The Math of Hua Luogeng – 华罗庚 – Another Game in Town

by Frank D. (Tony) Smith, Jr. - June 2009

Edward Witten said (Nature 438 (22/29 December 2005) 1085):
"… Albert Einstein famously devoted the latter part of his life to seeking a … 'unified field theory' … String theory is the only known generalization of relativistic quantum field theory that makes sense. … string theory … may well be the only way to reconcile gravity and quantum mechanics …".
Steven Weinberg (in a 22 Feb 2009 Telegraph article by Graham Farmelo) is quoted as saying that "… for a truly fundamental theory of physics … "strings are the only game in town" …".

Could Witten and Weinberg both be Wrong ?

Setting up Another Game in Town requires solving Three Tasks:

1 - Construct an EPR Physics Model that is consistent with Experimental Tests of Einstein-Podolsky-Rosen Reality;

2 - Use that EPR Physics Model to construct a Local Lagrangian that gives Gravity and the Standard Model with calculable Force Strengths and Particle Masses – such calculations will be seen to require the Math of Hua Luogeng;

3 - Quantize the Classical Local Lagrangian structure to get a Global Algebraic Quantum Field Theory (AQFT) which, as Bert Schroer said in hep-th/9608083, has "… emphasis on locality and … insistence of
separating local … properties ... resid[ing] in … the algebraic structure of local observables …
from global properties ... enter[ing] through … states and … representation spaces of local observables.

1 - An EPR Physics Model

Joy Christian in arXiv 0904.4259 “Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement” says:
“… a [geometrically] correct local-realistic framework … provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. … The alleged non-localities of these states … result from misidentified [geometries] of the EPR elements of reality. …
The correlations are … the classical correlations among the points of a 3 or 7-sphere … S3 and S7 … are … parallelizable …
The correlations … can be seen most transparently in the elegant language of Clifford algebra …”.

To go beyond the interesting but not completely physically realistic Bell, GHZ-3, GHZ-4, and Hardy states,
we must consider more complicated spaces than S3 and S7,
but still require that they be parallelizable
and be related to Clifford algebra structure.

As Martin Cederwall said in hep-th/9310115: “… The only simply connected compact parallelizable manifolds are
the Lie groups [including S3 = SU(2)] and S7 …”.

We know that S3 = SU(2) = Spin(4) / SU(2) so that it has global symmetry of Spin(4) transformations
and that 6-dimensional Spin(4) is the grade-2 part of the
16-dimensional Cl(4) Clifford algebra with graded structure
16 = 1 + 4 + 6 + 4 + 1 (where grades are 0,1,2, …).

We also know that S7 = Spin(8) / Spin(7) so that it has global symmetry of Spin(8) transformations
and that 28-dimensional Spin(8) is the grade-2 part of the
256-dimensional Cl(8) Clifford algebra with graded structure
256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1.

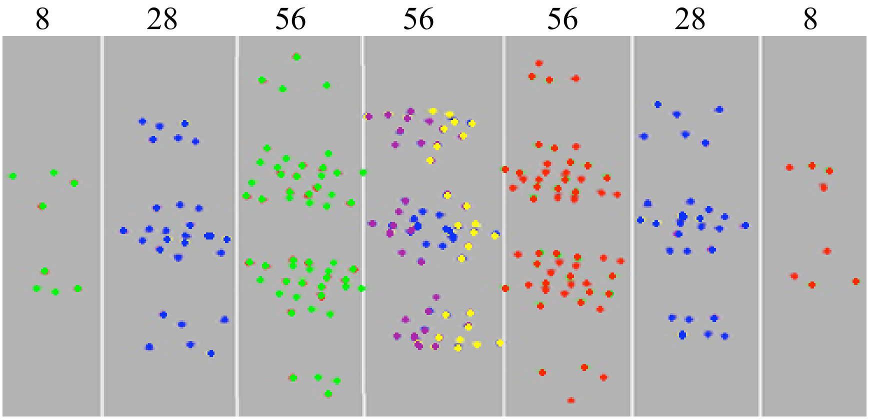
To get a Clifford algebra related parallelizable Lie group large enough to represent a realistic physics model, take the tensor product Cl(8) (x) Cl(8)
which by the 8-periodicity property of Real Clifford algebras is is 256x256 = 65,536-dimensional Cl(16) with graded structure
(1x1) + (1x8+8x1) + (1x28+28x1+8x8) + … = 1 + 16 + 120 + …
whose 28+28+64 = 120-dimensional grade-2 part is Spin(16)
and
whose spinor representation has 256 = 128+128 dimensions.

Spin(16) has Cl(16) Clifford algebra structure and is a Lie group, and therefore parallelizable,
but it has grade-2 bivector bosonic structure and so can only represent physical things like gauge bosons and vector spacetime, and cannot represent physical things like fermions with spinor structure.

However, if we add one of the two 128-dimensional Cl(16) half-spinor representations to the bosonic adjoint 120-dimensional representation of Spin(16) ,
we get the 120+128 = 248-dimensional exceptional Lie group E8 which Garrett Lisi in arXiv 0711.0770 used to try to describe a complete realistic physics model unifying Gravity and the Standard Model. Garrett Lisi acknowledges some flaws in his model of 0711.0770, but here we will use a variant of his approach that unifies Gravity and the Standard Model without such flaws.

248-dimensional E8 has a 7-grading (due to Thomas Larsson)
8 + 28 + 56 + 64 + 56 + 28 + 8
(where grades are -3,-2,-1,0,1,2,3)

If 8 of the 64 central grade-0 elements are assigned to an 8-dimensional Cartan subalgebra of E8, the remaining 248-8 = 240 elements are the 240 Root Vectors of E8 which have a graded structure



that indicates a physical interpretation for each of them:

The 128 odd-grade Root Vectors of E8 come from the 128 half-spinors and are shown as

0 + 0 + 0 + 0 + 56 + 0 + 8 = 64 red
The 8 grade +3 represent the basic state of 8 fermion particles.

8 + 0 + 56 + 0 + 0 + 0 + 0 = 64 green
The 8 grade -3 represent the basic state of 8 fermion antiparticles.

The 120-8 = 112 even-grade Root Vectors of E8 come from the 112 Root Vectors of Spin(16) and are shown as

0 + 28 + 0 + 8 + 0 + 28 + 0 = 64 blue
The 8 grade 0 represent 8 basic spacetime dimensions.

0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 purple
The 24 represent the Root Vectors of a gauge group Spin(8).

0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 gold
The 24 represent the Root Vectors of another gauge group Spin(8).

2 - A Local Lagrangian that gives General Relativity and the Standard Model with Calculable Force Strengths and Particle Masses

Use the E8 physical interpretations to construct a Lagrangian

by

integration over 8-dim spacetime base manifold

of

curvature terms from the two Spin(8) gauge groups

and

a Dirac fermion particle-antiparticle term.

This differs from conventional Gravity plus Standard Model in four respects:

1 - 8-dimensional spacetime
2 – two Spin(8) gauge groups
3 - no Higgs
4 - 1 generation of fermions

These differences can be reconciled as follows:

Introduce (freezing out at lower-than-Planck energies) a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime,
thus forming an 8-dim Kaluza-Klein spacetime M4xCP2
where M4 is 4-dim physical spacetime and CP2 is a 4-dim intrernal symmetry space.

Let the first Spin(8) gauge group act on the M4 physical spacetime through the SU(3) subgroup of its U(4) subgroup.
Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152):
"… each point of … the … fibre bundle … E consists of a four-dimensional spacetime point x [in M4] to which is attached

the homogeneous space G / H [SU(3) / U(2) = CP2] …
the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4])
…
the Yang-Mills action reduces to
a Yang-Mills action for the h-components [u(2) components] of the curvature over M [M4]
and
a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK … Brout-Englert-Higgs-Kibble … mechanism to work. …".

So, freezing out of a Kaluza-Klein M4xCP2 spacetime plus internal symmetry space produces a classical Lagrangian for the SU(3)xU(2) = SU(3)xSU(2)xU(1) Standard Model
Including a BEHK Higgs mechanism.

Let the second Spin(8) gauge group act on the M4 physical spacetime through its Conformal Subgroup U(2,2) = Spin(2,4). Rabindra Mohapatra said (section 14.6 of Unification and Supersymmetry, 2 nd edition, Springer-Verlag 1992):
"… gravitational theory can emerge from the gauging of conformal symmetry … we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. …".
See also MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739) and Chamseddine and West (Nucl. Phys. B 129 (1977) 39).

At this stage, we have reconciled the first 3 of the 4 differences between our E8 Physics Model and conventional Gravity plus the Standard Model. As to the fourth, the existence of 3 generations of

fermions, note that the 8 first generation fermion particles and the 8 first generation antiparticles can each be represented by the 8 basis elements of the Octonions O,
and that the second and third generations can be represented by Pairs of Octonions OxO and Triples of Octonions OxOxO, respectively.
When the unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein M4 x CP2, there are 3 possibilities for a fermion propagator from point A to point B:
1 – A and B are both in M4, so its path can be represented by the single O;
2 – Either A or B, but not both, is in CP2, so its path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O, giving a second generation OxO;
3 – Both A and B are in CP2, so its path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

Therefore, all four differences have been reconciled, and
our classical Lagrangian E8 Physics Model describes Gravity as well as the Standard Model with a BEHK Higgs mechanism.

However, for our classical Lagrangian E8 Physics Model to be said to be complete and realistic, it must allow us to calculate such things as Force Strengths and Particle Masses that are consistent with experimental and observational results.

This requires the use of the Math of Hua Luogeng,
about whom Wang Yuan ("Hua Loo-Keng" , translated by Peter Shiu, Springer 1999) said: "... A mathematician has to be judged by his research accomplishments and not by the number of university degrees earned. In Hua's case there are many of the former and none of the latter ... Teacher Hua only had a junior middle-school education ...

Hua visited the Soviet Union ...[in].... 1946 ... [Later]... in ... 1946 ... Hua Loo-Keng went to the Institute for Advanced Study at Princeton ... In the spring of 1948, Hua Loo-Keng was appointed full professor at the University of Illinois in Urbana. ...
Hua's decision to return to China ... in 1950 ... was based on his belief ... that the Chinese Communist Party and the Chinese Government ... would ... support ... his wish ... for ... mathematics in China ... to arrive at international level ... Besides this, he saw the racial prejudice in the United States …[and]… isolationist policy being implemented against the Chinese Communist Party and all its work ...
In 1935, E. Cartan proved that, under analytic mapping, there are precisely six types of irreducible homogeneous symmetric bounded domains. Of these there are two types of exceptional domains with dimensions 16 and ...[27]... and the remaining four types are called classical domains ... R_I = ... m x n matrix ... R_II = ... symmetric matrices ... R_III = skew-symmetric matrix ... R_IV [= a homogeneous space of 2xn real matrices]…
Classical domains can thus be regarded as the generalisation of the unit disc in the plane to higher dimensions [They are the subject of Hua's book]... Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains [Institute of Mathematics, Academica Sinica, No. 4 in the A-series, Russian translation 1959, English translation 1963] ... Now let R be a bounded and simply connected domain whose points ...[are]… made up of n complex variables, so that the corresponding Euclidean space has real dimension 2n. Suppose that L is a part of the boundary for R satisfying the following condition: Every analytic function in R takes its maximum modulus on L and that, for each point x in L, there is an analytic function f(z) in R which takes its maximum modulus at x. We then call L the characteristic manifold of R [also known as the Shilov Boundary of R], and it is a uniquely determined compact manifold. Generally speaking, L is only a part of the boundary for R ...
The following three kernels [and volumes of related geometric structures] can be computed: ... Bergman kernel ... Cauchy kernel ... Poisson kernel ...

Hua also generalized the notion of a bilinear (fractional linear) transformation in one complex variable to that for several complex variables ...
Hua ... seemed to have been the only person working on the subject ... but the work ... influenced ... the work on the theory of functions of several complex variables by the Russian mathematician I. I. Pyateskij-Shapiro ...".

Hua's calculated volumes related to the kernels and Shilov boundaries are the key to calculation of Force Strengths and Particle Masses. For example, the Lagrangian term for each of the Forces is integrated over the M4 physical spacetime base manifold, but each of the Four Forces sees M4 in terms of its own symmetry, consequently with its own measure which measure is proportional to Hua-calculated volumes. Since M4 was formed by a freezing out of a Quaternionic structure, M4 is a 4-dimensional manifold with Quaternionic structure and therefore can be seen as one of Joseph Wolf's 4 equivalence classes:
for Electromagnetism: T4 = U(1)^4
for Weak Force: S2 x S2 = SU(2) / U(1) x SU(2) / U(1)
for Color Force: CP2 = SU(3) / U(2)
for Gravity: S4 = Spin(5) / Spin(4) = Sp(2) / Sp(1)xSp(1)

When we also take into account the relevant volumes related to the curvature term in the Lagrangian for each force,
and the masses involved for forces with gauge bosons related to mass, the calculations produce results that are reasonably close to experimental observation. Full calculations of Force Srengths, the Dark Energy : Dark Matter : Ordinary Matter Ratio, Particle Masses, and Kobayashi-Maskawa Parameters, and discussion of details oversimplified here (such as signature and particle/spacetime polarizations beyond the basic level, etc.) are in my free pdf book "E8 and Cl(16) = Cl(8) x Cl(8)" which is available at
http://www.tony5m17h.net/E8physicsbook.pdf and
http://www.valdostamuseum.org/hamsmith/E8physicsbook.pdf

3 - A Global Algebraic Quantum Field Theory (AQFT)

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing E8 Global Time.

Mathematically, this is done by embedding E8 into Cl(16) and using a copy of Cl(16) to represent each Local Lagrangian Region.

A Global Structure is then formed by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, Cl(16) = Cl(8)xCl(8) and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16).

Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite II1 von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over C^(2n) (see John Baez's Week 175),
we can take the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8) to produce a generalized Hyperfinite II1 von Neumann factor that gives a natural Algebraic Quantum Field Theory structure for our E8 Physics model,
thus
making it a complete realistic theory that satisfies Einstein's criteria (quoted by Wilczek in the winter 2002 issue of Deadalus) :
"… a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no arbitrary constants …
that is to say,
nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occcur …".

EPR (Christian) => E8 (Lisi) + Cl(16) (Llull)

(Mayer) => M4xCP2 (Batakis) + Higgs + Standard Model

(MacDowell-Mansouri) => Conformal Gravity

(Segal) => Dark Energy : DM : OM

(HUA) => Force Strengths + Particle Masses = EXPERIMENTAL TESTS

Feynman:

The whole purpose of physics is to find a number, with decimal points, etc!

Otherwise you haven't done anything.

(Segal-Connes) => Clifford Real-Periodic HyperFinite Factor AQFT

Gradings and Triality

Frank Dodd (Tony) Smith, Jr. - August 2009

256-dimensional Clifford Algebra Cl(8) has graded structure

with grades 0,1,2,3,4,5,6,7,8

1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1

In E8 physics, the Cl(8) graded structure has physical interpretation:

8 = Vector Spacetime

28 = Spin(8) Gauge Bosons

1+70+1 contains 16 spinors(1+14+1) as components of Cl(16) Primitive Idempotent, which 16 spinors represent 8 Fermion Particles plus 8 Fermion AntiParticles

248-dimensional Lie algebra E8 has graded structure

with grades -3,-2,-1,0,+1,+2,+3

8 + 28 + 56 + 64 + 56 + 28 + 8

with even part 28 + 64 + 28 = 120 dim Adjoint(D8)

and odd part 8 + 56 + 56 + 8 = 128-dim Half-Spinor(D8)

In E8 physics, the E8 graded structure has physical interpretation:

8 = Fermion Particles

64 = 8 Spacetime + 28 Spin(8) Gauge Bosons + 28 (additional) Spin(8) Gauge Bosons

8 = Fermion AntiParticles

The Cl(8) and E8 Graded Structures and their Physical Interpretations are related by Triality.

First: Cl(8) Triality gives isomorphisms among

- **Cl(8) grade 1 Vector Spacetime**
- **Cl(8) grades 0 and 4 Fermion Particles**
- **Cl(8) grades 0 and 4 Fermion AntiParticles**

Second: Triality gives isomorphisms among

- Cl(8) grade 1 Vector Spacetime
- E8 grade -3 Fermion Particles
- E8 grade +3 Fermion AntiParticles

Third: Triality applied to Fermion Particle-AntiParticle Pairs and to BiVectors = Vector Pairs, along with Hodge Duality and taking into account Dirac Gamma - Spacetime Structures, gives isomorphisms among

- 28 of the E8 grade 0 Gauge Bosons
- the other 28 E8 grade 0 Gauge Bosons
- 28 Cl(8) grade 2 Gauge Bosons
- 28 Cl(8) grade 6 Dual-Gauge-Bosons
- 28 E8 grade -2 points representing 7 Dirac Gamma components of 4 Physical Spacetime dimensions of 8-dimensional Kaluza-Klein Vector Spacetime
- 28 E8 grade +2 points representing 7 Dirac Gamma components of 4 CP2 Internal Symmetry Space dimensions of 8-dimensional Kaluza-Klein Spacetime

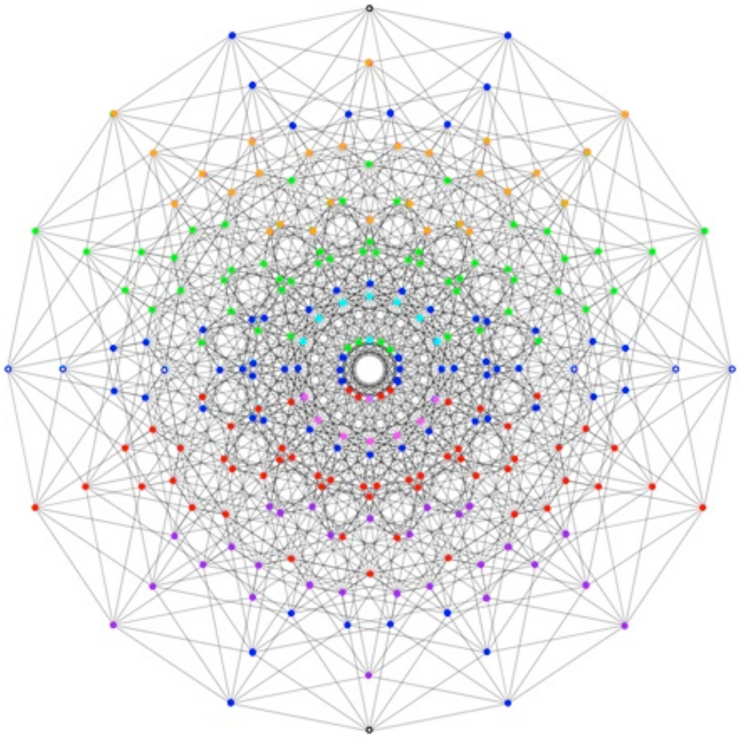
To see how all this works, start by looking at the Cl(8) Graded Structure

and a representation of 16x16 = 256-dimensional Cl(8) as the 2^8 = 256 vertices of an 8-dimesional HyperCube (images here are constructed from Mathematica and some graphics computer programs such as AppleWorks). This HyperCube image has 12 rings of points, with rings having 16,16,16,32,16,16,32,16,32,32,16,16 points respectively, for a total of 8x16 + 4x32 = 256 = 16x16 = 2^8 points.

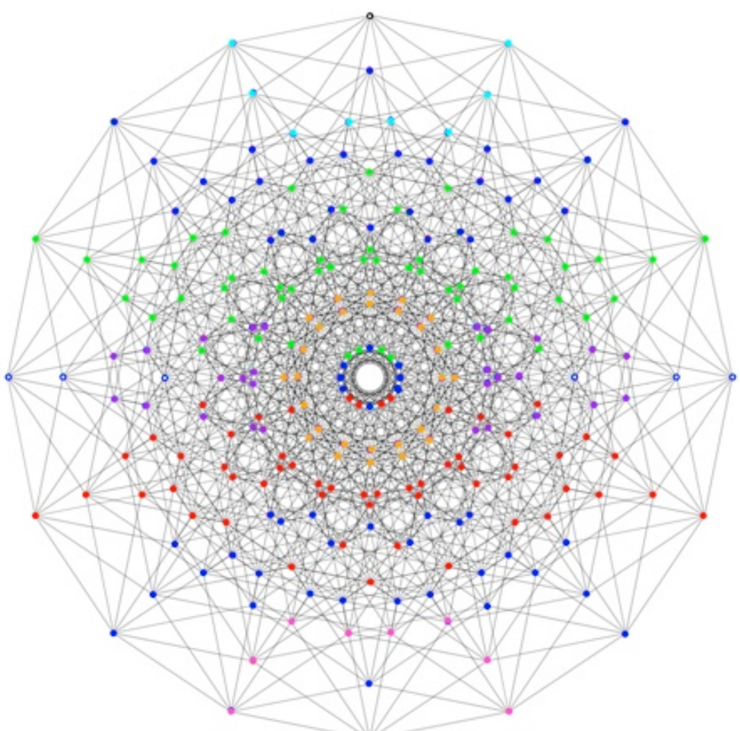
In the image the 256 basis elements of Cl(8) are represented as follows:

- the black point (with central white dot) at the top represents the grade 0 scalar Dirac Gamma
- the 8 blue points near the top represent grade 1 Vector Spacetime
- the 28 gold points near the top represent grade 2 Spin(8) Gauge Bosons
- the 56 green points above the center represent 7 polarization states of 8 Fermion Particles of grade 3
- the 70 points of grade 4 in the center are
    - 7 of the 8 cyan points represent the 7 Quark-Electron-type Fermion Particle Spinor elements which along with the scalar form half of a Cl(8) Primitive Idempotent
    - the 8th cyan point represents the Neutrino-type Fermion Particle Spinor element
    - 7 of the 8 magenta points represent the 7 Quark-Electron-type Fermion AntiParticle Spinor elements which along with the pseudo-scalar form the other half of a Cl(8) Primitive Idempotent
    - the 8th magenta point represents the Neutrino-type Fermion AntiParticle Spinor element
    - 48 blue points represent 6 Dirac Gamma components of 8-dimensional Vector Spacetime
    - 6 blue points (with central white dot) represent the Dirac Gammas beyond the scalar and pseudoscalar
- the 56 red points below the center represent 7 polarization states of 8 Fermion AntiParticles of grade 5
- the 28 purple points near the bottom represent grade 6 dual (position-momentum duality) Spin(8) Gauge Bosons
- the 8 blue points near the bottom represent grade 7 dual (position-momentum duality) Vector Spacetime
- the black point (with central white dot) at the bottom represents the grade 8 pseudo-scalar Dirac Gamma

The 1+6+1 = 8 black and blue points with central white dot, of grades 0,4,8, are the elements of 256-dimensional Cl(8) that are not found in 248-dimensional E8.



Applying the Triality relations to the Cl(8) Graded Structure gives the following picture of E8 Graded Structure (keep in mind that the 1+6+1 central white dot points are not present in 8+28+56+64+56+28+8 = 248-dimensional E8):



In the image the 248 basis elements of E8 are represented as follows

- **the 8 cyan grade -3 points near the top represent Fermion Particles**
- **the 28 blue grade -2 points near the top represent 7 Dirac Gamma components of 4 Physical**

Spacetime dimensions of 8-dimensional Kaluza-Klein Vector Spacetime

- **the 56 green grade -1 points above the center represent 7 polarization states of 8 Fermion Particles**
- **the 64 points of grade 0 in the center are**
  - **8 blue points represent 8-dimensional Kaluza-Klein Spacetime**
  - **28 gold points represent Spin(8) Gauge Bosons**
  - **28 purple points represent a second set of Spin(8) Gauge Bosons**
- **the 56 red grade +1 points below the center represent 7 polarization states of 8 Fermion AntiParticles**
- **the 28 blue grade +2 points near the bottom represent 7 Dirac Gamma components of 4 CP2 Internal Symmetry Space dimensions of 8-dimensional Kaluza-Klein Spacetime**
- **the 8 magenta grade + 3 points near the bottom represent Fermion AntiParticles**

Using the works of Conway and Sloane (Sphere Packings, Lattices and Groups) and Coxeter (Regular Complex Polytopes, Twelve Essays, Kaleidoscopes), we can see that: The basic figure of Cl(8) is the 8-real-dimensional HyperCube (8-cube) with 256 vertices at each of which are 8 edges for a total of 256x8/2 = 128x8 = 1024 edges. At each vertex are 28 squares, 56 cubes, 70 tesseracts, 56 5-cubes, 28 6-cubes, 8 7-cubes, and 1 8-cube for totals of 64x28 = 1792 squares, 32x56 = 1792 cubes, 16x70 = 1120 tesseracts, 8x56 = 448 5-cubes, 4x28 = 112 6-cubes, 2x8 = 16 7-cubes, and 1 8-cube (the 8-dimensional HyperCube itself) as described by Coxeter in his works (such as Regular Complex Polytopes

The 8-cube forms a Z8 lattice, the alternate checkerboard vertices of which form a D8 lattice that is the root vector lattice of the 120-dimensional D8 Lie algebra whose root vector polytope has 112 vertices. There are 128 alternate checkerboard vertices in each 8-cube of the Z8 lattice, and they correspond to a 128-dimensional half-spinor representation of D8.

The 256 vertices of 8-cube (+/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1) lie at distance sqrt(8) = 2 sqrt(2) from the origin among the 2160 vertices in that second layer out from the origin of an E8 lattice 5_21 which has 2160 = 112 + 256 + 1792 vertices with 112 = (+/-2, +/-2, 0, 0, 0, 0, 0, 0) and 256 = (+/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1) which correspond to 112 D8 root vectors + 128 D8 +half-spinors + 128 D8 -half-spinors + 1792. Note that the 128 D8 +half-spinors plus the 128 D8 -half-spinors form, as the two mirror image alternate checkerboards, the 256 vertices of an 8-cube, and also that E8 is made up of two D8 lattices, one containing the 112 D8 root vectors (the 112D8) and the other containing one of the two 128 D8 half-spinors (the 128D8) as described by Conway and Sloane: "... The covering radius ... the distance from a lattice point to a deep hole ... of Dn increases with n, and when n = 8 it is equal to the minimal distance between the lattice points. So when n >= 8 we can slide another copy of D8 in between the points of D8, doubling the number of points ... without reducing the distance between them ... When n = 8 this construction is especially important, the lattice ... being known as E8 ...". The 128D8 has by its defining 128 vertices one alternate checkerboard half of an 8-cube, and the 112D8 contains, by virtue of being a D8 lattice, 128 vertices that can be seen as another alternate checkerboard half of the 8-cube, so that the Triality of subtle supersymmetry in E8 physics can be seen as automorphisms (based on duality of the alternate checkerboard halves of the 8-cube) among 112D8 and each of the two 128D8 lattices. To see how E8 is made up of 112D8 plus a 128D8, note that for any dimension N, the DN root vectors can be shrunken and put around the origin to form a DN lattice. For the 112 D8 root vectors, there are enough deep holes to shrink and put one of the two 128 D8 half-spinor sets of alternate checkerboard parts of the 8-cube in the deep holes.

The 112+128 = 240-vertex inner first layer lies at distance 2 from the origin of the 5_21 lattice and corresponds to the 240 E8 root vectors:

112 = (+/-1, +/-1, 0, 0, 0, 0, 0, 0) with all possible permutations and signs for a total of 4 x 8x7/2 = 112

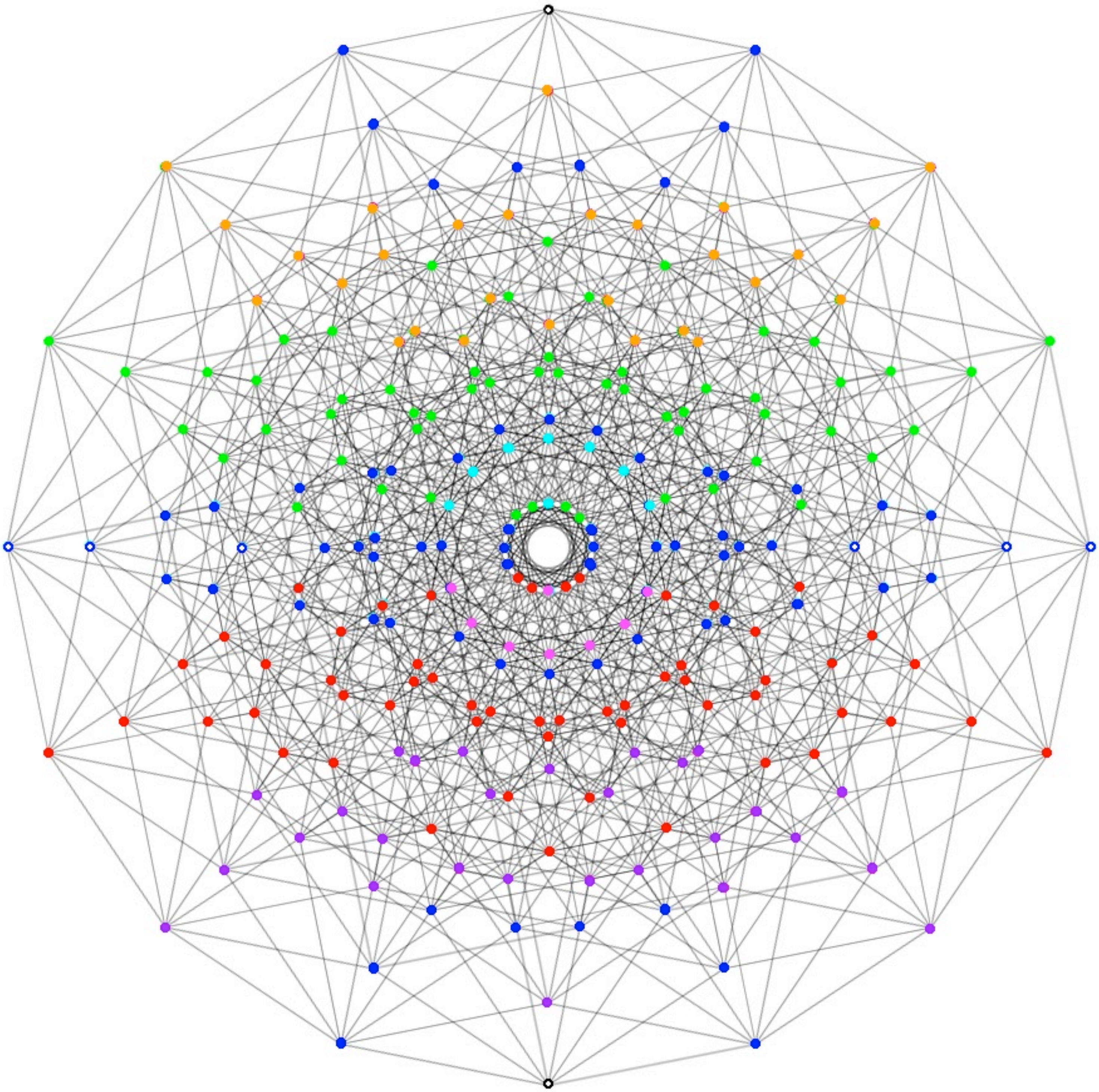
128 = (+/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2) with an even number of minus signs for a total of 2^8 / 2 = 128

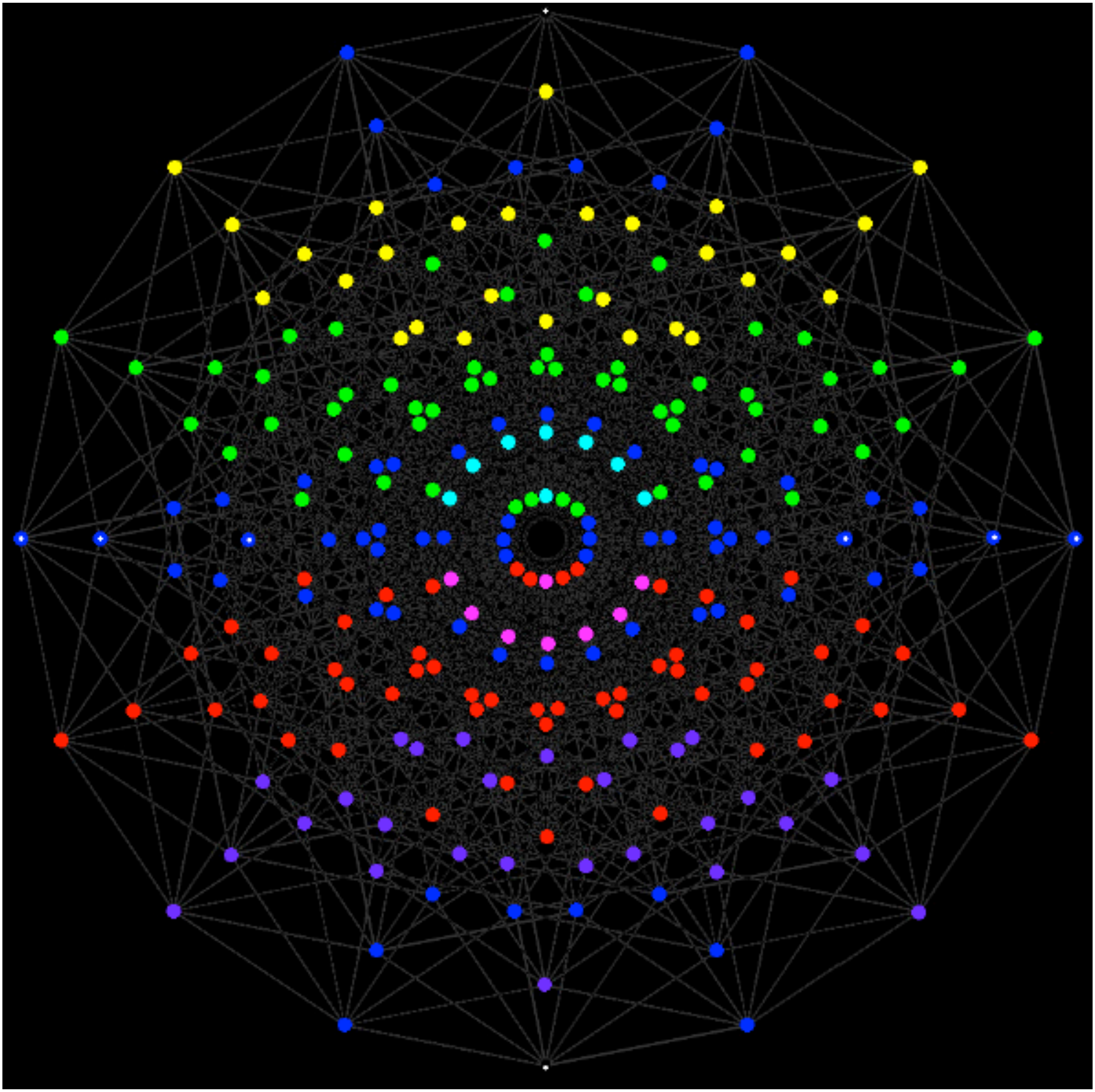
The 112 root vector polytope vertices of the original D8 plus the 128 alternate checkerboard 8-cube vertices of the second D8 form the 240 root vector vertices of the 248-dimensional Lie algebra E8, which can therefore be seen as the sum of the 120-dimensional Lie algebra D8 plus one of its 128-dimensional half-spinor representations.

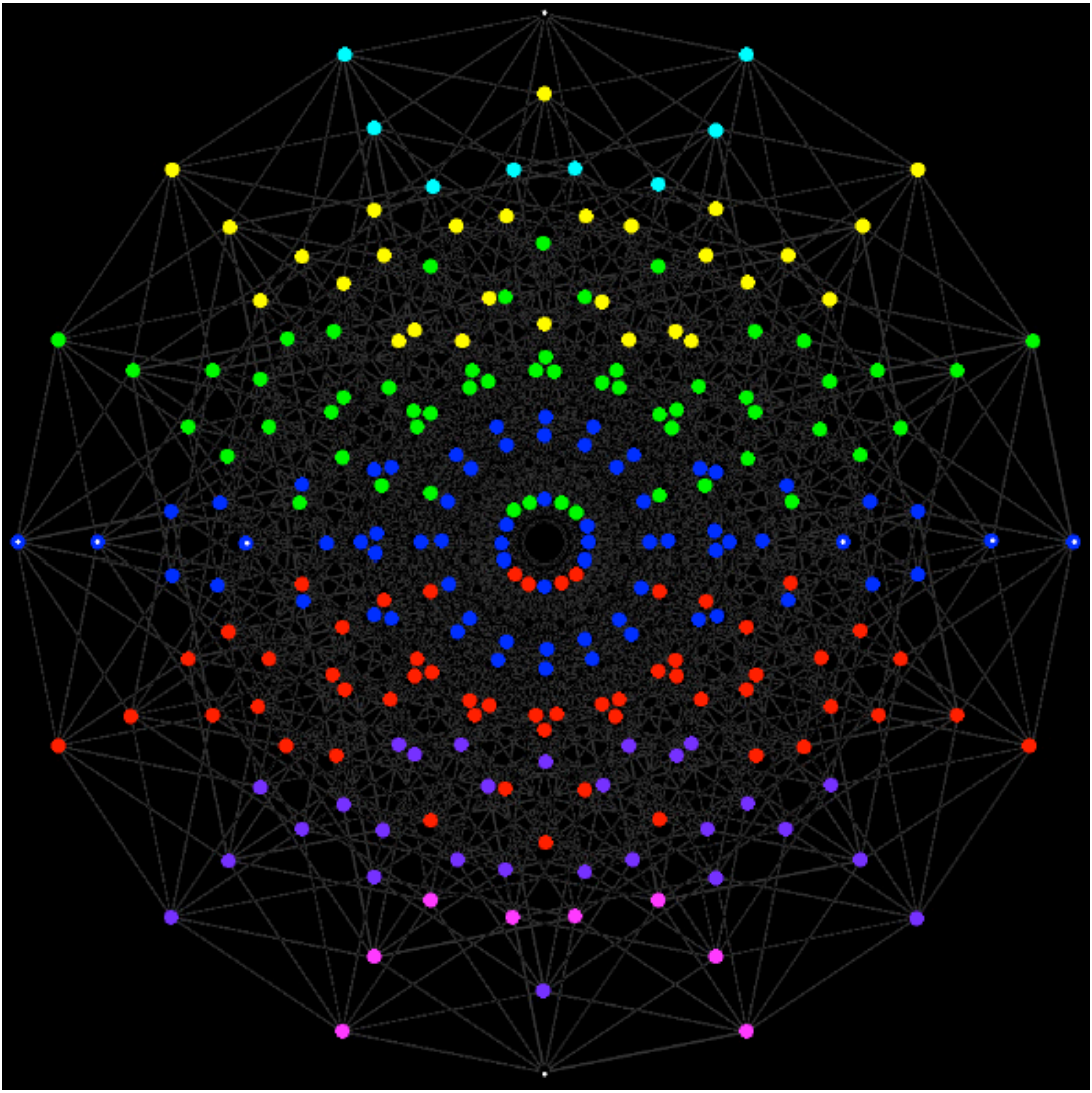
The basic real figure of E8 is the 8-real-dimensional Witting Polytope 4_21 with 240 vertices at each of which are 56 edges for a total of 240x56/2 = 6720 edges. The 56 real edges correspond to the 56 vertices of the 7-real-dimensional polytope 3_21 and to the 56-dimensional representation of E7 and to the 56-dimensional Freudenthal algebra Fr(3,O) of which E6 is its automorphism group.

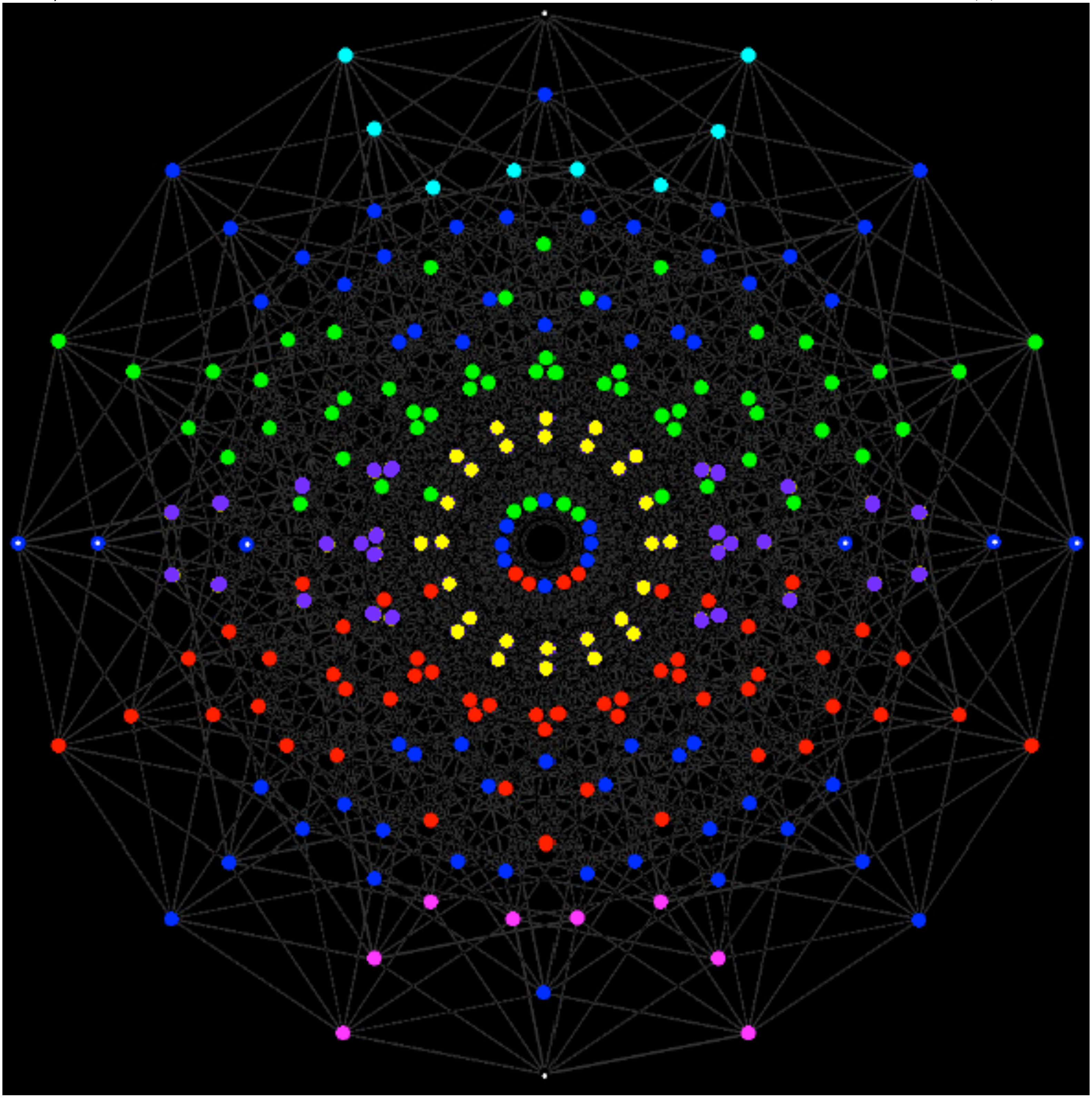
The basic complex figure of E8 is the 4-complex-dimensional Witting Polytope with 240 vertices at each of which are 27 complex (2-real-dimensional triangular) edges for a total of 240x27/3 = 2160 complex edges. The 27 complex edges correspond to the 27 lines on a general cubic surface and the 27 vertices of the 6-real-dimensional polytope 2_21 and to the 27-dimensional representation of E6 and to the 27-dimensional Jordan algebra J(3,O) of which F4 is its automorphism group.

Here is a higher-resolution version of the first above image (8-HyperCube Clifford Grading followed by an 8-HyperCube Clifford Grading image with black background and a black background image of 8-HyperCube after Vector Triality mapping and a black background image of 8-HyperCube after Vector and Bivector Triality mapping.



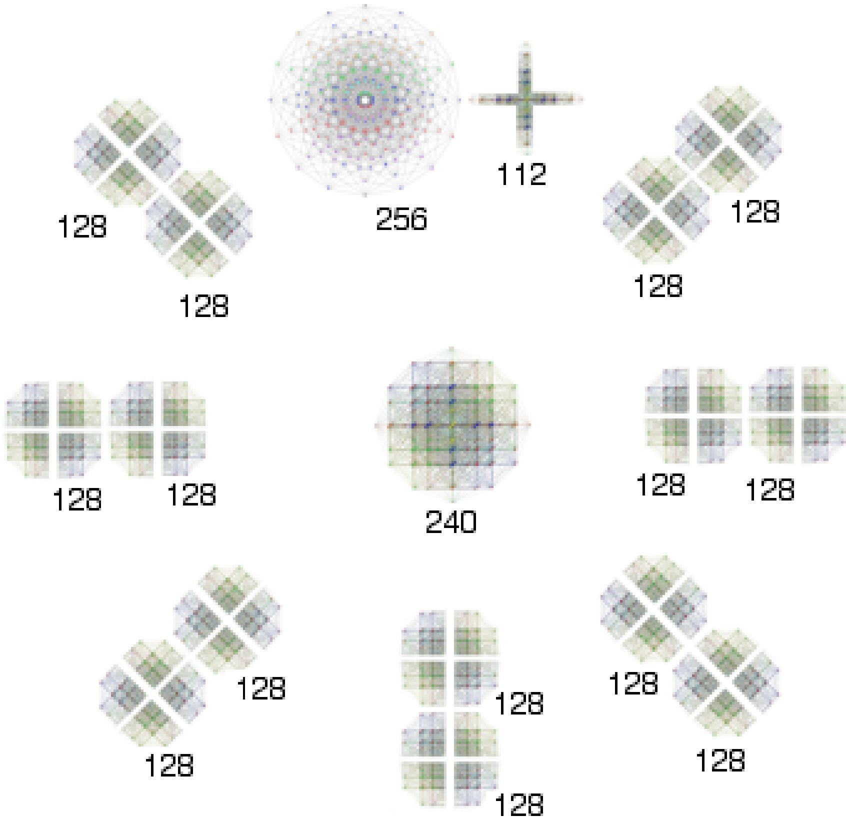






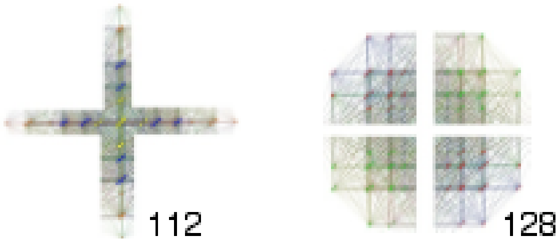
E8 Physics

by Frank D. (Tony) Smith, Jr. – September 2009

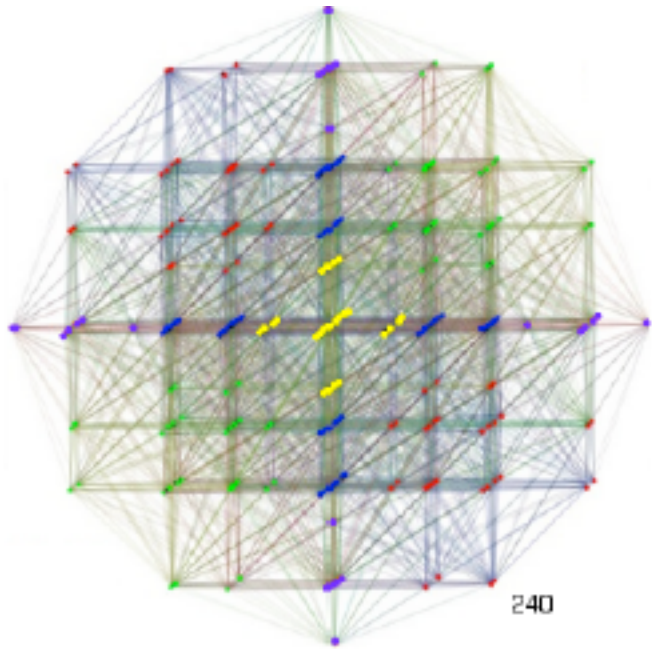


The First and Second shells of an E8 lattice have 240 and 2160 = 112 + 256 + 7(128+128) vertices.
The 256 is an 8-HyperCube with vertices
(+/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1) of which
one checkerboard half represents the 128 +half-spinors of D8 and the other mirror image checkerboard half represents the 128 -half-spinors of

D8.
The 112 represents the 112 root vectors of 120-dim D8.
Each of the 7 pairs of 128 are also representations of the 128 +half-spinors and 128 -half-spinors of D8.
The 112 can be combined with any of the 128



to form the 240 of the First Shell of an E8 latttice



that represents the 112 + 128 = 240 root vectors of E8.
There are 7 pairs of 128 in the Second Shell. Each choice of a pair from which to get a 128 to combine with the 112 produces one of the 7 independent E8 lattices.

You can also choose half of the 256 to combine with the 112 to form an 8^{th} E8 lattice. Although the 8^{th} E8 lattice is not independent of the 7, it is useful in constructing a physics model based on 8-Brane spacetime that in the continuum limit at low (compared to Planck) energies has M4 x CP2 Kaluza-Klein structure. Denote the 7 independent E8 lattices by 1E8, 2E8, 3E8, 4E8, 5E8, 6E8, and 7E8 and the 8^{th} E8 lattice by 8E8.

Note that each of the 8 E8 lattices uses only one of the 128 of a pair (or one half of the 256), and that it corresponds to one of the D8 half-spinor spaces.
Physically, the chosen 128 represents Fermion Particles and AntiParticles of one generation, so the E8 contains one generation of Fermion Particles and AntiParticles (the second and third generations emerge at low energies).
The 128 not chosen represents one antigeneration of Fermion Particles and AntiParticles, so the E8 does not
contain a Fermion antigeneration. Therefore,
the E8 model has realistic chirality properties.

My goal in this paper is to explain how this E8 model is realistic and overcomes the acknowledged shortcomings of Garrett Lisi's E8 model of arXiv 0711.0770 which model was the motivation for me to work on this E8 model. I think that Garrett Lisi should get full credit for doing the basic ground-work for the E8 model.
I hope that this paper shows to its readers that the E8 model and its AQFT constitute a complete realistic theory that satisfies Einstein's criteria (quoted by Wilczek in the winter 2002 issue of Deadalus) :
"… a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no arbitrary constants … that is to say,
nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occcur …".

The remainder of this paper consists of the following sections:

10 spacetime Dimensions of 26-dim Bosonic Strings
16 Fermionic Dimensions of 26-dim Bosonic Strings
Closed Bosonic String World-Lines
Quaternionic M4 x CP2 Kaluza-Klein
Calculations of Masses, Force Strengths, etc
-for detailed results of the calculations etc, see web book at
http://www.tony5m17h.net/E8physicsbook.pdf and
www.valdostamuseum.org/hamsmith/E8physicsbook.pdf
AQFT
EPR Entanglement

10 spacetime Dimensions of 26-dim Bosonic Strings:

An 8-Brane is constructed as a superposition of all of the 8 E8 lattices. Each 8-Brane represents a local neighborhood of spacetime.
Global spacetime is a collection of 8-Branes parameterized by two real variables a , b that are analagous to the conformal dimensions (1,1) that extend (1,3) Minkowski physical spacetime of Spin(1,3) to the (2,4) Conformal spacetime of Spin(2,4) = SU(2,2).
Physical Gauge Bosons link an 8-Brane to a successor 8-Brane along the World Line of that Gauge Boson as follows:
A Gauge Boson emanating from only the 8E8 lattice in the 8-Brane is a U(1) Electromagnetic Photon;
A Gauge Boson emanating from only the 8E8 and the 4E8 lattice in the 8-Brane is a U(2) Weak Boson (note that their common 8E8 unifies the Electromagnetic Photon with the Weak Bosons);
A Gauge Boson emanating from only the 5E8, 6E8, and 7E8 lattices in the 8-Brane is a U(3) Gluon;
A Gauge Boson emanating from only only the 8E8 lattice and the 1E8, 2E8, and 3E8 lattices in the 8-Brane is a U(2,2) = U(1)xSU(2,2) = U(1)xSpin(2,4) Conformal Gauge Boson that gives Gravity by the MacDowell-Mansouri mechanism.

16 Fermionic Dimensions of 26-dim Bosonic Strings:

We now have constructed the 10 dimensions of the base manifold of 26-dim Closed Unoriented Bosonic String Theory, as well as the Gauge Bosons of the Standard Model plus Gravity, in which Strings are physically interpreted as World-Lines,
with relatively large Closed Strings corresponding to World-Lines of particles that locally appear to be free
and relatively small Closed Strings corresponding to paths of virtual particles in the Path Integral Sum-Over-Histories picture.

To describe the one fundamental generation of Fermion Particles and AntiParticles of the E8 model add, to the 10 dimensions we already have, a 16-dimensional space that is discretized by Orbifolding it with respect to the 16-element discrete Octonionic multiplicative group
{+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K}
to reduce the 16-dim Fermionic representation space to 16 points {-1,-i,-j,-k,-E,-I,-J,-K;+1,+i,+j,+k,+E,+I,+J,+K}
for which Fermion Particles (nu, ru, gu, bu, e, rd, gd, bu)
are represented by {-1,-i,-j,-k,-E,-I,-J,-K}
and the corresponding Fermion AntiParticles
are represented by {+1,+i,+j,+k,+E,+I,+J,+K}.

Now our E8 model has realistic first-generation Fermions as well as a base manifold with the Standard Model plus Gravity (M4 x CP2 Kaluza-Klein spacetime, with its 4-dim physical spacetime, and the second and third generations of Fermions, emerge at low temperatures when a preferred Quaternionic substructure freezes out from the high-temperature Octonionic structure).

Closed Bosonic String World-Lines:

Interaction of Closed Bosonic Strings as World-Lines looks like Andrew Gray's idea in quant-ph/9712037
"… probabilities are … assigned to entire fine-grained histories … this new formulation makes the same experimental predictions as quantum

field theory …"
so it seems that physical results of Bosonic String Theory can be interpreted as:

String Tachyons can be physically interpreted as describing the virtual particle-antiparticle clouds that dress the orbifold Fermion particles (As Lubos Motl said in his on 13 July 2005: "... closed string tachyons ... can be localized if they appear in a twisted sector of an orbifold ... tachyons condense near the tip which smears out the tip of the cone which makes the tip nice and round. ..." and as Bert Schroer said in hep-th/9908021: "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles ...").

String spin-2 Gravitons can be physically interpreted as describing a Bohm-like Quantum Potential and what Penrose (in “Shadows of the Mind” (Oxford 1994) with respect to Quantum Consciousness) describes as “… the gravitational self-energy of that mass distribution which is the difference between the mass distributions of … states that are to be considered in quantum linear superposition …”.

The 128 in the 240 of the E8 model breaks up into two 64-element things.
One 64 = 8x8 represents the 8 Dirac gamma covariant components (with respect to high-energy 8-dim spacetime) of each of the 8 fundamental first-generation Fermion Particles;
the other 64 = 8x8 represents the 8 Dirac gamma covariant components (with respect to high-energy 8-dim spacetime) of each of the 8 fundamental first-generation Fermion AntiParticles.

The 112 in the 240 of the E8 model breaks up into three parts: a 64 plus a 24 plus a dual 24.
The 64 = 8x8 in the 112 represents 8 Dirac gammas for the 8 dimensions of high-energy spacetime;
the 24 represents the 24 root vectors of a 28-dim D4 Lie algebra whose generators include those of the Standard Model Gauge Bosons;
the dual 24 represents the 24 root vectors of a second 28-dim D4 Lie algebra whose generators include those of the conformal U(2,2) that

produces Gravity.

Quaternionic M4 x CP2 Kaluza-Klein:

At this stage, the E8 model differs from conventional Gravity plus Standard Model in four respects:
1 - 8-dimensional spacetime
2 – two Spin(8) gauge groups from the two D4 in 112
3 - no Higgs
4 - 1 generation of fermions

These differences can be reconciled as follows:

Introduce (freezing out at lower-than-Planck energies) a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime,
thus forming an 8-dim Kaluza-Klein spacetime M4xCP2
where M4 is 4-dim physical spacetime and CP2 is a 4-dim intrernal symmetry space.

Let the first Spin(8) gauge group act on the M4 physical spacetime through the SU(3) subgroup of its U(4) subgroup. As Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): "… each point of … the … fibre bundle … E consists of a four-dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] … the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4])
…
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4]
and
a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK … Brout-Englert-Higgs-Kibble … mechanism to work. …".

So, freezing out of a Kaluza-Klein M4xCP2 spacetime plus internal symmetry space produces a classical Lagrangian for the SU(3)xU(2) = SU(3)xSU(2)xU(1) Standard Model including a BEHK Higgs mechanism.

Let the second Spin(8) gauge group act on the M4 physical spacetime through its Conformal Subgroup U(2,2) = Spin(2,4). As Rabindra Mohapatra said (section 14.6 of Unification and Supersymmetry, 2nd edition, Springer-Verlag 1992): "… gravitational theory can emerge from the gauging of conformal symmetry … we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. …".
At this stage, we have reconciled the first 3 of the 4 differences between our E8 Physics Model and conventional Gravity plus the Standard Model. As to the fourth, the existence of 3 generations of fermions, note that the 8 first generation fermion particles and the 8 first generation antiparticles can each be represented by the 8 basis elements of the Octonions O, and that the second and third generations can be represented by
Pairs of Octonions OxO and
Triples of Octonions OxOxO, respectively.
When the unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein M4 x CP2, there are 3 possibilities for a fermion propagator from point A to point B:
1 – A and B are both in M4, so its path can be represented by the single O;
2 – Either A or B, but not both, is in CP2, so its path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O, giving a second generation OxO;
3 – Both A and B are in CP2, so its path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O, giving a third generation OxOxO.

Therefore, all four differences have been reconciled, and
our classical Lagrangian E8 Physics Model describes Gravity as well as the Standard Model with a BEHK Higgs mechanism.

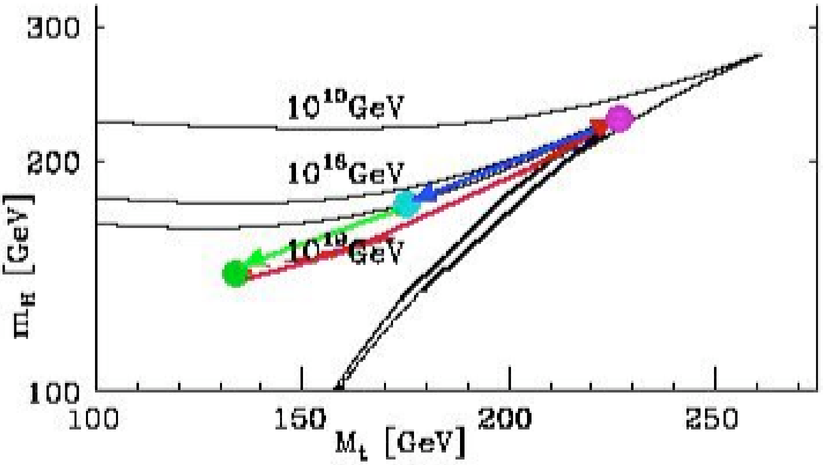
Calculations of Masses, Force Strengths, etc:

However, for our classical Lagrangian E8 Physics Model to be said to be complete and realistic, it must allow us to calculate such things as Force Strengths and Particle Masses that are consistent with experimental and observational results. To do that, we use the results of Hua in his book "Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains". (Similar use of the work of Hua was made years ago by Armand Wyler, and recently by a few others, such as Carlos Castro.)

Hua's calculated volumes related to kernels and Shilov boundaries are the key to calculation of Force Strengths and Particle Masses. For example, the Lagrangian term for each of the Forces is integrated over the M4 physical spacetime base manifold, but each of the Four Forces sees M4 in terms of its own symmetry, consequently with its own measure which measure is proportional to Hua-calculated volumes. Since M4 was formed by a freezing out of a Quaternionic structure, M4 is a 4-dimensional manifold with Quaternionic structure and therefore can be seen as one of Joseph Wolf's 4 equivalence classes:
for Electromagnetism: T4 = U(1)^4
for Weak Force: S2 x S2 = SU(2) / U(1) x SU(2) / U(1)
for Color Force: CP2 = SU(3) / U(2)
for Gravity: S4 = Spin(5) / Spin(4) = Sp(2) / Sp(1)xSp(1)
When we also take into account the relevant volumes related to the curvature term in the Lagrangian for each force,
and the masses involved for forces with gauge bosons related to mass,
the calculations produce results that are reasonably close to experimental observation:

Force Strengths:
Gravity = 5 x 10^-39
Electromagnetic = 1 / 137.03608
Weak = 1.05 x 10^-5
Color at 245 MeV = 0.6286
Renormalization gives Color at 91 GeV = 0.106
and including other effects gives Color at 91 Gev = 0.125

Tree-level fermion masses (Quark masses are constituent masses due to a Bohmian version of Many-Worlds Quantum Theory applied to a confined fermion, in which the fermion is at rest because its kinetic energy is transformed into Bohmian PSI-field potential energy.):
Neutrinos: Me-neutrino = Mmu-neutrino = Mtau-neutrino = 0 at tree-level (first order corrected masses are given below)
Electron/Positron Me = 0.5110 MeV
Up and Down Quarks Md = Mu = 312.8 MeV
Muon Mmu = 104.8 MeV
Strange Quark Ms = 625 MeV
Charm Quark Mc = 2.09 GeV
Tauon Mtau = 1.88 GeV
Beauty Quark Mb = 5.63 GeV
Truth Quark Mt = 130 GeV ground state - 8-dimensional Kaluza-Klein spacetime with Truth-Quark condensate Higgs gives a 3-state system with a renormalization line connecting the 3 states:



(see hep-ph/0307138 for background for chart immediately above)
Low ground state:
Higgs = 146 GeV and T-quark = 130 GeV
Medium Triviality Bound state:
Higgs = 176-188 GeV and T-quark = 172- 175 GeV
High Critical Point state:
Higgs = 239 +/- 3 GeV and T-quark = 218 +/- 3 GeV

Weak Boson Masses (based on a ground state Higgs mass of 146 GeV):
M_W+ = M_W- = 80.326 GeV;
M_Z0 = 80.326 + 11.536 = 91.862 GeV

Kobayashi-Maskawa parameter calculations use phase angle d13 =
= 1 radian (unit length on a phase circumference) to get the K-M matrix:

	d	s	b
u	0.975	0.222	0.00249-0.00388
c	-0.222-0.000161i	0.974-0.0000365i	0.0423
t	0.00698-0.00378i	-0.0418-0.00086i	0.999

Corrections to the tree-level neutrino calculations give neutrino masses
nu_1 = 0
nu_2 = 9 x 10^(-3)eV
nu_3 = 5.4 x 10^(-2) eV
and
the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.87	0.50	0
nu_m	-0.35	0.61	0.71
nu_t	0.35	-0.61	0.71

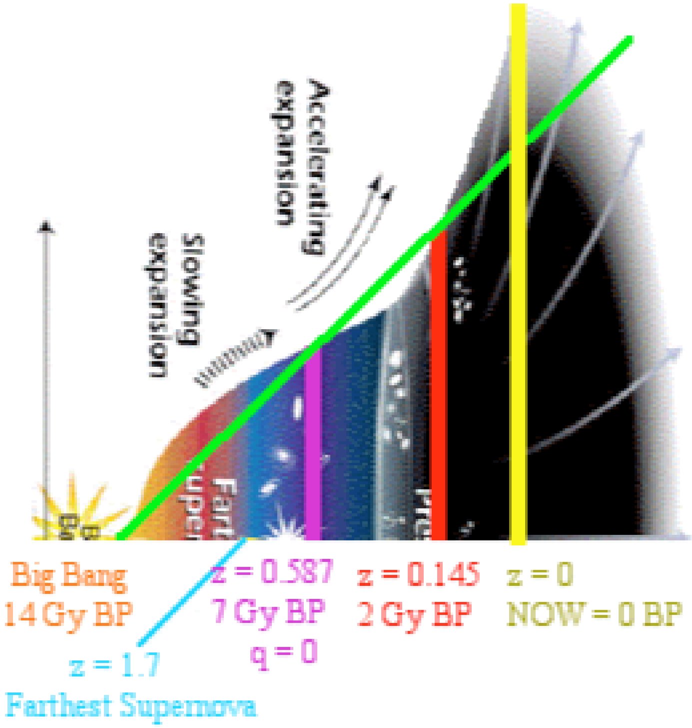
The mass of the charged pion is calculated to be 139 MeV
based on a Kerr-Newman Black Hole model of the pion and its constituent quark-antiquark pair. The pair of Black Holes form a Toroidal Black Hole for which the Torus is an Event Horizon that is (1+1)-dimensional with a timelike dimension which carries a Sine-Gordon Breather whose soliton and antisoliton are the quark and antiquark. The physically relevant Sine-Gordon solution for which the first-order weak coupling expansion is exact gives the ratio of quark constituent mass to the pion mass.

The Neutron-Proton mass difference is calculated to be 1.1 Mev
based on the down quark having virtual states related to the strange quark and the up quark having virtual states related to the charm quark, and the higher probability of strange quark states emerging from the nucleon sea.

The ratio Dark Energy : Dark Matter : Ordinary Matter
for our Universe at the present time is calculated to be:

0.75 : 0.21 : 0.04

based on the Conformal Gravity model of Irving Ezra Segal
and the 15 generators of the Conformal Group Spin(2,4) = SU(2,2)
10 = 6 Lorentz plus 4 Special Conformal = Dark Energy
4 Translations = Dark Matter Primordial Black Holes
1 Dilation = Ordinary Matter mass from Higgs
and the evolution of that basic ratio 10 : 4 : 1 = 0.67 : 0.27 : 0.06
as our universe has expanded



Details of calculations and discussion of some things that here are oversimplified can be found in my free pdf book "E8 and Cl(16) = Cl(8) x Cl(8)" which is available at
http://www.tony5m17h.net/E8physicsbook.pdf and
http://www.valdostamuseum.org/hamsmith/E8physicsbook.pdf

AQFT:

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT). Mathematically, this is done by using Clifford Algebras (others now using Clifford algebras in related ways include Carlos Castro and David Finkelstein) to embed E8 into Cl(16) and using a copy of Cl(16) to represent each Local Lagrangian Region. A Global Structure is then formed by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, Cl(16) = Cl(8)xCl(8) and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16). Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite II1 von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over C^(2n) (see John Baez's Week 175), we can take the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8) to produce a generalized Hyperfinite II1 von Neumann factor that gives a natural Algebraic Quantum Field Theory structure to the E8 model.

EPR Entanglement:

For the E8 model AQFT to be realistic, it must be consistent with EPR entanglement relations. Joy Christian in arXiv 0904.4259 "Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement" said: "… a [geometrically] correct local-realistic framework … provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. … The alleged non-localities of these states … result from misidentified [geometries] of the EPR elements of reality. … The correlations are … the classical correlations among the points of a 3 or 7-sphere … S3 and S7 … are … parallelizable …
The correlations … can be seen most transparently in the elegant language of Clifford algebra …". The E8 model AQFT is based on the parallelizable Lie group E8 and related Clifford algebras,
so the E8 model seems consistent with EPR.