NEW CALCULUSES

Vladislav Konovalov

Abstract

In the article the new calculuses are offered similar differential and integral, but differing, that in them the analysis of the previous and subsequent values of a function is made. The new calculuses allow to decide problems, the solution which one with usage customary differential and integral calculus is impossible.

The contents

1. To the READER

Now differential and integral calculus are unique. It studies, as the change of argument on velocity of that or diverse process influences or, on the contrary, as on an alteration of speed to find change of the function. Many problems from a point of view of a differential calculus are clear to us and easily are decided, for example problem of motion of a body with uniformly increased velocity. Many problems are easy are to translated in comfortable for a differential calculus by a kind many - difficult and this business requires high proficiency, and many - it is impossible. At the same time, frequently there are problems, the physical sense which one is clear to us, but "mathematize" them is rather complex. For example, for us is apparent, that if any microbe in particular time is doubled, after the lapse of this time the total number of microbes will be twice more initial number. In this case we would be helped by calculus as $y/y_{-1}=2$, where y - current value of a function, y_{-1} - previous value of a function. Our brains are arranged in such a manner that we can reasonably meditate, comparing current value of some value with its previous value. On it our prognostic abilities here again we are grounded we achieve large successes, even nothing knowing about the reasons of change of a considered function. The tendered calculuses are intended for support of these abilities and operate with current and previous values of any processes. And these values can vary not only in time, but also under activity of any arguments.

2. ACTUAL PROCESSES AND NECESSITY FOR NEW CALCULUSES

Now in practice the rather imperfect technique of the mathematical description any of process will be utilized. Make initial system equations, which one (under the guess of the explorer) more or less adequately reflect actual process. To this system equations set the starting conditions or parameters and then decide by its numerical methods, since the analytical expression frequently cannot be received. The received results compare with practically apparent and in case of considerable variance with the observation data introduce the correctives to initial system equations. Thus the initial system equations fast become complicated (to become simpler it in essence can not) and after several attempts to receive the reasonable solution, forces the explorer to go on the compromise, sacrificing both accuracy and common sense of initial system equations.

Any actual process in any given moment of the development "does not know" neither history, nor future. It develops on the basis of present conditions in this moment which was added up during given process or even up to its beginning. Apparently, that any process at change of conditions of its passing will go with other velocity or even in the other direction, therefore task of the starting conditions is not absolutely correct, since to actual process on them, by and large, not to care a button. Besides the passing and other, indirect processes accompanying explored process will change. The term of "conditions", in which one flow past process is not absolutely successful for the reason, that always of conditions it is possible to find or to number very much, and the given process is influenced only by some of them and in a miscellaneous degree. Therefore there is a sense to enter concept of resource of process. Resource of process is conditions directly influencing to given process, accelerating it or slowing down. At unlimited resource or resource having constant value, it can be not allow, since the course of process in these conditions does not vary. Let's consider some examples.

1. We shall suspect that we study growth of culture of micro-organisms in conditions of unlimited delivery of nutrient materials and removal of products of vital functions. Besides the volume of an incubator also is unrestricted. Apparently, that in this elementary for the analysis a case the external resources of process are unlimited; therefore given process do not influence also them it is possible to not take into consideration. In this case process limits internal resource - velocity of division of cells of micro-organisms. In conditions of a stationary value of temperature it also can be not allow, but at a temperature variation at once there is a temperature resource, which one can be both external, and internal, if the process is accompanied by heat liberation.

2. If in the previous example to limit a volume of an incubator, in a start some time the process will go precisely how in an example 1, but then occurs and starts to influence all stronger resource of restricted space. At the end it will cause to a self-destruction of culture of micro-organisms.

3. At presence of restricted resource of a feed, removal of products of vital functions etc., we shall come besides unfavorable conclusion.

From these examples it is visible, that any process is determined not initially by given regularity and start conditions (apparently, this notion was added up under influence of a mechanics with its stringent determinism), and the resources, which one are disposed by given process in a given instant.

Any process develops on the basis of momentary value of a function, for example, the law of a radioactive decay **is received** during decay only that each nucleus has particular probability of decay. Nobody will doubt that manages us in any moment to change probability of decay, and just from this moment the process will go in another way. In this sense the tendered below new calculuses have not formal - mathematical, and physical basis, suggesting new value of a function on the basis of its previous value. It enables considerably to perfect mathematical modeling of processes, allowing introducing the correctives to changes of a function on its course, instead of in initial system equations, thus it is possible to not know boundary conditions. Thus, the new calculuses allow as flexibly to operate change of a function, as the external conditions operates change of the most studied process. The mathematical methods sensitively reacting to change of a situation are necessary to applied science as it makes a brain of alive entities. We never shall create artificial reason on the basis of any laws, which one a priori guess a rigid determinism and eliminate any initiative and unpredictable behavior.

3. DEFINITION OF NEW CALCULUSES SIMILAR TO DIFFERENTIAL AND INTEGRAL (D-CALCULUS)

The author hopes, that the enunciated below principles of new calculuses will render the large help to the scientists engaging applied researches, and for the fans of mathematics here immense field of activity.

In a known differential calculus of derivative function:

$$y = f(x) \quad (1)$$

in a point *x* term a limit of the ratio of an increment of a function to increment of argument, at aspiration of increment of argument to zero point:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad (2).$$

Conditionally we shall term a differential calculus grounded on this definition by a derivative and the relevant integral calculus by *d*-calculus.

Let's extend definition (2), by accepting value Δx final, then from (2):

$$\frac{\Delta y^*}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad (3).$$

For further, is more convenient to copy (3) as:

$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(x-\Delta x)}{\Delta x} \quad (4),$$

whence, allowing (1) and by designating $f(x-\Delta x) = y_{-\Delta x}$:

$$\Delta y = y - y_{-\Delta x} \quad (5).$$

We go on generalizing, by noting, that at Δx final, Δy also final (for the majority of functions), therefore we shall define set of derivatives of a function $y = f(x)$ so:

$$Qy = Z(y, y_{-\Delta x}) \quad (6).$$

On the basis of generalized definition of a derivative of a function (6) it is possible to construct set Q-calculuses (we shall term them also differential and integral for the lack of other term). The *d*-calculus will be one of particular cases at $Z(y, y_{-\Delta x}) = f(x) - f(x-\Delta x)$ and $\Delta x \to 0$.

I select in set Q-calculuses a subset at $\Delta x = 1$, which one we shall term as *D*-calculuses. Thus it is necessary to take into account, that in actual applied problems for unit it is possible to receive any change of argument, as units of change of argument in applied problems are conditional (see examples of practical applied *D*-исчисления).

Thus, the *D*- derivative of a function $y = f(x)$ is determined by us by expression:

$$Dy = Z(y, y_{-1}) \quad (7),$$

i.e. it is determined by an arbitrary function from the subsequent and previous values of a function $y = f(x)$, at change of argument on unit (conditional). From (7) we shall discover y_{-1}:

$$y_{-1} = \varphi(Dy, y) \quad (8).$$

Let's assume: $$y = \psi(U, V) \quad (9),$$

where *U* and *V* - some functions from *x*.

By substituting (9) and (8) in (7), we shall discover:

$$Dy = Z\left\{\psi(U,V), \varphi\left[\varphi(DU,U), \varphi(DV,V)\right]\right\} \quad (10).$$

We have received main expression for finding of any derivative from set *D*, structure which one will be kept and at $y = \psi(U, V, W, \ldots)$.

For a composite function $y = U[V(x)]$ (function from a function), from (10) we shall receive:

$$Dy = Z\{U[V(x)], U(DV, V)\} \quad (11).$$

Let's contact between derivatives of a function $y = f(x)$ inside *D*-calculuses.

For particular calculus D_1 from (7) we shall discover:

$$D_1 y = Z_1\left(y, y_{-1}\right) \qquad (12).$$

For calculus D_2 from (8) we shall discover:

$$y_{-1} = \varphi_2(D_2 y, y) \qquad (13).$$

By substituting (13) in (12), we shall receive:

$$D_1 y = Z_1\left[y, \varphi_2\left(D_2 y, y\right)\right] \qquad (14).$$

The expression (14) establishes required connection.

It is interesting also to contact between *D*-calculuses and known *d*-calculus.

Let's assume: $Dy = f\left(x\right)$ (15), whence: $x = \varsigma(Dy)$ (16).

Let's assume: $\frac{dy}{dx} = \varphi\left(x\right)$ (17), whence: $x = \psi\left(\frac{dy}{dx}\right)$ (18).

From equality of the left-hand parts (16) and (18) we have:

$$\xi\left(Dy\right) = \psi\left(\frac{dy}{dx}\right) \qquad (19),$$

whence it is possible to express Dy through $\frac{dy}{dx}$ or on the contrary. By substituting (16) in (17) and (18) in (15), we shall discover connection between *d* and *D*-calculus in an explicit kind:

$$Dy = f\left[\psi\left(\frac{dy}{dx}\right)\right] \qquad (20),$$

$$\frac{dy}{dx} = \varphi\left[\xi\left(Dy\right)\right] \qquad (21).$$

As the *D*-derivatives of functions given, for example in points, exist, and *d*-derivatives is not present, the connection between *d* and *D*-calculus for such functions is not uncovered in an explicit kind by expressions (20) and (21), that is quite natural.

4. D-DIFFERENTIAL CALCULUS

For some (taken as an example) *D*-calculuses in the table the rules of differentiation are adduced, which one are received from (10) and (11). For an example the elementary functions $Dy = Z(y, y_{-1})$ are selected and title of particular *D*-calculuses conditionally.

Table (1) of *D*-derivatives.

Definition of some particular calculuses

Kind of *D*-calculus	$Dy = Z(y, y_{-1})$	$y_{-1} = \varphi(Dy, y)$
Δ	$\Delta y = y - y_{-1}$	$y_{-1} = y - \Delta y$
S	$Sy = y + y_{-1}$	$y_{-1} = Sy - y$
O	$Oy = \frac{y}{y_{-1}}$	$y_{-1} = \frac{y}{Oy}$
Θ	$\Theta y = \frac{y_{-1}}{y}$	$y_{-1} = y \cdot \Theta y$
Π	$\Pi y = y \cdot y_{-1}$	$y_{-1} = \frac{\Pi y}{y}$
L	$Ly = \ln y - \ln y_{-1}$	$y_{-1} = \frac{y}{e^{Ly}}$

Value of a *D*- derivative of a function $y = \psi(U, V)$

Type	$y = a(a = const)$	$y = aU$
Δ	0	$a \cdot \Delta U$

S	2a	$a \cdot SU$
O	1	OU
Θ	1	$\frac{1}{\Theta U}$
Π	a^2	$a^2 \cdot \Pi U$
L	0	LU

Type	$y = U \pm V$
Δ	$\Delta U \pm \Delta V$
S	$SU \pm SV$
O	$\frac{(U \pm V) OU \cdot OV}{U \cdot OV \pm V \cdot OU}$
Θ	$\frac{U \pm V}{U \cdot \Theta U \pm V \cdot \Theta V}$
Π	$\Pi U + \Pi V \pm \left(U \frac{\Pi V}{V} + V \frac{\Pi U}{U} \right)$
L	$\ln \frac{(U \pm V)(LU + LV)}{Ue^{LV} \pm Ve^{LU}}$

Type	$y = U \cdot V$
Δ	$U \cdot \Delta V + V \cdot \Delta U - \Delta U \cdot \Delta V$
S	$U \cdot SV + V \cdot SU - SU \cdot SV$
O	$OU \cdot OV$
Θ	$\frac{1}{\Theta U \cdot \Theta V}$
Π	$\Pi U \cdot \Pi V$
L	$LU + LV$

Type	$y = \frac{U}{V}$
Δ	$\frac{V \cdot \Delta U - U \cdot \Delta V}{V^2 - V \cdot \Delta V}$
S	$\frac{V \cdot SU + U \cdot SV - 2UV}{V \cdot SV - V^2}$
O	$\frac{OU}{OV}$
Θ	$\frac{\Theta V}{\Theta U}$
Π	$\frac{\Pi U}{\Pi V}$
L	$LU - LV$

Type	$y = U^V$
Δ	$U^V - (U - \Delta U)^{V - \Delta V}$
S	$U^V + (SU - U)^{SV - V}$
O	$U^{\frac{V(OV-1)}{OV}} \cdot OU^{\frac{V}{OV}}$
Θ	$\frac{1}{U^{V(\Theta V - 1)} \cdot \Theta U^{V \cdot \Theta V}}$
Π	$U^{\frac{V^2 - \Pi V}{V}} \cdot \Pi U^{\frac{\Pi V}{V}}$
L	$V\left[\left(1 - \frac{V}{e^{LV}}\right) \ln U + \frac{LU}{e^{LV}}\right]$

Type	$y = U[V(x)]$ function from a function

Δ	$U(V)-U(V-\Delta V)$
S	$U(V)+U(SV-V)$
O	$\dfrac{U(V)}{U\left(\dfrac{V}{OV}\right)}$
Θ	$\dfrac{U(V)}{U(V\cdot\Theta V)}$
Π	$U(V)\cdot U\left(\dfrac{\Pi V}{V}\right)$
L	$\ln U(V)-\left(\dfrac{V}{e^{LV}}\right)\ln U(V)$

Translation $D_i y$ in Δy : $\Delta y=\left[y-\varphi_i(D_i y, y)\right]$

Δ	Δy
S	$2y-Sy$
O	$y\left(1-\dfrac{1}{Oy}\right)$
Θ	$y(1-\Theta y)$
Π	$y-\dfrac{\Pi y}{y}$
L	$y\left(1-\dfrac{1}{e^{Ly}}\right)$

That was more understandable, as the cells of the table are stuffed, we shall consider a particular example of *O*-differentiation of a function

$$y=U+V \qquad (22),$$

where *U* and *V* some functions from x. Are apparent, that the previous term of this function will be

$$y_{-1}=U_{-1}+V_{-1} \qquad (23).$$

On definition,

$$Oy=\frac{y}{y_{-1}} \qquad (24).$$

On the strength from this definition

$$U_{-1}=\frac{U}{OU}\quad,\qquad V_{-1}=\frac{V}{OV} \qquad (25).$$

By substituting (22), (23) and (25) in (24), we shall discover value of a *O*-derivative of a function (22):

$$Oy=\frac{(U+V)OU\cdot OV}{UOV+VOU} \qquad (26),$$

which one corresponds to value, reduced in table (22) (second type of a function, third line).

In the last partition of table (22) the formulas of translation of any of considered *D*-calculuses in Δ-calculus are adduced, which one are directly received from definition of particular *D*-calculus and Δ-calculus, for example:

$$\Delta y=y-y_{-1} \qquad (27),$$

$$Ly=\ln y-\ln y_{-1} \qquad (28),$$

we exponentiates (28) to receive expression for y_{-1}, which one we shall substitute in (27) and we shall receive the formula of translation L-calculus in Δ-calculus:

$$\Delta y = y\left(1 - \frac{1}{e^{Ly}}\right) \qquad (29).$$

The formula (29) corresponds to the last cell of table 1.

Table *D*- derivatives of elementary functions is easy for compounding of expression (7) or, knowing D_i-derivative - from (14). For example, the *D*- derivatives of a function $y = e^x$ are those: $\Delta y = e^x\left(1 - \frac{1}{e}\right)$, $Sy = e^x\left(1 + \frac{1}{e}\right)$, $Oy = e$, $\Theta y = \frac{1}{e}$, $\Pi y = e^{2x-1}$, $Ly = 1$ etc.

To differentiate a function it is possible as directly, and using rules of table (22), for example, the function $y = \frac{x}{x-1}$ can be presented as the ratio of two functions: $U = x$ and $V = x - 1$. In both cases we shall receive (for *O*-calculus) $Oy = \frac{x(x-2)}{(x-1)^2}$.

The partial derivatives in any *D*-calculus are determined the same as and in *d*-calculus, for example, if there is a function several variables $y = f(x, z, t \ldots)$:

$$y = \frac{ze^x}{t^2} \qquad (30),$$

that the partial derivatives in *O*-calculus will be such:

$$Oy_z = \frac{z}{z-1},\ Oy_x = e,\ Oy_t = \frac{(t-1)^2}{t^2},$$

i.e. the differentiation on one variable is made at stationary values others variables.

The *D*-derivatives of the high orders are received by differentiation of a derivative of the lower order, for example the Δ-derivatives sequentially of heightened order from a function $y = e^x$ will be such: $\Delta' = e^x\left(1 - \frac{1}{e}\right)$, $\Delta'' = e^x\left(1 - \frac{1}{e}\right)^2$, …. $\Delta^n = e^x\left(1 - \frac{1}{e}\right)^n$

The *D*-derivatives of trigonometrical functions differ from remaining subjects, that unit of change of argument will be a π radian, for example:

$$y = \sin x \qquad (31),$$

then

$$y_{-1} = \sin(x - \pi) \qquad (32).$$

The *O*-derivative of a function (31) will be:

$$Oy = \frac{\sin x}{\sin(x - \pi)} = -1 \qquad (33),$$

and Δ-derivative:

$$\Delta y = \sin x - \sin(x - \pi) = \sin x - (\sin x \cos \pi - \cos x \sin \pi) = 2 \sin x \qquad (34).$$

The expression (34) is possible to receive and from (33) by taking advantage the formula of translation of *O*-calculus in Δ-calculus from table 1.

The rule of differentiation is universally for any function - argument of a function by decrease on unit (or on π for trigonometrical functions) also is noted a ratio between the y and y_{-1} pursuant to definition of particular calculus.

5. D-INTEGRAL CALCULUS

Primitive a function for a given function $y = f(x)$ we shall define similarly, how it is made in *d*-calculus, i.e. it is such function $F(x)$ a *D*-derivative from which one is peer $f(x)$, relevant, for example, Δ-integral:

$$\frac{\Delta}{}\int \Delta y = f(x) + C \qquad (35).$$

In a right part (35) constant *C* is present in Δ-calculus. In other calculuses in a right part the constant can miss, therefore *D*-integral can be defined so:

$$\frac{D}{}\int Z[A(y+C), A(y_{-1}+C)] = Af(x) + C \qquad (36),$$

or, allowing (8):

$$\frac{D}{}\int Z\{A(y+C), A[\varphi(Dy, y)+C]\} = Af(x) + C \qquad (37),$$

where *A* - constant factor.

The expression (36) is impossible to treat, how a uncertain *D*-integral, since, for example, in Δ-calculus the constant *C* in an integrand will vanish, and in a right part remain. For such calculuses the *D*-integral will be undefined then, as for example, the *S*-integral will be determined, as the constants *C* and *A* in an integrand are saved at differentiation of a primitive.

The difficulties of known *d*-integration are saved and in *D*-integration, as the *D*-integral are determined as not constructively, as well as *d*-integral.

Let's consider anyone uncertain O-integral, for example:

$$\frac{O}{}\int e^x = Ae^{\frac{1}{2}(x^2+x)} \text{ (4.4) or } \frac{O}{}\int x = A \cdot x! \qquad (38).$$

We see interesting feature encompassing therein, that as against set of primitives of a function e^x in *d*-calculus received by parallel carry lengthwise axis *y*, in O-calculus the set of primitives of a given function is received by stretching (squeezing) lengthwise axis *y*.

The rules of integration for each *D*-calculus as are various, as well as rule of differentiation, reflected in table (4.1). So, for example, the constant factor *k* is carried out for a sign of a *D*-integral $\frac{D}{}\int kf(x)$ so:

$$k\frac{\Delta}{}\int f(x),\ k\frac{S}{}\int f(x),\ k^x\frac{O}{}\int f(x),\ \frac{1}{k^x}\frac{\Theta}{}\int f(x),\ \sqrt{k}\frac{\Pi}{}\int f(x),\ \left[\frac{L}{}\int f(x)\right]^k \qquad (39).$$

Some indefinite integrals are adduced below. The simplest method of "integration" consists in differentiation of expression, which one is represented to you by useful. The received expression is an integrand, i.e.

$$\frac{D}{}\int Dy = y \qquad (40).$$

1. $\frac{o}{}\int x^n = (x!)^n$ is valid at ∞≥*n*≥-∞ and also not whole *n*., $\frac{o}{}\int \frac{a}{x} = \frac{a^x}{x!}$
2. $\frac{\Delta}{}\int (x-1)!(x-1) = x!$
3. $\frac{o}{}\int \frac{x(x-2)}{(x-1)^2} = \frac{x}{x-1}$

4. $\frac{\Delta}{}\!\int 2x-1 = x^2 \pm a^2$

5. $\frac{\Delta}{}\!\int 2x = x^2 + x$, $\frac{\Delta}{}\!\int x = \frac{x^2+x}{2}$

6. $\frac{O}{}\!\int \frac{x}{x-1} = ax$

If the value of an integral in any calculus is known, it is easy for translating in any other calculus, using the formulas of translation of table (22). For example, the formula (38) with the registration (40) and formulas of translation from one calculus in another under table (22) can be resulted in an integral № 2 (without a constant factor for simplicity).

The rules of integration are easy for formulating, analyzing rules of differentiation from table 1, regarding definition of an integral (40).

1. *Δ*- or the *S*-integral of the sum (difference) is peer to the sum (difference) of integrals of the separate terms.

2. *O*- or the *П*-integral of product is peer to product of integrals of the separate terms.

3. The *L*-integral of the sum of the separate terms is peer to product of integrals of these terms.

4. The *L*-integral of a difference of the separate terms is peer to the ratio of integrals of these terms.

5. The *Θ*-integral of product is peer to product of reverse values of integrals of the separate terms.

6. Rules of integration for more composite cases, when the function in an exponent of other function is integrated or the function from a function is integrated, directly follow from table (22) in which one the type of a function grows out integrations, and the particular *D*-derivative is an integrand.

Similarly it is possible to enter concept of *D*-differential equations, and also all what disposes the apparatus modern differential and integral calculus. Certainly, it is a huge transactions, but allowing above-stated, as the business of engineering is spoken, as the principle is clear.

6. D-DIFFERENTIAL EQUATIONS

The *D*-differential equations enter similarly to known *d*-equations. These equations are contained with unknowns of a function, arguments and derivatives of unknowns of functions (or their differentials) (example 1). The main difference of *D*-differential equations is that they can be contained derivatives and differentials of any of *D*-calculuses or even their mixture in one equation (example 1). If the unknowns of functions depend only on one argument, the *D*-differential equation will be termed ordinary (example 1). If it is some arguments, this equation with partial derivatives (example 2). The order of differential equation terms best of the orders of derivatives or differentials which are included in an equation (example 3). The author considers permissible to relate to *D*-differential equations as well equations containing integrals (example 4).

Example 1. $(\Delta y)^2 + L' - 2xy = 0$

Example 2. $Oy_z \cdot xyz = \Delta y_z$

Example 3. $\Pi'' y = a$ - second-degree equation.

Example 4. $\Theta y + \frac{O}{}\!\int x^2 - 1 = \Delta'' y$

The methods of the solution of *D*-differential equations are similar to known methods in *d*-calculus. The difference is that always it is possible to simplify the solution by selection of the most successful calculus ad-hoc.

Frequently there can be D-differential equations not relating to any of reviewed before D-numerations. In them the connection between current and previous value of a function is rather complex. Let's put examples of integrating of two similar D-differential equations.

Example 5. $y = 2y_{-1} - Ry_{-1}^2$, where *R* - constant. The integral of this equation is peer:

$$y = \frac{1 - a^{2^x}}{R} \quad (41),$$

where a - any constant. Really, from (41):

$$Ry = 1 - a^{2^x} \quad (42),$$

whence:

$$a^{2^x} = 1 - Ry \quad (43).$$

The previous value of a function:

$$a^{\frac{2^x}{2}} = 1 - Ry_{-1} \quad (44),$$

whence:

$$a^{2^x} = \left(1 - Ry_{-1}\right)^2 = 1 - 2Ry_{-1} + R^2 y_{-1}^2 \quad (45).$$

Equating (43) and (45), we shall discover:

$$y = 2y_{-1} - Ry_{-1}^2 \quad (46),$$

which one coincides with given equation of an example 5.
Verification.
Table 2.

x	0	1	2	3	4
y	$\frac{1-a}{R}$	$\frac{1-a^2}{R}$	$\frac{1-a^4}{R}$	$\frac{1-a^8}{R}$	$\frac{1-a^{16}}{R}$

The tabulated data 2 correspond to equations (41) and (46).
Example 6. More composite version of an equation (46):

$$y = (1+R)y_{-1} - Ry_{-1}^2 \quad (47).$$

The integral of first item in (47) will be peer:

$$y = \left(1+R\right)^x \quad (48).$$

Really, $y_{-1} = \left(1+R\right)^{x-1} = \frac{\left(1+R\right)^x}{1+R}$, whence $y = \left(1+R\right)y_{-1}$. The integral of an augend in (47) will be peer:

$$y = R^{2^x - 1} \quad (49).$$

Really, $y_{-1} = R^{2^{(x-1)}-1} = R^{\frac{2^x}{2}-1} = R^{\frac{2^x-2}{2}} = \sqrt{\frac{R^{2^x}}{R^2}}$, whence $y = Ry_{-1}^2$. Putting together (48) and (49), finally we shall discover:

$$y = \left(1+R\right)^x - R^{2^x - 1} \quad (50).$$

From reduced examples it is visible, that any ratio in an obvious or implicit kind linking two values of a function concern to D-differential equations. At forecasting we as a matter of fact always search for the solution of a D-differential equation.

7. EXAMPLES OF APPLYING OF NEW CALCULUSES

Let's consider some examples of practical use *D*-исчислений.
Problem about a radioactive decay.
It is known, that mass *M* of radioactive matter decreases twice in period τ_0 - time of a half-decay, i.e.:

$$M = \frac{M_{-1}}{2} \quad (51) \qquad \text{or } \Theta M = 2 \quad (52).$$

Integrating (52), we shall receive:

$$M = \overset{\Theta}{\int} 2 = A \cdot e^{-x \ln 2} \tag{53}$$

For unit of a time scale we picked τ_0. To receive relation from τ, it is necessary τ to result in a selected scale, i.e.:

$$x = \frac{\tau}{\tau_0} \tag{54}$$

By substituting (54) in (53), we shall receive:

$$M = Ae^{-\frac{\tau}{\tau_0}\ln 2} \tag{55}$$

At $\tau = 0,\ M = M_0$, therefore, we shall finally have:

$$M = M_0 e^{-\frac{\tau}{\tau_0}\ln 2} \tag{56}$$

As is an equation of a radioactive decay.

Let's consider one more problem resulting in to a Δ-differential equation. Let's suspect that we close down a certain surface by layers of the opaque chaotically arranged particles with mark-to-space ratio α in each layer. It is required to find a closed area *y*, as a function of number of layers *x*. Is apparent, that:

$$y_i = y_{i-1} + (1 - y_{i-1})\alpha \tag{57}$$

Whence it is easy to find a Δ-differential equation, by transferring y_{i-1} in the left-hand part (57):

$$\Delta y = (1 - y_{-1})\alpha \tag{58}$$

Integrating (58), we shall receive:

$$y = 1 - (1 - \alpha)^x \tag{59}$$

Let's check up (59), Δ-differentiate:

$$y = 1 - (1 - \alpha)^x \tag{60}$$

$$y_{-1} = 1 - (1 - \alpha)^{x-1} \tag{61}$$

$$\Delta y = y - y_{-1} = (1 - \alpha)^{x-1} \cdot \alpha \tag{62}$$

By dividing both parts (62) on α, we shall receive:

$$\frac{\Delta y}{\alpha} = (1 - \alpha)^{x-1} \tag{63}$$

From (61): $$(1 - \alpha)^{x-1} = 1 - y_{-1} \tag{64}$$

By substituting (64) in (63), we shall discover: $\Delta y = (1 - y_{-1})\alpha$, i.e. (58).

Let's look, as in Δ-calculus the problem of definition of a kind of a function will look, the second differences by which one are constant. As is known it is parabola. Apparently, that second the Δ-derivative thus is constant:

$$\Delta'' y = a \tag{65}$$

Integrates (65) once:

$$\Delta y = \overset{\Delta}{\int} a = ax + b \tag{66}$$

Integrates (66) once again:

$$y = \overset{\Delta}{\int}(ax + b) = a\overset{\Delta}{\int} x + \overset{\Delta}{\int} b = a\left(\frac{x^2}{2} + \frac{x}{2}\right) + bx + C = \frac{ax^2}{2} + \left(\frac{a}{2} + b\right)x + C = Ax^2 + Bx + c \tag{67}$$

Let's check up (57):

$$y_{-1} = \frac{a(x-1)^2}{2} + \left(\frac{a}{2} + b\right)(x - 1) + C \tag{68}$$

$$\Delta y = y - y_{-1} = ax + b \tag{69}$$

$$\Delta'' y = a \tag{70}$$

References:

http://www.new-physics.narod.ru