional geometry of what might be called world of classical worlds (WCW) -or more neutrally WCW of some higher-dimensional imbeddign space- might make sense. The requirement that the Hermitian conjugation of quantum theories has a geometric realization forces Kähler geometry for WCW. WCW defines the fixed arena of quantum physics and physical states are identified as spinor fields in WCW. These spinor fields are classical and no second quantization is needed at this level. The justification comes from the observation that infinite-dimensional Clifford algebra [A5] generated by gamma matrices allows a natural identification as fermionic oscillator algebra.

The basic challenges are following.

(a) Identify WCW.

(b) Provide WCW with Kähler metric and spinor structure

(c) Define what spinors and spinor fields in WCW are.

There is huge variety of finite-dimensional geometries and one might think that in infinite-dimensional case one might be drowned with the multitude of possibilities. The situation is however exactly opposite. The loop spaces associated with groups have a unique Kähler geometry due to the simple condition that Riemann connection exists mathematically [A71] . This condition requires that the metric possesses maximal symmetries. Thus raises the vision that infinite-dimensional Kähler geometric existence is unique once one poses the additional condition that the resulting geometry satisfies some basic constraints forced by physical considerations.

The observation about the uniqueness of loop geometries leads also to a concrete vision about what this geometry could be. Perhaps WCW could be regarded as a union of symmetric spaces [A31] for which every point is equivalent with any other. This would simplify the construction of the geometry immensely and would mean a generalization of cosmological principle to infinite-D context [K40] , [L12] .

This still requires an answer to the question why $M^4 \times CP_2$ is so unique. Something in the structure of this space must distinguish it in a unique manner from any other candidate. The uniqueness of M^4 factor can be understood from the miraculous conformal symmetries of the light-cone boundary but in the case of CP_2 there is no obvious mathematical argument of this kind although physically CP_2 is unique [L19] . The observation that $M^4 \times CP_2$ has dimension 8, the space-time surfaces have dimension 4, and partonic 2-surfaces, which are the fundamental objects by holography have dimension 2, suggests that classical number fields [A19, A7, A26] are involved and one can indeed end up to the choice $M^4 \times CP_2$ from physics as generalized number theory vision by simple arguments [K88] , [L15] . In particular, the choices M^8 -a subspace of complexified octonions (for octonions see [A19]), which I have used to call hyper-octonions- and $M^4 \times CP_2$ can be regarded as physically equivalent: this "number theoretical compactification" is analogous to spontaneous compactification in M-theory. No dynamical compactification takes place so that $M^8 - H$ duality is a more appropriate term.

Physics as generalized number theory

Physics as a generalized number theory (for an overview about number theory see [A18]) program consists of three separate threads: various p-adic physics and their fusion together with real number based physics to a larger structure [K87] , [L18] , the attempt to understand basic physics in terms of classical number fields [K88] , [L15] (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes [K86] , [L10] , whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. In this article a summary of the philosophical ideas behind this dream and a summary of the technical challenges and proposed means to meet them are discussed.

The construction of p-adic physics and real physics poses formidable looking technical challenges: p-adic physics should make sense both at the level of the imbedding space, the "world of classical worlds" (WCW), and space-time and these physics should allow a fusion to a larger coherent whole. This forces to generalize the notion of number by fusing reals and p-adics along rationals and common algebraic numbers. The basic problem that one encounters is definition of the definite integrals and harmonic analysis [A10] in the p-adic context [K55] . It turns out that the representability of WCW as a union of symmetric spaces [A31] provides a universal group theoretic solution not only to the construction of the Kähler geometry of WCW but also to this problem. The p-adic counterpart of a symmetric space is obtained from its discrete invariant by replacing discrete points with p-adic variants of the continuous symmetric space. Fourier analysis [A10] reduces integration to summation. If one wants to define also integrals at space-time level, one must pose additional strong constraints which effectively reduce the partonic 2-surfaces and perhaps even space-time surfaces to finite geometries and allow assign to a given partonic 2-surface a unique power of a unique p-adic prime characterizing the measurement resolution in angle variables. These integrals might make sense in the intersection of real and p-adic worlds defined by algebraic surfaces.

The dimensions of partonic 2-surface, space-time surface, and imbedding space suggest that classical number fields might be highly relevant for quantum TGD. The recent view about the connection is based on hyper-octonionic representation of the imbedding space gamma matrices, and the notions of associative and co-associative space-time regions defined as regions for which the modified gamma matrices span quaternionic or co-quaternionic plane at each point of the region. A further condition is that the tangent space at each point of space-time surface contains a preferred hyper-complex (and thus commutative) plane identifiable as the plane of non-physical polarizations so that gauge invariance has a purely number theoretic interpretation. WCW can be regarded as the space of sub-algebras of the local octonionic Clifford algebra [A5] of the imbedding space defined by space-time surfaces with the property that the local sub-Clifford algebra spanned by Clifford algebra valued functions restricted at them is associative or co-associative in a given region.

The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave fuctions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of imbedding space and space-time surface are subject to a number theoretic evolution.

Questions

The experience has shown repeatedly that a correct question and identification of some weakness of existing vision is what can only lead to a genuine progress. In the following I discuss the basic questions, which have stimulated progress in the challenge of constructing WCW geometry.

1. What is WCW?

Concerning the identification of WCW I have made several guesses and the progress has been basically due to the gradual realization of various physical constraints and the fact that standard physics ontology is not enough in TGD framework.

(a) The first guess was that WCW corresponds to all possible space-like 3-surfaces in $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 denotes complex projective space of two complex dimensions having also representation as coset space $SU(3)/U(2)$ (see the separate article summarizing the basic facts about CP_2 and how it codes for standard model symmetries [L5] , [L16, L5]). What led to the this particular choice H was the observation that the geometry of H codes for standard model quantum numbers and that the generalization of particle from point like particle to 3-surface allows to understand also remaining quantum numbers having no obvious explanation in standard model (family replication phenomenon). What is important to notice is that Poincare symmetries act as exact symmetries of M^4 rather than space-time surface itself: this realizes the basic vision about Poincare invariant theory of gravitation. This lifting of symmetries to the level of imbedding space and the new dynamical degrees of freedom brought by the sub-manifold geometry of space-time surface are absolutely essential for entire quantum TGD and distinguish it from general relativity and string models. There is however a problem: it is not obvious how to get cosmology.

(b) The second guess was that WCW consists of space-like 3-surfaces in $H_+ = M^4_+ \times CP2$, where M^4_+ future light-cone having interpretation as Big Bang cosmology at the limit of vanishing mass density with light-cone property time identified as the cosmic time. One obtains cosmology but loses exact Poincare invariance in cosmological scales since translations lead out of future light-cone. This as such has no practical significance but due to the metric 2-dimensionality of light-cone boundary δM^4_+ the conformal symmetries of string model assignable to finite-dimensional Lie group generalize to conformal symmetries assignable to an infinite-dimensional symplectic group of $S^2 \times CP_2$ and also localized with respect to the coordinates of 3-surface. These symmetries are simply too beautiful to be important only at the moment of Big Bang and must be present also in elementary particle length scales. Note that these symmetries are present only for 4-D Minkowski space so that a partial resolution of the old conundrum about why space-time dimension is just four emerges.

(c) The third guess was that the light-like 3-surfaces in H or H_+ are more attractive than space-like 3-surfaces. The reason is that the infinite-D conformal symmetries characterize also light-like 3-surfaces because they are metrically 2-dimensional. This leads to a generalization of Kac-Moody symmetries [A13] of super string models with finite-dimensional Lie group replaced with the group of isometries of H. The natural identification of light-like 3-surfaces is as 3-D surfaces defining the regions at which the signature of the induced metric changes from Minkowskian $(1,-1,-1,-1)$ to Euclidian $(-1-1-1-1)$- I will refer these surfaces as throats or wormhole throats in the sequel. Light-like 3-surfaces are analogous to blackhole horizons and are static because strong gravity makes them light-like. Therefore also the dimension 4 for the space-time surface is unique.

This identification leads also to a rather unexpected physical interpretation. Single light-like wormhole throat carriers elementary particle quantum numbers. Fermions and their superpartners are obtained by glueing Euclidian regions (deformations of so called CP_2 type vacuum extremals of Kähhler action) to the background with Minkowskian signature. Bosons are identified as wormhole contacts with two throats carrying fermion *resp.* anti-fermionic quantum numbers. These can be identified as deformations of CP_2 vacuum extremals between between two parallel Minkowskian space-time sheets. One can say that bosons and their superpartners emerge. This has dramatic implications for quantum TGD [K20] and QFT limit of TGD [K64] .

The question is whether one obtains also a generalization of Feynman diagrams. The answer is affirmative. Light-like 3-surfaces or corresponding Euclidian regions of space-time are analogous to the lines of Feynman diagram and vertices are replaced by 2-D surface at which these surfaces glued together. One can speak about Feynman diagrams with lines thickened to light-like 3-surfaces and vertices to 2-surfaces. The generalized Feynman diagrams are singular as 3-manifolds but the vertices are non-singular as 2-manifolds. Same applies to the corresponding space-time surfaces and space-like 3-surfaces. Therefore one can say that WCW consists of generalized Feynman diagrams-something rather different from the original identification as space-like 3-surfaces and one can wonder whether these identification could be equivalent.

(d) The fourth guess was a generalization of the WCW combining the nice aspects of the identifications $H = M^4 \times CP_2$ (exact Poincare invariance) and $H = M^4_+ \times CP_2$ (Big Bang cosmology). The idea was to generalize WCW to a union of basic building bricks -causal diamonds (CDs) - which themselves are analogous to Big Bang-Big Crunch cosmologies breaking Poincare invariance, which is however regained by the allowance of union of Poincare transforms of the causal diamonds.

The starting point is General Coordinate Invariance (GCI). It does not matter, which 3-D slice of the space-time surface one choose to represent physical data as long as slices are related by a diffeomorphism of the space-time surface. This condition implies holography in the sense that 3-D slices define holograms about 4-D reality.

The question is whether one could generalize GCI in the sense that the descriptions using space-like and light-like 3-surfaces would be equivalent physically. This requires that finite-sized space-like 3-surfaces are somehow equivalent with light-like 3-surfaces. This suggests that the light-like 3-surfaces must have ends. Same must be true for the space-time surfaces and must define preferred space-like 3-surfaces just like wormhole throats do. This makes sense only if the 2-D intersections of these two kinds of 3-surfaces -call them partonic 2-surfaces- and their 4-D tangent spaces carry the information about quantum physics. A strengthening of holography principle would be the outcome. The challenge is to understand, where the intersections defining the partonic 2-surfaces are located.

Zero energy ontology (ZEO) allows to meet this challenge.

i. Assume that WCW is union of sub-WCWs identified as the space of light-like 3-surfaces assignable to $CD \times CP_2$ with given CD defined as an intersection of future and past directed light-cones of M^4. The tips of CDs have localization in M^4 and one can perform for CD both translations and Lorentz boost for CDs. Space-time surfaces inside CD define the basic building brick of WCW. Also unions of CDs allowed and the CDs belonging to the union can intersect. One can of course consider the possibility of intersections and analogy with the set theoretic realization of topology.

ii. ZEO property means that the light-like boundaries of these objects carry positive and negative energy states, whose quantum numbers are opposite. Everything can be created from vacuum and can be regarded as quantum fluctuations in the standard vocabulary of quantum field theories.

iii. Space-time surfaces inside CDs begin from the lower boundary and end to the upper boundary and in ZEO it is natural to identify space-like 3-surfaces as pairs of space-like 3-surfaces at these boundaries. Light-like 3-surfaces connect these boundaries.

iv. The generalization of GCI states that the descriptions based on space-like 3-surfaces must be equivalent with that based on light-like 3-surfaces. Therefore only the 2-D intersections of light-like and space-like 3-surfaces - partonic 2-surfaces- and their 4-D tangent spaces (4-surface is there!) matter. Effective 2-dimensionality means a strengthened form of holography but does not imply exact 2-dimensionality, which would reduce the theory to a mere string model like theory. Once these data are given, the 4-D space-time surface is fixed and is analogous to a generalization of Bohr orbit to infinite-D context. This is the first guess. The situation is actually more delicate due to the non-determinism of Kähler action motivating the interaction of the hierarchy of CDs within CDs.

In this framework one obtains cosmology: CDs represent a fractal hierarchy of big bang-big crunch cosmologies. One obtains also Poincare invariance. One can also interpret the non-conservation of gravitational energy in cosmology which is an empirical fact but in conflict with exact Poincare invariance as it is realized in positive energy ontology [K93, K80] . The reason is that energy and four-momentum in zero energy ontology correspond to those assignable to the positive energy part of the zero energy state of a particular CD. The density of energy as cosmologist defines it is the statistical average for given CD: this includes the contributions of sub-CDs. This average density is expected to depend on the size scale of CD density is should therefore change as quantum dispersion in the moduli space of CDs takes place and leads to large time scale for any fixed sub-CD.

Even more, one obtains actually quantum cosmology! There is large variety of CDs since they have position in M^4 and Lorentz transformations change their shape. The first question is whether the M^4 positions of both tips of CD can be free so that one could assign to both tips of CD momentum eigenstates with opposite signs of four-momentum. The proposal, which might look somewhat strange, is that this not the case and that the proper time distance between the tips is quantized in octaves of a fundamental time scale $T = R/c$ defined by CP_2 size R. This would explains p-adic length scale hypothesis which is behind most quantitative predictions of TGD. That the time scales assignable to the CD of elementary particles correspond to biologically important time scales [K24] forces to take this hypothesis very seriously.

The interpretation for T could be as a cosmic time quantized in powers of two. Even more general quantization is proposed to take place. The relative position of the second tip with respect to the first defines a point of the proper time constant hyperboloid of the future light cone. The hypothesis is that one must replace this hyperboloid with a lattice like structure. This implies very powerful cosmological predictions finding experimental support from the quantization of redshifts for instance [K80] . For quite recent further empirical support see [E21] .

One should not take this argument without a grain of salt. Can one really realize zero energy ontology in this framework? The geometric picture is that translations correspond to translations of CDs. Translations should be done independently for the upper and lower tip of CD if one wants to speak about zero energy states but this is not possible if the proper time distance is quantized. If the relative M^4_+ coordinate is discrete, this pessimistic conclusion is strengthened further.

The manner to get rid of problem is to assume that translations are represented by quantum operators acting on states at the light-like boundaries. This is just what standard quantum theory assumes. An alternative- purely geometric- way out of difficulty is the Kac-Moody symmetry associated with light-like 3-surfaces meaning that local M^4 translations depending on the point of partonic 2-surface are gauge symmetries. For a given translation leading out of CD this gauge symmetry allows to make a compensating transformation which allows to satisfy the constraint.

This picture is roughly the recent view about WCW. What deserves to be emphasized is that a very concrete connection with basic structures of quantum field theory emerges already at the level of basic objects of the theory and GCI implies a strong form of holography and almost stringy picture.

2.Some Why's

In the following I try to summarize the basic motivations behind quantum TGD in form of various Why's.

(a) Why WCW?
Einstein's program has been extremely successful at the level of classical physics. Fusion of general relativity and quantum theory has however failed. The generalization of Einstein's geometrization program of physics from classical physics to quantum physics gives excellent hopes about the success in this project. Infinite-dimensional geometries are highly unique and this gives hopes about fixing the physics completely from the uniqueness of the infinite-dimensional Kähler geometric existence.

(b) Why spinor structure in WCW?
Gamma matrices defining the Clifford algebra [A5] of WCW are expressible in terms of fermionic oscillator operators. This is obviously something new as compared to the view about gamma matrices as bosonic objects. There is however no deep reason denying this kind of identification. As a consequence, a geometrization of fermionic oscillator operator algebra and fermionic statistics follows as also geometrization of super-conformal symmetries [A30, A13] since gamma matrices define super-generators of the algebra of WCW isometries extended to a super-algebra.

(c) Why Kähler geometry?
Geometrization of the bosonic oscillator operators in terms of WCW vector fields and fermionic oscillator operators in terms of gamma matrices spanning Clifford algebra.

Gamma matrices span hyper-finite factor of type II_1 and the extremely beautiful properties of these von Neuman algebras [A63] (one of the three von Neumann algebras that von Neumann suggests as possible mathematical frameworks behind quantum theory) lead to a direct connection with the basic structures of modern physics (quantum groups, non-commutative geometries,.. [A87]).

A further reason why is the finiteness of the theory.

i. In standard QFTs there are two kinds of divergences. Action is a local functional of fields in 4-D sense and one performs path integral over **all** 4-surfaces to construct S-matrix. Mathematically path integration is a poorly defined procedure and one obtains diverging Gaussian determinants and divergences due to the local interaction vertices. Regularization provides the manner to get rid of the infinities but makes the theory very ugly.

ii. Kähler function defining the Kähler geometry is a expected to be non-local functional of the partonic 2-surface (Kähler action for a preferred extremal having as its ends the positive and negative energy 3-surfaces). Path integral is replaced with a functional integral which is mathematically well-defined procedure and one performs functional integral only over the partonic 2-surfaces rather than all 4-surfaces (holography). The exponent of Kähler function defines a unique vacuum functional. The local divergences of local quantum field theories of local quantum field theories since there are no local interaction vertices. Also the divergences associated with the Gaussian determinant and metric determinant cancel since these two determinants cancel each other in the integration over WCW. As a matter fact, symmetric space property suggest a much more elegant manner to perform the functional integral by reducing it to harmonic analysis in infinite-dimensional symmetric space [K28] .

iii. One can imagine also the possibility of divergences in fermionic degrees of freedom but it has turned out that the generalized Feynman diagrams in ZEO are manifestly finite. Even more: it is quite possible that only finite number of these diagrams give non-vanishing contributions to the scattering amplitude. This is essentially due to the new view about virtual particles, which are identified as bound states of on mass shell states assigned with the throats of wormhole contacts so that the integration over loop momenta of virtual particles is extremely restricted [K28] .

(d) Why infinite-dimensional symmetries?

WCW must be a union of symmetric spaces in order that the Riemann connection exists (this generalizes the finding of Freed for loop groups [A71]). Since the points of symmetric spaces are metrically equivalent, the geometrization becomes tractable although the dimension is infinite. A union of symmetric spaces is required because 3-surfaces with a size of galaxy and electron cannot be metrically equivalent. Zero modes distinguish these surfaces and can be regarded as purely classical degrees of freedom whereas the degrees of freedom contributing to the WCW line element are quantum fluctuating degrees of freedom.

One immediate implication of the symmetric space property is constant curvature space property meaning that the Ricci tensor proportional to metric tensor. Infinite-dimensionality means that Ricci scalar either vanishes or is infinite. This implies vanishing of Ricci tensor and vacuum Einstein equations for WCW.

(e) Why $M^4 \times CP_2$?

This choice provides an explanation for standard model quantum numbers. The conjecture is that infinite-D geometry of 3-surfaces exists only for this choice. As noticed, the dimension of space-time surfaces and M^4 fixed by the requirement of generalized conformal invariance [A27] making possible to achieve symmetric space property. If $M^4 \times CP_2$ is so special, there must be a good reason for this. Number theoretical vision [K88] , [L15] indeed leads to the identification of this reason. One can assign the hierarchy of dimensions associated with partonic 2-surfaces, space-time surfaces and imbedding space to classical number fields and can assign to imbedding space what might be called hyper-octonionic structure. "Hyper" comes from the fact that the tangent space of H corresponds to the subspaces of complexified octonions with octonionic imaginary units multiplied by a commuting imaginary unit. The space-time regions would be either hyper-quaternionic or co-hyper-quaternionic so that associativity/co-associativity would become the basic dynamical principle at the level of space-time dynamics. Whether this dynamical principle is equivalent with the preferred extremal property of Kähler action remains an open conjecture.

(f) Why zero energy ontology and why causal diamonds?

The consistency between Poincare invariance and GRT requires ZEO. In positive energy ontology only one of the infinite number of classical solutions is realized and partially fixed by the values of conserved quantum numbers so that the theory becomes obsolete. Even in quantum theory conservation laws mean that only those solutions of field equations with the quantum numbers of the initial state of the Universe are interesting and one faces the problem of understanding what the the initial state of the universe was. In ZEO these problems disappear. Everything is creatable from vacuum: if the physical state is mathematically realizable it is in principle reachable by a sequence of quantum jumps. There are no physically non-reachable entities in the theory. Zero energy ontology leads also to a fusion of thermodynamics with quantum theory. Zero energy states ae defined as entangled states of positive and negative energy states and entanglement coefficients define what I call M-matrix identified as "complex square root" of density matrix expressible as a product of diagonal real and positive density matrix and unitary S-matrix [K20] .

There are several good reasons why for causal diamonds. ZEO requires CDs, the generalized form of GCI and strong form of holography (light-like and space-like 3-surfaces are physically equivalent representations) require CDs, and also the view about light-like 3-surfaces as generalized Feynman diagrams requires CDs. Also the classical non-determinism of Kähler action can be understood using the hierarchy CDs and the addition of CDs inside CDs to obtain a fractal hierarchy of them provides an elegant manner to undersand radiative corrections and coupling constant evolution in TGD framework.

A strong physical argument in favor of CDs is the finding that the quantized proper time distance between the tips of CD fixed to be an octave of a fundamental time scale defined by CP_2 happens to define fundamental biological time scale for electron, u quark and d quark [K24] : there would be a deep connection between elementary particle physics and living matter leading to testable predictions.

15.6.2 Overall view Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

1. What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignement of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

2. Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{15.6.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k \Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Associativity (co-associativity) and quantum criticality

Quantum criticality is one of the basic notions of TGD. It was originally introduced to fix the value(s) of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current: this current vanishes for Cartan algebra of isometries.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$.

Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The conjecture is that quantum critical space-time surfaces are associative (co-associative) in the sense that the tangent vectors span a associative (co-associative) subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The notion of octonionic tangent space can be expressed by introducing octonionic structure realized in terms of vielbein in manner completely analogous to that for the realization of gamma matrices.

One can also introduce octonionic representations of gamma matrices but this is not absolutely necessarily. The condition that both the modified gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the modified Dirac equation making sense also in the real context. The octonionic version of the modified Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

This condition is analogous to what happens for the spinor modes when they are restricted at string worlds sheets carrying vanishing induced W fields (and also Z^0 fields above weak length scale) to guarantee well-definedness of em charge and it might be that this strange looking condition makes sense. The possibility to define G_2 structure would thus be due to the well-definedness of em charge and in the generic case possible only for string world sheets and possibly also partonic 2-surfaces.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

The exponent of Kähler function as Dirac determinant for the Kähler Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. Since the definition of WCW geometry relies leads to a direct connection between the modes of Kähler Dirac operator and matrix elements of WCW metric the natural expectation is that Dirac determinant - if it can be defined - could be identified as exponent of Kähler function.

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

(a) Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

(b) Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

(c) The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

(d) The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

(e) Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu (dx^\mu/ds) ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

(f) The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

The formula would be

$$K \propto \oint J(x,y) J_T dx^1 \wedge dx^2 \quad . \tag{15.6.2}$$

This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

15.6.3 Three Dirac operators and their interpretation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the modified gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats. This applies in particular to condensed matter physics.

Three Dirac equations

To begin with, Dirac equation appears in three forms in TGD.

(a) The Dirac equation in world of classical worlds codes (WCW) for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. WCW Dirac operator generalizes the Dirac operator of 8-D imbedding space by bringing in vibrational degrees of freedom. This Dirac equation should give as its solutions zero energy states and corresponding M-matrices generalizing S-matrix and their collection defining the unitary U-matrix whose natural application appears in consciousness theory as a coder of what Penrose calls U-process. The ground states to which super-conformal algebras act correspond to imbedding space spinor modes in accordance with the idea that point like limit gives QFT in imbedding space.

(b) The analog of massless Dirac equation at the level of 8-D imbedding space and satisfied by fermionic ground states of super-conformal representations.

(c) Kähler Dirac equation is satisfied in the interior of space-time. In this equation the gamma matrices are replaced with modified gamma matrices defined by the contractions of canonical momentum currents $T^\alpha_k = \partial L/\partial_\alpha h^k$ with imbedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries. The well-definedness of em charge implies that the modes of induced spinor field are localized at 2-D surfaces so that a connection with string theory type approach emerges.

Kähler-Dirac equation defines Dirac equation at space-time level. Consider first K-D equation in the interior of space-time surface.

(a) The condition that electromagnetic charge operator defined in terms of em charge expressed in terms of Clifford algebra is well defined for spinor modes (completely analogous to spin defined in terms of sigma matrices) leads to the proposal that induced spinor fields are necessarily localized at 2-dimensional string worlds sheets [K105]. Only the covariantly constant right handed neutrino and its modes assignable to massless extremals (at least) generating super-symmetry (super-conformal symmetries) would form an exception since electroweak couplings would vanish. Note that the modified gamma matrices possess CP_2 and this must vanish in order to have de-localization.

(b) This picture implies stringy realization of super Kac-Moody symmetry elementary particles can be identified as string like objects albeit in different sense than in string models. At light-like 3-surfaces defining the orbits of partonic 2-surfaces spinor fields carrying electroweak quantum numbers would be located at braid strands as also the notion of finite measurement resolution requires. This picture is also consistent with the puzzling observation that the solutions of the Chern-Simons Dirac equation can be localized on light-like curves inside wormhole throat orbits.

(c) Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

(d) The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.

(e) Sound as a concept is usually assigned with a rather high level of description. Stringy world sheets could however dramatically raise the status of sound in this respect. The oscillations of string world sheets connecting wormhole throats describe non-local 2-particle interactions. Holography suggests that this interaction just "gravitational" dual for electroweak and color interactions. Could these oscillations inducing the oscillation of the distance between wormhole throats be interpreted at the limit of weak "gravitational" coupling as analogs of sound waves, and could sound velocity correspond to maximal signal velocity assignable to the effective metric?

Various arguments lead to the hypothesis that Kähler-Dirac action contains Chern-Simons-Dirac action localized at partonic orbits as additional term. This term cannot present at the space-like ends of the space-time surfaces. Also Kähler action contains Chern-Simons term and partonic orbits and reduces by field equations to Chern-Simons terms at the space-like ends of space-time surface.

(a) The variation of the Kähler-Dirac action gives rise to a boundary term, which is essentially contraction of the normal component of the vector Γ^n defined by Kähler-Dirac gamma matrices. Boundary condition gives $\sqrt{g_4}\Gamma^n\Psi = 0$. Therefore the incoming spinor modes at the boundaries of string world sheets must be massless. A further assumption is that the action of $\sqrt{g_4}\Gamma^n$ equals to that of a massless Dirac operator. By a suitable choice of coordinates this might be achieved. Thus massless Dirac equation in M^4 would emerge for on mass shell states.

(b) At parton orbits of wormhole one can assume that the spinors are generalized eigenstates of C-S-D operator reduces to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator and one would have good hopes that twistor Grassmannian approach works. In TGD based sringy variant of twistor Grassmann approach the integrals over virtual momenta as residue integrals reduce them to 3-D integrals over light-cone subject to momentum conservation constraints at vertices. Virtual fermions are massless but have unphysical polarization. This picture is discussed in detail in [K78].

15.6.4 Does energy metric provide the gravitational dual for condensed matter systems?

The modified gamma matrices define an effective metric via their anti-commutators quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of the modified gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. The energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anti-commutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0 has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it analogs of curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography could provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Could this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the analogs of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete determinism for Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity, which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [D18] . The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [K52]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteeing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a de-coherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

Preferred extremals as perfect fluids

15.6.5 Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [C14]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D14] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D6] .

(a) The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \quad . \qquad (15.6.3)$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \quad . \qquad (15.6.4)$$

From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

(b) The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \quad . \qquad (15.6.5)$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p) u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \quad . \qquad (15.6.6)$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric $((1,-1,-1,-1))$.

(c) If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p) g^{tt} \delta^\alpha_t \delta^\beta_t + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \quad . \qquad (15.6.7)$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.

The existence of a global flow parameter means that one has

$$v_i = \Psi \partial_i \Phi \quad . \qquad (15.6.8)$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \quad . \qquad (15.6.9)$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

(a) Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

(b) Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.

    Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

(a) Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At imbedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

(b) The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

    The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is feeded into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

(c) The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of $\hbar$ can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large $\hbar$ is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

(a) The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\parallel}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

(b) The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\parallel} v_{\perp} S \ .$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S \ .$$

From this one obtains

$$\eta = n_{abs} m v_{\perp} d \ .$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \ .$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

(c) Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_\perp$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv\ \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon}$$

Quantization condition would give

$$\epsilon = 4\pi \quad .$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

(d) One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

Is the effective metric one- or two-dimensional?

15.6.6 Is the effective metric effectively one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the modified gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. This localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces.

The argument is based on the following assumptions.

(a) The modified gamma matrices for Kähler action are contractions of the canonical momentum densities T^α_k with the gamma matrices of H.

(b) The strongest assumption is that the isometry currents

$$J^{A\alpha} = T^\alpha_k j^{Ak}$$

for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A (\nabla\Phi)^\alpha \tag{15.6.10}$$

with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

(c) A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A(\nabla\Phi_1)^\alpha \ , \\ J_2^{A\alpha} &= \Psi_2^A(\nabla\Phi_2)^\alpha \ . \end{aligned} \tag{15.6.11}$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [K41] . Isometry invariance does not allow more that two independent scalar functions Φ_i.

Consider now the argument.

(a) One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2. The tensor quantity $\sum_A j^{Ak}j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} \ , \tag{15.6.12}$$

where η_{AB} denotes the flat tangent space metric of H. In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

(b) In the most general case one obtains

$$\begin{aligned} T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_1)^\alpha \equiv f_1^k(\nabla\Phi_1)^\alpha \ , \\ T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_2)^\alpha \equiv f_2^k(\nabla\Phi_2)^\alpha \ . \end{aligned} \tag{15.6.13}$$

(c) The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla\Phi_1)^\alpha (\nabla\Phi_1)^\beta + s_{kl} f_2^k f_2^l (\nabla\Phi_2)^\alpha (\nabla\Phi_2)^\beta \ . \tag{15.6.14}$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

One can also develop an objection to effective 1- or 2-dimensionality. The proposal for what preferred extremals of Kähler action as deformations of the known extremals of Kähler action could be leads to a beautiful ansatz relying on generalization of conformal invariance and minimal surface equations of string model [K9]. The field equations of TGD reduce to those of classical string model generalized to 4-D context.

If the proposed picture is correct, field equations reduce to purely algebraically conditions stating that the Maxwellian energy momentum tensor for the Kähler action has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

There is however an important consistency condition involved. The Maxwell energy momentum tensor for Kähler action must have vanishing covariant divergence. This is satisfied if it is linear combination of Einstein tensor and metric. This gives Einstein's equations with cosmological term in the general case. By the algebraic character of field equations also minimal surface equations are satisfied and Einstein's General Relativity would be exact part of TGD.

In the case of modified Dirac equation the result means that modified gamma matrices are contractions of linear combination of Einstein tensor and metric tensor with the induced gamma matrices so that the TGD counterpart of ordinary Dirac equation would be modified by the addition of a term proportional to Einstein tensor. The condition of effective 1- or 2-dimensionality seems to pose too strong conditions on this combination.

15.7 Summary of generalized Feynman diagrammatics

This section gives a summary about the recent view abot generalized Feynman diagrammatics, which can be seen as a hybrid of Feynman diagrammatics and stringy diagrammatics. The analogs of Feynman diagrams are realized at the level of space-time topology and geometry and the lines of these diagrams are Euclidian space-time regions identifiable as wormhole contacts. For fundamental fermions one has the usual 1-D propagator lines.

Physical particles can be seen as bound state of massless fundamental fermions and involve two wormhole contacts forming parts of closed Kähler magnetic flux tubes carrying monople flux. The orbits of wormhole throats are connected by fermionic string world sheets whose boundaries correspond to massless fermion lines defining strands of braids. String world sheets in turn can form 2-braids.

It is a little bit matter of taste whether one refers to these diagrams generalized Feynman diagrams, generalized stringy diagrams, generalized Wilson loops or generalized twistor diagrams. All these labels are partly misleading.

In the sequel the basic action principles - Kähler action and Kähler-Dirac action are discussed first, and then a proposal for the diagrams describing M-matrix elements is discussed.

15.7.1 The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

1. What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized

Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignement of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

2. Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{15.7.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

15.7.2 A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

(a) At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

(b) In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

(c) Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

(d) Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

(e) The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

(f) Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??**, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

Chapter 16

Particle Massivation in TGD Universe

16.1 Introduction

This chapter represents the most recent view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters [K19, K48, K57, K52] of [K54] . In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the modified Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected.

Since 2010 a further progress took place. These steps of progress relate closely to ZEO, bosonic emergence, the discovery of the weak form of electric-magnetic duality, the realization of the importance of twistors in TGD, and the discovery that the well-definedness of em charge forces the modes of Kähler-Dirac operator to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This allows to assign to elementary particle closed string with pieces at two parallel space-time sheets and accompanying a Kähler magnetic flux tube carrying monopole flux.

Twistor approach and the understanding of the solutions of Kähler-Dirac Dirac operator served as a midwife in the process giving rise to the birth of the idea that all fundamental fermions are massless and that both ordinary elementary particles and string like objects emerge from them. Even more, one can interpret virtual particles as being composed of these massless on mass shell particles assignable to wormhole throats. Four-momentum conservation poses extremely powerful constraints on loop integrals but does not make them manifestly finite as believed first. String picture is necessary for getting rid of logarithmic divergences.

The weak form of electric-magnetic duality led to the realization that elementary particles correspond to bound states of two wormhole throats with opposite Kähler magnetic charges with second throat carrying weak isospin compensating that of the fermion state at second wormhole throat. Both fermions and bosons correspond to wormhole contacts: in the case of fermions topological condensation generates the second wormhole throat. This means that altogether four wormhole throats are involved with both fermions, gauge bosons, and gravitons (for gravitons this is unavoidable in any case). For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM. Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

One cannot avoid the question about the relation between p-adic mass calculations and Higgs mechanism. Higgs is predicted but does the analog of Higgs vacuum expectation emerge as the existence of QFT limit would suggest? Boundary conditions for Kähler-Dirac action with measurement interaction term for four-momentum lead to what looks like an as algebraic variant of masssless Dirac equation in Minkowski space coupled to the analog of Higgs vacuum expectation value restricted at fermionic strings. This equation does not however provide an analog of Higgs mechanism but a space-time correlate for the stringy mass formula coming from the vanishing of the scaling generator L_0 of superconformal algbra. It could also give a first principle explanation for the necessarily tachyonic ground state with half integer conformal weight.

For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM.

Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets. In fact, this contribution can be assigned to the additional conformal weight assignable to the stringy curve. The extension of this conformal algebra to Yangian brings in third integer characterizing the poly-locality of the Yangian generator (n-local generator acts on n partonic 2-surfaces simultaneously. Therefore three integers would characterize the generators of the full symmetry algebra as the very naive expectation on basis of 3-dimensionality of the fundamental objects would suggest. p-Adic mass calculations should be carried out for Yangian generalization of p-adic thermodynamics.

16.1.1 Physical states as representations of super-symplectic and Super Kac-Moody algebras

Physical states belong to the representations of super-symplectic algebra and Super Kac-Moody algebran. The precise identification of the two algebras has been rather tedious task but the recent progress in the construction of WCW geometry and spinor structure led to a considerable progress in this respect [K28, ?, K116].

(a) In the generic case the generators of both algebras receive information from 1-D ends of 2-D string world sheets at which the modes of induced spinor fields are localized by the condition that the modes are eigenstates of electromagnetic charge. Right-handed neutrino is an exception since it has no electroweak couplings. One must however require that right-handed neutrino does not mix with the left-handed one if the mode is de-localized at entire space-time sheet.

Either the preferred extremal is such that modified gamma matrices defined in terms of canonical momentum currents of Kähler action consist of only M^4 or CP_2 type flat space gammas so that there is no mixing with the left-handed neutrino. Or the CP_2 and M^4 parts of the Kähler Dirac operator annihilate the right-handed neutrino mode separately. One can of course have also modes which are mixtures of right- and left handed neutrinos but these are necessarily localized at string world sheets.

(b) The definition of super generator involves integration of string curve at the boundary of causal diamond (CD) so that the generators are labelled by *two* conformal weights: that associated with the radial light-like coordinate and that assignable with the string curve. This strongly suggests that the algebra extends to a 4-D Yangian involving multi-local generators (locus means partonic surface now) assignable to various partonic surfaces at the boundaries of CD - as indeed suggested [K101].

(c) As before, the symplectic algebra corresponds to a super-symplectic algebra assignable to symplectic transformations of $\delta M^4_\pm \times CP_2$. One can regard this algebra as a symplectic algebra of $S^2 \times CP_2$ localized with respect to the light-like radial coordinate r_M taking the role of complex variable z in conformal field theories. Super-generators are linear in the modes of right-handed neutrino. Covariantly constant mode and modes decoupling from left-handed neutrino define the most important modes.

(d) Second algebra corresponds to the Super Kac-Moody algebra. The corresponding Lie algebra generates symplectic isometries of $\delta M^4_\pm \times CP_2$. Fermionic generators are linear in the modes of induced spinor field with non-vanishing electroweak quantum numbers: that is left-hand neutrinos, charged leptons, and quarks.

(e) The overall important conclusion is that overall Super Virasoro algebra has five tensor factors corresponding to one tensor factor for super-symplectic algebra, and 4 tensor factors for Super Kac-Moody algebra $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ (CP_2 isometries, S^2 isometries, electroweak $SU(2)_{ew} \times U(1)$). This is essential for mass calculations.

What looks like the most plausible option relies on the generalization of a coset construction proposed already for years ago but badly mis-interpreted. The construction itself is strongly supported and perhaps even forced by the vision that WCW is union of homogenous or even symmetric spaces of form G/H [K116], where G is the isometry group of WCW and H its subgroup leaving invariant the chosen point of WCW (say the 3-surface corresponding to a maximum of Kähler function in Euclidian regions and stationary point of the Morse function defined by Kähler action for Minkowskian space-time regions). It seems clear that only the Super Virasoro associated with G can involve four-momentum so that the original idea that there are two identical four-momenta identifiable as gravitational and inertial four-momenta must be given up. This boils dow to the following picture.

(a) Assume a generalization of the coset construction so that the differences of G and H super-conformal generators O_n annihilate the physical states: $(O_n(G) - O_n(H))|phys\rangle = 0$.

(b) In zero energy ontology (ZEO) p-adic thermodynamics must be replaced with its square root so that one consideres genuine quantum states rather than thermodynamical states. Hence the system is quantum coherent. In the simplest situation this implies only that thermodynamical weights are replaced by their square roots possibly multiplied by square roots irrelevant for the mass squared expectation value.

(c) Construct first ground states with negative conformal weight annihilated by G and H generators G_n, L_n, $n < 0$. Apply to these states generators of tensor factors of Super Viraroso algebras to obtain states with vanishing G and H conformal weights. After this construct thermal states as superpositions of states obtained by applying H generators and corresponding G generators G_n,L_n, $n > 0$. Assume that these states are annihilated by G and H generators G_n, L_n,$n > 0$ and by the differences of *all* G and H generators.

(d) Super-symplectic algebra represents a completely new element and in the case of hadrons the non-perturbative contribution to the mass spectrum is easiest to understand in terms of super-symplectic thermal excitations contributing roughly 70 per cent to the p-adic thermal mass of the hadron.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the already generalized super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [K101]. The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [K101] .

16.1.2 Particle massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. The identification of elementary particles developed in three steps.

(a) Originally germions were identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian signature of the background space-time sheet. Gauge bosons and Higgs were identified as wormhole contacts with light-like throats carrying fermion and anti-fermion quantum numbers. Gravitons were identified as pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level. This picture generalizes to include super-symmetry. The fermionic oscillator operators associated with the partonic 2-surfaces act as generators of badly broken SUSY and right-handed neutrino gives to the not so badly broken $\mathcal{N} = 1$ SUSY consistent with empirical facts.

Of course, "badly" is relative notion. It is quite possible that the mixing of right-handed neutrino with left-handed one becomes important only in CP_2 scale and causes massivation. Hence spartners might well have mass of order CP_2 mass scale. The question about the mass scale of right-handed neutrino remains open.

(b) The next step was to realize that the topological condensation of fermion generates second wormhole throat which carries momentum and symplectic quantum numbers but no fermionic quantum numbers. This is also needed to the massivation by p-adic thermodynamics applied to the analogs of string like objects defined by wormhole throats with throats taking the role of string ends. p-Adic thermodynamics did not however allow a satisfactory understanding of the gauge bosons masses and it became clear that some additional contribution - maybe Higgsy or stringy contribution - dominates for weak gauge bosons. Gauge bosons should also somehow obtain their longitudinal polarizations and here Higgs like particles indeed predicted by the basic picture suggests itself strongly.

(c) A further step was the discovery of the weak form of electric-magnetic duality, which led to the realization that wormhole throats possess Kähler magnetic charge so that a wormole throat with opposite magnetic charge is needed to compensate this charge. This wormhole throat can also compensate the weak isospin of the second wormhole throat so that weak confinement and massivation results. In the case of quarks magnetic confinement might take place in hadronic rather than weak length scale. Second crucial observation was that gauge bosons are necessarily massive since the light-like momenta at two throats must correspond to opposite three-momenta so that no Higgs potential is needed. This leads to a picture in which gauge bosons eat the Higgs scalars and also photon, gluons, and gravitons develop small mass.

(d) A further step was the realization that although the existence of Higgs is established, it need not contribute to neither fermion or gauge boson masses. CP_2 geometry does not even allow covariantly constant holomorphic vector field as a representation for the vacuum expectation value of Higgs. Elementary particles are string like objects and string tension can give additional contribution to the mass squared. This would explain the large masses of weak bosons as compared to the mass of photon predicted also to be non-vanishing in principle. Also a small contribution to fermion masses is expected.

Higgs vacuum expectation would be replaced with the stringy contribution to the mass squared, which by perturbative argument should apart from normalization factor have the form $\Delta m^2 \propto g^2 T$, where g is the gauge coupling assignable to the weak boson, and T is the analog of hadronic string tension but in weak scale. This predicts correctly the ratio of W and Z boson masses in terms of Weinberg angle.

(e) The conformal weight characterizing fermionic masses in p-adic thermodynamics can be assigned to the very short piece of string connecting the opposite throats of wormhole contact. The conformal weight associated with the long string connecting the throats of two wormhole contacts should give the dominant contribution to the masses of weak gauge bosons. Five tensor factors are needed in super-conformal algebra and super-symplectic and super-Kac Moody contributions assignable to symplectic isometries give five factors.

One can assign conformal weights to both the light-like radial coordinate r_M of $\delta M^4_\pm$ and string. A third integer-valued quantum number comes from the extension of the extended super-conformal algebra to multi-local Yangian algebra. Yangian extension should take place for quark wormhole contacts inside hadrons and give non-perturbative multi-local contributions to hadron masses and might explain most of hadronic mass since quark contribution is very small. That three integers classify states conforms with the very naive first guess inspired by 3-dimensionality of the basic objects.

The details of the picture are however still fuzzy. Are the light-like radial and stringy conformal weights really independent quantum numbers as it seems? These conformal weights however must be additive in the expression for mass squared to get five tensor factors. Could one identify stringy coordinate with the light-like radial coordinate r_M in Minkowskian space-time regions to explain the additivity? The dominating contribution to the vacuum conformal weight must be negative and half-integer valued. What is the origin of this tachyonic contribution?

The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces.

(a) Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Chern-Simons action and its Dirac counterpart result as boundary terms of Kähler action and its Dirac counterpart for preferred extremals. This requires that $j \cdot A$ contribution to Kähler action vanishes for preferred extremals plus weak form of electric-magnetic duality.

The addition of 3-D measurement interaction term - essentially Dirac action associated with 3-D light-like orbits of partonic 2-surfaces implies that Chern-Simons Dirac operator plus Lagrangian multiplier term realizing the weak form of electric magnetic duality acts like massless M^4 Dirac operator assignable to the four-momentum propagating along the line of generalized Feynman diagram [K28]. This simplifies enormously the definition of the Dirac propagator needed in twistor Grassmannian approach [K78].

(b) That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is besides Loretnz invariance suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

Mass squared is identified as the p-adic thermal expectation value of mass squared operator m^2 appearing as M^4 contribution in the scaling generator $L_0(G)$ in the superposition of states with vanishing total conformal weight but with varying mass squared eigenvalues associated with the difference $L_0(G) - L_0(H)$ annihilating the physical state. This definition does not break Lorentz invariance in zero energy ontology. The states appearing in the superposition of different states with vanishing total conformal weight give different contribution to the p-adic thermodynamical expectation defining mass squared and the ability to physically observe this as massivation might be perhaps interpreted as breaking of conformal invariance.

(c) There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

(d) A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses via a generation of Higgs vacuum expectation having possibly interpretation in terms of a coherent state. Before the detailed model for elementary particles in terms of pairs of wormhole contacts at the ends of flux tubes the picture about the situation was as follows. From the beginning it was clear that is that ground state conformal weight must be negative. Then it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to stringy contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. The possible Higgs vacuum expectation makes sense only at QFT limit perhaps allowing to describe the Yangian aspects, and would be naturally proportional to Δh so that the coupling to Higgs would only apparently cause gauge boson massivation.

(e) A natural identification of the non-integer contribution to the conformal weight is as stringy contribution to the vacuum conformal weight. In twistor approach the generalized eigenvalues of Chern-Simons Dirac operator for external particles indeed correspond to light-like momenta and when the three-momenta are opposite this gives rise to non-vanishing mass. Higgs is necessary to give longitudinal polarizations for weak gauge bosons.

An important question concerns the justification of p-adic thermodynamics.

(a) The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. The physical justification for p-adic thermodynamics is effective p-adic topology characterizing the 3-surface: this is the case if real variant of light-like 3-surface has large number of common algebraic points with its p-adic counterpart obeying same algebraic equations but in different number field. In fact, there is a theorem stating that for rational surfaces the number of rational points is finite and rational (more generally algebraic points) would naturally define the notion of number theoretic braid essential for the realization of number theoretic universality.

(b) The most natural option is that the descriptions in terms of both real and p-adic thermodynamics make sense and are consistent. This option indeed makes if the number of generalized eigen modes of modified Dirac operator is finite. This is indeed the case if one accepts periodic boundary conditions for the Chern-Simons Dirac operator. In fact, the solutions are localized at the strands of braids [K28] . This makes sense because the theory has hydrodynamic interpretation [K28] . This reduces $\mathcal{N} = \infty$ to finite SUSY and realizes finite measurement resolution as an inherent property of dynamics.

The finite number of fermionic oscillator operators implies an effective cutoff in the number conformal weights so that conformal algebras reduce to finite-dimensional algebras. The first guess would be that integer label for oscillator operators becomes a number in finite field for some prime. This means that one can calculate mass squared also by using real thermodynamics but the consistency with p-adic thermodynamics gives extremely strong number theoretical constraints on mass scale. This consistency condition allows also to solve the problem how to map a negative ground state conformal weight to its p-adic counterpart. Negative conformal weight is divided into a negative half odd integer part plus positive part Δh, and negative part corresponds as such to p-adic integer whereas positive part is mapped to p-adic number by canonical identification.

p-Adic thermodynamics is what gives to this approach its predictive power.

(a) p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

(b) p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

16.1.3 What next?

The successes of p-adic mass calculations are basically due to the power of super-conformal symmetries and of number theory. One cannot deny that the description of the gauge boson and hadron massivation involves phenomenological elements. There are however excellent hopes that it might be possible some day to calculate everything from first principles. The non-local Yangian symmetry generalizing the super-conformal algebras suggests itself strongly as a fundamental symmetry of quantum TGD. The generalized of the Yangian symmetry replaces points with partonic 2-surfaces being multi-local with respect to them, and leads to general formulas for multi-local operators representing four-momenta and other conserved charges of composite states.

In TGD framework even elementary particles involve two wormhole contacts having each two wormhole throats identified as the fundamental partonic entities. Therefore Yangian approach would naturally define the first principle approach to the understanding of masses of elementary particles and their bound states (say hadrons). The power of this extended symmetry might be enough to deduce universal mass formulas. One of the future challenges would therefore be the mathematical and physical understanding of Yangian symmetry. This would however require the contributions of professional mathematicians.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD view about elementary particles [L78]
- p-Adic mass calculations [L56]
- Zero Energy Ontology (ZEO) [L85]
- Elementary particle vacuum functionals [L32]
- Emergence of bosons [L33]

- Leptohadron hypothesis [L48]
- M89 hadron physics [L50]
- SUSY and TGD [L69]

16.2 Identification of elementary particles

16.2.1 Family replication phenomenon topologically

One of the basic ideas of TGD approach has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of zero energy ontology this picture changed somewhat. It is the wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian, which correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components could allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface. The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and it is the moduli of these partonic 2-surfaces which appear in the elementary particle vacuum functionals naturally.

The first modification of the original simple picture comes from the identification of physical particles as bound states of pairs of wormhole contacts and from the assumption that for generalized Feynman diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats- also those appearing in internal lines- and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. This means that bosons and their super-partners correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. Free fermions and their superpartners could correspond to CP_2 type vacuum extremals with single wormhole throat. It turns however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as (g, g) type wormhole contacts.

Do free fermions correspond to single wormhole throat or (g, g) wormhole?

The original interpretation of genus-generation correspondence was that free fermions correspond to wormhole throats characterized by genus. The idea of $SU(3)$ as a dynamical symmetry suggested that gauge bosons correspond to octet and singlet representations of $SU(3)$. The further idea that all lines of generalized Feynman diagrams are massless poses a strong additional constraint and it is not clear whether this proposal as such survives.

(a) Twistorial program assumes that fundamental objects are massless wormhole throats carrying collinearly moving many-fermion states and also bosonic excitations generated by super-symplectic algebra. In the following consideration only purely bosonic and single fermion throats are considered since they are the basic building blocks of physical particles. The reason is that propagators for high excitations behave like p^{-n}, n the number of fermions associated with the wormhole throat. Therefore single throat allows only spins 0,1/2,1 as elementary particles in the usual sense of the word.

(b) The identification of massive fermions (as opposed to free massless fermions) as wormhole contacts follows if one requires that fundamental building blocks are massless since at least two massless throats are required to have a massive state. Therefore the conformal excitations with CP_2 mass scale should be assignable to wormhole contacts also in the case of fermions. As already noticed this is not the end of the story: weak strings are required by the weak form of electric-magnetic duality.

(c) If free fermions corresponding to single wormhole throat, topological condensation is an essential element of the formation of stringy states. The topological condensation of fermions by topological sum (fermionic CP_2 type vacuum extremal touches another space-time sheet) suggest $(g,0)$ wormhole contact. Note however that the identification of wormhole throat is as 3-surface at which the signature of the induced metric changes so that this conclusion might be wrong. One can indeed consider also the possibility of (g,g) pairs as an outcome of topological conensation. This is suggested also by the idea that wormhole throats are analogous to string like objects and only this option turns out to be consistent with the BFF vertex based on the requirement of dynamical $SU(3)$ symmetry to be discussed later. The structure of reaction vertices makes it possible to interpret (g,g) pairs as $SU(3)$ triplet. If bosons are obtained as fusion of fermionic and anti-fermionic throats (touching of corresponding CP_2 type vacuum extremals) they correspond naturally to (g_1,g_2) pairs.

(d) p-Adic mass calculations distinguish between fermions and bosons and the identification of fermions and bosons should be consistent with this difference. The maximal p-adic temperature $T=1$ for fermions could relate to the weakness of the interaction of the fermionic wormhole throat with the wormhole throat resulting in topological condensation. This wormhole throat would however carry momentum and 3-momentum would in general be non-parallel to that of the fermion, most naturally in the opposite direction.

p-Adic mass calculations suggest strongly that for bosons p-adic temperature $T=1/n$, $n>1$, so that thermodynamical contribution to the mass squared is negligible. The low p-adic temperature could be due to the strong interaction between fermionic and anti-fermionic wormhole throat leading to the "freezing" of the conformal degrees of freedom related to the relative motion of wormhole throats.

(e) The weak form of electric-magnetic duality forces second wormhole throat with opposite magnetic charge and the light-like momenta could sum up to massive momentum. In this case string tension corresponds to electroweak length scale. Therefore p-adic thermodynamics must be assigned to wormhole contacts and these appear as basic units connected by Kähler magnetic flux tube pairs at the two space-time sheets involved. Weak stringy degrees of freedom are however expected to give additional contribution to the mass, perhaps by modifying the ground state conformal weight.

Dynamical $SU(3)$ fixes the identification of fermions and bosons and fundamental interaction vertices

For 3 light fermion families $SU(3)$ suggests itself as a dynamical symmetry with fermions in fundamental $N=3$-dimensional representation and $N\times N=9$ bosons in the adjoint representation and singlet representation. The known gauge bosons have same couplings to fermionic families so that they must correspond to the singlet representation. The first challenge is to understand whether it is possible to have dynamical $SU(3)$ at the level of fundamental reaction vertices.

This is a highly non-trivial constraint. For instance, the vertices in which n wormhole throats with same (g_1,g_2) glued along the ends of lines are not consistent with this symmetry. The splitting of the fermionic worm-hole contacts before the proper vertices for throats might however allow the realization of dynamical $SU(3)$. The condition of $SU(3)$ symmetry combined with the requirement that virtual lines resulting also in the splitting of wormhole contacts are always massless, leads to the conclusion that massive fermions correspond to (g, g) type wormhole contacts transforming naturally like $SU(3)$ triplet. This picture conformsl with the identification of free fermions as throats but not with the naive expectation that their topological condensation gives rise to $(g, 0)$ wormhole contact.

The argument leading to these conclusions runs as follows.

(a) The question is what basic reaction vertices are allowed by dynamical $SU(3)$ symmetry. FFB vertices are in principle all that is needed and they should obey the dynamical symmetry. The meeting of entire wormhole contacts along their ends is certainly not possible. The splitting of fermionic wormhole contacts before the vertices might be however consistent with $SU(3)$ symmetry. This would give two a pair of 3-vertices at which three wormhole lines meet along partonic 2-surfaces (rather than along 3-D wormhole contacts).

(b) Note first that crossing gives all possible reaction vertices of this kind from $F(g_1)\overline{F}(g_2) \rightarrow B(g_1, g_2)$ annihilation vertex, which is relatively easy to visualize. In this reaction $F(g_1)$ and $\overline{F}(g_2)$ wormhole contacts split first. If one requires that all wormhole throats involved are massless, the two wormhole throats resulting in splitting and carrying no fermion number must carry light-like momentum so that they cannot just disappear. The ends of the wormhole throats of the boson must glued together with the end of the fermionic wormhole throat and its companion generated in the splitting of the wormhole. This means that fermionic wormhole first splits and the resulting throats meet at the partonic 2-surface.

his requires that topologically condensed fermions correspond to (g, g) pairs rather than $(g, 0)$ pairs. The reaction mechanism allows the interpretation of (g, g) pairs as a triplet of dynamical $SU(3)$. The fundamental vertices would be just the splitting of wormhole contact and 3-vertices for throats since $SU(3)$ symmetry would exclude more complex reaction vertices such as n-boson vertices corresponding the gluing of n wormhole contact lines along their 3-dimensional ends. The couplings of singlet representation for bosons would have same coupling to all fermion families so that the basic experimental constraint would be satisfied.

(c) Both fermions and bosons cannot correspond to octet and singlet of $SU(3)$. In this case reaction vertices should correspond algebraically to the multiplication of matrix elements e_{ij}: $e_{ij}e_{kl} = \delta_{jk}e_{il}$ allowing for instance $F(g_1, g_2) + \overline{F}(g_2, g_3) \rightarrow B(g_1, g_3)$. Neither the fusion of entire wormhole contacts along their ends nor the splitting of wormhole throats before the fusion of partonic 2-surfaces allows this kind of vertices so that BFF vertex is the only possible one. Also the construction of QFT limit starting from bosonic emergence led to the formulation of perturbation theory in terms of Dirac action allowing only BFF vertex as fundamental vertex [K29] .

(d) Weak electric-magnetic duality brings in an additional complication. $SU(3)$ symmetry poses also now strong constraints and it would seem that the reactions must involve copies of basic BFF vertices for the pairs of ends of weak strings. The string ends with the same Kähler magnetic charge should meet at the vertex and give rise to BFF vertices. For instance, $F\overline{F}B$ annihilation vertex would in this manner give rise to the analog of stringy diagram in which strings join along ends since two string ends disappear in the process.

If one accepts this picture the remaining question is why the number of genera is just three. Could this relate to the fact that $g \leq 2$ Riemann surfaces are always hyper-elliptic (have global Z_2 conformal symmetry) unlike $g > 2$ surfaces? Why the complete bosonic de-localization of the light families should be restricted inside the hyper-elliptic sector? Does the Z_2 conformal symmetry make these states light and make possible de-localization and dynamical $SU(3)$ symmetry? Could it be that for $g > 2$ elementary particle vacuum functionals vanish for hyper-elliptic surfaces? If this the case and if the time evolution for partonic 2-surfaces changing g commutes with Z_2 symmetry then the vacuum functionals localized to $g \leq 2$ surfaces do not disperse to $g > 2$ sectors.

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals, is made.

The basic assumptions underlying the construction are the following ones:

(a) Elementary particle vacuum functionals depend on the geometric properties of the two-surface X^2 representing elementary particle.

(b) Vacuum functionals possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting preferred extremal of the Kähler action as a generalized Bohr orbit [K40]) and determined in $Diff^3$ invariant manner.

(c) Zero energy ontology allows to select uniquely the partonic two surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower boundary of $CD \times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

(d) Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.

(e) Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.

(f) Elementary particle vacuum functionals are stable against the decay $g \to g_1 + g_2$ and one particle decay $g \to g - 1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized Feynman diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K19] the construction of elementary particle vacuum functionals is described in more detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered.

16.2.2 Basic facts about Riemann surfaces

In the following some basic aspects about Riemann surfaces will be summarized. The basic topological concepts, in particular the concept of the mapping class group, are introduced, and the Teichmueller parameters are defined as conformal invariants of the Riemann surface, which in fact specify the conformal equivalence class of the Riemann surface completely.

Mapping class group

The first homology group $H_1(X^2)$ of a Riemann surface of genus g contains $2g$ generators [A90, A69, A95] : this is easy to understand geometrically since each handle contributes two homology generators. The so called canonical homology basis can be .

One can define the so called intersection $J(a, b)$ for two elements a and b of the homology group as the number of intersection points for the curves a and b counting the orientation. Since $J(a, b)$ depends on the homology classes of a and b only, it defines an antisymmetric quadratic form in $H_1(X^2)$. In the canonical homology basis the non-vanishing elements of the intersection matrix are:

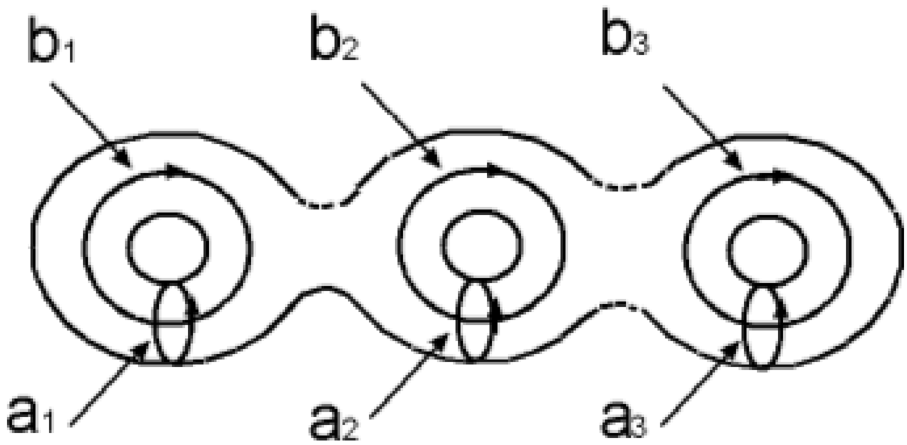

Figure 16.1: Definition of the canonical homology basis

$$J(a_i,b_j) \quad = \quad -J(b_j,a_i) = \delta_{i,j} \ . \tag{16.2.1}$$

J clearly defines symplectic structure in the homology group.

The dual to the canonical homology basis consists of the harmonic one-forms $\alpha_i, \beta_i, i = 1,..,g$ on X^2. These 1-forms satisfy the defining conditions

$$\begin{array}{ll} \int_{a_i} \alpha_j = \delta_{i,j} & \int_{b_i} \alpha_j = 0 \ , \\ \int_{a_i} \beta_j = 0 & \int_{b_i} \beta_j = \delta_{i,j} \ . \end{array} \tag{16.2.2}$$

The following identity helps to understand the basic properties of the Teichmueller parameters

$$\int_{X^2} \theta \wedge \eta \quad = \quad \sum_{i=1,..,g} [\int_{a_i} \theta \int_{b_i} \eta - \int_{b_i} \theta \int_{a_i} \eta] \ . \tag{16.2.3}$$

The existence of topologically nontrivial diffeomorphisms, when X^2 has genus $g > 0$, plays an important role in the sequel. Denoting by $Diff$ the group of the diffeomorphisms of X^2 and by $Diff_0$ the normal subgroup of the diffeomorphisms homotopic to identity, one can define the mapping class group M as the coset group

$$M \quad = \quad Diff/Diff_0 \ . \tag{16.2.4}$$

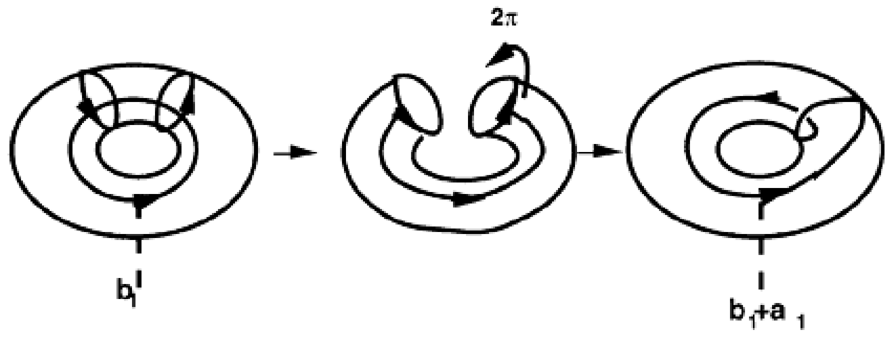

Figure 16.2: Definition of the Dehn twist

The generators of M are so called Dehn twists along closed curves a of X^2. Dehn twist is defined by excising a small tubular neighborhood of a, twisting one boundary of the resulting tube by 2π and gluing the tube back into the surface: see Fig. 16.2.2.

It can be shown that a minimal set of generators is defined by the following curves

$$a_1, b_1, a_1^{-1}a_2^{-1}, a_2, b_2, a_2^{-1}a_3^{-11}, ..., a_g, b_g \quad . \tag{16.2.5}$$

The action of these transformations in the homology group can be regarded as a symplectic linear transformation preserving the symplectic form defined by the intersection matrix. Therefore the matrix representing the action of $Diff$ on $H_1(X^2)$ is $2g \times 2g$ matrix M with integer entries leaving J invariant: $MJM^T = J$. Mapping class group is often referred also and denoted by $Sp(2g, Z)$. The matrix representing the action of M in the canonical homology basis decomposes into four $g \times g$ blocks A, B, C and D

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad , \tag{16.2.6}$$

where A and D operate in the subspaces spanned by the homology generators a_i and b_i respectively and C and D map these spaces to each other. The notation $D = [A, B; C, D]$ will be used in the sequel: in this notation the representation of the symplectic form J is $J = [0, 1; -1, 0]$.

Teichmueller parameters

The induced metric on the two-surface X^2 defines a unique complex structure. Locally the metric can always be written in the form

$$ds^2 \quad = \quad e^{2\phi}dzd\bar{z} \quad . \tag{16.2.7}$$

where z is local complex coordinate. When one covers X^2 by coordinate patches, where the line element has the above described form, the transition functions between coordinate patches are holomorphic and therefore define a complex structure.

The conformal transformations ξ of X^2 are defined as the transformations leaving invariant the angles between the vectors of X^2 tangent space invariant: the angle between the vectors

X and Y at point x is same as the angle between the images of the vectors under Jacobian map at the image point $\xi(x)$. These transformations need not be globally defined and in each coordinate patch they correspond to holomorphic (anti-holomorphic) mappings as is clear from the diagonal form of the metric in the local complex coordinates. A distinction should be made between local conformal transformations and globally defined conformal transformations, which will be referred to as conformal symmetries: for instance, for hyper-elliptic surfaces the group of the conformal symmetries contains two-element group Z_2.

Using the complex structure one can decompose one-forms to linear combinations of one-forms of type $(1,0)$ $(f(z,\bar{z})dz)$ and $(0,1)$ $(f(z,\bar{z})d\bar{z})$. $(1,0)$ form ω is holomorphic if the function f is holomorphic: $\omega = f(z)dz$ on each coordinate patch.

There are g independent holomorphic one forms ω_i known also as Abelian differentials Alvarez,Farkas,Mumford and one can fix their normalization by the condition

$$\int_{a_i} \omega_j = \delta_{ij} \ . \tag{16.2.8}$$

This condition completely specifies ω_i.

Teichmueller parameters Ω_{ij} are defined as the values of the forms ω_i for the homology generators b_j

$$\Omega_{ij} = \int_{b_j} \omega_i \ . \tag{16.2.9}$$

The basic properties of Teichmueller parameters are the following:

(a) The $g\times g$ matrix Ω is symmetric: this is seen by applying the formula (16.2.3) for $\theta = \omega_i$ and $\eta = \omega_j$.

(b) The imaginary part of Ω is positive: $Im(\Omega) > 0$. This is seen by the application of the same formula for $\theta = \eta$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.

(c) The space of Teichmueller parameters can be regarded as a coset space $Sp(2g,R)/U(g)$ [A95] : the action of $Sp(2g,R)$ is of the same form as the action of $Sp(2g,Z)$ and $U(g) \subset Sp(2g,R)$ is the isotropy group of a given point of Teichmueller space.

(d) Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.

(e) Teichmueller parameters specify completely the conformal structure of Riemann surface [A69] .

Although Teichmueller parameters fix the conformal structure of the 2-surface completely, they are not in one-to-one correspondence with the conformal equivalence classes of the two-surfaces:
i) The dimension for the space of the conformal equivalence classes is $D = 3g-3$, when $g > 1$ and smaller than the dimension of Teichmueller space given by $d = (g\times g + g)/2$ for $g > 3$: all Teichmueller matrices do not correspond to a Riemann surface. In TGD approach this does not produce any problems as will be found later.
ii) The action of the topologically nontrivial diffeomorphisms on Teichmueller parameters is nontrivial and can be deduced from the action of the diffeomorphisms on the homology ($Sp(2g,Z)$ transformation) and from the defining condition $\int_{a_i} \omega_j = \delta_{i,j}$: diffeomorphisms correspond to elements $[A,B;C,D]$ of $Sp(2g,Z)$ and act as generalized Möbius transformations

$$\Omega \to (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{16.2.10}$$

All Teichmueller parameters related by $Sp(2g, Z)$ transformations correspond to the same Riemann surface.
iii) The definition of the Teichmueller parameters is not unique since the definition of the canonical homology basis involves an arbitrary numbering of the homology basis. The permutation S of the handles is represented by same $g \times g$ orthogonal matrix both in the basis $\{a_i\}$ and $\{b_i\}$ and induces a similarity transformation in the space of the Teichmueller parameters

$$\Omega \to S\Omega S^{-1} \ . \tag{16.2.11}$$

Clearly, the Teichmueller matrices related by a similarity transformations correspond to the same conformal equivalence class. It is easy to show that handle permutations in fact correspond to $Sp(2g, Z)$ transformations.

Hyper-ellipticity

The motivation for considering hyper-elliptic surfaces comes from the fact, that $g > 2$ elementary particle vacuum functionals turn out to be vanishing for hyper-elliptic surfaces and this in turn will be later used to provide a possible explanation the non-observability of $g > 2$ particles.

Hyper-elliptic surface X can be defined abstractly as two-fold branched cover of the sphere having the group Z_2 as the group of conformal symmetries (see [A38, A69, A95] . Thus there exists a map $\pi : X \to S^2$ so that the inverse image $\pi^{-1}(z)$ for a given point z of S^2 contains two points except at a finite number (say p) of points z_i (branch points) for which the inverse image contains only one point. Z_2 acts as conformal symmetries permuting the two points in $\pi^{-1}(z)$ and branch points are fixed points of the involution.

The concept can be generalized [A38] : g-hyper-elliptic surface can be defined as a 2-fold covering of genus g surface with a finite number of branch points. One can consider also p-fold coverings instead of 2-fold coverings: a common feature of these Riemann surfaces is the existence of a discrete group of conformal symmetries.

A concrete representation for the hyper-elliptic surfaces [A95] is obtained by studying the surface of C^2 determined by the algebraic equation

$$w^2 - P_n(z) \ = \ 0 \ , \tag{16.2.12}$$

where w and z are complex variables and $P_n(z)$ is a complex polynomial. One can solve w from the above equation

$$w_{\pm} \ = \ \pm\sqrt{P_n(z)} \ , \tag{16.2.13}$$

where the square root is determined so that it has a cut along the positive real axis. What happens that w has in general two roots (two-fold covering property), which coincide at the roots z_i of $P_n(z)$ and if n is odd, also at $z = \infty$: these points correspond to branch points of the hyper-elliptic surface and their number r is always even: $r = 2k$. w is discontinuous at the cuts associated with the square root in general joining two roots of $P_n(z)$ or if n is odd, also some root of P_n and the point $z = \infty$. The representation of the hyper-elliptic surface is obtained by identifying the two branches of w along the cuts. From the construction it is clear that the surface obtained in this manner has genus $k-1$. Also it is clear that Z_2 permutes the different roots $w_\pm$ with each other and that $r=2k$ branch points correspond to fixed points of the involution.

The following facts about the hyper-elliptic surfaces [A69, A95] turn out to be important in the sequel:
i) All $g<3$ surfaces are hyper-elliptic.
ii) $g\geq 3$ hyper-elliptic surfaces are not in general hyper-elliptic and form a set of codimension 2 in the space of the conformal equivalence classes [A95] .

Theta functions

An extensive and detailed account of the theta functions and their applications can be found in the book of Mumford [A95] . Theta functions appear also in the loop calculations of string [J8] [A90] . In the following the so called Riemann theta function and theta functions with half integer characteristics will be defined as sections (not strictly speaking functions) of the so called Jacobian variety.

For a given Teichmueller matrix Ω, Jacobian variety is defined as the $2g$-dimensional torus obtained by identifying the points z of C^g (vectors with g complex components) under the equivalence

$$z\sim z+\Omega m+n \ , \tag{16.2.14}$$

where m and n are points of Z^g (vectors with g integer valued components) and Ω acts in Z^g by matrix multiplication.

The definition of Riemann theta function reads as

$$\Theta(z|\Omega) = \sum_n exp(i\pi n\cdot\Omega\cdot n+i2\pi n\cdot z) \ . \tag{16.2.15}$$

Here $\cdot$ denotes standard inner product in C^g. Theta functions with half integer characteristics are defined in the following manner. Let a and b denote vectors of C^g with half integer components (component either vanishes or equals to $1/2$). Theta function with characteristics $[a,b]$ is defined through the following formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a)+i2\pi(n+a)\cdot(z+b)\right] \ . \tag{16.2.16}$$

A brief calculation shows that the following identity is satisfied

$$\Theta[a,b](z|\Omega) = exp(i\pi a\cdot\Omega\cdot a+i2\pi a\cdot b)\times\Theta(z+\Omega a+b|\Omega) \tag{16.2.17}$$

Theta functions are not strictly speaking functions in the Jacobian variety but rather sections in an appropriate bundle as can be seen from the identities

$$\begin{aligned} \Theta[a,b](z+m|\Omega) &= exp(i2\pi a\cdot m)\Theta[a,b](z\Omega) \ , \\ \Theta[a,b](z+\Omega m|\Omega) &= exp(\alpha)\Theta[a,b](z|\Omega) \ , \\ exp(\alpha) &= exp(-i2\pi b\cdot m)exp(-i\pi m\cdot\Omega\cdot m - 2\pi m\cdot z) \ . \end{aligned} \quad (16.2.18)$$

The number of theta functions is 2^{2g} and same as the number of nonequivalent spinor structures defined on two-surfaces. This is not an accident [A90] : theta functions with given characteristics turn out to be in a close relation to the functional determinants associated with the Dirac operators defined on the two-surface. It is useful to divide the theta functions to even and odd theta functions according to whether the inner product $4a\cdot b$ is even or odd integer. The numbers of even and odd theta functions are $2^{g-1}(2^g+1)$ and $2^{g-1}(2^g-1)$ respectively.

The values of the theta functions at the origin of the Jacobian variety understood as functions of Teichmueller parameters turn out to be of special interest in the following and the following notation will be used:

$$\Theta[a,b](\Omega) \equiv \Theta[a,b](0|\Omega) \ , \quad (16.2.19)$$

$\Theta[a,b](\Omega)$ will be referred to as theta functions in the sequel. From the defining properties of odd theta functions it can be found that they are odd functions of z and therefore vanish at the origin of the Jacobian variety so that only even theta functions will be of interest in the sequel.

An important result is that also some *even* theta functions vanish for $g>2$ hyper-elliptic surfaces : in fact one can characterize $g>2$ hyper-elliptic surfaces by the vanishing properties of the theta functions [A69, A95] . The vanishing property derives from conformal symmetry (Z_2 in the case of hyper-elliptic surfaces) and the vanishing phenomenon is rather general [A38] : theta functions tend to vanish for Riemann surfaces possessing discrete conformal symmetries. It is not clear (to the author) whether the presence of a conformal symmetry is in fact equivalent with the vanishing of some theta functions. As already noticed, spinor structures and the theta functions with half integer characteristics are in one-to-one correspondence and the vanishing of theta function with given half integer characteristics is equivalent with the vanishing of the Dirac determinant associated with the corresponding spinor structure or equivalently: with the existence of a zero mode for the Dirac operator Alvarez . For odd characteristics zero mode exists always: for even characteristics zero modes exist, when the surface is hyper-elliptic or possesses more general conformal symmetries.

16.2.3 Elementary particle vacuum functionals

The basic assumption is that elementary particle families correspond to various elementary particle vacuum functionals associated with the 2-dimensional boundary components of the 3-surface. These functionals need not be localized to a single boundary topology. Neither need their dependence on the boundary component be local. An important role in the following considerations is played by the fact that the minimization requirement of the Kähler action associates a unique 3-surface to each boundary component, the "Bohr orbit" of the boundary and this surface provides a considerable (and necessarily needed) flexibility in the definition of the elementary particle vacuum functionals. There are several natural constraints to be satisfied by elementary particle vacuum functionals.

Extended Diff invariance and Lorentz invariance

Extended Diff invariance is completely analogous to the extension of 3-dimensional Diff invariance to four-dimensional Diff invariance in the interior of the 3-surface. Vacuum functional must be invariant not only under diffeomorphisms of the boundary component but also under the diffeomorphisms of the 3- dimensional "orbit" Y^3 of the boundary component. In other words: the value of the vacuum functional must be same for any time slice on the orbit the boundary component. This is guaranteed if vacuum functional is functional of some two-surface Y^2 belonging to the orbit and defined in $Diff^3$ invariant manner.

An additional natural requirement is Poincare invariance. In the original formulation of the theory only Lorentz transformations of the light cone were exact symmetries of the theory. In this framework the definition of Y^2 as the intersection of the orbit with the hyperboloid $\sqrt{m_{kl}m^km^l} = a$ is $Diff^3$ and Lorentz invariant.

1. Interaction vertices as generalization of stringy vertices

For stringy diagrams Poincare invariance of conformal equivalence class and general coordinate invariance are far from being a trivial issues. Vertices are now not completely unique since there is an infinite number of singular 3-manifolds which can be identified as vertices even if one assumes space-likeness. One should be able to select a unique singular 3-manifold to fix the conformal equivalence class.

One might hope that Lorentz invariant invariant and general coordinate invariant definition of Y^2 results by introducing light cone proper time a as a height function specifying uniquely the point at which 3-surface is singular (stringy diagrams help to visualize what is involved), and by restricting the singular 3-surface to be the intersection of $a = constant$ hyperboloid of M^4 containing the singular point with the space-time surface. There would be non-uniqueness of the conformal equivalence class due to the choice of the origin of the light cone but the decomposition of the configuration space of 3-surfaces to a union of WCWs characterized by unions of future and past light cones could resolve this difficulty.

2. Interaction vertices as generalization of ordinary ones

If the interaction vertices are identified as intersections for the ends of space-time sheets representing particles, the conformal equivalence class is naturally identified as the one associated with the intersection of the boundary component or light like causal determinant with the vertex. Poincare invariance of the conformal equivalence class and generalized general coordinate invariance follow trivially in this case.

Conformal invariance

Conformal invariance implies that vacuum functionals depend on the conformal equivalence class of the surface Y^2 only. What makes this idea so attractive is that for a given genus g WCW becomes effectively finite-dimensional. A second nice feature is that instead of trying to find coordinates for the space of the conformal equivalence classes one can construct vacuum functionals as functions of the Teichmueller parameters.

That one can construct this kind of functions as suitable functions of the Teichmueller parameters is not trivial. The essential point is that the boundary components can be regarded as sub-manifolds of $M^4_+ \times CP_2$: as a consequence vacuum functional can be regarded as a composite function:

2-surface $\rightarrow$ Teichmueller matrix Ω determined by the induced metric $\rightarrow \Omega_{vac}(\Omega)$

Therefore the fact that there are Teichmueller parameters which do not correspond to any Riemann surface, doesn't produce any trouble. It should be noticed that the situation differs from that in the Polyakov formulation of string models, where one doesn't assume that the metric of the two-surface is induced metric (although classical equations of motion imply this).

Diff invariance

Since several values of the Teichmueller parameters correspond to the same conformal equivalence class, one must pose additional conditions on the functions of the Teichmueller parameters in order to obtain single valued functions of the conformal equivalence class.

The first requirement of this kind is the invariance under topologically nontrivial Diff transformations inducing $Sp(2g,Z)$ transformation $(A,B;C,D)$ in the homology basis. The action of these transformations on Teichmueller parameters is deduced by requiring that holomorphic one-forms satisfy the defining conditions in the transformed homology basis. It turns out that the action of the topologically nontrivial diffeomorphism on Teichmueller parameters can be regarded as a generalized Möbius transformation:

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \quad . \tag{16.2.20}$$

Vacuum functional must be invariant under these transformations. It should be noticed that the situation differs from that encountered in the string models. In TGD the integration measure over WCW is Diff invariant: in string models the integration measure is the integration measure of the Teichmueller space and this is not invariant under $Sp(2g,Z)$ but transforms like a density: as a consequence the counterpart of the vacuum functional must be also modular covariant since it is the product of vacuum functional and integration measure, which must be modular invariant.

It is possible to show that the quantities

$$(\Theta[a,b]/\Theta[c,d])^4 \quad . \tag{16.2.21}$$

and their complex conjugates are $Sp(2g,Z)$ invariants [A95] and therefore can be regarded as basic building blocks of the vacuum functionals.

Teichmueller parameters are not uniquely determined since one can always perform a permutation of the g handles of the Riemann surface inducing a redefinition of the canonical homology basis (permutation of g generators). These transformations act as similarities of the Teichmueller matrix:

$$\Omega \rightarrow S\Omega S^{-1} \quad , \tag{16.2.22}$$

where S is the $g\times g$ matrix representing the permutation of the homology generators understood as orthonormal vectors in the g- dimensional vector space. Therefore the Teichmueller parameters related by these similarity transformations correspond to the same conformal equivalence class of the Riemann surfaces and vacuum functionals must be invariant under these similarities.

It is easy to find out that these similarities permute the components of the theta characteristics: $[a,b] \rightarrow [S(a),S(b)]$. Therefore the invariance requirement states that the handles of the Riemann surface behave like bosons: the vacuum functional constructed from the theta functions is invariant under the permutations of the theta characteristics. In fact, this requirement brings in nothing new. Handle permutations can be regarded as $Sp(2g,Z)$ transformations so that the modular invariance alone guarantees invariance under handle permutations.

Cluster decomposition property

Consider next the behavior of the vacuum functional in the limit, when boundary component with genus g splits to two separate boundary components of genera g_1 and g_2 respectively. The splitting into two separate boundary components corresponds to the reduction of the Teichmueller matrix Ω^g to a direct sum of $g_1 \times g_1$ and $g_2 \times g_2$ matrices ($g_1 + g_2 = g$):

$$\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2} \quad , \tag{16.2.23}$$

when a suitable definition of the Teichmueller parameters is adopted. The splitting can also take place without a reduction to a direct sum: the Teichmueller parameters obtained via $Sp(2g, Z)$ transformation from $\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2}$ do not possess direct sum property in general.

The physical interpretation is obvious: the non-diagonal elements of the Teichmueller matrix describe the geometric interaction between handles and at this limit the interaction between the handles belonging to the separate surfaces vanishes. On the physical grounds it is natural to require that vacuum functionals satisfy cluster decomposition property at this limit: that is they reduce to the product of appropriate vacuum functionals associated with the composite surfaces.

Theta functions satisfy cluster decomposition property [A90, A95] . Theta characteristics reduce to the direct sums of the theta characteristics associated with g_1 and g_2 ($a = a_1 \oplus a_2$, $b = b_1 \oplus b_2$) and the dependence on the Teichmueller parameters is essentially exponential so that the cluster decomposition property indeed results:

$$\Theta[a, b](\Omega^g) = \Theta[a_1, b_1](\Omega^{g_1})\Theta[a_2, b_2](\Omega^{g_2}) \quad . \tag{16.2.24}$$

Cluster decomposition property holds also true for the products of theta functions. This property is also satisfied by suitable homogenous polynomials of thetas. In particular, the following quantity playing central role in the construction of the vacuum functional obeys this property

$$Q_0 = \sum_{[a,b]} \Theta[a, b]^4 \bar{\Theta}[a, b]^4 \quad , \tag{16.2.25}$$

where the summation is over all even theta characteristics (recall that odd theta functions vanish at the origin of C^g).

Together with the $Sp(2g, Z)$ invariance the requirement of cluster decomposition property implies that the vacuum functional must be representable in the form

$$\Omega_{vac} = P_{M,N}(\Theta^4, \bar{\Theta}^4)/Q_{MN}(\Theta^4, \bar{\Theta}^4) \tag{16.2.26}$$

where the homogenous polynomials $P_{M,N}$ and $Q_{M,N}$ have same degrees (M and N as polynomials of $\Theta[a, b]^4$ and $\bar{\Theta}[a, b]^4$.

Finiteness requirement

Vacuum functional should be finite. Finiteness requirement is satisfied provided the numerator $Q_{M,N}$ of the vacuum functional is real and positive definite. The simplest quantity of this type is the quantity Q_0 defined previously and its various powers. $Sp(2g,Z)$ invariance and finiteness requirement are satisfied provided vacuum functionals are of the following general form

$$\Omega_{vac} = \frac{P_{N,N}(\Theta^4,\bar{\Theta}^4)}{Q_0^N} , \qquad (16.2.27)$$

where $P_{N,N}$ is homogenous polynomial of degree N with respect to $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$. In addition $P_{N,N}$ is invariant under the permutations of the theta characteristics and satisfies cluster decomposition property.

Stability against the decay $g \rightarrow g_1 + g_2$

Elementary particle vacuum functionals must be stable against the genus conserving decays $g \rightarrow g_1 + g_2$. This decay corresponds to the limit at which Teichmueller matrix reduces to a direct sum of the matrices associated with g_1 and g_2 (note however the presence of $Sp(2g,Z)$ degeneracy). In accordance with the topological description of the particle reactions one expects that this decay doesn't occur if the vacuum functional in question vanishes at this limit.

In general the theta functions are non-vanishing at this limit and vanish provided the theta characteristics reduce to a direct sum of the odd theta characteristics. For $g<2$ surfaces this condition is trivial and gives no constraints on the form of the vacuum functional. For $g=2$ surfaces the theta function $\Theta(a,b)$, with $a=b=(1/2,1/2)$ satisfies the stability criterion identically (odd theta functions vanish identically), when Teichmueller parameters separate into a direct sum. One can however perform $Sp(2g,Z)$ transformations giving new points of Teichmueller space describing the decay. Since these transformations transform theta characteristics in a nontrivial manner to each other and since all even theta characteristics belong to same $Sp(2g,Z)$ orbit [A90, A95] , the conclusion is that stability condition is satisfied provided $g=2$ vacuum functional is proportional to the product of fourth powers of all even theta functions multiplied by its complex conjugate.

If $g>2$ there always exists some theta functions, which vanish at this limit and the minimal vacuum functional satisfying this stability condition is of the same form as in $g=2$ case, that is proportional to the product of the fourth powers of all even Theta functions multiplied by its complex conjugate:

$$\Omega_{vac} = \prod_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 / Q_0^N , \qquad (16.2.28)$$

where N is the number of even theta functions. The results obtained imply that genus-generation correspondence is one to one for $g>1$ for the minimal vacuum functionals. Of course, the multiplication of the minimal vacuum functionals with functionals satisfying all criteria except stability criterion gives new elementary particle vacuum functionals: a possible physical identification of these vacuum functionals is most naturally as some kind of excited states.

One of the questions posed in the beginning was related to the experimental absence of $g>0$, possibly massless, elementary bosons. The proposed stability criterion suggests a nice explanation. The point is that elementary particles are stable against decays $g \rightarrow g_1 + g_2$ but not with respect to the decay $g \rightarrow g +$ *sphere*. As a consequence the direct emission of $g>0$ gauge bosons is impossible unlike the emission of $g=0$ bosons: for instance the decay muon $\rightarrow$ electron $+(g=1)$ photon is forbidden.

Stability against the decay $g \rightarrow g-1$

This stability criterion states that the vacuum functional is stable against single particle decay $g \rightarrow g-1$ and, if satisfied, implies that vacuum functional vanishes, when the genus of the surface is smaller than g. In stringy framework this criterion is equivalent to a separate conservation of various lepton numbers: for instance, the spontaneous transformation of muon to electron is forbidden. Notice that this condition doesn't imply that that the vacuum functional is localized to a single genus: rather the vacuum functional of genus g vanishes for all surfaces with genus smaller than g. This hierarchical structure should have a close relationship to Cabibbo-Kobayashi-Maskawa mixing of the quarks.

The stability criterion implies that the vacuum functional must vanish at the limit, when one of the handles of the Riemann surface suffers a pinch. To deduce the behavior of the theta functions at this limit, one must find the behavior of Teichmueller parameters, when i:th handle suffers a pinch. Pinch implies that a suitable representative of the homology generator a_i or b_i contracts to a point.

Consider first the case, when a_i contracts to a point. The normalization of the holomorphic one-form ω_i must be preserved so that that ω_i must behaves as $1/z$, where z is the complex coordinate vanishing at pinch. Since the homology generator b_i goes through the pinch it seems obvious that the imaginary part of the Teichmueller parameter $\Omega_{ii} = \int_{b_i} \omega_i$ diverges at this limit (this conclusion is made also in [A95]): $Im(\Omega_{ii}) \rightarrow \infty$.

Of course, this criterion doesn't cover all possible manners the pinch can occur: pinch might take place also, when the components of the Teichmueller matrix remain finite. In the case of torus topology one finds that $Sp(2g,Z)$ element $(A,B;C,D)$ takes $Im(\Omega)=\infty$ to the point C/D of real axis. This suggests that pinch occurs always at the boundary of the Teichmueller space: the imaginary part of Ω_{ij} either vanishes or some matrix element of $Im(\Omega)$ diverges.

Consider next the situation, when b_i contracts to a point. From the definition of the Teichmueller parameters it is clear that the matrix elements Ω_{kl}, with $k,l \neq i$ suffer no change. The matrix element Ω_{ki} obviously vanishes at this limit. The conclusion is that i:th row of Teichmueller matrix vanishes at this limit. This result is obtained also by deriving the $Sp(2g,Z)$ transformation permuting a_i and b_i with each other: in case of torus this transformation reads $\Omega \rightarrow -1/\Omega$.

Consider now the behavior of the theta functions, when pinch occurs. Consider first the limit, when $Im(\Omega_{ii})$ diverges. Using the general definition of $\Theta[a,b]$ it is easy to find out that all theta functions for which the i:th component a_i of the theta characteristic is non-vanishing (that is $a_i = 1/2$) are proportional to the exponent $exp(-\pi\Omega_{ii}/4)$ and therefore vanish at the limit. The theta functions with $a_i = 0$ reduce to $g-1$ dimensional theta functions with theta characteristic obtained by dropping i:th components of a_i and b_i and replacing Teichmueller matrix with Teichmueller matrix obtained by dropping i:th row and column. The conclusion is that all theta functions of type $\Theta(a,b)$ with $a = (1/2,1/2,....,1/2)$ satisfy the stability criterion in this case.

What happens for the $Sp(2g,Z)$ transformed points on the real axis? The transformation formula for theta function is given by [A90, A95]

$$\Theta[a,b]((A\Omega+B)(C\Omega+D)^{-1}) \quad = \quad exp(i\phi)det(C\Omega+D)^{1/2}\Theta[c,d](\Omega) \quad , \tag{16.2.29}$$

where

$$\left(\begin{array}{c} c \\ d \end{array} \right) = \left(\begin{array}{cc} A & B \\ C & D \end{array} \right) \left(\left(\begin{array}{c} a \\ b \end{array} \right) - \left(\begin{array}{c} (CD^T)_d/2 \\ (AB^T)_d/2 \end{array} \right) \right) \quad . \tag{16.2.30}$$

Here ϕ is a phase factor irrelevant for the recent purposes and the index d refers to the diagonal part of the matrix in question.

The first thing to notice is the appearance of the diverging square root factor, which however disappears from the vacuum functionals (P and Q have same degree with respect to thetas). The essential point is that theta characteristics transform to each other: as already noticed all even theta characteristics belong to the same $Sp(2g, Z)$ orbit. Therefore the theta functions vanishing at $Im(\Omega_{ii}) = \infty$ do not vanish at the transformed points. It is however clear that for a given Teichmueller parameterization of pinch some theta functions vanish always.

Similar considerations in the case $\Omega_{ik} = 0$, i fixed, show that all theta functions with $b = (1/2, ..., 1/2)$ vanish identically at the pinch. Also it is clear that for $Sp(2g, Z)$ transformed points one can always find some vanishing theta functions. The overall conclusion is that the elementary particle vacuum functionals obtained by using $g \to g_1 + g_2$ stability criterion satisfy also $g \to g-1$ stability criterion since they are proportional to the product of all even theta functions. Therefore the only nontrivial consequence of $g \to g-1$ criterion is that also $g = 1$ vacuum functionals are of the same general form as $g > 1$ vacuum functionals.

A second manner to deduce the same result is by restricting the consideration to the hyper-elliptic surfaces and using the representation of the theta functions in terms of the roots of the polynomial appearing in the definition of the hyper-elliptic surface [A95] . When the genus of the surface is smaller than three (the interesting case), this representation is all what is needed since all surfaces of genus $g < 3$ are hyper-elliptic.

Since hyper-elliptic surfaces can be regarded as surfaces obtained by gluing two compactified complex planes along the cuts connecting various roots of the defining polynomial it is obvious that the process $g \to g-1$ corresponds to the limit, when two roots of the defining polynomial coincide. This limit corresponds either to disappearance of a cut or the fusion of two cuts to a single cut. Theta functions are expressible as the products of differences of various roots (Thomae's formula [A95])

$$\Theta[a,b]^4 \propto \prod_{i<j\in T} (z_i - z_j) \prod_{k<l\in CT} (z_k - z_l) \ , \tag{16.2.31}$$

where T denotes some subset of $\{1, 2, ..., 2g\}$ containing $g+1$ elements and CT its complement. Hence the product of all even theta functions vanishes, when two roots coincide. Furthermore, stability criterion is satisfied only by the product of the theta functions.

Lowest dimensional vacuum functionals are worth of more detailed consideration.
i) $g = 0$ particle family corresponds to a constant vacuum functional: by continuity this vacuum functional is constant for all topologies.
ii) For $g = 1$ the degree of P and Q as polynomials of the theta functions is 24: the critical number of transversal degrees of freedom in bosonic string model! Probably this result is not an accident.
ii) For $g = 2$ the corresponding degree is 80 since there are 10 even genus 2 theta functions.

There are large numbers of vacuum functionals satisfying the relevant criteria, which do not satisfy the proposed stability criteria. These vacuum functionals correspond either to many particle states or to unstable single particle states.

Continuation of the vacuum functionals to higher genus topologies

From continuity it follows that vacuum functionals cannot be localized to single boundary topology. Besides continuity and the requirements listed above, a natural requirement is that the continuation of the vacuum functional from the sector g to the sector $g + k$ reduces to the product of the original vacuum functional associated with genus g and $g = 0$ vacuum functional at the limit when the surface with genus $g+k$ decays to surfaces with genus g and k: this requirement should guarantee the conservation of separate lepton numbers although different boundary topologies suffer mixing in the vacuum functional. These requirements are satisfied provided the continuation is constructed using the following rule:

Perform the replacement

$$\Theta[a,b]^4 \quad \rightarrow \quad \sum_{c,d} \Theta[a \oplus c, b \oplus d]^4 \tag{16.2.32}$$

for each fourth power of the theta function. Here c and d are Theta characteristics associated with a surface with genus k. The same replacement is performed for the complex conjugates of the theta function. It is straightforward to check that the continuations of elementary particle vacuum functionals indeed satisfy the cluster decomposition property and are continuous.

To summarize, the construction has provided hoped for answers to some questions stated in the beginning: stability requirements explain the separate conservation of lepton numbers and the experimental absence of $g > 0$ elementary bosons. What has not not been explained is the experimental absence of $g > 2$ fermion families. The vanishing of the $g > 2$ elementary particle vacuum functionals for the hyper-elliptic surfaces however suggest a possible explanation: under some conditions on the surface X^2 the surfaces Y^2 are hyper-elliptic or possess some conformal symmetry so that elementary particle vacuum functionals vanish for them. This conjecture indeed might make sense since the surfaces Y^2 are determined by the asymptotic dynamics and one might hope that the surfaces Y^2 are analogous to the final states of a dissipative system.

16.2.4 Explanations for the absence of the $g > 2$ elementary particles from spectrum

The decay properties of the intermediate gauge bosons [C33] are consistent with the assumption that the number of the light neutrinos is $N = 3$. Also cosmological considerations pose upper bounds on the number of the light neutrino families and $N = 3$ seems to be favored [C33]. It must be however emphasized that p-adic considerations [K52] encourage the consideration the existence of higher genera with neutrino masses such that they are not produced in the laboratory at present energies. In any case, for TGD approach the finite number of light fermion families is a potential difficulty since genus-generation correspondence suggests that the number of the fermion (and possibly also boson) families is infinite. Therefore one had better to find a good argument showing that the number of the observed neutrino families, or more generally, of the observed elementary particle families, is small also in the world described by TGD.

It will be later found that also TGD inspired cosmology requires that the number of the effectively massless fermion families must be small after Planck time. This suggests that boundary topologies with handle number $g > 2$ are unstable and/or very massive so that they, if present in the spectrum, disappear from it after Planck time, which correspond to the value of the light cone proper time $a \simeq 10^{-11}$ seconds.

In accordance with the spirit of TGD approach it is natural to wonder whether some geometric property differentiating between $g > 2$ and $g < 3$ boundary topologies might explain why only $g < 3$ boundary components are observable. One can indeed find a good candidate for this kind of property: namely hyper-ellipticity, which states that Riemann surface is a two-fold branched covering of sphere possessing two-element group Z_2 as conformal automorphisms. All $g < 3$ Riemann surfaces are hyper-elliptic unlike $g > 2$ Riemann surfaces, which in general do not posses this property. Thus it is natural to consider the possibility that hyper-ellipticity or more general conformal symmetries might explain why only $g < 2$ topologies correspond to the observed elementary particles.

As regards to the present problem the crucial observation is that some even theta functions vanish for the hyper-elliptic surfaces with genus $g > 2$ [A95] . What is essential is that these surfaces have the group Z_2 as conformal symmetries. Indeed, the vanishing phenomenon is more general. Theta functions tend to vanish for $g > 2$ two-surfaces possessing discrete group of conformal symmetries [A38] : for instance, instead of sphere one can consider branched coverings of higher genus surfaces.

From the general expression of the elementary particle vacuum functional it is clear that elementary particle vacuum functionals vanish, when Y^2 is hyper-elliptic surface with genus $g > 2$ and one might hope that this is enough to explain why the number of elementary particle families is three.

Hyper-ellipticity implies the separation of $g \leq 2$ and $g > 2$ sectors to separate worlds

If the vertices are defined as intersections of space-time sheets of elementary particles and if elementary particle vacuum functionals are required to have Z_2 symmetry, the localization of elementary particle vacuum functionals to $g \leq 2$ topologies occurs automatically. Even if one allows as limiting case vertices for which 2-manifolds are pinched to topologies intermediate between $g > 2$ and $g \leq 2$ topologies, Z_2 symmetry present for both topological interpretations implies the vanishing of this kind of vertices. This applies also in the case of stringy vertices so that also particle propagation would respect the effective number of particle families. $g > 2$ and $g \leq 2$ topologies would behave much like their own worlds in this approach. This is enough to explain the experimental findings if one can understand why the $g > 2$ particle families are absent as incoming and outgoing states or are very heavy.

What about $g > 2$ vacuum functionals which do not vanish for hyper-elliptic surfaces?

The vanishing of all $g \geq 2$ vacuum functionals for hyper-elliptic surfaces cannot hold true generally. There must exist vacuum functionals which do satisfy this condition. This suggest that elementary particle vacuum functionals for $g > 2$ states have interpretation as bound states of g handles and that the more general states which do not vanish for hyper-elliptic surfaces correspond to many-particle states composed of bound states $g \leq 2$ handles and cannot thus appear as incoming and outgoing states. Thus $g > 2$ elementary particles would decouple from $g \leq 2$ states.

Should higher elementary particle families be heavy?

TGD predicts an entire hierarchy of scaled up variants of standard model physics for which particles do not appear in the vertices containing the known elementary particles and thus behave like dark matter [K99] . Also $g > 2$ elementary particles would behave like dark matter and in principle there is no absolute need for them to be heavy.

The safest option would be that $g > 2$ elementary particles are heavy and the breaking of Z_2 symmetry for $g \geq 2$ states could guarantee this. p-Adic considerations lead to a general mass formula for elementary particles such that the mass of the particle is proportional to $\frac{1}{\sqrt{p}}$ [K54] . Also the dependence of the mass on particle genus is completely fixed by this formula. What remains however open is what determines the p-adic prime associated with a particle with given quantum numbers. Of course, it could quite well occur that p is much smaller for $g > 2$ genera than for $g \leq 2$ genera.

16.3 Non-topological contributions to particle masses from p-adic thermodynamics

In TGD framework p-adic thermodynamics provides a microscopic theory of particle massivation in the case of fermions. The idea is very simple. The mass of the particle results from a thermal mixing of the massless states with CP_2 mass excitations of super-conformal algebra. In p-adic thermodynamics the Boltzmann weight $exp(-E/T)$ does not exist in general and must be replaced with p^{L_0/T_p} which exists for Virasoro generator L_0 if the inverse of the p-adic temperature is integer valued $T_p = 1/n$. The expansion in powers of p converges extremely rapidly for physical values of p, which are rather large. Therefore the three lowest terms in expansion give practically exact results. Thermal massivation does not not necessarily lead to light states and this drops a large number of exotic states from the spectrum of light particles. The partition functions of N-S and Ramond type representations are not changed in TGD framework despite the fact that fermionic super generators carry fermion numbers and are not Hermitian. Thus the practical calculations are relatively straightforward albeit tedious.

In free fermion picture the p-adic thermodynamics in the boson sector is for fermion-anti-fermion states associated with the two throats of the bosonic wormhole. The question is whether the thermodynamical mass squared is just the sum of the two independent fermionic contributions for Ramond representations or should one use N-S type representation resulting as a tensor product of Ramond representations.

The overall conclusion about p-adic mass calculations is that fermionic mass spectrum is predicted in an excellent accuracy but that the thermal masses of the intermediate gauge bosons come 20-30 per cent to large for $T_p = 1$ and are completely negligible for $T_p = 1/2$. The bound state character of the boson states could be responsible for $T_p < 1$ and for extremely small thermodynamical contribution to the masses (present also for photon).

This forces to consider seriously the possibility that thermal contribution to the bosonic mass is negligible and that TGD can, contrary to the original expectations, provide dynamical Higgs field as a fundamental field and that even Higgs mechanism could contribute to the particle masses.

Higgs mechanism is probably the only viable description of Higgs mechanism in QFT approach, where particles are point-like but not in TGD, where particles are replaced by string like objects consisting of two wormhole contacts with monopole Kähler magnetic flux flowing between "upper" throats and returning back along "lower" space-time sheets. In this framework the assumption that fermion masses would result from p-adic thermodynamics but boson masses from Higgs couplings looks like an ugly idea. A more plausible vision is that the dominating contribution to gauge boson masses comes from the two flux tubes connecting the two wormhole contacts defining boson. This contribution would be present also for fermions but would be small. The correct W/Z mass ratio is obtained if the string tension is proportional to weak gauge coupling squared. The nice feature of this scenario is that naturalness is not lost: the dimensional gradient coupling of fermion to Higgs is same for all fermions.

The stringy contribution to mass squared could be expressed in terms of the deviation of the ground state conformal weight from negative half integer.

The problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.

16.3.1 Partition functions are not changed

One must write Super Virasoro conditions for L_n and *both* G_n and $G_n^\dagger$ rather than for L_n and G_n as in the case of the ordinary Super Virasoro algebra, and it is a priori not at all clear whether the partition functions for the Super Virasoro representations remain unchanged. This requirement is however crucial for the construction to work at all in the fermionic sector, since even the slightest changes for the degeneracies of the excited states can change light state to a state with mass of order m_0 in the p-adic thermodynamics.

Super conformal algebra

Super Virasoro algebra is generated by the bosonic the generators L_n (n is an integer valued index) and by the fermionic generators G_r, where r can be either integer (Ramond) or half odd integer (NS). G_r creates quark/lepton for $r > 0$ and antiquark/antilepton for $r < 0$. For $r = 0$, G_0 creates lepton and its Hermitian conjugate anti-lepton. The defining commutation and anti-commutation relations are the following:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{2}m(m^2-1)\delta_{m,-n} \ , \\
[L_m, G_r] &= (\frac{m}{2} - r)G_{m+r} \ , \\
\left[L_m, G_r^\dagger\right] &= (\frac{m}{2} - r)G_{m+r}^\dagger \ , \\
\{G_r, G_s^\dagger\} &= 2L_{r+s} + \frac{c}{3}(r^2 - \frac{1}{4})\delta_{m,-n} \ , \\
\{G_r, G_s\} &= 0 \ , \\
\{G_r^\dagger, G_s^\dagger\} &= 0 \ .
\end{aligned} \tag{16.3.1}$$

By the inspection of these relations one finds some results of a great practical importance.

(a) For the Ramond algebra G_0, G_1 and their Hermitian conjugates generate the $r \geq 0, n \geq 0$ part of the algebra via anti-commutations and commutations. Therefore all what is needed is to assume that Super Virasoro conditions are satisfied for these generators in case that G_0 and $G_0^\dagger$ annihilate the ground state. Situation changes if the states are *not* annihilated by G_0 and $G_0^\dagger$ since then one must assume the gauge conditions for both L_1, G_1 and $G_1^\dagger$ besides the mass shell conditions associated with G_0 and $G_0^\dagger$, which however do not affect the number of the Super Virasoro excitations but give mass shell condition and constraints on the state in the cm spin degrees of freedom. This will be assumed in the following. Note that for the ordinary Super Virasoro only the gauge conditions for L_1 and G_1 are needed.

(b) NS algebra is generated by $G_{1/2}$ and $G_{3/2}$ and their Hermitian conjugates (note that $G_{3/2}$ cannot be expressed as the commutator of L_1 and $G_{1/2}$) so that only the gauge conditions associated with these generators are needed. For the ordinary Super Virasoro only the conditions for $G_{1/2}$ and $G_{3/2}$ are needed.

Conditions guaranteeing that partition functions are not changed

The conditions guaranteeing the invariance of the partition functions in the transition to the modified algebra must be such that they reduce the number of the excitations and gauge conditions for a given conformal weight to the same number as in the case of the ordinary Super Virasoro.

(a) The requirement that physical states are invariant under $G \leftrightarrow G^\dagger$ corresponds to the charge conjugation symmetry and is very natural. As a consequence, the gauge conditions for G and $G^\dagger$ are not independent and their number reduces by a factor of one half and is the same as in the case of the ordinary Super Virasoro.

(b) As far as the number of the thermal excitations for a given conformal weight is considered, the only remaining problem are the operators $G_n G_n^\dagger$, which for the ordinary Super Virasoro reduce to $G_n G_n = L_{2n}$ and do not therefore correspond to independent degrees of freedom. In present case this situation is achieved only if one requires

$$(G_n G_n^\dagger - G_n^\dagger G_n)|phys\rangle = 0 \ . \tag{16.3.2}$$

It is not clear whether this condition must be posed separately or whether it actually follows from the representation of the Super Virasoro algebra automatically.

Partition function for Ramond algebra

Under the assumptions just stated, the partition function for the Ramond states not satisfying any gauge conditions

$$Z(t) = 1+2t+4t^2+8t^3+14t^4+.... \ , \tag{16.3.3}$$

which is identical to that associated with the ordinary Ramond type Super Virasoro.

For a Super Virasoro representation with $N=5$ sectors, of main interest in TGD, one has

$$\begin{aligned} Z_N(t) &= Z^{N=5}(t) = \sum D(n)t^n \\ &= 1+10t+60t^2+280t^3+... \ . \end{aligned} \tag{16.3.4}$$

The degeneracies for the states satisfying gauge conditions are given by

$$d(n) = D(n)-2D(n-1) \ . \tag{16.3.5}$$

corresponding to the gauge conditions for L_1 and G_1. Applying this formula one obtains for $N=5$ sectors

$$d(0)=1 \ , \quad d(1)=8 \ , \quad d(2)=40 \ , \quad d(3)=160 \ . \tag{16.3.6}$$

The lowest order contribution to the p-adic mass squared is determined by the ratio

$$r(n) = \frac{D(n+1)}{D(n)} \ ,$$

where the value of n depends on the effective vacuum weight of the ground state fermion. Light state is obtained only provided the ratio is integer. The remarkable result is that for lowest lying states the ratio is integer and given by

$$r(1)=8 \ , \quad r(2)=5 \ , \quad r(3)=4 \ . \tag{16.3.7}$$

It turns out that $r(2)=5$ gives the best possible lowest order prediction for the charged lepton masses and in this manner one ends up with the condition $h_{vac}=-3$ for the tachyonic vacuum weight of Super Virasoro.

Partition function for NS algebra

For NS representations the calculation of the degeneracies of the physical states reduces to the calculation of the partition function for a single particle Super Virasoro

$$Z_{NS}(t) = \sum_n z(n/2)t^{n/2} \ . \tag{16.3.8}$$

Here $z(n/2)$ gives the number of Super Virasoro generators having conformal weight $n/2$. For a state with N active sectors (the sectors with a non-vanishing weight for a given ground state) the degeneracies can be read from the N-particle partition function expressible as

$$Z_N(t) \quad = \quad Z^N(t) \ . \tag{16.3.9}$$

Single particle partition function is given by the expression

$$Z(t) \quad = \quad 1 + t^{1/2} + t + 2t^{3/2} + 3t^2 + 4t^{5/2} + 5t^3 + ... \ . \tag{16.3.10}$$

Using this representation it is an easy task to calculate the degeneracies for the operators of conformal weight Δ acting on a state having N active sectors.

One can also derive explicit formulas for the degeneracies and calculation gives

$$\begin{array}{ll} D(0,N) = 1 \ , & D(1/2,N) = N \ , \\ D(1,N) = \frac{N(N+1)}{2} \ , & D(3/2,N) = \frac{N}{6}(N^2+3N+8) \ , \\ D(2,N) = \frac{N}{2}(N^2+2N+3) \ , & D(5/2,N) = 9N(N-1) \ , \\ D(3,N) = 12N(N-1) + 2N(N-1) & . \end{array} \tag{16.3.11}$$

as a function of the conformal weight $\Delta = 0, 1/2, ..., 3$.

The number of states satisfying Super Virasoro gauge conditions created by the operators of a conformal weight Δ, when the number of the active sectors is N, is given by

$$d(\Delta,N) \quad = \quad D(\Delta,N) - D(\Delta - 1/2, N) - D(\Delta - 3/2, N) \ . \tag{16.3.12}$$

The expression derives from the observation that the physical states satisfying gauge conditions for $G^{1/2}$, $G^{3/2}$ satisfy the conditions for all Super Virasoro generators. For $T_p = 1$ light bosons correspond to the integer values of $d(\Delta+1,N)/d(\Delta,N)$ in case that massless states correspond to thermal excitations of conformal weight Δ: they are obtained for $\Delta = 0$ only (massless ground state). This is what is required since the thermal degeneracy of the light boson ground state would imply a corresponding factor in the energy density of the black body radiation at very high temperatures. For the physically most interesting nontrivial case with $N = 2$ two active sectors the degeneracies are

$$d(0,2) = 1 \ , \quad d(1,2) = 1 \ , \quad d(2,2) = 3 \ , \quad d(3,2) = 4 \ . \tag{16.3.13}$$

N, Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 3. Degeneracies $d(\Delta, N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation for bosons assuming that they correspond to N-S representations are listed.

16.3.2 Fundamental length and mass scales

The basic difference between quantum TGD and super-string models is that the size of CP_2 is not of order Planck length but much larger: of order $10^{3.5}$ Planck lengths. This conclusion is forced by several consistency arguments, the mass scale of electron, and by the cosmological data allowing to fix the string tension of the cosmic strings which are basic structures in TGD inspired cosmology.

The relationship between CP_2 radius and fundamental p-adic length scale

One can relate CP_2 'cosmological constant' to the p-adic mass scale: for $k_L = 1$ one has

$$m_0^2 = \frac{m_1^2}{k_L} = m_1^2 = 2\Lambda \ . \tag{16.3.14}$$

$k_L = 1$ results also by requiring that p-adic thermodynamics leaves charged leptons light and leads to optimal lowest order prediction for the charged lepton masses. Λ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$ with cosmological term).

The real counterpart of the p-adic thermal expectation for the mass squared is sensitive to the choice of the unit of p-adic mass squared which is by definition mapped as such to the real unit in canonical identification. Thus an important factor in the p-adic mass calculations is the correct identification of the p-adic mass squared scale, which corresponds to the mass squared unit and hence to the unit of the p-adic numbers. This choice does not affect the spectrum of massless states but can affect the spectrum of light states in case of intermediate gauge bosons.

(a) For the choice

$$M^2 = m_0^2 \leftrightarrow 1 \tag{16.3.15}$$

the spectrum of L_0 is integer valued.

(b) The requirement that all sufficiently small mass squared values for the color partial waves are mapped to real integers, would fix the value of p-adic mass squared unit to

$$M^2 = \frac{m_0^2}{3} \leftrightarrow 1 \ . \tag{16.3.16}$$

For this choice the spectrum of L_0 comes in multiples of 3 and it is possible to have a first order contribution to the mass which cannot be of thermal origin (say $m^2 = p$). This indeed seems to happen for electro-weak gauge bosons.

p-Adic mass calculations allow to relate m_0 to electron mass and to Planck mass by the formula

$$\begin{aligned} \frac{m_0}{m_{Pl}} &= \frac{1}{\sqrt{5+Y_e}} \times 2^{127/2} \times \frac{m_e}{m_{Pl}} \ , \\ m_{Pl} &= \frac{1}{\sqrt{\hbar G}} \ . \end{aligned} \tag{16.3.17}$$

For $Y_e = 0$ this gives $m_0 = .2437 \times 10^{-3} m_{Pl}$.

This means that CP_2 radius R defined by the length $L = 2\pi R$ of CP_2 geodesic is roughly $10^{3.5}$ times the Planck length. More precisely, using the relationship

$$\Lambda = \frac{3}{2R^2} = M^2 = m_0^2 \ ,$$

one obtains for

$$L = 2\pi R \quad = \quad 2\pi\sqrt{\frac{3}{2}}\frac{1}{m_0} \simeq 3.1167 \times 10^4\sqrt{\hbar G} \ \text{ for } \ Y_e = 0 \ . \qquad (16.3.18)$$

The result came as a surprise: the first belief was that CP_2 radius is of order Planck length. It has however turned out that the new identification solved elegantly some long standing problems of TGD.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{\hbar G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{\hbar G}) \times 10^{-4}$	3.1167	3.1167	3.1167
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23\times 2^{-3} * (15/17)$ is rational number producing R^2/G approximately is given.

The value of top quark mass favors $Y_e = 0$ and $Y_e = .5$ is largest value of Y_e marginally consistent with the limits on the value of top quark mass.

CP_2 radius as the fundamental p-adic length scale

The identification of CP_2 radius as the fundamental p-adic length scale is forced by the Super Virasoro invariance. The pleasant surprise was that the identification of the CP_2 size as the fundamental p-adic length scale rather than Planck length solved many long standing problems of older TGD.

(a) The earliest formulation predicted cosmic strings with a string tension larger than the critical value giving the angle deficit 2π in Einstein's equations and thus excluded by General Relativity. The corrected value of CP_2 radius predicts the value k/G for the cosmic string tension with k in the range $10^{-7} - 10^{-6}$ as required by the TGD inspired model for the galaxy formation solving the galactic dark matter problem.

(b) In the earlier formulation there was no idea as how to derive the p-adic length scale $L \sim 10^{3.5}\sqrt{\hbar G}$ from the basic theory. Now this problem becomes trivial and one has to predict gravitational constant in terms of the p-adic length scale. This follows in principle as a prediction of quantum TGD. In fact, one can deduce G in terms of the p-adic length scale and the action exponential associated with the CP_2 extremal and gets a correct value if α_K approaches fine structure constant at electron length scale (due to the fact that electromagnetic field equals to the Kähler field if Z^0 field vanishes).

Besides this, one obtains a precise prediction for the dependence of the Kähler coupling strength on the p-adic length scale by requiring that the gravitational coupling does not depend on the p-adic length scale. p-Adic prime p in turn has a nice physical interpretation: the critical value of α_K is same for the zero modes with given p. As already found, the construction of graviton state allows to understand the small value of the gravitational constant in terms of a de-coherence caused by multi-p fractality reducing the value of the gravitational constant from L_p^2 to G.

(c) p-Adic length scale is also the length scale at which super-symmetry should be restored in standard super-symmetric theories. In TGD this scale corresponds to the transition to Euclidian field theory for CP_2 type extremals. There are strong reasons to believe that sparticles are however absent and that super-symmetry is present only in the sense that super-generators have complex conformal weights with $Re(h) = \pm 1/2$ rather than $h = 0$. The action of this super-symmetry changes the mass of the state by an amount of order CP_2 mass.

16.4 Color degrees of freedom

The ground states for the Super Virasoro representations correspond to spinor harmonics in $M^4 \times CP_2$ characterized by momentum and color quantum numbers. The correlation between color and electro-weak quantum numbers is wrong for the spinor harmonics and these states would be also hyper-massive. The super-symplectic generators allow to build color triplet states having negative vacuum conformal weights, and their values are such that p-adic massivation is consistent with the predictions of the earlier model differing from the recent one in the quark sector. In the following the construction and the properties of the color partial waves for fermions and bosons are considered. The discussion follows closely to the discussion of [A99] .

16.4.1 SKM algebra and counterpart of Super Virasoro conditions

There have been a considerable progress also in the understanding of super-conformal symmetries [K17, K21].

(a) Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW.

(b) One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one. Note that the geometric part of SKM algebra must respect the light-likeness of the partonic 3-surface.

(c) An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

The requirement that symplectic generators have well defined radial conformal weight with respect to the light-like coordinate r of X^3 restricts M^4 conformal transformations to the group $SO(3) \times E^3$. This involves choice of preferred time coordinate. If the preferred M^4 coordinate is chosen to correspond to a preferred light-like direction in $\delta M^4_\pm$ characterizing the theory, a reduction to $SO(2) \times E^2$ more familiar from string models occurs. SKM algebra contains also $U(2)_{ew}$ Kac-Moody algebra acting as holonomies of CP_2 and having no bosonic counterpart.

p-Adic mass calculations require $N = 5$ sectors of super-conformal algebra. These sectors correspond to the 5 tensor factors for the $SO(3) \times E^3 \times SU(3) \times U(2)_{ew}$ (or $SO(2) \times E^2 \times$

$SU(3) \times U(2)_{ew}$) decomposition of the SKM algebra to gauge symmetries of gravitation, color and electro-weak interactions.

For symplectic isometries (Super-Kac-Moody algebra) fermionic algebra is realized in terms second quantized induced spinor field Ψ and spinor modes with well-defined em charge restricted to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. The full symplectic algebra is realized in terms of Ψ and covariantly constant right handed neutrino mode. One can consider also the possiblity of extended the symplectic isometries of $\delta M^4_\pm = S^2 \times R_+$ to include all isometries which act as conformal transformations of S^2 and for which conformal scaling of the metric is compensated by S^2 local scaling of the light-like radial coordinate r_M of R_+.

The algebra differs from the standard one in that super generators $G(z)$ carry lepton and quark numbers are not Hermitian as in super-string models (Majorana conditions are not satisfied). The counterparts of Ramond representations correspond to zero modes of a second quantized spinor field with vanishing radial conformal weight.

The Ramond or N-S type Virasoro conditions satisfied by the physical states in string model approach are replaced by the formulas expressing mass squared as a conformal weight. The condition is not equivalent with super Virasoro conditions since four-momentum does not appear in super Virasoro generators. It seems possible to assume that the commutator algebra $[SKM, SC]$ and the commutator of $[SKMV, SSV]$ of corresponding Super Virasoro algebras annihilate physical states. This would give rise to the analog of Super Virasoro conditions which could be seen as a Dirac equation in the world of classical worlds.

CP_2 CM degrees of freedom

Important element in the discussion are center of mass degrees of freedom parameterized by imbedding space coordinates. By the effective 2-dimensionality it is indeed possible to assign to partons momenta and color partial waves and they behave effectively as free particles. In fact, the technical problem of the earlier scenario was that it was not possible to assign symmetry transformations acting only on the light-like 3-surfaces at which the signature of the induced metric transforms from Minkowskian to Euclidian.

The original assumption was that 3-surface has boundary components to which elementary particle quantum numbers were assigned. It however became clear that boundary conditions at boundaries probably fail to be satisfied. Hence the above described light-like 3-surfaces took the role the boundary components. Space-time sheets were replaced with surfaces looking like double-sheeted (at least) structures from M^4 perspective with sheets meeting along 3-D surfaces. Sphere in Euclidian 3-space is the simplest analog for this kind of structure.

One can assign to each eigen state of color quantum numbers a color partial wave in CP_2 degrees of freedom. Thus color quantum numbers are not spin like quantum numbers in TGD framework except effectively in the length scales much longer than CP_2 length scale. The correlation between color partial waves and electro-weak quantum numbers is not physical in general: only the covariantly constant right handed neutrino has vanishing color.

Mass formula, and condition determining the effective string tension

Mass squared eigenvalues are given by

$$M^2 = m^2_{CP_2} + kL_0 \ . \tag{16.4.1}$$

The contribution of CP_2 spinor Laplacian to the mass squared operator is in general not integer valued.

The requirement that mass squared spectrum is integer valued for color partial waves possibly representing light states fixes the possible values of k determining the effective string tension modulo integer. The value $k = 1$ is the only possible choice. The earlier choice $k_L = 1$ and $k_q = 2/3$, $k_B = 1$ gave integer conformal weights for the lowest possible color partial waves. The assumption that the total vacuum weight h_{vac} is conserved in particle vertices implied $k_B = 1$.

16.4.2 General construction of solutions of Dirac operator of H

The construction of the solutions of massless spinor and other d'Alembertians in $M^4_+ \times CP_2$ is based on the following observations.

(a) d'Alembertian corresponds to a massless wave equation $M^4 \times CP_2$ and thus Kaluza-Klein picture applies, that is M^4_+ mass is generated from the momentum in CP_2 degrees of freedom. This implies mass quantization:

$$M^2 = M_n^2 \ ,$$

where M_n^2 are eigenvalues of CP_2 Laplacian. Here of course, ordinary field theory is considered. In TGD the vacuum weight changes mass squared spectrum.

(b) In order to get a respectable spinor structure in CP_2 one must couple CP_2 spinors to an odd integer multiple of the Kähler potential. Leptons and quarks correspond to $n = 3$ and $n = 1$ couplings respectively. The spectrum of the electromagnetic charge comes out correctly for leptons and quarks.

(c) Right handed neutrino is covariantly constant solution of CP_2 Laplacian for $n = 3$ coupling to Kähler potential whereas right handed 'electron' corresponds to the covariantly constant solution for $n = -3$. From the covariant constancy it follows that all solutions of the spinor Laplacian are obtained from these two basic solutions by multiplying with an appropriate solution of the scalar Laplacian coupled to Kähler potential with such a coupling that a correct total Kähler charge results. Left handed solutions of spinor Laplacian are obtained simply by multiplying right handed solutions with CP_2 Dirac operator: in this operation the eigenvalues of the mass squared operator are obviously preserved.

(d) The remaining task is to solve scalar Laplacian coupled to an arbitrary integer multiple of Kähler potential. This can be achieved by noticing that the solutions of the massive CP_2 Laplacian can be regarded as solutions of S^5 scalar Laplacian. S^5 can indeed be regarded as a circle bundle over CP_2 and massive solutions of CP_2 Laplacian correspond to the solutions of S^5 Laplacian with $exp(is\tau)$ dependence on S^1 coordinate such that s corresponds to the coupling to the Kähler potential:

$$s = n/2 \ .$$

Thus one obtains

$$D_5^2 = (D_\mu - iA_\mu\partial_\tau)(D^\mu - iA^\mu\partial_\tau) + \partial_\tau^2 \qquad (16.4.2)$$

so that the eigen values of CP_2 scalar Laplacian are

$$m^2(s) = m_5^2 + s^2 \qquad (16.4.3)$$

for the assumed dependence on τ.

(e) What remains to do, is to find the spectrum of S^5 Laplacian and this is an easy task. All solutions of S^5 Laplacian can be written as homogenous polynomial functions of C^3 complex coordinates Z^k and their complex conjugates and have a decomposition into the representations of $SU(3)$ acting in natural manner in C^3.

(f) The solutions of the scalar Laplacian belong to the representations $(p, p+s)$ for $s \geq 0$ and to the representations $(p+|s|, p)$ of $SU(3)$ for $s \leq 0$. The eigenvalues $m^2(s)$ and degeneracies d are

$$\begin{aligned} m^2(s) &= \frac{2\Lambda}{3}[p^2 + (|s|+2)p + |s|] \ , \ \ p > 0 \ , \\ d &= \frac{1}{2}(p+1)(p+|s|+1)(2p+|s|+2) \ . \end{aligned} \quad (16.4.4)$$

Λ denotes the 'cosmological constant' of CP_2 ($R_{ij} = \Lambda s_{ij}$).

16.4.3 Solutions of the leptonic spinor Laplacian

Right handed solutions of the leptonic spinor Laplacian are obtained from the asatz of form

$$\nu_R = \Phi_{s=0}\nu_R^0 \ ,$$

where u_R is covariantly constant right handed neutrino and Φ scalar with vanishing Kähler charge. Right handed 'electron' is obtained from the ansats

$$e_R = \Phi_{s=3}e_R^0 \ ,$$

where e_R^0 is covariantly constant for $n = -3$ coupling to Kähler potential so that scalar function must have Kähler coupling $s = n/2 = 3$ a in order to get a correct Kähler charge. The d'Alembert equation reduces to

$$\begin{aligned} & (D_\mu D^\mu - (1-\epsilon)\Lambda)\Phi = -m^2\Phi \ , \\ & \qquad \epsilon(\nu) = 1 \ , \quad \epsilon(e) = -1 \ . \end{aligned} \quad (16.4.5)$$

The two additional terms correspond to the curvature scalar term and $J_{kl}\Sigma^{kl}$ terms in spinor Laplacian. The latter term is proportional to Kähler coupling and of different sign for ν and e, which explains the presence of the sign factor ϵ in the formula.

Right handed neutrinos correspond to (p, p) states with $p \geq 0$ with mass spectrum

$$\begin{aligned} m^2(\nu) &= \frac{m_1^2}{3}\left[p^2 + 2p\right] \ , \ \ p \geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \quad (16.4.6)$$

Right handed 'electrons' correspond to $(p, p+3)$ states with mass spectrum

$$m^2(e) = \frac{m_1^2}{3}\left[p^2 + 5p + 6\right] \ , \ \ p \geq 0 \ . \quad (16.4.7)$$

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions and have the same mass spectrum and representational content as right handed leptons with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino (($p = 0, p = 0$) state) annihilates it.

16.4.4 Quark spectrum

Quarks correspond to the second conserved H-chirality of H-spinors. The construction of the color partial waves for quarks proceeds along similar lines as for leptons. The Kähler coupling corresponds to $n = 1$ (and $s = 1/2$) and right handed U type quark corresponds to a right handed neutrino. U quark type solutions are constructed as solutions of form

$$U_R = u_R \Phi_{s==1} \ ,$$

where u_R possesses the quantum numbers of covariantly constant right handed neutrino with Kähler charge $n = 3$ ($s = 3/2$). Hence Φ_s has $s = -1$. For D_R one has

$$D_R = d_r \Phi_{s=2} \ .$$

d_R has $s = -3/2$ so that one must have $s = 2$. For U_R the representations $(p+1, p)$ with triality one are obtained and $p = 0$ corresponds to color triplet. For D_R the representations $(p, p+2)$ are obtained and color triplet is missing from the spectrum ($p = 0$ corresponds to $\bar{6}$).

The CP_2 contributions to masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \ , \ p \geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] \ , \ p \geq 0 \ . \end{aligned} \quad (16.4.8)$$

Left handed quarks are obtained by applying Dirac operator to right handed quark states and mass formulas and color partial wave spectrum are the same as for right handed quarks.

The color contributions to p-adic mass squared are integer valued if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since canonical identification does not commute with a division by integer. More precisely, the images of number xp in canonical identification has a value of order 1 when x is a non-trivial rational whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes. This choice does not however affect the spectrum of massless states but can affect the spectrum of light states in case of electro-weak gauge bosons.

16.4.5 Spectrum of elementary particles

The assumption that $k = 1$ holds true for all particles forces to modify the earlier construction of quark states. This turns out to be possible without affecting the p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (which can be negative).

Leptonic spectrum

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod \ 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p+3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2 + 2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Right handed neutrino remains essentially massless. $p = 0$ right handed neutrino does not however generate $N = 1$ space-time (or rather, imbedding space) super symmetry so that no sparticles are predicted. The breaking of the electro-weak symmetry at the level of the masses comes out basically from the anomalous color electro-weak correlation for the Kaluza-Klein partial waves implying that the weights for the ground states of the fermions depend on the electromagnetic charge of the fermion. Interestingly, TGD predicts lepto-hadron physics based on color excitations of leptons and color bound states of these excitations could correspond topologically condensed on string like objects but not fundamental string like objects.

Spectrum of quarks

Earlier arguments [K57] related to a model of CKM matrix as a rational unitary matrix suggested that the string tension parameter k is different for quarks, leptons, and bosons. The basic mass formula read as

$$M^2 = m^2_{CP_2} + kL_0 \quad .$$

The values of k were $k_q = 2/3$ and $k_L = k_B = 1$. The general theory however predicts that $k = 1$ for all particles.

(a) By earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as unit, color conformal weight $h_c \equiv m^2_{CP_2}$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1, p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2 + 4p + 4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

(b) In the recent case $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This forces the super-symplectic operator O to compensate the anomalous color to have a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SSV: this would also explain why h_{sc} would be same for all fermions.

(c) It would seem that the tensor product of the spinor harmonic of quarks (as also leptons) with Hamiltonians gives rise to a large number of exotic colored states which have same thermodynamical mass as ordinary quarks (and leptons). Why these states have smaller values of p-adic prime that ordinary quarks and leptons, remains a challenge for the theory. Note that the decay widths of intermediate gauge bosons pose strong restrictions on the possible color excitations of quarks. On the other hand, the large number of fermionic color exotics can spoil the asymptotic freedom, and it is possible to have and entire p-adic length scale hierarchy of QCDs existing only in a finite length scale range without affecting the decay widths of gauge bosons.

The following table summarizes the color conformal weights and super-symplectic vacuum conformal weights for the elementary particles.

	L	ν_L	U	D	W	γ, G, g
h_{vac}	-3	-3	-3	-3	-2	0
h_c	2	1	2	3	2	0

Table 2. The values of the parameters h_{vac} and h_c assuming that $k = 1$. The value of $h_{vac} \leq -h_c$ is determined from the requirement that p-adic mass calculations give best possible fit to the mass spectrum.

Photon, graviton and gluon

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac} = 0$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain several physically more or less identical photons and this would be seen in the spectrum of black-body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

Masslessness or approximate masslessness requires low enough temperature $T_p = 1/n$, $n > 1$ at least and small enough value of the possible contribution coming from the ground state conformal weight.

In NS thermodynamics the only possibility to get exactly massless states in thermal sense is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. For neutral gauge bosons this is indeed achieved. For $T_p = 1/2$, which is required by the mass spectrum of intermediate gauge bosons, the thermal contribution to the mass squared is however extremely small even for W boson.

16.5 Modular contribution to the mass squared

The success of the p-adic mass calculations gives convincing support for the generation-genus correspondence. The basic physical picture is following.

(a) Fermionic mass squared is dominated by partonic contribution, which is sum of cm and modular contributions: $M^2 = M^2(cm) + M^2(mod)$. Here 'cm' refers to the thermal contribution. Modular contribution can be assumed to depend on the genus of the boundary component only.

(b) If Higgs contribution for diagonal (g, g) bosons (singlets with respect to "topological" $SU(3)$) dominates, the genus dependent contribution can be assumed to be negligible. This should be due to the bound state character of the wormhole contacts reducing thermal motion and thus the p-adic temperature.

(c) Modular contribution to the mass squared can be estimated apart from an overall proportionality constant. The mass scale of the contribution is fixed by the p-adic length scale hypothesis. Elementary particle vacuum functionals are proportional to a product of all even theta functions and their conjugates, the number of even theta functions and their conjugates being $2N(g) = 2^g(2^g + 1)$. Also the thermal partition function must also be proportional to $2N(g)$:th power of some elementary partition function. This implies that thermal/ quantum expectation $M^2(mod)$ must be proportional to $2N(g)$. Since single handle behaves effectively as particle, the contribution must be proportional to genus g also. The success of the resulting mass formula encourages the belief that the argument is essentially correct.

The challenge is to construct theoretical framework reproducing the modular contribution to mass squared. There are two alternative manners to understand the origin modular contribution.

(a) The realization that super-symplectic algebra is relevant for elementary particle physics leads to the idea that two thermodynamics are involved with the calculation of the vacuum conformal weight as a thermal expectation. The first thermodynamics corresponds to Super Kac-Moody algebra and second thermodynamics to super-symplectic algebra. This approach allows a first principle understanding of the origin and general form of the modular contribution without any need to introduce additional structures in modular degrees of freedom. The very fact that super-symplectic algebra does not commute with the modular degrees of freedom explains the dependence of the super-symplectic contribution on moduli.

(b) The earlier approach was based on the idea that he modular contribution could be regarded as a quantum mechanical expectation value of the Virasoro generator L_0 for the elementary particle vacuum functional. Quantum treatment would require generalization the concepts of the moduli space and theta function to the p-adic context and finding an acceptable definition of the Virasoro generator L_0 in modular degrees of freedom. The problem with this interpretation is that it forces to introduce, not only Virasoro generator L_0, but the entire super Virasoro algebra in modular degrees of freedom. One could also consider of interpreting the contribution of modular degrees of freedom to vacuum conformal weight as being analogous to that of CP_2 Laplacian but also this would raise the challenge of constructing corresponding Dirac operator. Obviously this approach has become obsolete.

The thermodynamical treatment taking into account the constraints from that p-adicization is possible might go along following lines.

(a) In the real case the basic quantity is the thermal expectation value $h(M)$ of the conformal weight as a function of moduli. The average value of the deviation $\Delta h(M) = h(M) - h(M_0)$ over moduli space $\mathcal{M}$ must be calculated using elementary particle vacuum functional as a modular invariant partition function. Modular invariance is achieved if this function is proportional to the logarithm of elementary particle vacuum functional: this reproduces the qualitative features basic formula for the modular contribution to the conformal weight. p-Adicization leads to a slight modification of this formula.

(b) The challenge of algebraically continuing this calculation to the p-adic context involves several sub-tasks. The notions of moduli space $\mathcal{M}_p$ and theta function must be defined in the p-adic context. An appropriately defined logarithm of the p-adic elementary particle vacuum functional should determine $\Delta h(M)$. The average of $\Delta h(M)$ requires an integration over $\mathcal{M}_p$. The problems related to the definition of this integral could be circumvented if the integral in the real case could be reduced to an algebraic expression, or if the moduli space is discrete in which case integral could be replaced by a sum.

(c) The number theoretic existence of the p-adic Θ function leads to the quantization of the moduli so that the p-adic moduli space is discretized. Accepting the sharpened form of Riemann hypothesis [K76] , the quantization means that the imaginary *resp.* real parts of the moduli are proportional to integers *resp.* combinations of imaginary parts of zeros of Riemann Zeta. This quantization could occur also for the real moduli for the maxima of Kähler function. This reduces the problematic p-adic integration to a sum and the resulting sum defining $\langle \Delta h \rangle$ converges extremely rapidly for physically interesting primes so that only the few lowest terms are needed.

16.5.1 Conformal symmetries and modular invariance

The full SKM invariance means that the super-conformal fields depend only on the conformal moduli of 2-surface characterizing the conformal equivalence class of the 2-surface. This means that all induced metrics differing by a mere Weyl scaling have same moduli. This symmetry is extremely powerful since the space of moduli is finite-dimensional and means that the entire infinite-dimensional space of deformations of parton 2-surface X^2 degenerates to a finite-dimensional moduli spaces under conformal equivalence. Obviously, the configurations of given parton correspond to a fiber space having moduli space as a base space. Super-symplectic degrees of freedom could break conformal invariance in some appropriate sense.

Conformal and SKM symmetries leave moduli invariant

Conformal transformations and super Kac Moody symmetries must leave the moduli invariant. This means that they induce a mere Weyl scaling of the induced metric of X^2 and thus preserve its non-diagonal character $ds^2 = g_{z\overline{z}}dzd\overline{z}$. This is indeed true if

(a) the Super Kac Moody symmetries are holomorphic isometries of $X^7 = \delta M^4_\pm \times CP_2$ made local with respect to the complex coordinate z of X^2, and

(b) the complex coordinates of X^7 are holomorphic functions of z.

Using complex coordinates for X^7 the infinitesimal generators can be written in the form

$$J^{An} = z^n j^{Ak} D_k + \overline{z}^n j^{A\overline{k}} D_{\overline{k}} \ . \tag{16.5.1}$$

The intuitive picture is that it should be possible to choose X^2 freely. It is however not always possible to choose the coordinate z of X^2 in such a manner that X^7 coordinates are holomorphic functions of z since a consistency of inherent complex structure of X^2 with that induced from X^7 is required. Geometrically this is like meeting of two points in the space of moduli.

Lorentz boosts produce new inequivalent choices of S^2 with their own complex coordinate: this set of complex structures is parameterized by the hyperboloid of future light cone (Lobatchevski space or mass shell), but even this is not enough. The most plausible manner to circumvent the problem is that only the maxima of Kähler function correspond to the holomorphic situation so that super-symplectic algebra representing quantum fluctuations would induce conformal anomaly.

The isometries of δM^4_+ are in one-one correspondence with conformal transformations

For CP_2 factor the isometries reduce to $SU(3)$ group acting also as symplectic transformations. For $\delta M^4_+ = S^2 \times R_+$ one might expect that isometries reduce to Lorentz group containing rotation group of $SO(3)$ as conformal isometries. If r_M corresponds to a macroscopic length scale, then X^2 has a finite sized S^2 projection which spans a rather small solid angle so that group $SO(3)$ reduces in a good approximation to the group $E^2 \times SO(2)$ of translations and rotations of plane.

This expectation is however wrong! The light-likeness of δM^4_+ allows a dramatic generalization of the notion of isometry. The point is that the conformal transformations of S^2 induce a conformal factor $|df/dw|^2$ to the metric of δM^4_+ and the local radial scaling $r_M \rightarrow r_M/|df/dw|$ compensates it. Hence the group of conformal isometries consists of conformal transformations of S^2 with compensating radial scalings. This compensation of two kinds of conformal transformations is the deep geometric phenomenon which translates to the condition $L_{SC} - L_{SKM} = 0$ in the sub-space of physical states. Note that an analogous phenomenon occurs also for the light-like CDs X^3_l with respect to the metrically 2-dimensional induced metric.

The X^2-local radial scalings $r_M \rightarrow r_M(z, \overline{z})$ respect the conditions $g_{zz} = g_{\overline{z}\overline{z}} = 0$ so that a mere Weyl scaling leaving moduli invariant results. By multiplying the conformal isometries of δM^4_+ by z^n (z is used as a complex coordinate for X^2 and w as a complex coordinate for S^2) a conformal localization of conformal isometries would result. Kind of double conformal transformations would be in question. Note however that this requires that X^7 coordinates are holomorphic functions of X^2 coordinate. These transformations deform X^2 unlike the conformal transformations of X^2. For X^3_l similar local scalings of the light like coordinate leave the moduli invariant but lead out of X^7.

Symplectic transformations break the conformal invariance

In general, infinitesimal symplectic transformations induce non-vanishing components $g_{zz}, g_{\overline{zz}}$ of the induced metric and can thus change the moduli of X^2. Thus the quantum fluctuations represented by super-symplectic algebra and contributing to the WCW metric are in general moduli changing. It would be interesting to know explicitly the conditions (the number of which is the dimension of moduli space for a given genus), which guarantee that the infinitesimal symplectic transformation is moduli preserving.

16.5.2 The physical origin of the genus dependent contribution to the mass squared

Different p-adic length scales are not enough to explain the charged lepton mass ratios and an additional genus dependent contribution in the fermionic mass formula is required. The general form of this contribution can be guessed by regarding elementary particle vacuum functionals in the modular degrees of freedom as an analog of partition function and the modular contribution to the conformal weight as an analog of thermal energy obtained by averaging over moduli. p-Adic length scale hypothesis determines the overall scale of the contribution.

The exact physical origin of this contribution has remained mysterious but super-symplectic degrees of freedom represent a good candidate for the physical origin of this contribution. This would mean a sigh of relief since there would be no need to assign conformal weights, super-algebra, Dirac operators, Laplacians, etc.. with these degrees of freedom.

Thermodynamics in super-symplectic degrees of freedom as the origin of the modular contribution to the mass squared

The following general picture is the simplest found hitherto.

(a) Elementary particle vacuum functionals are defined in the space of moduli of surfaces X^2 corresponding to the maxima of Kähler function. There some restrictions on X^2. In particular, p-adic length scale poses restrictions on the size of X^2. There is an infinite hierarchy of elementary particle vacuum functionals satisfying the general constraints but only the lowest elementary particle vacuum functionals are assumed to contribute significantly to the vacuum expectation value of conformal weight determining the mass squared value.

(b) The contribution of Super-Kac Moody thermodynamics to the vacuum conformal weight h coming from Virasoro excitations of the $h = 0$ massless state is estimated in the previous calculations and does not depend on moduli. The new element is that for a partonic 2-surface X^2 with given moduli, Virasoro thermodynamics is present also in super-symplectic degrees of freedom.

Super-symplectic thermodynamics means that, besides the ground state with $h_{gr} = -h_{SC}$ with minimal value of super-symplectic conformal weight h_{SC}, also thermal excitations of this state by super-symplectic Virasoro algebra having $h_{gr} = -h_{SC} - n$ are possible. For these ground states the SKM Virasoro generators creating states with net conformal weight $h = h_{SKM} - h_{SC} - n \geq 0$ have larger conformal weight so that the SKM thermal average h depends on n. It depends also on the moduli M of X^2 since the Beltrami differentials representing a tangent space basis for the moduli space $\mathcal{M}$ do not commute with the super-symplectic algebra. Hence the thermally averaged SKM conformal weight h_{SKM} for given values of moduli satisfies

$$h_{SKM} = h(n,M) \ . \tag{16.5.2}$$

(c) The average conformal weight induced by this double thermodynamics can be expressed as a super-symplectic thermal average $\langle\cdot\rangle_{SC}$ of the SKM thermal average $h(n,M)$:

$$h(M) = \langle h(n,M)\rangle_{SC} = \sum p_n(M)h(n) \ , \tag{16.5.3}$$

where the moduli dependent probability $p_n(M)$ of the super-symplectic Virasoro excitation with conformal weight n should be consistent with the p-adic thermodynamics. It is convenient to write $h(M)$ as

$$h(M) = h_0 + \Delta h(M) \ , \tag{16.5.4}$$

where h_0 is the minimum value of $h(M)$ in the space of moduli. The form of the elementary particle vacuum functionals suggest that h_0 corresponds to moduli with $Im(\Omega_{ij}) = 0$ and thus to singular configurations for which handles degenerate to one-dimensional lines attached to a sphere.

(d) There is a further averaging of $\Delta h(M)$ over the moduli space $\mathcal{M}$ by using the modulus squared of elementary particle vacuum functional so that one has

$$h = h_0 + \langle\Delta h(M)\rangle_{\mathcal{M}} \ . \tag{16.5.5}$$

Modular invariance allows to pose very strong conditions on the functional form of $\Delta h(M)$. The simplest assumption guaranteeing this and thermodynamical interpretation is that $\Delta h(M)$ is proportional to the logarithm of the vacuum functional Ω:

$$\Delta h(M) \propto -log(\frac{\Omega(M)}{\Omega_{max}}) \ . \tag{16.5.6}$$

Here Ω_{max} corresponds to the maximum of Ω for which $\Delta h(M)$ vanishes.

Justification for the general form of the mass formula

The proposed general ansatz for $\Delta h(M)$ provides a justification for the general form of the mass formula deduced by intuitive arguments.

(a) The factorization of the elementary particle vacuum functional Ω into a product of $2N(g) = 2^g(2^g + 1)$ terms and the logarithmic expression for $\Delta h(M)$ imply that the thermal expectation values is a sum over thermal expectation values over $2N(g)$ terms associated with various even characteristics (a, b), where a and b are g-dimensional vectors with components equal to 1/2 or 0 and the inner product $4a \cdot b$ is an even integer. If each term gives the same result in the averaging using Ω_{vac} as a partition function, the proportionality to $2N_g$ follows.

(b) For genus $g \geq 2$ the partition function defines an average in $3g-3$ complex-dimensional space of moduli. The analogy of $\langle\Delta h\rangle$ and thermal energy suggests that the contribution is proportional to the complex dimension $3g-3$ of this space. For $g \leq 1$ the contribution the complex dimension of moduli space is g and the contribution would be proportional to g.

$$\begin{aligned} \langle \Delta h \rangle &\propto g \times X(g) \text{ for } g \leq 1 \ , \\ \langle \Delta h \rangle &\propto (3g-3) \times X(g) \text{ for } g \geq 2 \ , \\ X(g) &= 2^g(2^g+1) \ . \end{aligned} \quad (16.5.7)$$

If X^2 is hyper-elliptic for the maxima of Kähler function, this expression makes sense only for $g \leq 2$ since vacuum functionals vanish for hyper-elliptic surfaces.

(c) The earlier argument, inspired by the interpretation of elementary particle vacuum functional as a partition function, was that each factor of the elementary particle vacuum functional gives the same contribution to $\langle \Delta h \rangle$, and that this contribution is proportional to g since each handle behaves like a particle:

$$\langle \Delta h \rangle \propto g \times X(g) \ . \quad (16.5.8)$$

The prediction following from the previous differs by a factor $(3g-3)/g$ for $g \geq 2$. This would scale up the dominant modular contribution to the masses of the third $g = 2$ fermionic generation by a factor $\sqrt{3/2} \simeq 1.22$. One must of course remember, that these rough arguments allow $g-$ dependent numerical factors of order one so that it is not possible to exclude either argument.

16.5.3 Generalization of Θ functions and quantization of p-adic moduli

The task is to find p-adic counterparts for theta functions and elementary particle vacuum functionals. The constraints come from the p-adic existence of the exponentials appearing as the summands of the theta functions and from the convergence of the sum. The exponentials must be proportional to powers of p just as the Boltzmann weights defining the p-adic partition function. The outcome is a quantization of moduli so that integration can be replaced with a summation and the average of $\Delta h(M)$ over moduli is well defined.

It is instructive to study the problem for torus in parallel with the general case. The ordinary moduli space of torus is parameterized by single complex number τ. The points related by $SL(2,Z)$ are equivalent, which means that the transformation $\tau \rightarrow (A\tau+B)/(C\tau+D)$ produces a point equivalent with τ. These transformations are generated by the shift $\tau \rightarrow \tau+1$ and $\tau \rightarrow -1/\tau$. One can choose the fundamental domain of moduli space to be the intersection of the slice $Re(\tau) \in [-1/2, 1/2]$ with the exterior of unit circle $|\tau| = 1$. The idea is to start directly from physics and to look whether one might some define p-adic version of elementary particle vacuum functionals in the p-adic counterpart of this set or in some modular invariant subset of this set.

Elementary particle vacuum functionals are expressible in terms of theta functions using the functions $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ as a building block. The general expression for the theta function reads as

$$\Theta[a,b](\Omega) = \sum_n exp(i\pi(n+a)\cdot\Omega\cdot(n+a))exp(2i\pi(n+a)\cdot b) \ . \quad (16.5.9)$$

The latter exponential phase gives only a factor $\pm i$ or ± 1 since $4a\cdot b$ is integer. For $p \ mod \ 4 = 3$ imaginary unit exists in an algebraic extension of p-adic numbers. In the case of torus (a,b) has the values $(0,0)$, $(1/2,0)$ and $(0,1/2)$ for torus since only even characteristics are allowed.

Concerning the p-adicization of the first exponential appearing in the summands in Eq. 16.5.9, the obvious problem is that π does not exists p-adically unless one allows infinite-dimensional extension.

(a) Consider first the real part of Ω. In this case the proper manner to treat the situation is to introduce and algebraic extension involving roots of unity so that $Re(\Omega)$ rational. This approach is proposed as a general approach to the p-adicization of quantum TGD in terms of harmonic analysis in symmetric spaces allowing to define integration also in p-adic context in a physically acceptable manner by reducing it to Fourier analysis. The simplest situation corresponds to integer values for $Re(\Omega)$ and in this case the phase are equal to $\pm i$ or ± 1 since a is half-integer valued. One can consider a hierarchy of variants of moduli space characterized by the allowed roots of unity. The physical interpretation for this hierarchy would be in terms of a hierarchy of measurement resolutions. Note that the real parts of Ω can be assumed to be rationals of form m/n where n is constructed as a product of finite number of primes and therefore the allowed rationals are linear combinations of inverses $1/p_i$ for a subset $\{p_i\}$ of primes.

(b) For the imaginary part of Ω different approach is required. One wants a rapid convergence of the sum formula and this requires that the exponents reduces in this case to positive powers of p. This is achieved if one has

$$Im(\Omega) = -n\frac{log(p)}{\pi)} , \tag{16.5.10}$$

Unfortunately this condition is not consistent with the condition $Im(\Omega) > 0$. A manner to circumvent the difficulty is to replace Ω with its complex conjugate. Second approach is to define the real discretized variant of theta function first and then map it by canonical identification to its p-adic counterpart: this would map phase to phases and powers of p to their inverses. Note that a similar change of sign must be performed in p-adic thermodynamics for powers of p to map p-adic probabilities to real ones. By rescaling $Im(\Omega) \rightarrow \frac{log(p)}{\pi)} Im(\Omega)$ one has non-negative integer valued spectrum for $Im(\Omega)$ making possible to reduce integration in moduli space to a summation over finite number of rationals associated with the real part of Ω and powers of p associated with the imaginary part of Ω.

(c) Since the exponents appearing in

$$p^{(n+a)\cdot Im(\Omega_{ij,p})\cdot(n+a)} = p^{a\cdot Im(\Omega)\cdot a} \times p^{2a\cdot Im(\Omega\cdot n} \times p^{+n\cdot Im(\Omega_{ij,p})\cdot n}$$

are positive integers valued, $\Theta_{[a,b]}$ exist in R_p and converges. The problematic factor is the first exponent since the components of the vector a can have values 1/2 and 0 and its existence implies a quantization of $Im(\Omega_{ij})$ as

$$Im(\Omega) = -Kn\frac{log(p)}{p} , \quad n \in Z , \quad n \geq 1 , \tag{16.5.11}$$

In p-adic context this condition must be formulated for the exponent of Ω defining the natural coordinate. $K = 4$ guarantees the existence of Θ functions and $K = 1$ the existence of the building blocks $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ of elementary particle vacuum functionals in R_p. The extension to higher genera means only replacement of Ω with the elements of a matrix.

(d) One can criticize this approach for the loss of the full modular covariance in the definition of theta functions. The modular transformations $\Omega \rightarrow \Omega + n$ are consistent with the number theoretic constraints but the transformations $\Omega \rightarrow -1/\Omega$ do not respect them. It seem that one can circumvent the difficulty by restricting the consideration to a fundamental domain satisfying the number theoretic constraints.

This variant of moduli space is discrete and p-adicity is reflected only in the sense that the moduli space makes sense also p-adically. One can consider also a continuum variant of the p-adic moduli space using the same prescription as in the construction of p-adic symmetric spaces [K87] .

(a) One can introduce $exp(i\pi Re(\Omega))$ as the counterpart of $Re(\Omega)$ as a coordinate of the Teichmueller space. This coordinate makes sense only as a local coordinate since it does not differentiate between $Re(\Omega)$ and $Re(\Omega+2n)$. On the other hand, modular invariance states that Ω abd $\Omega + n$ correspond to the same moduli so that nothing is lost. In the similar manner one can introduce $exp^(\pi Im(\Omega)) \in \{p^n, n > 0\}$ as the counterpart of discretized version of $Im(\Omega)$.

(b) The extension to continuum would mean in the case of $Re(\Omega)$ the extension of the phase $exp(i\pi Re(\Omega))$ to a product $exp(i\pi Re(\Omega))exp(ipx) = exp(i\pi Re(\Omega) + exp(ipx)$, where x is p-adic integer which can be also infinite as a real integer. This would mean that each root of unity representing allowed value $Re(\Omega)$ would have a p-adic neighborhood consisting of p-adic integers. This neighborhood would be the p-adic counterpart for the angular integral $\Delta\phi$ for a given root of unity and would not make itself visible in p-adic integration.

(c) For the imaginary part one can also consider the extension of $exp(\pi Im(\Omega))$ to $p^n \times exp(npx)$ where x is a p-adic integer. This would assign to each point p^n a p-adic neighborhood defined by p-adic integers. This neighborhood is same all integers n with same p-adic norm. When n is proportional to p^k one has $exp(npx) - 1 \propto p^k$.

The quantization of moduli characterizes precisely the conformal properties of the partonic 2-surfaces corresponding to different p-adic primes. In the real context -that is in the intersection of real and p-adic worlds- the quantization of moduli of torus would correspond to

$$\tau = K\left[\sum q + i \times n\frac{log(p)}{\pi}\right] , \tag{16.5.12}$$

where q is a rational number expressible as linear combination of inverses of a finite fixed set of primes defining the allowed roots of unity. $K = 1$ guarantees the existence of elementary particle vacuum functionals and $K = 4$ the existence of Theta functions. The ratio for the complex vectors defining the sides of the plane parallelogram defining torus via the identification of the parallel sides is quantized. In other words, the angles Φ between the sides and the ratios of the sides given by $|\tau|$ have quantized values.

The quantization rules for the moduli of the higher genera is of exactly same form

$$\Omega_{ij} = K\left[\sum q_{ij} + i \times n_{ij} \times \frac{log(p)}{\pi}\right] , \tag{16.5.13}$$

If the quantization rules hold true also for the maxima of Kähler function in the real context or more precisely- in the intersection of real and p-adic variants of the "world of classical worlds" identified as partonic 2-surfaces at the boundaries of causal diamond plus the data about their 4-D tangent space, there are good hopes that the p-adicized expression for Δh is obtained by a simple algebraic continuation of the real formula. Thus p-adic length scale would characterize partonic surface X^2 rather than the light like causal determinant X^3_l containing X^2. Therefore the idea that various p-adic primes label various X^3_l connecting fixed partonic surfaces X^2_i would not be correct.

Quite generally, the quantization of moduli means that the allowed 2-dimensional shapes form a lattice and are thus additive. It also means that the maxima of Kähler function would obey a linear superposition in an extreme abstract sense. The proposed number theoretical quantization is expected to apply for any complex space allowing some preferred complex coordinates. In particular, WCW of 2-surfaces could allow this kind of quantization in the complex coordinates naturally associated with isometries and this could allow to define

WCW integration, at least the counterpart of integration in zero mode degrees of freedom, as a summation.

Number theoretic vision leads to the notion of multi-p-p-adicity in the sense that the same partonic 2-surface can correspond to several p-adic primes and that infinite primes code for these primes [K28, K86] . At the level of the moduli space this corresponds to the replacement of p with an integer in the formulas so that one can interpret the formulas both in real sense and p-adic sense for the primes p dividing the integer. Also the exponent of given prime in the integer matters.

16.5.4 The calculation of the modular contribution $\langle \Delta h\rangle$ to the conformal weight

The quantization of the moduli implies that the integral over moduli can be defined as a sum over moduli. The theta function $\Theta[a,b](\Omega)_p(\tau_p)$ is proportional to $p^{a\cdot aIm(\Omega_{ij,p})} = p^{Kn_{ij}m(a)/4}$ for $a\cdot a = m(a)/4$, where $K=1$ *resp.* $K=4$ corresponds to the existence existence of elementary particle vacuum functionals *resp.* theta functions in R_p. These powers of p can be extracted from the thetas defining the vacuum functional. The numerator of the vacuum functional gives $(p^n)^{2K\sum_{a,b}m(a)}$. The numerator gives $(p^n)^{2K\sum_{a,b}m(a_0)}$, where a_0 corresponds to the minimum value of $m(a)$. $a_0=(0,0,..,0)$ is allowed and gives $m(a_0)=0$ so that the p-adic norm of the denominator equals to one. Hence one has

$$|\Omega_{vac}(\Omega_p)|_p \quad = \quad p^{-2nK\sum_{a,b}m(a)} \tag{16.5.14}$$

The sum converges extremely rapidly for large values of p as function of n so that in practice only few moduli contribute.

The definition of $log(\Omega_{vac})$ poses however problems since in $log(p)$ does not exist as a p-adic number in any p-adic number field. The argument of the logarithm should have a unit p-adic norm. The simplest manner to circumvent the difficulty is to use the fact that the p-adic norm $|\Omega_p|_p$ is also a modular invariant, and assume that the contribution to conformal weight depends on moduli as

$$\Delta h_p(\Omega_p) \quad \propto \quad log(\frac{\Omega_{vac}}{|\Omega_{vac}|_p}) \ . \tag{16.5.15}$$

The sum defining $\langle\Delta h_p\rangle$ converges extremely rapidly and gives a result of order $O(p)$ p-adically as required.

The p-adic expression for $\langle\Delta h_p\rangle$ should result from the corresponding real expression by an algebraic continuation. This encourages the conjecture that the allowed moduli are quantized for the maxima of Kähler function, so that the integral over the moduli space is replaced with a sum also in the real case, and that Δh given by the double thermodynamics as a function of moduli can be defined as in the p-adic case. The positive power of p multiplying the numerator could be interpreted as a degeneracy factor. In fact, the moduli are not primary dynamical variables in the case of the induced metric, and there must be a modular invariant weight factor telling how many 2-surfaces correspond to given values of moduli. The power of p could correspond to this factor.

16.6 The contributions of p-adic thermodynamics to particle masses

In the sequel various contributions to the mass squared are discussed.

16.6.1 General mass squared formula

The thermal independence of Super Virasoro and modular degrees of freedom implies that mass squared for elementary particle is the sum of Super Virasoro, modular and Higgsy contributions:

$$M^2 = M^2(color) + M^2(SV) + M^2(mod) + M^2(Higgsy) \ . \tag{16.6.1}$$

Also small renormalization correction contributions might be possible.

16.6.2 Color contribution to the mass squared

The mass squared contains a non-thermal color contribution to the ground state conformal weight coming from the mass squared of CP_2 spinor harmonic. The color contribution is an integer multiple of $m_0^2/3$, where $m_0^2 = 2\Lambda$ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$).

The color contribution to the p-adic mass squared is integer valued only if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since the simplest form of the canonical identification does not commute with a division by integer. More precisely, the image of number xp in canonical identification has a value of order 1 when x is a non-trivial rational number whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes.

The choice of the p-adic mass squared unit are no effects on zeroth order contribution which must vanish for light states: this requirement eliminates quark and lepton states for which the CP_2 contribution to the mass squared is not integer valued using m_0^2 as a unit. There can be a dramatic effect on the first order contribution. The mass squared $m^2 = p/3$ using $m_0^2/3$ means that the particle is light. The mass squared becomes $m^2 = p/3$ when m_0^2 is used as a unit and the particle has mass of order 10^{-4} Planck masses. In the case of W and Z^0 bosons this problem is actually encountered. For light states using $m_0^2/3$ as a unit only the second order contribution to the mass squared is affected by this choice.

16.6.3 Modular contribution to the mass of elementary particle

The general form of the modular contribution is derivable from p-adic partition function for conformally invariant degrees of freedom associated with the boundary components. The general form of the vacuum functionals as modular invariant functions of Teichmueller parameters was derived in [K19] and the square of the elementary particle vacuum functional can be identified as a partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g > 0$ is given by

$$N(g) = 2^{g-1}(2^g + 1) \ . \tag{16.6.2}$$

One has $N(1) = 3$ for muon and $N(2) = 10$ for τ.

(a) Single theta function is analogous to a partition function. This implies that the modular contribution to the mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in the partition function.

(b) The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g-g_1$ handles, the partition function reduces to a product of g_1 and $g-g_1$ partition functions. This implies that the contribution to the mass squared is proportional to the genus of the surface. Altogether one has

$$\begin{aligned} M^2(mod,g) &= 2k(mod)N(g)g\frac{m_0^2}{p} \ , \\ k(mod) &= 1 \ . \end{aligned} \tag{16.6.3}$$

Here $k(mod)$ is some integer valued constant (in order to avoid ultra heavy mass) to be determined. $k(mod)=1$ turns out to be the correct choice for this parameter.

Summarizing, the real counterpart of the modular contribution to the mass of a particle belonging to $g+1$:th generation reads as

$$\begin{aligned} M^2(mod) &= 0 \ for \ e,\nu_e,u,d \ , \\ M^2(mod) &= 9\frac{m_0^2}{p(X))} \ for \ X=\mu,\nu_\mu,c,s \ , \\ M^2(mod) &= 60\frac{m_0^2}{p(X)} \ for \ X=\tau,\nu_\tau,t,b \ . \end{aligned} \tag{16.6.4}$$

The requirement that hadronic mass spectrum and CKM matrix are sensible however forces the modular contribution to be the same for quarks, leptons and bosons. The higher order modular contributions to the mass squared are completely negligible if the degeneracy of massless state is $D(0,mod,g)=1$ in the modular degrees of freedom as is in fact required by $k(mod)=1$.

16.6.4 Thermal contribution to the mass squared

One can deduce the value of the thermal mass squared in order $O(p^2)$ (an excellent approximation) using the general mass formula given by p-adic thermodynamics. Assuming maximal p-adic temperature $T_p=1$ one has

$$\begin{aligned} M^2 &= k(sp+Xp^2+O(p^3)) \ , \\ s_\Delta &= \frac{D(\Delta+1)}{D(\Delta)} \ , \\ X_\Delta &= 2\frac{D(\Delta+2)}{D(\Delta)}-\frac{D^2(\Delta+1)}{D^2(\Delta)} \ , \\ k &= 1 \ . \end{aligned} \tag{16.6.5}$$

Δ is the conformal weight of the operator creating massless state from the ground state.

The ratios $r_n=D(n+1)/D(n)$ allowing to deduce the values of s and X have been deduced from p-adic thermodynamics in [K48] . Light state is obtained only provided $r(\Delta)$ is an integer. The remarkable result is that for lowest lying states this is the case. For instance, for Ramond representations the values of r_n are given by

$$(r_0,r_1,r_2,r_3)=(8,5,4,\frac{55}{16}) \ . \tag{16.6.6}$$

The values of s and X are

$$(s_0, s_1, s_2) = (8, 5, 4) \ ,$$
$$(X_0, X_1, X_2) = (16, 15, 11 + 1/2)) \ . \qquad (16.6.7)$$

The result means that second order contribution is extremely small for quarks and charged leptons having $\Delta < 2$. For neutrinos having $\Delta = 2$ the second order contribution is non-vanishing.

16.6.5 The contribution from the deviation of ground state conformal weight from negative integer

The interpretation inspired by p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of the modified Dirac operator correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. A good guess is that induced Kähler magnetic field B_K dictates the magnitude of the eigenvalues which is thus of order $h_0 = \sqrt{B_K R}$, R CP_2 radius. The first guess is that eigenvalues in the first approximation come as $(n + 1/2)h_0$. Each region where induced Kähler field is non-vanishing would correspond to different scale mass scale h_0.

(a) The vacuum expectation value of Higgs is only proportional to an eigenvalue λ, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and anti-fermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this).

(b) Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of modified Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

(c) p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign if the effective metric of X_l^3 defined by the inner products $T_K^{k\alpha} T_K^{l\beta} h_{kl}$ of the Kähler energy momentum tensor $T^{k\alpha} = h^{kl}\partial L_K/\partial h^l_\alpha$ and appearing in the modified Dirac operator D_K has Minkowskian signature.

The situation changes if the effective metric has Euclidian signature. This seems to be the case for the light-like surfaces assignable to the known extremals such as MEs and cosmic strings. In this kind of situation light-like coordinate possesses Euclidian signature and real eigenvalue spectrum is replaced with a purely imaginary one. Since

Dirac operator is in question both signs for eigenvalues are possible and one obtains both exponentially increasing and decreasing solutions. This is essential for having solutions extending from the past end of X_l^3 to its future end. Non-unitary time evolution is possible because X_l^3 does not strictly speaking represent the time evolution of 2-D dynamical object but actual dynamical objects (by light-likeness both interpretation as dynamical evolution and dynamical object are present). The Euclidian signature of the effective metric would be a direct analog for the tachyonicity of the Higgs in unstable minimum and the generation of Higgs vacuum expectation would correspond to the compensation of ground state conformal weight by conformal weights of Super Virasoro generators.

(d) In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = \lambda_i^2 = -1/2 - n + \Delta h_c$ so that lowest ground state conformal weight would be $h_c = -1/2$ in the first approximation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

(e) p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

16.6.6 General mass formula for Ramond representations

By taking the modular contribution from the boundaries into account the general p-adic mass formulas for the Ramond type states read for states for which the color contribution to the conformal weight is integer valued as

$$\begin{aligned}
\frac{m^2(\Delta=0)}{m_0^2} &= (8+n(g))p+Yp^2 \ , \\
\frac{m^2(\Delta=1)}{m_0^2} &= (5+n(g)p+Yp^2 \ , \\
\frac{m^2(\Delta=2)}{m_0^2} &= (4+n(g))p+(Y+\frac{23}{2})p^2 \ , \\
n(g) &= 3g\cdot 2^{g-1}(2^g+1) \ .
\end{aligned} \tag{16.6.8}$$

Here Δ denotes the conformal weight of the operators creating massless states from the ground state and g denotes the genus of the boundary component. The values of $n(g)$ for the three lowest generations are $n(0)=0$, $n(1)=9$ and $n(2)=60$. The value of second order thermal contribution is nontrivial for neutrinos only. The value of the rational number Y can, which corresponds to the renormalization correction to the mass, can be determined using experimental inputs.

Using m_0^2 as a unit, the expression for the mass of a Ramond type state reads in terms of the electron mass as

$$\begin{aligned}
M(\Delta,g,p)_R &= K(\Delta,g,p)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0,g,p) &= \sqrt{\frac{n(g)+8+Y_R}{X}} \\
K(1,g,p) &= \sqrt{\frac{n(g)+5+Y_R}{X}} \\
K(2,g,p) &= \sqrt{\frac{n(g)+4+Y_R}{X}} \ , \\
X &= \sqrt{5+Y(e)_R} \ .
\end{aligned} \tag{16.6.9}$$

Y can be assumed to depend on the electromagnetic charge and color representation of the state and is therefore same for all fermion families. Mathematica provides modules for calculating the real counterpart of the second order contribution and for finding realistic values of Y.

16.6.7 General mass formulas for NS representations

Using $m_0^2/3$ as a unit, the expression for the mass of a light NS type state for $T_p = 1$ ad $k_B = 1$ reads in terms of the electron mass as

$$\begin{aligned}
M(\Delta,g,p,N)_R &= K(\Delta,g,p,N)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0,g,p,1) &= \sqrt{\frac{n(g)+Y_R}{X}} \ , \\
K(0,g,p,2) &= \sqrt{\frac{n(g)+1+Y_R}{X}} \ , \\
K(1,g,p,3) &= \sqrt{\frac{n(g)+3+Y_R}{X}} \ , \\
K(2,g,p,4) &= \sqrt{\frac{n(g)+5+Y_R}{X}} \ , \\
K(2,g,p,5) &= \sqrt{\frac{n(g)+10+Y_R}{X}} \ , \\
X &= \sqrt{5+Y(e)_R} \ .
\end{aligned} \tag{16.6.10}$$

Here N is the number of the 'active' NS sectors (sectors for which the conformal weight of the massless state is non-vanishing). Y denotes the renormalization correction to the boson mass and in general depends on the electro-weak and color quantum numbers of the boson.

The thermal contribution to the mass of W boson is too large by roughly a factor $\sqrt{3}$ for $T_p = 1$. Hence $T_p = 1/2$ must hold true for gauge bosons and their masses must have a non-thermal origin perhaps analogous to Higgs mechanism. Alternatively, the non-covariant constancy of charge matrices could induce the boson mass [K48] .

It is interesting to notice that the minimum mass squared for gauge boson corresponds to the p-adic mass unit $M^2 = m_0^2 p/3$ and this just what is needed in the case of W boson. This forces to ask whether $m_0^2/3$ is the correct choice for the mass squared unit so that non-thermally induced W mass would be the minimal $m_W^2 = p$ in the lowest order. This choice would mean the replacement

$$Y_R \rightarrow \frac{(3Y)_R}{3}$$

in the preceding formulas and would affect only neutrino mass in the fermionic sector. $m_0^2/3$ option is excluded by charged lepton mass calculation. This point will be discussed later.

16.6.8 Primary condensation levels from p-adic length scale hypothesis

p-Adic length scale hypothesis states that the primary condensation levels correspond to primes near prime powers of two $p \simeq 2^k$, k integer with prime values preferred. Black hole-elementary particle analogy [K59] suggests a generalization of this hypothesis by allowing k to be a power of prime. The general number theoretical vision discussed in [K87] provides a first principle justification for p-adic length scale hypothesis in its most general form. The best fit for the neutrino mass squared differences is obtained for $k = 13^2 = 169$ so that the generalization of the hypothesis might be necessary.

A particle primarily condensed on the level k can suffer secondary condensation on a level with the same value of k: for instance, electron ($k = 127$) suffers secondary condensation on $k = 127$ level. u, d, s quarks ($k = 107$) suffer secondary condensation on nuclear space-time sheet having $k = 113$). All quarks feed their color gauge fluxes at $k = 107$ space-time sheet. There is no deep reason forbidding the condensation of p on p. Primary and secondary condensation levels could also correspond to different but nearly identical values of p with the same value of k.

16.7 Fermion masses

In the earlier model the coefficient of $M^2 = kL_0$ had to be assumed to be different for various particle states. $k = 1$ was assumed for bosons and leptons and $k = 2/3$ for quarks. The fact that $k = 1$ holds true for all particles in the model including also super-symplectic invariance forces to modify the earlier construction of quark states. This turns out to be possible without affecting the earlier p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (h_{gr} can be negative). The structure of lepton and quark states in color degrees of freedom was discussed in [K48] .

16.7.1 Charged lepton mass ratios

The overall mass scale for lepton and quark masses is determined by the condensation level given by prime $p \simeq 2^k$, k prime by length scale hypothesis. For charged leptons k must correspond to $k = 127$ for electron, $k = 113$ for muon and $k = 107$ for τ. For muon $p = 2^{113} - 1 - 4 * 378$ is assumed (smallest prime below 2^{113} allowing $\sqrt{2}$ but not $\sqrt{3}$). So called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne so that all charged leptons correspond to generalized Mersenne primes.

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \; mod \; 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p + 3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2+2p)/3$, $p \geq 1$, for neutrinos and $p=1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2+5p+6)/3$ gives $h_c = 2$ for $p=0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Using CP_2 mass scale m_0^2 [K48] as a p-adic unit, the mass formulas for the charged leptons read as

$$\begin{aligned} M^2(L) &= A(\nu)\frac{m_0^2}{p(L)} \ , \\ A(e) &= 5 + X(p(e)) \ , \\ A(\mu) &= 14 + X(p(\mu)) \ , \\ A(\tau) &= 65 + X(p(\tau)) \ . \end{aligned} \tag{16.7.1}$$

$X(\cdot)$ corresponds to the yet unknown second order corrections to the mass squared.

The following table gives the basic parameters as determined from the mass of electron for some values of Y_e. The mass of top quark favors as maximal value of CP_2 mass which corresponds to $Y_e = 0$.

Y_e	0	.5	.7798
$(m_0/m_{Pl}) \times 10^3$	.2437	.2323	.2266
$K \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954

Table 1. Table gives the values of CP_2 mass m_0 using Planck mass $m_{Pl} = 1/\sqrt{G}$ as unit, the ratio $K = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$.

The following table lists the lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value. The values of lepton masses are $m_e = .510999$ MeV, $m_\mu = 105.76583$ MeV, $m_\tau = 1775$ MeV.

$$\begin{aligned} \frac{m(\mu)_+}{m(\mu)} &= \sqrt{\frac{15}{5}}2^7\frac{m_e}{(\mu)} \simeq 1.0722 \ , \\ \frac{m(\mu)_-}{m(\mu)} &= \sqrt{\frac{14}{6}}2^7\frac{m_e}{m(\mu)} \simeq 0.9456 \ , \\ \frac{m(\tau)_+}{m(\tau)} &= \sqrt{\frac{66}{5}}2^{10}\frac{m_e}{m(\tau)} \simeq 1.0710 \ , \\ \frac{m(\tau)_-}{m(\tau)} &= \sqrt{\frac{65}{6}}2^{10}\frac{m_e}{m(\tau)} \simeq .9703 \ . \end{aligned} \tag{16.7.2}$$

For the maximal value of CP_2 mass the predictions for the mass ratio are systematically too large by a few per cent. From the formulas above it is clear that the second order corrections to mass squared can be such that correct masses result.

τ mass is least sensitive to $X(p(e)) \equiv Y_e$ and the maximum value of $Y_e \equiv Y_{e,max}$ consistent with τ mass corresponds to $Y_{e,max} = .7357$ and $Y_\tau = 1$. This means that the CP_2 mass is at least a fraction .9337 of its maximal value. If Y_L is same for all charged leptons and has the maximal value $Y_{e,max} = .7357$, the predictions for the mass ratios are

$$\begin{aligned} \frac{m(\mu)_{pr}}{m(\mu)} &= \sqrt{\frac{14+Y_{e,max}}{5+Y_{e,max}}} \times 2^7 \frac{m_e}{m(\mu)} \simeq .9922 \ , \\ \frac{m(\tau)_{pr}}{m(\tau)} &= \sqrt{\frac{65+Y_{e,max}}{5+Y_e(max}} \times 2^{10} \frac{m_e}{m(\tau)} \simeq .9980 \ . \end{aligned} \tag{16.7.3}$$

The error is .8 per cent *resp.* .2 per cent for muon *resp.* τ.

The argument leading to estimate for the modular contribution to the mass squared [K48] leaves two options for the coefficient of the modular contribution for $g = 2$ fermions: the value of coefficient is either $X = g$ for $g \leq 1$, $X = 3g - 3$ for $g \geq 2$ or $X = g$ always. For $g = 2$ the predictions are $X = 2$ and $X = 3$ in the two cases. The option $X = 3$ allows slightly larger maximal value of Y_e equal to $Y^{1)}_{e,max} = Y_{e,max} + (5 + Y_{e,max})/66$.

16.7.2 Neutrino masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible and neutrino mixing cannot yet be predicted from the basic principles. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the LSND data on neutrino mass measurement of [C56, C30] and the explanation of the atmospheric ν-deficit in terms of $\nu_\mu - \nu_\tau$ mixing [C45, C36] one can deduce that the most plausible condensation level of μ and τ neutrinos is $k = 167$ or $k = 13^2 = 169$ allowed by the more general form of the p-adic length scale hypothesis suggested by the blackhole-elementary particle analogy. One can also deduce information about the mixing matrix associated with the neutrinos so that mass predictions become rather precise. In particular, the mass splitting of μ and τ neutrinos is predicted correctly if one assumes that the mixing matrix is a rational unitary matrix.

Super Virasoro contribution

Using $m_0^2/3$ as a p-adic unit, the expression for the Super Virasoro contribution to the mass squared of neutrinos is given by the formula

$$\begin{aligned} M^2(SV) &= (s + (3Yp)_R/3)\frac{m_0^2}{p} \ , \\ s &= 4 \text{ or } 5 \ , \\ Y &= \frac{23}{2} + Y_1 \ , \end{aligned} \tag{16.7.4}$$

where m_0^2 is universal mass scale. One can consider two possible identifications of neutrinos corresponding to $s(\nu) = 4$ with $\Delta = 2$ and $s(\nu) = 5$ with $\Delta = 1$. The requirement that CKM matrix is sensible forces the asymmetric scenario in which quarks and, by symmetry, also leptons correspond to lowest possible excitation so that one must have $s(\nu) = 4$. Y_1 represents second order contribution to the neutrino mass coming from renormalization effects coming from self energy diagrams involving intermediate gauge bosons. Physical intuition suggest that this contribution is very small so that the precise measurement of the neutrino masses should give an excellent test for the theory.

With the above described assumptions and for $s = 4$, one has the following mass formula for neutrinos

$$\begin{aligned} M^2(\nu) &= A(\nu)\frac{m_0^2}{p(\nu))} \ , \\ A(\nu_e) &= 4 + \frac{(3Y(p(\nu_e)))_R}{3} \ , \\ A(\nu_\mu) &= 13 + \frac{(3Y(p(\nu_\mu)))_R}{3} \ , \\ A(\nu_\tau) &= 64 + \frac{(3Y(p(\nu_\tau)))_R}{3} \ , \\ 3Y &\simeq \frac{1}{2} \ . \end{aligned} \quad (16.7.5)$$

The predictions must be consistent with the recent upper bounds [C23] of order 10 eV, 270 keV and 0.3 MeV for ν_e, ν_μ and ν_τ respectively. The recently reported results of LSND measurement [C30] for $\nu_e -> \nu_\mu$ mixing gives string limits for $\Delta m^2(\nu_e,\nu_\mu)$ and the parameter $sin^2(2\theta)$ characterizing the mixing: the limits are given in the figure 30 of [C30]. The results suggests that the masses of both electron and muon neutrinos are below 5 eV and that mass squared difference $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$ is between $.25 - 25\ eV^2$. The simplest possibility is that ν_μ and ν_e have common condensation level (in analogy with d and s quarks). There are three candidates for the primary condensation level: namely $k = 163, 167$ and $k = 169$. The p-adic prime associated with the primary condensation level is assumed to be the nearest prime below 2^k allowing p-adic $\sqrt{2}$ but not $\sqrt{3}$ and satisfying $p \ mod\ 4 = 3$. The following table gives the values of various parameters and unmixed neutrino masses in various cases of interest.

k	p	$(3Y)_R/3$	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau)/eV$
163	$2^{163} - 4*144 - 1$	1.36	1.78	3.16	6.98
167	$2^{167} - 4*144 - 1$	.34	.45	.79	1.75
169	$2^{169} - 4*210 - 1$	.17	.22	.40	.87

Could neutrino topologically condense also in other p-adic length scales than $k = 169$?

One must keep mind open for the possibility that there are several p-adic length scales at which neutrinos can condense topologically. Biological length scales are especially interesting in this respect. In fact, all intermediate p-adic length scales $k = 151, 157, 163, 167$ could correspond to metastable neutrino states. The point is that these p-adic lengths scales are number theoretically completely exceptional in the sense that there exist Gaussian Mersenne $2^k \pm i$ (prime in the ring of complex integers) for all these values of k. Since charged leptons, atomic nuclei ($k = 113$) , hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes, it would not be surprising if the biologically important Gaussian Mersennes would correspond to length scales giving rise to metastable neutrino states. Of course, one can keep mind open for the possibility that $k = 167$ rather than $k = 13^2 = 169$ is the length scale defining the stable neutrino physics.

Neutrino mixing

Consider next the neutrino mixing. A quite general form of the neutrino mixing matrix D given by the table below will be considered.

	ν_e	ν_μ	ν_τ
ν_e	c_1	s_1c_3	s_1s_3
ν_μ	$-s_1c_2$	$c_1c_2c_3 - s_2s_3exp(i\delta)$	$c_1c_2s_3 + s_2c_3exp(i\delta)$
ν_τ	$-s_1s_2$	$c_1s_2c_3 + c_2s_3exp(i\delta)$	$c_1s_2s_3 - c_2c_3exp(i\delta)$

Physical intuition suggests that the angle δ related to CP breaking is small and will be assumed to be vanishing. Topological mixing is active only in modular degrees of freedom and one obtains for the first order terms of mixed masses the expressions

$$\begin{aligned} s(\nu_e) &= 4 + 9|U_{12}|^2 + 60|U_{13}|^2 = 4 + n_1 \ , \\ s(\nu_\mu) &= 4 + 9|U_{22}|^2 + 60|U_{23}|^2 = 4 + n_2 \ , \\ s(\nu_\tau) &= 4 + 9|U_{32}|^2 + 60|U_{33}|^2 = 4 + n_3 \ . \end{aligned} \tag{16.7.6}$$

The requirement that resulting masses are not ultra heavy implies that $s(\nu)$ must be small integers. The condition $n_1 + n_2 + n_3 = 69$ follows from unitarity. The simplest possibility is that the mixing matrix is a rational unitary matrix. The same ansatz was used successfully to deduce information about the mixing matrices of quarks. If neutrinos are condensed on the same condensation level, rationality implies that $\nu_\mu - \nu_\tau$ mass squared difference must come from the first order contribution to the mass squared and is therefore quantized and bounded from below.

The first piece of information is the atmospheric ν_μ/ν_e ratio, which is roughly by a factor 2 smaller than predicted by standard model [C45]. A possible explanation is the CKM mixing of muon neutrino with τ-neutrino, whereas the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [C45] are in accordance with the mixing $m^2(\nu_\tau) - m^2(\nu_\mu) \simeq 1.6 \cdot 10^{-2}\ eV^2$ and mixing angle $sin^2(2\theta) = 1.0$: also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If mixing matrix is assumed to be rational then only $k = 169$ condensation level is allowed for ν_μ and ν_τ. For this level $\nu_\mu - \nu_\tau$ mass squared difference turns out to be $\Delta m^2 \simeq 10^{-2}\ eV^2$ for $\Delta s \equiv s(\nu_\tau) - s(\nu_\mu) = 1$, which is the only acceptable possibility and predicts $\nu_\mu - \nu_\tau$ mass squared difference correctly within experimental uncertainties! The fact that the predictions for mass squared differences are practically exact, provides a precision test for the rationality assumption.

What is measured in LSND experiment is the probability $P(t,E)$ that ν_μ transforms to ν_e in time t after its production in muon decay as a function of energy E of ν_μ. In the limit that ν_τ and ν_μ masses are identical, the expression of $P(t,E)$ is given by

$$\begin{aligned} P(t,E) &= sin^2(2\theta)sin^2(\frac{\Delta Et}{2}) \ , \\ sin^2(2\theta) &= 4c_1^2s_1^2c_2^2 \ , \end{aligned} \tag{16.7.7}$$

where ΔE is energy difference of ν_μ and ν_e neutrinos and t denotes time. LSND experiment gives stringent conditions on the value of $\sin^2(2\theta)$ as the figure 30 of [C30] shows. In particular, it seems that $sin^2(2\theta)$ must be considerably below 10^{-1} and this implies that s_1^2 must be small enough.

The study of the mass formulas shows that the only possibility to satisfy the constraints for the mass squared and $sin^2(2\theta)$ given by LSND experiment is to assume that the mixing of the electron neutrino with the tau neutrino is much larger than its mixing with the muon neutrino. This means that s_3 is quite near to unity. At the limit $s_3 = 1$ one obtains the following (nonrational) solution of the mass squared conditions for $n_3 = n_2 + 1$ (forced by the atmospheric neutrino data)

$$\begin{aligned} s_1^2 &= \frac{69 - 2n_2 - 1}{60} \ , \\ c_2^2 &= \frac{n_2 - 9}{2n_2 - 17} \ , \\ sin^2(2\theta) &= \frac{4(n_2 - 9)}{51}\frac{(34 - n_2)(n_2 - 4)}{30^2} \ , \\ s(\nu_\mu) - s(\nu_e) &= 3n_2 - 68 \ . \end{aligned} \tag{16.7.8}$$

The study of the LSND data shows that there is only one acceptable solution to the conditions obtained by assuming maximal mass squared difference for ν_e and ν_μ

$$\begin{aligned} n_1 &= 2 \ \ n_2 = 33 \ \ n_3 = 34 \ , \\ s_1^2 &= \frac{1}{30} \ \ c_2^2 = \frac{24}{49} \ , \\ sin^2(2\theta) &= \frac{24}{49}\frac{2}{15}\frac{29}{30} \simeq .0631 \ , \\ s(\nu_\mu) - s(\nu_e)) &= 31 \leftrightarrow .32 \ eV^2 \ . \end{aligned} \tag{16.7.9}$$

That c_2^2 is near 1/2 is not surprise taking into account the almost mass degeneracy of ν_{mu} and ν_τ. From the figure 30 of [C30] it is clear that this solution belongs to 90 per cent likelihood region of LSND experiment but $sin^2(2\theta)$ is about two times larger than the value allowed by Bugey reactor experiment. The study of various constraints given in [C30] shows that the solution is consistent with bounds from all other experiments. If one assumes that $k > 169$ for ν_e $\nu_\mu - \nu_e$ mass difference increases, implying slightly poorer consistency with LSND data.

There are reasons to hope that the actual rational solution can be regarded as a small deformation of this solution obtained by assuming that c_3 is non-vanishing. $s_1^2 = \frac{69-2n_2-1}{60-51c_3^2}$ increases in the deformation by $O(c_3^2)$ term but if c_3 is positive the value of $c_2^2 \simeq \frac{24-102c_1^0c_2^0s_2^0c_3}{49} \sim \frac{24-61c_3}{49}$ decreases by $O(c_3)$ term so that it should be possible to reduce the value of $sin^2(2\theta)$. Consistency with Bugey reactor experiment requires $.030 \leq sin^2(2\theta) < .033$. $sin^2(2\theta) = .032$ is achieved for $s_1^2 \simeq .035$,$s_2^2 \simeq .51$ and $c_3^2 \simeq .068$. The construction of U and D matrices for quarks shows that very stringent number theoretic conditions are obtained and as in case of quarks it might be necessary to allow complex CP breaking phase in the mixing matrix. One might even hope that the solution to the conditions is unique.

For the minimal rational mixing one has $s(\nu_e) = 5$, $s(\nu_\mu) = 36$ and $s(\nu_\tau) = 37$ if unmixed ν_e corresponds to $s = 4$. For $s = 5$ first order contributions are shifted by one unit. The masses ($s = 4$ case) and mass squared differences are given by the following table.

k	$m(\nu_e)$	$m(\nu_\mu)$	$m(\nu_\tau)$	$\Delta m^2(\nu_\mu - \nu_e)$	$\Delta m^2(\nu_\tau - \nu_\mu)$
169	.27 eV	.66 eV	.67 eV	.32 eV^2	.01 eV^2

Predictions for neutrino masses and mass squared splittings for $k = 169$ case.

Evidence for the dynamical mass scale of neutrinos

In recent years (I am writing this towards the end of year 2004 and much later than previous lines) a great progress has been made in the understanding of neutrino masses and neutrino mixing. The pleasant news from TGD perspective is that there is a strong evidence that neutrino masses depend on environment [C21]. In TGD framework this translates to the statement that neutrinos can suffer topological condensation in several p-adic length scales. Not only in the p-adic length scales suggested by the number theoretical considerations but also in longer length scales, as will be found.

The experiments giving information about mass squared differences can be divided into three categories [C21].

(a) There along baseline experiments, which include solar neutrino experiments [C41, C46, C51] and [C67] as well as earlier studies of solar neutrinos. These experiments see evidence for the neutrino mixing and involve significant propagation through dense matter. For the solar neutrinos and KamLAND the mass splittings are estimated to be of order $O(8\times 10^{-5})$ eV^2 or more cautiously 8×10^{-5} eV^2 $<\delta m^2<2\times 10^{-3}$ eV^2. For K2K and atmospheric neutrinos the mass splittings are of order $O(2\times 10^{-3})eV^2$ or more cautiously $\delta m^2 > 10^{-3}$eV^2. Thus the scale of mass splitting seems to be smaller for neutrinos in matter than in air, which would suggest that neutrinos able to propagate through a dense matter travel at space-time sheets corresponding to a larger p-adic length scale than in air.

(b) There are null short baseline experiments including CHOOZ, Bugey, and Palo Verde reactor experiments, and the higher energy CDHS, JARME, CHORUS, and NOMAD experiments, which involve muonic neutrinos (for references see [C21]. No evidence for neutrino oscillations have been seen in these experiments.

(c) The results of LSND experiment [C30] are consistent with oscillations with a mass splitting greater than $3\times 10^{-2}eV^2$. LSND has been generally been interpreted as necessitating a mixing with sterile neutrino. If neutrino mass scale is dynamical, situation however changes.

If one assumes that the p-adic length scale for the space-time sheets at which neutrinos can propagate is different for matter and air, the situation changes. According to [C21] a mass 3×10^{-2} eV in air could explain the atmospheric results whereas mass of of order .1 eV and $.07eV^2<\delta m^2<.26eV^2$ would explain the LSND result. These limits are of the same order as the order of magnitude predicted by $k=169$ topological condensation.

Assuming that the scale of the mass splitting is proportional to the p-adic mass scale squared, one can consider candidates for the topological condensation levels involved.

(a) Suppose that $k=169=13^2$ is indeed the condensation level for LSND neutrinos. $k=173$ would predict $m_{\nu_e}\sim 7\times 10^{-2}$ eV and $\delta m^2\sim .02$ eV^2. This could correspond to the masses of neutrinos propagating through air. For $k=179$ one has $m_{\nu_e}\sim .8\times 10^{-2}$ eV and $\delta m^2\sim 3\times 10^{-4}$ eV^2 which could be associated with solar neutrinos and KamLAND neutrinos.

(b) The primes $k=157,163,167$ associated with Gaussian Mersennes would give $\delta m^2(157)=2^6\delta m^2(163)=2^{10}\delta m^2(167)=2^{12}\delta m^2(169)$ and mass scales $m(157)\sim 22.8$ eV, $m(163)\sim 3.6$ eV, $m(167)\sim .54$ eV. These mass scales are unrealistic or propagating neutrinos. The interpretation consistent with TGD inspired model of condensed matter in which neutrinos screen the classical Z^0 force generated by nucleons would be that condensed matter neutrinos are confined inside these space-time sheets whereas the neutrinos able to propagate through condensed matter travel along $k>167$ space-time sheets.

The results of MiniBooNE group as a support for the energy dependence of p-adic mass scale of neutrino

The basic prediction of TGD is that neutrino mass scale can depend on neutrino energy and the experimental determinations of neutrino mixing parameters support this prediction.

The newest results (11 April 2007) about neutrino oscillations come from MiniBooNE group which has published its first findings [C19] concerning neutrino oscillations in the mass range studied in LSND experiments [C18].

1. The motivation for MiniBooNE

Neutrino oscillations are not well-understood. Three experiments LSND, atmospheric neutrinos, and solar neutrinos show oscillations but in widely different mass regions (1 eV^2 , 3×10^{-3} eV^2, and 8×10^{-5} eV^2).

In TGD framework the explanation would be that neutrinos can appear in several p-adically scaled up variants with different mass scales and therefore different scales for the differences Δm^2 for neutrino masses so that one should not try to try to explain the results of these experiments using single neutrino mass scale. In single-sheeted space-time it is very difficult to imagine that neutrino mass scale would depend on neutrino energy since neutrinos interact so extremely weakly with matter. The best known attempt to assign single mass to all neutrinos has been based on the use of so called sterile neutrinos which do not have electro-weak couplings. This approach is an ad hoc trick and rather ugly mathematically and excluded by the results of MiniBooNE experiments.

2. The result of MiniBooNE experiment

The purpose of the MiniBooNE experiment was to check whether LSND result $\Delta m^2 = 1eV^2$ is genuine. The group used muon neutrino beam and looked whether the transformations of muonic neutrinos to electron neutrinos occur in the mass squared region $\Delta m^2 \simeq 1$ eV^2. No such transitions were found but there was evidence for transformations at low neutrino energies.

What looks first as an over-diplomatic formulation of the result was *MiniBooNE researchers showed conclusively that the LSND results could not be due to simple neutrino oscillation, a phenomenon in which one type of neutrino transforms into another type and back again.* rather than direct refutation of LSND results.

3. LSND and MiniBooNE are consistent in TGD Universe

The habitant of the many-sheeted space-time would not regard the previous statement as a mere diplomatic use of language. It is quite possible that neutrinos studied in MiniBooNE have suffered topological condensation at different space-time sheet than those in LSND if they are in different energy range (the preferred rest system fixed by the space-time sheet of the laboratory or Earth). To see whether this is the case let us look more carefully the experimental arrangements.

(a) In LSND experiment 800 MeV proton beam entering in water target and the muon neutrinos resulted in the decay of produced pions. Muonic neutrinos had energies in 60-200 MeV range [C18].

(b) In MiniBooNE experiment [C19] 8 GeV muon beam entered Beryllium target and muon neutrinos resulted in the decay of resulting pions and kaons. The resulting muonic neutrinos had energies the range 300-1500 GeV to be compared with 60-200 MeV.

Let us try to make this more explicit.

(a) Neutrino energy ranges are quite different so that the experiments need not be directly comparable. The mixing obeys the analog of Schrödinger equation for free particle with energy replaced with $\Delta m^2/E$, where E is neutrino energy. The mixing probability as a function of distance L from the source of muon neutrinos is in 2-component model given by

$$P = sin^2(\theta)sin^2(1.27\Delta m^2 L/E) \ .$$

The characteristic length scale for mixing is $L = E/\Delta m^2$. If L is sufficiently small, the mixing is fifty-fifty already before the muon neutrinos enter the system, where the measurement is carried out and no mixing is detected. If L is considerably longer than the size of the measuring system, no mixing is observed either. Therefore the result can be understood if Δm^2 is much larger or much smaller than E/L, where L is the size of the measuring system and E is the typical neutrino energy.

(b) MiniBooNE experiment found evidence for the appearance of electron neutrinos at low neutrino energies (below 500 MeV) which means direct support for the LSND findings and for the dependence of neutron mass scale on its energy relative to the rest system defined by the space-time sheet of laboratory.

(c) Uncertainty Principle inspires the guess $L_p \propto 1/E$ implying $m_p \propto E$. Here E is the energy of the neutrino with respect to the rest system defined by the space-time sheet of the laboratory. Solar neutrinos indeed have the lowest energy (below 20 MeV) and the lowest value of Δm^2. However, atmospheric neutrinos have energies starting from few hundreds of MeV and $\Delta; m^2$ is by a factor of order 10 higher. This suggests that the the growth of Δm^2 with E^2 is slower than linear. It is perhaps not the energy alone which matters but the space-time sheet at which neutrinos topologically condense. For instance, MiniBooNE neutrinos above 500 MeV would topologically condense at space-time sheets for which the p-adic mass scale is higher than in LSND experiments and one would have $\Delta m^2 >> 1$ eV^2 implying maximal mixing in length scale much shorter than the size of experimental apparatus.

(d) One could also argue that topological condensation occurs in condensed matter and that no topological condensation occurs for high enough neutrino energies so that neutrinos remain massless. One can even consider the possibility that the p-adic length scale L_p is proportional to E/m_0^2, where m_0 is proportional to the mass scale associated with non-relativistic neutrinos. The p-adic mass scale would obey $m_p \propto m_0^2/E$ so that the characteristic mixing length would be by a factor of order 100 longer in MiniBooNE experiment than in LSND.

Comments

Some comments on the proposed scenario are in order: some of the are written much later than the previous text.

(a) Mass predictions are consistent with the bound $\Delta m(\nu_\mu, \nu_e) < 2\ eV^2$ coming from the requirement that neutrino mixing does not spoil the so called r-process producing heavy elements in Super Novae [C63].

(b) TGD neutrinos cannot solve the dark matter problem: the total neutrino mass required by the cold+hot dark matter models would be about 5 eV. In [K22] a model of galaxies based on string like objects of galaxy size and providing a more exotic source of dark matter, is discussed.

(c) One could also consider the explanation of LSND data in terms of the interaction of ν_μ and nucleon via the exchange of $g = 1$ W boson. The fraction of the reactions $\bar{\nu}_\mu + p \rightarrow e^+ + n$ is at low neutrino energies $P \sim \frac{m_W^4(g=0)}{m_W^4(g=1)} sin^2(\theta_c)$, where θ_c denotes Cabibbo angle. Even if the condensation level of $W(g = 1)$ is $k = 89$, the ratio is by a factor of order .05 too small to explain the average $\nu_\mu \rightarrow \nu_e$ transformation probability $P \simeq .003$ extracted from LSND data.

(d) The predicted masses exclude MSW and vacuum oscillation solutions to the solar neutrino problem unless one assumes that several condensation levels and thus mass scales are possible for neutrinos. This is indeed suggested by the previous considerations.

16.7.3 Quark masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

Basic mass formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p,p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-symplectic operators O with a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

Consider now the mass squared values for quarks. For $h(D) = 0$ and $h(U) = -1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D) = 8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{16.7.10}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark. When m_0^2 is used as a unit X is replaced by $X = (Y_p)_R$.

With the above described assumptions one has the following mass formula for quarks

$$\begin{array}{l} M^2(q) = A(q)\frac{m_0^2}{p(q)} \ , \\ \\ \begin{array}{lll} A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\ A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ . \end{array} \end{array} \tag{16.7.11}$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulomb energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$. The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

(a) $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$(n_d, n_s, n_b) = (5,5,59) \ , \quad (n_u, n_c, n_t) = (5,6,58) \ . \tag{16.7.12}$$

fixing completely the quark masses apart possible Higgs contribution [K57] . Note that top quark mass is still rather near to its maximal value.

(b) The constraint that valence quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. $u-d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

The p-adic length scales associated with quarks and quark masses

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model [K57]) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

(a) Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\bar{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2} m_q$ must hold true. This gives strong constraints on quark masses.

(b) The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

(a) The nuclear p-adic length scale $L_e(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *all* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux feeded to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is feeded to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be feeded at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

(b) The requirement that the masses of diagonal pseudo-scalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.

Top quark mass tends to be slightly too low as compared to the most recent experimental value of $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [C24] . The optimal situation corresponds to $Y_e = 0$ and $Y_t = 1$ and happens to give top mass exactly equal to the most probable experimental value. It must be emphasized that top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top.

In the case of light quarks there are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula [K57] .

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. The table below summarizes constituent quark masses as predicted by this model.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [C6] gives the value ranges for current quark masses depicted in the table below together with TGD predictions for the spectrum of current quark masses.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8×10^3

Table 3. The experimental value ranges for current quark masses [C6] and TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, and $Y_e = 0$. For top quark $Y_t = 0$ is assumed. $Y_t = 1$ would give 169.2 GeV.

Some comments are in order.

(a) The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.

(b) u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4 \times 31, 3 \times 41, 2 \times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2 \times 61, 11^2, 3 \times 5 \times 8)$.

(c) The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [K84, L6] , [L6] .

(d) The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [C6] but not that of b.

(e) The mass of top quark has been slightly below the experimental estimate for long time. The experimental value has been coming down slowly and the most recent value obtained by CDF [C25] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV and consistent with the TGD prediction for $Y_e = Y_t = 0$.

One can talk about constituent and current quark masses simultaneously only if they correspond to dual descriptions. $M^8 - H$ duality [K48] has been indeed suggested to relate the old fashioned low energy description of hadrons in terms of $SO(4)$ symmetry (Skyrme model) and higher energy description of hadrons based on QCD. In QCD description the mass of say baryon would be dominated by the mass associated with super-symplectic quanta carrying color. In $SO(4)$ description constituent quarks would carry most of the hadron mass.

Can Higgs field develop a vacuum expectation in fermionic sector at all?

An important conclusion following from the calculation of lepton and quark masses is that if Higgs contribution is present, it can be of second order p-adically and even negligible, perhaps even vanishing. There is indeed an argument forcing to consider this possibility seriously. The recent view about elementary particles is following.

(a) Fermions correspond to CP_2 type vacuum extremals topologically condensed at positive/negative energy space-time sheets carrying quantum numbers at light-like wormhole throat. Higgs and gauge bosons correspond to wormhole contacts connecting positive and negative energy space-time sheets and carrying fermion and anti-fermion quantum numbers at the two light-like wormhole throats.

(b) If the values of p-adic temperature are $T_p = 1$ and $T_p = 1/n$, $n > 1$f or fermions and bosons the thermodynamical contribution to the gauge boson mass is negligible.

(c) Different p-adic temperatures and Kähler coupling strengths for fermions and bosons make sense if bosonic and fermionic partonic 3-surfaces meet only along their ends at the vertices of generalized Feynman diagrams but have no other common points [K20] . This forces to consider the possibility that fermions cannot develop Higgs vacuum expectation value although they can couple to Higgs. This is not in contradiction with the modification of sigma model of hadrons based on the assumption that vacuum expectation of σ field gives a small contribution to hadron mass [K52] since this field can be assigned to some bosonic space-time sheet pair associated with hadron.

(d) Perhaps the most elegant interpretation is that ground state conformal is equal to the square of the eigenvalue of the modified Dirac operator. The ground state conformal weight is negative and its deviation from half odd integer value gives contribution to both fermion and boson masses. The Higgs expectation associated with coherent state of Higgs like wormhole contacts is naturally proportional to this parameter since no other parameter with dimensions of mass is present. Higgs vacuum expectation determines gauge boson masses only apparently if this interpretation is correct. The contribution of the ground state conformal weight to fermion mass square is near to zero. This means that λ is very near to negative half odd integer and therefore no significant difference between fermions and gauge bosons is implied.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 2. Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

16.8 About the microscopic description of gauge boson massivation

The conjectured QFT limit allows to estimate the quantitative predictions of the theory. This is not however enough. One should identify the microscopic TGD counterparts for various aspects of gauge boson massivation. There is also the question about the consistency of the gauge theory limit with the ZEO inspired view about massivation. The basic challenge are obvious: one should translate notions like Higgs vacuum expectation, massivation of gauge bosons, and finite range of weak interactions to the language of wormhole throats, Kähler magnetic flux tubes, and string world sheets. The proposal is that generalization of super-conformal symmetries to their Yangian counterparts is needed to meet this challenge in mathematically satisfactory manner.

16.8.1 Can p-adic thermodynamics explain the masses of intermediate gauge bosons?

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \ .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Contrary to the original optimistic beliefs based on calculational error, it turned out impossible to predict W/e and Z/e mass ratios correctly in the original p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they seemed to exclude the explanation for the massivation of gauge bosons using p-adic thermodynamics.

(a) The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

(b) Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. For $T_p = 1/2$ thermal masses are completely negligible.

(c) The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$.

The conclusion is that p-adic thermodynamics does not produce a natural description for the massivation of weak bosons. For $p = M_{89}$ the mass scale is somewhat too small even if the second order contribution is maximal. If it is characterized by small integer, the contribution is extremely small. An explanation for the value of Weinberg angle is also missing. Hence some additional contribution to mass must be present. Higgsy contribution is not natural in TGD framework but stringy contribution looks very natural.

16.8.2 The counterpart of Higgs vacuum expectation in TGD

The development of the TGD view about Higgs involves several wrong tracks involving a lot of useless calculation. All this could have been avoided with more precise definition of basic notions. The following view has distilled through several failures and might be taken as starting point.

The basic challenge is to translate the QFT description of gauge boson massivation to microscopic description.

(a) One can say that gauge bosons "eat" the components of Higgs. In unitary gauge one gauge rotates Higgs field to electromagnetically neutral direction defined by the vacuum expectation value of Higgs. The rotation matrix codes for the degrees of freedom assignable to non-neutral part of Higgs and they are transferred to the longitudinal components of Higgs in gauge transformation. This gives rise to the third polarization direction for gauge boson. Photon remains massless because em charge commutes with Higgs.

(b) The generation of vacuum expectation value has two functions: to make weak gauge bosons massive and to define the electromagnetically neutral direction to which Higgs field is rotated in the transition to the unitary gauge. In TGD framework only the latter function remains for Higgs if p-adic thermodynamics takes care of massivation. The notion of induced gauge field together with CP_2 geometry uniquely defines the electromagnetically neutral direction so that vacuum expectation is not needed. Of course, the essential element is gauge invariance of the Higgs gauge boson couplings. In twistor Grassmann approach gauge invariance is replaced with Yangian symmetry, which is excellent candidate also for basic symmetry of TGD.

(c) The massivation of gauge bosons (all particles) involves two contributions. The contribution from p-adic thermodynamics in CP_2 scale (wormhole throat) and the stringy contribution in weak scale more generally, in hadronic scale. The latter contribution cannot be calculated yet. The generalization of p-adic thermodynamics to that for Yangian symmetry instead of mere super-conformal symmetry is probably necessary to achieve this and the construction WCW geometry and spinor structure strongly supports the interpretation in terms of Yangian.

One can look at the situation also at quantitative level.

(a) W/Z mass ratio is extremely sensitive test for any model for massivation. In the recent case this requires that string tension for weak gauge boson depends on boson and is proportional to the appropriate gauge coupling strength depending on Weinberg angle. This is natural if the contribution to mass squared can be regarded as perturbative.

(b) Higgs mechanism is characterized by the parameter m_0^2 defining the originally tachyonic mass of Higgs, the dimensionless coupling constant λ defining quartic self-interaction of Higgs. Higgs vacuum expectation is given by $\mu^2 = m_0^2/\lambda$, Higgs mass squared by $m_0^2 = \mu^2\lambda$, and weak boson mass squared is proportional $g^2\mu^2$. In TGD framework λ takes the role of g^2 in stringy picture and the string tensions of bosons are proportional to g_w^2, g_Z^2, λ respectively.

(c) Whether λ in TGD framework actually corresponds to the quartic self-couplingof Higgs or just to the numerical factor in Higgs string tension, is not clear. The problem of Higgs mechanism is that the mass of observed Higgs is somewhat too low. This requires fine tuning of the parameters of the theory and SUSY, which was hoped to come in rescue, did not solve the problem. TGD approach promises to solve the problem.

16.8.3 Elementary particles in ZEO

Let us first summarize what kind of picture ZEO suggests about elementary particles.

(a) Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [K29].

(b) The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

(c) A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the modified Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is spartner of left-handed neutrino.

It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

16.8.4 Virtual and real particles and gauge conditions in ZEO

ZEO and twistor Grassmann approach force to build a detailed view about real and virtual particles. ZEO suggests also new approaches to gauge conditions in the attempts to build detailed connection between QFT picture and that provided by TGD. The following is the most conservative one. Of course, also this proposal must be taken with extreme cautiousness.

(a) In ZEO all wormhole throats - also those associated with virtual particles - can be regarded as massless. In twistor Grassmann approach [K78] this means that the fermionic propagators can be by residue integration transformed to their inverses which correspond to online massless states but having an unphysical polarization so that the internal lines do not vanish identically.

(b) This picture inspired by twistorial considerations is consistent with the simplest picture about Kähler-Dirac action. The boundary term for K-D action is $\sqrt{g_4}\overline{\Psi}\Gamma^n_{K-D}\Psi d^3x$ and due to the localization of spinor modes to 2-D surfaces reduces to a term localized at the boundaries of string world sheets. The normal component Γ^n_{K-D} of the modified gamma matrices defined by the canonical momentum currents of Kähler action should define the inverse of massless fermionic propagator. If the action of this operator on the induced spinor mode at stringy curves satisfies

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi \ ,$$

this reduction is achieved. One can pose the condition $g_4 = constant$ as a coordinate condition on stringy curves at the boundaries of CD and the condition would correlate the spinor modes at stringy curve with incoming quantum numbers. This is extremely powerful simplification giving hopes about calculable theory. The residue integral for virtual momenta reduces the situation to integral over on mass shell momenta and only non-physical helicities contribute in internal lines. This would generalize twistorial formulas to fermionic context.

One however ends up with an unexpected prediction which has bothered me for a long time. Consider the representation of massless spin 1 gauge bosons as pairs as wormhole throat carrying fermion and antifermion having net quantum numbers of the boson. Neglect the effects of the second wormhole throat. The problem is that for on-mass shell massless fermion and antifermion with physical helicities the boson has spin 0. Helicity 1 state would require that second fermion has unphysical helicity. What does this mean?

(a) Are all on mass shell gauge bosons - including photon - massive? Or is on mass shell massless propagation impossible? Massivation is achieved if the fermion and antifermion have different momentum directions: for instance opposite 3-momen but same sign of energy. Higher order contributions in p-adic thermodynamics could make also photon massive. The 4-D world-lines of fermion and antifermion would not be however parallel, which does not conform with the geometric optics based prejudices.

(b) Or could on mass shell gauge bosons have opposite four-momenta so that the second gauge boson would have negative energy? In this manner one could have massless on mass shell states. ZEO ontology certainly allows the identification massless gauge bosons as on mass shell states with opposite directions of four-momenta. This would however require the weakening of the hypothesis that all incoming (outgoing) fundamental fermions have positive (negative) energies to the assumption that only the incoming (outgoing) particles have positive (negative) energies. In the case of massless gauge boson the gauge condition $p \cdot \epsilon = 0$ would be satisfied by the momenta of both fermion and antifermion. With opposite 3-momenta (massivation) but same energy the condition $p_{tot} \cdot \epsilon = 0$ is satisfied for three polarization since in cm system p_{tot} has only time component.

(c) The problem is present also for internal lines. Since by residue argument only the unphysical fermion helicities contribute in internal lines, both fermion and antifermion must have unphysical helicity. For the same sign of energy the wormhole throat would behave as scalar particle. Therefore it seems that the energies must have different sign or momenta cannot be strictly parallel. This is required also by the possibility of space-like momenta for virtual bosons.

16.8.5 The role of string world sheets and magnetic flux tubes in massivation

What is the role of string world sheets and flux tubes in the massivation? At the fundamental level one studies correlation functions for particles and finite correlation length means massivation.

(a) String world sheets define as essential element in 4-D description. All particles are basically bi-local objects: pairs of string at parallel space-time sheets extremely near to each other and connected by wormhole contacts at ends. String world sheets are expected to represent correlations between wormhole throats.

(b) Correlation length for the propagator of the gauge boson characterizes its mass. Correlation length can be estimated by calculating the correlation function. For bosons this reduces to the calculation of fermionic correlations functions assignable to string world sheets connecting the upper and lower boundaries of CD and having four external fermions at the ends of CD. The perturbation theory reduces to functional integral over space-time sheets and deformation of the space-time sheet inducing the deformation of the induced spinor field expressible as convolution of the propagator associated with the modified Dirac operator with vertex factor defined by the deformation multiplying the spinor field. The external vertices are braid ends at partonic 2-surfaces and internal vertices are in the interior of string world sheet. Recall that the conjecture is that the restriction to the wormhole throat orbits implies the reduction to diagrams involving only propagators connecting braid ends. The challenge is to understand how the coherent state assigned to the Euclidian pion field induces the finite correlation length in the case of gauge bosons other than photon.

(c) The non-vanishing commutator of the gauge boson charge matrix with the vacuum expectation assigned to the Euclidian pion must play a key role. The study of the modified Dirac operator suggests that the braid strands contain the Abelianized variant of non-integrable phase factor defined as $exp(i\int Adx)$. If A is identified as string world sheet Hodge dual of Kac-Moody charge the opposite edges of string world sheet with geometry of square given contributions which compensate each other by conservation of Kac-Moody charge if A commutes with the operators building the coherent Higgs state. For photon this would be true. For weak gauge bosons this would not be the case and this gives hopes about obtaining destructive interference leading to a finite correlation length.

One can also consider try to build more concrete manners to understand the finite correlation length.

(a) Quantum classical correspondence suggests that string with length of order $L \sim \hbar/E$, $E = \sqrt{p^2 + m^2}$ serves as a correlate for particle defined by a pair of wormhole contacts. For massive particle wave length satisfies $L \leq \hbar/m$. Here (p, m) must be replaced with (p_L, m_L) if one takes the notion of longitudinal mass seriously. For photon standard option gives $L = \lambda$ or $L = \lambda_L$ and photon can be a bi-local object connecting arbitrarily distant objects. For the second option small longitudinal mass of photon gives an upper bound for the range of the interaction. Also gluon would have longitudinal mass: this makes sense in QCD where the decomposition $M^4 = M^2 \times E^2$ is basic element of the theory.

(b) The magnetic flux tube associated with the particle carries magnetic energy. Magnetic energy grows as the length of flux tube increases. If the flux is quantized magnetic field behaves like $1/S$, where S is the area of the cross section of the flux tube, the total magnetic energy behaves like L/S. The dependence of S on L determines how the magnetic energy depends on L. If the magnetic energy increases as function of L the probability of long flux tubes is small and the particle cannot have large size and therefore mediates short range interactions. For $S \propto L^\alpha \sim \lambda^\alpha$, $\alpha > 1$, the magnetic energy behaves like $\lambda^{-\alpha+1}$ and the thickness of the flux tube scales like $\sqrt{\lambda^\alpha}$. In case of photon one might expect this option to be true. Note that for photon string world sheet one can argue that the natural choice of string is as light-like string so that its length vanishes.

What kind of string world sheets are possible? One can imagine two options.

(a) All strings could connect only the wormhole contacts defining a particle as a bi-local object so that particle would be literally the geometric correlate for the interaction between two objects. The notion of free particle would be figment of imagination. This would lead to a rather stringy picture about gauge interactions. The gauge interaction between systems S_1 and S_2 would mean the emission of gauge bosons as flux tubes with charge carrying end at S_1 and neutral end. Absorption of the gauge boson would mean that the neutral end of boson and neutral end of charge particle fuse together line the lines of Feynman diagram at 3-vertex.

(b) Second option allows also string world sheets connecting wormhole contacts of different particles so that there is no flux tube accompanying the string world sheet. In this case particles would be independent entities interacting via string world sheets. In this case one could consider the possibility that photon corresponds to string world sheet (or actually parallel pair of them) not accompanied by a magnetic flux tube and that this makes the photon massless at least in excellent approximation.

The first option represents the ontological minimum.

Super-conformal symmetry involves two conformal weight like integers and these correspond to the conformal weight assignable to the radial light-like coordinate appearing in the role of complex coordiante in super-symplectic Hamiltonians and to the spinorial conformal weight assignable to the solutions of Kähler Dirac equation localized to string world sheets. These conformal weights are independent quantum numbers unless one can use the light-like radial coordinate as string coordinate, which is certainly not possible always. The latter conformal weight should correspond to the stringy contribution to the masses of elementary particles and hadron like states. In fact, it is difficult to distinguish between elementary particles and hadrons at the fundamental level since both involve the stringy aspect.

The Yangian symmetry variant of conformal symmetry is highly suggestive and brings in poly-locality with respect to partonic 2-surfaces. This integer would count the number of partonic 2-surfaces to which the generator acts and need not correspond to spinorial conformal weight as one might think first. In any case, Yangian variant of p-adic termodynamics provides an attractive approach concerning the mathematical realization of this vision.

16.8.6 Weak Regge trajectories

The weak form of electric-magnetic duality suggests strongly the existence of weak Regge trajectories.

(a) The most general mass squared formula with spin-orbit interaction term $M^2_{L-S} L \cdot S$ reads as

$$M^2 = nM_1^2 + M_0^2 + M^2_{L-S} L \cdot S \ , \ n = 0, 2, 4 \ \text{or} \ n = 1, 3, 5, ..., \ . \tag{16.8.1}$$

M_1^2 corresponds to string tension and M_0^2 corresponds to the thermodynamical mass squared and possible other contributions. For a given trajectory even (odd) values of n have same parity and can correspond to excitations of same ground state. From ancient books written about hadronic string model one vaguely recalls that one can have several trajectories (satellites) and if one has something called exchange degeneracy, the even and odd trajectories define single line in $M^2 - J$ plane. As already noticed TGD variant of Higgs mechanism combines together $n = 0$ states and $n = 1$ states to form massive gauge bosons so that the trajectories are not independent.

(b) For fermions, possible Higgs, and pseudo-scalar Higgs and their super partners also p-adic thermodynamical contributions are present. M_0^2 must be non-vanishing also for gauge bosons and be equal to the mass squared for the $n = L = 1$ spin singlet. By applying the formula to $h = \pm 1$ states one obtains

$$M_0^2 = M^2(boson) \ . \tag{16.8.2}$$

The mass squared for transversal polarizations with $(h, n, L) = (\pm 1, n = L = 0, S = 1)$ should be same as for the longitudinal polarization with $(h = 0, n = L = 1, S = 1, J = 0)$ state. This gives

$$M_1^2 + M_0^2 + M^2_{L-S} L \cdot S = M_0^2 \ . \tag{16.8.3}$$

From $L \cdot S = [J(J+1) - L(L+1) - S(S+1)]/2 = -2$ for $J = 0, L = S = 1$ one has

$$M^2_{L-S} = -\frac{M_1^2}{2} \ . \tag{16.8.4}$$

Only the value of weak string tension M_1^2 remains open.

(c) If one applies this formula to arbitrary $n = L$ one obtains total spins $J = L + 1$ and $L - 1$ from the tensor product. For $J = L - 1$ one obtains

$$M^2 = (2n + 1)M_1^2 + M_0^2 \ .$$

For $J = L + 1$ only M_0^2 contribution remains so that one would have infinite degeneracy of the lightest states. Therefore stringy mass formula must contain a non-linear term making Regge trajectory curved. The simplest possible generalization which does not affect n=0 and n=1 states is of from

$$M^2 = n(n-1)M_2^2 + (n - \frac{L \cdot S}{2})M_1^2 + M_0^2 \ . \tag{16.8.5}$$

The challenge is to understand the ratio of W and Z^0 masses, which is purely group theoretic and provides a strong support for the massivation by Higgs mechanism.

(a) The above formula and empirical facts require

$$\frac{M_0^2(W)}{M_0^2(Z)} = \frac{M^2(W)}{M^2(Z)} = cos^2(\theta_W) \ . \tag{16.8.6}$$

in excellent approximation. Since this parameter measures the interaction energy of the fermion and anti-fermion decomposing the gauge boson depending on the net quantum numbers of the pair, it would look very natural that one would have

$$M_0^2(W) = g_W^2 M_{SU(2)}^2 \ , \ M_0^2(Z) = g_Z^2 M_{SU(2)}^2 \ . \tag{16.8.7}$$

Here $M_{SU(2)}^2$ would be the fundamental mass squared parameter for $SU(2)$ gauge bosons. p-Adic thermodynamics of course gives additional contribution which is vanishing or very small for gauge bosons.

(b) The required mass ratio would result in an excellent approximation if one assumes that the mass scales associated with $SU(2)$ and $U(1)$ factors suffer a mixing completely analogous to the mixing of $U(1)$ gauge boson and neutral $SU(2)$ gauge boson W_3 leading to γ and Z^0. Also Higgs, which consists of $SU(2)$ triplet and singlet in TGD Universe, would very naturally suffer similar mixing. Hence $M_0(B)$ for gauge boson B would be analogous to the vacuum expectation of corresponding mixed Higgs component. More precisely, one would have

$$\begin{aligned} M_0(W) &= M_{SU(2)} \ , \\ M_0(Z) &= cos(\theta_W)M_{SU(2)} + sin(\theta_W)M_{U(1)} \ , \\ M_0(\gamma) &= -sin(\theta_W)M_{SU(2)} + cos(\theta_W)M_{U(1)} \ . \end{aligned} \tag{16.8.8}$$

The condition that photon mass is very small and corresponds to IR cutoff mass scale gives $M_0(\gamma) = \epsilon cos(\theta_W)M_{SU(2)}$, where ϵ is very small number, and implies

$$\begin{aligned} \frac{M_{U(1)}}{M(W)} &= tan(\theta_W) + \epsilon \ , \\ \frac{M(\gamma)}{M(W)} &= \epsilon \times cos(\theta_W) \ , \\ \frac{M(Z)}{M(W)} &= \frac{1 + \epsilon \times sin(\theta_W)cos(\theta_W)}{cos(\theta_W)} \ . \end{aligned} \tag{16.8.9}$$

There is a small deviation from the prediction of the standard model for W/Z mass ratio but by the smallness of photon mass the deviation is so small that there is no hope of measuring it. One can of course keep mind open for $\epsilon = 0$. The formulas allow also an interpretation in terms of Higgs vacuum expectations as it must. The vacuum expectation would most naturally correspond to interaction energy between the massless fermion and anti-fermion with opposite 3-momenta at the throats of the wormhole contact and the challenge is to show that the proposed formulas characterize this interaction energy. Since CP_2 geometry codes for standard model symmetries and their breaking, it woul not be surprising if this would happen. One cannot exclude the possibility that p-adic thermodynamics contributes to $M_0^2(boson)$. For instance, ϵ might characterize the p-adic thermal mass of photon.

If the mixing applies to the entire Regge trajectories, the above formulas would apply also to weak string tensions, and also photons would belong to Regge trajectories containing high spin excitations.

(c) What one can one say about the value of the weak string tension M_1^2? The naive order of magnitude estimate is $M_1^2 \simeq m_W^2 \simeq 10^4$ GeV^2 is by a factor 1/25 smaller than the direct scaling up of the hadronic string tension about 1 GeV^2 scaled up by a factor

2^{18}. The above argument however allows also the identification as the scaled up variant of hadronic string tension in which case the higher states at weak Regge trajectories would not be easy to discover since the mass scale defined by string tension would be 512 GeV to be compared with the recent beam energy 7 TeV. Weak string tension need of course not be equal to the scaled up hadronic string tension. Weak string tension - unlike its hadronic counterpart- could also depend on the electromagnetic charge and other characteristics of the particle.

16.8.7 Low mass exotic mesonic structures as evidence for dark scaled down variants of weak bosons?

During last years reports about low mass exotic mesonic structures have appeared. It is interesting to combine these bits of data with the recent view about TGD analog of Higgs mechanism and find whether new predictions become possible. The basic idea is to derive understanding of the low mass exotic structures from LHC data by scaling and understanding of LHC data from data about mesonic structures by scaling back.

(a) The article *Search for low-mass exotic mesonic structures: II. attempts to understand the experimental results* by Taticheff and Tomasi-Gustafsson [C68] mentions evidence for exotic mesonic structures. The motivation came from the observation of a narrow range of dimuon masses in $\Sigma^+ \rightarrow pP^0$, $P^0 \rightarrow \mu^-\mu^+$ in the decays of P^0 with mass of $214.3 \pm .5$ MeV: muon mass is 105.7 MeV giving $2m_\mu = 211.4$ MeV. Mesonlike exotic states with masses $M = 62, 80, 100, 181, 198, 215, 227.5$, and 235 MeV are reported. This fine structure of states with mass difference 20-40 MeV between nearby states is reported for also for some baryons.

(b) The preprint *Observation of the E(38) boson* by Kh.U. Abraamyan et al [C71, C72, C35] reports the observation of what they call E(38) boson decaying to gamma pair observed in d(2.0 GeV/n)+C,d(3.0 GeV/n)+Cu and p(4.6 GeV)+C reactions in experiments carried in JINR Nuclotron.

If these results can be replicated they mean a revolution in nuclear and hadron physics. What strongly suggests itself is a fine structure for ordinary hadron states in much smaller energy scale than characterizing hadronic states. Unfortunately the main stream, in particular the theoreticians interested in beyond standard model physics, regard the physics of strong interactions and weak interactions as closed chapters of physics, and are not interested on results obtained in nuclear collisions.

In TGD framework situation is different. The basic characteristic of TGD Universe is fractality. This predicts new physics in all scales although standard model symmetries are fundamental unlike in GUTs and are reduced to number theory. p-Adic length scale hypothesis characterizes the fractality.

(a) In TGD Universe p-adic length scale hypothesis predicts the possibility of scaled versions of both strong and weak interactions. The basic objection against new light bosons is that the decay widths of weak bosons do not allow them. A possible manner to circumvent the objection is that the new light states correspond to dark matter in the sense that the value of Planck constant is not the standard one but its integer multiple [K27].

The assumption that only particles with the same value of Planck constant can appear in the vertex, would explain why weak bosons do not decay directly to light dark particles. One must however allow the transformation of gauge bosons to their dark counterparts. The 2-particle vertex is characterized by a coupling having dimensions of mass squared in the case of bosons, and p-adic length scale hypothesis suggests that the primary p-adic mass scale characterizes the parameter (the secondary p-adic mass scale is lower by factor $1/\sqrt{p}$ and would give extremely small transformation rate).

(b) Ordinary strong interactions correspond to Mersenne prime M_n, $n = 2^{107} - 1$, in the sense that hadronic space-time sheets correspond to this p-adic prime. Light quarks correspond to space-time sheets identifiable as color magnetic flux tubes, which are much larger than hadron itself. M_{89} hadron physics has hadronic mass scale 512 times higher than ordinary hadron physics and should be observed at LHC. There exist some pieces of evidence for the mesons of this hadron physics but masked by the Higgsteria. The expectation is that Minkowskian M_{89} pion has mass around 140 GeV assigned to CDF bump [C16].

(c) In the leptonic sector there is evidence for lepto-hadron physics for all charged leptons labelled by Mersenne primes M_{127}, $M_{G,113}$ (Gaussian Mersenne), and M_{107} [K92]. One can ask whether the above mentioned resonance P^0 decaying to $\mu^-\mu^+$ pair motivating the work described in [C68] could correspond to pion of muon-hadron physics consisting of a pair of color octet excitations of muon. Its production would presumably take place via production of virtual gluon pair decaying to a pair of color octet muons.

Returning to the observations of [C68]: the reported meson-like exotic states seem to be arranged along Regge trajectories but with string tension lower than that for the ordinary Regge trajectories with string tension $T = .9$ GeV^2. String tension increases slowly with mass of meson like state and has three values $T/GeV^2 \in \{1/390, 1/149.7, 1/32.5\}$ in the piecewise linear fit discussed in the article. The TGD inspired proposal is that IR Regge trajectories assignable to the color magnetic flux tubes accompanying quarks are in question. For instance, in hadrons u and d quarks - understood as constituent quarks - would have $k = 113$ quarks and string tension would be by naive scaling by a factor $2^{107-113} = 1/64$ lower: as a matter of fact, the largest value of the string tension is twice this value. For current quark with mass scale around 5 MeV the string tension would be by a factor of order $2^{107-121} = 2^{-16}$ lower.

Clearly, a lot of new physics is predicted and it begins to look that fractality - one of the key predictions of TGD - might be realized both in the sense of hierarchy of Planck constants (scaled variants with same mass) and p-adic length scale hypothesis (scaled variants with varying masses). Both hierarchies would represent dark matter if one assumes that the values of Planck constant and p-adic length scale are same in given vertex. The testing of predictions is not however expected to be easy since one must understand how ordinary matter transforms to dark matter and vice versa. Consider only the fact, that only recently the exotic meson like states have been observed and modern nuclear physics regarded often as more or less trivial low energy phenomenology was born born about 80 years ago when Chadwick discovered neutron.

16.8.8 Weak Regge trajectories

The weak form of electric-magnetic duality suggests strongly the existence of weak Regge trajectories.

(a) The most general mass squared formula with spin-orbit interaction term $M^2_{L-S} L\cdot S$ reads as

$$M^2 = nM_1^2 + M_0^2 + M^2_{L-S} L\cdot S \ , \ n = 0, 2, 4 \text{ or } n = 1, 3, 5, ..., \ . \quad (16.8.10)$$

M_1^2 corresponds to string tension and M_0^2 corresponds to the thermodynamical mass squared and possible other contributions. For a given trajectory even (odd) values of n have same parity and can correspond to excitations of same ground state. From ancient books written about hadronic string model one vaguely recalls that one can have several trajectories (satellites) and if one has something called exchange degeneracy, the even and odd trajectories define single line in $M^2 - J$ plane. As already noticed TGD variant of Higgs mechanism combines together $n = 0$ states and $n = 1$ states to form massive gauge bosons so that the trajectories are not independent.

(b) For fermions, possible Higgs, and pseudo-scalar Higgs and their super partners also p-adic thermodynamical contributions are present. M_0^2 must be non-vanishing also for gauge bosons and be equal to the mass squared for the $n = L = 1$ spin singlet. By applying the formula to $h = \pm 1$ states one obtains

$$M_0^2 = M^2(boson) . \tag{16.8.11}$$

The mass squared for transversal polarizations with $(h, n, L) = (\pm 1, n = L = 0, S = 1)$ should be same as for the longitudinal polarization with $(h = 0, n = L = 1, S = 1, J = 0)$ state. This gives

$$M_1^2 + M_0^2 + M_{L-S}^2 L \cdot S = M_0^2 . \tag{16.8.12}$$

From $L \cdot S = [J(J+1) - L(L+1) - S(S+1)]/2 = -2$ for $J = 0, L = S = 1$ one has

$$M_{L-S}^2 = -\frac{M_1^2}{2} . \tag{16.8.13}$$

Only the value of weak string tension M_1^2 remains open.

(c) If one applies this formula to arbitrary $n = L$ one obtains total spins $J = L + 1$ and $L - 1$ from the tensor product. For $J = L - 1$ one obtains

$$M^2 = (2n+1)M_1^2 + M_0^2 .$$

For $J = L+1$ only M_0^2 contribution remains so that one would have infinite degeneracy of the lightest states. Therefore stringy mass formula must contain a non-linear term making Regge trajectory curved. The simplest possible generalization which does not affect n=0 and n=1 states is of from

$$M^2 = n(n-1)M_2^2 + (n - \frac{L \cdot S}{2})M_1^2 + M_0^2 . \tag{16.8.14}$$

The challenge is to understand the ratio of W and Z^0 masses, which is purely group theoretic and provides a strong support for the massivation by Higgs mechanism.

(a) The above formula and empirical facts require

$$\frac{M_0^2(W)}{M_0^2(Z)} = \frac{M^2(W)}{M^2(Z)} = cos^2(\theta_W) . \tag{16.8.15}$$

in excellent approximation. Since this parameter measures the interaction energy of the fermion and anti-fermion decomposing the gauge boson depending on the net quantum numbers of the pair, it would look very natural that one would have

$$M_0^2(W) = g_W^2 M_{SU(2)}^2 , \quad M_0^2(Z) = g_Z^2 M_{SU(2)}^2 . \tag{16.8.16}$$

Here $M_{SU(2)}^2$ would be the fundamental mass squared parameter for $SU(2)$ gauge bosons. p-Adic thermodynamics of course gives additional contribution which is vanishing or very small for gauge bosons.

(b) The required mass ratio would result in an excellent approximation if one assumes that the mass scales associated with $SU(2)$ and $U(1)$ factors suffer a mixing completely analogous to the mixing of $U(1)$ gauge boson and neutral $SU(2)$ gauge boson W_3 leading to γ and Z^0. Also Higgs, which consists of $SU(2)$ triplet and singlet in TGD Universe, would very naturally suffer similar mixing. Hence $M_0(B)$ for gauge boson B would be analogous to the vacuum expectation of corresponding mixed Higgs component. More precisely, one would have

$$
\begin{aligned}
M_0(W) &= M_{SU(2)} \ , \\
M_0(Z) &= cos(\theta_W)M_{SU(2)} + sin(\theta_W)M_{U(1)} \ , \\
M_0(\gamma) &= -sin(\theta_W)M_{SU(2)} + cos(\theta_W)M_{U(1)} \ .
\end{aligned} \tag{16.8.17}
$$

The condition that photon mass is very small and corresponds to IR cutoff mass scale gives $M_0(\gamma) = \epsilon cos(\theta_W)M_{SU(2)}$, where ϵ is very small number, and implies

$$
\begin{aligned}
\frac{M_{U(1)}}{M(W)} &= tan(\theta_W) + \epsilon \ , \\
\frac{M(\gamma)}{M(W)} &= \epsilon \times cos(\theta_W) \ , \\
\frac{M(Z)}{M(W)} &= \frac{1 + \epsilon \times sin(\theta_W)cos(\theta_W)}{cos(\theta_W)} \ .
\end{aligned} \tag{16.8.18}
$$

There is a small deviation from the prediction of the standard model for W/Z mass ratio but by the smallness of photon mass the deviation is so small that there is no hope of measuring it. One can of course keep mind open for $\epsilon = 0$. The formulas allow also an interpretation in terms of Higgs vacuum expectations as it must. The vacuum expectation would most naturally correspond to interaction energy between the massless fermion and anti-fermion with opposite 3-momenta at the throats of the wormhole contact and the challenge is to show that the proposed formulas characterize this interaction energy. Since CP_2 geometry codes for standard model symmetries and their breaking, it woul not be surprising if this would happen. One cannot exclude the possibility that p-adic thermodynamics contributes to $M_0^2(boson)$. For instance, ϵ might characterize the p-adic thermal mass of photon.

If the mixing applies to the entire Regge trajectories, the above formulas would apply also to weak string tensions, and also photons would belong to Regge trajectories containing high spin excitations.

(c) What one can one say about the value of the weak string tension M_1^2? The naive order of magnitude estimate is $M_1^2 \simeq m_W^2 \simeq 10^4$ GeV^2 is by a factor 1/25 smaller than the direct scaling up of the hadronic string tension about 1 GeV^2 scaled up by a factor 2^{18}. The above argument however allows also the identification as the scaled up variant of hadronic string tension in which case the higher states at weak Regge trajectories would not be easy to discover since the mass scale defined by string tension would be 512 GeV to be compared with the recent beam energy 7 TeV. Weak string tension need of course not be equal to the scaled up hadronic string tension. Weak string tension - unlike its hadronic counterpart- could also depend on the electromagnetic charge and other characteristics of the particle.

16.8.9 Cautious conclusions

The discussion of TGD counterpart of Higgs mechanism gives support for the following general picture.

(a) p-Adic thermodynamics for wormhole contacts contributes to the masses of all particles including photon and gluons: in these cases the contributions are however small. For fermions they dominate. For weak bosons the contribution from string tension of string connecting wormhole contacts as the correct group theoretical prediction for the W/Z mass ratio demonstrates. The mere spin 1 character for gauge bosons implies that they are massive in 4-D sense unless massless fermion and anti-fermion have opposite signs of energy. Higgs provides the longitudinal components of weak bosons by gauge invariance and CP_2 geometry defines unitary gauge so that Higgs vacuum expectation value is not needed. The non-existence of covariantly constant CP_2 vector field does not mean absence of Higgs like particle as believed first but only the impossibility of Higgs vacuum expectation value.

The usual space-time SUSY associated with imbedding space in TGD framework is not needed, and there are strong arguments suggesting that it is not present [?] For space-time regarded as 4-surfaces one obtains 2-D super-conformal invariance for fermions localized at 2-surfaces and for right-handed neutrino it extends to 4-D superconformal symmetry generalizing ordinary SUSY to infinite-D symmetry.

(b) The basic predictions to LHC are following. M_{89} hadron physics, whose pion was first proposed to be identifiable as Higgs like particle, will be discovered. The findings from RHIC and LHC concerning collisions of heavy ions and protons and heavy ions already provide support for the existence of string like objects identifiable as mesons of M_{89} physics. Fermi satellite has produced evidence for a particle with mass around 140 GeV and this particle could correspond to the pion of M_{89} physics. This particle should be observed also at LHC and CDF reported already earlier evidence for it. There has been also indications for other mesons of M_{89} physics from LHC discussed in [K52].

(c) Fermion and boson massivation by Higgs mechanism could emerge unavoidably as a theoretical artefact if one requires the existence of QFT limit leading unavoidably to a description in terms of Higgs mechanism. In the real microscopic theory p-adic thermodynamics for wormhole contacts and strings connecting them would describe fermion massivation, and might describe even boson massivation in terms of long parts of flux tubes. Situation remains open in this respect. Therefore the observation of decays of Higgs at expected rate to fermion pairs cannot kill TGD based vision.

The new view about Higgs combined with the stringy vision about twistor Grassmannian [?] allows to see several conjectures related to ZEO in new light and also throw away some conjectures such as the idea about restriction of virtual momenta to plane $M^2 \subset M^4$.

(a) The basic conjecture related to the perturbation theory is that wormhole throats are massless on mass shell states in imbedding space sense: this would hold true also for virtual particles and brings in mind what happens in twistor program. The recent progress [K105] in the construction of n-point functions leads to explicit general formulas for them expressing them in terms of a functional integral over four-surfaces. The deformation of the space-time surface fixes the deformation of basis for induced spinor fields and one obtains a perturbation theory in which correlation functions for imbedding space coordinates and fermionic propagator defined by the inverse of the modified Dirac operator appear as building bricks and the electroweak gauge coupling of the modified Dirac operator define the basic vertex. This operator is indeed 2-D for all other fermions than right-handed neutrino.

(b) The functional integral gives some expressions for amplitudes which resemble twistor amplitudes in the sense that the vertices define polygons and external fermions are massless although gauge bosons as their bound states are massive. This suggests a stringy generalization of twistor Grassmannian approach [K78]. The residue integral would replace 4-D integrations of virtual fermion momenta to integrals over massless momenta. The outcome would be non-vanishing for non-physical helicities of virtual fermion. Also the problem due to the fact that fermionic Super Virasoro generator carries fermion number in TGD framework disappears.

(c) There are two conformal weights involved. The conformal weight associated with the light-like radial coordinate of $\delta M^4_\pm$ and the spinorial conformal weight associated with the fermionic string connecting wormhole throats and throats of wormhole contact. Are these conformal weights independent or not? For instance, could one use radial light-like coordinate as string coordinate in the generic situation so that the conformal weights would not define independent quantum numbers? This does not look feasible. The Yangian variant of conformal algebra involves two integers. Second integer would naturally be the number of partonic 2-surfaces acted by the generator characterizing the poly-locality of Yangian generators, and it is not clear whether it has anything to do with the spinorial conformal weight. One can of course consider also three integers! This would be in accordance with the idea that the basic objects are 3-dimensional.

If the conjecture that Yangian invariance realized in terms of Grassmannians makes sense, it could allow to deduce the outcome of the functional integral over four-surfaces and one could hope that TGD can be transformed to a calculable theory. Also p-adic mass calculations should be formulated using p-adic thermodynamics assuming Yangian invariance and enlargened conformal algebra.

16.9 Calculation of hadron masses and topological mixing of quarks

The calculation of quark masses is not enough since one must also understand CKM mixing of quarks in order to calculate hadron massess. A model for CKM matrix and hadron masses is constructed in [K57] and here only a brief summary about basic ideas involved is given.

16.9.1 Topological mixing of quarks

In TGD framework CKM mixing is induced by topological mixing of quarks (that is 2-dimensional topologies characterized by genus). The strongest number theoretical constraint on mixing matrices would be that they are rational. Perhaps a more natural constraint is that they are expressible in terms of roots of unity for some finite dimensional algebraic extension of rationals and therefore also p-adic numbers.

Number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium subject to constraints from the integer valued modular contributions remaing integer valued in the mixing. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as direct sum of essentially unique 2×2 and 1×1 matrices. The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of integers n_1 and n_2 characterizing the lowest order contribution to quark mass. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n(u), n(c)) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n(u), n(c)) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n(d) = n(s) = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

16.9.2 Higgsy contribution to fermion masses is negligible

There are good reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. Thus p-adic thermodynamics would explain fermion masses completely. This together with the fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it, fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very strong constraint on the model.

16.9.3 The p-adic length scale of quark is dynamical

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses using only quark masses and the assumption mass squared is additive for quarks with same p-adic length scale and mass for quarks labelled by different primes p. This conforms with the idea that pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between hadrons containing different numbers of strange and heavy quarks can be understood if s, b and c quarks appear as several scaled up versions.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in the case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

It should be however made clear that the notion of quark mass is problematic. One can speak about current quark masses and constituent quark masses. For u and d quarks constituent quark masses have scale 10^2 GeV are much higher than current quark masses having scale 10 GeV. For current quarks the dominating contribution to hadron mass would come from super-symplectic bosons at quantum level and at more phenomenological level from hadronic string tension. The open question is which option to choose or whether one should regard the two descriptions as duals of each other based on $M^8 - H$ duality. M^8 description would be natural at low energies since $SO(4)$ takes the role of color group. One could also say that current quarks are created in de-confinement phase transition which involves change of the p-adic length scale characterizing the quark. Somewhat counter intuitively but in accordance with Uncertainty Principle this length scale would increase but one could assign it the color magnetic field body of the quark.

16.9.4 Super-symplectic bosons at hadronic space-time sheet can explain the constant contribution to baryonic masses

Current quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD which could be characterized in terms of constituent quark masses in M^8 picture and in terms of current quark masses and string tension or super-symplectic bosons in $M^4 \times CP_2$ picture.

Super-symplectic gluons provide an attractive description of this contribution. They need not exclude more phenomenological description in terms of string tension. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-symplectic gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-symplectic boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-symplectic boson need not to be assumed. The preferred role of pion would relate to the fact that its mass scale is below QCD Λ.

16.9.5 Description of color magnetic spin-spin splitting in terms of conformal weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are contributions coming from both ordinary gluons and super-symplectic gluons and the latter is expected to dominate by the large value of color coupling strength.

Conformal weight replaces energy as the basic variable but group theoretical structure of color magnetic contribution to the conformal weight associated with hadronic space-time sheet ($k = 107$) is same as in case of energy. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

The comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-symplectic particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-symplectic gluons. The next challenge would be to predict the correlation of hadron spin with super-symplectic particle content in case of long-lived hadrons.

Chapter 17

New Physics Predicted by TGD

17.1 Introduction

TGD predicts a lot of new physics and it is quite possible that this new physics becomes visible at LHC. Although calculational formalism is still lacking, p-adic length scale hypothesis allows to make precise quantitative predictions for particle masses by using simple scaling arguments. Actually there is already now evidence for effects providing further support for TGD based view about QCD and first rumors about super-symmetric particles have appeared.

Before detailed discussion it is good to summarize what elements of quantum TGD are responsible for new physics.

(a) The new view about particles relies on their identification as partonic 2-surfaces (plus 4-D tangent space data to be precise). This effective metric 2-dimensionality implies generalization of the notion of Feynman diagram and holography in strong sense. One implication is the notion of field identity or field body making sense also for elementary particles and the Lamb shift anomaly of muonic hydrogen could be explained in terms of field bodies of quarks.

(b) The topological explanation for family replication phenomenon implies genus generation correspondence and predicts in principle infinite number of fermion families. One can however develop a rather general argument based on the notion of conformal symmetry known as hyper-ellipticity stating that only the genera $g = 0, 1, 2$ are light [**?**] What "light" means is however an open question. If light means something below CP_2 mass there is no hope of observing new fermion families at LHC. If it means weak mass scale situation changes.

   For bosons the implications of family replication phenomenon can be understood from the fact that they can be regarded as pairs of fermion and anti-fermion assignable to the opposite wormhole throats of wormhole throat. This means that bosons formally belong to octet and singlet representations of dynamical SU(3) for which 3 fermion families define 3-D representation. Singlet would correspond to ordinary gauge bosons. Also interacting fermions suffer topological condensation and correspond to wormhole contact. One can either assume that the resulting wormhole throat has the topology of sphere or that the genus is same for both throats.

(c) The view about space-time supersymmetry differs from the standard view in many respects. First of all, the super symmetries are not associated with Majorana spinors. Super generators correspond to the fermionic oscillator operators assignable to leptonic and quark-like induced spinors and there is in principle infinite number of them so that formally one would have $\mathcal{N} = \infty$ SUSY. I have discussed the required modification of the formalism of SUSY theories in [**?**]nd it turns out that effectively one obtains just $\mathcal{N} = 1$ SUSY required by experimental constraints. The reason is that the fermion states with higher fermion number define only short range interactions analogous to van der Waals forces. Right handed neutrino generates this super-symmetry broken by the mixing of the M^4 chiralities implied by the mixing of M^4 and CP_2 gamma matrices for induced gamma matrices. The simplest assumption is that particles and their superpartners obey the same mass formula but that the p-adic length scale can be different for them.

(d) The new view about particle massivation based on p-adic thermodynamics raises the question about the role of Higgs field. The vacuum expectation value (VEV) of Higgs is not feasible in TGD since CP_2 does not allow covariantly constant holomorphic vector fields. The original too strong conclusion from this was that TGD does not allow Higgs. Higgs VEV is not needed for the selection of preferred electromagnetic direction in electro-weak gauge algebra (unitary gauge) since CP_2 geometry does that. p-Adic thermodynamics explains fermion masses bout the masses of weak bosons cannot be understood on basis of p-adic thermodynamics alone giving extremely small second order contribution only and failing to explain W/Z mass ratio. Weak boson mass can be associated to the string tension of the strings connecting the throats of two wormhole contacts associated with elementary particle (two of them are needed since the monopole magnetic flux must have closed field lines).

At M^4 QFT limit Higgs VEV is the only possible description of massivation. Dimensional gradient coupling to Higgs field developing VEV explains fermion masses at this limit. The dimensional coupling is same for all fermions so that one avoids the loss of "naturalness" due to the huge variation of Higgs-fermion couplings in the usual description.

The stringy contribution to elementary particle mass cannot be calculated from the first principles. A generalization of p-adic thermodynamics based on the generalization of super-conformal algebra is highly suggestive. There would be two conformal weights corresponding the the conformal weight assignable to the radial light-like coordinate of light-cone boundary and to the stringy coordinate and third integer characterizing the poly-locality of the generator of Yangian associated with this algebra (n-local generator acts on n partonic 2-surfaces simultaneously).

(e) One of the basic distinctions between TGD and standard model is the new view about color.

i. The first implication is separate conservation of quark and lepton quantum numbers implying the stability of proton against the decay via the channels predicted by GUTs. This does not mean that proton would be absolutely stable. p-Adic and dark length scale hierarchies indeed predict the existence of scale variants of quarks and leptons and proton could decay to hadons of some zoomed up copy of hadrons physics. These decays should be slow and presumably they would involve phase transition changing the value of Planck constant characterizing proton. It might be that the simultaneous increase of Planck constant for all quarks occurs with very low rate.

ii. Also color excitations of leptons and quarks are in principle possible. Detailed calculations would be required to see whether their mass scale is given by CP_2 mass scale. The so called lepto-hadron physics proposed to explain certain anomalies associated with both electron, muon, and τ lepton could be understood in terms of color octet excitations of leptons [?]

(f) Fractal hierarchies of weak and hadronic physics labelled by p-adic primes and by the levels of dark matter hierarchy are highly suggestive. Ordinary hadron physics corresponds to $M_{107} = 2^{107} - 1$ One especially interesting candidate would be scaled up hadronic physics which would correspond to $M_{89} = 2^{89} - 1$ defining the p-adic prime of weak bosons. The corresponding string tension is about 512 GeV and it might be possible to see the first signatures of this physics at LHC. Nuclear string model in turn predicts that nuclei correspond to nuclear strings of nucleons connected by colored flux tubes having light quarks at their ends. The interpretation might be in terms of M_{127} hadron physics. In biologically most interesting length scale range 10 nm-2.5 μm contains four electron Compton lengths $L_e(k) = \sqrt{5}L)k)$ associated with Gaussian Mersennes and the conjecture is that these and other Gaussian Mersennes are associated with zoomed up variants of hadron physics relevant for living matter. Cosmic rays might also reveal copies of hadron physics corresponding to M_{61} and M_{31}

The well-definedness of em charge for the modes of induced spinor fields localizes them at 2-D surfaces with vanishing W fields and also Z^0 field above weak scale. This allows to avoid undesirable parity breaking effects.

(g) Weak form of electric magnetic duality implies that the fermions and anti-fermions associated with both leptons and bosons are Kähler magnetic monopoles accompanied by monopoles of opposite magnetic charge and with opposite weak isospin. For quarks Kähler magnetic charge need not cancel and cancellation might occur only in hadronic length scale. The magnetic flux tubes behave like string like objects and if the string tension is determined by weak length scale, these string aspects should become visible at LHC. If the string tension is 512 GeV the situation becomes less promising.

In this chapter the predicted new elementary particle physics and possible indications for it are discussed. Second chapter is devoted to new hadron physics and scaled up variants of hardon physics in both quark and lepton sector.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L21]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L22]. The topics relevant to this chapter are given by the following list.

- TGD view about elementary particles [L78]
- p-Adic length scale hypothesis [L57]
- p-Adic mass calculations [L56]
- Geometrization of fields [L37]
- Magnetic body [L51]
- Emergent ideas and notions [L34]
- Elementary particle vacuum functionals [L32]
- Emergence of bosons [L33]
- Leptohadron hypothesis [L48]
- M89 hadron physics [L50]
- SUSY and TGD [L69]

17.2 Scaled variants of quarks and leptons

17.2.1 Fractally scaled up versions of quarks

The strange anomalies of neutrino oscillations [C21] suggesting that neutrino mass scale depends on environment can be understood if neutrinos can suffer topological condensation in several p-adic length scales [K48] . The obvious question whether this could occur also in the case of quarks led to a very fruitful developments leading to the understanding of hadronic mass spectrum in terms of scaled up variants of quarks. Also the mass distribution of top quark candidate exhibits structure which could be interpreted in terms of heavy variants of light quarks. The ALEPH anomaly [C73], which I first erratically explained in terms of a light top quark has a nice explanation in terms of b quark condensed at $k = 97$ level and having mass ~ 55 GeV. These points are discussed in detail in [K57] .

The emergence of ALEPH results [C73] meant a an important twist in the development of ideas related to the identification of top quark. In the LEP 1.5 run with $E_{cm} = 130-140\ GeV$, ALEPH found 14 e^+e^- annihilation events, which pass their 4-jet criteria whereas 7.1 events are expected from standard model physics. Pairs of dijets with vanishing mass difference are in question and dijets could result from the decay of a new particle with mass about 55 GeV.

The data do not allow to conclude whether the new particle candidate is a fermion or boson. Top quark pairs produced in e^+e^- annihilation could produce 4-jets via gluon emission but this mechanism does not lead to an enhancement of 4-jet fraction. No $b\bar{b}b\bar{b}$ jets have been observed and only one event containing b has been identified so that the interpretation in terms of top quark is not possible unless there exists some new decay channel, which dominates in decays and leads to hadronic jets not initiated by b quarks. For option 2), which seems to be the only sensible option, this kind of decay channels are absent.

Super symmetrized standard model suggests the interpretation in terms of super partners of quarks or/and gauge bosons [C47] . It seems now safe to conclude that TGD does not predict sparticles. If the exotic particles are gluons their presence does not affect Z^0 and W decay widths. If the condensation level of gluons is $k = 97$ and mixing is absent the gluon masses are given by $m_g(0) = 0$, $m_g(1) = 19.2\ GeV$ and $m_g(2) = 49.5\ GeV$ for option 1) and assuming $k = 97$ and hadronic mass renormalization. It is however very difficult to understand how a pair of $g = 2$ gluons could be created in e^+e^- annihilation. Moreover, for option 2), which seems to be the only sensible option, the gluon masses are $m_g(0) = 0$, $m_g(1) = m_g(2) = 30.6\ GeV$ for $k = 97$. In this case also other values of k are possible since strong decays of quarks are not possible.

The strong variations in the order of magnitude of mass squared differences between neutrino families [C21] can be understood if they can suffer a topological condensation in several p-adic length scales. One can ask whether also t and b quark could do the same. In absence of mixing effects the masses of $k = 97$ t and b quarks would be given by $m_t \simeq 48.7$ GeV and $m_b \simeq 52.3$ GeV taking into account the hadronic mass renormalization. Topological mixing reduces the masses somewhat. The fact that b quarks are not observed in the final state leaves only $b(97)$ as a realistic option. Since Z^0 boson mass is ~ 94 GeV, $b(97)$ does not appreciably affect Z^0 boson decay width. The observed anomalies concentrate at cm energy about 105 GeV. This energy is 15 percent smaller than the total mass of top pair. The discrepancy could be understood as resulting from the binding energy of the $b(97)\bar{b}(97)$ bound states. Binding energy should be a fraction of order $\alpha_s \simeq .1$ of the total energy and about ten per cent so that consistency is achieved.

17.2.2 Could neutrinos appear in several p-adic mass scales?

There are some indications that neutrinos can appear in several mass scales from neutrino oscillations [C5]. These oscillations can be classified to vacuum oscillations and to solar neutrino oscillations believed to be due to the so called MSW effect in the dense matter of Sun. There are also indications that the mixing is different for neutrinos and antineutrinos [C22, C4].

In TGD framework p-adic length scale hypothesis might explain these findings. The basic vision is that the p-adic length scale of neutrino can vary so that the mass squared scale comes as octaves. Mixing matrices would be universal. The large discrepancy between LSND and MiniBoone results [C22] contra solar neutrino results could be understood if electron and muon neutrinos have same p-adic mass scale for solar neutrinos but for LSND and MiniBoone the mass scale of either neutrino type is scaled up. The existence of a sterile neutrino [C52] suggested as an explanation of the findings would be replaced by p-adically scaled up variant of ordinary neutrino having standard weak interactions. This scaling up can be different for neutrinos and antineutrinos as suggested by the fact that the anomaly is present only for antineutrinos.

The different values of Δm^2 for neutrinos and antineutrinos in MINOS experiment [C4] can be understood if the p-adic mass scale for neutrinos increases by one unit. The breaking of CP and CPT would be spontaneous and realized as a choice of different p-adic mass scales and could be understood in zero energy ontology. Similar mechanism would break supersymmetry and explain large differences between the mass scales of elementary fermions, which for same p-adic prime would have mass scales differing not too much.

Experimental results

There several different type of experimental approaches to study the oscillations. One can study the deficit of electron type solar electron neutrinos (Kamiokande, Super-Kamiokande); one can measure the deficit of muon to electron flux ratio measuring the rate for the transformation of ν_μ to ν_τ (super-Kamiokande); one can study directly the deficit of ν_e ($\overline{\nu}_e$) neutrinos due to transformation to ν_μ $\nu_{\overline{\mu}}$ coming from nuclear reactor with energies in the same range as for solar neutrinos (KamLAND); and one can also study neutrinos from particle accelerators in much higher energy range such as solar neutrino oscillations (K2K,LSND,Miniboone,Minos).

1. Solar neutrino experiments and atmospheric neutrino experiments

The rate of neutrino oscillations is sensitive to the mass squared differences Δm_{12}^2, Δm_{12}^2, Δm_{13}^2 and corresponding mixing angles θ_{12}, θ_{13}, θ_{23} between ν_e, ν_μ, and ν_τ (ordered in obvious manner). Solar neutrino experiments allow to determine $sin^2(2\theta_{12})$ and Δm_{12}^2. The experiments involving atmospheric neutrino oscillations allow to determine $sin^2(2\theta_{23})$ and Δm_{23}^2.

The estimates of the mixing parameters obtained from solar neutrino experiments and atmospheric neutrino experiments are $sin^2(2\theta_{13}) = 0.08$, $sin^2(2\theta_{23}) = 0.95$, and $sin^2(2\theta_{12}) = 0.86$. The mixing between ν_e and ν_τ is very small. The mixing between ν_e and ν_μ, and ν_μ and ν_τ tends is rather near to maximal. The estimates for the mass squared differences are $\Delta m_{12}^2 = 8\times 10^{-5}$ eV^2, $\Delta m_{23}^2 \simeq \Delta m_{13}^2 = 2.4\times 10^{-3}$ eV^2. The mass squared differences have obviously very different scale but this need not means that the same is true for mass squared values.

2. The results of LSND and MiniBoone

LSND experiment measuring the transformation of $\overline{\nu}_\mu$ to $\overline{\nu}_e$ gave a totally different estimate for Δm_{12}^2 than solar neutrino experiments MiniBoone [C52]. If one assumes same value of $sin^2(\theta_{12})^2 \simeq .86$ one obtains $\Delta m_{23}^2 \sim .1$ eV^2 to be compared with $\Delta m_{12}^2 = 8\times 10^{-5}$ eV^2. This result is known as LSND anomaly and led to the hypothesis that there exists a sterile neutrino having no weak interactions and mixing with the ordinary electron neutrino and inducing a rapid mixing caused by the large value of Δm^2. The purpose of MiniBoone experiment [C22] was to test LSND anomaly.

(a) It was found that the two-neutrino fit for the oscillations for $\nu_\mu \to \nu_e$ is not consistent with LSND results. There is an unexplained 3σ electron excess for $E < 475$ MeV. For $E > 475$ MeV the two-neutrino fit is not consistent with LSND fit. The estimate for Δm^2 is in the range $.1 - 1$ eV^2 and differs dramatically from the solar neutrino data.

(b) For antineutrinos there is a small 1.3σ electron excess for $E < 475$ MeV. For $E >$ 475 MeV the excess is 3 per cent consistent with null. Two-neutrino oscillation fits are consistent with LSND. The best fit gives $(\Delta m_{12}^2, sin^2(2\theta_{12}) = (0.064\ eV^2, 0.96)$. The value of Δm_{12}^2 is by a factor 800 larger than that estimated from solar neutrino experiments.

All other experiments (see the table of the summary of [C52] about sterile neutrino hypothesis) are consistent with the absence of $\nu_\mu \to n_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ mixing and only LSND and MiniBoone report an indication for a signal. If one however takes these findings seriously they suggest that neutrinos and antineutrinos behave differently in the experimental situations considered. Two-neutrino scenarios for the mixing (no sterile neutrinos) are consistent with data for either neutrinos or antineutrinos but not both [C52].

3. The results of MINOS group

The MINOS group at Fermi National Accelerator Laboratory has reported evidence that the mass squared differences between neutrinos are not same for neutrinos and antineutrinos [C4]. In this case one measures the disappearance of ν_μ and $\overline{\nu}_\mu$ neutrinos from high energy beam

beam in the range .5-1 GeV and the dominating contribution comes from the transformation to τ neutrinos. Δm_{23}^2 is reported to be about 40 percent larger for antineutrinos than for neutrinos. There is 5 percent probability that the mass squared differences are same. The best fits for the basic parameters are ($\Delta m_{23}^2 = 2.35\times 10^{-3}, sin^2(2\theta_{23}=1)$ for neutrinos with error margin for Δm^2 being about 5 per cent and ($\Delta m_{23}^2 = 3.36\times 10^{-3}, sin^2(2\theta_{23}) = .86$) for antineutrinos with errors margin around 10 per cent. The ratio of mass squared differences is $r \equiv \Delta m^2(\overline{\nu})/\Delta m^2(\nu) = 1.42$. If one assumes $sin^2(2\theta_{23}) = 1$ in both cases the ratio comes as $r = 1.3$.

Explanation of findings in terms of p-adic length scale hypothesis

p-Adic length scale hypothesis predicts that fermions can correspond to several values of p-adic prime meaning that the mass squared comes as octaves (powers of two). The simplest model for the neutrino mixing assumes universal topological mixing matrices and therefore for CKM matrices so that the results should be understood in terms of different p-adic mass scales. Even CP breaking and CPT breaking at fundamental level is un-necessary although it would occur spontaneously in the experimental situation selecting different p-adic mass scales for neutrinos and antineutrinos. The expression for the mixing probability a function of neutrino energy in two-neutrino model for the mixing is of form

$$P(E) = sin^2(2\theta)sin^2(X) \ \ , \ \ X = k\times \Delta m^2 \times \frac{L}{E} \ .$$

Here k is a numerical constant, L is the length travelled, and E is neutrino energy.

1. LSND and MiniBoone results

LSND and MiniBoone results are inconsistent with solar neutrino data since the value of Δm_{12}^2 is by a factor 800 larger than that estimated from solar neutrino experiments. This could be understood if in solar neutrino experiments ν_μ and ν_w correspond to the same p-adic mass scale $k = k_0$ and have very nearly identical masses so that Δm^2 scale is much smaller than the mass squared scale. If either p-adic scale is changed from k_0 to $k_0 + k$, the mass squared difference increases dramatically. The counterpart of the sterile neutrino would be a p-adically scaled up version of the ordinary neutrino having standard electro-weak interactions. The p-adic mass scale would correspond to the mass scale defined by Δm^2 in LSND and MiniBoone experiments and therefore a mass scale in the range .3-1 eV. The electron Compton scale assignable to eV mass scale could correspond to $k = 167$, which corresponds to cell length scale of 2.5 μm. $k = 167$ defines one of the Gaussian Mersennes $M_{G,k} = (1 + i)^k - 1$. $L_e(k) = \sqrt{5}L(k)$, $k = 151, 157, 163, 167$, varies in the range 10 nm (cell membrane thickness) and 2.5 μm defining the size of cell nucleus. These scales could be fundamental for the understanding of living matter [K24] .

2. MINOS results

One must assume also now that the p-adic mass scales for ν_τ and $\overline{\nu}_\tau$ are near to each other in the "normal" experimental situation. Assuming that the mass squared scales of ν_μ or $\overline{\nu}_\mu$ come as 2^{-k} powers of $m_{\nu_\mu}^2 = m_{\nu_\tau}^2 + \Delta m^2$, one obtains

$$m_{\nu_\tau}^2(k_0) - m_{\overline{\nu}_\mu}^2(k_0 + k) = (1 - 2^{-k})m^2_{nu_\tau} - 2^{-k}\Delta m_0^2 \ .$$

For $k = 1$ this gives

$$r \ = \ \frac{\Delta m^2(k=2)}{\Delta m^2(k=1)} = \frac{\frac{3}{2} - \frac{2r}{3}}{1-r} \ \ , \ \ r = \frac{\Delta m_0^2}{m_{\nu_\tau}^2} \ . \qquad (17.2.1)$$

One has $r \geq 3/2$ for $r > 0$ if one has $m_{\nu_\tau} > m_{\nu_\mu}$ for the same p-adic length scale. The experimental ratio $r \simeq 1.3$ could be understood for $r \simeq -.31$. The experimental uncertainties certainly allow the value $r = 1.5$ for $k(\overline{\nu}_\mu) = 1$ and $k(\nu_\mu) = 2$.

This result implies that the mass scale of ν_μ and ν_τ differ by a factor 1/2 in the "normal" situation so that mass squared scale of ν_τ would be of order 5×10^{-3} eV^2. The mass scales for $\overline{\nu}_\tau$ and ν_τ would about .07 eV and .05 eV. In the LSND and MiniBoone experiments the p-adic mass scale of other neutrino would be around .1-1 eV so that different p-adic mass scale large by a factor $2^{k/2}$, $2 \leq 2 \leq 7$ would be in question. The different resuts from various experiments could be perhaps understood in terms of the sensitivity of the p-adic mass scale to the experimental situation. Neutrino energy could serve as a control parameter.

CPT breaking [B3] requires the breaking of Lorentz invariance. Zero energy ontology could therefore allow a spontaneous breaking of CP and CPT. This might relate to matter anti-matter asymmetry at the level of given CD.

There is some evidence that the mixing matrices for neutrinos and antineutrinos are different in the experimental situations considered [C4, C22]. This would require CPT breaking in the standard QFT framework. In TGD p-adic length scale hypothesis allowing neutrinos to reside in several p-adic mass scales. Hence one could have apparent CPT breaking if the measurement arrangements for neutrinos and antineutrinos select different p-adic length scales for them [K52] .

Is CP and T breaking possible in zero energy ontology?

The CKM matrices for quarks and possibly also leptons break CP and T. Could one understand the breaking of CP and T at fundamental level in TGD framework?

(a) In standard QFT framework Chern-Simons term breaks CP and T. Kähler action indeed reduces to Chern-Simons terms for the proposed ansatz for preferred extremals assuming that weak form of electric-magnetic duality holds true.

In TGD framework one must however distinguish between space-time coordinates and imbedding space coordinates. CP breaking occurs at the imbedding space level but instanton term and Chern-Simons term are odd under P and T only at the space-time level and thus distinguish between different orientations of space-time surface. Only if one identifies P and T at space-time level with these transformations at imbedding space level, one has hope of interpreting CP and T breaking as spontaneous breaking of these symmetries for Kähler action and basically due to the weak form of electric-magnetic duality and vanishing of $j \cdot A$ term for the preferred extremals. This identification is possible for space-time regions allowing representation as graphs of maps $M^4 \rightarrow CP_2$.

Chern-Simons Dirac action is assigned with the light-like parton orbits as the only non-singular action principle and the condition that the action of C-S-D operator equals to that of massless Dirac operator is posed as additional condition allowing to obtain perturbation theory and connection with twistor Grassmannian approach. It is natural to add also to the Kähler action Chern-Simons term restricted to tge partonic orbits so that the action reduces to the Chern-Simons contributions from the ends the space-time surfaces by weak form of electric magnetic duality. Chern-Simons Dirac terms could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

(b) The GRT-QFT limit of TGD obtained by lumping together various space-time sheets to a region of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of sheets from Minkowski metric. Gauge potentials for the effective space-time would idenfied as sums of gauge potentials for space-time sheets. At this limit the identification of P and T at space-time level and imbedding space level would be natural. Could the resulting effective theory in Minkowski space or GRT space-time break CP and T slightly? If so, CKM matrices for quarks and fermions would emerge as a result of representing different topologies for wormhole throats with different topologies as single point like particle with additional genus quantum number.

(c) Could the breaking of CP and T relate to the generation of the arrow of time? The arrow of time relates to the fact that state function reduction can occur at either boundary of CD [K6]. Zero energy states do not change at the boundary at which reduction occurs repeatedly but the change at the other boundary and also the wave function for the position of the second boundary of CD changes in each quantum jump so that the average temporal distance between the tips of CD increases. This gives to the arrow of psychological time, and in TGD inspired theory of consciousness "self" as a counterpart of observed can be identified as sequence of quantum jumps for which the state function reduction occurs at a fixed boundary of CD. The sequence of reductions at fixed boundary breaks T-invariance and has interpretation as irreversibility. The standard view is that the irreversibility has nothing to do with breaking of T-invariance but it might be that in elementary particle scales irreversibility might manifest as small breaking of T-invariance.

Is CPT breaking needed/possible?

Different values of Δm_{ij}^2 for neutrinos and antineutrinos would require in standard QFT framework not only the violation of CP but also CP [B3] which is the cherished symmetry of quantum field theories. CPT symmetry states that when one reverses time's arrow, reverses the signs of momenta and replaces particles with their antiparticles, the resulting Universe obeys the same laws as the original one. CPT invariance follows from Lorentz invariance, Lorentz invariance of vacuum state, and from the assumption that energy is bounded from below. On the other hand, CPT violation requires the breaking of Lorentz invariance.

In TGD framework this kind of violation does not seem to be necessary at fundamental level since p-adic scale hypothesis allowing neutrinos and also other fermions to have several mass scales coming as half-octaves of a basic mass scale for given quantum numbers. In fact, even in TGD inspired low energy hadron physics quarks appear in several mass scales. One could explain the different choice of the p-adic mass scales as being due to the experimental arrangement which selects different p-adic length scales for neutrinos and antineutrinos so that one could speak about spontaneous breaking of CP and possibly CPT. The CP breaking at the fundamental level which is however expected to be small in the case considered. The basic prediction of TGD and relates to the CP breaking of Chern-Simons action inducing CP breaking in the modified Dirac action defining the fermionic propagator [L13]. For preferred extremals Kähler action would indeed reduce to Chern-Simons terms.

In TGD one has breaking of translational invariance and the symmetry group reduces to Lorentz group leaving the tip of CD invariant. Positive and negative energy parts of zero energy states correspond to different Lorentz groups and zero energy states are superpositions of state pairs with differen values of mass squared. Is the breaking of Lorentz invariance in this sense enough for breaking of CPT is not clear.

One can indeed consider the possibility of a spontaneous breaking of CPT symmetry in TGD framework since for a given CD (causal diamond defined as the intersection of future and past directed light-cones whose size scales are assumed to come as octaves) the Lorentz invariance is broken due to the preferred time direction (rest system) defined by the time-like line connecting the tips of CD. Since the world of classical worlds is union of CDs with all boosts included the Lorentz invariance is not violated at the level of WCW. Spontaneous symmetry breaking would be analogous to that for the solutions of field equations possessing the symmetry themselves. The mechanism of breaking would be same as that for supersymmetry. For same p-adic length scale particles and their super-partners would have same masses and only the selection of the p-adic mass scale would induces the mass splitting.

17.3 Family replication phenomenon and super-symmetry

17.3.1 Family replication phenomenon for bosons

17.3.2 Higher gauge boson families

TGD predicts that also gauge bosons, with gravitons included, should be characterized by family replication phenomenon but not quite in the expected manner. The first expectation was that these gauge bosons would have at least 3 light generations just like quarks and leptons.

Only within last years it has become clear that there is a deep difference between fermions and gauge bosons. Elementary fermions and particles super-conformally related to elementary fermions correspond to single throat of a wormhole contact assignable to a topologically condensed CP_2 type vacuum extremal whereas gauge bosons would correspond to a wormhole throat pair assignable to wormhole contact connecting two space-time sheets. Wormhole throats correspond to light-like partonic 3-surfaces at which the signature of the induced metric changes.

In the case of 3 generations gauge bosons can be arranged to octet and singlet representations of a dynamical SU(3) and octet bosons for which wormhole throats have different genus could be massive and effectively absent from the spectrum.

Exotic gauge boson octet would induce particle reactions in which conserved handle number would be exchanged between incoming particles such that total handle number of boson would be difference of the handle numbers of positive and negative energy throat. These gauge bosons would induce flavor changing but genus conserving neutral current. There is no evidence for this kind of currents at low energies which suggests that octet mesons are heavy. Typical reaction would be $\mu + e \rightarrow e + \mu$ scattering by exchange of $\Delta g = 1$ exotic photon.

17.3.3 Masses of super partners and first rumors about supersymmetric partners from LHC

Experimental indication for space-time super-symmetry

Experimental indication for space-time super-symmetry

There is experimental indication for super-symmetry dating back to 1995 [C62] . The event involves $e^+e^-\gamma\gamma$ plus missing transverse energy $\not{E}_T$. The electron-positron pair has transversal energies $E_T = (36, 59)$ GeV and invariant mass $M_{ee} = 165$ GeV. The two photons have transversal energies (30,38) GeV. The missing transverse energy is $\not{E}_T = 53$ GeV. The cross sections for these events in standard model are too small to be observed. Statistical fluctuation could be in question but one could also consider the event as an indication for super-symmetry.

In [C40] an explanation of the event in terms of minimal super-symmetric standard model (MSSM) was proposed.

(a) The collision of proton and antiproton would induce an annihilation of quark and antiquark to selectron pair $\tilde{e}^-\tilde{e}^+$ via virtual photon or Z^0 boson with the mass of $\tilde{e}$ in the range (80,130) GeV (the upper bound comes from the total energy of the particles involved.

(b) $\tilde{e}^{\pm}$ would in turn decay to $e^{\pm}$ and neutralino χ_2^0 and χ_2^0 in turn to the lightest supersymmetric particle χ_1^0 and photon. The neutralinos are in principle mixtures of the super partners associated with γ, Z^0, and neutral higgs h (there are two of them in minimal super-symmetric generalization of standard model). The highest probability for the chain is obtained if χ_2^0 is zino and χ_1^0 is higgsino.

(c) The kinematics of the event allows to deduce the bounds

$$
\begin{aligned}
80 &< m(\tilde{e})/GeV < 130 \ , \\
38 &\leq m(\chi_2^0)/GeV \leq min\left[1.12m(\tilde{e})/GeV - 37, 95 + 0.17m(\chi_1^0)/GeV\right] \ , \\
m(\chi_1^0)/GeV &\leq m(\chi_2^0)/GeV \leq min\left[1.4m(\tilde{e})/GeV - 105, 1.6m(\chi_2^0)/GeV - 60\right] \ .
\end{aligned}
\tag{17.3.1}
$$

Note that the bounds give no lower bound for $m(\chi_1^0)$ so that it could correspond to neutrino.

(d) Sfermion production rate depends only on masses of the sfermions, so that slepton production cross section decouples from the analysis of particular scenarios. The cross section is at the level of $\sigma = 10$ fb and consistent with data (one event!). The parameters of MSSM are super-symmetric soft-breaking parameters, super-potential parameters, and the parameter $tan(\beta)$. This allows to derive more stringent limits on the masses and parameters of MSSM.

Consider now the explanation of the event in TGD framework.

(a) For the simplest TGD inspired option both Higgs and higgsino would disappear from the spectrum in the massivation and χ_2^0 would decay to photon and neutrino so that the missing energy would consist of neutrinos.

(b) By the properties of super-partners the production rate for $\tilde{e}^-\tilde{e}^+$ is predicted to be same as in MSSM for $\tilde{e} = e_R\overline{\nu}_R$. Same order of magnitude is predicted also for more exotic super-partners such as $e_L\overline{\nu}_R$ with spin 1.

(c) In TGD framework it is safest to use just the kinematical bounds on the masses and p-adic length scale hypothesis. If super-symmetry breaking means same mass formula from p-adic thermodynamics but in a different p-adic mass scale, $m(\tilde{e})$ is related by a power of $\sqrt{2}$ to $m(e)$. Using $m(\tilde{e}) = 2^{(127-k(\tilde{e}))/2}m(e)$ one finds that the mass range $[80, 130]$ GeV allows two possible masses for selectron corresponding to $p \simeq 2^k$, $k = 91$ with $m(\tilde{e}) = 131.1$ GeV and $k = 92$ with $m(\tilde{e}) = 92.7$ GeV. The bounds on $m(Z)$ leave only the option $m(\tilde{Z}) = m(Z) = 91.2$ GeV and $m(\tilde{e}) = 131.1$ GeV.

(d) In the earlier variant of the TGD inspired model the existence of Higgs was considered as a realistic option. The indirect determinations of Higgs masses from experimental data seemed to converge to two different values. The first one seemed to correspond to $m(h) =$ 129 GeV and $k(h) = 94$ and second one to $m(h) = 91$ GeV with $k(h) = 95$ [K48] . The fact that already the TGD counterpart for the Gell-Mann-Okubo mass formula in TGD framework requires quarks to exist at several p-adic mass scales [K57], suggests that Higgs can exist in both of these mass scales depending on the experimental situation. The mass of Higgsino would correspond to some half octave of $m(h)$. Note that the model allows to conclude that Higgs indeed exists also in TGD Universe although it does not seem to play the same role in particle massivation as in the standard model. The bounds allow only $k(\tilde{h}) = k(h) + 3 = 97$ and $m(\tilde{h}) = 45.6$ GeV for $m(h) = 129$ GeV . The same same mass is obtained for $m(h) = 91$ GeV. Therefore the kinematic limits plus super-symmetry breaking at the level of p-adic mass scale fix completely the masses of the super-particles involved in absence of mixing effects for sneutralinos.

To sum up, the masses of sparticles involved for the option allowing Higgs are predicted to be

$$
m(\tilde{e}) = 131 \ \ GeV \ \ , \quad m(\tilde{Z}^0) = 91.2 \ \ GeV \ \ , \quad m(\tilde{h}) = 45.6 \ \ GeV \ \ . \tag{17.3.2}
$$

If Higgs and Higgsino are both eaten in the massivation, the third condition drops off. The argument to be represented below suggests that also sleptons could correspond to Mersennes and Gaussian Mersennes: this option predictions $k(\tilde{e}) = 89$ so that the mass would be 250 GeV: this excludes the proposed interpretation of the strange event.

First rumors about supersymmetric partners from LHC

First rumors about supersymmetric partners from LHC

Lubos Motl reported the first rumors from LHC concerning super-partners [C7] . The estimates for the masses are 200 GeV for a scalar super partner of fermion and 160 GeV for a fermionic superpartner of gauge boson or Higgs. Being an incurable optimist I supposed that the rumors from LHC are more trustworthy than the physics blog rumors usually. Therefore I asked whether one could understand these masses in TGD framework? It was not possible to achieve consistency with the strange CDF event and it turned later that the rumour suffered the usual fate of rumours. I however decided to keep my reaction to it.

Consider first the theoretical background in light p-adic mass calculations, the weak form of electric-magnetic duality, and TGD based view about supersymmetry.

(a) The simplest possibility is that the p-adic length scale of the super-partner differs from that of partner but the p-adic thermodynamical contributions to the mass squared obey the same formula.

(b) If the p-adic prime $p \simeq 2^k$ of super-partner is smaller than $M_{89} = 2^{89} - 1$, the weak length scale must be scaled down and $M_{61} = 2^{89} - 1$ is the next Mersenne prime. Scaled up variant of QCD for M_{89} would naturally correspond to M_{61} weak physics and would have hadronic string tension about 2^{18} GeV^2 by scaling the ordinary hadronic string tension of about 1 GeV^2. This scaled up variant of hadronic physics is an old prediction of TGD. As noticed, also weak string tension could have the same value. Quite generally, the pairs of weak and hadronic scales predicted to form a hierarchy could correspond to pairs of subsequent (possibly Gaussian) Mersenne primes.

(c) What happens for $k = 89$? Can the particle topologically condense at the same p-adic scale that characterizes its weak flux tube? Or should one assume that the p-adic prime corresponds to $k \leq 89$assuming that the particle has standard weak interactions. If so then the superpartners of light fermions would have $k \leq 89$. This is a strong prediction if superpartners obey the same mass formula as particles. In the case of weak gluinos and also QCD gluinos the bound would be $k \leq 89$ and even stronger bound would be $k = 89$ so that the masses of wino and zino would be same as W and Z^0.

One must be however very cautious with this kind of arguments since one is dealing with quantum theory. For instance, quarks inside proton have masses in 10 MeV scale and their Compton lengths are much larger than the Compton size of proton and even atomic nucleus. The interpretation is that for the corresponding space-time sheets is in terms of the color magnetic body of quark. These large space-time sheets are essential in the model of the Lamb shift anomaly of muonic hydrogen discussed in the section "The incredibly shrinking proton".

(d) In TGD framework Higgs and its pseudo-scalar companion define electroweak triplet and singlet and Higgs could be eaten completely by electro-weak gauge bosons if the TGD based mechanism of massivation is correct. The condition of exact Yangian symmetry demands the cancellation of IR divergences requiring a small mass for all gauge bosons and graviton. The twistorially natural assumption that gauge bosons are bound states of massless fermion and anti-fermion implies that the three-momenta of fermion and anti-fermion are in opposite directions so that all gauge bosons -even photon- and graviton would be massive. Super-symmetry strongly suggests that gauginos eat Higgsinos as they become massive so that only massive gauge bosons and gauginos and possible pseudo-scalar Higgs and its superpartner would remain to be discovered at LHC. Similar mechanism can indeed work also in the case of gluons expected to have colored scalar counterparts. Gluon would be massless below the scale corresponding to QCD Λ and massive above this scale.

What does this picture give when compared with the rumors about super-partners of fermion and scalar. If selectron corresponds to the not necessarily allowed $M_{89} = 2^{89} - 1$, and obeys otherwise the same mass formula as electron, the mass should be 250 GeV, which seems too large. For $k = 88$ which is the smallest value allowed by the above argument, one would obtain 177 GeV. It remains unclear whether the interpretation as selectron could make sense.

In the case of super-partner of scalar one can consider several options.

(a) 160 GeV mass does not satisfy the proposed upper bound $k \geq 89$ for higgsinos and gauginos suggested by the condition that the weak string cannot have p-adic length scale longer than the p-adic length scale at which the particle condensed topologically. Hence neither higgsino nor longitudinal polarization of gaugino can be in question.

(b) If one gives up the upper bound $m_Z = 91.2$ GeV on mass but takes the twistorially motivated and mathematically beautiful horror scenario for LHC seriously, the 160 GeV particle can only correspond to a longitudinal polarization of Zino or photino.

One can of course forget the upper bound on mass and give up the horror scenario for a moment and look what one obtains.

(a) If photonic Higgs is not eaten by photon, one would obtain $k(Higgs) = k(Higgsino)+n$. $n = 1, 2, 3$ would give Higgs mass equal to $(141, 100, 71)$ GeV for $m(Higgsino) = 200$ GeV. On basis of experimental data mildly suggesting that neutral Higgs appears in two mass scales I have considered the possibility that Higgs indeed appears at two p-adic length scales corresponding to about 130 GeV and 92 GeV related by square root of two factor. 130 GeV would give $m(Higgsino) = 184$ GeV: I dare guess that this is consistent with the estimate 200 GeV.

(b) For W and Z^0 Higgsinos the mass mass would be p-adically scaled up variant of W or Z^0 mass and for Z^0 mass about 91.2 GeV Z^0 Higgsino mass would be 182.4 GeV for $n = 2$. For W Higgsino the mass would be around 160.8 GeV.

I have already earlier considered the predictions of p-adic length scale hypothesis for super partners on basis of single very strange scattering event (see the section "Experimental indication for space-time supersymmetry"). This kind of considerations must of course be taken as a mere blog entertainment. The hypothesis assuming that the mass formulas for particles and sparticles are same but p-adic length scale is possibly different, combined with kinematical constraints fixes the masses of TGD counterparts of selectron, higgsino, and Z^0-gluino to be 131 GeV (just at the upper bound allowed kinematically), 45.6 GeV, and 91.2 GeV (Z^0 mass) respectively. The masses are consistent with the bounds predicted by the MSSM inspired model.

Selectron mass would be by a factor factor $2^{-1/2}$ smaller than 177 GeV and inconsistent with LHC rumour. Higgsino mass would be one half of Z^0 mass and would satisfy the proposed constraint $k \leq 89$. Z^0 gluino mass would be equal to Z^0 mass also in accordance with the proposed constraint. W gluino is predicted to have same mass as W. In the case of photino the upper bound to the mass would be given by weak boson mass scale. Could it be that the life would be so simple? Could these predictions make it easy to discover super partners at LHC? Well-informed reader might be able to answer these questions.

17.4 New hadron physics

17.4.1 Leptohadron physics

TGD suggest strongly ('predicts' is perhaps too strong expression) the existence of color excited leptons. The mass calculations based on p-adic thermodynamics and p-adic conformal invariance lead to a rather detailed picture about color excited leptons.

(a) The simplest color excited neutrinos and charged leptons belong to the color octets ν_8 and L_{10} and $L_{\overline{10}}$ decouplet representations respectively and lepto-hadrons are formed as the color singlet bound states of these and possible other representations. Electroweak symmetry suggests strongly that the minimal representation content is octet and decouplets for both neutrinos and charged leptons.

(b) The basic mass scale for lepto-hadron physics is completely fixed by p-adic length scale hypothesis. The first guess is that color excited leptons have the levels $k = 127, 113, 107, ...$ ($p \simeq 2^k$, k prime or power of prime) associated with charged leptons as primary condensation levels. p-Adic length scale hypothesis allows however also the level $k = 11^2 = 121$ in case of electronic lepto-hadrons. Thus both $k = 127$ and $k = 121$ must be considered as a candidate for the level associated with the observed lepto-hadrons. If also lepto-hadrons correspond non-perturbatively to exotic Super Virasoro representations, lepto-pion mass relates to pion mass by the scaling factor $L(107)/L(k) = k^{(107-k)/2}$. For $k = 121$ one has $m_{\pi_L} \simeq 1.057$ MeV which compares favorably with the mass $m_{\pi_L} \simeq 1.062$ MeV of the lowest observed state: thus $k = 121$ is the best candidate contrary to the earlier beliefs. The mass spectrum of lepto-hadrons is expected to have same general characteristics as hadronic mass spectrum and a satisfactory description should be based on string tension concept. Regge slope is predicted to be of order $\alpha' \simeq 1.02/MeV^2$ for $k = 121$. The masses of ground state lepto-hadrons are calculable once primary condensation levels for colored leptons and the CKM matrix describing the mixing of color excited lepton families is known.

The strongest counter arguments against color excited leptons are the following ones.

(a) The decay widths of Z^0 and W boson allow only $N = 3$ light particles with neutrino quantum numbers. The introduction of new light elementary particles seems to make the decay widths of Z^0 and W intolerably large.

(b) Lepto-hadrons should have been seen in e^+e^- scattering at energies above few MeV. In particular, lepto-hadronic counterparts of hadron jets should have been observed.

A possible resolution of these problems is provided by the loss of asymptotic freedom in lepto-hadron physics. Lepto-hadron physics would effectively exist in a rather limited energy range about one MeV.

The development of the ideas about dark matter hierarchy [K32, K84, K25, K23] led however to a much more elegant solution of the problem.

(a) TGD predicts an infinite hierarchy of various kinds of dark matters which in particular means a hierarchy of color and electro-weak physics with weak mass scales labelled by appropriate p-adic primes different from M_{89}: the simplest option is that also ordinary photons and gluons are labelled by M_{89}.

(b) There are number theoretical selection rules telling which particles can interact with each other. The assignment of a collection of primes to elementary particle as characterizer of p-adic primes characterizing the particles coupling directly to it, is inspired by the notion of infinite primes [K86] , and discussed in [K32] . Only particles characterized by integers having common prime factors can interact by the exchange of elementary bosons: the p-adic length scale of boson corresponds to a common primes.

(c) Also the physics characterized by different values of $\hbar$ are dark with respect to each other as far quantum coherent gauge interactions are considered. Laser beams might well correspond to photons characterized by p-adic prime different from M_{89} and decoherence for the beam would mean decay to ordinary photons. De-coherence interaction involves scaling down of the Compton length characterizing the size of the space-time of particle implying that particles do not anymore overlap so that macroscopic quantum coherence is lost.

(d) Those dark physics which are dark relative to each other can interact only via graviton exchange. If lepto-hadrons correspond to a physics for which weak bosons correspond to a p-adic prime different from M_{89}, intermediate gauge bosons cannot have direct decays to colored excitations of leptons irrespective of whether the QCD in question is asymptotically free or not. Neither are there direct interactions between the QED:s and QCD:s in question if M_{89} characterizes also ordinary photons and gluons. These ideas are discussed and applied in detail in [K32, K84, K25] .

Skeptic reader might stop the reading after these counter arguments unless there were definite experimental evidence supporting the lepto-hadron hypothesis.

(a) The production of anomalous e^+e^- pairs in heavy ion collisions (energies just above the Coulomb barrier) suggests the existence of pseudo-scalar particles decaying to e^+e^- pairs. A natural identification is as lepto-pions that is bound states of color octet excitations of e^+ and e^-.

(b) The second puzzle, Karmen anomaly, is quite recent [C53] . It has been found that in charge pion decay the distribution for the number of neutrinos accompanying muon in decay $\pi \rightarrow \mu + \nu_\mu$ as a function of time seems to have a small shoulder at $t_0 \sim ms$. A possible explanation is the decay of charged pion to muon plus some new weakly interacting particle with mass of order 30 MeV [C70] : the production and decay of this particle would proceed via mixing with muon neutrino. TGD suggests the identification of this state as color singlet leptobaryon of, say type $L_B = f_{abc}L_8^a L_8^b \bar{L}_8^c$, having electro-weak quantum numbers of neutrino.

(c) The third puzzle is the anomalously high decay rate of orto-positronium. [C31] . e^+e^- annihilation to virtual photon followed by the decay to real photon plus virtual lepto-pion followed by the decay of the virtual lepto-pion to real photon pair, $\pi_L\gamma\gamma$ coupling being determined by axial anomaly, provides a possible explanation of the puzzle.

(d) There exists also evidence for anomalously large production of low energy e^+e^- pairs [C42, C27, C38, C10] in hadronic collisions, which might be basically due to the production of lepto-hadrons via the decay of virtual photons to colored leptons.

In this chapter a revised form of lepto-hadron hypothesis is described.

(a) Sigma model realization of PCAC hypothesis allows to determine the decay widths of lepto-pion and lepto-sigma to photon pairs and e^+e^- pairs. Ortopositronium anomaly determines the value of $f(\pi_L)$ and therefore the value of lepto-pion-lepto-nucleon coupling and the decay rate of lepto-pion to two photons. Various decay widths are in accordance with the experimental data and corrections to electro-weak decay rates of neutron and muon are small.

(b) One can consider several alternative interpretations for the resonances.

Option 1: For the minimal color representation content, three lepto-pions are predicted corresponding to $8, 10, \overline{10}$ representations of the color group. If the lightest lepto-nucleons e_{ex} have masses only slightly larger than electron mass, the anomalous e^+e^- could be actually $e^+_{ex} + e^-_{ex}$ pairs produced in the decays of lepto-pions. One could identify 1.062, 1.63 and 1.77 MeV states as the three lepto-pions corresponding to $8, 10, \overline{10}$ representations and also understand why the latter two resonances have nearly degenerate masses. Since d and s quarks have same primary condensation level and same weak quantum numbers as colored e and μ, one might argue that also colored e and μ correspond to $k = 121$. From the mass ratio of the colored e and μ, as predicted by TGD, the mass of the muonic lepto-pion should be about 1.8 MeV in the absence of topological mixing. This suggests that 1.83 MeV state corresponds to the lightest $g = 1$ lepto-pion.

Option 2: If one believes sigma model (in ordinary hadron physics the existence of sigma meson is not established and its width is certainly very large if it exists), then lepto-pions are accompanied by sigma scalars. If lepto-sigmas decay dominantly to e^+e^- pairs (this might be forced by kinematics) then one could adopt the previous sceneario and could identify 1.062 state as lepto-pion and 1.63, 1.77 and 1.83 MeV states as lepto-sigmas rather than lepto-pions. The fact that muonic lepto-pion should have mass about 1.8 MeV in the absence of topological mixing, suggests that the masses of lepto-sigma and lepto-pion should be rather close to each other.

Option 3: One could also interpret the resonances as string model 'satellite states' having interpretation as radial excitations of the ground state lepto-pion and lepto-sigma. This identification is not however so plausible as the genuinely TGD based identification and will not be discussed in the sequel.

(c) PCAC hypothesis and sigma model leads to a general model for lepto-hadron production in the electromagnetic fields of the colliding nuclei and production rates for lepto-pion and other lepto-hadrons are closely related to the Fourier transform of the instanton density $\bar{E}\cdot\bar{B}$ of the electromagnetic field created by nuclei. The first source of anomalous e^+e^- pairs is the production of $\sigma_L\pi_L$ pairs from vacuum followed by $\sigma_L \rightarrow e^+e^-$ decay. If $e^+_{ex}e^-_{ex}$ pairs rather than genuine e^+e^- pairs are in question, the production is production of lepto-pions from vacuum followed by lepto-pion decay to lepto-nucleon pair.

Option 1: For the production of lepto-nucleon pairs the cross section is only slightly below the experimental upper bound for the production of the anomalous e^+e^- pairs and the decay rate of lepto-pion to lepto-nucleon pair is of correct order of magnitude.

Option 2: The rough order of magnitude estimate for the production cross section of anomalous e^+e^- pairs via $\sigma_l\pi_l$ pair creation followed by $\sigma_L \rightarrow e^+e^-$ decay, is by a factor of order $1/\sum N_c^2$ (N_c is the total number of states for a given colour representation and sum over the representations contributing to the ortopositronium anomaly appears) smaller than the reported cross section in case of 1.8 MeV resonance. The discrepancy could be due to the neglect of the large radiative corrections (the coupling $g(\pi_L\pi_L\sigma_L) = g(\sigma_L\sigma_L\sigma_L)$ is very large) and also due to the uncertainties in the value of the measured cross section.

Given the unclear status of sigma in hadron physics, one has a temptation to conclude that anomalous e^+e^- pairs actually correspond to lepto-nucleon pairs.

(d) The vision about dark matter suggests that direct couplings between leptons and lepto-hadrons are absent in which case no new effects in the direct interactions of ordinary leptons are predicted. If colored leptons couple directly to ordinary leptons, several new physics effects such as resonances in photon-photon scattering at cm energy equal to lepto-pion masses and the production of $e_{ex}\bar{e}_{ex}$ (e_{ex} is leptobaryon with quantum numbers of electron) and $e_{ex}\bar{e}$ pairs in heavy ion collisions, are possible. Lepto-pion exchange would give dominating contribution to $\nu-e$ and $\bar{\nu}-e$ scattering at low energies. Lepto-hadron jets should be observed in e^+e^- annihilation at energies above few MeV:s unless the loss of asymptotic freedom restricts lepto-hadronic physics to a very narrow energy range and perhaps to entirely non-perturbative regime of lepto-hadronic QCD.

During 18 years after the first published version of the model also evidence for colored μ has emerged. Towards the end of 2008 CDF anomaly gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

17.4.2 Evidence for TGD view about QCD plasma

The emergence of the first interesting findings from LHC by CMS collaboration [C17, C2] provide new insights to the TGD picture about the phase transition from QCD plasma to hadronic phase and inspired also the updating of the model of RHIC events (mainly elimination of some remnants from the time when the ideas about hierarchy of Planck constants had just born).

In some proton-proton collisions more than hundred particles are produced suggesting a single object from which they are produced. Since the density of matter approaches to that observed in heavy ion collisions for five years ago at RHIC, a formation of quark gluon plasma and its subsequent decay is what one would expect. The observations are not however quite what QCD plasma picture would allow to expect. Of course, already the RHIC results disagreed with what QCD expectations. What is so striking is the evolution of long range correlations between particles in events containing more than 90 particles as the transverse momentum of the particles increases in the range 1-3 GeV (see the excellent description of the correlations by Lubos Motl in his blog [C8]).

One studies correlation function for two particles as a function of two variables. The first variable is the difference $\Delta\phi$ for the emission angles and second is essentially the difference for the velocities described relativistically by the difference $\Delta\eta$ for hyperbolic angles. As the transverse momentum p_T increases the correlation function develops structure. Around origin of $\Delta\eta$ axis a widening plateau develops near $\Delta\phi = 0$. Also a wide ridge with almost constant value as function of $\Delta\eta$ develops near $\Delta\phi = \pi$. The interpretation is that particles tend to move collinearly and or in opposite directions. In the latter case their velocity differences are large since they move in opposite directions so that a long ridge develops in $\Delta\eta$ direction in the graph.

Ideal QCD plasma would predict no correlations between particles and therefore no structures like this. The radiation of particles would be like blackbody radiation with no correlations between photons. The description in terms of string like object proposed also by Lubos on basis of analysis of the graph showing the distributions as an explanation of correlations looks attractive. The decay of a string like structure producing particles at its both ends moving nearly parallel to the string to opposite directions could be in question.

Since the densities of particles approach those at RHIC, I would bet that the explanation (whatever it is!) of the hydrodynamical behavior observed at RHIC for some years ago should apply also now. The introduction of string like objects in this model was natural since in TGD framework even ordinary nuclei are string like objects with nucleons connected by color flux tubes [L6] , [L6] : this predicts a lot of new nuclear physics for which there is evidence. The basic idea was that in the high density hadronic color flux tubes associated with the colliding nucleon connect to form long highly entangled hadronic strings containing quark gluon plasma. The decay of these structures would explain the strange correlations. It must be however emphasized that in the recent case the initial state consists of two protons rather than heavy nuclei so that the long hadronic string could form from the QCD like quark gluon plasma at criticality when long range fluctuations emerge.

The main assumptions of the model for the RHIC events and those observed now deserve to be summarized. Consider first the "macroscopic description".

(a) A critical system associated with confinement-deconfinement transition of the quark-gluon plasma formed in the collision and inhibiting long range correlations would be in question.

(b) The proposed hydrodynamic space-time description was in terms of a scaled variant of what I call critical cosmology defining a universal space-time correlate for criticality: the specific property of this cosmology is that the mass contained by comoving volume approaches to zero at the the initial moment so that Big Bang begins as a silent whisper and is not so scaring;-). Criticality means flat 3-space instead of Lobatchevski space and means breaking of Lorentz invariance to SO(4). Breaking of Lorentz invariance was indeed observed for particle distributions but now I am not so sure whether it has much to do with this.

The microscopic level the description would be like follows.

(a) A highly entangled long hadronic string like object (color-magnetic flux tube) would be formed at high density of nucleons via the fusion of ordinary hadronic color-magnetic flux tubes to much longer one and containing quark gluon plasma. In QCD world plasma would not be at flux tube.

(b) This geometrically (and perhaps also quantally!) entangled string like object would straighten and split to hadrons in the subsequent "cosmological evolution" and yield large numbers of almost collinear particles. The initial situation should be apart from scaling similar as in cosmology where a highly entangled soup of cosmic strings (magnetic flux tubes) precedes the space-time as we understand it. Maybe ordinary cosmology could provide analogy as galaxies arranged to form linear structures?

(c) This structure would have also black hole like aspects but in totally different sense as the 10-D hadronic black-hole proposed by Nastase to describe the findings. Note that M-theorists identify black holes as highly entangled strings: in TGD 1-D strings are replaced by 3-D string like objects.

17.4.3 New view about space-time and particles and Lamb shift anomaly of muonium

17.4.4 The incredibly shrinking proton

The discovery that the charge radius of proton deduced from the muonic version of hydrogen atom is about 4 per cent smaller than from the radius deduced from hydrogen atom [C48, C59] is in complete conflict with the cherished belief that atomic physics belongs to the museum of science. The title of the article *Quantum electrodynamics-a chink in the armour?* of the article published in Nature [C39] expresses well the possible implications, which might actually go well extend beyond QED.

The finding is a problem of QED or to the standard view about what proton is. Lamb shift [C3] is the effect distinguishing between the states hydrogen atom having otherwise the same energy but different angular momentum. The effect is due to the quantum fluctuations of the electromagnetic field. The energy shift factorizes to a product of two expressions. The first one describes the effect of these zero point fluctuations on the position of electron or muon and the second one characterizes the average of nuclear charge density as "seen" by electron or muon. The latter one should be same as in the case of ordinary hydrogen atom but it is not. Does this mean that the presence of muon reduces the charge radius of proton as determined from muon wave function? This of course looks implausible since the radius of proton is so small. Note that the compression of the muon's wave function has the same effect.

Before continuing it is good to recall that QED and quantum field theories in general have difficulties with the description of bound states: something which has not received too much attention. For instance, van der Waals force at molecular scales is a problem. A possible TGD based explanation and a possible solution of difficulties proposed for two decades ago is that for bound states the two charged particles (say nucleus and electron or two atoms) correspond to two 3-D surfaces glued by flux tubes rather than being idealized to points of Minkowski space. This would make the non-relativistic description based on Schrödinger amplitude natural and replace the description based on Bethe-Salpeter equation having horrible mathematical properties.

Basic facts and notions

In this section the basic TGD inspired ideas and notions - in particular the notion of field body- are introduced and the general mechanism possibly explaining the reduction of the effective charge radius relying on the leakage of muon wave function to the flux tubes associated with u quarks is introduced. After this the value of leakage probability is estimated from the standard formula for the Lamb shift in the experimental situation considered.

1. Basic notions of TGD which might be relevant for the problem

Can one say anything interesting about the possible mechanism behind the anomaly if one accepts TGD framework? How the presence of muon could reduce the charge radius of proton? Let us first list the basic facts and notions.

(a) One can say that the size of muonic hydrogen characterized by Bohr radius is by factor $m_e/m_\mu = 1/211.4 = 4.7\times 10^{-4}$ smaller than for hydrogen atom and equals to 250 fm. Hydrogen atom Bohr radius is .53 Angstroms.

(b) Proton contains 2 quarks with charge 2e/3 and one d quark which charge -e/3. These quarks are light. The last determination of u and d quark masses [C32] gives masses, which are $m_u = 2$ MeV and $m_d = 5$ MeV (I leave out the error bars). The standard view is that the contribution of quarks to proton mass is of same order of magnitude. This would mean that quarks are not too relativistic meaning that one can assign to them a size of order Compton wave length of order $4 \times r_e \simeq 600$ fm in the case of u quark (roughly twice the Bohr radius of muonic hydrogen) and $10 \times r_e \simeq 24$ fm in the case of d quark. These wavelengths are much longer than the proton charge radius and for u quark more than twice longer than the Bohr radius of the muonic hydrogen. That parts of proton would be hundreds of times larger than proton itself sounds a rather weird idea. One could of course argue that the scales in question do not correspond to anything geometric. In TGD framework this is not the way out since quantum classical correspondence requires this geometric correlate.

(c) There is also the notion of classical radius of electron and quark. It is given by $r = \alpha\hbar/m$ and is in the case of electron this radius is 2.8 fm whereas proton charge radius is .877 fm and smaller. The dependence on Planck constant is only apparent as it should be since classical radius is in question. For u quark the classical radius is .52 fm and smaller than proton charge radius. The constraint that the classical radii of quarks are smaller than proton charge radius gives a lower bound of quark masses: p-adic scaling of u quark mass by $2^{-1/2}$ would give classical radius .73 fm which still satisfies the bound. TGD framework the proper generalization would be $r = \alpha_K\hbar/m$, where α_K is Kähler coupling strength defining the fundamental coupling constant of the theory and quantized from quantum criticality. Its value is very near or equal to fine structure constant in electron length scale.

(d) The intuitive picture is that light-like 3-surfaces assignable to quarks describe random motion of partonic 2-surfaces with light-velocity. This is analogous to zitterbewegung assigned classically to the ordinary Dirac equation. The notion of braid emerges from the localization of the modes of the induced spinor field to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces carrying vanishing W fields and Z^0 field at least above weak scale. It is implied by well-definedness of em charge for the modes of Kähler-Dirac action. The orbits of partonic 2-surface effectively reduces to braids carrying fermionic quantum numbers. These braids in turn define higher level braids which would move inside a structure characterizing the particle geometrically. Internal consistency suggests that the classical radius $r = \alpha_K\hbar/m$ characterizes the size scale of the zitterbewegung orbits of quarks.

I cannot resist the temptation to emphasize the fact that Bohr orbitology is now reasonably well understood. The solutions of field equations with higher than 3-D CP_2 projection describing radiation fields allow only generalizations of plane waves but not their superpositions in accordance with the fact it is these modes that are observed. For massless extremals with 2-D CP_2 projection superposition is possible only for parallel light-like wave vectors. Furthermore, the restriction of the solutions of the Chern-Simons Dirac equation at light-like 3-surfaces to braid strands gives the analogs of Bohr orbits. Wave functions of -say electron in atom- are wave functions for the position of wormhole throat and thus for braid strands so that Bohr's theory becomes part of quantum theory.

(e) In TGD framework quantum classical correspondence requires -or at least strongly suggests- that also the p-adic length scales assignable to u and d quarks have geometrical correlates. That quarks would have sizes much larger than proton itself how sounds rather paradoxical and could be used as an objection against p-adic length scale hypothesis. Topological field quantization however leads to the notion of field body as a structure consisting of flux tubes and and the identification of this geometric correlate would be in terms of Kähler (or color-, or electro-) magnetic body of proton consisting of color flux tubes beginning from space-time sheets of valence quarks and having length scale of order Compton wavelength much longer than the size of proton itself. Magnetic loops and electric flux tubes would be in question. Also secondary p-adic length scale characterizes field body. For instance, in the case of electron the causal diamond assigned to electron would correspond to the time scale of .1 seconds defining an important bio-rhythm.

2. Could the notion of field body explain the anomaly?

The large Compton radii of quarks and the notion of field body encourage the attempt to imagine a mechanism affecting the charge radius of proton as determined from electron's or muon's wave function.

(a) Muon's wave function is compressed to a volume which is about 8 million times smaller than the corresponding volume in the case of electron. The Compton radius of u quark more that twice larger than the Bohr radius of muonic hydrogen so that muon should interact directly with the field body of u quark. The field body of d quark would have size 24 fm which is about ten times smaller than the Bohr radius so that one can say that the volume in which muons sees the field body of d quark is only one thousandth of the total volume. The main effect would be therefore due to the two u quarks having total charge of 4e/3.

One can say that muon begins to "see" the field bodies of u quarks and interacts directly with u quarks rather than with proton via its electromagnetic field body. With d quarks it would still interact via protons field body to which d quark should feed its electromagnetic flux. This could be quite enough to explain why the charge radius of proton determined from the expectation value defined by its wave function wave function is smaller than for electron. One must of course notice that this brings in also direct magnetic interactions with u quarks.

(b) What could be the basic mechanism for the reduction of charge radius? Could it be that the electron is caught with some probability into the flux tubes of u quarks and that Schrödinger amplitude for this kind state vanishes near the origin? If so, this portion of state would not contribute to the charge radius and the since the portion ordinary state would smaller, this would imply an effective reduction of the charge radius determined from experimental data using the standard theory since the reduction of the norm of the standard part of the state would be erratically interpreted as a reduction of the charge radius.

(c) This effect would be of course present also in the case of electron but in this case the u quarks correspond to a volume which million times smaller than the volume defined by Bohr radius so that electron does not in practice "see" the quark sub-structure of proton. The probability P for getting caught would be in a good approximation proportional to the value of $|\Psi(r_u)|^2$ and in the first approximation one would have

$$\frac{P_e}{P_\mu} \sim (a_\mu/a_e)^3 = (m_e/m_\mu)^3 \sim 10^{-7} \quad .$$

from the proportionality $\Psi_i \propto 1/a_i^{3/2}$, i=e,$\mu$.

3. A general formula for Lamb shift in terms of proton charge radius

The charge radius of proton is determined from the Lamb shift between 2S- and 2P states of muonic hydrogen. Without this effect resulting from vacuum polarization of photon Dirac equation for hydrogen would predict identical energies for these states. The calculation reduces to the calculation of vacuum polarization of photon inducing to the Coulomb potential and an additional vacuum polarization term. Besides this effect one must also take into account the finite size of the proton which can be coded in terms of the form factor deducible from scattering data. It is just this correction which makes it possible to determine the charge radius of proton from the Lamb shift.

(a) In the article [C9] the basic theoretical results related to the Lamb shift in terms of the vacuum polarization of photon are discussed. Proton's charge density is in this representation is expressed in terms of proton form factor in principle deducible from the scattering data. Two special cases can be distinguished corresponding to the point like proton for which Lamb shift is non-vanishing only for S wave states and non-point like proton for which energy shift is present also for other states. The theoretical expression for the Lamb shift involves very refined calculations. Between 2P and 2S states the expression for the Lamb shift is of form

$$\Delta E(2P^{F=2}_{3/2}2S^{F=1}_{1/2} = a - br_p^2 + cr_p^3 = 209.968(5)5.2248 \times r_p^2 + 0.0347 \times r_p^3 \ meV \quad (17.4.1)$$

where the charge radius $r_p = .8750$ is expressed in femtometers and energy in meVs.

(b) The general expression of Lamb shift is given in terms of the form factor by

$$E(2P-2S) = \int \frac{d^3q}{2\pi)^3} \times (-4\pi\alpha)\frac{F(q^2)}{q^2}\frac{\Pi(q^2)}{q^2} \times \int (|\Psi_{2P}(r)|^2 - |\Psi_{2S}(r)|^2) exp(iq\cdot r)dV \ . \quad (17.4.2)$$

Here Π is is a scalar representing vacuum polarization due to decay of photon to virtual pairs.

The model to be discussed predicts that the effect is due to a leakage from "standard" state to what I call flux tube state. This means a multiplication of $|\Psi_{2P}|^2$ with the normalization factor $1/N$ of the standard state orthogonalized with respect to flux tube state. It is essential that $1/N$ is larger than unity so that the effect is a genuine quantum effect not understandable in terms of classical probability.

The modification of the formula is due to the normalization of the 2P and 2S states. These are in general different. The normalization factor $1/N$ is same for all terms in the expression of Lamb shift for a given state but in general different for 2S and 2P states. Since the lowest order term dominates by a factor of ~ 40 over the second one, one one can conclude that the modification should affect the lowest order term by about 4 per cent. Since the second term is negative and the modification of the first term is interpreted as a modification of the second term when r_p is estimated from the standard formula, the first term must increase by about 4 per cent. This is achieved if this state is orthogonalized with respect to the flux tube state. For states Ψ_0 and Ψ_{tube} with unit norm this means the modification

$$\begin{aligned} \Psi_0 &\rightarrow \frac{1}{1-|C|^2} \times (\Psi_i - C\Psi_{tube}) \ , \\ C &= \langle \Psi_{tube}|\Psi_0\rangle \ . \end{aligned} \quad (17.4.3)$$

In the lowest order approximation one obtains

$$a - br_p^2 + cr_p^3 \rightarrow (1+|C|^2)a - br_p^2 + cr_p^3 \ . \quad (17.4.4)$$

Using instead of this expression the standard formula gives a wrong estimate r_p from the condition

$$a - b\hat{r}_p^2 + c\hat{r}_p^3 \rightarrow (1+|C|^2)a - br_p^2 + cr_p^3 \ . \quad (17.4.5)$$

This gives the equivalent conditions

$$\begin{aligned} \hat{r_p^2} &= r_p^2 - \frac{|C|^2 a}{b} \ , \\ P_{tube} &\equiv |C|^2 \simeq 2\frac{b}{a} \times r_p^2 \times \frac{(r_p - \hat{r_p}}{r_p}) \ . \end{aligned} \quad (17.4.6)$$

The resulting estimate for the leakage probability is $P_{tube} \simeq .0015$. The model should be able to reproduce this probability.

A model for the coupling between standard states and flux tube states

Just for fun one can look whether the idea about confinement of muon to quark flux tube carrying electric flux could make sense.

(a) Assume that the quark is accompanied by a flux tube carrying electric flux $\int EdS = -\int \nabla\Phi \cdot dS = q$, where $q = 2e/3 = ke$ is the u quark charge. The potential created by the u quark at the proton end of the flux tube with transversal area $S = \pi R^2$ idealized as effectively 1-D structure is

$$\Phi = -\frac{ke}{\pi R^2}|x| + \Phi_0 \ . \quad (17.4.7)$$

The normalization factor comes from the condition that the total electric flux is q. The value of the additive constant V_0 is fixed by the condition that the potential coincides with Coulomb potential at $r = r_u$, where r_u is u quark Compton length. This gives

$$e\Phi_0 = \frac{e^2}{r_u} + Kr_u \ , \ K = \frac{ke^2}{\pi R^2} \ . \quad (17.4.8)$$

(b) Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parameterization is

$$R = z\alpha r_u \ . \quad (17.4.9)$$

This gives

$$K = \frac{4k}{\alpha r_u^2} \ , \ e\Phi_0 = 4(\pi\alpha + \frac{k}{\alpha})\frac{1}{r_u} \ . \quad (17.4.10)$$

(c) The requirement that electron with four times larger charge radius that u quark can topologically condensed inside the flux tube without a change in the average radius of the flux tube (and thus in a reduction in p-adic length scale increasing its mass by a factor 4!) suggests that $z \geq 4$ holds true at least far away from proton. Near proton the condition that the radius of the flux tube is smaller than electron's charge radius is satisfied for $z = 1$.

1. Reduction of Schrödinger equation at flux tube to Airy equation

The 1-D Schrödinger equation at flux tube has as its solutions Airy functions and the related functions known as "Bairy" functions.

(a) What one has is a one-dimensional Schrödinger equation of general form

$$-\frac{\hbar^2}{2m_\mu}\frac{d^2\Psi}{dx^2}+(Kx-e\Phi_0)\Psi \;=\; E\Psi \;\; , \;\; K=\frac{ke^2}{\pi R^2} \;\; . \qquad (17.4.11)$$

By performing a linear coordinate change

$$u \;=\; (\frac{2m_\mu K}{\hbar^2})^{1/3}(x-x_E) \;\; , \;\; x_E=\frac{-|E|+e\Phi_0}{K} \;\; , \qquad (17.4.12)$$

one obtains

$$\frac{d^2\Psi}{du^2}-u\Psi \;=\; 0 \;\; . \qquad (17.4.13)$$

This differential equation is known as Airy equation (or Stokes equation) and defines special functions $Ai(x)$ known as Airy functions and related functions $Bi(x)$ referred to as "Bairy" functions [B1] . Airy functions characterize the intensity near an optical directional caustic such as that of rainbow.

(b) The explicit expressions for $A_i(u)$ and $Bi(u)$ are is given by

$$\begin{aligned} Ai(u) &= \frac{1}{\pi}\int_0^\infty cos(\frac{1}{3}t^3+ut)dt \;\; , \\ Bi(u) &= \frac{1}{\pi}\int_0^\infty \left[exp(-\frac{1}{3}t^3)+sin(\frac{1}{3}t^3+ut)dt\right] \;\; . \end{aligned} \qquad (17.4.14)$$

$Ai(u)$ oscillates rapidly for negative values of u having interpretation in terms of real wave vector and goes exponentially to zero for $u>0$. $Bi(u)$ oscillates also for negative values of x but increases exponentially for positive values of u. The oscillatory behavior and its character become obvious by noticing that stationary phase approximation is possible for $x<0$.

The approximate expressions of $Ai(u)$ and $Bi(u)$ for $u>0$ are given by

$$\begin{aligned} Ai(u) &\sim \frac{1}{2\pi^{1/2}}exp(-\frac{2}{3}u^{3/2})u^{-1/4} \;\; , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}exp(\frac{2}{3}u^{3/2})u^{-1/4} \;\; . \end{aligned} \qquad (17.4.15)$$

For $u<0$ one has

$$\begin{aligned} Ai(u) &\sim \frac{1}{\pi^{1/2}}sin(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \;\; , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}cos(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \;\; . \end{aligned} \qquad (17.4.16)$$

(c) $u=0$ corresponds to the turning point of the classical motion where the kinetic energy changes sign. $x=0$ and $x=r_u$ correspond to the points

$$\begin{aligned} u_{min}\equiv u(0) &= -(\frac{2m_\mu K}{\hbar^2})^{1/3}x_E \;\; , \\ u_{max}\equiv u(r_u) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(r_u-x_E) \;\; , \\ x_E &= \frac{-|E|+e\Phi_0}{K} \;\; . \end{aligned} \qquad (17.4.17)$$

(d) The general solution is

$$\Psi = aAi(u) + bBi(u) \ . \tag{17.4.18}$$

The natural boundary condition is the vanishing of Ψ at the lower end of the flux tube giving

$$\frac{b}{a} = -\frac{Ai(u(0)}{Bi(u(0))} \ . \tag{17.4.19}$$

A non-vanishing value of b implies that the solution increases exponentially for positive values of the argument and the solution can be regarded as being concentrated in an excellent approximation near the upper end of the flux tube.

Second boundary condition is perhaps most naturally the condition that the energy is same for the flux tube amplitude as for the standard solution. Alternative boundary conditions would require the vanishing of the solution at both ends of the flux tube and in this case one obtains very large number of solutions as WKB approximation demonstrates. The normalization of the state so that it has a unit norm fixes the magnitude of the coefficients a and b since one can choose them to be real.

2. Estimate for the probability that muon is caught to the flux tube

The simplest estimate for the muon to be caught to the flux tube state characterized by the same energy as standard state is the overlap integral of the ordinary hydrogen wave function of muon and of the effectively one-dimensional flux tube. What one means with overlap integral is however not quite obvious.

(a) The basic condition is that the modified "standard" state is orthogonal to the flux tube state. One can write the expression of a general state as

$$\begin{aligned} \Psi_{nlm} &\rightarrow N \times (\Psi_{nlm} - C(E,nlm)\Phi_{nlm}) \ , \\ \Phi_{nlm} &= Y_{lm}\Psi_E \ , \\ C(E,nlm) &= \langle \Psi_E | \Psi_{nlm} \rangle \ . \end{aligned} \tag{17.4.20}$$

Here Φ_{nlm} depends a flux tube state in which spherical harmonics is wave function in the space of orientations of the flux tube and Ψ_E is flux tube state with same energy as standard state. Here an inner product between standard states and flux tube states is introduced.

(b) Assuming same energy for flux tube state and standard state, the expression for the total total probability for ending up to single flux tube would be determined from the orthogonality condition as

$$P_{nlm} = \frac{|C(E,nlm)|^2}{1 - |C(E,lmn)|^2} \ . \tag{17.4.21}$$

Here E refers to the common energy of flux tube state and standard state. The fact that flux tube states vanish at the lower end of the flux tube implies that they do not contribute to the expression for average charge density. The reduced contribution of the standard part implies that the attempt to interpret the experimental results in "standard model" gives a reduced value of the charge radius. The size of the contribution is given by P_{nlm} whose value should be about 4 per cent.

One can consider two alternative forms for the inner product between standard states and flux tube states. Intuitively it is clear that an overlap between the two wave functions must be in question.

(a) The simplest possibility is that one takes only overlap at the upper end of the flux tube which defines 2-D surface. Second possibility is that that the overlap is over entire flux tube projection at the space-time sheet of atom.

$$\begin{aligned} \langle\Psi_E|\Psi_{nlm}\rangle &= \int_{end} \overline{\Psi}_r \Psi_{nlm} dS \ \ (\text{Option I}) \ , \\ \langle\Psi_E|\Psi_{nlm}\rangle &= \int_{tube} \overline{\Psi}_r \Psi_{nlm} dV \ \ (\text{Option II}) \ . \end{aligned} \quad (17.4.22)$$

(b) For option I the inner product is non-vanishing only if Ψ_E is non-vanishing at the end of the flux tube. This would mean that electron ends up to the flux tube through its end. The inner product is dimensionless without introduction of a dimensional coupling parameter if the inner product for flux tube states is defined by 1-dimensional integral: one might criticize this assumption as illogical. Unitarity might be a problem since the local behaviour of the flux tube wave function at the end of the flux tube could imply that the contribution of the flux tube state in the quantum state dominates and this does not look plausible. One can of course consider the introduction to the inner product a coefficient representing coupling constant but this would mean loss of predictivity. Schrödinger equation at the end of the flux tubes guarantees the conservation of the probability current only if the energy of flux tube state is same as that of standard state or if the flux tube Schrödinger amplitude vanishes at the end of the flux tube.

(c) For option II there are no problems with unitary since the overlap probability is always smaller than unity. Option II however involves overlap between standard states and flux tube states even when the wave function at the upper end of the flux tube vanishes. One can however consider the possibility that the possible flux tube states are orthogonalized with respect to standard states with leakage to flux tubes. The interpretation for the overlap integral would be that electron ends up to the flux tube via the formation of wormhole contact.

3. Option I fails

The considerations will be first restricted to the simpler option I. The generalization of the results of calculation to option II is rather straighforward. It turns out that option II gives correct order of magnitude for the reduction of charge radius for reasonable parameter values.

(a) In a good approximation one can express the overlap integrals over the flux tube end (option I) as

$$\begin{aligned} C(E,nlm) &= \int_{tube} \overline{\Psi}_E \Psi_{nlm} dS \simeq \pi R^2 \times Y_{lm} \times C(E,nl) \ , \\ C(E,nl) &= \overline{\Psi}_E(r_u) R_{nl}(r_u) \ . \end{aligned} \quad (17.4.23)$$

An explicit expression for the coefficients can be deduced by using expression for Ψ_E as a superposition of Airy and Bairy functions. This gives

$$\begin{aligned} C(E,nl) &= \overline{\Psi}_E(r_u) R_{nl}(r_u) \ , \\ \Psi_E(x) &= a_E Ai(u_E) + b Bi(u_E) \ , \ \frac{a_E}{b_E} = -\frac{Bi(u_E(0))}{Ai(u_E(0)} \ , \\ u_E(x) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(x - x_E) \ , \ x_E = \frac{|E| - e\Phi_0}{K} \ , \\ K &= \frac{ke^2}{\pi R^2} \ , \ R = z\alpha_K r_u \ , \ k = \frac{2}{3} \ . \end{aligned} \quad (17.4.24)$$

The normalization of the coefficients is fixed from the condition that a and b chosen in such a manner that Ψ has unit norm. For these boundary conditions Bi is expected to dominate completely in the sum and the solution can be regarded as exponentially decreasing function concentrated around the upper end of the flux tube.

In order to get a quantitative view about the situation one can express the parameters u_{min} and u_{max} in terms of the basic dimensionless parameters of the problem.

(a) One obtains

$$\begin{aligned} u_{min} \equiv u(0) &= -2(\frac{k}{z\alpha})^{1/3}\left[1+\pi\frac{z}{k}\alpha^2(1-\frac{1}{2}\alpha r)\right]\times r^{1/3} \ , \\ u_{max} \equiv u(r_u) &= u(0)+2\frac{k}{z\alpha}\times r^{1/3} \ , \\ r &= \frac{m_\mu}{m_u} \ , \quad R = z\alpha r_u \ . \end{aligned} \tag{17.4.25}$$

Using the numerical values of the parameters one obtains for $z = 1$ and $\alpha = 1/137$ the values $u_{min} = -33.807$ and $u_{max} = 651.69$. The value of u_{max} is so large that the normalization is in practice fixed by the exponential behavior of Bi for the suggested boundary conditions.

(b) The normalization constant is in good approximation defined by the integral of the approximate form of Bi^2 over positive values of u and one has

$$N^2 \simeq \frac{dx}{du}\times\int_{u_{min}}^{u_{max}} Bi(u)^2du \ , \quad \frac{dx}{du} = \frac{1}{2}(\frac{z^2\alpha}{k})^{1/3}\times r^{1/3}r_u \ , \tag{17.4.26}$$

By taking $t = exp(\frac{4}{3}u^{3/2})$ as integration variable one obtains

$$\begin{aligned} \int_{u_{min}}^{u_{max}} Bi(u)^2du &\simeq \pi^{-1}\int_{u_{min}}^{u_{max}} exp(\frac{4}{3}u^{3/2})u^{-1/2}du \\ &= (\frac{4}{3})^{2/3}\pi^{-1}\int_{t_{min}}^{t_{max}}\frac{dt}{log(t)^{2/3}} \simeq \frac{1}{\pi}\frac{exp(\frac{4}{3}u_{max}^{3/2})}{u_{max}} \ . \end{aligned} \tag{17.4.27}$$

This gives for the normalization factor the expression

$$N \simeq \frac{1}{2}(\frac{z^2\alpha}{k})^{2/3}r^{1/3}r_u^{1/2}exp(\frac{2}{3}u_{max}^{3/2}) \ . \tag{17.4.28}$$

(c) One obtains for the value of Ψ_E at the end of the flux tube the estimate

$$\Psi_E(r_u) = \frac{Bi(u_{max})}{N} \simeq 2\pi^{-1/2}\times(\frac{k}{z^2\alpha})^{2/3}r^{1/3}r_u^{-1/2} \ , \quad r = \frac{r_u}{r_\mu} \ . \tag{17.4.29}$$

(d) The inner product defined as overlap integral gives for the ground state

$$\begin{aligned} C_{E,00} &= \Psi_E(r_u)\times\Psi_{1,0,0}(r_u)\times\pi R^2 \\ &= 2\pi^{-1/2}(\frac{k}{z^2\alpha})^{2/3}r^{1/3}r_u^{-1/2}\times(\frac{1}{\pi a(\mu)^3})^{1/2}\times exp(-\alpha r)\times\pi z^2\alpha^2r_u^2 \\ &= 2\pi^{1/2}k^{2/3}z^{2/3}r^{11/6}\alpha^{17/6}exp(-\alpha r) \ . \end{aligned} \tag{17.4.30}$$

The relative reduction of charge radius equals to $P = C_{E,00}^2$. For $z = 1$ one obtains $P = C_{E,00}^2 = 5.5\times10^{-6}$, which is by three orders of magnitude smaller than the value needed for $P_{tube} = C_{E,20}^2 = .0015$. The obvious explanation for the smallness is the α^2 factor coming from the area of flux tube in the inner product.

4. Option II could work

The failure of the simplest model is essentially due to the inner product. For option II the inner product for the flux tube states involves the integral over the area of flux tube so that the normalization factor for the state is obtained from the previous one by the replacement $N \rightarrow N/\sqrt{\pi R^2}$. In the integral over the flux tube the exponent function is is in the first approximation equal to constant since the wave function for ground state is at the end of the flux tube only by a factor .678 smaller than at the origin and the wave function is strongly concentrated near the end of the flux tube. The inner product defined by the overlap integral over the flux tube implies $N \rightarrow NS^{1/2}$, $S = \pi R^2 = z^2\alpha^2 r_u^2$. In good approximation the inner product for option II means the replacement

$$\begin{aligned} C_{E,n0} &\rightarrow A \times B \times C_{E,n0} \ , \\ A &= \frac{\frac{dx}{du}}{\sqrt{\pi R^2}} = \frac{1}{2\sqrt{\pi}} z^{-1/3}k^{-1/3}\alpha^{-2/3}r^{1/3} \ , \\ B &= \frac{\int Bi(u)du}{\sqrt{Bi(u_{max})}} = u_{max}^{-1/4} = 2^{-1/4}z^{1/2}k^{-1/4}\alpha^{1/4}r^{-1/12} \ . \end{aligned} \tag{17.4.31}$$

Using the expression

$$R_{20}(r_u) = \frac{1}{2\sqrt{2}} \times (\frac{1}{a_\mu})^{3/2} \times (2 - r\alpha) \times exp(-r\alpha) \ , \ r = \frac{r_u}{r_\mu} \tag{17.4.32}$$

one obtains for $C_{E,20}$ the expression

$$C_{E,20} = 2^{-3/4}z^{5/6}k^{1/12}\alpha^{29/12}r^{25/12} \times (2 - r\alpha) \times exp(-r\alpha) \ . \tag{17.4.33}$$

By the earlier general argument one should have $P_{tube} = |C_{E,20}|^2 \simeq .0015$. $P_{tube} = .0015$ is obtained for $z = 1$ and $N = 2$ corresponding to single flux tube per u quark. If the flux tubes are in opposite directions, the leakage into 2P state vanishes. Note that this leakage does not affect the value of the coefficient a in the general formula for the Lamb shift. The radius of the flux tube is by a factor 1/4 smaller than the classical radius of electron and one could argue that this makes it impossible for electron to topologically condense at the flux tube. For $z = 4$ one would have $P_{tube} = .015$ which is 10 times too large a value. Note that the nucleus possess a wave function for the orientation of the flux tube. If this corresponds to S-wave state then only the leakage beween S-wave states and standard states is possible.

Are exotic flux tube bound states possible?

There seems to be no deep reason forbidding the possibility of genuine flux tube states decoupling from the standard states completely. To get some idea about the energy eigenvalues one can apply WKB approximation. This approach should work now: in fact, the study on WKB approximation near turning point by using linearization of the the potential leads always to Airy equation so that the linear potential represents an ideal situation for WKB approximation. As noticed these states do not seem to be directly relevant for the recent situation. The fact that these states have larger binding energies than the ordinary states of hydrogen atom might make possible to liberate energy by inducing transitions to these states.

(a) Assume that a bound state with a negative energy E is formed inside the flux tube. This means that the condition $p^2 = 2m(E-V) \geq 0$, $V = -e\Phi$, holds true in the region $x \leq x_{max} < r_u$ and $p^2 = 2m(E-V) < 0$ in the region $r_u > x \geq x_{max}$. The expression for x_{max} is

$$x_{max} = \frac{\pi R^2}{k}\left(-\frac{|E|}{e^2}+\frac{1}{r_u}+\frac{kr_u}{\pi R^2}\right)\hbar \ . \quad (17.4.34)$$

$x_{max} < r_u$ holds true if one has

$$|E| < \frac{e^2}{r_u} = E_{max} \ . \quad (17.4.35)$$

The ratio of this energy to the ground state energy of muonic hydrogen is from $E(1) = e^2/2a(\mu)$ and $a = \hbar/\alpha m$ given by

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_\mu} \simeq 5.185 \ . \quad (17.4.36)$$

This encourages to think that the ground state energy could be reduced by the formation of this kind of bound state if it is possible to find a value of n in the allowed range. The physical state would of course contain only a small fraction of this state. In the case of electron the increase of the binding energy is even more dramatic since one has

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_e} = \frac{8}{\alpha} \simeq 1096 \ . \quad (17.4.37)$$

Obviously the formation of this kind of states could provide a new source of energy. There have been claims about anomalous energy production in hydrogen [D19] . I have discussed these claims from TGD viewpoint in [K90]

(b) One can apply WKB quantization in the region where the momentum is real to get the condition

$$I = \int_0^{x_{max}} \sqrt{2m(E+e\Phi)}\frac{dx}{\hbar} = n+\frac{1}{2} \ . \quad (17.4.38)$$

By performing the integral one obtains the quantization condition

$$\begin{aligned} I &= k^{-1}(8\pi\alpha)^{1/2} \times \frac{R^2}{r_u^{3/2} r_\mu} \times A^{3/2} = n+\frac{1}{2} \ , \\ A &= 1 + kx^2 - \frac{|E| r_u}{e^2} \ , \\ x &= \frac{r_u}{R} \ , \ k = \frac{2}{3\pi} \ , \ r_i = \frac{\hbar}{m_i} \ . \end{aligned} \quad (17.4.39)$$

(c) Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parameterization is

$$R = z\alpha r_u \ . \quad (17.4.40)$$

This gives for the binding energy the general expression in terms of the ground state binging energy $E(1,\mu)$ of muonic hydrogen as

$$\begin{array}{rcl} |E| & = & C\times E(1,\mu) \ , \\ C & = & D\times (1+Kz^{-2}\alpha^{-2}-(\frac{y}{z^2})^{2/3}\times (n+1/2)^{2/3}) \ , \\ D & = & 2y\times (\frac{K^2}{8\pi\alpha})^{1/3} \ , \\ y & = & \frac{m_u}{m_\mu} \ , \ K=\frac{2}{3\pi} \ . \end{array} \quad (17.4.41)$$

(d) There is a finite number of bound states. The above mentioned consistency conditions coming from $0 < x_{max} < r_\mu$ give $0 < C < C_{max} = 5.185$ restricting the allowed value of n to some interval. One obtains the estimates

$$\begin{array}{rcl} n_{min} & \simeq & \frac{z^2}{y}(1+Kz^{-2}\alpha^{-2}-\frac{C_{max}}{D})^{3/2}-\frac{1}{2} \ , \\ n_{max} & = & \frac{z^2}{y}(1+Kz^{-2}\alpha^{-2})^{3/2}-\frac{1}{2} \ . \end{array} \quad (17.4.42)$$

Very large value of n is required by the consistency condition. The calculation gives $n_{min} \in \{1.22\times 10^7, 4.59\times 10^6, 1.48\times 10^5\}$ and $n_{max} \in \{1.33\times 10^7, 6.66\times 10^6, 3.34\times 10^6\}$ for $z \in \{1,2,4\}$. This would be a very large number of allowed bound states -about 3.2×10^6 for $z=1$.

The WKB state behaves as a plane wave below x_{max} and sum of exponentially decaying and increasing amplitudes above x_{max}:

$$\begin{array}{c} \frac{1}{\sqrt{k(x)}}\left[Aexp(i\int_0^x k(y)dy)+Bexp(-i\int_0^x k(y)dy)\right] \ , \\ \frac{1}{\sqrt{\kappa(x)}}\left[Cexp(-\int_{x_{max}}^x \kappa(y)dy+Dexp(\int_{x_{max}}^x \kappa(y)dy\right] \ , \\ k(x)=\sqrt{2m(-|E|+e\Phi} \ , \ \kappa(x)\sqrt{2m(|E|-e\Phi} \ . \end{array} \quad (17.4.43)$$

At the classical turning point these two amplitudes must be identical.

The next task is to decide about natural boundary conditions. Two types of boundary conditions must be considered. The basic condition is that genuine flux tube states are in question. This requires that the inner product between flux tube states and standard states defined by the integral over flux tube ends vanishes. This is guaranteed if the Schrödinger amplitude for the flux tube state vanishes at the ends of the flux tube so that flux tube behaves like an infinite potential well. The condition $\Psi(0)=0$ at the lower end of the flux tube would give $A=-B$. Combined with the continuity condition at the turning point these conditions imply that Ψ can be assumed to be real. The $\Psi(r_u)=0$ gives a condition leading to the quantization of energy.

The wave function over the directions of flux tube with a given value of n is given by the spherical harmonics assigned to the state (n,l,m).

17.4.5 Dark nucleons and genetic code

17.4.6 Dark nuclear strings as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code?

Water memory is one of the ugly words in the vocabulary of a main stream scientist. The work of pioneers is however now carrying fruit. The group led by Jean-Luc Montagnier, who received Nobel prize for discovering HIV virus, has found strong evidence for water memory and detailed information about the mechanism involved [K39, K91] , [I6] . The work leading to the discovery was motivated by the following mysterious finding. When the water solution containing human cells infected by bacteria was filtered in purpose of sterilizing it, it indeed satisfied the criteria for the absence of infected cells immediately after the procedure. When one however adds human cells to the filtrate, infected cells appear within few weeks. If this is really the case and if the filter does what it is believed to do, this raises the question whether there might be a representation of genetic code based on nano-structures able to leak through the filter with pores size below 200 nm.

The question is whether dark nuclear strings might provide a representation of the genetic code. In fact, I posed this question year before the results of the experiment came with motivation coming from attempts to understand water memory. The outcome was a totally unexpected finding: the states of dark nucleons formed from three quarks can be naturally grouped to multiplets in one-one correspondence with 64 DNAs, 64 RNAS, and 20 amino-acids and there is natural mapping of DNA and RNA type states to amino-acid type states such that the numbers of DNAs/RNAs mapped to given amino-acid are same as for the vertebrate genetic code.

The basic idea is simple. Since baryons consist of 3 quarks just as DNA codons consist of three nucleotides, one might ask whether codons could correspond to baryons obtained as open strings with quarks connected by two color flux tubes. This representation would be based on entanglement rather than letter sequences. The question is therefore whether the dark baryons constructed as string of 3 quarks using color flux tubes could realize 64 codons and whether 20 amino-acids could be identified as equivalence classes of some equivalence relation between 64 fundamental codons in a natural manner.

The following model indeed reproduces the genetic code directly from a model of dark neutral baryons as strings of 3 quarks connected by color flux tubes.

(a) Dark nuclear baryons are considered as a fundamental realization of DNA codons and constructed as open strings of 3 dark quarks connected by two colored flux tubes, which can be also charged. The baryonic strings cannot combine to form a strictly linear structure since strict rotational invariance would not allow the quark strings to have angular momentum with respect to the quantization axis defined by the nuclear string. The independent rotation of quark strings and breaking of rotational symmetry from SO(3) to SO(2) induced by the direction of the nuclear string is essential for the model.

   i. Baryonic strings could form a helical nuclear string (stability might require this) locally parallel to DNA, RNA, or amino-acid) helix with rotations acting either along the axis of the DNA or along the local axis of DNA along helix. The rotation of a flux tube portion around an axis parallel to the local axis along DNA helix requires that magnetic flux tube has a kink in this portion. An interesting question is whether this kink has correlate at the level of DNA too. Notice that color bonds appear in two scales corresponding to these two strings. The model of DNA as topological quantum computer [K26] allows a modification in which dark nuclear string of this kind is parallel to DNA and each codon has a flux tube connection to the lipid of cell membrane or possibly to some other bio-molecule.

   ii. The analogs of DNA -, RNA -, and of amino-acid sequences could also correspond to sequences of dark baryons in which baryons would be 3-quark strings in the plane transversal to the dark nuclear string and expected to rotate by stringy boundary conditions. Thus one would have nuclear string consisting of short baryonic strings not connected along their ends. In this case all baryons would be free to rotate.

(b) The new element as compared to the standard quark model is that between both dark quarks and dark baryons can be charged carrying charge $0, \pm 1$. This is assumed also in nuclear string model and there is empirical support for the existence of exotic nuclei containing charged color bonds between nuclei.

(c) The net charge of the dark baryons in question is assumed to vanish to minimize Coulomb repulsion:

$$\sum_q Q_{em}(q) = -\sum_{flux\ tubes} Q_{em}(flux\ tube) \ . \quad (17.4.44)$$

This kind of selection is natural taking into account the breaking of isospin symmetry. In the recent case the breaking cannot however be as large as for ordinary baryons (implying large mass difference between Δ and nucleon states).

(d) One can classify the states of the open 3-quark string by the total charges and spins associated with 3 quarks and to the two color bonds. Total em charges of quarks vary in the range $Z_B \in \{2,1,0,-1\}$ and total color bond charges in the range $Z_b \in \{2,1,0,-1,-2\}$. Only neutral states are allowed. Total quark spin projection varies in the range $J_B = 3/2, 1/2, -1/2, -3/2$ and the total flux tube spin projection in the range $J_b = 2,1,-1,-2$. If one takes for a given total charge assumed to be vanishing one representative from each class (J_B, J_b), one obtains $4\times 5 = 20$ states which is the number of amino-acids. Thus genetic code might be realized at the level of baryons by mapping the neutral states with a given spin projection to single representative state with the same spin projection. The problem is to find whether one can identify the analogs of DNA, RNA and amino-acids as baryon like states.

States in the quark degrees of freedom

One must construct many-particle states both in quark and flux tube degrees of freedom. These states can be constructed as representations of rotation group SU(2) and strong isospin group SU(2) by using the standard tensor product rule $j_1\times j_2 = j_1+j_2\oplus j_1+j_2-1\oplus ...\oplus |j_1-j_2|$ for the representation of SU(2) and Fermi statistics and Bose-Einstein statistics are used to deduce correlations between total spin and total isospin (for instance, $J = I$ rule holds true in quark degrees of freedom). Charge neutrality is assumed and the breaking of rotational symmetry in the direction of nuclear string is assumed.

Consider first the states of dark baryons in quark degrees of freedom.

(a) The tensor product $2\otimes 2\otimes 2$ is involved in both cases. Without any additional constraints this tensor product decomposes as $(3\oplus 1)\otimes 2 = 4\oplus 2\oplus 2$: 8 states altogether. This is what one should have for DNA and RNA candidates. If one has only identical quarks uuu or ddd, Pauli exclusion rule allows only the 4-D spin 3/2 representation corresponding to completely symmetric representation -just as in standard quark model. These 4 states correspond to a candidate for amino-acids. Thus RNA and DNA should correspond to states of type uud and ddu and amino-acids to states of type uuu or ddd. What this means physically will be considered later.

(b) Due to spin-statistics constraint only the representations with $(J,I) = (3/2,3/2)$ (Δ resonance) and the second $(J,I) = (1/2,1/2)$ (proton and neutron) are realized as free baryons. Now of course a dark -possibly p-adically scaled up - variant of QCD is considered so that more general baryonic states are possible. By the way, the spin statistics problem which forced to introduce quark color strongly suggests that the construction of the codons as sequences of 3 nucleons - which one might also consider - is not a good idea.

(c) Second nucleon like spin doublet - call it 2_{odd} - has wrong parity in the sense that it would require $L = 1$ ground state for two identical quarks (uu or dd pair). Dropping 2_{odd} and using only $4\oplus 2$ for the rotation group would give degeneracies $(1,2,2,1)$ and 6 states only. All the representations in $4\oplus 2\oplus 2_{odd}$ are needed to get 8 states with a given quark charge and one should transform the wrong parity doublet to positive parity doublet somehow. Since open string geometry breaks rotational symmetry to a subgroup SO(2) of rotations acting along the direction of the string and since the boundary conditions on baryonic strings force their ends to rotate with light velocity, the attractive possibility is to add a baryonic stringy excitation with angular momentum projection $L_z = -1$ to the wrong parity doublet so that the parity comes out correctly.

$L_z = -1$ orbital angular momentum for the relative motion of uu or dd quark pair in the open 3-quark string would be in question. The degeneracies for spin projection value $J_z = 3/2, ..., -3/2$ are $(1, 2, 3, 2)$. Genetic code means spin projection mapping the states in $4 \oplus 2 \oplus 2_{odd}$ to 4.

States in the flux tube degrees of freedom

Consider next the states in flux tube degrees of freedom.

(a) The situation is analogous to a construction of mesons from quarks and antiquarks and one obtains the analogs of π meson (pion) with spin 0 and ρ meson with spin 1 since spin statistics forces $J = I$ condition also now. States of a given charge for a flux tube correspond to the tensor product $2 \otimes 2 = 3 \oplus 1$ for the rotation group.

(b) Without any further constraints the tensor product $3 \otimes 3 = 5 \oplus 3 \oplus 1$ for the flux tubes states gives 8+1 states. By dropping the scalar state this gives 8 states required by DNA and RNA analogs. The degeneracies of the states for DNA/RNA type realization with a given spin projection for $5 \oplus 3$ are $(1, 2, 2, 2, 1)$. 8× 8 states result altogether for both uud and udd for which color bonds have different charges. Also for ddd state with quark charge -1 one obtains $5 \oplus 3$ states giving 40 states altogether.

(c) If the charges of the color bonds are identical as the are for uuu type states serving as candidates for the counterparts of amino-acids bosonic statistics allows only 5 states ($J = 2$ state). Hence 20 counterparts of amino-acids are obtained for uuu. Genetic code means the projection of the states of $5 \oplus 3$ to those of 5 with the same spin projection and same total charge.

Analogs of DNA,RNA, amino-acids, and of translation and transcription mechanisms

Consider next the identification of analogs of DNA, RNA and amino-acids and the baryonic realization of the genetic code, translation and transcription.

(a) The analogs of DNA and RNA can be identified dark baryons with quark content uud, ddu with color bonds having different charges. There are 3 color bond pairs corresponding to charge pairs $(q_1, q_2) = (-1, 0), (-1, 1), (0, 1)$ (the order of charges does not matter). The condition that the total charge of dark baryon vanishes allows for uud only the bond pair $(-1, 0)$ and for udd only the pair $(-1, 1)$. These thus only single neutral dark baryon of type *uud resp. udd*: these would be the analogous of DNA and RNA codons. Amino-acids would correspond to uuu states with identical color bonds with charges $(-1, -1)$, $(0, 0)$, or $(1, 1)$. uuu with color bond charges (-1,-1) is the only neutral state. Hence only the analogs of DNA, RNA, and amino-acids are obtained, which is rather remarkable result.

(b) The basic transcription and translation machinery could be realized as processes in which the analog of DNA can replicate, and can be transcribed to the analog of mRNA in turn translated to the analogs of amino-acids. In terms of flux tube connections the realization of genetic code, transcription, and translation, would mean that only dark baryons with same total quark spin and same total color bond spin can be connected by flux tubes. Charges are of course identical since they vanish.

(c) Genetic code maps of $(4 \oplus 2 \oplus 2) \otimes (5 \oplus 3)$ to the states of 4×5. The most natural map takes the states with a given spin to a state with the same spin so that the code is unique. This would give the degeneracies $D(k)$ as products of numbers $D_B \in \{1, 2, 3, 2\}$ and $D_b \in \{1, 2, 2, 2, 1\}$: $D = D_B \times D_b$. Only the observed degeneracies $D = 1, 2, 3, 4, 6$ are predicted. The numbers $N(k)$ of amino-acids coded by D codons would be

$$[N(1), N(2), N(3), N(4), N(6)] = [2, 7, 2, 6, 3] \ .$$

The correct numbers for vertebrate nuclear code are $(N(1), N(2), N(3), N(4), N(6)) = (2, 9, 1, 5, 3)$. Some kind of symmetry breaking must take place and should relate to the emergence of stopping codons. If one codon in second 3-plet becomes stopping codon, the 3-plet becomes doublet. If 2 codons in 4-plet become stopping codons it also becomes doublet and one obtains the correct result $(2, 9, 1, 5, 3)$!

(d) Stopping codons would most naturally correspond to the codons, which involve the $L_z = -1$ relative rotational excitation of uu or dd type quark pair. For the 3-plet the two candidates for the stopping codon state are $|1/2, -1/2\rangle \otimes \{|2, k\rangle\}$, $k = 2, -2$. The total spins are $J_z = 3/2$ and $J_z = -7/2$. The three candidates for the 4-plet from which two states are thrown out are $|1/2, -3/2\rangle \otimes \{|2, k\rangle, |1, k\rangle\}$, $k = 1, 0, -1$. The total spins are now $J_z = -1/2, -3/2, -5/2$. One guess is that the states with smallest value of J_z are dropped which would mean that $J_z = -7/2$ states in 3-plet and $J_z = -5/2$ states 4-plet become stopping codons.

(e) One can ask why just vertebrate code? Why not vertebrate mitochondrial code, which has unbroken $A - G$ and $T - C$ symmetries with respect to the third nucleotide. And is it possible to understand the rarely occurring variants of the genetic code in this framework? One explanation is that the baryonic realization is the fundamental one and biochemical realization has gradually evolved from non-faithful realization to a faithful one as kind of emulation of dark nuclear physics. Also the role of tRNA in the realization of the code is crucial and could explain the fact that the code can be context sensitive for some codons.

Understanding the symmetries of the code

Quantum entanglement between quarks and color flux tubes would be essential for the baryonic realization of the genetic code whereas chemical realization could be said to be classical. Quantal aspect means that one cannot decompose to codon to letters anymore. This raises questions concerning the symmetries of the code.

(a) What is the counterpart for the conjugation $ZYZ \rightarrow X_cY_cZ_c$ for the codons?

(b) The conjugation of the second nucleotide Y having chemical interpretation in terms of hydrophoby-hydrophily dichotomy in biology. In DNA as TQC model it corresponds to matter-antimatter conjugation for quarks associated with flux tubes connecting DNA nucleotides to the lipids of the cell membrane. What is the interpretation in now?

(c) The A-G, T-C symmetries with respect to the third nucleotide Z allow an interpretation as weak isospin symmetry in DNA as TQC model. Can one identify counterpart of this symmetry when the decomposition into individual nucleotides does not make sense?

Natural candidates for the building blocks of the analogs of these symmetries are the change of the sign of the spin direction for quarks and for flux tubes.

(a) For quarks the spin projections are always non-vanishing so that the map has no fixed points. For flux tube spin the states of spin $S_z = 0$ are fixed points. The change of the sign of quark spin projection must therefore be present for both $XYZ \rightarrow X_cY_cZ_c$ and $Y \rightarrow Y_c$ but also something else might be needed. Note that without the symmetry breaking $(1, 3, 3, 1) \rightarrow (1, 2, 3, 2)$ the code table would be symmetric in the permutation of 2 first and 2 last columns of the code table induced by both full conjugation and conjugation of Y.

(b) The analogs of the approximate $A - G$ and $T - C$ symmetries cannot involve the change of spin direction in neither quark nor flux tube sector. These symmetries act inside the A-G and T-C sub-2-columns of the 4-columns defining the rows of the code table. Hence this symmetry must permute the states of same spin inside 5 and 3 for flux tubes and 4 and 2 for quarks but leave 2_{odd} invariant. This guarantees that for the two non-degenerate codons coding for only single amino-acid and one of the codons inside triplet the action is trivial. Hence the baryonic analog of the approximate $A - G$ and $T - C$

symmetry would be exact symmetry and be due to the basic definition of the genetic code as a mapping states of same flux tube spin and quark spin to single representative state. The existence of full 4-columns coding for the same amino-acid would be due to the fact that states with same quark spin inside $(2,3,2)$ code for the same amino-acid.

(c) A detailed comparison of the code table with the code table in spin representation should allow to fix their correspondence uniquely apart from permutations of n-plets and thus also the representation of the conjugations. What is clear that Y conjugation must involve the change of quark spin direction whereas Z conjugation which maps typically 2-plets to each other must involve the permutation of states with same J_z for the flux tubes. It is not quite clear what X conjugation correspond to.

Some comments about the physics behind the code

Consider next some particle physicist's objections against this picture.

(a) The realization of the code requires the dark scaled variants of spin 3/2 baryons known as Δ resonance and the analogs (and only the analogs) of spin 1 mesons known as ρ mesons. The lifetime of these states is very short in ordinary hadron physics. Now one has a scaled up variant of hadron physics: possibly in both dark and p-adic senses with latter allowing arbitrarily small overall mass scales. Hence the lifetimes of states can be scaled up.

(b) Both the absolute and relative mass differences between Δ and N *resp.* ρ and π are large in ordinary hadron physics and this makes the decays of Δ and ρ possible kinematically. This is due to color magnetic spin-spin splitting proportional to the color coupling strength $\alpha_s \sim .1$, which is large. In the recent case α_s could be considerably smaller - say of the same order of magnitude as fine structure constant 1/137 - so that the mass splittings could be so small as to make decays impossible.

(c) Dark hadrons could have lower mass scale than the ordinary ones if scaled up variants of quarks in p-adic sense are in question. Note that the model for cold fusion that inspired the idea about genetic code requires that dark nuclear strings have the same mass scale as ordinary baryons. In any case, the most general option inspired by the vision about hierarchy of conscious entities extended to a hierarchy of life forms is that several dark and p-adic scaled up variants of baryons realizing genetic code are possible.

(d) The heaviest objection relates to the addition of $L_z = -1$ excitation to $S_z = |1/2, \pm 1/2\rangle_{odd}$ states which transforms the degeneracies of the quark spin states from $(1,3,3,1)$ to $(1,2,3,2)$. The only reasonable answer is that the breaking of the full rotation symmetry reduces $SO(3)$ to $SO(2)$. Also the fact that the states of massless particles are labeled by the representation of $SO(2)$ might be of some relevance. The deeper level explanation in TGD framework might be as follows. The generalized imbedding space is constructed by gluing almost copies of the 8-D imbedding space with different Planck constants together along a 4-D subspace like pages of book along a common back. The construction involves symmetry breaking in both rotational and color degrees of freedom to Cartan sub-group and the interpretation is as a geometric representation for the selection of the quantization axis. Quantum TGD is indeed meant to be a geometrization of the entire quantum physics as a physics of the classical spinor fields in the "world of classical worlds" so that also the choice of measurement axis must have a geometric description.

The conclusion is that genetic code can be understand as a map of stringy baryonic states induced by the projection of all states with same spin projection to a representative state with the same spin projection. Genetic code would be realized at the level of dark nuclear physics and biochemical representation would be only one particular higher level representation of the code. A hierarchy of dark baryon realizations corresponding to p-adic and dark matter hierarchies can be considered. Translation and transcription machinery would be realized by flux tubes connecting only states with same quark spin and flux tube spin. Charge neutrality is essential for having only the analogs of DNA, RNA and amino-acids and would guarantee the em stability of the states.

17.5 Cosmic rays and Mersenne Primes

Sabine Hossenfelder has written two excellent blog postings about cosmic rays. The first one is about the GKZ cutoff for cosmic ray energies and second one about possible indications for new physics above 100 TeV. This inspired me to read what I have said about cosmic rays and Mersenne primes- this was around 1996 - immediately after performing for the first time p-adic mass calculations. It was unpleasant to find that some pieces of the text contained a stupid mistake related to the notion of cosmic ray energy. I had forgotten to take into account the fact that the cosmic ray energies are in the rest system of Earth- what a shame! The recent version should be free of worst kind of blunders. Before continuing it should be noticed I am now living year 2012 and this section was written for the first time for around 1996 - and as it became clear - contained some blunders due to the confusion with what one means with cosmic ray energy. The recent version should be free of worst kind of blunders.

TGD suggests the existence of a scaled up copy of hadron physics associated with each Mersenne prime $M_n = 2^n - 1$, n prime: M_{107} corresponds to ordinary hadron physics. Also lepto-hadrons are predicted. Also Gaussian Mersennes $(1+i)^k - 1$, could correspond to hadron physics. Four of them ($k = 151, 157, 163, 167$) are in the biologically interesting length scale range between cell membrane thickness and the size of cell nucleus. Also leptonic counterparts of hadron physics assignable to certain Mersennes are predicted and there is evidence for them [K92].

The scaled up variants of hadron physics corresponding to $k < 107$ are of special interest. $k = 89$ defines the interesting Mersenne prime at LHC, and the near future will probably tell whether the 125 GeV signal corresponds to Higgs or a pion of M_{89} physics. Also cosmic ray spectrum could provide support for M_{89} hadrons and quite recent cosmic ray observations [C66] are claimed to provide support for new physics around 100 TeV. M_{89} proton would correspond to .5 TeV mass considerably below 100 TeV but this mass scale could correspond to a mass scale of a scaled up copy of a heavy quark of M_{107} hadron physics: a naive scaling of top quark mass by factor 512 would give mass about 87 TeV. Also the lighter hadrons of M_{89} hadron physics should contribute to cosmic ray spectrum and there are indeed indications for this.

The mechanisms giving rise to ultra high energy cosmic rays are poorly understood. The standard explanation would be acceleration in huge magnetic fields. TGD suggests a new mechanism based on the decay cascade of cosmic strings. The basis idea is that cosmic string decays *cosmic string* $\rightarrow$ M_2 *hadrons* $\rightarrow$ M_3 *hadrons* ....$\rightarrow$ M_{61} $\rightarrow$ M_{89} $\rightarrow$ M_{107} *hadrons* could be a new source of cosmic rays. Also variants of this scenario with decay cascade beginning from larger Mersenne prime can be considered. One expects that the decay cascade leads rapidly to extremely energetic ordinary hadrons, which can collide with ordinary hadrons in atmosphere and create hadrons of scaled variants of ordinary hadron physics. These cosmic ray events could serve as a signature for the existence of these scale up variants of hadron physics.

(a) Centauro events and the peculiar events associated with $E > 10^5$ GeV radiation from Cygnus X-3. E refers to energy in Earth's rest frame and for a collision with proton the cm energy would be $E_{cm} = \sqrt{2EM} > 10$ TeV in good approximation whereas M_{89} variant of proton would have mass of .5 TeV. These events be understood as being due to the collisions of energetic M_{89} hadrons with ordinary hadrons (nucleons) in the atmosphere.

(b) The decay $\pi_n \rightarrow \gamma\gamma$ produces a peak in the spectrum of the cosmic gamma rays at energy $\frac{m(\pi_n)}{2}$. These produce peaks in cosmic gamma ray spectrum at energies which depend on the energy of π_n in the rest system of Earth. If the pion is at rest in the cm system of incoming proton and atmospheric proton one can estimate the energy of the peak if the total energy of the shower can be estimated reliably.

(c) The slope in the hadronic cosmic ray spectrum changes at $E = 3 \cdot 10^6$ GeV. This corresponds to the energy $E_{cm} = 2.5$ TeV in the cm system of cosmic ray hadron and atmospheric proton. This is not very far from M_{89} proton mass .5 TeV. The creation of M_{89} hadrons in atmospheric collisions could explain the change of the slope.

(d) The ultra-higher energy cosmic ray radiation having energies of order 10^9 GeV in Earth's rest system apparently consisting of protons and nuclei not lighter than Fe might be actually dominated by gamma rays: at these energies γ and p induced showers have same muon content. $E = 10^9$ GeV corresponds to $E_{cm} = \sqrt{2Em_p} = 4 \times 10^4$ GeV. M_{89} nucleon would correspond to mass scale 512 GeV.

(e) So called GKZ cutoff should take place for cosmic gamma ray spectrum due to the collisions with the cosmic microwave background. This should occur around $E = 6\times 10^{10}$ GeV, which corresponds to $E_{cm} = 3.5 \times 10^5$ GeV. Cosmic ray events above this cutoff are however claimed. There should be some mechanism allowing for ultra high energy cosmic rays to propagate over much longer distances as allowed by the limits. Cosmic rays should be able to propagate without collisions. Many-sheeted space-time suggests manners for how gamma rays could avoid collisions with microwave background. For instance, gamma rays could be dark in TGD sense and therefore have large value of Planck constant. One can even imagine exotic variants of hadrons, which differ from ordinary hadrons in that they do not have quarks and therefore no interactions with the microwave background.

(f) The highest energies of cosmic rays are around $E = 10^{11}$ GeV, which corresponds to $E_{cm} = 4 \times 10^5$ GeV. M_{61} nucleon and pion correspond to the mass scale of 6×10^6 GeV and 8.4×10^5 GeV. These events might correspond to the creation of M_{61} hadrons in atmosphere.

The identification of the hadronic space-time sheet as super-symplectic mini black-hole [K57] suggests the science fictive possibility that part of ultra-high energy cosmic rays could be also protons which have lost their valence quarks. These particles would have essentially same mass as proton and would behave like mini black-holes consisting of dark matter. They could even give a large contribution to the dark matter. Since electro-weak interactions are absent, the scattering from microwave background is absent, and they could propagate over much longer distances than ordinary particles. An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV in the rest system of Earth are super-symplectic mini black-holes associated with M_{107} hadron physics or some other copy of hadron physics.

17.5.1 Mersenne primes and mass scales

p-Adic mass calculations lead to quite detailed predictions for elementary particle masses. In particular, there are reasons to believe that the most important fundamental elementary particle mass scales correspond to Mersenne primes $M_n = 2^n - 1$, $n = 2, 3, 7, 13, 17, 19, ...$

$$\begin{aligned} m_n^2 &= \frac{m_0^2}{M_n} \ , \\ m_0 &\simeq 1.41 \cdot \frac{10^{-4}}{\sqrt{G}} \ , \end{aligned} \tag{17.5.1}$$

where $\sqrt{G}$ is Planck length. The lower bound for n can be of course larger than $n = 2$. The known elementary particle mass scales were identified as mass scales associated identified with Mersenne primes $M_{127} \simeq 10^{38}$ (leptons), M_{107} (hadrons) and M_{89} (intermediate gauge bosons). Of course, also other p-adic length scales are possible and it is quite possible that not all Mersenne primes are realized. On the other hand, also Gaussian Mersennes could be important (muon and atomic nuclei corresponds to Gaussian Mersenne $(1 + i)^k - 1$ with $k = 113$).

Theory predicts also some higher mass scales corresponding to the Mersenne primes M_n for $n = 89, 61, 31, 19, 17, 13, 7, 3$ and suggests the existence of a scaled up copy of hadron physics with each of these mass scales. In particular, masses should be related by simple scalings to the masses of the ordinary hadrons.

An attractive first working hypothesis hypothesis is that the color interactions of the particles of level M_n can be described using the ordinary QCD scaled up to the level M_n so that that masses and the confinement mass scale Λ is scaled up by the factor $\sqrt{M_n/M_{107}}$.

$$\Lambda_n = \sqrt{\frac{M_n}{M_{107}}}\Lambda \ . \tag{17.5.2}$$

In particular, the naive scaling prediction for the masses of the exotic pions associated with M_n is given by

$$m(\pi_n) = \sqrt{\frac{M_n}{M_{107}}}m_\pi \ . \tag{17.5.3}$$

Here $m_\pi \simeq 135\ MeV$ is the mass of the ordinary pion. This estimte is of course extremely naive and the recent LHC data suggests that the 125 GeV Higgs candidate could be M_{89} pion. The mass would be two times higher than the naive estimate gives. p-Adic scalings by small powers of $\sqrt{2}$ must be considered in these estimates.

The interactions between the different level hadrons are mediated by the emission of electroweak gauge bosons and by gluons with cm energies larger than the energy defined by the confinement scale of level with smaller p. The decay of the exotic hadrons at level M_{n_k} to exotic hadrons at level $M_{n_{k+1}}$ must take place by a transition sequence leading from the effective M_{n_k}-adic space-time topology to effective $M_{n_{k+1}}$-adic topology. All intermediate p-adic topologies might be involved.

17.5.2 Cosmic strings and cosmic rays

Cosmic strings are fundamental objects in quantum TGD and dominated during early cosmology.

Cosmic strings

Cosmic strings (not quite the same thing in TGD as in GUTs) are basic objects in TGD inspired cosmology [K22, K80] .

(a) In TGD inspired galaxy model galaxies are regarded as mass concentrations around cosmic strings and the energy of the string corresponds to the dark energy whereas the particles condensed at cosmic strings and magnetic flux tubes resulting from them during cosmic expansion correspond to dark matter [K22, K80] . The galactic nuclei, often regarded as candidates for black holes, are the most probable seats for decaying highly entangled cosmic strings.

(b) Galaxies are known to organize to form larger linear structures. This can be understood if the highly entangled galactic strings organize around long strings like pearls in necklace. Long strings could correspond to galactic jets and their gravitational field could explain the constant velocity spectrum of distant stars in the galactic halo.

(c) In [K22, K80, K79] it is suggested that decaying cosmic strings might provide a common explanation for the energy production of quasars, galactic jets and gamma ray bursters and that the visible matter in galaxies could be regarded as decay products of cosmic strings. The magnetic and Z^0 magnetic flux tubes resulting during the cosmic expansion from cosmic strings allow to assign at least part of gamma ray bursts to neutron stars. Hot spots (with temperature even as high as $T \sim \frac{10^{-3,5}}{\sqrt{G}}$) in the cosmic string emitting ultra high energy cosmic rays might be created under the violent conditions prevailing in the galactic nucleus.

The decay of the cosmic strings provides a possible mechanism for the production of the exotic hadrons and in particular, exotic pions. In [C44] the idea that cosmic strings might produce gamma rays by decaying first into 'X' particles with mass of order 10^{15} GeV and then to gamma rays, was proposed. As authors notice this model has some potential difficulties resulting from the direct production of gamma rays in the source region and the presence of intensive electromagnetic fields near the source. These difficulties are overcome if cosmic strings decay first into exotic hadrons of type M_{n_0}, $n_0 \geq 3$ of energy of order $2^{-n_0+2}10^{25}$ GeV , which in turn decay to exotic hadrons corresponding to M_k, $k > n_0$ via ordinary color interaction, and so on so that a sequence of M_k:s starting some value of n_0 in $n = 2, 3, 7, 13, 17, 19, 31, 61, 89, 107$ is obtained. The value of n remains open at this stage and depends on the temperature of the hot spot and much smaller temperatures than the $T \sim m_0$ are possible: favored temperatures are the temperatures $T_n \sim m_n$ at which M_n hadrons become unstable against thermal decay.

Decays of cosmic strings as producer of high energy cosmic gamma rays

In [C58] the gamma ray signatures from ordinary cosmic strings were considered and a dynamical QCD based model for the decay of cosmic string was developed. In this model the final state particles were assumed to be ordinary hadrons and final state interactions were neglected. In the recent case the string decays first to M_{n_0} hadrons and the time scale of for color interaction between M_{n_0} hadrons is extremely short (given by the length scale defined by the inverse of π_{n_0} mass) as compared to the time time scale in case of ordinary hadrons. Therefore the interactions between the final state particles must be taken into account and there are good reasons to expect that thermal equilibrium sets on and much simpler thermodynamic description of the process becomes possible.

A possible description for the decaying part of the highly tangled cosmic string is as a 'fireball' containing various M_{n_0} $(n \geq 3)$ partons in thermal equilibrium at Hagedorn temperature T_{n_0} of order $T_{n_0} \sim m_{n_0} = 2^{-2+n_0} \frac{10^{-4}}{k\sqrt{G}}$, $k \simeq 1.288$. The experimental discoveries made in RHIC suggest [C57] that high energy nuclear collisions create instead of quark gluon plasma a liquid like phase involving gluonic BE condensate christened as color glass condensate. Also black hole like behavior is suggested by the experiments.

RHIC findings inspire a TGD based model for this phase as a macroscopic quantum phase condensed on a highly tangled color magnetic string at Hagedorn temperature. The model relies also on the notion of dynamical but quantized $\hbar$ [K23] and its recent form to the realization that super-symplectic many-particle states at hadronic space-time sheets give dominating contribution to the baryonic mass and explain hadronic masses with an excellent accuracy.

This phase has no direct gauge interactions with ordinary matter and is identified in TGD framework as a particular instance of dark matter. Quite generally, quantum coherent dark matter would reside at magnetic flux tubes idealizable as string like objects with string tension determined by the p-adic length scale and thus outside the "ordinary" space-time. This suggests that color glass condensate forms when hadronic space-time sheets fuse to single long string like object containing large number of super-symplectic bosons.

Color glass condensate has black-hole like properties by its electro-weak darkness and there are excellent reasons to believe that also ordinary black holes could by their large density correspond to states in which super-symplectic matter would form single connected string like structure (if Planck constant is larger for super-symplectic hadrons, this fusion is even more probable).

This inspires the following mechanism for the decay of exotic boson.

(a) The tangled cosmic string begins to cool down and when the temperature becomes smaller than $m(\pi_{n_0})$ mass it has decayed to M_{n_1} matter which in turn continues to decay to M_{n_2} matter. The decay to M_{n_1} matter could occur via a sequence $n_0 \rightarrow n_0 - 1 \rightarrow ...n_1$ of phase transitions corresponding to the intermediate p-adic length scales $p \simeq 2^k$, $n_1 \geq k > n_0$. Of course, all intermediate p-adic length scales are in principle possible so that the process would be practically continuous and analogous to p-adic length scale evolution with $p \simeq 2^k$ representing more stable intermediate states.

(b) The first possibility is that virtual hadrons decay to virtual hadrons in the transition $k \rightarrow k-1$. The alternative option is that the density of final state hadrons is so high that they fuse to form a single highly entangled hadronic string at Hagedorn temperature T_{k-1} so that the process would resemble an evaporation of a hadronic black hole staying in quark plasma phase without freezing to hadrons in the intermediate states. This entangled string would contain partons as "color glass condensate".

(c) The process continues until all particles have decayed to ordinary hadrons. Part of the M_n low energy thermal pions decay to gamma ray pairs and produce a characteristic peak in cosmic gamma ray spectrum at energies $E_n = \frac{m(\pi_n)}{2}$ (possibly red-shifted by the expansion of the Universe). The decay of the cosmic string generates also ultra high energy hadronic cosmic rays, say protons. Since the creation of ordinary hadron with ultra high energy is certainly a rare process there are good hopes of avoiding the problems related to the direct production of protons by cosmic strings (these protons produce two high flux of low energy gamma rays, when interacting with cosmic microwave background [C44]).

Topologically condensed cosmic strings as analogs super-symplectic black-holes?

Super-symplectic matter has very stringy character. For instance, it obeys stringy mass formula due the additivity and quantization of mass squared as multiples of p-adic mass scale squared [K57] . The ensuing additivity of mass squared defines a universal formula for binding energy having no independence on interaction mechanism. Highly entangled strings carrying super-symplectic dark matter are indeed excellent candidates for TGD variants of black-holes. The space-time sheet containing the highly entangled cosmic string is separated from environment by a wormhole contact with a radius of black-hole horizon. Schwartschild radius has also interpretation as Compton length with Planck constant equal to gravitational Planck constant $\hbar/\hbar_0 = 2GM^2$. In this framework the proposed decay of cosmic strings would represent nothing but the TGD counterpart of Hawking radiation. Presumably the value of p-adic prime in primordial stage was as small as possible, even $p = 2$ can be considered.

Exotic cosmic ray events and exotic hadrons

One signature of the exotic hadrons is related to the interaction of the ultra high energy gamma rays with the atmosphere. What can happen is that gamma rays in the presence of an atmospheric nucleus decay to virtual exotic quark pair associated with M_{n_k}, which in turn produces a cascade of exotic hadrons associated with M_{n_k} through the ordinary scaled up color interaction. These hadrons in turn decay $M_{n_{k+1}}$ type hadrons via mechanisms to be discussed later. At the last step ordinary hadrons are produced. The collision creates in the atmospheric nucleus the analog of quark gluon plasma which forms a second kind of fireball decaying to ordinary hadrons. RHIC experiments have already discovered these fireballs and identified them as color glass condensates [C57]. It must be emphasized that it is far from clear whether QCD really predicts this phase.

These showers differ from ordinary gamma ray showers in several respects.

(a) Exotic hadrons can have small momenta and the decay products can have isotropic angular distribution so that the shower created by gamma rays looks like that created by a massive particle.

(b) The muon content is expected to be similar to that of a typical hadronic shower generated by proton and larger than the muon content of ordinary gamma ray shower [C74] .

(c) Due to the kinematics of the reactions of type $\gamma + p \rightarrow H_{M_n} + ... + p$ the only possibility at the available gamma ray energies is that M_{89} hadrons are produced at gamma ray energies above 10 TeV. The masses of these hadrons are predicted to be above 70 GeV

and this suggests that these hadrons might be identified incorrectly as heavy nuclei (heavier than ^{56}Fe). These signatures will be discussed in more detail in the sequel in relation to Centauro type events, Cygnus X-3 events and other exotic cosmic ray events. For a good review for these events and models form them see the review article [C1] .

Some cosmic ray events [C49, C20] have total laboratory energy as high as 3000 TeV which suggests that the shower contains hadron like particles, which are more penetrating than ordinary hadrons.

(a) One might argue that exotic hadrons corresponding M_k, $k > 107$with interact only electro-weakly (color is confined in the length scale associated with M_n) with the atmosphere one might argue that they are more penetrating than the ordinary hadrons.

(b) The observed highly penetrating fireballs could also correspond super-symplectic dark matter part of incoming, possibly exotic, hadron fused with that for a hadron of atmosphere. Both hadrons would have lost their valence quarks in the collision just as in the case of Pomeron events. Large fraction of the collision energy would be transformed to super-symplectic quanta in the process and give rise to a large color spin glass condensate. These condensates would have no direct electro-weak interactions with ordinary matter which would explain their long penetration lengths in the atmosphere. Sooner or later the color glass condensate would decay to hadrons by the analog of blackhole evaporation. This process is different from QCD type hadronization process occurring in hadronic collisions and this might allow to understand the anomalously low production of neutral pions.

Exotic mesons can also decay to lepton pairs and neutral exotic pions produce gamma pairs. These gamma pairs in principle provide a signature for the presence of exotic pions in the cosmic ray shower. If M_{89} proton is sufficiently long-lived enough they might be detectable.The properties of Centauro type events however suggest that M_{89} protons are short lived.

17.5.3 Centauro type events, Cygnus X-3 and M_{89} hadrons

The results reported by Brazil-Japan Emulsion Chamber Collaboration [C49, C34] on multiple production of hadrons induced by cosmic rays with energies $E_{lab} > 10^5\ GeV$ provide evidence for new Physics. The distributions for the transverse momentum p_T and longitudinal momentum fraction x for pions were found to differ from the distributions extrapolated from lower energies. The widening of the transversal momentum distributions has also been observed at accelerator energies (ISR above $\sqrt{s} = 63\ GeV$ and CERN SPS-p$\bar{p}$ Collider at $\sqrt{s} = 540\ GeV$). Furthermore, exotic events called Geminion, Centauro, Chiron with emission of $n_B \leq 100$ hundred baryons but practically no pions were detected. There are also peculiar events associated with the radiation coming from Cygnus X-3. A recent summary about peculiar events is given in the review article [C1] .

Mirim, Acu and Quacu

The exotic cosmic ray events are described in the review article of [C49] . In [C49] the multiple production of pions is classified into 3 jet types called Mirim, Acu and Quacu. Although the transverse momentum distributions for pions observed at low energies are universal, Acu and Quacu jets are characterized by wider transverse momentum distributions with larger value of average transverse momentum p_T than in low energy pionization: this widening is in accordance with accelerator results. The distributions for the longitudinal momentum fraction x scale but differ from the low energy situation for Acu and Quacu jets.

In [C49, C60] a description of these events in terms of 'fireballs' decaying into ordinary hadrons were considered. The p_T distribution associated with Mirim is just the ordinary low energy transverse momentum distribution whereas the distributions associated with Acu and Quacu are wider. The masses of the fireballs were assumed to be discrete and were found to be $M_0 \sim 2-3\ GeV$ (Mirim), $M_1 \sim 15-30\ GeV$ (Acu) , $M_2 \sim 100-300\ GeV$ (Quacu).

It should be noticed that the upper bounds for the masses associated with Acu and Quacu fireballs are roughly by a factor of two smaller than the naive mass estimates 69 GeV and 481 GeV associated with M_{89} pion and M_{89} proton. The temperatures were found to be in range $0.4 - 10\ GeV$ for Acu and Quacu fireball and to be substantially larger than the ordinary Hagedorn temperature $T_H \simeq 0.16\ GeV$.

Chirons, Centauros, anti-Centauros, and Geminions

For the second class of events consisting of Chirons, Centauros and Geminions observed at laboratory energies $100 - 1000\ TeV$ pion production is strongly suppressed (gamma pairs resulting from the decay of neutral pions are almost absent) [C49] . The primary event takes place few hundred meters above the detector and decay products are known to be hadrons and mostly baryons: about 15 (100) for Mini-Centauros (Centauros). This excludes the possibility that exotic hadrons decay in emulsion chamber and implies also that the decay mechanism of the primary particle is such that very few mesons are produced.

The fireball hypothesis has been applied also to Centauro type events assuming that fireballs corresponds to a different phase than in the case of Mirim, Acu and Quacu [C49] . The fireball masses associated with Mini-Centauro and Centauro are according to the estimate of [C49] $M_{mini} = 35$ GeV and $M_{Centauro} = 230$ GeV. These masses are almost exactly one half of the masses of the M_{89} pion (70 GeV) and proton (470 GeV) respectively!

$$\begin{aligned} M_{Mini} &\simeq \frac{m(\pi_{89})}{2} , \\ M_{Centauro} &\simeq \frac{m(p_{89})}{2} . \end{aligned} \tag{17.5.4}$$

This suggests that the decay of cosmic gamma ray to M_{89} quark pair which in turn hadronizes to (possibly virtual) M_{89} hadrons induced by the interaction with the nucleon of atmosphere is the origin of Mini-Centauro/Centauro events.

The basic difference between the decaying fireballs in Acu/Quacu events and Centauro type events is that Acu/Quacu decays produce neutral pions unlike Centauros.

The appearance of the factor of 1/2 in the mass estimates needs an explanation. One explanation is systematic error in the evaluation of hadronic energy: for instance, the gamma inelasticity k_γ telling which fraction of hadronic energy is transformed to electromagnetic energy might be actually smaller than believed by a factor of order two. An alternative explanation is related to the decay mechanism of M_{89} particle: if the decay takes place via a decay to two off mass shell M_{89} hadrons decaying in turn to hadrons then the average rest energy of the fireball is indeed one half of the mass of the decaying on mass shell particle. The reason for the necessity of off mass shell intermediate states is perhaps the stability of the on mass shell exotic hadrons against the direct decay to ordinary hadrons.

Anti-Centauros are much like Centauros except that neutral pions are over-abundant [C1] . The speculative model [C11] relies on the notion of chiral condensates consisting of neutral pions in the case of Centauros and charged pions in the case of anti-Centauros. If one wants to explain Anti-Centauros in terms of M_{89} physics should be able to explain the over abundance of neutral pions in terms of decay products of ordinary hadrons at later stages of the decay cascade.

The case of Cygnus X-3

There are peculiar events associated with the cosmic rays coming from Cygnus X-3 at gamma ray energies above $10^5\ GeV$ [C12] . The primary particle must be massless particle and is most probably ordinary gamma ray. The structure of the shower however suggests that the decaying particle is very massive! Furthermore, the muon content of the shower is larger than that associated with gamma ray shower. A possible explanation is that the gamma rays coming from Cygnus X-3 with energy above the threshold 10^4 GeV produce M_{89} hadrons, which in turn create the cosmic ray shower through the decay to M_{89} hadrons and the decay of these to the ordinary M_{107} hadrons: this indeed means that the gamma rays behave like a massive particles in the atmosphere.

17.5.4 TGD based explanation of the exotic events

The TGD based model for exotic events involve p-adic length scale hierarchy, many-sheeted space-time, and TGD inspired view about dark matter. A decisive empirical input comes from RHIC events suggesting that quark gluon plasma is actually a liquid like "macroscopic" quantum phase identifiable as a particular instance of dark matter.

General considerations

The mass estimates for the fireballs and the absence of neutral pions suggest that Mini-Centauro/Centauro type events correspond to the decay of M_{89} hadrons (pion/proton) to ordinary hadrons. The general model for the exotic events would be following.

(a) Cosmic gamma ray decays first into M_{89} quark pair via electromagnetic interaction with the nucleon of the atmosphere. Pairs of Centauros/anti-Centauros and quark-gluon-plasma blobs explaing Mirim/Qcu/Quacu events would be naturally created in these collisions.

(b) The quark pair in turn hadronizes to M_{89} hadrons decaying to virtual $k > 89$ hadrons which in turn end up via a sequential decay process to ordinary hadrons. This process is kinematically possible if the condition $E_{tot} > 2M^2/m_p$, is satisfied (M is the mass of the exotic hadron). For example, the energy of the gamma ray must be larger than 500 TeV for exotic proton pair production. For the exotic pion the corresponding lower bound is about 10 TeV. The energies of the exotic events are indeed above 100 TeV in accordance with these bounds. The average total energy is about $E_{tot} = 1740$ TeV for Centauros and $E_{tot} \simeq 903$ TeV for Mini-Centauros [C49]. The mechanism implies that two M_{89} fireballs are produced. 'Binocular' events (Geminions) consisting of two widely separated fireballs have indeed been observed [C49] .

(c) If anti-Centauros result via the same mechanism there must be a mechanism explaining why the production of neural pions varies from event to event. One proposal is that the difference is due to a formation of pion condensates consisting of neural *resp.* charged pions in the two situations [C11] . This hypothesis would unify Centauro events with anti-Centauro events in which the production of neutral pions is abnormally high [C1] .

(d) Mirim/Acu/Quacu events could correspond to the decay of a high temperature quark-gluon plasma blob, or rather color glass condensate, to hadrons (recall that the estimated plasma temperatures are much lower than for Centauros). The collision of M_{89} hadron possibly generated in the interaction of the cosmic gamma ray with ordinary nucleon could induce both the decay of M_{89} hadron to virtual hadrons and generate quark-gluon plasma blob in the atmospheric target nucleus. Hagedorn temperature $T(k)$, $89 < k \leq 107$ is a good guess for the temperature of this plasma blob. RHIC findings [C57] suggest that the blob corresponds to highly tangled hadronic string containing super-symplectic dark matter and decaying by de-coherence to ordinary hadrons [K23] .

Connection with TGD based model for RHIC events

The counterparts of Centauros and other exotic events have not been observed in accelerator experiments. More than a decade after writing the first version of the model for Centauros came however data from RHIC experiment [C57], which seems to provide a connection between laboratory and cosmic ray data. In RHIC collisions of very energetic Gold nuclei are studied. The collisions were expected to create a quark gluon plasma freezing to ordinary hadrons. The surprise was that the resulting state behaves like an ideal liquid and has also black hole like properties [C57].

Recall that the TGD based model [K79, K23] for RHIC findings is following.

(a) The state in question corresponds to a highly entangled hadronic string at Hagedorn temperature defining the analog of black hole and decaying by evaporation. The gravitational constant defined by Planck length is effectively replaced by a hadronic gravitational constant defined by the hadronic length scale. p-Adic length scale hypothesis predicts entire hierarchy of Hagedorn temperatures.

(b) Bose-Einstein condensate of gluons referred to as color glass condensate has been proposed as an explanation for the liquid like behavior of the quark-gluon phase. TGD based explanation for the liquid like state is that that the state in question corresponds to a large Bose-Einstein condensate like state of super-symplectic particles resulting as hadronic space-time sheets fuse. Super-symplectic bosons have vanishing electro-weak quantum numbers since super-symplectic generators are either purely bosonic or possess quantum numbers of right handed neutrino. Dark matter is in question.

(c) LHC has already produce evidence for quark gluon plasma possessing anomalous properties but created in collisions of protons rather than those of heavy nuclei. The TGD based explanation is in formation of long highly entangled color flux tube producing hadrons as it decays [K52] . It might be that the creation of these objects in the decays of M_{89} hadrons are responsible for some aspects of the exotic cosmic ray events.

A more precise model for exotic events

A more detailed formulation necessitates a rough model for the transformation of M_{89} hadrons to M_{107} hadrons.

(a) On mass shell exotic hadrons can be assumed to be stable against direct decay to ordinary hadrons so that their decay must take place via a sequential decay to off mass shell exotic hadrons characterized by $107 > k > 89$, which eventually decay to ordinary hadrons. The simplest decay mode is the decay to two virtual exotic hadrons with average mass, which is one half of the mass of the decaying exotic hadron in accordance with observations.

(b) M_{89} hadron decays to virtual hadrons with $p \simeq 2^k > M_{89}$ dominate over electro-weak decays since the characteristic time scale is defined by $\Lambda(QCD, M_{89}) = 512\Lambda(QCD, 107)$. This means that most of the energy in the process goes to virtual $k > 89$ virtual mesons. Neutral $k > 89$ virtual pions, if created, can decay to gamma pairs so that the problem of understanding the absence of neutral pions remains.

(c) M_{89} hadronic space-time sheet suffers a topological phase transition to M_{107} hadronic space-time sheet via several steps $k = 89 \rightarrow k_1 > 89.. \rightarrow k_n = 107$. In the process the size of hadronic surface suffers a $2^9 = 512$-fold expansion meaning the increase of volume by a factor for $2^{27} \sim 10^9/8$ so that a small scale Big Bang is really in question! The expansion brings in mind liquid-vapor phase transition but the freezing to hadrons (due to the properties of color coupling constant evolution) makes the transition more like a liquid-solid phase transition.

As noticed, all p-adic length scales in the range involved could be present but $p \simeq 2^k$ would define more stable intermediate states. A possible experimental signature for the sequence of the phase transitions labeled by $89 \leq k \leq 107$ is a bumpy structure of the detected hadronic cascades with a maximum of 17 maxima. This kind of structure with a constant distance between maxima and 11 maxima has been indeed observed for some cascades (see Fig. 8 of [C1]).

A good guess for the critical temperature of the Big Bang like phase transition to occur is $T_{cr}(89) = km_{89}$, where k is some numerical factor. TGD inspired model for the early cosmology provides a universal hydrodynamics model for this period as a mini Big Bang, or rather "a soft whisper amplified to a relatively big bang", containing the duration of the period as the only parameter [K80] .

(d) If the decay process is fast enough, the density of virtual hadrons in the final state becomes so high that they form single highly tangled cosmic string in Hagedorn temperature $T(k)$. An entire sequence of $T(k) = km_k$, $107 > k > 89$ of phase transition temperatures could be involved without intermediate freezing to hadrons. Since the transformation of $k = 89$ hadrons to $k = 107$ hadrons would be essentially a decay process, the distribution of decay products is isotropic in the center of mass frame of $k = 89$ hadron (Centauros/anti-Centauros). The same conclusion holds true for the decay of quark gluon plasma (Mirim/Qcu/Quacu).

How to understand the anomalous production of pions?

One can imagine two different explanations for the varying number of pions in the events.

1. M_{89} hadrons produce M_{89} pions

This model would explain the special features of Centauros. To Anti-Centauros the model does not apply. One could hope that the decay cascade of Centauro leads at later stages to color glass phases for ordinary hadrons producing surplus of neutral pions.

2. Restoration of electro-weak symmetry?

The anomalous production of pions might relate to the restoration of electro-weak symmetry in case of M_{89} hadrons. For M_{89} hadrons the restoration of the electro-weak symmetry would be natural since in TGD framework classical induced gauge fields are massless for known non-vacuum extremals below the p-adic length scale $L(89)$ defining the fundamental electro-weak length scale. The finite size of the space-time sheet carrying these fields brings in the length scale determining the boson mass when the space-time sheet in question looks point like in the length scale resolution used. The model of elementary particles as weak strings (Kähler magnetic flux tubes) suggests that electroweak symmetry restoration takes place inside weak magnetic flux tubes and that one might have Bose-Einstein condensate with negative and positive net charges in turn implying the abundance of charged pions. One might argue argue that for particles topologically condensed to space-time sheets with $k > 89$ M_{61} defines the weak scale so that weak interactions effectively disappear.

In zero energy ontology zero energy states are characterized by time-like entanglement coefficients defining M-matrices in turn identifiable as the rows of the unitary U-matrix coding for physics in TGD Universe. The superposition of zero energy states for which positive energy parts have varying values of conserved charges (say electromagnetic charge) do not break conservation laws. Note that also in super-conductors coherent states of Cooper pairs make sense in zero energy ontology without breaking the conservation of fermion number. Therefore one can consider generation of coherent states of pions with non-standard direction of isospin in the collisions of cosmic rays with the nuclei of atmosphere. The TGD inspired model for lepto-hadrons [K92] assumes that the coherent states of lepto-pions consisting of pionlike bound states of colored excitations of leptons are created in the strong non-orthogonal magnetic and electric fields of the colliding heavy nuclei or other charged particles. Similar situation might be encountered in the collision of high energy cosmic rays with the nuclei of the atmosphere.

Both Centauros and anti-Centauros could be understood if the transformation of M_{89} hadrons to ordinary hadrons generates "mis-aligned" pionic BE condensates. $U(2)_{ew}$ symmetry is restored for M_{89} hadrons and there is no preferred isospin direction for the order parameter of M_{89} pionic BE condensate. This BE condensate is however excluded by energetic considerations. The sequence of phase transitions leading to M_{107} hadrons involving intermediate p-adic length scales could however generate this kind of BE condensate.

If an overcooling occurs in the sense that electro-weak symmetry is not lost, the first intermediate pion condensate can correspond to π_+,π_- or π_0. Charged π condensates would be created in pairs with opposite charges. In this kind of situation the number of gamma rays produced in the decay to ordinary hadrons would vary from event to event.

The presence of pionic BE condensates favors the decay to M_{107} hadrons via hadronic intermediate states rather than via the cooling of partonic phase condensed on single tangled string whose length grows. This and the idea that $U(2)_{ew}$ symmetry could be exact for the dark matter phase, encourages to consider also the possibility that M_{89} hadron decays to a state consisting of dark M_{107} hadrons forming a BE condensate like state behaving like single coherent unit and interacting with the ordinary matter only via emission of dark gauge boson BE condensates de-cohering to ordinary gauge bosons.

Dark pionic BE condensates with various charges could be present. These dark π condensates would decay coherently to pairs of dark ew boson "laser beams", which can interact with the ordinary matter only after they have de-cohered to ordinary ew gauge bosons and remain undetected if the de-coherence time for dark bosons is long enough, probably not so. Dark hadron option could thus explain also the abnormally long penetration lengths.

3. Is long range charge entanglement involved?

The variation for the number of pions could involve electromagnetic charge entanglement between particles produced in the event and ordinary matter. This would guarantee strict charge conservation when the quantization axis for weak isospin for the resulting hadrons differs from that for the ordinary matter. The decay of the pion to gamma pair becomes possible only after the entanglement is reduced and if de-coherence time is long enough it is possible to understand the variation.

17.5.5 Cosmic ray spectrum and exotic hadrons

The hierarchy of M_n hadron physics provides also a mechanism producing ultra high energy cosmic gamma rays and hadrons.

Do gamma rays dominate the spectrum at ultrahigh energies?

A possible piece of evidence for M_{89} hadrons is related to the analysis [C43] of the cosmic ray composition near $E = 10^9\ GeV$ (note that the energy is in the rest frame of Earth). The analysis was based on the assumption that the spectrum consists of nuclei. The assumptions and conclusions of the analysis can be criticized:

(a) There is argument [C50] , which states that the interaction of protons having energy above $10^9\ GeV$ with the cosmic microwave background implies pion pair creation and a rapid loss of proton energy so that the contribution of protons should be strongly suppressed in the cosmic ray spectrum above $E = 7 \cdot 10^{10}\ GeV$. If protons dominate, cosmic ray spectrum should effectively terminate at energy of order $7 \cdot 10^{10}\ GeV$: some events above $E = 10^{11}\ GeV$ have been however detected [C61] .

(b) It is not obvious whether one can distinguish between protons and gamma rays at these energies since the muon content of the photon and proton showers are near to each other at these energies [C44] . Therefore the particles identified as protons might well be gamma rays.

(c) The spectrum can be fitted assuming that cosmic ray spectrum has two components. Light component ('protons') can be identified as protons and He nuclei. The heavy component ('Fe') corresponds to Fe and heavier nuclei. The nuclei between He and Fe seem to be peculiarly absent. Furthermore, there are also indications that spectrum contains only light nuclei in the range $3 \cdot 10^7 - 10^{11}\ GeV$ [C55] .

An alternative interpretation suggested also in [C44] is that cosmic ray flux is dominated by gamma rays at these energies. 'Protons' could correspond to gamma rays interacting ordinarily with matter. 'Fe nuclei' correspond to the fraction of gamma rays decaying first into

M_{89} exotic quark pair producing corresponding exotic hadrons, which then decay to ordinary hadrons and produce showers resembling ordinary heavy nucleus shower. Super-symplectic vision allows to consider the possibility that 'protons' correspond to super-symplectic part of proton having essentially the same mass.

Hadronic component of the cosmic ray spectrum

The properties of the hadronic cosmic ray spectrum above $4 \cdot 10^5$ GeV are not well understood. This energy correspond for a collision with atmospheric proton to cm energy of about $E_{cm} = 10^3$ GeV which suggests that the production of M_{89} hadrons in atmosphere is involved.

(a) It has turned out difficult to invent acceleration mechanisms producing hadronic cosmic rays having energies above 10^5 GeV [C43] .

(b) The spectrum contains a 'knee' (power $E^{-2.7}$ changes to about E^{-3} at the knee), which is at the energy $E = 3 \cdot 10^6$ GeV corresponding to $E_{cm} = 2.5 \times 10^3$ GeV [C43] . This could relate to production of M_{89} hadrons: the mass of M_{89} proton is 512 GeV by naive scaling. It is difficult to understand how the knee is generated although several explanations have been proposed (these are reviewed shortly in [C43]).

A possible solution of the problems is that part of the hadronic cosmic rays are generated in the decay of string like objects rather than by some acceleration mechanism. Assume that M_{n_k} hadron is created in the decay cascade. Since $M_{n_{k+m}}$, $m = 1, 2, ..$ hadrons can have rest masses above M_{n_k} threshold mass, one can consider the possibility that M_{n_k} hadron decays sequentially to ordinary M_{107} hadron with arbitrary large rest mass (even larger than M_{n_k} pion mass) and that this ordinary hadron in turn produces some very energetic low mass hadrons, say proton and antiproton, identifiable as cosmic rays. The most efficient producers of hadrons are M_{n_k} pions since these are produced most abundantly in the decay of $M_{n_{k+1}}$ hadrons. M_{n_k} pion at rest cannot however decay to ordinary hadrons with energy above M_{n_k} pion mass. Therefore the slope of the cosmic ray energy flux should become steeper above M_{n_k}, in particular M_{61}, threshold.

The incoming hadrons would outcome of the decay sequence and therefore ordinary hadrons. They would collide with the hadrons of atmosphere and collisions would create M_{89} hadrons if sufficiently energetic.

The problem of relic quarks and hierarchy of QCD:s

Baryon and lepton numbers are conserved separately in TGD and one of the basic problems of the gauge theories with conserved baryon number is the problem of relic quarks. Hadronization starts in temperature of the order of quark mass and since hadronization is basically many quark process it continues until the expansion rate of the Universe becomes larger than the rate of the hadronization. As a consequence the number density of relic quarks is much larger than the upper bound $n_{relic} < \rho_B/m_q = 10^{-9} n_\gamma m_p/m_q$ obtained from the requirement that the contribution of relic quarks to mass density is smaller than the baryonic mass density. There is also an experimental upper bound $n_{relic} < 10^{-28} n_\gamma$.

The assumption about the existence of QCD:s with a hierarchy of increasing scales $\Lambda_{QCD}(M_n)$ implies that the length scale $L(n) \sim 1/\sqrt{\Lambda_{QCD}(M_n)}$ below which quarks are free, decreases with increasing cosmic temperature and therefore the problem of the relic quarks disappears.

17.5.6 Ultrahigh energy cosmic rays as super-symplectic quanta?

Near the end of year 2007 Pierre Auger Collaboration made a very important announcement relating to ultrahigh energy cosmic rays. I glue below a popular summary of the findings [E14] .

Scientists of the Pierre Auger Collaboration announced today (8 Nov. 2007) that active galactic nuclei are the most likely candidate for the source of the highest-energy cosmic rays *that hit Earth. Using the Pierre Auger Observatory in Argentina, the largest cosmic-ray observatory in the world, a team of scientists from 17 countries found that the sources of the highest-energy particles are not distributed uniformly across the sky. Instead, the Auger results link the origins of these mysterious particles to the locations of nearby galaxies that have active nuclei in their centers. The results appear in the Nov. 9 issue of the journal Science.*

Active Galactic Nuclei (AGN) are thought to be powered by supermassive black holes that are devouring large amounts of matter. They have long been considered sites where high-energy particle production might take place. They swallow gas, dust and other matter from their host galaxies and spew out particles and energy. While most galaxies have black holes at their center, only a fraction of all galaxies have an AGN. The exact mechanism of how AGNs can accelerate particles to energies 100 million times higher than the most powerful particle accelerator on Earth is still a mystery.

What has been found?

About million cosmic ray events have been recorded and 80 of them correspond to particles with energy above the so called GKZ bound, which is $.54 \times 10^{11}$ GeV. Electromagnetically interacting particles with these energies from distant galaxies should not be able to reach Earth. This would be due to the scattering from the photons of the microwave background. About 20 particles of this kind however comes from the direction of distant active galactic nuclei and the probability that this is an accident is about 1 per cent. Particles having only strong interactions would be in question. The problem is that this kind of particles are not predicted by the standard model (gluons are confined).

What could TGD say about the finding?

TGD provides a possible explanation for the new kind of particles.

(a) The original TGD based model for the galactic nucleus is as a highly tangled cosmic string (in TGD sense of course [K22] . Much later it became clear that also TGD based model for black-hole is as this kind of string like object near Hagedorn temperature [K22] . Ultrahigh energy particles could result as decay products of a decaying split cosmic string as an extremely energetic galactic jet. Kind of cosmic fire cracker would be in question. Originally I proposed this decay as an explanation for the gamma ray bursts. It seems that gamma ray bursts however come from thickened cosmic strings having weaker magnetic field and much lower energy density [K79] .

(b) TGD predicts particles having only strong interactions [K48] . I have christened these particles super-symplectic quanta. These particles correspond to the vibrational degrees of freedom of partonic 2-surface and are not visible at the quantum field theory limit for which partonic 2-surfaces become points.

What super-symplectic quanta are?

Super-symplectic quanta are created by the elements of super-symplectic algebra, which creates quantum states besides the super Kac-Moody algebra present also in super string model. Both algebras relate closely to the conformal invariance of light-like 3-surfaces.

(a) The elements of super-symplectic algebra are in one-one correspondence with the Hamiltonians generating symplectic transformations of $\delta M^4_+ \times CP_2$. Note that the 3-D light-cone boundary is metrically 2-dimensional and possesses degenerate symplectic and Kähler structures so that one can indeed speak about symplectic (canonical) transformations.

(b) This algebra is the analog of Kac-Moody algebra with finite-dimensional Lie group replaced with the infinite-dimensional group of symplectic transformations [K18] . This should give an idea about how gigantic a symmetry is in question. This is as it should be since these symmetries act as the largest possible symmetry group for the Kähler geometry of the world of classical worlds (WCW) consisting of light-like 3-surfaces in 8-D imbedding space for given values of zero modes (labeling the spaces in the union of infinite-dimensional symmetric spaces). This implies that for the given values of zero modes all points of WCW are metrically equivalent: a generalization of the perfect cosmological principle making theory calculable and guaranteeing that WCW metric exists mathematically. Super-symplectic generators correspond to gamma matrices of WCW and have the quantum numbers of right handed neutrino (no electro-weak interactions). Note that a geometrization of fermionic statistics is achieved.

(c) The Hamiltonians and super-Hamiltonians have only color and angular momentum quantum numbers and no electro-weak quantum numbers so that electro-weak interactions are absent. Super-symplectic quanta however interact strongly.

Also hadrons contain super-symplectic quanta

One can say that TGD based model for hadron is at space-time level kind of combination of QCD and old fashioned string model forgotten when QCD came in fashion and then transformed to the highly unsuccessful but equally fashionable theory of everything.

(a) At quantum level the energy corresponding to string tension explaining about 70 per cent of proton mass corresponds to super-symplectic quanta [K57] . Super-symplectic quanta allow to understand hadron masses with a precision better than 1 per cent.

(b) Super-symplectic degrees of freedom allow also to solve spin puzzle of the proton: the average quark spin would be zero since same net angular momentum of hadron can be obtained by coupling quarks of opposite spin with angular momentum eigen states with different projection to the direction of quantization axis.

(c) If one considers proton without valence quarks and gluons, one obtains a boson with mass very nearly equal to that of proton (for proton super-symplectic binding energy compensates quark masses with high precision). These kind of pseudo protons might be created in high energy collisions when the space-time sheets carrying valence quarks and super-symplectic space-time sheet separate from each other. Super-symplectic quanta might be produced in accelerators in this manner and there is actually experimental support for this from Hera.

(d) The exotic particles could correspond to some p-adic copy of hadron physics predicted by TGD and have very large mass smaller however than the energy. Mersenne primes $M_n = 2^n - 1$ define excellent candidates for these copies. Ordinary hadrons correspond to M_{107}. The protons of M_{61} hadron physics would have the mass of proton scaled up by a factor $2^{(107-61)/2} = 2^{23} \simeq 8 \times 10^6$. GKZ limit $E = .54 \times 10^{11}$ GeV corresponds to cm energy $E_{cm} = 3.3 \times 10^5$ GeV and is below 8×10^6 GeV. Super-symplectic M_{89} protons having no valence quarks can propagate without interactions with cosmic microwave background. Note that CP_2 mass corresponds roughly to about 10^{14} proton masses.

(e) Ideal blackholes would be very long highly tangled string like objects, scaled up hadrons, containing only super-symplectic quanta. Hence it would not be surprising if they would emit super-symplectic quanta. The transformation of supernovas to neutron stars and possibly blackholes would involve the fusion of hadronic strings to longer strings and eventual annihilation and evaporation of the ordinary matter so that only super-symplectic matter would remain eventually. A wide variety of intermediate states with different values of string tension would be possible and the ultimate blackhole would correspond to highly tangled cosmic string. Dark matter would be in question in the sense that Planck constant could be very large.

Chapter 1

Appendix

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of CP_2 to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

A-1 Imbedding space $M^4 \times CP_2$ and related notions

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that imbedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Imbedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://www.tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://www.tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://www.tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

A-2.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A117] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://www.tgdtheory.fi/appfigures/cp2.jpg`

A-2.2 Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \to exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , \quad e^1 = \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \tfrac{r\sigma_3}{F} \ . \end{aligned} \quad \text{(A-2.10)}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \tfrac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \tfrac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \quad \text{(A-2.11)}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \quad \text{(A-2.12)}$$

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \quad \text{(A-2.13)}$$

is given by

$$\begin{aligned} V_{01} &= -\tfrac{e^1}{r} \ , & V_{23} &= \tfrac{e^1}{r} \ , \\ V_{02} &= -\tfrac{e^2}{r} \ , & V_{31} &= \tfrac{e^2}{r} \ , \\ V_{03} &= (r - \tfrac{1}{r})e^3 \ , & V_{12} &= (2r + \tfrac{1}{r})e^3 \ . \end{aligned} \quad \text{(A-2.14)}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{aligned} R_{01} &= e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} &= e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} &= e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} &= -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} &= 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} &= 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{aligned} \quad \text{(A-2.15)}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \quad \text{(A-2.16)}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \quad \text{(A-2.17)}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F} sin\Theta d\Theta d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

A-2.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A99] . However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-2.4 Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A81] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \text{ for } X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \text{ or equivalently } (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \text{ or equivalently } (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-3 CP_2 geometry and standard model symmetries

A-3.1 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B38] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-3.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-3.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{A-3.3}$$

and

$$B = 2re^3 \ , \tag{A-3.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-3.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-3.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-3.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{A-3.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{A-3.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-3.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1\times\gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-3.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-3.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-3.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-3.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{A-3.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-3.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-3.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-3.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-3.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-3.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9} \right] \ , \tag{A-3.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18 + n_i^2))} \ . \tag{A-3.24}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-3.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B75] .

A-3.2 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

(a) Symmetries must be realized as purely geometric transformations.

(b) Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B16] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-3.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-3.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^{\dagger}\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-3.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-4 The relationship of TGD to QFT and string models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://www.tgdtheory.fi/appfigures/particletgd.jpg`

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situtation in Minkowskian regions of space-time surface. http://www.tgdtheory.fi/appfigures/fermistring.jpg

TGD based view about elementary particles has two aspects.

(a) The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

(b) Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacs carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompnied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. http://www.tgdtheory.fi/appfigures/elparticletgd.jpg

Particle interactions involve both stringy and QFT aspects.

(a) The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

(b) Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats beheave like virtual bosons so that one has BFF type vertices typically.

(c) In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. http://www.tgdtheory.fi/appfigures/tgdgraphs.jpg

A-5 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

Induction procedure for gauge fields

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as base space the imbedded manifold. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induce gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface. Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://www.tgdtheory.fi/appfigures/induct.jpg`

Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-5.1 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. http://www.tgdtheory.fi/appfigures/manysheeted.jpg

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. http://www.tgdtheory.fi/appfigures/wormholecontact.jpg

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)) as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://www.tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-5.2 Imbedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

(a) Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

(b) Spinor harmonics of imbedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

(a) Although the imbedding space spinor connection carries W gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

(b) The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

(c) Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

(d) One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

(e) This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-5.3 Space-time surfaces with vanishing em, Z^0, or Kähler fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-5.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-5.2}$$

where Θ_W denotes Weinberg angle.

(a) The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k+cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-5.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-5.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u+k| = [(1+r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.
The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p}Xdu \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p}J \ . \end{aligned} \tag{A-5.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

(b) The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

(c) The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2}Z^0 \ . \end{aligned} \tag{A-5.6}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$
\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-5.7)}
\end{aligned}
$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$
\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \qquad \text{(A-5.8)}
\end{aligned}
$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-5.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-6 p-Adic numbers and TGD

A-6.1 p-Adic number fields

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A48] . p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k \geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \quad \text{(A-6.1)}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \quad \text{(A-6.2)}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \quad \text{(A-6.3)}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \ . \quad \text{(A-6.4)}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \ . \quad \text{(A-6.5)}$$

This division of the metric space into classes has following properties:

(a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

(b) Distances of points x and y inside single class are smaller than distances between different classes.

(c) Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B54] . The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-6.2 Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} \quad , \\ y_k &\in \{0, 1, .., p-1\} \quad . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \quad , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \quad . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \quad , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \quad , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. 11.2.4) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://www.tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R^n_p of R^n each of which projects to a copy of R^n_+ (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://www.tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

A-6.3 The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles" with reverse map interpreted as a transformation of intention to action and would be realized in terms of canonical identification or some of its variants.

Fig. 16. The basic idea between p-adic manifold. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

(a) Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

(b) Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

(c) Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-7 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://www.tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. http://www.tgdtheory.fi/appfigures/reconnect1.jpg

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. http://www.tgdtheory.fi/appfigures/reconect2.jpg

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://www.tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://www.tgdtheory.fi/appfigures/cat.jpg

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://www.tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://www.tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see fig. http://www.tgdtheory.fi/appfigures/timemirror.jpg or fig. 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://www.tgdtheory.fi/appfigures/timemirror.jpg

REFERENCES

Mathematics

[A1] http://www.ams.org/mathweb/mi-mathbyclass.html.

[A2] A quantum octonion algebra. http://arxiv.org/abs/math/9801141.

[A3] Atyiah-Singer index-theorem. http://en.wikipedia.org/wiki/Atiyah-Singer_index_theorem.

[A4] Catastrophe Theory. http://en.wikipedia.org/wiki/Catastrophe_theory#Cusp_catastrophe.

[A5] Clifford algebra. http://en.wikipedia.org/wiki/Clifford_algebra.

[A6] Complex projective space. http://en.wikipedia.org/wiki/Complex_projective_space.

[A7] Fields. http://en.wikipedia.org/wiki/Field_(mathematics).

[A8] Frames and locales. http://en.wikipedia.org/wiki/Frames_and_locales.

[A9] Groupoid. http://en.wikipedia.org/wiki/Groupoid.

[A10] Harmonic analysis. http://en.wikipedia.org/wiki/Harmonic_analysis.

[A11] Hyper-Kähler manifold. http://en.wikipedia.org/wiki/Hyper-Khler_manifold.

[A12] Hyperfinite type II factor. http://en.wikipedia.org/wiki/Hyperfinite_type_II-1_factor.

[A13] Kac-Moody algebra. http://en.wikipedia.org/wiki/KacMoody_algebra.

[A14] Kähler manifold. http://en.wikipedia.org/wiki/Khler_manifold.

[A15] KMS state. http://en.wikipedia.org/wiki/KMS_state.

[A16] Lagrangian sub-manifold. http://en.wikipedia.org/wiki/Lagrangian_submanifold.

[A17] Langlands program. http://en.wikipedia.org/wiki/Langlands_program.

[A18] Number theory. http://en.wikipedia.org/wiki/Number_theory.

[A19] Octonions. http://en.wikipedia.org/wiki/Octonions.

[A20] Particular point topology. http://en.wikipedia.org/wiki/Particular_point_topology.

[A21] Planar algebra. http://en.wikipedia.org/wiki/Planar_algebra.

[A22] Pointless topology. http://en.wikipedia.org/wiki/Pointless_topology.

[A23] Pythagorean triangles. http://en.wikipedia.org/wiki/Pythagorean_triangle.

[A24] Quasicrystals. http://en.wikipedia.org/wiki/Quasicrystal.

[A25] Quaternion-Kähler symmetric space. http://en.wikipedia.org/wiki/Quaternion-Khler_symmetric_space.

[A26] Quaternions. http://en.wikipedia.org/wiki/Quaternion.

[A27] Scale invariance vs. conformal invariance. http://en.wikipedia.org/wiki/Conformal_field_theory#Scale_invariance_vs._conformal_invariance.

[A28] Sierpinski space. http://en.wikipedia.org/wiki/Sierpinski_space.

[A29] Stiefel-Whitney class. http://en.wikipedia.org/wiki/StiefelWhitney_class.

[A30] Super Virasoro algebra. http://en.wikipedia.org/wiki/Super_Virasoro_algebra.

[A31] Symmetric space. http://en.wikipedia.org/wiki/Symmetric_space.

[A32] Symplectic geometry. http://en.wikipedia.org/wiki/Symplectic_geometry.

[A33] Symplectic manifold. http://en.wikipedia.org/wiki/Symplectic_manifold.

[A34] Twisted K-theory. http://en.wikipedia.org/wiki/Twisted_K-theory.

[A35] Von Neumann algebra. http://en.wikipedia.org/wiki/Von_Neumann_algebra.

[A36] Yangian symmetry. http://en.wikipedia.org/wiki/Yangian.

[A37] S. S. Abhyankar. Algebraic Geometry for Scientists and Engineers. *Mathematical Surveys and Monographs*, 35, 1980.

[A38] R. Accola. *Riemann Surfaces, Theta functions and Abelian Automorphism Groups. Lecture Notes in Mathematics 483*. Springer Verlag, 1975.

[A39] K. Appel and W.Haken. Every Planar Map is Four Colorable. *American Mathematical Society*, 98, 1989.

[A40] M. Atyiah and N. Hitschin. *The Geometry and Dynamics of Magnetic Monopoles.* Princeton University Press, 1988.

[A41] J. Baez. Higher-dimensional algebra II: 2-Hilbert spaces. http://arxiv.org/abs/q-alg/9609018, 1997.

[A42] John Baez. Quantum Quandaries. http://math.ucr.edu/home/baez/quantum/node1.html, 2007.

[A43] John C. Baez. The Octonions. *Bull. Amer. Math. Soc.*. *http://math.ucr.edu/home/baez/Octonions/octonions.html*, 39(2002), 2001.

[A44] A. L. Besse. *Einstein Manifolds*. Springer Verlag, 1987.

[A45] D. Bisch. Subfactors and Planar Algebras. http://arxiv.org/abs/math/0304340, 2003.

[A46] H. J. Borchers. On Revolutionizing QFT with Tomita's Modular Theory. *J. Math. Phys.*. *http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html*, pages 3604–3673, 2000.

[A47] Z. I. Borevich and I. R. Shafarevich. *Number Theory*. Academic Press, 1966.

[A48] Z. I. Borevich and I. R. Shafarevich. *Number Theory*. Academic Press, 1966.

[A49] O. Bratteli and D. W. Robinson. *Operator Algebras and Quantum Statistical Mechanics*. Springer Verlag, New York, 1979.

[A50] L. Brekke and P. G. O. Freund. p-Adic Numbers in Physics. *Phys. Rep.*, 233(1), 1993.

[A51] M. Brenner. Quantum octonions. *Communications in Algebra*. *http://math.usask.ca/~bremner/research/publications/qo.pdf*, 27, 1999.

[A52] V. Chari and A. Pressley. *A Guide to Quantum Groups*. Cambridge University Press, Cambridge, 1994.

[A53] M. Choda. Conjugate but non inner conjugate subfactors. *Proceedings of the American Mathematical Society*. *http://links.jstor.org/sici?sici=0002-9939(199601)124%3A1%3C147%3ACBNICS%3E2.0.CO%3B2-0*, 124(1), January 1996.

[A54] A. Connes. Une classification des facteurs de type III. *Ann. Sci. Ecole Norm. Sup.*, 6, 1973.

[A55] A. Connes. *Non-commutative Geometry*. Academic Press, San Diego, 1994.

[A56] A. Connes. Quantum Fields and Motives. http://arxiv.org/abs/hep-th/0504085, 2005.

[A57] A. Connes. Heart bit #1. http://noncommutativegeometry.blogspot.com/2007/10/heart-bit-1.html, 2007.

[A58] A. Connes and D. Kreimer. *Hofp algebras, renormalization, and non-commutative geometry*, volume 1999. Kluwer, 1998.

[A59] A. Connes and C. Rovelli. Von Neumann algebra automorphisms and time-thermodynamics relation in general covariant quantum theories. http://arxiv.org/PS_cache/gr-qc/pdf/9406/9406019v1.pdf, 1994.

[A60] A. C. da Silva. Symplectic geometry. http://www.math.princeton.edu/~acannas/symplectic.pdf, 2004.

[A61] J. Daboul and R. Delborough. Matrix Representations of Octonions and Generalizations. http://arxiv.org/abs/hep-th/9906065, 1999.

[A62] P. A. M. Dirac. A New Notation for Quantum Mechanics. *Proceedings of the Cambridge Philosophical Society*, 35:416–418, 1939.

[A63] J. Dixmier. *Von Neumann Algebras*. North-Holland, Amsterdam, 1981.

[A64] J. J. Duistermaat and G. J. Heckmann. *Inv. Math.*, 69, 1982.

[A65] Eisenhart. *Riemannian Geometry*. Princeton University Press, 1964.

[A66] J. Esmonde and M. Ram Murty. *Problems in Algebraic Number Theory*. Springer Verlag, New York, 1991.

[A67] D. Evans and Y. Kawahigashi. *Quantum symmetries on operator algebras*. Oxford University Press, New York, 1998.

[A68] P. de la Harpe F. M. Goodman and V. F. R. Jones. *Coxeter graphs and towers of algebras*. Springer Verlag, 1989.

[A69] H. M. Farkas and I. Kra. *Riemann Surfaces*. Springer Verlag, 1980.

[A70] R. P. Feynman. Space-Time Approach to Non-Relativistic Quantum Mechanics. *Reviews of Modern Physics*, 20:367–387, 1948.

[A71] D. S. Freed. The Geometry of Loop Groups, 1985.

[A72] D. Gaiotto and E. Witten. Knot Invariants from Four-Dimensional Gauge Theory. http://arxiv.org/abs/1106.4789v1, 2011.

[A73] G. W. Gibbons and C. N. Pope. CP_2 as gravitational instanton. *Comm. Math. Phys.*, 55, 1977.

[A74] P. Goddard and D. Olive. Kac-Moody and Virasoro algebras in relation to Quantum Physics, 1986.

[A75] F. Q. Gouvêa. *p-Adic Numbers: An Introduction.* Springer Verlag, 1997.

[A76] R. Haag. *Local Quantum Physics.* Springer, Berlin, 1992.

[A77] R. Haag and D. Kastler. An Algebraic Approach to Quantum Field Theory. *Journal of Mathematical Physics*, 5:848–861, 1964.

[A78] H. P. Halvorson. Locality, Localization, and the Particle Concept: Topics in the Foundations of Quantum Field Theory. http://philsci-archive.pitt.edu/archive/00000346/00/main-new.pdf, 2001.

[A79] R. Harvey. *Spinors and Calibrations.* Academic Press, New York, 1990.

[A80] S. W. Hawking and C. N. Pope. Generalized Spin Structures in Quantum Gravity. *Phys. Lett.*, (1), 1978.

[A81] S. Helgason. *Differential Geometry and Symmetric Spaces.* Academic Press, New York, 1962.

[A82] L. P. Horwitz. Hypercomplex quantum mechanics. http://arxiv.org/abs/quant-ph/9602001, 1996.

[A83] R. A. Minklos I. M. Gelfand and Z. Ya. Shapiro. *Representations of the rotation and Lorentz groups and their applications.* Pergamon Press, 1963.

[A84] F. R. Jones. *Braid groups, Hecke algebras and type II_1 factors.* 1983.

[A85] V. Jones. In and around the origin of quantum groups. http://arxiv.org/abs/math/0309199, 2003.

[A86] V. F. R. Jones. *The planar algebra of a bipartite graph*, pages 94–117. World Science Publishing, 2000.

[A87] C. Kassel. *Quantum Groups.* Springer Verlag, 1995.

[A88] A. Khrennikov. *p-Adic Valued Distributions in Mathematical Physics.* Kluwer, Dordrecht, 1994.

[A89] A. Yu. Khrennikov. p-Adic Probability and Statistics. *Dokl. Akad Nauk*, (6), 1992.

[A90] G. Moore L. Alvarez-Gaume and C. Vafa. Theta functions, Modular invariance and Strings. *Comm. Math. Phys.*, 106, 1986.

[A91] S. De Leo and K. Abdel-Khalek. Octonionic quantum mechanics and complex geometry. *Prog. Theor. Phys. http://arxiv.org/abs/hep-th/9609032*, 96:823–832, 1996.

[A92] R. Longo. Operators algebras and Index Theorems in Quantum Field Theory, 2004.

[A93] J. Mickelson. *Current Algebras and Groups.* Plenum, New York, 1989.

[A94] J. Milnor. *Topology form Differential Point of View.* The University Press of Virginia, Virginia, 1965.

[A95] D. Mumford. *Tata Lectures on Theta I,II,III.* Birkhäuser, 1983.

[A96] F. J. Murray and J. von Neumann. On Rings of Operators. *Ann. Math.*, pages 37116–229, 1936.

[A97] S. Okubo. Angular momentum, quaternion, octonion, and Lie-super algebra $Osp(1,2)$. http://arxiv.org/pdf/physics/9710038, 1997.

[A98] A. Kent P. Goddard and D. Olive. Unitary representations of the Virasoro and super-Virasoro algebras. *Comm. Math. Phys.*, 103(1), 1986.

[A99] C. N. Pope. Eigenfunctions and $Spin^c$ Structures on CP_2, 1980.

[A100] M. Rainer. Algebraic Quantum Field Theory on Manifolds: A Haag-Kastler Setting for Quantum Geometry. http://arxiv.org/abs/gr-qc/9911076, 2000.

[A101] M. Redei and M. Stöltzner (eds). *John von Neumann and the Foundations of Quantum Physics. Vol. 8, Dordrecht.* Kluwer, 2001.

[A102] R. J. Rivers. *Path Integral Methods in Quantum Field Theory.* Cambridge University Press, Cambridge, 1987.

[A103] A. Robinson. *Nonstandard Analysis.* North-Holland, Amsterdam, 1974.

[A104] D. Ruberman. Comment in discussion about unitary cobordisms. http://math.ucr.edu/home/baez/quantum/ruberman.html.

[A105] R. Haag S. Doplicher and J. E. Roberts. Local Observables and Particle Statistics I. *Ann. Math.*, 23(1974):75–119, 1971.

[A106] G. Sorkin S. Solla and S. White. *Configuration space analysis for optimization problems.* Springer Verlag, Berlin, 1986.

[A107] S. Salamon. Quaternionic Kähler manifolds. *Invent. Math.*, 67:143, 1982.

[A108] H. Saleur. Zeroes of chromatic polynomials: a new approach to the Beraha conjecture using quantum groups. *Comm. Math. Phys.*, 132, 1990.

[A109] S. Sawin. Links, Quantum Groups, and TQFT's. http://arxiv.org/abs/q-alg/9506002, 1995.

[A110] J. Schray and C. A. Manogue. Octonionic representations of Clifford algebras and triality. http://arxiv.org/abs/hep-th/9407179, 1994.

[A111] B. Schroer. Lectures on Algebraic Quantum Field Theory and Operator Algebras. http://xxx.lanl.gov/abs/math-ph/0102018, 2001.

[A112] L. Schwartz. Generalisation de la Notion de Fonction, de Derivation, de Transformation de Fourier et Applications Mathematiques et Physiques. *Publications de l'Institut de Mathematique de l'Universite de Strasbourg, Vols 9-10, Paris: Hermann*, 1945.

[A113] T. Smith. D4-D5-E6 Physics. http://galaxy.cau.edu/tsmith/d4d5e6hist.html, 1997.

[A114] M. Spivak. *Differential Geometry I,II,III,IV.* Publish or Perish, Boston, 1970.

[A115] A. Stern. *Matrix Logic.* North-Holland, 1988.

[A116] H. Sugawara. A field theory of currents. *Phys. Rev.*, 176, 1968.

[A117] B. Gilkey T. Eguchi and J. Hanson. *Phys. Rep.*, 66:1980, 1980.

[A118] M. Takesaki. *Tomita's Theory of Modular Hilbert Algebras and Its Applications*, volume 128. Springer, Berlin, 1970.

[A119] N. H. V. Temperley and E. H. Lieb. Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices:some exact results for the percolation problem. *Proc. Roy. Soc. London*, 322(1971), 1971.

[A120] R. Thom. *Commentarii Math. Helvet.*, 28, 1954.

[A121] J. von Neumann. Quantum Mechanics of Infinite Systems, 1937.

[A122] Wallace. *Differential Topology.* W. A. Benjamin, New York, 1968.

[A123] A. Wassermann. Operator algebras and conformal field theory. III. Fusion of positive energy representations of LSU(N) using bounded operators. *Invent. Math.*, 133(3), 1998.

[A124] H. Wenzl. Hecke algebra of type A_n and subfactors. *Invent. Math.*, 92, 1988.

[A125] E. Witten. Quantum field theory and the Jones polynomial. *Comm. Math. Phys.*, 121:351–399, 1989.

[A126] C. N. Yang and M. L. Ge. *Braid Group, Knot Theory, and Statistical Mechanics.* World Scientific, 1989.

[A127] J. Yngvason. The role of Type III Factors in Quantum Field Theory. http://arxiv.org/abs/math-ph/0411058, 2004.

[A128] E. C. Zeeman. *Catastrophe Theory.* Addison-Wessley Publishing Company, 1977.

Theoretical Physics

[B1] Airy functions. http://en.wikipedia.org/wiki/Airy_function.

[B2] Chern-Simons theory. http://en.wikipedia.org/wiki/ChernSimons_theory.

[B3] CPT symmetry. http://en.wikipedia.org/wiki/CPT_symmetry.

[B4] Global Scaling. http://www.dr-nawrocki.de/globalscalingengl2.html.

[B5] K-theory (physics). http://en.wikipedia.org/wiki/K-theory_(physics).

[B6] Monster group. http://en.wikipedia.org/wiki/Monster_group.

[B7] Montonen Olive Duality. http://en.wikipedia.org/wiki/Montonen-Olive_duality.

[B8] Not Even Wrong. http://www.math.columbia.edu/~woit/blog/.

[B9] Self organization. http://en.wikipedia.org/wiki/Self_organization.

[B10] Self-Organized Criticality. http://en.wikipedia.org/wiki/Self-organized_criticality.

[B11] Super symmetry. http://en.wikipedia.org/wiki/SUSY.

[B12] U. Lindström A. Karlhede and M. Rocek. Hyper Kähler Metrics and Super Symmmetry. *Comm. Math. Phys.*, 108(4), 1987.

[B13] T. Adamo. Twistor actions for gauge theory and gravity. http://arxiv.org/pdf/1308.2820.pdf, 2013.

[B14] L. Alvarez-Gaume and D. Z. Freedman. Geometrical Structure and Ultraviolet Finiteness in the Super-symmetric σ-Model. *Comm. Math. Phys.*, 80, 1981.

[B15] N. Arkani-Hamed and Y. Trnka. The Amplituhedron. http://arxiv.org/abs/1312.2007, 2013.

[B16] J. Björken and S. Drell. *Relativistic Quantum Fields.* Mc Graw-Hill, New York, 1965.

[B17] R. Boels. On BCFW shifts of integrands and integrals. http://arxiv.org/abs/1008.3101, 2010.

[B18] O. I Bogoyavlenskij. Exact unsteady solutions to the Navier-Stokes equations and viscous MHD equations. *Phys. Lett. A*, pages 281–286, 2003.

[B19] H. J. Borchers. On Revolutionizing QFT with Tomita's Modular Theory. *J. Math. Phys.. http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html*, 41:3604–3673, 2000.

[B20] M. M. Bowick and S. G. Rajeev. The holomorphic geometry of closed bosonic string theory and $Diff(S^1)/S^1$. *Nucl. Phys. B*, 293, 1987.

[B21] D. Chowdbury. *Spin Glasses and other Frustrated Systems.* World Scientific, 1986.

[B22] R. E. Cutkosky. *J. Math. Phys.*, (1):429–433, 1960.

[B23] G. Moore D.-E. Diaconescu and E. Witten. E8 Gauge Theory, and a Derivation of K-Theory from M-Theory). http://arXiv.org/abs/hep-th/0005090v3, 2005.

[B24] S. de Haro Olle. Quantum Gravity and the Holographic Principle. http://arxiv.org/abs/hep-th/0107032, 2001.

[B25] N. Arkani-Hamed et al. A duality for the S-matrix. http://arxiv.org/abs/0907.5418, 2009.

[B26] Nima Arkani-Hamed et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM. http://arxiv.org/find/hep-th/1/au:+Bourjaily_J/0/1/0/all/0/1, 2010.

[B27] S. Ansoldi et al. p-Branes ElectricMagnetic Duality and Stueckelberg/Higgs Mechanism: a PathIntegral Approach. http://arxiv.org/abs/hep-th/0004044v2, 2000.

[B28] T. Banks et al. M-Theory as a Matrix Model: A Conjecture. *Phys. Rev. D*, 55:5112–5128, 1997.

[B29] J. Etnyre and G. Ghrist. Generic hydrodynamic instability of curl eigen fields. http://arxiv.org/abs/math/0306310, 2003.

[B30] H. Evslin. What doesn't K-theory classify? http://arxiv.org/abs/hep-th/0610328, 2006.

[B31] P. Svrcek F. Cachazo and E. Witten. MHV Vertices and Tree Amplitudes In Gauge Theory. http://arxiv.org/abs/hep-th/0403047, 2004.

[B32] D. S. Freed and E. Witten. Anomalies in string theory with D-branes. http://arXiv.org/abs/hep-th/9907189v2, 1999.

[B33] M. B. Green and J. H. Schwarz. *Nucl. Phys. B*, 117(1985):54, 1984.

[B34] R. Haag and D. Kastler. An Algebraic Approach to Quantum Field Theory. *Journal of Mathematical Physics*, 5:, 1964.

[B35] H. Haken. *Information and Self-Organization.* Springer Verlag, Berlin, 1988.

[B36] L.P. Horwitz. Hypercomplex quantum mechanics. http://arxiv.org/abs/quant-ph/9602001, 1996.

[B37] S. Hossenfelder. Comments on and comments on comments on verlindes paper on the origin of gravity and the laws of newton. http://www.scribd.com/doc/34724732/Verlinde-Gravity-From-Entropy-Explained-by-Sabine-Hossenfelder, 2011.

[B38] K. Huang. *Quarks,Leptons & Gauge Fields.* World Scientific, 1982.

[B39] Y. Ito and I. Nakamura. Hilbert schemes and simple singularities. *Proc. of EuroConference on Algebraic Geometry, Warwick. http://www.math.sci.hokudai.ac.jp/~nakamura/ADEHilb.pdf*, pages 151–233, 1996.

[B40] J. Henn J. Drummond and J. Plefka. Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory. http://cdsweb.cern.ch/record/1162372/files/jhep052009046.pdf, 2009.

[B41] G. Moore J. Maldacena and N. Seiberg. D-Brane Instantons and K-Theory Charges). http://arxiv.org/abs/hep-th/010810, 2001.

[B42] P. Ginsparg L. Dixon and J. Harvey. Beauty and the Beast; Superconformal Symmetry in a Monster Module. http://ccdb4fs.kek.jp/cgi-bin/img_index?8806247, 1988.

[B43] C. R. Nappi L. Dolan and E. Witten. Yangian Symmetry in $D = 4$ superconformal Yang-Mills theory. http://arxiv.org/abs/hep-th/0401243, 2004.

[B44] A. Lakthakia. *Beltrami Fields in Chiral Media*, volume 2. World Scientific, Singapore, 1994.

[B45] J. H. Schartz M. B. Green and E. Witten. Cambridge University Press, Cambridge, 1987.

[B46] J. H. Schwartz M. B. Green and E. Witten. *Superstring Theory.* Cambridge University Press, Cambridge, 1987.

[B47] H. Larsen M. Freedman and Z. Wang. A modular functor which is universal for quantum computation. *Comm. Math. Phys.. http://arxiv.org/abs/quant-ph/0001108*, 1(2):605–622, 2002.

[B48] J. T. Liu M. J. Duff and R. Minasian. Eleven-Dimensional Origin of String/String Duality: A One-Loop Test. *Nucl. Phys. B*, 452:261, 1995.

[B49] J. M. Maldacena. The Large N Limit of Superconformal Field Theories and Supergravity. http://arxiv.org/abs/hep-th/9711200, 1997.

[B50] G. E. Marsh. *Helicity and Electromagnetic Field Topology.* World Scientific, 1995.

[B51] L. Motl. Monstrous symmetry of black holes. http://motls.blogspot.com/2007/05/monstrous-symmetry-of-black-holes.html, 2007.

[B52] N. Ohta. Introduction to branes and M-theory for relativists and cosmologists. http://arxiv.org/abs/gr-qc/0205036, 2002.

[B53] B. R. Greene P. S. Aspinwall and D. R. Morrison. Calabi-Yau Moduli Space, Mirror Manifolds, and Space-time Topology Change in String Theory. http://arxiv.org/abs/hep-th/9309097, 1993.

[B54] G. Parisi. *Field Theory, Disorder and Simulations.* World Scientific, 1992.

[B55] S. Parke and T. Taylor. An Amplitude for N gluon Scattering. *Phys. Rev.*, 56, 1986.

[B56] R. Penrose. The Central Programme of Twistor Theory. *Chaos, Solitons and Fractals*, 10, 1999.

[B57] R. Penrose. Fantasy, Fashion, and Faith in Theoretical Physics. http://www.princeton.edu/WebMedia/lectures/, 2004.

[B58] R. Penrose. Strings with a twist. *New Scientist*, 183(2458), 2004.

[B59] J. Polchinski. Dirichlet-Branes and Ramond-Ramond Charges. http://arxiv.org/abs/hep-th/9510017, 1995.

[B60] B. Feng R. Britto, F. Cachazo and E. Witten. Direct Proof of Tree-Level Recursion Relation in Yang- Mills Theory. *Phys. Rev.. http://arxiv.org/abs/hep-th/0501052*, 94:181602, 2005.

[B61] P. Ramond. *Nucl. Phys. B*, 3(1971):31, 1971.

[B62] J. Scherk and J. H. Schwarz. *Nucl. Phys. B*, 81, 1974.

[B63] B. Schroer. Lectures on Algebraic Quantum Field Theory and Operator Algebras. http://arxiv.org/abs/math-ph/0102018, 2001.

[B64] J. H. Schwartz. *Super strings. The first 15 years of Superstring Theory.* World Scientific, 1985.

[B65] A. Sen. Tachyon Condensation on the Brane Antibrane system). http://arxiv.org/abs/hep-th/9805170, 1998.

[B66] L. Smolin. Scientific alternatives to the anthropic principle. http://arxiv.org/abs/hep-th/0407213, 2004.

[B67] A. Strominger and C. Vafa. *Nucl. Phys. B. http://arxiv.org/abs/hep-th/9601029*, 99, 1996.

[B68] L. Susskind. The Anthropic Landscape of String Theory. http://arxiv.org/abs/hep-th/0302219, 2003.

[B69] P. Townsend. p-Brane democracy). http://xxx.lanl.gov/abs/hep-th/9507048, 1995.

[B70] A. Verlinde. On the Origin of Gravity and the Laws of Newton. http://arxiv.org/abs/1001.0785, 2010.

[B71] E. Verlinde. Global Aspects of Electric-Magnetic Duality). http://arxiv.org/abs/hep-th/9506011v3, 1995.

[B72] E. Witten. Coadjoint orbits of the Virasoro Group. PUPT-1061 preprint, 1987.

[B73] E. Witten. Perturbative Gauge Theory As a String Theory In Twistor Space. http://arxiv.org/abs/hep-th/0312171, 2003.

[B74] P. Woit. String Theory: Evaluation. http://arxiv.org/abs/hep-th/0102051, 2001.

[B75] A. Zee. *The Unity of Forces in the Universe*. World Science Press, Singapore, 1982.

Particle and Nuclear Physics

[C1] Are Centauros Exotic Signals of Quark-Gluon Plasma. http://www1.jinr.ru/Archive/Pepan/v-34-3/v-34-3-3.pdf.

[C2] CMS observes a potentially new and interesting effect. http://user.web.cern.ch/user/news/2010/100921.html.

[C3] Lamb shift. http://en.wikipedia.org/wiki/Lamb_shift.

[C4] Minos for Scientists. http://www-numi.fnal.gov/PublicInfo/forscientists.html.

[C5] Neutrino oscillation. http://en.wikipedia.org/wiki/Neutrino_oscillation.

[C6] Quark. http://en.wikipedia.org/wiki/Current_quark_mass.

[C7] CMS SUSY group working hard. http://motls.blogspot.com/2010/10/cms-susy-group-working-hard.html, 2010.

[C8] LHC probably sees new shocking physics. http://motls.blogspot.com/2010/09/lhc-probably-sees-new-shocking-physics.html, 2010.

[C9] A. S. Antognini. The Lamb shift Experiment in Muonic Hydrogen. http://edoc.ub.uni-muenchen.de/5044/1/Antognini_Aldo.pdf, 2005.

[C10] S. Barshay. *Mod. Phys. Lett. A*, 7(20):1843, 1992.

[C11] J. D. Bjorken. *Acta Phys. Polonica B*, 28:2773, 1997.

[C12] J. M. Bonnet-Bidaud and G. Chardin. Cygnus X-3. a critical review. *Phys. Rep.*, (6), 1988.

[C13] M. Chown. Quantum Rebel. *New Scientist*, 2004.

[C14] Alice Collaboration. Charged-particle multiplicity density at mid-rapidity in central Pb-Pb collisions at $\sqrt{s_{NN}}$= 2.76 TeV. http://arxiv.org/abs/1011.3916, 2010.

[C15] CDF Collaboration. Study of multi-muon events produced in p-pbar collisions at sqrt(s)=1.96 TeV. http://arxiv.org/PS_cache/arxiv/pdf/0810/0810.0714v1.pdf, 2008.

[C16] CDF Collaboration. Searches for the Higgs Boson. http://blois.in2p3.fr/2011/transparencies/punzi.pdf, 2011.

[C17] CMS Collaboration. Observation of Long-Range, Near-Side Angular Correlations in Proton-Proton Collisions at the LHC. http://cms.web.cern.ch/cms/News/2010/QCD-10-002/QCD-10-002.pdf, 2010.

[C18] LSND Collaboration. Evidence for $\nu_\mu - \nu_e$ oscillations from LSND. http://arxiv.org/absnucl-ex/9709006, 1997.

[C19] MiniBooNe collaboration. A Search for Electron Neutrino in $\Delta m^2 = 1$ eV^2 scale. http://www-boone.fnal.gov/publicpages/prl8.pdf. See also the press release at http://www.fnal.gov/pub/presspass/press_releases/BooNE-box.html., 2007.

[C20] Pamir Collaboration. In *Proc. 16:th Intern. Cosmic Ray Conf.*, volume 7, page 279, 1979.

[C21] A. E. Nelson D. B. Kaplan and N. Weiner. Neutrino Oscillations as a Probe of Dark Energy. http://arxiv.org/abs/hep-ph/0401099, 2004.

[C22] R. Van de Water. Updated Anti-neutrino Oscillation Results from MiniBooNE. http://indico.cern.ch/getFile.py/access?contribId=208&sessionId=3&resId=0&materialId=slides&confId=73981, 2010.

[C23] A. D. Dolgov and I. Z. Rothstein. *Phys. Rev. Lett.*, 71(4), 1993.

[C24] T. Dorigo. Rumsfeld hadrons. http://dorigo.wordpress.com/2007/06/20/rumsfeld-hadrons/, 2007.

[C25] T. Dorigo. Top quark mass measured with neutrino phi weighting. http://dorigo.wordpress.com/2008/12/08/top-quark-mass-measured-with-neutrino-phi-weighting/, 2008.

[C26] U. Egede. A theoretical limit on Higgs mass. http://www.hep.lu.se/atlas//thesis/egede/thesis-node20.html, 1998.

[C27] A.T. Goshaw et al. *Phys. Rev.*, 43, 1979.

[C28] B. B. Back et al. *Phys. Rev. Lett.*. *http://arxiv.org/abs/hep-ex/0208004*, 89(22), November 2002.

[C29] B. R. Barber et al. *Phys. Rev.*, 72(9):1380, 1994.

[C30] C. Athanassopoulos et al. Evidence for Neutrino Oscillations from Muon Decay at Rest. http://arxiv.org/abs/nucl-ex/9605001, 1996.

[C31] C. I. Westbrook et al. *Phys. Rev.*, 58:1328, 1987.

[C32] D. T. H. Davies et al. Precise Charm to Strange Mass Ratio and Light Quark Masses from Full Lattice QCD. *Phys. Rev.*. *http://prl.aps.org/abstract/PRL/v104/i13/e132003*, 104, 2010.

[C33] Decamp et al. The number of neutrino species. Aleph Collaboration,CERN-EP/89-141, 1989.

[C34] J. A. Chinellato et al. In *Proton-Antiproton Collider Physics, 1981*, New York, 1981. Madison, Wis.

[C35] Kh. U. Abraamyan et al. Observation of E(38)boson. http://arxiv.org/pdf/1208.3829.pdf, 2012.

[C36] L. Borodovsky et al. *Phys. Rev. Lett.*, 68:274, 1992.

[C37] M. Derrick et al. *Phys. Lett B*, 315:481, 1993.

[C38] P.V. Chliapnikov et al. *Phys. Lett. B*, 141, 1984.

[C39] R. Pohl et al. The size of proton. *Nature*. *http://www.nature.com/nature/journal/v466/n7303/full/nature09250.html*, 466, 2010.

[C40] S. Ambrosanio et al. Supersymmetric analysis and predictions based on the CDF $ee\gamma\gamma + \not{E}_T$ event. http://arxiv.org/abs/hep-ph/9602239, 1996.

[C41] SNO: Q. R Ahmad et al. *Phys. Rev. Lett.*. *http://arxiv.org/abs/nucl-ex/0204008*, 89:11301, 2002.

[C42] T. Akesson et al. *Phys. Lett. B*, 463:36, 1987.

[C43] T. K. Gaiser et al. Cosmic ray composition around 10^{18} eV . *Phys. Rev. D*, (5), 1993.

[C44] X. Chi et al. Cosmic rays and cosmic strings. *Gamma*, 1:129–131, 1993.

[C45] Y. Fukuda et al. *Phys. Lett. B*, 335:237, 1994.

[C46] K. Eguchi et al (KamLAND). *Phys. Rev. Lett.*. *http://arxiv.org/abs/hep-ex/0212021*, 90:21802, 2003.

[C47] G. R. Farrar. New Signatures of Squarks. http://arxiv.org/abs/hep-ph/9512306, 1995.

[C48] J. Flowers. Quantum electrodynamics: A chink in the armour? *Nature. http://www.nature.com/nature/journal/v466/n7303/pdf/466195a.pdf*, 466, 2010.

[C49] C. M. G. Lattes. Y. Fujimoto and S. Hasegava. *Phys. Rep.*, 65(3), 1980.

[C50] K. Greisen. *Phys. Rev.*, 16, 1966.

[C51] R. J. Wilkes (K2K). http://arxiv.org/abs/hep-ex/0212035, 2002.

[C52] G. Karagiorgi. Towards Solution of MiniBoone-LSND anomalies. http://indico.cern.ch/contributionDisplay.py?contribId=209&sessionId=3&confId=73981, 2010.

[C53] B. Armbruster et al KARMEN Collaboration. *Phys. Lett. B*, 348, 1995.

[C54] C. L. Kervran. *Biological transmutations, and their applications in chemistry, physics, biology, ecology, medicine, nutrition, agriculture, geology.* Swan House Publishing Co., 1972.

[C55] J. Linsley and A. A. Watson. *Phys. Rev.*, 436, 1981.

[C56] W. C. Louis. In *Proceedings of the XVI Conference on Neutrino Physics and Astrophysics. Eilat, Israel*, 1994.

[C57] T. Ludham and L. McLerran. What Have We Learned From the Relativistic Heavy Ion Collider? *Physics Today. http://www.physicstoday.org/vol-56/iss-10/p48.html*, October 2003.

[C58] J. H. MacGibbon and R. H. Brandenberger. Gamma-ray signatures for ordinary cosmic strings. *Phys. Rev. D*, (6):2883, 1993.

[C59] K. McAlpine. Incredible shrinking proton raises eyebrows. http://www.newscientist.com/article/dn19141-incredible-shrinking-proton-raises-eyebrows.html, 2010.

[C60] W. A. Rodriguez Jr. P. Ammiraju, E. Recami. Chirons, Geminions, Centauros, Decays into Pions:a Phenomenological and Theoretical Analysis. *Il Nuovo Cimento*, 78(2):173, 1983.

[C61] P. Sommers P. Sokolsky and B. R. Dawson. Extremely High Energy Cosmic Rays. *Phys. Rep.*, 217(5), 1992.

[C62] S. Park. Search for New Phenomena in CDF-I: Z, W, and leptoquarks. *Proceedings for the 10th Topical Workshop on Proton-Antiproton Collider Physics. Fermilab, Batavia, Illinois, 1995. Fermilab-Conf-95/155-E. http://lss.fnal.gov/archive/1995/conf/Conf-95-155-E.pdf*, 1995.

[C63] Q. Z. Qian and G. M. Fuller. *Phys. Rev. D*, 51:1479, 1995.

[C64] E. S. Reich. Black hole like phenomenon created by collider. *New Scientist*, 19(2491), 2005.

[C65] E. Samuel. Ghost in the Atom. *New Scientist*, (2366):30, October 2002.

[C66] Nimrod Shaham and Tsvi Piran. The UHECRs Composition Problem: Evidence for a New Physics at 100 TeV? http://arxiv.org/abs/1204.1488, 2012.

[C67] M. B. Smy (Super-Kamiokande). *Nucl. Phys. Proc. Suppl.. http://arxiv.org/abs/hep-ex/0208004*, 118:25, 2003.

[C68] B. Taticheff and E. Tomasi-Gustafsson. Search for low-mass exotic mesonic structures: II. Attempts to understand the experimental results. *Part. Nucl. Lett.*, 5(5):709–713, 2008.

[C69] P. Tompkins and C. Bird. *The secret life of plants.* Harper & Row, New York, 1973.

[C70] R. J. N. Phillips V. Barger and S. Sarkar. *Phys. Lett. B*, 352:365–371, 1995.

[C71] E. van Beveren and G. Rupp. First indications of the existence of a 38 MeV light scalar boson. http://arxiv.org/pdf/1102.1863v2.pdf, 2011.

[C72] E. van Beveren and G. Rupp. Material evidence of a 38 MeV boson. http://arxiv.org/pdf/1202.1739.pdf, 2011.

[C73] H. Waschmuth. Results from e^+e^- collisions at 130, 136 and 140 GeV center of mass energies in the ALEPH Experiment. http://alephwww.cern.ch/ALPUB/pub/pub_96.html, 1996.

[C74] J. Wdowczyk. In *Proc. 9th Int. Conf. Cosmic Rays*, volume 2, page 691, 1965.

[C75] P. Woit. A new long-lived particle by CDF experiment. http://www.math.columbia.edu/~woit/wordpress/?p=1045, 2008.

[C76] J. Tandean X.-G. He and G. Valencia. Has HyperCP Observed a Light Higgs Boson? *Phys. Rev. D. http://arxiv.org/abs/hep-ph/0610274*, 74, 2007.

Condensed Matter Physics

[D1] Burning salt water. http://www.youtube.com/watch?v=aGg0ATfoBgo.

[D2] Fractional quantum Hall Effect. http://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect.

[D3] Majorana fermion. http://en.wikipedia.org/wiki/Majorana_fermion.

[D4] Observation of the fractional ac Josephson effect: the signature of Majorana particles. http://arxiv.org/abs/1204.4212.

[D5] Phase conjugation. http://www.usc.edu/dept/ee/People/Faculty/feinberg.html.

[D6] Viscosity. http://en.wikipedia.org/wiki/Viscosity.

[D7] Ferromagnetism. http://www.tcd.ie/Physics/Magnetism/Lectures/5006/5006-5.pdf, 2007.

[D8] J. K. Borchardt. The chemical formula H2O - a misnomer. *The Alchemist*, August 2003.

[D9] M. Chaplin. Water Structure and Behavior. http://www.lsbu.ac.uk/water/index.html, 2005.

[D10] M. Chaplin. Water as a Network of Icosahedral Water Clusters. http://www.lsbu.ac.uk/water/clusters.html, 2006.

[D11] M. Chown. Quantum Rebel. *New Scientist*, (2457), 2004.

[D12] R. A. Cowley. Neutron-scattering experiments and quantum entanglement. *Physica B*, 350:243–245, 2004.

[D13] B. R. Barber et al. *Phys. Rev.*, 72(9):1380, 1994.

[D14] C. Cao et al. Universal Quantum Viscosity in a Unitary Fermi Gas. http://www.sciencemag.org/content/early/2010/12/08/science.1195219, 2010.

[D15] D. J. P. Morris et al. Dirac Strings and Magnetic Monopoles in Spin Ice Dy2Ti2O7. *Physics World*, 326(5951):411–414, 2009.

[D16] J. B. Miller et al. Fractional Quantum Hall effect in a quantum point contact at filling fraction 5/2. http://arxiv.org/abs/cond-mat/0703161v2, 2007.

[D17] K. S. Novoselov et al. Two-dimensional gas of massless Dirac fermions in graphene. *Nature*, 438, November 2005.

[D18] P. K. Kotvun et al. Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics. http://arxiv.org/abs/hep-th/0405231, 2010.

[D19] R. Mills et al. Spectroscopic and NMR identification of novel hybrid ions in fractional quantum energy states formed by an exothermic reaction of atomic hydrogen with certain catalysts. http://www.blacklightpower.com/techpapers.html, 2003.

[D20] R. Moreh et al. Search for anomalous scattering of keV neutrons from H2O-D2O mixtures. *Phys. Rev.*, 94, 2005.

[D21] S. L. Glashow. Can Science Save the World? http://www.hypothesis.it/nobel/nobel99/eng/pro/pro_2.htm, 1999.

[D22] J. K. Jain. *Phys. Rev.*, 63, 1989.

[D23] P. Kanarev and T. Mizuno. Cold fusion by plasma electrolysis of water. http://www.guns.connect.fi/innoplaza/energy/story/Kanarev/codlfusion/, 2002.

[D24] R. B. Laughlin. *Phys. Rev.*, 50, 1990.

[D25] R. Mackenzie and F. Wilczek. *Rev. Mod. Phys. A*, 3:2827, 1988.

[D26] D. Monroe. Know Your Anyons. *New Scientist*, (2676), 2008.

[D27] D. M. Pepper. Nonlinear Optical Phase Conjugation. *Optical Engineering*, 21(2), March 1982.

[D28] S. A. Kivelson V. J. Emery and J. M. Tranquada. Stripe phases in high-temperature superconductors. *Perspective. http://www.pnas.org/cgi/reprint/96/16/8814.pdf*, 96(16), August 1999.

[D29] J. Zaanen. Superconductivity: Quantum Stripe Search. *Nature. http://www.lorentz.leidenuniv.nl/~jan/nature03/qustripes06.pdf*, April 2006.

Cosmology and Astro-Physics

[E1] Allais effect. http://en.wikipedia.org/wiki/Allais_effect.

[E2] Cosmic Microwave Background. http://en.wikipedia.org/wiki/Microwave_background.

[E3] Dark flow. http://en.wikipedia.org/wiki/Dark_flow.

[E4] Gamma ray burst. http://en.wikipedia.org/wiki/Gamma-ray_burst.

[E5] Λ-CDM model. http://en.wikipedia.org/wiki/Lambda-CDM_model.

[E6] Mars. http://en.wikipedia.org/wiki/Mars.

[E7] Quintessence. http://en.wikipedia.org/wiki/Quintessence.

[E8] Relativistic jet. http://en.wikipedia.org/wiki/http://en.wikipedia.org/wiki/Relativistic_jet.

[E9] Solar data. http://hyperphysics.phy-astr.gsu.edu/hbase/solar/soldata2.html.

[E10] Dark energy. *Physics World. http://physicsworld.com/cws/article/print/19419*, May 2004.

[E11] M. Allais. Should the Laws of Gravitation Be Reconsidered: Part I,II,III? http://home.t01.itscom.net/allais/blackprior/allais/lawgravit/lawgrav-one.pdf, 1959.

[E12] F. R. Bouchet and D. P. Bennet. Preprint PUPT-89-1128, 1989.

[E13] C. Charbonnel and F. Primas. The lithium content of the Galactic Halo stars. http://arxiv.org/abs/astro-ph/0505247, 2005.

[E14] The Pierre Auger Collaboration. Correlation of Highest Energy Cosmic Rays with Nearby Galactic Objects. *Science*, 9, November 2007.

[E15] A. G. Riess et al. Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. http://arxiv.org/abs/astro-ph/0402512, 2004.

[E16] A. G. Riess et al. Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. http://arxiv.org/abs/astro-ph/0402512, 2004.

[E17] M. Tajmar et al. Experimental Detection of Gravimagnetic London Moment. http://arxiv.org/abs/gr-gc0603033, 2006.

[E18] S. Perlmutter et al. *Ap. J.*, 483, 1997.

[E19] D. A. Frail. Gamma-Ray Bursts: Jets and Energetics. http://arxiv.org/abs/astro-ph/0311301, 2003.

[E20] G. I. Rusu G. T. Jeverdan and V. Antonescu. Experiments using the Foucault pendulum during the solar eclipse of 15 February, 1961. *Biblical Astronomer*, 1(55), 1981.

[E21] M. R. S. Hawkins. On time dilation in quasar light curves. *Monthly Notices of the Royal Astronomical Society. http://www3.interscience.wiley.com/journal/123345710/abstract?CRETRY=1&SRETRY=0*, 2010.

[E22] T. W. B. Kibble. *Nucl. Phys. B*, 252, 1985.

[E23] T. W. B. Kibble. Cosmic strings reborn? http://arxiv.org/abs/astro-ph/0410073, 2004.

[E24] M. Milgrom. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. http://www.astro.umd.edu/~ssm/mond/astronow.html, 1983.

[E25] V. A. Popescu and D. Olenici. A confirmation of the Allais and Jeverdan-Rusu-Antonescu effects during the solar eclipse from 22 September 2006, and the quantization behavior of pendulum. http://www.hessdalen.org/sse/program/Articol.pdf, 2007.

[E26] A. G. Riess. *PASP*, 112, 2000.

[E27] D. Da Roacha and L. Nottale. Gravitational Structure Formation in Scale Relativity. http://arxiv.org/abs/astro-ph/0310036, 2003.

[E28] A. Rubric and J. Rubric. The Quantization of the Solar-like Gravitational Systems. *Fizika B*, 7:1–13, 1998.

[E29] W. C. Saslaw. *Gravitational Physics of Stellar and Galactic Systems.* Cambridge University Press, Cambridge, 1985.

[E30] B. Schaefer. The hubble diagram to z = 6.3 with swift gamma ray bursts. http://www.phys.lsu.edu/GRBHD/, January 2006.

[E31] M. Tajmar and C. J. de Matos. Local Photon and Graviton Mass and Its Consequences. http://arxiv.org/abs/gr-gc0603032, 2006.

[E32] W. G. Tifft. Discrete States Of Redshift And Galaxy Dynamics I. *Astrophysical Journal*, 206:38–56, May 1976.

[E33] A. Vilenkin. *Cosmic Strings and Domain Walls*, volume 121. North-Holland, Amsterdam, 1985.

[E34] Y. Wang and M. Tegmark. New Light on Dark Energy. *Physics Web. http://physicsweb.org/articles/news/8/6/14*, 2005.

[E35] S. Weinberg. *Gravitation and Cosmology.* Wiley, New York, 1967.

[E36] M. A. Jimenez X. Hernandez and C. Allen. The Breakdown of Classical Gravity? http://arxiv.org/abs/1105.1873, 2011.

[E37] J. Einasto Ya. V. Zeldovich and S. F. Shandarin. Giant Voids in the Universe. *Nature*, 300, 1982.

Physics of Earth

[F1] A challenge to all geologists of Earth. http://www.nealadams.com/challenge.html.

[F2] Expanding Earth Theory. http://en.wikipedia.org/wiki/Expanding_earth_theory.

[F3] Oceanic trench. http://en.wikipedia.org/wiki/Oceanic_trench.

[F4] Orogenies. http://en.wikipedia.org/wiki/Orogenies.

[F5] Plate tectonics. http://en.wikipedia.org/wiki/Plate_tectonics.

[F6] Volcano. http://en.wikipedia.org/wiki/Volcano.

[F7] Quakes reveal 'core within a core'. *Nature. http://news.bbc.co.uk/1/hi/sci/tech/2290551.stm*, October 2002.

[F8] N. Adams. Conspiracy of Science, Earth is in fact growing. http://www.youtube.com/watch?v=VjgidAICoQI, 2006.

Fringe Physics

[H1] V. V. Roshchin and S. M. Godin. An Experimental Investigation of the Physical Effects in a Dynamic Magnetic System. *New Energy Technologies*, 1, 2001.

[H2] J. Searl. *The Searl effect generator and the levity disc.* Jupiter Verlag, 2001.

Biology

[I1] Brief introduction into WaveGenetics. Its scope and opportunities. http://www.wavegenetics.jino-net.ru.

[I2] Dinosaurs. http://en.wikipedia.org/wiki/Dinosaur.

[I3] High energy phosphate. http://en.wikipedia.org/wiki/High-energy_phosphate.

[I4] S. Comorosan. On a possible biological spectroscopy. *Bull. of Math. Biol.*, page 419, 1975.

[I5] F. A. Popp et al. Emission of Visible and Ultraviolet Radiation by Active Biological Systems. *Collective Phenomena*, 3, 1981.

[I6] L. Montagnier et al. Electromagnetic Signals Are Produced by Aqueous Nanostructures Derived from Bacterial DNA Sequences. *Interdiscip. Sci. Comput. Life Sci.*. *http://www.springerlink.com/content/0557v31188m3766x/*, 2009.

[I7] P. Gariaev et al. *The DNA-wave biocomputer*, volume 10. CHAOS, 2001.

[I8] P. P. Gariaev et al. The spectroscopy of bio-photons in non-local genetic regulation. *Journal of Non-Locality and Remote Mental Interactions*. *http://www.emergentmind.org/gariaevI3.htm*, (3), 2002.

[I9] S. J. Gould. *Wonderful Life*. Penguin Books, 1991.

[I10] M. W. Ho. *The Rainbow and the Worm*. World Scientific, Singapore, 1993.

[I11] E. Lozneanu and M. Sanduloviciu. Minimal-cell system created in laboratory by self-organization. *Chaos, Solitons & Fractals*, 18(2):335, September 2003.

[I12] J. O′Donoghue. How trees changed the world? *New Scientist*. *http://www.newscientist.com/channel/life/mg19626311.500-how-trees-changed-the-world.html*, 2631, November 2007.

[I13] G. G. Tertishni P. P. Gariaev and A.V. Tovmash. Experimental investigation in vitro of holographic mapping and holographic transposition of DNA in conjuction with the information pool encircling DNA. *New Medical Tehcnologies*, 9:42–53, 2007.

[I14] M. Hristea S. Comorosan and P. Murogoki. On a new symmetry in biological systems. *Bull. of Math. Biol.*, page 107, 1980.

[I15] M. Poschmann S. J. Braddy and O. E. Tetlie. Giant claw reveals the largest ever arthropod. *Biology Letters*. *http://www.journals.royalsoc.ac.uk/content/t15r2588mn27n0w1*, November 2007.

[I16] R. Sheldrake. *A New Science of Life: The Hypothesis of Formative Causation*. Inner Traditions Intl Ltd., 1995.

Neuroscience and Consciousness

[J1] G. Adenier and A. Khrennikov. Is the Fair Sampling Assumption Supported by EPR Experiments? http://arxiv.org/abs/quantum-ph/0606122, 2006.

[J2] D. J. Benor. *Spiritual Healing: scientific validation of a healing revolution Vol. I.* Vision publications, Southfield MI, 2001.

[J3] C. F. Blackman. *Effect of Electrical and Magnetic Fields on the Nervous System*, pages 331–355. Plenum, New York, 1994.

[J4] S. J. Blackmore. Near death experiences: in or out of the body? *Skeptical Inquirer**http://www.uwe.ac.uk/fas/staff/sb/si91nde.html*, 1991:34–45, 1991.

[J5] M. Chown. Quantum Rebel. *New Scientist*, 2004.

[J6] B. Libet et al. Subjective referral of the timing for a conscious sensory experience. *Brain*, 102, 1979.

[J7] G. Weihs et al. *Phys. Rev.*, 81:5039, 1998.

[J8] H. Abdelmeik et al. Changes in electrical properties of frog sciatic nerves at low temperature: superconductor-like behaviour. http://tgdtheory.com/public_html/articles/hafpaper.pdf, 2003.

[J9] M. Germine. Scientific Validation of Planetary Consciousness. *Journal of Non-Locality and Remote Mental Interactions. http://www.emergentmind.org/germineI3.htm*, 1(3), 2002.

[J10] A. Khrennikov. Bell's inequality for conditional probabilities as a test for quantum like behaviour of mind. http://arxiv.org/abs/quant-ph/0402169, 2004.

[J11] S. Klein. Libet's Research on Timing of Conscious Intention to Act: A Commentary. *Consciousness and Cognition. http://cornea.berkeley.edu/pubs/ccog_2002_0580-Klein-Commentary.pdf*, 11, 2002.

[J12] S. Klein. Libet's Research on Timing of Conscious Intention to Act: A Commentary. *Consciousness and Cognition. http://cornea.berkeley.edu/pubs/ccog_2002_0580-Klein-Commentary.pdf*, 11, 2002.

Books related to TGD

[K1] M. Pitkänen. 1983.

[K2] M. Pitkänen. Cosmic Strings. In *Topological Geometrodynamics*. Onlinebook. http://tgdtheory.fi/public_html/tgd.html#cstrings, 2000.

[K3] M. Pitkänen. Luniver Press, 2006.

[K4] M. Pitkänen. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Towards M-matrix*. Onlinebook. http://tgdtheory.fi/public_html/pdfpool/append.pdf, 2006.

[K5] M. Pitkänen. Is it Possible to Understand Coupling Constant Evolution at Space-Time Level? In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#rgflow, 2006.

[K6] M. Pitkänen. About Nature of Time. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#timenature, 2006.

[K7] M. Pitkänen. About the New Physics Behind Qualia. In *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html#newphys, 2006.

[K8] M. Pitkänen. Anomalies Related to the Classical Z^0 Force and Gravitation. In *TGD and Fringe Physics*. Onlinebook. http://tgdtheory.fi/public_html/freenergy/freenergy.html#Zanom, 2006.

[K9] M. Pitkänen. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#class, 2006.

[K10] M. Pitkänen. *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html, 2006.

[K11] M. Pitkänen. Bio-Systems as Conscious Holograms. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#hologram, 2006.

[K12] M. Pitkänen. *Bio-Systems as Self-Organizing Quantum Systems*. Onlinebook. http://tgdtheory.fi/public_html/bioselforg/bioselforg.html, 2006.

[K13] M. Pitkänen. Bio-Systems as Super-Conductors: part I. In *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html#superc1, 2006.

[K14] M. Pitkänen. Bio-Systems as Super-Conductors: part II. In *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html#superc2, 2006.

[K15] M. Pitkänen. Category Theory and Quantum TGD. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#categorynew, 2006.

[K16] M. Pitkänen. Category Theory, Quantum TGD, and TGD inspired theory of consciousness. In *Mathematical Aspect of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/mathconsc/mathconsc.html#categoryc, 2006.

[K17] M. Pitkänen. Configuration Space Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#cspin, 2006.

[K18] M. Pitkänen. Construction of Configuration Space Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#compl1, 2006.

[K19] M. Pitkänen. Construction of elementary particle vacuum functionals. In *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#elvafu, 2006.

[K20] M. Pitkänen. Construction of Quantum Theory: M-matrix. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#towards, 2006.

[K21] M. Pitkänen. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#quthe, 2006.

[K22] M. Pitkänen. Cosmic Strings. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cstrings, 2006.

[K23] M. Pitkänen. Dark Forces and Living Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#darkforces, 2006.

[K24] M. Pitkänen. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#eegdark, 2006.

[K25] M. Pitkänen. Dark Nuclear Physics and Condensed Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#exonuclear, 2006.

[K26] M. Pitkänen. DNA as Topological Quantum Computer. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#dnatqc, 2006.

[K27] M. Pitkänen. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#Planck, 2006.

[K28] M. Pitkänen. Does the Modified Dirac Equation Define the Fundamental Action Principle? In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#Dirac, 2006.

[K29] M. Pitkänen. Does the QFT Limit of TGD Have Space-Time Super-Symmetry? In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#susy, 2006.

[K30] M. Pitkänen. Evolution in Many-Sheeted Space-Time. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#prebio, 2006.

[K31] M. Pitkänen. Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#mblocks, 2006.

[K32] M. Pitkänen. General Ideas about Many-Sheeted Space-Time: Part I. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#topcond, 2006.

[K33] M. Pitkänen. General Ideas about Many-Sheeted Space-Time: Part II. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#newviews, 2006.

[K34] M. Pitkänen. General Theory of Qualia. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#qualia, 2006.

[K35] M. Pitkänen. Generalized Feynman Diagrams as Generalized Braids. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#braidfeynman, 2006.

[K36] M. Pitkänen. Generalized Feynman Diagrams as Generalized Braids. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#braidfeynman, 2006.

[K37] M. Pitkänen. *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html, 2006.

[K38] M. Pitkänen. Genes and Memes. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#genememec, 2006.

[K39] M. Pitkänen. Homeopathy in Many-Sheeted Space-Time. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#homeoc, 2006.

[K40] M. Pitkänen. Identification of the Configuration Space Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#kahler, 2006.

[K41] M. Pitkänen. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#knotstgd, 2006.

[K42] M. Pitkänen. Langlands Program and TGD. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdeeg/tgdnumber.html#Langlandia, 2006.

[K43] M. Pitkänen. Macro-Temporal Quantum Coherence and Spin Glass Degeneracy. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#macro, 2006.

[K44] M. Pitkänen. Macroscopic Quantum Coherence and Quantum Metabolism as Different Sides of the Same Coin: Part I. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#metab, 2006.

[K45] M. Pitkänen. Macroscopic Quantum Phenomena and CP_2 Geometry. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#super, 2006.

[K46] M. Pitkänen. *Magnetospheric Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/magnconsc/magnconsc.html, 2006.

[K47] M. Pitkänen. Many-Sheeted DNA. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#genecodec, 2006.

[K48] M. Pitkänen. Massless states and particle massivation. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#mless, 2006.

[K49] M. Pitkänen. *Mathematical Aspects of Consciousness Theory*. Onlinebook. http://tgdtheory.fi/public_html/mathconsc/mathconsc.html, 2006.

[K50] M. Pitkänen. Matter, Mind, Quantum. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#conscic, 2006.

[K51] M. Pitkänen. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#nmpc, 2006.

[K52] M. Pitkänen. New Particle Physics Predicted by TGD: Part I. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#mass4, 2006.

[K53] M. Pitkänen. Nuclear String Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#nuclstring, 2006.

[K54] M. Pitkänen. *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html, 2006.

[K55] M. Pitkänen. p-Adic Numbers and Generalization of Number Concept. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#padmat, 2006.

[K56] M. Pitkänen. p-Adic Particle Massivation: Elementary Particle Masses. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#mass2, 2006.

[K57] M. Pitkänen. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#mass3, 2006.

[K58] M. Pitkänen. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#cognic, 2006.

[K59] M. Pitkänen. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#phblocks, 2006.

[K60] M. Pitkänen. *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html, 2006.

[K61] M. Pitkänen. Quantum Antenna Hypothesis. In *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html#tubuc, 2006.

[K62] M. Pitkänen. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#qastro, 2006.

[K63] M. Pitkänen. Quantum Control and Coordination in Bio-Systems: Part II. In *Bio-Systems as Self-Organizing Quantum Systems*. Onlinebook. http://tgdtheory.fi/public_html/bioselforg/bioselforg.html#qcococII, 2006.

[K64] M. Pitkänen. Quantum Field Theory Limit of TGD from Bosonic Emergence. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#emergence, 2006.

[K65] M. Pitkänen. Quantum Hall effect and Hierarchy of Planck Constants. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.#anyontgd, 2006.

[K66] M. Pitkänen. *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html, 2006.

[K67] M. Pitkänen. Quantum Model for Bio-Superconductivity: I. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#biosupercondI, 2006.

[K68] M. Pitkänen. Quantum Model for Bio-Superconductivity: II. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#biosupercondII, 2006.

[K69] M. Pitkänen. Quantum Model for Hearing. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#hearing, 2006.

[K70] M. Pitkänen. Quantum Model for Nerve Pulse. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#pulse, 2006.

[K71] M. Pitkänen. Quantum Model of EEG. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#eegII, 2006.

[K72] M. Pitkänen. Quantum Model of Memory. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#memoryc, 2006.

[K73] M. Pitkänen. *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook.http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html, 2006.

[K74] M. Pitkänen. *Quantum TGD*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquant.html, 2006.

[K75] M. Pitkänen. Quantum Theory of Self-Organization. In *Bio-Systems as Self-Organizing Quantum Systems*. Onlinebook. http://tgdtheory.fi/public_html/bioselforg/bioselforg.html#selforgac, 2006.

[K76] M. Pitkänen. Riemann Hypothesis and Physics. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#riema, 2006.

[K77] M. Pitkänen. Self and Binding. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#selfbindc, 2006.

[K78] M. Pitkänen. Some Fresh Ideas about Twistorialization of TGD. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#twistornotices, 2006.

[K79] M. Pitkänen. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#astro, 2006.

[K80] M. Pitkänen. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cosmo, 2006.

[K81] M. Pitkänen. *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html, 2006.

[K82] M. Pitkänen. *TGD and Fringe Physics*. Onlinebook. http://tgdtheory.fi/public_html/freenergy/freenergy.html, 2006.

[K83] M. Pitkänen. TGD and M-Theory. In *Topological Geometrodynamics: Overview*. Onlinebook. http://tgdtheory.fi/public_html/tgdview/tgdview.html#MTGD, 2006.

[K84] M. Pitkänen. TGD and Nuclear Physics. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#padnucl, 2006.

[K85] M. Pitkänen. *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html, 2006.

[K86] M. Pitkänen. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionc, 2006.

[K87] M. Pitkänen. TGD as a Generalized Number Theory: p-Adicization Program. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visiona, 2006.

[K88] M. Pitkänen. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionb, 2006.

[K89] M. Pitkänen. *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html, 2006.

[K90] M. Pitkänen. The Notion of Free Energy and Many-Sheeted Space-Time Concept. In *TGD and Fringe Physics*. Onlinebook. http://tgdtheory.fi/public_html/freenergy/freenergy.html#freenergy, 2006.

[K91] M. Pitkänen. The Notion of Wave-Genome and DNA as Topological Quantum Computer. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#gari, 2006.

[K92] M. Pitkänen. The Recent Status of Lepto-hadron Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#leptc, 2006.

[K93] M. Pitkänen. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#tgdgrt, 2006.

[K94] M. Pitkänen. Three new physics realizations of the genetic code and the role of dark matter in bio-systems. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#dnatqccodes, 2006.

[K95] M. Pitkänen. Time, Spacetime and Consciousness. In *Bio-Systems as Conscious Holograms*. Onlinebook. http://tgdtheory.fi/public_html/hologram/hologram.html#time, 2006.

[K96] M. Pitkänen. *Topological Geometrodynamics: an Overview*. Onlinebook.http://tgdtheory.fi/public_html/tgdview/tgdview.html, 2006.

[K97] M. Pitkänen. Topological Quantum Computation in TGD Universe. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public_html/genememe/genememe.html#tqc, 2006.

[K98] M. Pitkänen. Twistors, N=4 Super-Conformal Symmetry, and Quantum TGD. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#twistor, 2006.

[K99] M. Pitkänen. Was von Neumann Right After All. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

[K100] M. Pitkänen. Wormhole Magnetic Fields. In *Quantum Hardware of Living Matter*. Onlinebook. http://tgdtheory.fi/public_html/bioware/bioware.html#wormc, 2006.

[K101] M. Pitkänen. Yangian Symmetry, Twistors, and TGD. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#Yangian, 2006.

[K102] M. Pitkänen. Motives and Infinite Primes. In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#infmotives, 2011.

[K103] M. Pitkänen. Quantum Mind and Neuroscience. In *TGD based view about living matter and remote mental interactions*. Onlinebook. http://tgdtheory.fi/public_html/pdfpool/lianPN.pdf, 2012.

[K104] M. Pitkänen. *TGD Based View About Living Matter and Remote Mental Interactions*. Onlinebook. http://tgdtheory.fi/public_html/tgdlian/tgdlian.html, 2012.

[K105] M. Pitkänen. The Recent Vision About Preferred Extremals and Solutions of the Modified Dirac Equation. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#dirasvira, 2012.

[K106] M. Pitkänen. Topological Geometrodynamics: Basic Visions. In *TGD based view about living matter and remote mental interactions*. Onlinebook. http://tgdtheory.fi/public_html/pdfpool/lianPTGD.pdf, 2012.

[K107] M. Pitkänen. Are dark photons behind biophotons. In *TGD based view about living matter and remote mental interactions*. Onlinebook. http://tgdtheory.fi/public_html/pdfpool/biophotonslian.pdf, 2013.

[K108] M. Pitkänen. Comments on the recent experiments by the group of Michael Persinger. In *TGD based view about living matter and remote mental interactions*. Onlinebook. http://tgdtheory.fi/public_html/pdfpool/persconsc.pdf, 2013.

[K109] M. Pitkänen. *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public_html/neuplanck/neuplanck.html, 2013.

[K110] M. Pitkänen. *p-Adic length Scale Hypothesis*. Onlinebook. http://tgdtheory.fi/public_html/padphys/padphys.html, 2013.

[K111] M. Pitkänen. *Quantum TGD*. Onlinebook. http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html, 2013.

[K112] M. Pitkänen. TGD and Potential Anomalies of GRT. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#granomalies, 2013.

[K113] M. Pitkänen. Various General Ideas Related to TGD. In *Quantum TGD as Infinite-dimensional Spinor Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#tgdmisc, 2013.

[K114] M. Pitkänen. What are the counterparts of Einstein's equations in TGD? In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#EinsteinTGD, 2013.

[K115] M. Pitkänen. What p-Adic Icosahedron Could Mean? And What about p-Adic Manifold? In *TGD as a Generalized Number Theory*. Onlinebook. http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#picosahedron, 2013.

[K116] M. Pitkänen. Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds" . In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#wcwnew, 2014.

[K117] M. Pitkänen. Unified Number Theoretical Vision. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#numbervision, 2014.

Articles about TGD

[L1] M. Pitkänen. A Strategy for Proving Riemann Hypothesis. http://arxiv.org/abs/math/0111262, 2002.

[L2] M. Pitkänen. Macro-temporal quantum coherence, quantum spin glass degeneracy, and number theoretic information concept, 2003.

[L3] M. Pitkänen. Time, Space-Time, and Consciousness. http://www.emergentmind.org/journal.htm, 2003.

[L4] M. Pitkänen. Gravitational Schrödinger equation as a quantum model for the formation of astrophysical structures and dark matter? http://tgdtheory.fi/articles/nottale.pdf, 2004.

[L5] M. Pitkänen. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. http://tgdtheory.fi/pdfpool/append.pdf, 2006.

[L6] M. Pitkänen. Further Progress in Nuclear String Hypothesis. http://tgdtheory.fi/articles/nuclstring.pdf, 2007.

[L7] M. Pitkänen. About the Nature of Time. http://tgdtheory.fi/articles/time.pdf, 2008.

[L8] M. Pitkänen. TGD Inspired Theory of Consciousness. http://tgdtheory.fi/articles/tgdconsc.pdf, 2008.

[L9] M. Pitkänen. Topological Geometrodynamics: What Might Be The Basic Principles. http://tgdtheory.fi/articles/tgd2008.pdf, 2008.

[L10] M. Pitkänen. Physics as Generalized Number Theory II: Classical Number Fields. https://www.createspace.com/3569411, July 2010.

[L11] M. Pitkänen. Physics as Infinite-dimensional Geometry I: Identification of the Configuration Space Kähler Function. https://www.createspace.com/3569411, July 2010.

[L12] M. Pitkänen. Physics as Infinite-dimensional Geometry II: Configuration Space Kähler Geometry from Symmetry Principles. https://www.createspace.com/3569411, July 2010.

[L13] M. Pitkänen. Construction of Configuration Space Spinor Structure. http://tgdtheory.fi/articles/spinstructure.pdf, 2010.

[L14] M. Pitkänen. How to Define Generalized Feynman Diagrams?, 2010.

[L15] M. Pitkänen. Physics as Generalized Number Theory I: p-Adic Physics and Number Theoretic Universality. https://www.createspace.com/3569411, July 2010.

[L16] M. Pitkänen. Physics as Generalized Number Theory III: Infinite Primes. https://www.createspace.com/3569411, July 2010.

[L17] M. Pitkänen. Physics as Infinite-dimensional Geometry III: Configuration Space Spinor Structure. https://www.createspace.com/3569411, July 2010.

[L18] M. Pitkänen. Physics as Infinite-dimensional Geometry IV: Weak Form of Electric-Magnetic Duality and Its Implications. https://www.createspace.com/3569411, July 2010.

[L19] M. Pitkänen. The Geometry of CP_2 and its Relationship to Standard Model. https://www.createspace.com/3569411, July 2010.

[L20] M. Pitkänen. Two attempts to understand PK. http://tgdtheory.fi/public_html/articles/PKoptions.pdf, 2012.

[L21] M. Pitkänen. CMAP representations about TGD. http://www.tgdtheory.fi/cmaphtml.html, 2014.

[L22] M. Pitkänen. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. http://www.tgdtheory.fi/tgdglossary.pdf, 2014.

[L23] Matti Pitkänen. 4-D spin glass degeneracy.http://www.tgdtheory.fi/webCMAPs/4-Dspinglassdegeneracy.html. 2014.

[L24] Matti Pitkänen. Astrophysics and TGD.http://www.tgdtheory.fi/webCMAPs/AstrophysicsandTGD.html. 2014.

[L25] Matti Pitkänen. Basic notions behind $M^8 - Hduality$.http://www.tgdtheory.fi/webCMAPs/BasicnotionsbehindM^8-Hduality.html. 2014.

Matti Pitkänen. Basic TGD.http://www.tgdtheory.fi/webCMAPs/BasicTGD.html. 2014.

Matti Pitkänen. Basic TGD.http://www.tgdtheory.fi/webCMAPs/BasicTGD.html. 2014.

Matti Pitkänen. Classical TGD.http://www.tgdtheory.fi/webCMAPs/ClassicalTGD.html. 2014.

Matti Pitkänen. Comparison with other theories.http://www.tgdtheory.fi/webCMAPs/Comparisonwithotherth html. 2014.

Matti Pitkänen. Cosmic strings.http://www.tgdtheory.fi/webCMAPs/Cosmicstrings.html. 2014.

Matti Pitkänen. Differences between TGD and string models.http://www.tgdtheory.fi/webCMAPs/DifferencesbetweenTGDandstringmodels.html. 2014.

Matti Pitkänen. Elementary particle vacuum functionals.http://www.tgdtheory.fi/webCMAPs/Elementaryparticlevacuumfunctionals.html. 2014.

Matti Pitkänen. Emergence of bosons.http://www.tgdtheory.fi/webCMAPs/Emergenceofbosons.html. 2014.

Matti Pitkänen. Emergent ideas and notions.http://www.tgdtheory.fi/webCMAPs/Emergentideasandnotions.html. 2014.

Matti Pitkänen. Equivalence Principle.http://www.tgdtheory.fi/webCMAPs/EquivalencePrinciple.html. 2014.

Matti Pitkänen. Generalized Feynman diagrams.http://www.tgdtheory.fi/webCMAPs/GeneralizedFeynmandiagrams.html. 2014.

Matti Pitkänen. Geometrization of fields.http://www.tgdtheory.fi/webCMAPs/Geometrizationoffields.html. 2014.

Matti Pitkänen. Geometry of WCW.http://www.tgdtheory.fi/webCMAPs/GeometryofWCW.html. 2014.

Matti Pitkänen. Hierarchy of Planck constants.http://www.tgdtheory.fi/webCMAPs/HierarchyofPlanckconstants.html. 2014.

Matti Pitkänen. How quantum TGD differs from standard quantum physics.http://www.tgdtheory.fi/webCMAPs/HowquantumTGDdiffersfromstandardquantumphysics.html. 2014.

Matti Pitkänen. How TGD differs from standard model.http://www.tgdtheory.fi/webCMAPs/HowTGDdiffersfromstandardmodel.html. 2014.

Matti Pitkänen. Hyperfinite factors and TGD.http://www.tgdtheory.fi/webCMAPs/HyperfinitefactorsandTGD.html. 2014.

Matti Pitkänen. Identification of preferred extremals of Kaehler action.http://www.tgdtheory.fi/webCMAPs/IdentificationofpreferredextremalsofKaehleraction.html. 2014.

Matti Pitkänen. Intelligence and hierarchy of Planck constants.http://www.tgdtheory.fi/webCMAPs/IntelligenceandhierarchyofPlanckconstants.html. 2014.

Matti Pitkänen. Kaehler-Dirac action.http://www.tgdtheory.fi/webCMAPs/Kaehler-Diracaction.html. 2014.

Matti Pitkänen. KD equation.http://www.tgdtheory.fi/webCMAPs/KDequation.html. 2014.

Matti Pitkänen. Key notions and ideas of TGD.http://www.tgdtheory.fi/webCMAPs/KeynotionsandideasofTGD.html. 2014.

Matti Pitkänen. Leptohadron hypothesis.http://www.tgdtheory.fi/webCMAPs/Leptohadronhypothesis.html. 2014.

Matti Pitkänen. $M^8-Hduality$.http://www.tgdtheory.fi/webCMAPs/M^8-Hduality.html. 2014.

Matti Pitkänen. M89 hadron physics.http://www.tgdtheory.fi/webCMAPs/M89hadronphysics.html. 2014.

Matti Pitkänen. Magnetic body.http://www.tgdtheory.fi/webCMAPs/Magneticbody.html. 2014.

Matti Pitkänen. Manysheeted space-time.http://www.tgdtheory.fi/webCMAPs/Manysheetedspace-time.html. 2014.

Matti Pitkänen. Nature of time.http://www.tgdtheory.fi/webCMAPs/Natureoftime.html. 2014.

Matti Pitkänen. Negentropy Maximization Principle.http://www.tgdtheory.fi/webCMAPs/NegentropyMaximizationPr
html. 2014.

Matti Pitkänen. Overall view about TGD.http://www.tgdtheory.fi/webCMAPs/OverallviewaboutTGD.html. 2014.

Matti Pitkänen. p-Adic mass calculations.http://www.tgdtheory.fi/webCMAPs/p-Adicmasscalculations.html. 2014.

Matti Pitkänen. p-Adice length scale hypothesis.http://www.tgdtheory.fi/webCMAPs/p-Adicelengthscalehyp html. 2014.

Matti Pitkänen. Physics as generalized number theory.http://www.tgdtheory.fi/webCMAPs/Physicsasgeneralizednumbertheory.html. 2014.

Matti Pitkänen. Quantum Classical Correspondence.http://www.tgdtheory.fi/webCMAPs/QuantumClassicalCo html. 2014.

Matti Pitkänen. Quantum consciousness.http://www.tgdtheory.fi/webCMAPs/Quantumconsciousness.html. 2014.

Matti Pitkänen. Quantum criticality.http://www.tgdtheory.fi/webCMAPs/Quantumcriticality.html. 2014.

Matti Pitkänen. Quantum intelligence.http://www.tgdtheory.fi/webCMAPs/Quantumintelligence.html. 2014.

Matti Pitkänen. Quantum model of qualia.http://www.tgdtheory.fi/webCMAPs/Quantummodelofqualia.html. 2014.

Matti Pitkänen. Quantum physics as generalized number theory.http://www.tgdtheory.fi/webCMAPs/Quantumphysicsasgeneralizednumbertheory.html. 2014.

Matti Pitkänen. Quaternionic planes of octonions.http://www.tgdtheory.fi/webCMAPs/Quaternionicplanesof html. 2014.

Matti Pitkänen. Similarities between TGD and string models.http://www.tgdtheory.fi/webCMAPs/SimilaritiesbetweenTGDandstringmodels.html. 2014.

Matti Pitkänen. Space-time as 4-surface in M^4xCP_2.http://www.tgdtheory.fi/webCMAPs/Space-timeas4-surfaceinM^4xCP_2.html. 2014.

Matti Pitkänen. Structure of WCW.http://www.tgdtheory.fi/webCMAPs/StructureofWCW.html. 2014.

Matti Pitkänen. SUSY and TGD.http://www.tgdtheory.fi/webCMAPs/SUSYandTGD.html. 2014.

Matti Pitkänen. Symmetries of WCW.http://www.tgdtheory.fi/webCMAPs/SymmetriesofWCW.html. 2014.

Matti Pitkänen. TGD and classical number fields.http://www.tgdtheory.fi/webCMAPs/TGDandclassicalnumber html. 2014.

Matti Pitkänen. TGD and GRT.http://www.tgdtheory.fi/webCMAPs/TGDandGRT.html. 2014.

Matti Pitkänen. TGD as ATQFT.http://www.tgdtheory.fi/webCMAPs/TGDasATQFT.html. 2014.

Matti Pitkänen. TGD as infinite-dimensional geometry.http://www.tgdtheory.fi/webCMAPs/TGDasinfinite-dimensionalgeometry.html. 2014.

Matti Pitkänen. TGD as unified theory of fundamental interactions.http://www.tgdtheory.fi/webCMAPs/TGDasunifiedtheoryoffundamentalinteractions.html. 2014.

Matti Pitkänen. TGD inspired cosmology.http://www.tgdtheory.fi/webCMAPs/TGDinspiredcosmology.html. 2014.

Matti Pitkänen. TGD inspired theory of consciousness.http://www.tgdtheory.fi/webCMAPs/TGDinspiredtheoryofconsciousness.html. 2014.

Matti Pitkänen. TGD view about elementary particles.http://www.tgdtheory.fi/webCMAPs/TGDviewaboutelementaryparticles.html. 2014.

Matti Pitkänen. The unique role of twistors in TGD.http://www.tgdtheory.fi/webCMAPs/TheuniqueroleoftwistorsinTGD.html. 2014.

Matti Pitkänen. Topological field quantization.http://www.tgdtheory.fi/webCMAPs/Topologicalfieldquantiz html. 2014.

Matti Pitkänen. Twistors and TGD.http://www.tgdtheory.fi/webCMAPs/TwistorsandTGD.html. 2014.

Matti Pitkänen. Vacuum functional in TGD.http://www.tgdtheory.fi/webCMAPs/VacuumfunctionalinTGD.html. 2014.

Matti Pitkänen. Weak form of electric-magnetic duality.http://www.tgdtheory.fi/webCMAPs/Weakformofelectric-magneticduality.html. 2014.

Matti Pitkänen. What TGD is.http://www.tgdtheory.fi/webCMAPs/WhatTGDis.html. 2014.

Matti Pitkänen. Zero Energy Ontology (ZEO).http://www.tgdtheory.fi/webCMAPs/ZeroEnergyOntology(ZEO).html. 2014.

Index

TOPOLOGICAL GEOMETRODYNAMICS: AN OVERVIEW

Topological Geometrodynamics (TGD) is a modification of general relativity inspired by the problems related to the definition of inertial and gravitational energies in general relativity. TGD is also a generalization of super string models. Physical space-times are seen as four-dimensional surfaces in certain 8-dimensional space H. The choice of H is fixed by symmetries of standard model and leads to a geometrization of known classical fields and elementary particle numbers. In fermionic sector strings indeed emerge.

Many-sheeted space-time replaces Einsteinian space-time, which follows as a long length scale approximation in which sheets of the many-sheeted space-time are lumped together. The extension of number concept based on the fusion of real numbers and p-adic number fields implies a further generalisation of the space-time concept allowing to identify space-time correlates of cognition and intentionality.

Zero energy ontology forces an extension of quantum measurement theory to a theory of consciousness and a hierarchy of phases identified as dark matter is predicted with far reaching implications for the understanding of consciousness and living systems. This all implies an elegant theoretical projection of our reality honoring the work by renowned scientists (such as Wheeler, Feynman, Penrose, Einstein, Josephson to name a few) and creating a solid foundation for modeling our Universe in terms of geometry.

Ph.D. Matti Pitkänen • *Hanko, Finland*

Matti Pitkänen started to work with the basic idea of TGD at 1977, published his thesis work about TGD at 1982, and has since then worked to transform the basic vision to a consistent predictive mathematical framework, to solve various interpretational issues, and understand the relationship of TGD with existing theories.

TGD Web Pages: http://www.tgdtheory.com
TGD Diary and Blog: http://matpitka.blogspot.com