FLORENTIN SMARANDACHE & V. CHRISTIANTO

NEUTROSOPHIC LOGIC, WAVE MECHANICS, AND OTHER STORIES

SELECTED WORKS 2005-2008

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The front cover image represents the first author at a conference in Indonesia in 2006 showing the journals "Progress in Physics" and "Infinite Energy".

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FOREWORD

There is beginning for anything; we used to hear that phrase. The same wisdom word applies to us too. What began in 2005 as a short email on some ideas related to interpretation of the Wave Mechanics results in a number of papers and books up to now. Some of these papers can be found in *Progress in Physics* or elsewhere.

It is often recognized that when a mathematician meets a physics-inclined mind then the result is either a series of endless debates or publication. In our story, we prefer to publish rather than perish.

Therefore, our purpose with this book is to present a selection of published papers in a compilation which enable the readers to find some coherent ideas which appear in those articles. For this reason, the ordering of the papers here is based on categories of ideas.

While some of these articles have been published in book format elsewhere, we hope that reading this book will give the readers an impression of the progress of our thoughts. A few other papers are not yet published elsewhere, or being planned to publish in other journal.

We wish to extend our sincere gratitude to plenty of colleagues, friends and relatives all around the world for sharing their ideas, insightful discussions etc. Special thanks to D. Rabounski, S. Crothers, L. Borissova for their great service in *Progress in Physics* journal.

One of these authors (VC) would like to thank to Profs. A. Yefremov and M. Fil'chenkov for all hospitality extended to him in the *Institute of Gravitation and Cosmology* of PFUR, where this book is prepared. Discussions with Prof. V.V. Kassandrov, Prof. V. Ivashchuk, & Prof. Yu P. Rybakov are appreciated. Many thanks also to Dr. S. Trihandaru and others from UKSW, Central

Java,Indonesia. Sincere thanks to good friends in PFUR, especially to D. Kermite, Y. Umniyati, Anastasia Golubtsova & Serguey– all other friends are of course worth mentioning here, but the margin of this book is quite limited to mention all of you.

And to all other scientist colleagues, allow us to say: Full speed ahead*!*

FS & VC, March 2009

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CONTENT

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NEUTROSOPHIC LOGIC, WAVE MECHANICS, AND OTHER STORIES

SELECTED WORKS 2005-2008

A New Form of Matter — Unmatter, Composed of Particles and Anti-Particles

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Besides *matter* and *antimatter* there must exist *unmatter* (as a new form of matter) in accordance with the neutrosophy theory that between an entity <A> and its opposite <AntiA> there exist intermediate entities <NeutA>. Unmatter is neither matter nor antimatter, but something in between. An atom of unmatter is formed either by (1): electrons, protons, and antineutrons, or by (2): antielectrons, antiprotons, and neutrons. At CERN it will be possible to test the production of unmatter. The existence of unmatter in the universe has a similar chance to that of the antimatter, and its production also difficult for present technologies.

1 Introduction

This article is an improved version of an old manuscript [1]. This is a theoretical assumption about the possible existence of a new form of matter. Up to day the unmatter was not checked in the lab.

According to the neutrosophy theory in philosophy [2], between an entity <A> and its opposite <AntiA> there exist intermediate entities <NeutA> which are neither <A> nor <AntiA>.

Thus, between "matter" and "antimatter" there must exist something which is neither matter nor antimatter, let's call it UNMATTER.

In neutrosophy, <NonA> is what is not <A>, i. e. <NonA> = <AntiA> ∪ <NeutA>. Then, in physics, NONMATTER is what is not matter, i. e. nonmatter means antimatter together with unmatter.

2 Classification

A. **Matter** is made out of electrons, protons, and neutrons.

Each matter atom has electrons, protons, and neutrons, except the atom of ordinary hydrogen which has no neutron.

The number of electrons is equal to the number of protons, and thus the matter atom is neutral.

B. Oppositely, the **antimatter** is made out of antielectrons, antiprotons, and antineutrons.

Each antimatter atom has antielectrons (positrons), antiprotons, and antineutrons, except the antiatom of ordinary hydrogen which has no antineutron.

The number of antielectrons is equal to the number of antiprotons, and thus the antimatter atom is neutral.

C. **Unmatter** means neither matter nor antimatter, but in between, an entity which has common parts from both of them.

Etymologically "un-matter" comes from [ME < OE, akin to Gr. *an-*, *a-*, Latin *in-*, and to the negative elements in *no*, *not*, *nor*] and [ME *matière* < OFr < Latin *material*] matter (see [3]), signifying no/without/off the matter.

There are two types of unmatter atoms, that we call unatoms:

u1. The first type is derived from matter; and a such unmatter atom is formed by electrons, protons, and antineutrons;

u2. The second type is derived from antimatter, and a such unmatter atom is formed by antielectrons, antiprotons, and neutrons.

One unmatter type is oppositely charged with respect to the other, so when they meet they annihilate.

The unmatter nucleus, called **unnucleus**, is formed either by protons and antineutrons in the first type, or by antiprotons and neutrons in the second type.

The charge of unmatter should be neutral, as that of matter or antimatter.

The charge of un-isotopes will also be neutral, as that of isotopes and anti-isotopes. But, if we are interested in a negative or positive charge of un-matter, we can consider an un-ion. For example an anion is negative, then its corresponding unmatter of type 1 will also be negative. While taking a cation, which is positive, its corresponding unmatter of type 1 will also be positive.

Sure, it might be the question of how much *stable* the unmatter is, as J. Murphy pointed out in a private e-mail. But Dirac also theoretically supposed the existence of antimatter in 1928 which resulted from Dirac's mathematical equation, and finally the antimatter was discovered/produced in large accelerators in 1996 when it was created the first atom of antihydrogen which lasted for 37 nanoseconds only.

There does not exist an unmatter atom of ordinary hydrogen, neither an unnucleus of ordinary hydrogen since the ordinary hydrogen has no neutron. Yet, two isotopes of the hydrogen, *deuterium* (^{2}H) which has one neutron, and artificially made *tritium* (^{3}H) which has two neutrons have corresponding unmatter atoms of both types, *un-deuterium* and *un-tritium* respectively. The isotopes of an element X differ in the number of neutrons, thus their nuclear mass is different, but their nuclear charges are the same.

For all other matter atom X, there is corresponding an antimatter atom and two unmatter atoms

The unmatter atoms are also neutral for the same reason that either the number of electrons is equal to the number of protons in the first type, or the number of antielectrons is equal to the number of antiprotons in the second type.

If antimatter exists then a higher probability would be for the unmatter to exist, and reciprocally.

Unmatter atoms of the same type stick together form an **unmatter molecule** (we call it **unmolecule**), and so on. Similarly one has two types of unmatter molecules.

The *isotopes* of an atom or element X have the same atomic number (same number of protons in the nucleus) but different atomic masses because the different number of neutrons.

Therefore, similarly the **un-isotopes of type 1** of X will be formed by electrons, protons, and antineutrons, while the **un-isotopes of type 2** of X will be formed by antielectrons, antiprotons, and neutrons.

An *ion* is an atom (or group of atoms) X which has last one or more electrons (and as a consequence carries a negative charge, called *anion*, or has gained one or more electrons (and as a consequence carries a positive charge, called *cation*).

Similarly to isotopes, the **un-ion of type 1** (also called **un-anion 1** or **un-cation 1** if resulted from a negatively or respectively positive charge ion) of X will be formed by electrons, protons, and antineutrons, while the **un-ion of type 2** of X (also called **un-anion 2** or **un-cation 2** if resulted from a negatively or respectively positive charge ion) will be formed by antielectrons, antiprotons, and neutrons.

The ion and the un-ion of type 1 have the same charges, while the ion and un-ion of type 2 have opposite charges.

D. Nonmatter means what is not matter, therefore nonmatter actually comprises antimatter and unmatter. Similarly one defines a nonnucleus.

3 Unmatter propulsion

We think (as a prediction or supposition) it could be possible at using unmatter as fuel for space rockets or for weapons platforms because, in a similar way as antimatter is presupposed to do [4, 5], its mass converted into energy will be fuel for propulsion.

It seems to be a little easier to build unmatter than antimatter because we need say antielectrons and antiprotons only (no need for antineutrons), but the resulting energy might be less than in matter-antimatter collision.

We can collide unmatter 1 with unmatter 2, or unmatter 1 with antimatter, or unmatter 2 with matter.

When two, three, or four of them (unmatter 1, unmatter 2, matter, antimatter) collide together, they annihilate and turn into energy which can materialize at high energy into new particles and antiparticles.

4 Existence of unmatter

The existence of unmatter in the universe has a similar chance to that of the antimatter, and its production also difficult for present technologies. At CERN it will be possible to test the production of unmatter.

If antimatter exists then a higher probability would be for the unmatter to exist, and reciprocally.

The 1998 Alpha Magnetic Spectrometer (AMS) flown on the International Space Station orbiting the Earth would be able to detect, besides cosmic antimatter, unmatter if any.

5 Experiments

Besides colliding electrons, or protons, would be interesting in colliding neutrons. Also, colliding a neutron with an antineutron in accelerators.

We think it might be easier to produce in an experiment an unmatter atom of deuterium (we can call it un-deuterium of type 1). The deuterium, which is an isotope of the ordinary hydrogen, has an electron, a proton, and a neutron. The idea would be to convert/transform in a deuterium atom the neutron into an antineutron, then study the properties of the resulting un-deuterium 1.

Or, similarly for un-deuterium 2, to convert/transform in a deuterium atom the electron into an antielectron, and the proton into an antiproton (we can call it un-deuterium of type 2).

Or maybe choose another chemical element for which any of the previous conversions/transformations might be possible.

6 Neutrons and antineutrons

Hadrons consist of baryons and mesons and interact via strong force.

Protons, neutrons, and many other hadrons are composed from quarks, which are a class of fermions that possess a fractional electric charge. For each type of quark there exists a corresponding antiquark. Quarks are characterized by properties such as *flavor* (up, down, charm, strange, top, or bottom) and *color* (red, blue, or green).

A neutron is made up of quarks, while an antineutron is made up of antiquarks.

A neutron (see [9]) has one Up quark (with the charge of $+\frac{2}{3}\times 1.606\times 10^{19}$ C) and two Down quarks (each with the charge of $-\frac{1}{3}\times 1.606\times 10^{19}$ C), while an antineutron has one anti Up quark (with the charge of $-\frac{2}{3}\times 1.606\times 10^{19}$ C) and two anti Down quarks (each with the charge of $+\frac{1}{3}\times 1.606\times\times 10^{19}$ C).

An antineutron has also a neutral charge, through it is opposite to a neutron, and they annihilate each other when meeting.

Both, the neutron and the antineutron, are neither attracted to nor repelling from charges particles.

7 Characteristics of unmatter

Unmatter should look identical to antimatter and matter, also the gravitation should similarly act on all three of them. Unmatter may have, analogously to antimatter, utility in medicine and may be stored in vacuum in traps which have the required configuration of electric and magnetic fields for several months.

8 Open Questions

8.a Can a matter atom and an unmatter atom of first type stick together to form a molecule?

8.b Can an antimatter atom and an unmatter atom of second type stick together to form a molecule?

8.c There might be not only a You and an anti-You, but some versions of an un-You in between You and anti-You. There might exist un-planets, un-stars, un-galaxies? There might be, besides our universe, an anti-universe, and more un-universes?

8.d Could this unmatter explain why we see such an imbalance between matter and antimatter in our corner of the universe? (Jeff Farinacci)

8.e If matter is thought to create gravity, is there any way that antimatter or unmatter can create antigravity or ungravity? (Mike Shafer from Cornell University)

I assume that since the magnetic field or the gravitons generate gravitation for the matter, then for antimatter and unmatter the corresponding magnetic fields or gravitons would look different since the charges of subatomic particles are different. . .

I wonder how would the universal law of attraction be for antimmater and unmatter?

References

1. Smarandache F. Unmatter, mss., 1980, Archives Vâlcea.
2. Smarandache F. A unifying field in logics: neutrosophic logic. Neutrosophy, Neutrosophic Probability, Set, and Logic. American Research Press, Rehoboth, 2002, 144 p. (see it in e-print: http://www.gallup.unm.edu/∼smarandache).
3. Webster's New World Dictionary. Third College Edition, Simon and Schuster Inc., 1988.
4. Mondardini R. The history of antimatter. CERN Laboratory, Genève, on-line http://livefromcern.web.cern.ch/livefromcern/antimatter/history/AM-history00.html.
5. De Rújula A. and Landua R. Antimatter — frequently asked questions. CERN Laboratory, Genève, http://livefromcern.web.cern.ch/livefromcern/antimatter/FAQ1.html.
6. Pompos A. Inquiring minds — questions about physics. Fermilab, see on-line http://www.fnal.gov/pub/inquiring/questions/antineuron.html.

Verifying Unmatter by Experiments, More Types of Unmatter, and a Quantum Chromodynamics Formula

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As shown, experiments registered unmatter: a new kind of matter whose atoms include both nucleons and anti-nucleons, while their life span was very short, no more than 10^{-20}sec. Stable states of unmatter can be built on quarks and anti-quarks: applying the unmatter principle here it is obtained a quantum chromodynamics formula that gives many combinations of unmatter built on quarks and anti-quarks.

In the last time, before the apparition of my articles defining "matter, antimatter, and unmatter" [1, 2], and Dr. S. Chubb's pertinent comment [3] on unmatter, new development has been made to the unmatter topic.

1 Definition of Unmatter

In short, unmatter is formed by matter and antimatter that bind together [1, 2]. The building blocks (most elementary particles known today) are 6 quarks and 6 leptons; their 12 antiparticles also exist. Then *unmatter* will be formed by at least a building block and at least an antibuilding block which can bind together.

2 Exotic atom

If in an atom we substitute one or more particles by other particles of the same charge (constituents) we obtain an exotic atom whose particles are held together due to the electric charge. For example, we can substitute in an ordinary atom one or more electrons by other negative particles (say π^-, anti-ρ-meson, D^-, D_s^- - muon, τ, Ω^-, Δ^-, etc., generally clusters of quarks and antiquarks whose total charge is negative), or the positively charged nucleus replaced by other positive particle (say clusters of quarks and antiquarks whose total charge is positive, etc).

3 Unmatter atom

It is possible to define the unmatter in a more general way, using the exotic atom. The classical unmatter atoms were formed by particles like:

(a) electrons, protons, and antineutrons, or

(b) antielectrons, antiprotons, and neutrons.

In a more general definition, an unmatter atom is a system of particles as above, or such that one or more particles are replaces by other particles of the same charge. Other categories would be:

(c) a matter atom with where one or more (but not all) of the electrons and/or protons are replaced by antimatter particles of the same corresponding charges, and

(d) an antimatter atom such that one or more (but not all) of the antielectrons and/or antiprotons are replaced by matter particles of the same corresponding charges.

In a more composed system we can substitute a particle by an unmatter particle and form an unmatter atom.

Of course, not all of these combinations are stable, semi-stable, or quasi-stable, especially when their time to bind together might be longer than their lifespan.

4 Examples of unmatter

During 1970-1975 numerous pure experimental verifications were obtained proving that "atom-like" systems built on nucleons (protons and neutrons) and anti-nucleons (anti-protons and anti-neutrons) are real. Such "atoms", where nucleon and anti-nucleon are moving at the opposite sides of the same orbit around the common centre of mass, are very unstable, their life span is no more than 10^{-20} sec. Then nucleon and anti-nucleon annihilate into gamma-quanta and more light particles (pions) which can not be connected with one another, see [6, 7, 8]. The experiments were done in mainly Brookhaven National Laboratory (USA) and, partially, CERN (Switzerland), where "proton — anti-proton" and "anti-proton — neutron" atoms were observed, called them $\bar{p}p$ and $\bar{p}n$ respectively, see Fig. 1 and Fig. 2.

After the experiments were done, the life span of such "atoms" was calculated in theoretical way in Chapiro's works [9, 10, 11]. His main idea was that nuclear forces, acting between nucleon and anti-nucleon, can keep them far way from each other, hindering their annihilation. For instance, a proton and anti-proton are located at the opposite sides in the same orbit and they are moved around the orbit centre. If the diameter of their orbit is much more than the diameter of "annihilation area", they can be kept out of annihilation (see Fig. 3). But because the orbit, according to Quantum Mechanics, is an actual cloud spreading far around the average radius, at any radius between the proton and the anti-proton there is a probability that they can meet one another at the annihilation distance. Therefore "nucleon — anti-nucleon" system annihilates in any case, this system

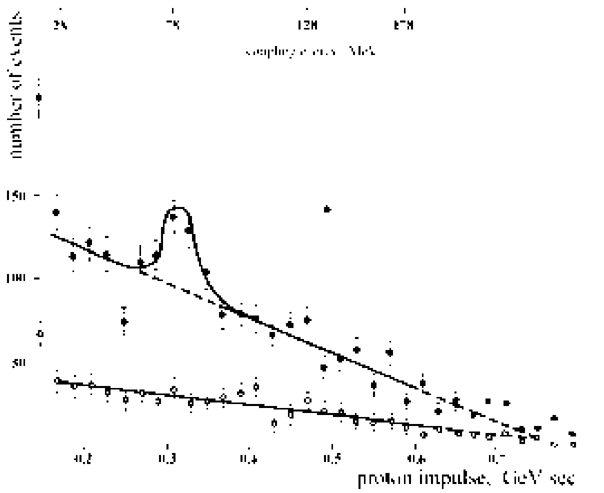

Fig. 1: Spectra of proton impulses in the reaction $\bar{p}+d \to (\bar{p}n)+p$. The upper arc — annihilation of $\bar{p}n$ into even number of pions, the lower arc — its annihilation into odd number of pions. The observed maximum points out that there is a connected system$\bar{p}n$. Abscissa axis represents the proton impulse in GeV/sec (and the connection energy of the system $\bar{p}n$). Ordinate axis — the number of events. Cited from [6].

is unstable by definition having life span no more than 10^{-20} sec.

Unfortunately, the researchers limited the research to the consideration of $\bar{p}p$ and $\bar{p}n$ "atoms" only. The reason was that they, in the absence of a theory, considered $\bar{p}p$ and $\bar{p}n$ "atoms" as only a rare exception, which gives no classes of matter.

Despite Benn Tannenbaum's and Randall J. Scalise's rejections of unmatter and Scalise's personal attack on me in a true Ancient Inquisitionist style under MadSci moderator John Link's tolerance (MadSci web site, June-July 2005), the unmatter does exists, for example some messons and antimessons, through for a trifling of a second lifetime, so the pions are unmatter*, the kaon K^+ (usˆ), K^- (uˆs), Phi (ssˆ), D^+ (cdˆ), D^0 (cuˆ), D_s^+ (csˆ), J/Psi (ccˆ), B^- (buˆ), B^0 (dbˆ), B_s^0 (sbˆ), Upsilon (bbˆ), etc. are unmatter too†.

Also, the pentaquark theta-plus Θ^+, of charge $+1$, uuddsˆ (i. e. two quarks up, two quarks down, and one anti-strange quark), at a mass of 1.54 GeV and a narrow width of 22 MeV, is unmatter, observed in 2003 at the Jefferson Lab in Newport News, Virginia, in the experiments that involved multi-GeV photons impacting a deuterium target. Similar pentaquark evidence was obtained by Takashi Nakano of Osaka University in 2002, by researchers at the ELSA accelerator in Bonn in 1997-1998, and by researchers at ITEP in Moscow in 1986. Besides theta-plus, evidence has been

*Which have the composition uˆd and udˆ, where by uˆ we mean anti-up quark, d = down quark, and analogously u = up quark and dˆ = anti-down quark, while by ˆ we mean "anti".

†Here c = charm quark, s = strange quark, b = bottom quark.

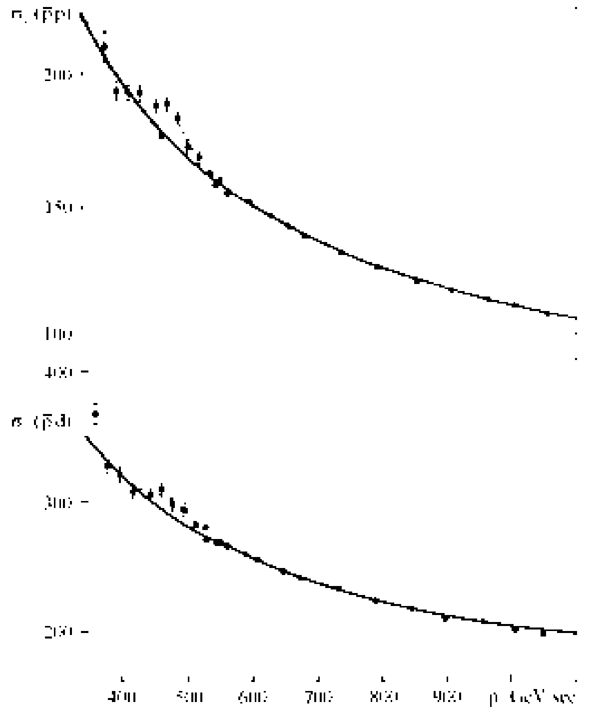

Fig. 2: Probability σ of interaction between $\bar{p}$, p and deutrons d (cited from [7]). The presence of maximum stands out the existence of the resonance state of "nucleon — anti-nucleon".

found in one experiment [4] for other pentaquarks, Ξ_s^- (ddssuˆ) and Ξ_s^+ (uussdˆ).

In order for the paper to be self-contained let's recall that the *pionium* is formed by a π^+ and π^- mesons, the *positronium* is formed by an antielectron (positron) and an electron in a semi-stable arrangement, the *protonium* is formed by a proton and an antiproton also semi-stable, the *antiprotonic helium* is formed by an antiproton and electron together with the helium nucleus (semi-stable), and *muonium* is formed by a positive muon and an electron. Also, the *mesonic atom* is an ordinary atom with one or more of its electrons replaced by negative mesons. The *strange matter* is a ultra-dense matter formed by a big number of strange quarks bounded together with an electron atmosphere (this strange matter is hypothetical).

From the exotic atom, the pionium, positronium, protonium, antiprotonic helium, and muonium are unmatter. The mesonic atom is unmatter if the electron(s) are replaced by negatively-charged antimessons. Also we can define a mesonic antiatom as an ordinary antiatomic nucleous with one or more of its antielectrons replaced by positively-charged mesons. Hence, this mesonic antiatom is unmatter if the antielectron(s) are replaced by positively-charged messons. The strange matter can be unmatter if these exists at least an antiquark together with so many quarks in the nucleous. Also, we can define the strange antimatter as formed by

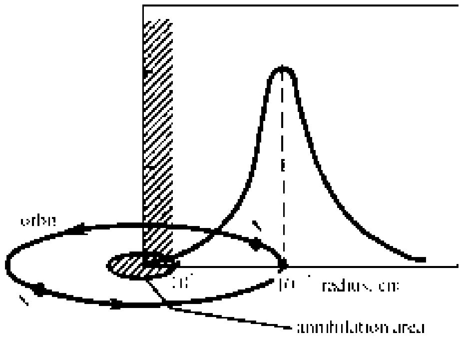

Fig. 3: Annihilation area and the probability arc in "nucleon — anti-nucleon" system (cited from [11]).

a large number of antiquarks bound together with an anti-electron around them. Similarly, the strange antimatter can be unmatter if there exists at least one quark together with so many antiquarks in its nucleous.

The bosons and antibosons help in the decay of unmatter. There are 13 + 1 (Higgs boson) known bosons and 14 anti-bosons in present.

5 Quantum Chromodynamics formula

In order to save the colorless combinations prevailed in the Theory of Quantum Chromodynamics (QCD) of quarks and antiquarks in their combinations when binding, we devise the following formula:

$$\mathrm{Q} - \mathrm{A} \in \pm \mathrm{M3}\,, \tag{1}$$

where M3 means multiple of three, i. e. $\pm\mathrm{M3} = \{3k \mid k \in Z\} = \{\ldots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \ldots\}$, and Q = number of quarks, A = number of antiquarks. But (1) is equivalent to

$$\mathrm{Q} \equiv \mathrm{A} \pmod 3 \tag{2}$$

(Q is congruent to A modulo 3).

To justify this formula we mention that 3 quarks form a colorless combination, and any multiple of three (M3) combination of quarks too, i. e. 6, 9, 12, etc. quarks. In a similar way, 3 antiquarks form a colorless combination, and any multiple of three (M3) combination of antiquarks too, i. e. 6, 9, 12, etc. antiquarks. Hence, when we have hybrid combinations of quarks and antiquarks, a quark and an antiquark will annihilate their colors and, therefore, what's left should be a multiple of three number of quarks (in the case when the number of quarks is bigger, and the difference in the formula is positive), or a multiple of three number of antiquarks (in the case when the number of antiquarks is bigger, and the difference in the formula is negative).

6 Quark-antiquark combinations

Let's note by q = quark ∈ {Up, Down, Top, Bottom, Strange, Charm}, and by a = antiquark ∈ {Upˆ, Downˆ, Topˆ, Bottomˆ, Strangeˆ, Charmˆ}. Hence, for combinations of n quarks and antiquarks, $n \geqslant 2$, prevailing the colorless, we have the following possibilities:

- if n = 2, we have: qa (biquark — for example the mesons and antimessons);
- if n = 3, we have qqq, aaa (triquark — for example the baryons and antibaryons);
- if n = 4, we have qqaa (tetraquark);
- if n = 5, we have qqqqa, aaaaq (pentaquark);
- if n = 6, we have qqqaaa, qqqqqq, aaaaaa (hexaquark);
- if n = 7, we have qqqqqaa, qqaaaaa (septiquark);
- if n = 8, we have qqqqaaaa, qqqqqqaa, qqaaaaaa (octoquark);
- if n = 9, we have qqqqqqqqq, qqqqqqaaa, qqqaaaaaa, aaaaaaaaa (nonaquark);
- if n = 10, we have qqqqqaaaaa, qqqqqqqqaa, qqaaaaaaaa (decaquark); etc.

7 Unmatter combinations

From the above general case we extract the unmatter combinations:

- For combinations of 2 we have: qa (unmatter biquark), mesons and antimesons; the number of all possible unmatter combinations will be 6×6 = 36, but not all of them will bind together.

It is possible to combine an entity with its mirror opposite and still bound them, such as: uuˆ, ddˆ, ssˆ, ccˆ, bbˆ which form mesons. It is possible to combine, unmatter + unmatter = unmatter, as in udˆ + usˆ = uudˆsˆ (of course if they bind together).

- For combinations of 3 (unmatter triquark) we can not form unmatter since the colorless can not hold.
- For combinations of 4 we have: qqaa (unmatter tetraquark); the number of all possible unmatter combinations will be $6^2 \times 6^2$ = 1,296, but not all of them will bind together.
- For combinations of 5 we have: qqqqa, or aaaaq (unmatter pentaquarks); the number of all possible unmatter combinations will be $6^4 \times 6 + 6^4 \times 6$ = 15,552, but not all of them will bind together.
- For combinations of 6 we have: qqqaaa (unmatter hexaquarks); the number of all possible unmatter combinations will be $6^3 \times 6^3$ = 46,656, but not all of them will bind together.
- For combinations of 7 we have: qqqqqaa, qqaaaaa (unmatter septiquarks); the number of all possible unmatter combinations will be $6^5 \times 6^2 + 6^2 \times 6^5$ = 559,872, but not all of them will bind together.

- For combinations of 8 we have: qqqqaaaa, qqqqqqqa, qaaaaaaa (unmatter octoquarks); the number of all the unmatter combinations will be $6^4{\times}6^4+6^7{\times}6^1+6^1\times6^7=$ $=5{,}038{,}848$, but not all of them will bind together.
- For combinations of 9 we have types: qqqqqqaaa, qqqaaaaaa (unmatter nonaquarks); the number of all the unmatter combinations will be $6^6{\times}6^3+6^3{\times}6^6{=}2{\times}6^9$ $=20{,}155{,}392$, but not all of them will bind together.
- For combinations of 10 we have types: qqqqqqqqaa, qqqqqaaaaa, qqaaaaaaaa (unmatter decaquarks); the number of all the unmatter combinations will be $3{\times}6^{10}=181{,}398{,}528$, but not all of them will bind together. Etc.

I wonder if it is possible to make infinitely many combinations of quarks/antiquarks and leptons/antileptons... Unmatter can combine with matter and/or antimatter and the result may be any of these three. Some unmatter could be in the strong force, hence part of hadrons.

8 Unmatter charge

The charge of unmatter may be positive as in the pentaquark theta-plus, 0 (as in positronium), or negative as in anti-ρ-meson (uˆd) (M. Jordan).

9 Containment

I think for the containment of antimatter and unmatter it would be possible to use electromagnetic fields (a container whose walls are electromagnetic fields). But its duration is unknown.

10 Further research

Let's start from neutrosophy [13], which is a generalization of dialectics, i. e. not only the opposites are combined but also the neutralities. Why? Because when an idea is launched, a category of people will accept it, others will reject it, and a third one will ignore it (don't care). But the dynamics between these three categories changes, so somebody accepting it might later reject or ignore it, or an ignorant will accept it or reject it, and so on. Similarly the dynamicity of <A>, <antiA>, <neutA>, where <neutA> means neither <A> nor <antiA>, but in between (neutral). Neutrosophy considers a kind not of di-alectics but tri-alectics (based on three components: <A>, <antiA>, <neutA>). Hence unmatter is a kind of neutrality (not referring to the charge) between matter and antimatter, i. e. neither one, nor the other.

Upon the model of unmatter we may look at ungravity, unforce, unenergy, etc.

Ungravity would be a mixture between gravity and antigravity (for example attracting and rejecting simultaneously or alternatively; or a magnet which changes the + and − poles frequently).

Unforce. We may consider positive force (in the direction we want), and negative force (repulsive, opposed to the previous). There could be a combination of both positive and negative forces in the same time, or alternating positive and negative, etc.

Unenergy would similarly be a combination between positive and negative energies (as the alternating current, a. c., which periodically reverses its direction in a circuit and whose frequency, f, is independent of the circuit's constants). Would it be possible to construct an alternating-energy generator?

To conclusion: According to the Universal Dialectic the unity is manifested in duality and the duality in unity. "Thus, Unmatter (unity) is experienced as duality (matter vs antimatter). Ungravity (unity) as duality (gravity vs antigravity). Unenergy (unity) as duality (positive energy vs negative energy) and thus also ... between duality of being (existence) vs nothingness (antiexistence) must be 'unexistence' (or pure unity)" (R. Davic).

References

1. Smarandache F. A new form of matter — unmatter, composed of particles and anti-particles. *Progr. in Phys.*, 2005, v. 1, 9–11.
2. Smarandache F. Matter, antimatter, and unmatter. *Infinite Energy*, v. 11, No. 62, 50–51, (July/August 2005).
3. Chubb S. Breaking through editorial. *Infinite Energy*, v. 11, No. 62, 6–7 (July/August 2005).
4. Alt C. et al., (NA49 Collaboration). *Phys. Rev. Lett.*, 2004, v. 92, 042003.
5. Carman D. S., Experimental evidence for the pentaquark. *Eur. Phys. A*, 2005, v. 24, 15–20.
6. Gray L., Hagerty P., Kalogeropoulos T. E. Evidence for the existence of a narrow p-barn bound state. *Phys. Rev. Lett.*, 1971, v. 26, 1491–1494.
7. Carrol A. S. et al. Observation of structure in $\bar{p}p$ and $\bar{p}d$ total cross sections below 1.1 GeV/s. *Phys. Rev. Lett.*, 1974, v.32, 247–250.
8. Kalogeropoulos T. E., Vayaki A., Grammatikakis G., Tsilimigras T., Simopoulou E. Observation of excessive and direct gamma production in $\bar{p}d$ annihilations at rest. *Phys. Rev. Lett.*, 1974, v.33, 1635–1637.
9. Chapiro I. S. *Physics-Uspekhi*, 1973, v.109, 431.
10. Bogdanova L. N., Dalkarov O. D., Chapiro I. S. Quasinuclear systems of nucleons and antinucleons. *Annals of Physics*, 1974, v.84, 261–284.
11. Chapiro I. S. New "nuclei" built on nucleons and anti-nucleons. *Nature (Russian)*, 1975, No. 12, 68–73.
12. Davic R., John K., Jordan M., Rabounski D., Borissova L., Levin B., Panchelyuga V., Shnoll S., Private communications with author, June-July, 2005.
13. Smarandache F. A unifying field in logics, neutrosophic logic / neutrosophy, neutrosophic set, neutrosophic probability. Amer. Research Press, 1998.

Unmatter Entities inside Nuclei, Predicted by the Brightsen Nucleon Cluster Model

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Applying the R. A. Brightsen Nucleon Cluster Model of the atomic nucleus we discuss how unmatter entities (the conjugations of matter and antimatter) may be formed as clusters inside a nucleus. The model supports a hypothesis that antimatter nucleon clusters are present as a parton (*sensu* Feynman) superposition within the spatial confinement of the proton ($^1H_1$), the neutron, and the deuteron ($^1H_2$). If model predictions can be confirmed both mathematically and experimentally, a new physics is suggested. A proposed experiment is connected to othopositronium annihilation anomalies, which, being related to one of known unmatter entity, orthopositronium (built on electron and positron), opens a way to expand the Standard Model.

1 Introduction

According to Smarandache [1, 2, 3], following neutrosophy theory in philosophy and set theory in mathematics, the union of matter <A> and its antimatter opposite <AntiA> can form a neutral entity <NeutA> that is neither <A> nor <AntiA>. The <NeutA> entity was termed "unmatter" by Smarandache [1] in order to highlight its intermediate physical constitution between matter and antimatter. Unmatter is formed when matter and antimatter baryons intermingle, regardless of the amount of time before the conjugation undergoes decay. Already Bohr long ago predicted the possibility of unmatter with his principle of complementarity, which holds that nature can be understood in terms of concepts that come in complementary pairs of opposites that are inextricably connected by a Heisenberg-like uncertainty principle. However, not all physical union of <A> with <AntiA> must form unmatter. For instance, the charge quantum number for the electron (e^-) and its antimatter opposite positron (e^+) make impossible the formation of a charge neutral state — the quantum situation must be either (e^-) or (e^+).

Although the terminology "unmatter" is unconventional, unstable entities that contain a neutral union of matter and antimatter are well known experimentally for many years (e. g, pions, pentaquarks, positronium, etc.). Smarandache [3] presents numerous additional examples of unmatter that conform to formalism of quark quantum chromodynamics, already known since the 1970's. The basis that unmatter does exists comes from the 1970's experiments done at Brookhaven and CERN [4–8], where unstable unmatter-like entities were found. Recently "physicists suspect they have created the first molecules from atoms that meld matter with antimatter. Allen Mills of the University of California, Riverside, and his colleagues say they have seen telltale signs of positronium molecules, made from two positronium atoms" [9, 10]. A bound and quasi-stable unmatter baryonium has been verified experimentally as a weak resonance between a proton and antiproton using a Skyrme-type model potential. Further evidence that neutral entities derive from union of opposites comes from the spin induced magnetic moment of atoms, which can exist in a quantum state of both spin up and spin down at the same time, a quantum condition that follows the superposition principal of physics. In quantum physics, virtual and physical states that are mutually exclusive while simultaneously entangled, can form a unity of opposites <NeutA> via the principle of superposition.

Our motivation for this communication is to the question: would the superposition principal hold when mass symmetrical and asymmetrical matter and antimatter nucleon wavefunctions become entangled, thus allowing for possible formation of macroscopic "unmatter" nucleon entities, either stable or unstable? Here we introduce how the novel Nucleon Cluster Model of the late R. A. Brightsen [11–17] does predict formation of unmatter as the product of such a superposition between matter and antimatter nucleon clusters. The model suggests a radical hypothesis that antimatter nucleon clusters are present as a hidden parton type variable (*sensu* Feynman) superposed within the spatial confinement of the proton ($^1H_1$), the neutron, and the deuteron ($^1H_2$). Because the mathematics involving interactions between matter and antimatter nucleon clusters is not developed, theoretical work will be needed to test model predictions. If model predictions can be experimentally confirmed, a new physics is suggested.

2 The Brightsen Nucleon Cluster Model to unmatter entities inside nuclei

Of fundamental importance to the study of nuclear physics is the attempt to explain the macroscopic structural phenomena of the atomic nucleus. Classically, nuclear structure mathematically derives from two opposing views: (1) that the proton [P] and neutron [N] are independent (unbound) interacting

Matter Clusters ⟶ Antimatter Clusters ↓	[NP] Deuteron i Stable	[NPN] Triton j Beta-unstable	[PNP] Helium-3 k Stable	[NN] Di-Neutron l	[PP] Di-Proton m	[NNN] Tri-Neutron n	[PPP] Tri-Proton o
[NˆPˆ] a Stable		[N] |NP| |NˆPˆ|	[P] |NP| |NˆPˆ|	Pions (q qˆ)	Pions (q qˆ)	[N] |NN| |NˆPˆ|	[P] |NˆPˆ| |PP|
[NˆPˆNˆ] b Beta-unstable	[Nˆ] |NP| |NˆPˆ|		Pions (q qˆ)	[Pˆ] |NN| |NˆNˆ|	[Nˆ] |NˆPˆ| |PP|	Pions (q qˆ)	Tetraquarks (q q qˆqˆ)
[PˆNˆPˆ] c Stable	[Pˆ] |NP| |NˆPˆ|	Pions (q qˆ)		[Pˆ] |NˆPˆ| |NN|	[Nˆ] |PP| |PˆPˆ|	Tetraquarks (q q qˆqˆ)	Pions (q qˆ)
[NˆNˆ] d	Pions (q qˆ)	[N] |NN| |NˆNˆ|	[P] |NP| |NˆNˆ|		Tetraquarks (q q qˆqˆ)	[N] |NN| |NˆNˆ|	[P] |PP| |NˆNˆ|
[PˆPˆ] e	Pions (q qˆ)	[N] |NP| |PˆPˆ|	[P] |NP| |PˆPˆ|	Tetraquarks (q q qˆqˆ)		[N] |PˆPˆ| |NN|	[P] |PP| |PˆPˆ|
[NˆNˆNˆ] f	[Nˆ] |NP| |NˆNˆ|	Pions (q qˆ)	Tetraquarks (q q qˆqˆ)	[Nˆ] |NN| |NˆNˆ|	[Nˆ] |NˆNˆ| |PP|		Hexaquarks (q q q qˆqˆqˆ)
[PˆPˆPˆ] g	[Pˆ] |NP| |PˆPˆ|	Tetraquarks (q q qˆqˆ)	Pions (q qˆ)	[Pˆ] |PˆPˆ| |NN|	[Pˆ] |PˆPˆ| |PP|	Hexaquarks (q q q qˆqˆqˆ)	

Table 1: Unmatter entities (stable, quasi-stable, unstable) created from union of matter and antimatter nucleon clusters as predicted by the gravity-antigravity formalism of the Brightsen Nucleon Cluster Model. Shaded cells represent interactions that result in annihilation of mirror opposite two- and three- body clusters. Top nucleons within cells show superposed state comprised of three valance quarks; bottom structures show superposed state of hidden unmatter in the form of nucleon clusters. Unstable pions, tetraquarks, and hexaquark unmatter are predicted from union of mass symmetrical clusters that are not mirror opposites. The symbol ˆ = antimatter, N = neutron, P = proton, q = quark. (Communication with R. D. Davic).

fermions within nuclear shells, or (2) that nucleons interact collectively in the form of a liquid-drop. Compromise models attempt to cluster nucleons into interacting [NP] boson pairs (e.g., Interacting Boson Model-IBM), or, as in the case of the Interacting Boson-Fermion Model (IBFM), link boson clusters [NP] with un-paired and independent nucleons [P] and [N] acting as fermions.

However, an alternative view, at least since the 1937 Resonating Group Method of Wheeler, and the 1965 Close-Packed Spheron Model of Pauling, holds that the macroscopic structure of atomic nuclei is best described as being composed of a small number of interacting boson-fermion nucleon "clusters" (e. g., helium-3 [PNP], triton [NPN], deuteron [NP]), as opposed to independent [N] and [P] nucleons acting as fermions, either independently or collectively. Mathematically, such clusters represent a spatially localized mass-charge-spin subsystem composed of strongly correlated nucleons, for which realistic two- and three body wave functions can be written. In this view, quark-gluon dynamics are confined within the formalism of 6-quark bags [NP] and 9-quark bags ([PNP] and [NPN]), as opposed to valance quarks forming free nucleons. The experimental evidence in support of nucleons interacting as boson-fermion clusters is now extensive and well reviewed.

One novel nucleon cluster model is that of R. A. Brightsen, which was derived from the identification of mass-charge symmetry systems of isotopes along the Z-N Serge plot. According to Brightsen, all beta-stable matter and antimatter isotopes are formed by potential combinations of two- and three nucleon clusters; e.g., ([NP], [PNP], [NPN], [NN], [PP], [NNN], [PPP], and/or their mirror antimatter clusters [NˆPˆ], [PˆNˆPˆ], [NˆPˆNˆ], [NˆNˆ], [PˆPˆ], [PˆPˆPˆ], [NˆNˆNˆ], where the symbol ˆ here is used to denote antimatter. A unique prediction of the Brightsen model is that a stable union must result between interaction of mass asymmetrical matter (positive mass) and antimatter (negative mass) nucleon clusters to form protons and neutrons, for example the interaction between matter [PNP] + antimatter

[NˆPˆ]. Why union and not annihilation of mass asymmetrical matter and antimatter entities? As explained by Brightsen, independent (unbound) neutron and protons do not exist in nuclear shells, and the nature of the mathematical series of cluster interactions (3 [NP] clusters = 1[NPN] cluster + 1 [PNP] cluster), makes it impossible for matter and antimatter clusters of identical mass to coexist in stable isotopes. Thus, annihilation cannot take place between mass asymmetrical two- and three matter and antimatter nucleon clusters, only strong bonding (attraction).

Here is the Table that tells how unmatter may be formed from nucleon clusters according to the Brightsen model.

3 A proposed experimental test

As known, Standard Model of Quantum Electrodynamics explains all known phenomena with high precision, aside for anomalies in orthopositronium annihilation, discovered in 1987.

The Brightsen model, like many other models (see References), is outside the Standard Model. They all pretend to expand the Standard Model in one or another way. Therefore today, in order to judge the alternative models as true or false, we should compare their predictions to orthopositronium annihilation anomalies, the solely unexplained by the Standard Model. Of those models the Brightsen model has a chance to be tested in such way, because it includes unmatter entities (the conjugations of particles and anti-particles) inside an atomic nucleus that could produce effect in the forming of orthopositronium by β^+-decay positrons and its annihilation.

In brief, the anomalies in orthopositronium annihilation are as follows.

Positronium is an atom-like orbital system that includes an electron and its anti-particle, positron, coupled by electrostatic forces. There are two kinds of that: parapositronium ^{S}Ps, in which the spins of electron and positron are oppositely directed and the summary spin is zero, and orthopositronium ^{T}Ps, in which the spins are co-directed and the summary spin is one. Because a particle-antiparticle (unmatter) system is unstable, life span of positronium is rather small. In vacuum, parapositronium decays in $\tau \simeq 1.25\times10^{-10}$ s, while orthopositronium is $\tau \simeq 1.4\times10^{-7}$ s after the birth. In a medium the life span is even shorter because positronium tends to annihilate with electrons of the media.

In laboratory environment positronium can be obtained by placing a source of free positrons into a matter, for instance, one-atom gas. The source of positrons is β^+-decay, self-triggered decays of protons in neutron-deficient atoms*

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_{\mathrm{e}}.$$

Some of free positrons released from β^+-decay source into gas quite soon annihilate with free electrons and electrons in the container's walls. Other positrons capture electrons from gas atoms thus producing orthopositronium and parapositronium (in 3:1 statistical ratio). Time spectrum of positrons (number of positrons vs. life span) is the basic characteristic of their annihilation in matter.

In inert gases the time spectrum of annihilation of free positrons generally reminds of exponential curve with a plateau in its central part, known as "shoulder" [29, 30]. In 1965 Osmon published [29] pictures of observed time spectra of annihilation of positrons in inert gases (He, Ne, Ar, Kr, Xe). In his experiments he used ^{22}NaCl as a source of β^+-decay positrons. Analyzing the results of the experiments, Levin noted that the spectrum in neon was peculiar compared to those in other one-atom gases: in neon points in the curve were so widely scattered, that presence of a "shoulder" was unsure. Repeated measurements of temporal spectra of annihilation of positrons in He, Ne, and Ar, later accomplished by Levin [31, 32], have proven existence of anomaly in neon. Specific feature of the experiments done by Osmon, Levin and some other researchers in the UK, Canada, and Japan is that the source of positrons was ^{22}Na, while the moment of birth of positron was registered according to γ_{n}-quantum of decay of excited ^{22*}Ne

$$^{22*}\mathrm{Ne} \rightarrow {}^{22}\mathrm{Ne} + \gamma_{\mathrm{n}}\,,$$

from one of products of β^+-decay of ^{22*}Na.

In his experiments [33, 34] Levin discovered that the peculiarity of annihilation spectrum in neon (abnormally wide scattered points) is linked to presence in natural neon of substantial quantity of its isotope ^{22}Ne (around 9%). Levin called this effect *isotope anomaly*. Temporal spectra were measured in neon environments of two isotopic compositions: (1) natural neon (90.88% of ^{20}Ne, 0.26% of ^{21}Ne, and 8.86% of ^{22}Ne); (2) neon with reduced content of ^{22}Ne (94.83% of ^{20}Ne, 0.22% of ^{21}Ne, and 4.91% of ^{22}Ne). Comparison of temporal spectra of positron decay revealed: in natural neon (the 1st composition) the shoulder is fuzzy, while in neon poor with ^{22}Ne (the 2nd composition) the shoulder is always clearly pronounced. In the part of spectrum, to which ^{T}Ps-decay mostly contributes, the ratio between intensity of decay in poor neon and that in natural neon (with much isotope ^{22}Ne) is 1.85 ± 0.1 [34].

Another anomaly is substantially higher measured rate of annihilation of orthopositronium (the value reciprocal to its life span) compared to that predicted by QED.

Measurement of orthopositronium annihilation rate is among the main tests aimed to experimental verification of QED laws of conservation. In 1987 thanks to new precision technology a group of researchers based in the University of Michigan (Ann Arbor) made a breakthrough in this area. The obtained results showed substantial gap between experiment and theory. The anomaly that the Michigan group revealed

*It is also known as positron β^+-decay. During β^--decay in nucleus neutron decays $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \tilde{\nu}_{\mathrm{e}}$.

was that measured rates of annihilation at $\lambda_{\mathrm{T(exp)}} = 7.0514\pm$ ± 0.0014 μs^{-1} and $\lambda_{\mathrm{T(exp)}} = 7.0482 \pm 0.0016$ μs^{-1} (with unseen-before precision of 0.02% and 0.023% using vacuum and gas methods [35–38]) were much higher compared to $\lambda_{\mathrm{T(theor)}} = 7.00383 \pm 0.00005$ μs^{-1} as predicted by QED [39–42]. The effect was later called λ_{T}-anomaly [43].

Theorists foresaw possible annihilation rate anomaly not long before the first experiments were accomplished in Michigan. In 1986 Holdom [44] suggested that "mixed type" particles may exist, which being in the state of oscillation stay for some time in our world and for some time in the mirror Universe, possessing negative masses and energies. In the same year Glashow [45] gave further development to the idea and showed that in case of 3-photon annihilation ${}^{\mathrm{T}}$Ps will "mix up" with its mirror twin thus producing two effects: (1) higher annihilation rate due to additional mode of decay ${}^{\mathrm{T}}$Ps $\rightarrow$ nothing, because products of decay passed into the mirror Universe can not be detected; (2) the ratio between orthopositronium and parapositronium numbers will decrease from ${}^{\mathrm{T}}\mathrm{Ps} : {}^{\mathrm{S}}\mathrm{Ps}$ = 3:1 to 1.5 : 1. But at that time (in 1986) Glashow concluded that no interaction is possible between our-world and mirror-world particles.

On the other hand, by the early 1990's these theoretic studies encouraged many researchers worldwide for experimental search of various "exotic" (unexplained in QED) modes of ${}^{\mathrm{T}}$Ps-decay, which could lit some light on abnormally high rate of decay. These were, to name just a few, search for ${}^{\mathrm{T}}$Ps $\rightarrow$ nothing mode [46], check of possible contribution from 2-photon mode [47–49] or from other exotic modes [50–52]. As a result it has been shown that no exotic modes can contribute to the anomaly, while contribution of ${}^{\mathrm{T}}$Ps $\rightarrow$ nothing mode is limited to $5.8{\times}10^{-4}$ of the regular decay.

The absence of theoretical explanation of λ_{T}-anomaly encouraged Adkins et al. [53] to suggest experiments made in Japan [54] in 1995 as an alternative to the basic Michigan experiments. No doubt, high statistical accuracy of the Japanese measurements puts them on the same level with the basic experiments [35–38]. But all Michigan measurements possessed the property of a "full experiment", which in this particular case means no external influence could affect wave function of positronium. Such influence is inevitable due to electrodynamic nature of positronium and can be avoided only using special technique. In Japanese measurements [54] this was not taken into account and thus they do not possess property of "full experiment". Latest experiments of the Michigans [55], so-called *Resolution of Orthopositronium-Lifetime Pussle*, as well do not possess property of "full experiment", because the qualitative another statement included external influence of electromagnetic field [56, 57].

As early as in 1993 Karshenboim [58] showed that QED had actually run out of any of its theoretical capabilities to explain orthopositronium anomaly.

Electric interactions and weak interactions were joined into a common electroweak interaction in the 1960's by commonly Salam, Glashow, Weinberg, etc. Today's physicists attempt to join electroweak interaction and strong interaction (unfinished yet). They follow an intuitive idea that forces, connecting electrons and a nucleus, and forces, connecting nucleons inside a nucleus, are particular cases of a common interaction. That is the basis of our claim. If that is true, our claim is that orthopositronium atoms born in neon of different isotope contents (^{22}Ne, ^{21}Ne, ^{20}Ne) should be different from each other. There should be an effect of "inner" structure of neon nuclei if built by the Brightsen scheme, because the different proton-neutron contents built by different compositions of nucleon pairs. As soon as a free positron drags an electron from a neon atom, the potential of electro-weak interactions have changed in the atom. Accordingly, there in the nucleus itself should be re-distribution of strong interactions, than could be once as the re-building of the Brightsen pairs of nucleons there. So, lost electron of ^{22}Ne should have a different "inner" structure than that of ^{21}Ne or ^{20}Ne. Then the life span of orthopositronium built on such electrons should be as well different.

Of course, we can only qualitatively predict that difference, because we have no exact picture of what really happens inside a "structurized" nucleus. Yet only principal predictions are possible there. However even in such case we vote for continuation of "isotope anomaly" experiments with orthopositronium in neon of different isotope contents. If further experiments will be positive, it could be considered as one more auxiliary proof that the Brightsen model is true.

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References

1. Smarandache F. Matter, antimatter, and unmatter. *Infinite Energy*, 2005, v. 11, issue 62, 50–51.
2. Smarandache F. A new form of matter — unmatter, composed of particles and anti-particles. *Progress in Physics*, 2005, v. 1, 9–11.
3. Smarandache F. Verifying unmatter by experiments, more types of unmatter, and a quantum chromodynamics formula. *Progress in Physics*, 2005, v. 2, 113–116.

UNMATTER BASIS EXPERIMENTS

4. Gray L., Hagerty P., Kalogeropoulos T. E. *Phys. Rev. Lett.*, 1971, v. 26, 1491–1494.
5. Carrol A.S., Chiang I.-H., Kucia T. F., Li K. K., Mazur P. O., Michael D. N., Mockett P., Rahm D. C., Rubinstein R. *Phys. Rev. Lett.*, 1974, v. 32, 247–250.
6. Kalogeropoulos T. E., Vayaki A., Grammatikakis G., Tsilimigras T., Simopoulou E. *Phys. Rev. Lett.*, 1974, v. 33, 1635–1637.
7. Chapiro I. S. *Physics-Uspekhi*, 1973, v. 109, 431.

8. Bogdanova L. N., Dalkarov O. D., Chapiro I. S. *Annals of Physics*, 1974, v. 84, 261–284.
9. Cassidy D. B., Deng S. H. M., Greaves R. G., Maruo T., Nishiyama N., Snyder J. B., Tanaka H. K. M., Mills A. P. Jr. *Phys. Rev. Lett.*, 2005, v. 95, No. 19, 195006.
10. Ball Ph. *News Nature*, 22 November 2005.

THE BRIGHTSEN MODEL

11. Brightsen R. A. Nucleon cluster structures in beta-stable nuclides. *Infinite Energy*, 1995, v. 1, no. 4, 55–56.
12. Brightsen R. A. Correspondence of the Nucleon Cluster Model with the Periodic Table of Elements. *Infinite Energy*, 1995/96, v. 1(5/6), 73–74.
13. Brightsen R. A. Correspondence of the Nucleon Cluster Model with the Classical Periodic Table of Elements. *J. New Energy*, 1996, v. 1(1), 75–78.
14. Brightsen R. A. The Nucleon Cluster Model and the Periodic Table of Beta-Stable Nuclides. 1996 (available online at http://www.brightsenmodel.phoenixrising-web.net).
15. Brightsen R. A. The nucleon cluster model and thermal neutron fission. *Infinite Energy*, 2000, v. 6(31), 55–63.
16. Brightsen R. A., Davis R. Appl. of the Nucleon Cluster Model to experimental results. *Infinite Energy*, 1995, v. 1(3), 13–15.
17. Bass R. W. Experimental evidence favoring Brightsen's nucleon cluster model. *Infinite Energy*, 1996, v. 2(11), 78–79.

SOME REVIEW PAPERS ON CLUSTER MODELS

18. Buck B., Merchant A. C., Perez S. M. *Phys. Rev. C*, 2005, v. 71(1), 014311–15.
19. Akimune H., Yamagata T., Nakayama S., Fujiwara M., Fushimi K., Hara K., Hara K. Y., Ichihara K., Kawase K., Matsui K., Nakanishi K., Shiokawa A., Tanaka M., Utsunomiya H., and Yosoi M. *Physics of Atomic Nuclei*, 2004, v. 67(9), 1721–1725.
20. *Clustering Aspects of Nuclear Structure and Dynamics, Cluster'99, Proc. of the 7th Intern. Conf.*, 1999, Rab (Croatia).
21. Wuosmaa A. H., Betts R. R., Freer M., and Fulton B. R. *Annual Review of Nuclear and Particle Science*. 1995, v. 45, 89–131.
22. Bromley D. A. *Clust. Aspects of Nucl. Structure, Proc. of the 4th Intern. Conf.*, Chester (UK), D. Reidel Publ., Dordrecht.
23. Horiuchi H. and Ikeda K. *Cluster Models and Other Topics, Intern. Rev. of Nucl. Physics*, 1986, v. 4, World Scientific, Singapore, 1–259.

CLUSTER MODELS THAT THE BRIGHTSEN MODEL BUILDS ON

24. Wheeler J. A. *Phys. Rev.*, 1937, v. 52, 1083.
25. Wheeler J. A. *Phys. Rev.*, 1937, v. 52, 1107.
26. Pauling L. *Proc. Natl. Acad. Sci. USA*, 1965, v. 54, no. 4, 989.
27. Pauling L. *Science*, 1965, v. 150, no. 3694, 297.
28. Pauling L. *Revue Roumain de Physique*, 1966, v. 11, no. 9/10, 825–833.

ANOMALIES OF ORTHOPOSITRONIUM ANNIHILATION

29. Osmon P. E. *Physical Review B*, 1965, v. 138, 216.
30. Tao S. J., Bell J., and Green J. H. *Proceedings of the Physical Society*, 1964, v. 83, 453.
31. Levin B. M. and Shantarovich V. P. *High Energy Chemistry*, 1977, v. 11(4), 322–323.
32. Levin B. M. *Soviet J. Nucl. Physics*, 1981, v. 34(6), 917–918.
33. Levin B. M. and Shantarovich V. P. *Soviet J. Nucl. Physics*, 1984, v. 39(6), 855–856.
34. Levin B. M., Kochenda L. M., Markov A. A., and Shantarovich V. P. *Soviet J. Nucl. Physics*, 1987, v. 45(6), 1119–1120.
35. Gidley D. W., Rich A., Sweetman E., and West D. *Physical Review Letters*, 1982, v. 49, 525–528.
36. Westbrook C. I., Gidley D. W., Conti R. S., and Rich A. *Physical Review Letters*, 1987, v. 58, 1328–1331.
37. Westbrook C. I., Gidley D. W., Conti R. S., and Rich A. *Physical Review A*, 1989, v. 40, 5489–5499.
38. Nico J. S., Gidley D. W., Rich A., and Zitzewitz P. W. *Physical Review Letters*, 1990, v. 65, 1344–1347.
39. Caswell W. E. and Lepage G. P. *Phys. Rev. A*, 1979, v. 20, 36.
40. Adkins G. S. *Ann. Phys. (N.Y.)*, 1983, v. 146, 78.
41. Adkins G. S., Salahuddin A. A., and Schalm K. E. *Physical Review A*, 1992, v. 45, 3333–3335.
42. Adkins G. S., Salahuddin A. A., and Schalm K. E. *Physical Review A*, 1992, v. 45, 7774–7780.
43. Levin B. M. *Physics of Atomic Nuclei*, 1995, v. 58(2), 332–334.
44. Holdom B. *Physics Letters B*, 1986, v. 166, 196–198.
45. Glashow S. L. *Physics Letters B*, 1986, v. 167, 35–36.
46. Atoyan G. S., Gninenko S. N., Razin V. I., and Ryabov Yu. V. *Physics Letters B*, 1989, v. 220, 317–320.
47. Asai S., Orito S., Sanuki T., Yasuda M., and Yokoi T. *Physical Review Letters*, 1991, v. 66, 1298–1301.
48. Gidley D. W., Nico J. S., and Skalsey M. *Physical Review Letters*, 1991, v. 66, 1302–1305.
49. Al-Ramadhan A. H. and Gidley D. W. *Physical Review Letters*, 1994, v. 72, 1632–1635.
50. Orito S., Yoshimura K., Haga T., Minowa M., and Tsuchiaki M. *Physical Review Letters*, 1989, v. 63, 597–600.
51. Mitsui T., Fujimoto R., Ishisaki Y., Ueda Y., Yamazaki Y., Asai S., and Orito S. *Phys. Rev. Lett.*, 1993, v. 70, 2265–2268.
52. Skalsey M. and Conti R.S. *Phys. Rev. A*, 1997, v. 55(2), 984.
53. Adkins G.S., Melnikov K., and Yelkhovsky A. *Phys. Rev. A*, 1999, v. 60(4), 3306–3307.
54. Asai S., Orito S., and Shinohara N. *Physics Letters B*, 1995, v. 357, 475–480.
55. Vallery R. S., Zitzewitz P. W., and Gidley D. W. *Phys. Rev. Lett.*, 2003, v. 90, 203402.
56. Levin B. M. arXiv: quant-ph/0303166.
57. Levin B. M. CERN EXT-2004-016.
58. Karshenboim S. G. *Yadern. Fizika*, 1993, v. 56(12), 155–171.

On Emergent Physics, "Unparticles" and Exotic "Unmatter" States

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Emergent physics refers to the formation and evolution of collective patterns in systems that are nonlinear and out-of-equilibrium. This type of large-scale behavior often develops as a result of simple interactions at the component level and involves a dynamic interplay between order and randomness. On account of its universality, there are credible hints that emergence may play a leading role in the Tera-ElectronVolt (TeV) sector of particle physics. Following this path, we examine the possibility of hypothetical high-energy states that have fractional number of quanta per state and consist of arbitrary mixtures of particles and antiparticles. These states are similar to "un-particles", massless fields of non-integral scaling dimensions that were recently conjectured to emerge in the TeV sector of particle physics. They are also linked to "unmatter", exotic clusters of matter and antimatter introduced few years ago in the context of Neutrosophy.

1 Introduction

Quantum Field Theory (QFT) is a framework whose methods and ideas have found numerous applications in various domains, from particle physics and condensed matter to cosmology, statistical physics and critical phenomena [1, 2]. As successful synthesis of Quantum Mechanics and Special Relativity, QFT represents a collection of *equilibrium* field theories and forms the foundation for the Standard Model (SM), a body of knowledge that describes the behavior of all known particles and their interactions, except gravity. Many broken symmetries in QFT, such as violation of parity and CP invariance, are linked to either the electroweak interaction or the physics beyond SM [3–5]. This observation suggests that unitary evolution postulated by QFT no longer holds near or above the energy scale of electroweak interaction ($\approx$ 300GeV) [6,7]. It also suggests that progress on the theoretical front requires a framework that can properly handle *non-unitary evolution* of phenomena beyond SM. We believe that fractional dynamics naturally fits this description. It operates with derivatives of non-integer order called *fractal operators* and is suitable for analyzing many complex processes with long-range interactions [6–9]. Building on the current understanding of fractal operators, we take the dimensional parameter of the regularization program $\varepsilon = 4-d$ to represent the order of fractional differentiation in physical space-time (alternatively, $\varepsilon = 1-d$ in one-dimensional space) [10, 11]. It can be shown that ε is related to the reciprocal of the cutoff scale $\varepsilon \approx (\mu_0/\Lambda)$, where μ_0 stands for a finite and arbitrary reference mass and Λ is the cutoff energy scale. Under these circumstances, ε may be thought as an infinitesimal parameter that can be continuously tuned and drives the departure from equilibrium. The approach to scale invariance demands that the choice of this parameter is completely arbitrary, as long as $\varepsilon \ll 1$. Full scale invariance and equilibrium field theory are asymptotically recovered in the limit of physical space-time ($d = 4$) as $\varepsilon \to 0$ or $\Lambda \to \infty$ [11, 12].

2 Definitions

We use below the Riemann-Liouville definition for the one-dimensional left and right fractal operators [13]. Consider for simplicity a space-independent scalar field $\varphi(t)$. Taking the time coordinate to be the representative variable, one writes

$$ {}_0D_L^{\alpha}\,\varphi(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha}\varphi(\tau)d\tau\,, \tag{1}$$

$$ {}_0D_R^{\alpha}\,\varphi(t) = \frac{1}{\Gamma(1-\alpha)}\Big(-\frac{d}{dt}\Big)\int_t^0 (\tau-t)^{-\alpha}\varphi(\tau)d\tau\,. \tag{2}$$

Here, fractional dimension $0 < \alpha < 1$ denotes the order of fractional differentiation. In general, it can be shown that α is linearly dependent on the dimensionality of the space-time support [8]. By definition, α assumes a continuous spectrum of values on fractal supports [11].

3 Fractional dynamics and 'unparticle' physics

The classical Lagrangian for the free scalar field theory in 3+1 dimensions reads [1–2, 14]

$$ L = \partial^{\mu}\varphi\partial_{\mu}\varphi - m^2\varphi^2, \tag{3}$$

and yields the following expression for the field momentum

$$ \pi = \frac{\partial L}{\partial(\frac{\partial\varphi}{\partial t})} = \frac{\partial\varphi}{\partial t}\,. \tag{4}$$

It is known that the standard technique of canonical quantization promotes a classical field theory to a quantum field theory by converting the field and momentum variables into operators. To gain full physical insight with minimal complications in formalism, we work below in 0+1 dimensions. Ignoring the left/right labels for the time being, we define the field and momentum operators as

$$\varphi \to \widehat{\varphi} = \varphi\,, \tag{5}$$

$$\pi \to \widehat{\pi}^{\alpha} = -\,i\,\frac{\partial^{\alpha}}{\partial\,|\varphi|^{\alpha}} \equiv -iD^{\alpha}. \tag{6}$$

Without the loss of generality, we set $m = 1$ in (3). The Hamiltonian becomes

$$H \to \widehat{H}^{\alpha} = -\frac{1}{2}\,D^{2\alpha} + \frac{1}{2}\,\varphi^{2} = \frac{1}{2}\left(\widehat{\pi}^{2\alpha} + \varphi^{2}\right). \tag{7}$$

By analogy with the standard treatment of harmonic oscillator in quantum mechanics, it is convenient to work with the destruction and creation operators defined through [1–2, 14]

$$\widehat{a}^{\alpha} \doteq \frac{1}{\sqrt{2}}\left[\widehat{\varphi} + i\widehat{\pi}^{\alpha}\right], \tag{8}$$

$$\widehat{a}^{+\alpha} \doteq \frac{1}{\sqrt{2}}\left[\widehat{\varphi} - i\,\widehat{\pi}^{\alpha}\right]. \tag{9}$$

Straightforward algebra shows that these operators satisfy the following commutation rules

$$[\widehat{a},\,\widehat{a}] = [\widehat{a}^{+\alpha},\,\widehat{a}^{+\alpha}] = 0\,, \tag{10}$$

$$[\widehat{a}^{+\alpha},\,\widehat{a}^{\alpha}] = i\,[\widehat{\varphi},\,\widehat{\pi}^{\alpha}] = -\alpha\,\widehat{\pi}^{(\alpha-1)}. \tag{11}$$

The second relation of these leads to

$$\widehat{H}^{\alpha} = \widehat{a}^{+\alpha}\,\widehat{a}^{\alpha} + \frac{1}{2}\,\alpha\widehat{\pi}^{(\alpha-1)}. \tag{12}$$

In the limit $\alpha = 1$ we recover the quantum mechanics of the harmonic oscillator, namely

$$\widehat{H} = \widehat{a}^{+}\widehat{a} + \frac{1}{2}\,. \tag{13}$$

It was shown in [6] that the fractional Hamiltonian (12) leads to a continuous spectrum of states having non-integer numbers of quanta per state. These unusual flavors of particles and antiparticles emerging as fractional objects were named "*complexons*". Similar conclusions have recently surfaced in a number of papers where the possibility of a scale-invariant "hidden" sector of particle physics extending beyond SM has been investigated. A direct consequence of this setting is a continuous spectrum of massless fields having non-integral scaling dimensions called "un-particles". The reader is directed to [15–21] for an in-depth discussion of "un-particle" physics.

4 Mixing properties of fractal operators

Left and *right* fractal operators (L/R) are natural analogues of *chiral* components associated with the structure of quantum fields [8, 9]. The goal of this section is to show that there is an inherent mixing of (L/R) operators induced by the fractional dynamics, as described below. An equivalent representation of (1) is given by

$${}_{0}D^{\alpha}_{L}\,\varphi(t) = \frac{1}{\Gamma(1-\alpha)}\left(-\frac{d}{dt}\right)\int\limits_{t}^{0}\left[-(\tau-t)\right]^{-\alpha}\varphi(\tau)\,d\tau\,, \tag{14}$$

or

$$\begin{aligned}{}_{0}D^{\alpha}_{L}\,\varphi(t) \;&=\; \frac{(-1)^{-\alpha}}{\Gamma(1-\alpha)}\left(-\frac{d}{dt}\right)\int\limits_{t}^{0}(\tau-t)^{-\alpha}\varphi(\tau)\,d\tau = \\ &=\; (-1)^{-\alpha}\,{}_{0}D^{\alpha}_{R}\,\varphi(t)\,,\end{aligned} \tag{15}$$

$${}_{0}D^{\alpha}_{R} = (-1)^{\alpha}\,{}_{0}D^{\alpha}_{L} = \exp(i\pi\alpha)\,{}_{0}D^{\alpha}_{L}\,. \tag{16}$$

Starting from (2) instead, we find

$${}_{0}D^{\alpha}_{L} = (-1)^{\alpha}\,{}_{0}D^{\alpha}_{R} = \exp(i\pi\alpha)\,{}_{0}D^{\alpha}_{R}\,. \tag{17}$$

Consider now the one-dimensional case $d = 1$, take $\alpha = \varepsilon = 1 - d$ and recall that continuous tuning of ε does not impact the physics as a consequence of scale invariance. Let us iterate (16) and (17) a finite number of times ($n \geqslant 1$) under the assumption that $n\varepsilon \ll 1$. It follows that the fractal operator of any infinitesimal order may be only defined up to an arbitrary dimensional factor $\exp(i\pi n\varepsilon) \approx 1 + (i\pi n\varepsilon) = 1 - i\widetilde{\varepsilon}$, that is,

$${}_{0}D^{\varepsilon}_{L,R}\,\varphi(t) \approx \left[{}_{0}D^{0}_{L,R} - i\,\widetilde{\varepsilon}\right]\varphi(t) \tag{18}$$

or

$$i_{0}D^{\varepsilon}_{L,R}\,\varphi(t) = \left[i\,{}_{0}D^{0}_{L,R} + \widetilde{\varepsilon}\right]\varphi(t)\,, \tag{19}$$

where

$$\lim_{\varepsilon\to 0} D^{\varepsilon}_{L,R}\,\varphi(t) = \varphi(t)\,. \tag{20}$$

Relations (18–20) indicate that fractional dimension $\widetilde{\varepsilon}$ induces: (a) a new type of mixing between chiral components of the field and (b) an ambiguity in the very definition of the field, fundamentally different from measurement uncertainties associated with Heisenberg principle. Both effects are *irreversible* (since fractional dynamics describes irreversible processes) and of *topological nature* (being based on the concept of continuous dimension). They do not have a counterpart in conventional QFT.

5 Emergence of "unmatter" states

Using the operator language of QFT and taking into account (6), (18) can be presented as

$$\widehat{\pi}^{\varepsilon}\varphi(t) = \widehat{\pi}^{\varepsilon}\varphi(t) - \widetilde{\varepsilon}\,\widehat{\varphi}(t)\,. \tag{21}$$

Relation (21) shows that the fractional momentum operator $\widehat{\pi}^{\varepsilon}$ and the field operator $\widehat{\varphi}(t) = \varphi(t)$ are no longer independent entities but linearly coupled through fractional dimension $\widetilde{\varepsilon}$. From (11) it follows that the destruction and creation operators are also coupled to each other. As a result, particles and antiparticles can no longer exist as linearly independent objects. Because $\widetilde{\varepsilon}$ is continuous, they emerge as an *infinite spectrum of mixed states*. This surprising finding is counterintuitive as it does not have an equivalent in conventional QFT. Moreover, arbitrary mixtures of particles and antiparticles may be regarded as a manifestation of "unmatter", a concept launched in the context of Neutrosophic Logic [22–24].

6 Definition of unmatter

In short, unmatter is formed by matter and antimatter that bind together [23, 24].

The building blocks (most elementary particles known today) are 6 quarks and 6 leptons; their 12 antiparticles also exist.

Then *unmatter* will be formed by at least a building block and at least an antibuilding block which can bind together.

Let's start from neutrosophy [22], which is a generalization of dialectics, i.e. not only the opposites are combined but also the neutralities. Why? Because when an idea is launched, a category of people will accept it, others will reject it, and a third one will ignore it (don't care). But the dynamics between these three categories changes, so somebody accepting it might later reject or ignore it, or an ignorant will accept it or reject it, and so on. Similarly the dynamicity of <A>, <antiA>, <neutA>, where <neutA> means neither <A> nor <antiA>, but in between (neutral). Neutrosophy considers a kind not of di-alectics but tri-alectics (based on three components: <A>, <antiA>, <neutA>).

Hence unmatter is a kind of intermediary (not referring to the charge) between matter and antimatter, i.e. neither one, nor the other.

Neutrosophic Logic (NL) is a generalization of fuzzy logic (especially of intuitionistic fuzzy logic) in which a proposition has a degree of truth, a degree of falsity, and a degree of neutrality (neither true nor false); in the normalized NL the sum of these degrees is 1.

7 Exotic atom

If in an atom we substitute one or more particles by other particles of the same charge (constituents) we obtain an exotic atom whose particles are held together due to the electric charge. For example, we can substitute in an ordinary atom one or more electrons by other negative particles (say π^-, anti-Rho meson, D^-, D_s^-, muon, tau, Ω^-, Δ^-, etc., generally clusters of quarks and antiquarks whose total charge is negative), or the positively charged nucleus replaced by other positive particle (say clusters of quarks and antiquarks whose total charge is positive, etc.).

8 Unmatter atom

It is possible to define the unmatter in a more general way, using the exotic atom.

The classical unmatter atoms were formed by particles like (a) electrons, protons, and antineutrons, or (b) antielectrons, antiprotons, and neutrons.

In a more general definition, an unmatter atom is a system of particles as above, or such that one or more particles are replaces by other particles of the same charge.

Other categories would be (c) a matter atom with where one or more (but not all) of the electrons and/or protons are replaced by antimatter particles of the same corresponding charges, and (d) an antimatter atom such that one or more (but not all) of the antielectrons and/or antiprotons are replaced by matter particles of the same corresponding charges.

In a more composed system we can substitute a particle by an unmatter particle and form an unmatter atom.

Of course, not all of these combinations are stable, semi-stable, or quasi-stable, especially when their time to bind together might be longer than their lifespan.

9 Examples of unmatter

During 1970–1975 numerous pure experimental verifications were obtained proving that "atom-like" systems built on nucleons (protons and neutrons) and anti-nucleons (anti-protons and anti-neutrons) are real. Such "atoms", where nucleon and anti-nucleon are moving at the opposite sides of the same orbit around the common centre of mass, are very unstable, their life span is no more than 10^{-20} sec. Then nucleon and anti-nucleon annihilate into gamma-quanta and more light particles (pions) which can not be connected with one another, see [6, 7, 8]. The experiments were done in mainly Brookhaven National Laboratory (USA) and, partially, CERN (Switzerland), where "proton–anti-proton" and "anti-proton–neutron" atoms were observed, called them $\bar{p}p$ and $\bar{p}n$ respectively.

After the experiments were done, the life span of such "atoms" was calculated in theoretical way in Chapiro's works [9, 10, 11]. His main idea was that nuclear forces, acting between nucleon and anti-nucleon, can keep them far way from each other, hindering their annihilation. For instance, a proton and anti-proton are located at the opposite sides in the same orbit and they are moved around the orbit centre. If the diameter of their orbit is much more than the diameter of "annihilation area", they are kept out of annihilation. But because the orbit, according to Quantum Mechanics, is an actual cloud spreading far around the average radius, at any radius between the proton and the anti-proton there is a probability that they can meet one another at the annihilation distance. Therefore nucleon—anti-nucleon system annihilates in any case, this system is unstable by definition having life span no more than 10^{-20} sec.

Unfortunately, the researchers limited the research to the consideration of $\bar{\mathrm{p}}\mathrm{p}$ and $\bar{\mathrm{p}}\mathrm{n}$ nuclei only. The reason was that they, in the absence of a theory, considered $\bar{\mathrm{p}}\mathrm{p}$ and $\bar{\mathrm{p}}\mathrm{n}$ "atoms" as only a rare exception, which gives no classes of matter.

The unmatter does exists, for example some messons and antimessons, through for a trifling of a second lifetime, so the pions are unmatter (which have the composition u^d and ud^, where by u^ we mean anti-up quark, d = down quark, and analogously u = up quark and d^ = anti-down quark, while by ^ means anti), the kaon K^+ (us^), K^- (u^s), Phi (ss^), D^+ (cd^), D^0(cu^), D_s^+ (cs^), J/Psi (cc^), B^- (bu^), B^0 (db^), B_s^0 (sb^), Upsilon (bb^), where c = charm quark, s = strange quark, b = bottom quark, etc. are unmatter too.

Also, the pentaquark Theta-plus (Θ^+), of charge $^+1$, uudds^ (i.e. two quarks up, two quarks down, and one anti-strange quark), at a mass of 1.54 GeV and a narrow width of 22 MeV, is unmatter, observed in 2003 at the Jefferson Lab in Newport News, Virginia, in the experiments that involved multi-GeV photons impacting a deuterium target. Similar pentaquark evidence was obtained by Takashi Nakano of Osaka University in 2002, by researchers at the ELSA accelerator in Bonn in 1997–1998, and by researchers at ITEP in Moscow in 1986.

Besides Theta-plus, evidence has been found in one experiment [25] for other pentaquarks, Ξ_5^-(ddssu^) and Ξ_5^+(uussd^).

D. S. Carman [26] has reviewed the positive and null evidence for these pentaquarks and their existence is still under investigation.

In order for the paper to be self-contained let's recall that the *pionium* is formed by a π^+ and π^- mesons, the *positronium* is formed by an antielectron (positron) and an electron in a semi-stable arrangement, the *protonium* is formed by a proton and an antiproton also semi-stable, the *antiprotonic helium* is formed by an antiproton and electron together with the helium nucleus (semi-stable), and *muonium* is formed by a positive muon and an electron.

Also, the *mesonic atom* is an ordinary atom with one or more of its electrons replaced by negative mesons.

The *strange matter* is a ultra-dense matter formed by a big number of strange quarks bounded together with an electron atmosphere (this strange matter is hypothetical).

From the exotic atom, the pionium, positronium, protonium, antiprotonic helium, and muonium are unmatter.

The mesonic atom is unmatter if the electron(s) are replaced by negatively-charged antimessons.

Also we can define a mesonic antiatom as an ordinary antiatomic nucleous with one or more of its antielectrons replaced by positively-charged mesons. Hence, this mesonic antiatom is unmatter if the antielectron(s) are replaced by positively-charged messons.

The strange matter can be unmatter if these exists at least an antiquark together with so many quarks in the nucleous. Also, we can define the strange antimatter as formed by a large number of antiquarks bound together with an antielectron around them. Similarly, the strange antimatter can be unmatter if there exists at least one quark together with so many antiquarks in its nucleous.

The bosons and antibosons help in the decay of unmatter. There are 13+1 (Higgs boson) known bosons and 14 antibosons in present.

10 Chromodynamics formula

In order to save the colorless combinations prevailed in the Theory of Quantum Chromodynamics (QCD) of quarks and antiquarks in their combinations when binding, we devise the following formula:

$$\mathbf{Q} - \mathbf{A} \in \pm \boldsymbol{M3}\,, \tag{22}$$

where $\boldsymbol{M3}$ means multiple of three, i.e. $\pm\boldsymbol{M3}=\{3\cdot k|k\in Z\}=\{\ldots,-12,-9,-6,-3,0,3,6,9,12,\ldots\}$, and $\mathbf{Q}$ = number of quarks, $\mathbf{A}$ = number of antiquarks.

But (22) is equivalent to:

$$\mathbf{Q} \equiv \mathbf{A}(\mathbf{mod 3}) \tag{23}$$

($\mathbf{Q}$ is congruent to $\mathbf{A}$ modulo 3).

To justify this formula we mention that 3 quarks form a colorless combination, and any multiple of three ($\boldsymbol{M3}$) combination of quarks too, i.e. 6, 9, 12, etc. quarks. In a similar way, 3 antiquarks form a colorless combination, and any multiple of three ($\boldsymbol{M3}$) combination of antiquarks too, i.e. 6, 9, 12, etc. antiquarks. Hence, when we have hybrid combinations of quarks and antiquarks, a quark and an antiquark will annihilate their colors and, therefore, what's left should be a multiple of three number of quarks (in the case when the number of quarks is bigger, and the difference in the formula is positive), or a multiple of three number of antiquarks (in the case when the number of antiquarks is bigger, and the difference in the formula is negative).

11 Quantum chromodynamics unmatter formula

In order to save the colorless combinations prevailed in the Theory of Quantum Chromodynamics (QCD) of quarks and antiquarks in their combinations when binding, we devise the following formula:

$$\mathbf{Q} - \mathbf{A} \in \pm \boldsymbol{M3}\,, \tag{24}$$

where $\boldsymbol{M3}$ means multiple of three, i.e. $\pm\boldsymbol{M3}=\{3\cdot k|k\in Z\}=\{\ldots,-12,-9,-6,-3,0,3,6,9,12,\ldots\}$, and $\mathbf{Q}$ = number of quarks, $\mathbf{A}$ = number of antiquarks, with $\mathbf{Q} \geqslant 1$ and $\mathbf{A} \geqslant 1$.

But (24) is equivalent to:

$$\mathbf{Q} \equiv \mathbf{A}(\mathbf{mod3}) \tag{25}$$

($\mathbf{Q}$ is congruent to $\mathbf{A}$ modulo 3), and also $\mathbf{Q} \geqslant 1$ and $\mathbf{A} \geqslant 1$.

12 Quark-antiquark combinations

Let's note by q = quark ∈ {Up, Down, Top, Bottom, Strange, Charm}, and by a = antiquark ∈ {Up, Down, Top, Bottom, Strange, Charm}.

Hence, for combinations of n quarks and antiquarks, $n \geqslant 2$, prevailing the colorless, we have the following possibilities:

— if n = 2, we have: qa (biquark — for example the mesons and antimessons);

— if n = 3, we have qqq, aaa (triquark — for example the baryons and antibaryons);

— if n = 4, we have qqaa (tetraquark);

— if n = 5, we have qqqqa, aaaaq (pentaquark);

— if n = 6, we have qqqaaa, qqqqqq, aaaaaa (hexaquark);

— if n = 7, we have qqqqqaa, qqaaaaa (septiquark);

— if n = 8, we have qqqqaaaa, qqqqqqaa, qqaaaaaa (octoquark);

— if n = 9, we have qqqqqqqqq, qqqqqqaaa, qqqaaaaaa, aaaaaaaaa (nonaquark);

— if n = 10, obtain qqqqqaaaaa, qqqqqqqqaa, qqaaaaaaaa (decaquark);

— etc.

13 Unmatter combinations

From the above general case we extract the unmatter combinations:

— For combinations of 2 we have: qa (unmatter biquark), (mesons and antimesons); the number of all possible unmatter combinations will be $6 \cdot 6 = 36$, but not all of them will bind together.

It is possible to combine an entity with its mirror opposite and still bound them, such as: uuˆ, ddˆ, ssˆ, ccˆ, bbˆ which form mesons.

It is possible to combine, unmatter + unmatter = unmatter, as in udˆ + usˆ = uudˆsˆ (of course if they bind together);

— For combinations of 3 (unmatter triquark) we can not form unmatter since the colorless can not hold.

— For combinations of 4 we have: qqaa (unmatter tetraquark); the number of all possible unmatter combinations will be $6^2 \cdot 6^2 = 1{,}296$, but not all of them will bind together;

— For combinations of 5 we have: qqqqa, or aaaaq (unmatter pentaquarks); the number of all possible unmatter combinations will be $6^4 \cdot 6 + 6^4 \cdot 6 = 15{,}552$, but not all of them will bind together;

— For combinations of 6 we have: qqqaaa (unmatter hexaquarks); the number of all possible unmatter combinations will be $6^3 \cdot 6^3 = 46{,}656$, but not all of them will bind together;

— For combinations of 7 we have: qqqqqaa, qqaaaaa (unmatter septiquarks); the number of all possible unmatter combinations will be $6^5 \cdot 6^2 + 6^2 \cdot 6^5 = 559{,}872$, but not all of them will bind together;

— For combinations of 8 we have: qqqqaaaa, qqqqqqqa, qaaaaaaa (unmatter octoquarks); the number of all possible unmatter combinations will be $6^4 \cdot 6^4 + 6^7 \cdot 6^1 + 6^1 \cdot 6^7 = 5{,}038{,}848$, but not all of them will bind together;

— For combinations of 9 we have: qqqqqqaaa, qqqaaaaaa (unmatter nonaquarks); the number of all possible unmatter combinations will be $6^6 \cdot 6^3 + 6^3 \cdot 6^6 = 2 \cdot 6^9 = 20{,}155{,}392$, but not all of them will bind together;

— For combinations of 10: qqqqqqqqaa, qqqqqaaaaa, qqaaaaaaaa (unmatter decaquarks); the number of all possible unmatter combinations will be $3 \cdot 6^{10} = 181{,}398{,}528$, but not all of them will bind together;

— etc.

I wonder if it is possible to make infinitely many combinations of quarks/antiquarks and leptons/antileptons... Unmatter can combine with matter and/or antimatter and the result may be any of these three.

Some unmatter could be in the strong force, hence part of hadrons.

14 Unmatter charge

The charge of unmatter may be positive as in the pentaquark Theta-plus, 0 (as in positronium), or negative as in anti-Rho meson, i.e. uˆd, (M. Jordan).

15 Containment

I think for the containment of antimatter and unmatter it would be possible to use electromagnetic fields (a container whose walls are electromagnetic fields). But its duration is unknown.

16 Summary and conclusions

It is apparent from these considerations that, in general, both "unmatter" and "unparticles" are non-trivial states that may become possible under conditions that substantially deviate from our current laboratory settings. Unmatter can be thought as *arbitrary* clusters of ordinary matter and antimatter, unparticles contain *fractional numbers of quanta* per state and carry *arbitrary spin* [6]. They both display a much richer dynamics than conventional SM doublets, for example mesons (quark-antiquark states) or lepton pairs (electron-electron antineutrino). Due to their unusual properties, "unmatter" and "unparticles" are presumed to be highly unstable and may lead to a wide range of symmetry breaking scenarios. In particular, they may violate well established conservation principles such as electric charge, weak isospin and color. Future observational evidence and analytic studies are needed to confirm, expand or falsify these tentative findings.

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References

1. Weinberg S. The quantum theory of fields. Volume 1: foundations. Cambridge University Press, London, 1995.
2. Zinn-Justin J. Quantum field theory and critical phenomena. Oxford University Press, Oxford, 2002.
3. Hagiwara K. et al. *Phys. Rev. D*, 2002, v. 66, 010001-1; Nir Y. CP violation and beyond the Standard Model. XXVII SLAC Summer Institute, 1999; Rogalyov R.N. arXiv: hep-ph/0204099.
4. Bigi I.I. and Sanda A.I. CP violation. Cambridge University Press, 2000; Yao W.M. et al. *Journal of Phys. G*, 2006, v. 33(1).
5. Donoghue J.F. et al.. Dynamics of the Standard Model. Cambridge University Press, 1996.
6. Goldfain E. *Chaos, Solitons and Fractals*, 2006, v. 28(4), 913.
7. Goldfain E. *Intern. Journal of Nonlinear Sciences and Numerical Simulation*, 2005, v. 6(3), 223.
8. Goldfain E. *Comm. in Nonlinear Science and Numer. Simulation*, 2008, v. 13, 666.
9. Goldfain E. *Comm. in Nonlinear Science and Numer. Simulation*, 2008, v. 13, 1397.
10. Ballhausen H. et al. *Phys. Lett. B*, 2004, v. 582, 144.
11. Goldfain E. *Chaos, Solitons and Fractals*, 2008, v. 38(4), 928.
12. Goldfain E. *Intern. Journal of Nonlinear Science*, 2007, v. 3(3), 170.
13. Samko S.G. et al. Fractional integrals and derivatives. Theory and applications. Gordon and Breach, New York, 1993.
14. Ryder L. Quantum field theory. Cambridge University Press, 1996.
15. Georgi H. *Phys. Rev. Lett.*, 2007, v. 98, 221601.
16. Georgi H. *Phys. Lett. B*, 2007, v. 650, 275.
17. Cheung K. et al. *Phys. Rev. D*, 2007, v. 76, 055003.
18. Bander M. et al. *Phys. Rev. D*, 2007, v. 76, 115002.
19. Zwicky R. *Phys. Rev. D*, 2008, v. 77, 036004.
20. Kikuchi T. et al. *Phys. Rev. D*, 2008, v. 77, 094012.
21. Goldfain E. A brief note on "un-particle" physics. *Progress in Physics*, 2008, v. 28, v. 3, 93.
22. Smarandache F. A unifying field in logics, neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. Am. Res. Press, 1998.
23. Smarandache F. Matter, antimatter, and unmatter. CDS-CERN, EXT-2005-142, 2004; *Bulletin of Pure and Applied Sciences*, 2004, v. 23D, no. 2, 173–177.
24. Smarandache F. Verifying unmatter by experiments, more types of unmatter, and a quantum chromodynamics formula. *Progress in Physics*, 2005, v. 2, 113–116; *Infinite Energy*, 2006, v. 12, no. 67, 36–39.
25. Chubb S. Breaking through editorial. *Infinite Energy*, 2005, v. 11, no. 62, 6–7.
26. Carman D.S. Experimental evidence for the pentaquark. *Euro Phys. A*, 2005, v. 24, 15–20.
27. Gray L., Hagerty P., and Kalogeropoulos T.E. Evidence for the existence of a narrow p-barn bound state. *Phys. Rev. Lett.*, 1971, v. 26, 1491–1494.
28. Carrol A.S., Chiang I.-H., Kucia T.F., Li K.K., Mazur P.O., Michael D.N., Mockett P., Rahm D.C., and Rubinstein R. Observation of structure in p-barp and p-bard total cross cections below 1.1 GeV/c. *Phys. Rev. Lett.*, 1974, v. 32, 247–250.
29. Kalogeropoulos T.E., Vayaki A., Grammatikakis G., Tsilimigras T., and Simopoulou E. Observation of excessive and direct gamma production in p-bard annihilations at rest. *Phys. Rev. Lett.*, 1974, v. 33, 1635–1637.
30. Chapiro I.S. *Physics-Uspekhi* (*Uspekhi Fizicheskikh Nauk*), 1973, v. 109, 431.
31. Bogdanova L.N., Dalkarov O.D., and Chapiro I.S. Quasinuclear systems of nucleons and antinucleons. *Annals of Physics*, 1974, v. 84, 261–284.
32. Chapiro I.S. New "nuclei" built on nucleons and anti-nucleons. *Nature-USSR*, 1975, no. 12, 68–73.
33. Barmin V.V. et al. (DIANA Collaboration). *Phys. Atom. Nucl.*, 2003, v. 66, 1715.
34. Ostrick M. (SAPHIR Collaboration). *Pentaquark 2003 Workshop*, Newport News, VA, Nov. 6–8, 2003.
35. Smith T.P. Hidden worlds, hunting for quarks in ordinary matter. Princeton University Press, 2003.
36. Wang M.J. (CDF Collaboration). In: *Quarks and Nuclear Physics 2004*, Bloomington, IN, May 23–28, 2004.

Thirty Unsolved Problems in the Physics of Elementary Particles

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Unlike what some physicists and graduate students used to think, that physics science has come to the point that the only improvement needed is merely like adding more numbers in decimal place for the masses of elementary particles or gravitational constant, there is a number of unsolved problems in this field that may require that the whole theory shall be reassessed. In the present article we discuss thirty of those unsolved problems and their likely implications. In the first section we will discuss some well-known problems in cosmology and particle physics, and then other unsolved problems will be discussed in next section.

1 Unsolved problems related to cosmology

In the present article we discuss some unsolved problems in the physics of elementary particles, and their likely implications. In the first section we will discuss some well-known problems in cosmology and particle physics, and then other unsolved problems will be discussed in next section. Some of these problems were inspired by and expanded from Ginzburg's paper [1]. The problems are:

1. The problem of the three origins. According to Marcelo Gleiser (Darthmouth College) there are three unsolved questions which are likely to play significant role in 21st-century science: the origin of the universe, the origin of life, and the origin of mind;
2. The problem of symmetry and antimatter observation. This could be one of the biggest puzzle in cosmology: If it's true according to theoretical physics (Dirac equation etc.) that there should be equal amounts of matter and antimatter in the universe, then why our observation only display vast amounts of matter and very little antimatter?
3. The problem of dark matter in cosmology model. Do we need to introduce dark matter to describe galaxy rotation curves? Or do we need a revised method in our cosmology model? Is it possible to develop a new theory of galaxy rotation which agrees with observations but without invoking dark matter? For example of such a new theory without dark matter, see Moffat and Brownstein [2, 3];
4. Cosmological constant problem. This problem represents one of the major unresolved issues in contemporary physics. It is presumed that a presently unknown symmetry operates in such a way to enable a vanishingly small constant while remaining consistent with all accepted field theoretic principles [4];
5. Antimatter hydrogen observation. Is it possible to find isolated antimatter hydrogen (antihydrogen) in astrophysics (stellar or galaxies) observation? Is there antihydrogen star in our galaxy?

Now we are going to discuss other seemingly interesting problems in the physics of elementary particles, in particular those questions which may be related to the New Energy science.

2 Unsolved problems in the physics of elementary particles

We discuss first unsolved problems in the Standard Model of elementary particles. Despite the fact that Standard Model apparently comply with most experimental data up to this day, the majority of particle physicists feel that SM is not a complete framework. E. Goldfain has listed some of the most cited reasons for this belief [5], as follows:

6. The neutrino mass problem. Some recent discovery indicates that neutrino oscillates which implies that neutrino has mass, while QM theories since Pauli predict that neutrino should have no mass [6]. Furthermore it is not yet clear that neutrino (oscillation) phenomena correspond to Dirac or Majorana neutrino [7];
7. SM does not include the contribution of gravity and gravitational corrections to both quantum field theory and renormalization group (RG) equations;
8. SM does not fix the large number of parameters that enter the theory (in particular the spectra of masses, gauge couplings, and fermion mixing angles). Some physicists have also expressed their objections that in the QCD scheme the number of quarks have increased to more than 30 particles, therefore they assert that QCD-quark model cease to be a useful model for elementary particles;

9. SM has a gauge hierarchy problem, which requires fine tuning. Another known fine-tuning problem in SM is "strong CP problem" [8, p. 18];
10. SM postulates that the origin of electroweak symmetry breaking is the Higgs mechanism. Unfortunately Higgs particle has never been found; therefore recently some physicists feel they ought to introduce more speculative theories in order to save their Higgs mechanism [9];
11. SM does not clarify the origin of its gauge group SU(3)×SU(2)×U(1) and why quarks and lepton occur as representations of this group;
12. SM does not explain why (only) the electroweak interactions are chiral (parity-violating) [8, p. 16];
13. Charge quantization problem. SM does not explain another fundamental fact in nature, i.e. why all particles have charges which are multiples of $e/3$ [8, p. 16].

Other than the known problems with SM as described above, there are other quite fundamental problems related to the physics of elementary particles and mathematical physics in general, for instance [10]:

14. Is there dynamical explanation of quark confinement problem? This problem corresponds to the fact that quarks cannot be isolated. See also homepage by Clay Institute on this problem;
15. What is the dynamical mechanism behind Koide's mixing matrix of the lepton mass formula [11]?
16. Does neutrino mass correspond to the Koide mixing matrix [12]?
17. Does Dirac's new electron theory in 1951 reconcile the quantum mechanical view with the classical electrodynamics view of the electron [13]?
18. Is it possible to explain anomalous ultraviolet hydrogen spectrum?
19. Is there quaternion-type symmetry to describe neutrino masses?
20. Is it possible to describe neutrino oscillation dynamics with Bogoliubov-deGennes theory, in lieu of using standard Schrödinger-type wave equation [6]?
21. Solar neutrino problem — i.e. the seeming deficit of observed solar neutrinos [14]. The Sun through fusion, send us neutrinos, and the Earth through fission, antineutrinos. But observation in SuperKamiokande etc. discovers that the observed solar neutrinos are not as expected. In SuperKamiokande Lab, it is found that the number of electron neutrinos which is observed is 0.46 that which is expected [15]. One proposed explanation for the lack of electron neutrinos is that they may have oscillated into muon neutrinos;
22. Neutrino geology problem. Is it possible to observe terrestrial neutrino? The flux of terrestrial neutrino is a direct reflection of the rate of radioactive decays in the Earth and so of the associated energy production, which is presumably the main source of Earth's heat [14];
23. Is it possible to explain the origin of electroweak symmetry breaking without the Higgs mechanism or Higgs particles? For an example of such alternative theory to derive boson masses of electroweak interaction without introducing Higgs particles, see E. Goldfain [16];
24. Is it possible to write quaternionic formulation [17] of quantum Hall effect? If yes, then how?
25. Orthopositronium problem [18]. What is the dynamics behind orthopositronium observation?
26. Is it possible to conceive New Energy generation method from orthopositronium-based reaction? If yes, then how?
27. Muonium problem. Muonium is atom consisting of muon and electron, discovered by a team led by Vernon Hughes in 1960 [19]. What is the dynamics behind muonium observation?
28. Is it possible to conceive New Energy generation method from muonium-based reaction? If yes, then how?
29. Antihydrogen problem [20]. Is it possible to conceive New Energy generation method from antihydrogen-based reaction? If yes, then how?
30. Unmatter problem [21]. Would unmatter be more useful to conceiving New Energy than antimatter? If yes, then how?

It is our hope that perhaps some of these questions may be found interesting to motivate further study of elementary particles.

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References

1. Ginzburg V. L. What problems of physics and astrophysics seem now to be especially important and interesting (thirty years later, already on the verge of XXI century)? *Physics-Uspekhi*, 1999, v. 42(2), 353–373.
2. Moffat J. W. Scalar-Tensor-Vector gravity theory. To be published in *J. Cosmol. Astropart. Phys.*, 2006; preprint arXiv: gr-qc/0506021.
3. Moffat J. W. Spectrum of cosmic microwave fluctuations and the formation of galaxies in a modified gravity theory. arXiv: astro-ph/0602607.

4. Goldfain E. Dynamics of neutrino oscillations and the cosmological constant problem. To appear at *Far East J. Dynamical Systems*, 2007.
5. Goldfain E. Fractional dynamics in the Standard Model for particle physics. To appear at *Comm. Nonlin. Science and Numer. Simul.*, 2007; see preprint in http://www.sciencedirect.com.
6. Giunti C. Theory of neutrino oscillations. arXiv: hep-ph/0401244.
7. Singh D., *et al*. Can gravity distinguish between Dirac and Majorana neutrinos? arXiv: gr-qc/0605133.
8. Langacker P. Structure of the Standard Model. arXiv: hep-ph/0304186, p.16.
9. Djouadi A., *et al*. Higgs particles. arXiv: hep-ph/9605437.
10. Smarandache F., Christianto V., Fu Yuhua, Khrapko R., and Hutchison J. In: *Unfolding Labyrinth: Open Problems in Physics, Mathematics, Astrophysics and Other Areas of Science*, Phoenix (AZ), Hexis, 2006, p. 8–9; arxiv: math/0609238.
11. Koide Y. arXiv: hep-ph/0506247; hep-ph/0303256.
12. Krolikowski W. Towards a realistics neutrino mass formula. arXiv: hep-ph/0609187.
13. deHaas P. J. A renewed theory of electrodynamics in the framework of Dirac ether. London PIRT Conference 2004.
14. Stodolsky L. Neutrino and dark matter detection at low temperature. *Physics-Today*, August 1991, p. 3.
15. Jaffe R. L. Two state systems in QM: applications to neutrino oscillations and neutral kaons. *MIT Quantum Theory Notes, Supplementary Notes for MIT's Quantum Theory Sequence*, (August 2006), p. 26–28.
16. Goldfain E. Derivation of gauge boson masses from the dynamics of Levy flows. *Nonlin. Phenomena in Complex Systems*, 2005, v. 8, no. 4.
17. Balatsky A. V. Quaternion generalization of Laughlin state and the three dimensional fractional QHE. arXiv: cond-mat/9205006.
18. Kotov B. A., Levin B. M. and Sokolov V. I. *et al*. On the possibility of nuclear synthesis during orthopositronium formation. *Progress in Physics*, 2007, v. 3.
19. Jungmann K. Past, present and future of muonium. arXiv: nucl-ex/040401.
20. Voronin A. and Carbonell J. Antihydrogen-hydrogen annihilation at sub-kelvin temperatures. arXiv: physics/0209044.
21. Smarandache F. Matter, antimatter, and unmatter. *Infinite Energy*, 2005, v. 11, issue 62, 50–51.

There Is No Speed Barrier for a Wave Phase Nor for Entangled Particles

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In this short paper, as an extension and consequence of Einstein-Podolski-Rosen paradox and Bell's inequality, one promotes the hypothesis (it has been called the Smarandache Hypothesis [1, 2, 3]) that: There is no speed barrier in the Universe and one can construct arbitrary speeds, and also one asks if it is possible to have an infinite speed (instantaneous transmission)? Future research: to study the composition of faster-than-light velocities and what happens with the laws of physics at faster-than-light velocities?

This is the new version of an early article. That early version, based on a 1972 paper [4], was presented at the Universidad de Blumenau, Brazil, May–June 1993, in the Conference on "Paradoxism in Literature and Science"; and at the University of Kishinev, in December 1994. See that early version in [5].

1 Introduction

What is new in science (physics)?

According to researchers from the common group of the University of Innsbruck in Austria and US National Institute of Standards and Technology (starting from December 1997, Rainer Blatt, David Wineland et al.):

- Photon is a bit of light, the quantum of electromagnetic radiation (quantum is the smallest amount of energy that a system can gain or lose);
- Polarization refers to the direction and characteristics of the light wave vibration;
- If one uses the entanglement phenomenon, in order to transfer the polarization between two photons, then: whatever happens to one is the opposite of what happens to the other; hence, their polarizations are opposite of each other;
- In quantum mechanics, objects such as subatomic particles do not have specific, fixed characteristic at any given instant in time until they are measured;
- Suppose a certain physical process produces a pair of entangled particles A and B (having opposite or complementary characteristics), which fly off into space in the opposite direction and, when they are billions of miles apart, one measures particle A; because B is the opposite, the act of measuring A instantaneously tells B what to be; therefore those instructions would somehow have to travel between A and B faster than the speed of light; hence, one can extend the Einstein-Podolsky-Rosen paradox and Bell's inequality and assert that the light speed is not a speed barrier in the Universe.

Such results were also obtained by: Nicolas Gisin at the University of Geneva, Switzerland, who successfully teleported quantum bits, or qubits, between two labs over 2 km of coiled cable. But the actual distance between the two labs was about 55 m; researchers from the University of Vienna and the Austrian Academy of Science (Rupert Ursin et al. have carried out successful teleportation with particles of light over a distance of 600 m across the River Danube in Austria); researchers from Australia National University and many others [6, 7, 8].

2 Scientific hypothesis

We even promote the hypothesis that:

> There is no speed barrier in the Universe, which would theoretically be proved by increasing, in the previous example, the distance between particles A and B as much as the Universe allows it, and then measuring particle A.

It has been called the *Smarandache Hypotesis* [1, 2, 3].

3 An open question now

If the space is infinite, is the maximum speed infinite?

"This Smarandache hypothesis is controversially interpreted by scientists. Some say that it violates the theory of relativity and the principle of causality, others support the ideas that this hypothesis works for particles with no mass or imaginary mass, in non-locality, through tunneling effect, or in other (extra-) dimension(s)." Kamla John, [9].

Scott Owens' answer [10] to Hans Gunter in an e-mail from January 22, 2001 (the last one forwarded it to the author): "It appears that the only things the Smarandache hypothesis can be applied to are entities that do not have real mass or energy or information. The best example I can come up with is the difference between the wavefront velocity of a photon and the phase velocity. It is common for the phase velocity to exceed the wavefront velocity c, but that does not mean that any real energy is traveling faster than c. So, while it is possible to construct arbitrary speeds from zero in infinite, the superluminal speeds can only apply to purely imaginary entities or components."

Would be possible to accelerate a photon (or another particle traveling at, say, $0.99c$ and thus to get speed greater than c (where c is the speed of light)?

4 Future possible research

It would be interesting to study the composition of two velocities v and u in the cases when:

$v < c$ and $u = c$;

$v = c$ and $u = c$;

$v > c$ and $u = c$;

$v > c$ and $u > c$;

$v < c$ and $u = \infty$;

$v = c$ and $u = \infty$;

$v > c$ and $u = \infty$;

$v = \infty$ and $u = \infty$.

What happens with the laws of physics in each of these cases?

References

1. Motta L. Smarandache Hypothesis: Evidences, Implications, and Applications. *Proceedings of the Second International Conference on Smarandache Type Notions In Mathematics and Quantum Physics*, University of Craiova, Craiova, Romania, December 21–24, 2000 (see the e-print version in the web site at York University, Canada, http://at.yorku.ca/cgi-bin/amca/caft-03).
2. Motta L. and Niculescu G., editors. *Proceedings of the Second International Conference on Smarandache Type Notions In Mathematics and Quantum Physics*, American Research Press, 2000.
3. Weisstein E. W. Smarandache Hypothesis. *The Encyclopedia of Physics*, Wolfram Research (http://scienceworld.wolfram.com/physics/SmarandacheHypothesis.htm).
4. Smarandache F. Collected Papers. Vol. III, Abaddaba Publ. Hse., Oradea, Romania, 2000, 158.
5. Smarandache F. There is no speed barrier in the Universe. *Bulletin of Pure and Applied Sciences*, Delhi, India, v. 17D (Physics), No. 1, 1998, 61.
6. Rincon P. Teleportation breakthrough made. BBC News Online, 2004/06/16.
7. Rincon P. Teleportation goes long distance. BBC News Online, 2004/08/18.
8. Whitehouse D. Australian teleport breakthrough. BBC News Online, 2002/06/17.
9. Kamla J. Private communications. 2001.
10. Owens S. Private communications. 2001.

Entangled States and Quantum Causality Threshold in the General Theory of Relativity

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This article shows, Synge-Weber's classical problem statement about two particles interacting by a signal can be reduced to the case where the same particle is located in two different points A and B of the basic space-time in the same moment of time, so the states A and B are entangled. This particle, being actual two particles in the entangled states A and B, can interact with itself radiating a photon (signal) in the point A and absorbing it in the point B. That is our goal, to introduce entangled states into General Relativity. Under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state Quantum Causality Threshold.

1 Disentangled and entangled particles in General Relativity. Problem statement

In his article of 2000, dedicated to the 100th anniversary of the discovery of quanta, Belavkin [1] generalizes definitions assumed *de facto* in Quantum Mechanics for entangled and disentangled particles. He writes:

> "The only distinction of the classical theory from quantum is that the prior mixed states cannot be dynamically achieved from pure initial states without a procedure of either statistical or chaotic mixing. In quantum theory, however, the mixed, or decoherent states can be dynamically induced on a subsystem from the initial pure disentangled states of a composed system simply by a unitary transformation.
>
> Motivated by Eintein-Podolsky-Rosen paper, in 1935 Schrödinger published a three part essay* on *The Present Situation in Quantum Mechanics*. He turns to EPR paradox and analyses completeness of the description by the wave function for the entangled parts of the system. (The word *entangled* was introduced by Schrödinger for the description of nonseparable states.) He notes that if one has pure states $\psi(\sigma)$ and $\chi(\upsilon)$ for each of two completely separated bodies, one has maximal knowledge, $\psi_1(\sigma,\upsilon) = \psi(\sigma)\chi(\upsilon)$, for two taken together. But the converse is not true for the entangled bodies, described by a non-separable wave function $\psi_1(\sigma,\upsilon)\neq\psi(\sigma)\chi(\upsilon)$: Maximal knowledge of a total system does not necessary imply maximal knowledge of all its parts, not even when these are completely separated one from another, and at the time can not influence one another at all."

In other word, because Quantum Mechanics considers particles as stochastic clouds, there can be entangled particles — particles whose states are entangled, they build a whole system so that if the state of one particle changes the state of the other particles changes immediately as they are far located one from the other.

In particular, because of the permission for entangled states, Quantum Mechanics permits quantum teleportation — the experimentally discovered phenomenon. The term "quantum teleportation" had been introduced into theory in 1993 [2]. First experiment teleporting massless particles (quantum teleportation of photons) was done five years later, in 1998 [3]. Experiments teleporting mass-bearing particles (atoms as a whole) were done in 2004 by two independent groups of scientists: quantum teleportation of the ion of Calcium atom [4] and of the ion of Beryllium atom [5].

There are many followers who continue experiments with quantum teleportation, see [6–16] for instance.

It should be noted, the experimental statement on quantum teleportation has two channels in which information (the quantum state) transfers between two entangled particles: "teleportation channel" where information is transferred instantly, and "synchronization channel" — classical channel where information is transferred in regular way at the light speed or lower of it (the classical channel is targeted to inform the receiving particle about the initial state of the first one). After teleportation the state of the first particle destroys, so there is data transfer (not data copying).

General Relativity draws another picture of data transfer: the particles are considered as point-masses or waves, not stochastic clouds. This statement is true for both mass-bearing particles and massless ones (photons). Data transfer between any two particles is realized as well by point-mass particles, so in General Relativity this process is not of stochastic origin.

In the classical problem statement accepted in General Relativity [17, 18, 19], two mass-bearing particles are con-

*Schrödinger E. *Naturwissenschaften*, 1935, Band 23, 807–812, 823–828, 844–849.

sidered which are moved along neighbour world-lines, a signal is transferred between them by a photon. One of the particles radiates the photon at the other, where the photon is absorbed realizing data transfer between the particles. Of course, the signal can as well be carried by a mass-bearing particle.

If there are two free mass-bering particles, they fall freely along neighbour geodesic lines in a gravitational field. This classical problem has been developed in Synge's book [20] where he has deduced the geodesic lines deviation equation (Synge's equation, 1950's). If these are two particles connected by a non-gravitational force (for instance, by a spring), they are moved along neighbour non-geodesic world-lines. This classical statement has been developed a few years later by Weber [21], who has obtained the world-lines deviation equation (Synge-Weber's equation).

Anyway in this classical problem of General Relativity two interacting particles moved along both neighbour geodesic and non-geodesic world-lines are *disentangled*. This happens, because of two reasons:

1. In this problem statement a signal moves between two interacting particles at the velocity no faster than light, so their states are absolutely separated — these are *disentangled states*;
2. Any particle, being considered in General Relativity's space-time, has its own four-dimensional trajectory (world-line) which is the set of the particle's states from its birth to decay. Two different particles can not occupy the same world-line, so they are in absolutely separated states — they are *disentangled* particles.

The second reason is much stronger than the first one. In particular, the second reason leads to the fact that, in General Relativity, *entangled* are only neighbour states of the same particle along its own world-line — its own states separated in time, not in the three-dimensional space. No two different particles could be entangled. Any two different particles, both mass-bearing and massless ones, are *disentangled* in General Relativity.

On the other hand, experiments on teleportation evident that *entanglement* is really an existing state that happens with particles if they reach specific physical conditions. This is the fact, that should be taken into account by General Relativity.

Therefore our task in this research is to introduce entangled states into General Relativity. Of course, because of the above reasons, two particles can not be in entangled state if they are located in the basic space-time of General Relativity — the four-dimensional pseudo-Riemannian space with sign-alternating label $(+---)$ or $(-+++)$. Its metric is strictly non-degenerated as of any space of Riemannian space family, namely — there the determinant $g = \det\|g_{\alpha\beta}\|$ of the fundamental metric tensor $g_{\alpha\beta}$ is strictly negative $g < 0$. We expand the Synge-Weber problem statement, considering it in a *generalized space-time* whose metric can become degenerated $g = 0$ under specific physical conditions. This space is one of Smarandache geometry spaces [22–28], because its geometry is partially Riemannian, partially not.

As it was shown in [29, 30] (Borissova and Rabounski, 2001), when General Relativity's basic space-time degenerates physical conditions can imply *observable teleportation* of both a mass-bearing and massless particle — its instant displacement from one point of the space to another, although it moves no faster than light in the degenerated space-time area, outside the basic space-time. In the generalized space-time the Synge-Weber problem statement about two particles interacting by a signal (see fig. 1) can be reduced to the case where the same particle is located in two different points A and B of the basic space-time in the same moment of time, so the states A and B are entangled (see fig. 2). This particle, being actual two particles in the entangled states A and B, can interact with itself radiating a photon (signal) in the point A and absorbing it in the point B. That is our goal, to introduce entangled states into General Relativity.

Moreover, as we will see, under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state *Quantum Causality Threshold*.

2 Introducing entangled states into General Relativity

In the classical problem statement, Synge [20] considered two free-particles (fig. 1) moving along neighbour geodesic world-lines $\Gamma(v)$ and $\Gamma(v+dv)$, where v is a parameter along the direction orthogonal to the geodesics (it is taken in the plane normal to the geodesics). There is $v = \text{const}$ along each the geodesic line.

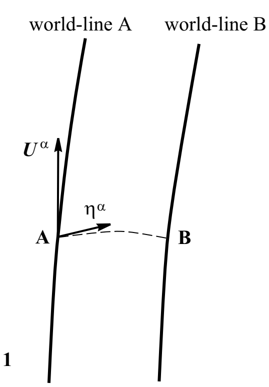

Fig. 1

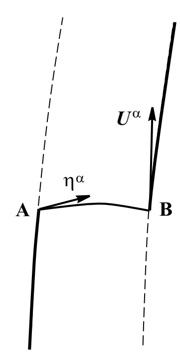

Fig. 2

Motion of the particles is determined by the well-known geodesic equation

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}\, U^\mu \frac{dx^\nu}{ds} = 0\,, \tag{1}$$

which is the actual fact that the absolute differential $\mathrm{D}U^\alpha = dU^\alpha + \Gamma^\alpha_{\mu\nu}\, U^\mu dx^\nu$ of a tangential vector U^α (the velocity world-vector $U^\alpha = \frac{dx^\alpha}{ds}$, in this case), transferred along that geodesic line to where it is tangential, is zero. Here s is an invariant parameter along the geodesic (we assume it the space-time interval), and $\Gamma^\alpha_{\mu\nu}$ are Christoffel's symbols of the 2nd kind. Greek $\alpha = 0, 1, 2, 3$ sign for four-dimensional (space-time) indices.

The parameter v is different for the neighbour geodesics, the difference is dv. Therefore, in order to study relative displacements of two geodesics $\Gamma(v)$ and $\Gamma(v + dv)$, we shall study the vector of their infinitesimal relative displacement

$$\eta^\alpha = \frac{\partial x^\alpha}{\partial v}\, dv\,, \tag{2}$$

As Synge had deduced, a deviation of the geodesic line $\Gamma(v + dv)$ from the geodesic line $\Gamma(v)$ can be found as the solution of his obtained equation

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = 0\,, \tag{3}$$

that describes relative accelerations of two neighbour free-particles ($R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}$ is Riemann-Chrostoffel's curvature tensor). This formula is known as the geodesic lines deviation equation or the *Synge equation*.

In Weber's statement [21] the difference is that he considers two particles connected by a non-gravitational force Φ^α, a spring for instance. So their world-trajectories are non-geodesic, they are determined by the equation

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}\, U^\mu \frac{dx^\nu}{ds} = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{4}$$

which is different from the geodesic equation in that the right part in not zero here. His deduced improved equation of the world lines deviation

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = \frac{1}{m_0 c^2}\frac{\mathrm{D}\Phi^\alpha}{dv}\, dv\,, \tag{5}$$

describes relative accelerations of two particles (of the same rest-mass m_0), connected by a spring. His deviation equation is that of Synge, except of that non-gravitational force Φ^α in the right part. This formula is known as the *Synge-Weber equation*. In this case the angle between the vectors U^α and η^α does not remain unchanged along the trajectories

$$\frac{\partial}{\partial s}(U_\alpha \eta^\alpha) = \frac{1}{m_0 c^2}\, \Phi_\alpha \eta^\alpha. \tag{6}$$

Now, proceeding from this problem statement, we are going to introduce entangled states into General Relativity. At first we determine such states in the space-time of General Relativity, then we find specific physical conditions under which two particles reach a state to be entangled.

Definition Two particles A and B, located in the same spatial section* at the distance $dx^i \neq 0$ from each other, are filled in non-separable states if the observable time interval $d\tau$ between linked events in the particles† is zero $d\tau = 0$. If only $d\tau = 0$, the states become non-separated one from the other, so the particles A and B become entangled.

So we will refer to $d\tau = 0$ as the *entanglement condition* in General Relativity.

Let us consider the entanglement condition $d\tau = 0$ in connection with the world-lines deviation equations.

In General Relativity, the interval of physical observable time $d\tau$ between two events distant at dx^i one from the other is determined through components of the fundamental metric tensor as

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i, \tag{7}$$

see §84 in the well-known *The Classical Theory of Fields* by Landau and Lifshitz [19]. The mathematical apparatus of physical observable quantities (Zelmanov's theory of chronometric invariants [31, 32], see also the brief account in [30, 29]) transforms this formula to

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i, \tag{8}$$

where $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential of an acting gravitational field, and $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation.

So, following the theory of physical observable quantities, in real observations where the observer accompanies his references the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ is

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \tag{9}$$

where $d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i dx^k$ is a three-dimensional (spatial) invariant, built on the metric three-dimensional observable tensor $h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}$. This metric observable tensor, in real observations where the observer accompanies his references, is the same that the analogous built general covariant tensor $h_{\alpha\beta}$. So, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the spatial observable interval for any observer who accompanies his references.

As it is easy to see from (9), there are two possible cases where the entanglement condition $d\tau = 0$ occurs:

(1) $ds = 0$ and $d\sigma = 0$,

(2) $ds^2 = -d\sigma^2 \neq 0$, so $d\sigma$ becomes imaginary,

we will refer to them as the *1st kind and 2nd kind entanglement auxiliary conditions*.

Let us get back to the Synge equation and the Synge-Weber equation.

According to Zelmanov's theory of physical observable quantities [31, 32], if an observer accompanies his references

*A three-dimensional section of the four-dimensional space-time, placed in a given point in the time line. In the space-time there are infinitely many spatial sections, one of which is our three-dimensional space.

†Such linked events in the particles A and B can be radiation of a signal in one and its absorbtion in the other, for instance.

the projection of a general covariant quantity on the observer's spatial section is its spatial observable projection.

Following this way, Borissova has deduced (see eqs. 7.16–7.28 in [33]) that the spatial observable projection of the Synge equation is*

$$\frac{d^2\eta^i}{d\tau^2} + 2\big(D_k^i + A_{k\cdot}^{\cdot i}\big)\frac{d\eta^k}{d\tau} = 0\,, \qquad (10)$$

she called it the *Synge equation in chronometrically invariant form*. The Weber equation is different in its right part containing the non-gravitational force that connects the particles (of course, the force should be filled in the spatially projected form). For this reason, conclusions obtained for the Synge equation will be the same that for the Weber one.

In order to make the results of General Relativity applicable to practice, we should consider tensor quantities and equations designed in chronometrically invariant form, because in such way they contain only chronometrically invariant quantities — physical quantities and geometrical properties of space, measurable in real experiment [31, 32].

Let us look at our problem under consideration from this viewpoint.

As it easy to see, the Synge equation in its chronometrically invariant form (10) under the entanglement condition $d\tau = 0$ becomes nonsense. The Weber equation becomes nonsense as well. So, the classical problem statement becomes senseless as soon as particles reach entangled states.

At the same time, in the recent theoretical research [29] two authors of the paper (Borissova and Rabounski, 2005) have found two groups of physical conditions under which particles can be teleported in non-quantum way. They have been called the *teleportation conditions*:

(1) $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$, the conditions of photon teleportation;

(2) $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$, the conditions of substantial (mass-bearing) particles teleportation.

There also were theoretically deduced physical conditions†, which should be reached in a laboratory in order to teleport particles in the non-quantum way [29].

As it is easy to see the non-quantum teleportation condition is identical to introduce here the entanglement main condition $d\tau = 0$ in couple with the 1st kind and 2nd kind auxiliary entanglement conditions!

*In this formula, according to Zelmanov's mathematical apparatus of physical observable quantities [31, 32], $D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t} = \frac{1}{2\sqrt{g_{00}}}\frac{\partial h_{ik}}{\partial t}$ is the three-dimensional symmetric tensor of the space deformation observable rate while $A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\big(F_i v_k - F_k v_i\big)$ is the three-dimensional antisymmetric tensor of the space rotation observable angular velocities, which indices can be lifted/lowered by the metric observable tensor so that $D_k^i = h^{im} D_{km}$ and $A_{k\cdot}^{\cdot i} = h^{im} A_{km}$. See brief account of the Zelmanov mathematical apparatus in also [30, 33, 34, 35].

†A specific correlation between the gravitational potential w, the space rotation linear velocity v_i and the teleported particle's velocity u^i.

Taking this one into account, we transform the classical Synge and Weber problem statement into another. In our statement the world-line of a particle, being entangled to itself by definition, splits into two different world-lines under teleportation conditions. In other word, as soon as the teleportation conditions occur in a research laboratory, the world-line of a teleported particle breaks in one world-point A and immediately starts in the other world-point B (fig. 2). Both particles A and B, being actually two different states of the same teleported particle at a remote distance one from the other, are in *entangled states*. So, in this statement, the particles A and B themselves are *entangled*.

Of course, this entanglement exists in only the moment of the teleportation when the particle exists in two different states simultaneously. As soon as the teleportation process has been finished, only one particle of them remains so the entanglement disappears.

It should be noted, it follows from the entanglement conditions, that only substantial particles can reach entangled states in the basic space-time of General Relativity — the four-dimensional pseudo-Riemannian space. Not photons. Here is why.

As it is known, the interval $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ can not be fully degenerated in a Riemannian space‡: the condition is that the determinant of the metric fundamental tensor $g_{\alpha\beta}$ must be strictly negative $g = \det\|g_{\alpha\beta}\| < 0$ by definition of Riemannian spaces. In other word, in the basic space-time of General Relativity the fundamental metric tensor must be strictly non-degenerated as $g < 0$.

The observable three-dimensional (spatial) interval $d\sigma^2 = h_{ik}\,dx^i dx^k$ is positive determined [31, 32], proceeding from physical sense. It fully degenerates $d\sigma^2 = 0$ if only the space compresses into point (the senseless case) or the determinant of the metric observable tensor becomes zero $h = \det\|h_{ik}\| = 0$.

As it was shown by Zelmanov [31, 32], in real observations where an observer accompanies his references, the determinant of the metric observable tensor is connected with the determinant of the fundamental one by the relationship $h = -\frac{g}{g_{00}}$. From here we see, if the three-dimensional observable metric fully degenerates $h = 0$, the four-dimensional metric degenerates as well $g = 0$.

We have obtained that states of two substantial particles can be entangled, if $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$ in the space neighbourhood. So $h > 0$ and $g < 0$ in the neighbourhood, hence the four-dimensional pseudo-Riemannian space is not degenerated.

Conclusion Substantial particles can reach entangled states in the basic space-time of General Relativity (the four-dimensional pseudo-Riemannian space) under specific conditions in the neighbourhood.

‡It can only be partially degenerated. For instance, a four-dimensional Riemannian space can be degenerated into a three-dimensional one.

Although $ds^2 = -d\sigma^2$ in the neighbourhood ($d\sigma$ should be imaginary), the substantial particles remain in regular sub-light area, they do not become super-light tachyons. It is easy to see, from the definition of physical observable time (8), the entanglement condition $d\tau = 0$ occurs only if the specific relationship holds

$$\mathrm{w} + v_i u^i = c^2 \tag{11}$$

between the gravitational potential w, the space rotation linear velocity v_i and the particles' true velocity $u^i = dx^i/dt$ in the observer's laboratory. For this reason, in the neighbourhood the space-time metric is

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k, \tag{12}$$

so the substantial particles can become entangled if the space initial signature $(+---)$ becomes inverted $(-+++)$ in the neighbourhood, while the particles' velocities u^i remain no faster than light.

Another case — massless particles (photons). States of two phonos can be entangled, only if there is in the space neighbourhood $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$. In this case the determinant of the metric observable tensor becomes $h = 0$, so the space-time metric as well degenerates $g = -g_{00}\, h = 0$. This is not the four-dimensional pseudo-Riemannian space.

Where is that area? In the previous works (Borissova and Rabounski, 2001 [30, 29]) a generalization to the basic space-time of General Relativity was introduced — the four-dimensional space which, having General Relativity's sign-alternating label $(+---)$, permits the space-time metric to be fully degenerated so that there is $g \leqslant 0$.

As it was shown in those works, as soon as the specific condition $\mathrm{w} + v_i u^i = c^2$ occurs, the space-time metric becomes fully degenerated: there are $ds = 0$, $d\sigma = 0$, $d\tau = 0$ (it can be easy derived from the above definition for the quantities) and, hence $h = 0$ and $g = 0$. Therefore, in a space-time where the *degeneration condition* $\mathrm{w} + v_i u^i = c^2$ is permitted the determinant of the fundamental metric tensor is $g \leqslant 0$. This case includes both Riemannian geometry case $g < 0$ and non-Riemannian, fully degenerated one $g = 0$. For this reason a such space is one of Smarandache geometry spaces [22–28], because its geometry is partially Riemannian, partially not*. In the such generalized space-time the 1st kind entanglement conditions $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$ (the entanglement conditions for photons) are permitted in that area where the space metric fully degenerates (there $h = 0$ and, hence $g = 0$).

Conclusion Massless particles (photons) can reach entangled states, only if the basic space-time fully degenerates $g = \det \| g_{\alpha\beta} \| = 0$ in the neighbourhood. It is permitted in the generalized four-dimensional space-time which metric can be fully degenerated $g \leqslant 0$ in that area where the degeneration conditions occur. The generalized space-time is attributed to Smarandache geometry spaces, because its geometry is partially Riemannian, partially not.

So, entangled states have been introduced into General Relativity for both substantial particles and photons.

3 Quantum Causality Threshold in General Relativity

This term was introduced by one of the authors two years ago (Smarandache, 2003) in our common correspondence [36] on the theme:

Definition Considering two particles A and B located in the same spatial section, *Quantum Causality Threshold* was introduced as a special state in which neither A nor B can be the cause of events located "over" the spatial section on the Minkowski diagram.

The term *Quantum* has been added to the *Causality Threshold*, because in this problem statement an interaction is considered between two infinitely far away particles (in infinitesimal vicinities of each particle) so this statement is applicable to only quantum scale interactions that occur in the scale of elementary particles.

Now, we are going to find physical conditions under which particles can reach the threshold in the space-time of General Relativity.

Because in this problem statement we look at causal relations in General Relativity's space-time from "outside", it is required to use an "outer viewpoint" — a point of view located outside the space-time.

We introduce a such point of outlook in an Euclidean flat space, which is tangential to our's in that world-point, where the observer is located. In this problem statement we have a possibility to compare the absolute cause relations in that tangential flat space with those in ours. As a matter, a tangential Euclidean flat space can be introduced at any point of the pseudo-Riemannian space.

At the same time, according to Zelmanov [31, 32], within infinitesimal vicinities of any point located in the pseudo-Riemannian space a *locally geodesic reference frame* can be introduced. In a such reference frame, within infinitesimal vicinities of the point, components of the metric fundamental tensor (marked by tilde)

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial \tilde{x}^\mu \partial \tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)(\tilde{x}^\nu - x^\nu) + \ldots \tag{13}$$

*In foundations of geometry it is known the *S-denying* of an axiom [22–25], i. e. in the same space an "axiom is false in at least two different ways, or is false and also true. Such axiom is said to be Smarandachely denied, or S-denied for short" [26]. As a result, it is possible to introduce geometries, which have common points bearing mixed properties of Euclidean, Lobachevsky-Bolyai-Gauss, and Riemann geometry in the same time. Such geometries has been called paradoxist geometries or Smarandache geometries. For instance, Iseri in his book *Smarandache Manifolds* [26] and articles [27, 28] introduced manifolds that support particular cases of such geometries.

are different from those $g_{\alpha\beta}$ at the point of reflection to within only the higher order terms, which can be neglected. So, in a locally geodesic reference frame the fundamental metric tensor can be accepted constant, while its first derivatives (Christoffel's symbols) are zeroes. The fundamental metric tensor of an Euclidean space is as well a constant, so values of $\tilde{g}_{\mu\nu}$, taken in the vicinities of a point of the pseudo-Riemannian space, converge to values of $g_{\mu\nu}$ in the flat space tangential at this point. Actually, we have a system of the flat space's basic vectors $\vec{e}_{(\alpha)}$ tangential to curved coordinate lines of the pseudo-Riemannian space. Coordinate lines in Riemannian spaces are curved, inhomogeneous, and are not orthogonal to each other (the latest is true if the space rotates). Therefore the lengths of the basic vectors may be very different from the unit.

Writing the world-vector of an infinitesimal displacement as $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$, we obtain $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components of the basic vectors $\vec{e}_{(\alpha)}$ tangential to the coordinate lines are $\vec{e}_{(0)} = \{e^0_{(0)}, 0, 0, 0\}$, $\vec{e}_{(1)} = \{0, e^1_{(1)}, 0, 0\}$, $\vec{e}_{(2)} = \{0, 0, e^2_{(2)}, 0\}$, $\vec{e}_{(3)} = \{0, 0, 0, e^2_{(3)}\}$. Scalar product of $d\vec{r}$ with itself is $d\vec{r}d\vec{r} = ds^2$ or, in another $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, so $g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)} e_{(\beta)} \cos\,(x^\alpha; x^\beta)$. We obtain

$$g_{00} = e^2_{(0)}\,, \qquad g_{0i} = e_{(0)} e_{(i)} \cos\,(x^0; x^i)\,, \tag{14}$$

$$g_{ik} = e_{(i)} e_{(k)} \cos\,(x^i; x^k)\,, \qquad i, k = 1, 2, 3\,. \tag{15}$$

Then, substituting g_{00} and g_{0i} from formulas that determine the gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ and the space rotation linear velocity $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$, we obtain

$$v_i = -c\, e_{(i)} \cos\,(x^0; x^i)\,, \tag{16}$$

$$h_{ik} = e_{(i)} e_{(k)} \Big[\cos(x^0; x^i)\cos(x^0; x^k) - \cos(x^i; x^k)\Big]\,. \tag{17}$$

From here we see: if the pseudo-Riemannian space is free of rotation, $\cos\,(x^0; x^i) = 0$ so the observer's spatial section is strictly orthogonal to time lines. As soon as the space starts to do rotation, the cosine becomes different from zero so the spatial section becomes non-orthogonal to time lines (fig. 3). Having this process, the light hypercone inclines with the time line to the spatial section. In this inclination the light hypercone does not remain unchanged, it "compresses" because of hyperbolic transformations in pseudo-Riemannian space. The more the light hypercone inclines, the more it symmetrically "compresses" because the space-time's geometrical structure changes according to the inclination.

In the ultimate case, where the cosine reach the ultimate value $\cos\,(x^0; x^i) = 1$, time lines coincide the spatial section: time "has fallen" into the three-dimensional space. Of course, in this case the light hypercone overflows time lines and the spatial section: the light hypercone "has as well fallen" into the three-dimensional space.

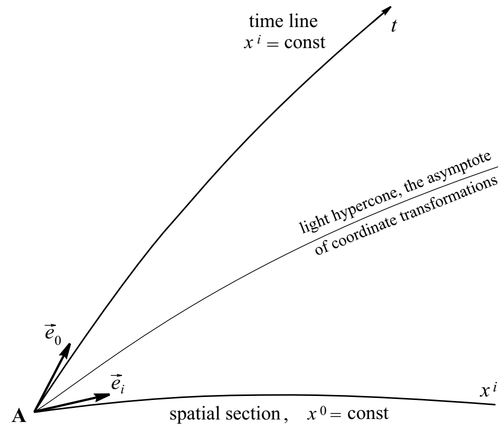

Fig. 3

As it is easy to see from formula (16), this ultimate case occurs as soon as the space rotation velocity v_i reaches the light velocity. If particles A and B are located in the space filled into this ultimate state, neither A nor B can be the cause of events located "over" the spatial section in the Minkowski diagrams we use in the pictures. So, in this ultimate case the space-time is filled into a special state called Quantum Causality Threshold.

Conclusion Particles, located in General Relativity's space-time, reach Quantum Causality Threshold as soon as the space rotation reaches the light velocity. Quantum Causality Threshold is impossible if the space does not rotate (holonomic space), or if it rotates at a sub-light speed.

So, Quantum Causality Threshold has been introduced into General Relativity.

References

1. Belavkin V. P. Quantum causality, decoherence, trajectories and information. arXiv: quant-ph/0208087, 76 pages.
2. Bennett C. H., Brassard G., Crepeau C., Jozsa R., Peres A., and Wootters W. K. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.*, 1993, v. 70, 1895–1899.
3. Boschi D., Branca S., De Martini F., Hardy L., and Popescu S. Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen Channels. *Phys. Rev. Lett.*, 1998, v. 80, 1121–1125.
4. Riebe M., Häffner H., Roos C. F., Hänsel W., Benhelm J., Lancaster G. P. T., Korber T. W., Becher C., Schmidt-Kaler F., James D. F. V., and Blatt R. Deterministic quantum teleportation with atoms. *Nature*, 2004, v. 429 (June, 17), 734–736.
5. Barrett M. D., Chiaverini J., Schaetz T., Britton J., Itano W. M., Jost J. D., Knill E., Langer C., Leibfried D., Ozeri R., Wineland D. J. Deterministic quantum teleportation of atomic qubits. *Nature*, 2004, v. 429 (June, 17), 737–739.

6. Pan J.-W., Bouwmeester D., Daniell M., Weinfurter H., Zeilinger A. Experimental test of quantum nonlocality in three-photon Greenberger-Horne-Zeilinger entanglement. *Nature*, 2000, v. 403 (03 Feb 2000), 515–519.
7. Mair A., Vaziri A., Weihs G., Zeilinger A. Entanglement of the orbital angular momentum states of photons. *Nature*, v. 412 (19 July 2001), 313–316.
8. Lukin M. D., Imamoglu A. Controlling photons using electromagnetically induced transparency *Nature*, v. 413 (20 Sep 2001), 273–276.
9. Julsgaard B., Kozhekin A., Polzik E. S. Experimental long-lived entanglement of two macroscopic objects. *Nature*, v. 413 (27 Sep 2001), 400–403.
10. Duan L.-M., Lukin M. D., Cirac J. I., Zoller P. Long-distance quantum communication with atomic ensembles and linear optics. *Nature*, v. 414 (22 Nov 2001), 413–418.
11. Yamamoto T., Koashi M., Özdemir Ş.K., Imoto N. Experimental extraction of an entangled photon pair from two identically decohered pairs. *Nature*, v. 421 (23 Jan 2003), 343–346.
12. Pan J.-W., Gasparoni S., Aspelmeyer M., Jennewein T., Zeilinger A. Experimental realization of freely propagating teleported qubits. *Nature*, v. 421 (13 Feb 2003), 721–725.
13. Pan J.-W., Gasparoni S., Ursin R., Weihs G., Zeilinger A. Experimental entanglement purification of arbitrary unknown states. *Nature*, v. 423 (22 May 2003), 417–422.
14. Zhao Zhi, Chen Yu-Ao, Zhang An-Ning, Yang T., Briegel H. J., Pan J.-W. Experimental demonstration of five-photon entanglement and open-destination teleportation. *Nature*, v. 430 (01 July 2004), 54–58.
15. Blinov B. B., Moehring D. L., Duan L.-M., Monroe C. Observation of entanglement between a single trapped atom and a single photon. *Nature*, v. 428 (11 Mar 2004), 153–157.
16. Ursin R., Jennewein T., Aspelmeyer M., Kaltenbaek R., Lindenthal M., Walther P., Zeilinger A. Communications: Quantum teleportation across the Danube. *Nature*, v. 430 (19 Aug 2004), 849–849.
17. Pauli W. Relativitätstheorie. *Encyclopäedie der mathematischen Wissenschaften*, Band V, Heft IV, Art. 19, 1921 (Pauli W. Theory of Relativity. Pergamon Press, 1958).
18. Eddington A. S. The mathematical theory of relativity. Cambridge University Press, Cambridge, 1924 (referred with the 3rd expanded edition, GTTI, Moscow, 1934, 508 pages).
19. Landau L. D. and Lifshitz E. M. The classical theory of fields. GITTL, Moscow, 1939 (ref. with the 4th final exp. edition, Butterworth–Heinemann, 1980).
20. Synge J. L. Relativity: the General Theory. North Holland, Amsterdam, 1960 (referred with the 2nd expanded edition, Foreign Literature, Moscow, 1963, 432 pages).
21. Weber J. General Relativity and gravitational waves. R. Marshak, New York, 1961 (referred with the 2nd edition, Foreign Literature, Moscow, 1962, 271 pages).
22. Smarandache F. Paradoxist mathematics. *Collected papers*, v. II, Kishinev University Press, Kishinev, 1997, 5–29.
23. Ashbacher C. Smarandache geometries. *Smarandache Notions*, book series, v. 8, ed. by C. Dumitrescu and V. Seleacu, American Research Press, Rehoboth, 1997, 212–215.
24. Chimienti S. P., Bencze M. Smarandache paradoxist geometry. *Bulletin of Pure and Applied Sciences*, 1998, v. 17E, No. 1, 123–124. See also *Smarandache Notions*, book series, v. 9, ed. by C. Dumitrescu and V. Seleacu, American Research Press, Rehoboth, 1998, 42–43.
25. Kuciuk L. and Antholy M. An introduction to Smarandache geometries. *New Zealand Math. Coll.*, Massey Univ., Palmerston North, New Zealand, Dec 3–6, 2001 (on-line http://atlas-conferences.com/c/a/h/f/09.htm).
26. Iseri H. Smarandache manifolds. American Research Press, Rehoboth, 2002.
27. Iseri H. Partially paradoxist Smarandache geometry. *Smarandache Notions*, book series, v. 13, ed. by J. Allen, F. Liu, D. Costantinescu, Am. Res. Press, Rehoboth, 2002, 5–12.
28. Iseri H. A finitely hyperbolic point in a smooth manifold. *JP Journal on Geometry and Topology*, 2002, v. 2 (3), 245–257.
29. Borissova L. B. and Rabounski D. D. On the possibility of instant displacements in the space-time of General Relativity. *Progress in Physics*, 2005, v. 1, 17–19.
30. Borissova L. B. and Rabounski D. D. Fields, vacuum, and the mirror Universe. Editorial URSS, Moscow, 2001, 272 pages (the 2nd revised ed.: CERN, EXT-2003-025).
31. Zelmanov A. L. Chronometric invariants. Dissertation, 1944. First published: CERN, EXT-2004-117, 236 pages.
32. Zelmanov A. L. Chronometric invariants and co-moving coordinates in the general relativity theory. *Doklady Acad. Nauk USSR*, 1956, v. 107 (6), 815–818.
33. Borissova L. Gravitational waves and gravitational inertial waves in the General Theory of Relativity: A theory and experiments. *Progress in Physics*, 2005, v. 2, 30–62.
34. Rabounski D. A new method to measure the speed of gravitation. *Progress in Physics*, 2005, v. 1, 3–6.
35. Rabounski D. A theory of gravity like electrodynamics. *Progress in Physics*, 2005, v. 2, 15–29.
36. Smarandache F. Private communications with D. Rabounski and L. Borissova, May 2005.

Quantum Quasi-Paradoxes and Quantum Sorites Paradoxes

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There can be generated many paradoxes or quasi-paradoxes that may occur from the combination of quantum and non-quantum worlds in physics. Even the passage from the micro-cosmos to the macro-cosmos, and reciprocally, can generate unsolved questions or counter-intuitive ideas. We define a quasi-paradox as a statement which has a *prima facie* self-contradictory support or an explicit contradiction, but which is not completely proven as a paradox. We present herein four elementary quantum quasi-paradoxes and their corresponding quantum Sorites paradoxes, which form a class of quantum quasi-paradoxes.

1 Introduction

According to the Dictionary of Mathematics (Borowski and Borwein, 1991 [1]), the **paradox** is "an apparently absurd or self-contradictory statement for which there is *prima facie* support, or an explicit contradiction derived from apparently unexceptionable premises". Some paradoxes require the revision of their intuitive conception (Russell's paradox, Cantor's paradox), others depend on the inadmissibility of their description (Grelling's paradox), others show counter-intuitive features of formal theories (Material implication paradox, Skolem Paradox), others are self-contradictory — Smarandache Paradox: "All is <A> the <Non-A> too!", where <A> is an attribute and <Non-A> its opposite; for example "All is possible the impossible too!" (Weisstein, 1998 [2]).

Paradoxes are normally true and false in the same time.

The **Sorites paradoxes** are associated with Eubulides of Miletus (fourth century B. C.) and they say that there is not a clear frontier between visible and invisible matter, determinist and indeterminist principle, stable and unstable matter, long time living and short time living matter.

Generally, between <A> and <Non-A> there is no clear distinction, no exact frontier. Where does <A> really end and <Non-A> begin? One extends Zadeh's "fuzzy set" concept to the "neutrosophic set" concept.

Let's now introduce the notion of quasi-paradox:

A **quasi-paradox** is a statement which has a *prima facia* self-contradictory support or an explicit contradiction, but which is not completely proven as a paradox. A quasi-paradox is an *informal* contradictory statement, while a paradox is a *formal* contradictory statement.

Some of the below quantum quasi-paradoxes can later be proven as real quantum paradoxes.

2 Quantum Quasi-Paradoxes and Quantum Sorites Paradoxes

The below quasi-paradoxes and Sorites paradoxes are based on the antinomies: visible/invisible, determinist/indeterminist, stable/unstable, long time living/short time living, as well as on the fact that there is not a clear separation between these pairs of antinomies.

2.1.1 **Invisible Quasi-Paradox:** Our visible world is composed of a totality of invisible particles.

2.1.2 **Invisible Sorites Paradox:** There is not a clear frontier between visible matter and invisible matter.

(a) An invisible particle does not form a visible object, nor do two invisible particles, three invisible particles, etc. However, at some point, the collection of invisible particles becomes large enough to form a visible object, but there is apparently no definite point where this occurs.

(b) A similar paradox is developed in an opposite direction. It is always possible to remove a particle from an object in such a way that what is left is still a visible object. However, repeating and repeating this process, at some point, the visible object is decomposed so that the left part becomes invisible, but there is no definite point where this occurs.

2.2.1 **Uncertainty Quasi-Paradox:** Large matter, which is at some degree under the "determinist principle", is formed by a totality of elementary particles, which are under Heisenberg's "indeterminacy principle".

2.2.2 **Uncertainty Sorites Paradox:** Similarly, there is not a clear frontier between the matter under the "determinist principle" and the matter under "indeterminist principle".

2.3.1 **Unstable Quasi-Paradox:** "Stable" matter is formed by "unstable" elementary particles (elementary particles decay when free).

2.3.2 **Unstable Sorites Paradox:** Similarly, there is not a clear frontier between the "stable matter" and the "unstable matter".

2.4.1 **Short-Time-Living Quasi-Paradox:** "Long-time-living” matter is formed by very “short-time-living” elementary particles.

2.4.2 **Short-Time-Living Sorites Paradox:** Similarly, there is not a clear frontier between the “long-time-living” matter and the “short-time-living” matter.

3 Conclusion

“More such quantum quasi-paradoxes and paradoxes can be designed, all of them forming a class of Smarandache quantum quasi-paradoxes.” (Dr. M. Khoshnevisan, Griffith University, Gold Coast, Queensland, Australia [3])

References

1. Borowski E. J. and Borwein J. M. The Harper Collins Dictionary of Mathematics. Harper Perennial, A Division of Harper Collins Publishers, New York, 1991.
2. Weisstein E. W. Smarandache Paradox. *CRC Concise Encyclopaedia of Mathematics*, CRC Press, Boca Raton, Florida, 1998, 1661, (see the e-print version of this article in http://mathworld.wolfram.com/SmarandacheParadox.html).
3. Khoshnevisan M. Private communications, 1997.
4. Motta L. A Look at the Smarandache Sorites Paradox. *Proceedings of the Second International Conference on Smarandache Type Notions In Mathematics and Quantum Physics*, University of Craiova, Craiova, Romania, December 21–24, 2000 (see the e-print version in the web site at York University, Canada, http://at.yorku.ca/cgi-bin/amca/caft-04).
5. Niculescu G. On Quantum Smarandache Paradoxes. *Proceedings of the Second International Conference on Smarandache Type Notions In Mathematics and Quantum Physics*, University of Craiova, Craiova, Romania, December 21–24, 2000 (see the e-print version in the web site at York University, Canada, http://at.yorku.ca/cgi-bin/amca/caft-20).
6. Smarandache F. Invisible paradox. *Neutrosophy, Neutrosophic Probability, Set, and Logic*, American Research Press, Rehoboth, 1998 (see the third e-print edition of the book in the web site http://www.gallup.unm.edu/∼smarandache).
7. Smarandache F. Sorites paradoxes. *Definitions, Solved and Unsolved Problems, Conjectures, and Theorems in Number Theory and Geometry* (edited by M. L. Perez), Xiquan Publishing House, Phoenix, 2000.

The Neutrosophic Logic View to Schrödinger's Cat Paradox

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This article discusses Neutrosophic Logic interpretation of the Schrodinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic Logic could take into consideration, whereas other methods including Fuzzy Logic could not. For a balanced discussion, other interpretations have also been discussed.

1 Schrödinger equation

As already known, Schrödinger equation is the most used equation to describe non-relativistic quantum systems. Its relativistic version was developed by Klein-Gordon and Dirac, but Schrödinger equation has wide applicability in particular because it resembles classical wave dynamics. For introduction to non-relativistic quantum mechanics, see [1].

Schrödinger equation begins with definition of total energy $E = \vec{p}^{\,2}/2m$. Then, by using a substitution

$$E = i\hbar\frac{\partial}{\partial t}, \qquad P = \frac{\hbar}{i}\nabla, \tag{1}$$

one gets [2]

$$\left[i\hbar\frac{\partial}{\partial t} + \hbar\frac{\bar{\nabla}^2}{2m} - U(x)\right]\psi = 0 \tag{2}$$

or

$$\frac{i\partial}{\partial t}\psi = H\psi\,. \tag{3}$$

While this equation seems quite clear to represent quantum dynamics, the physical meaning of the wavefunction itself is not so clear. Soon thereafter Born came up with hypothesis that the square of the wavefunction has the meaning of chance to find the electron in the region defined by dx (Copenhagen School). While so far his idea was quickly adopted as "standard interpretation", his original "guiding field" interpretation has been dropped after criticism by Heisenberg over its physical meaning [3]. Nonetheless, a definition of "Copenhagen interpretation" is that it gives the wavefunction a role in the actions of something else, namely of certain macroscopic objects, called "measurement apparatus", therefore it could be related to phenomenological formalism [3].

Nonetheless, we should also note here that there are other approaches different from Born hypothesis, including:

- The square of the wavefunction represents a measure of the density of matter in region defined by dx (Determinism school [3, 4, 5]). Schrödinger apparently preferred this argument, albeit his attempt to demonstrate this idea has proven to be unfruitful;
- The square of wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6];
- The wavefunction in Schrödinger equation represents tendency to make structures;
- The wavemechanics can also be described in terms of topological Aharonov effect, which then it could be related to the notion of topological quantization [7, 8]. Aharonov himself apparently argues in favour of "realistic" meaning of Schrödinger wave equation, whose interpretation perhaps could also be related to Kron's work [9].

So forth we will discuss solution of this paradox.

2 Solution to Schrödinger's cat paradox

2.1 Standard interpretation

It is known that Quantum Mechanics could be regarded more as a "mathematical theory" rather than a physical theory [1, p. 2]. It is wave mechanics allowing a corpuscular duality. Already here one could find problematic difficulties: i. e. while the quantity of wavefunction itself could be computed, the physical meaning of wavefunction itself remains *indefinable* [1]. Furthermore, this notion of wavefunction corresponds to another fundamental indefinable in Euclidean geometry: the point [1, p. 2]. It is always a baffling question for decades, whether the electron could be regarded as wave, a point, or we should introduce a *non-zero* finite entity [4]. Attempts have been made to describe wave equation in such non-zero entity but the question of the physical meaning of wavefunction itself remains mystery.

The standard Copenhagen interpretation advertised by Bohr and colleagues (see DeBroglie, Einstein, Schrödinger who advocated "realistic" interpretation) asserts that it is practically *impossible* to know what really happens in quantum scale. The quantum measurement itself only represents reading in *measurement apparatus*, and therefore it is difficult to separate the object to be measured and the measurement apparatus itself. Bohr's phenomenological viewpoint perhaps could be regarded as pragmatic approach, starting with the request not to attribute a deep meaning to the wave function but immediately go over to statistical likelihood [10]. Consequently, how the process of "wave collapse" could happen remains mystery.

Heisenberg himself once emphasized this viewpoint when asked directly the question: Is there a fundamental level of reality? He replied as follows:

> "This is just the point: I do not know what the words fundamental reality mean. They are taken from our daily life situation where they have a good meaning, but when we use such terms we are usually extrapolating from our daily lives into an area very remote from it, where we cannot expect the words to have a meaning. This is perhaps one of the fundamental difficulties of philosophy: that our thinking hangs in the language. Anyway, we are forced to use the words so far as we can; we try to extend their use to the utmost, and then we get into situations in which they have no meaning" [11].

A modern version of this interpretation suggests that at the time of measurement, the wave collapses instantaneously into certain localized object corresponding to the action of measurement. In other words, the measurement processes define how the wave should define itself. At this point, the wave ceases to become coherent, and the process is known as "decoherence". Decoherence may be thought of as a way of making real for an observer in the large scale world only one possible history of the universe which has a likelihood that it will occur. Each possible history must in addition obey the laws of logic of this large-scale world. The existence of the phenomenon of decoherence is now supported by laboratory experiments [12]. It is worthnoting here, that there are also other versions of decoherence hypothesis, for instance by Tegmark [13] and Vitiello [14].

In the meantime, the "standard" Copenhagen interpretation emphasizes the role of observer where the "decoherence viewpoint" may not. The problem becomes more adverse because the axioms of standard statistical theory themselves are not fixed forever [15, 16]. And here is perhaps the source of numerous debates concerning the interpretation and philosophical questions implied by Quantum Mechanics. From this viewpoint, Neutrosophic Logic offers a new viewpoint to problems where indeterminacy exists. We will discuss this subsequently. For a sense of balance, we also discuss a number of alternative interpretations. Nonetheless this article will not discuss all existing interpretations of the quantum wavefunction in the literature.

2.2 Schrödinger's cat paradox

To make the viewpoint on this paradox a bit clearer, let us reformulate the paradox in its original form.

According to Uncertainty Principle, any measurement of a system must disturb the system under investigation, with a resulting lack of precision in the measurement. Soon after reading Einstein-Podolsky-Rosen's paper discussing incompleteness of Quantum Mechanics, Schrödinger in 1935 came up with a series of papers in which he used the "cat paradox" to give an illustration of the problem of viewing these particles in a "thought experiment" [15, 17]:

> "One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The wave-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared into equal parts."

In principle, Schrödinger's thought experiment asks whether the cat is dead or alive after an hour. The most logical solution would be to wait an hour, open the box, and see if the cat is still alive. However once you open the box to determine the state of the cat you have viewed and hence disturbed the system and introduced a level of uncertainty into the results. The answer, in quantum mechanical terms, is that before you open the box the cat is in a state of being half-dead and half-alive.

Of course, at this point one could ask whether it is possible to find out the state of the cat without having to disturb its wavefunction via action of "observation".

If the meaning of word "observation" here is defined by *to open the box and see the cat*, and then it seems that we could argue whether it is possible to propose another equally possible experiment where we introduce a pair of twin cats, instead of only one. A cat is put in the box while another cat is located in a separate distance, let say 1 meter from the box. If the state of the cat inside the box altered because of poison reaction, it is likely that we could also observe its effect to its twin, perhaps something like "sixth sense" test (perhaps via monitoring frequency of the twin cat's brain).

This plausible experiment could be viewed as an alternative "thought experiment" of well-known Bell-Aspect-type experiment. One could also consider an entangled pair of photons instead of twin cats to conduct this "modified" cat paradox. Of course, for this case then one would get a bit complicated problem because now he/she should consider two probable state: the decaying atom and the photon pair.

We could also say that using this alternative configuration, we know exact information about the Cat outside, while indeterminate information about the Cat inside. However, because both Cats are entangled (twin) we are sure of all the properties of the Cat inside "knows" the state of the Cat outside the box, via a kind of "spooky action at distance" reason (in Einstein's own word)*.

Therefore, for experimental purpose, perhaps it would be useful to simplify the problem by using "modified" Aspect-type experiment [16]. Here it is proposed to consider a decaying atom of Cesium which emits two correlated photons, whose polarization is then measured by Alice (A) on the left and by Bob (B) on the right (see Fig. 1). To include the probable state as in the original cat paradox, we will use a switch instead of Alice A. If a photon comes to this switch, then it will turn on a coffee-maker machine, therefore the observer will get a cup of coffee†. Another switch and coffee-maker set also replace Bob position (see Fig. 2). Then we encapsulate the whole system of decaying atom, switch, and coffee-maker at A, while keeping the system at B side open. Now we can be sure, that by the time the decaying atom of Cesium emits photon to B side and triggers the switch at this side which then turns on the coffee-maker, it is "likely" that we could also observe the same cup of coffee at A side, even if we do not open the box.

We use term "likely" here because now we encounter a "quasi-deterministic" state where there is also small chance that the photon is shifted different from -0.0116, which is indeed what the Aspect, Dalibard and Roger experiment demonstrated in 1982 using a system of two correlated photons [16]. At this "shifted" phase, it could be that the switch will not turn on the coffee-maker at all, so when an observer opens the box at A side he will not get a cup of coffee.

If this hypothetical experiment could be verified in real world, then it would result in some wonderful implications, like prediction of ensembles of multi-particles system, — or a colony of cats.

Another version of this cat paradox is known as GHZ paradox: "The Greenberger-Horne-Zeilinger paradox exhibits some of the most surprising aspects of multiparticle entanglement" [18]. But we limit our discussion here on the original cat paradox.

2.3 Hidden-variable hypothesis

It would be incomplete to discuss quantum paradoxes, in particular Schrödinger's cat paradox, without mentioning hidden-variable hypothesis. There are various versions of this argument, but it could be summarised as an assertion that there is "something else" which should be included in the Quantum Mechanical equations in order to explain thoroughly all quantum phenomena. Sometimes this assertion can be formulated in question form [19]: Can Quantum Mechanics be considered complete? Interestingly, however, the meaning of "complete" itself remains quite abstract (fuzzy).

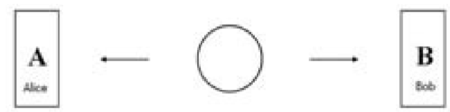

Figure 1: Aspect-type experiment

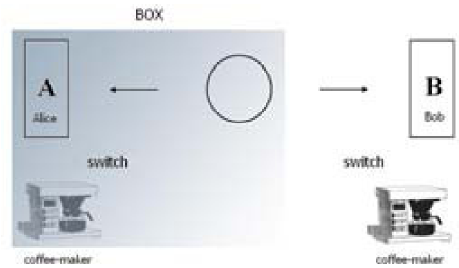

Figure 2: Aspect-type experiment in box

An interpretation of this cat paradox suggests that the problem arises because we mix up the macroscopic systems (observer's wavefunction and apparatus' wavefunction) from microscopic system to be observed. In order to clarify this, it is proposed that "... the measurement apparatus should be described by a classical model in our approach, and the physical system eventually by a quantum model" [20].

2.4 Hydrodynamic viewpoint and diffusion interpretation

In attempt to clarify the meaning of wave collapse and decoherence phenomenon, one could consider the process from (dissipative) hydrodynamic viewpoint [21]. Historically, the hydrodynamic/diffusion viewpoint of Quantum Mechanics has been considered by some physicists since the early years of wave mechanics. Already in 1933, Fuerth showed that Schrödinger equation could be written as a diffusion equation with an imaginary diffusion coefficient [1]

$$D_{qm} = \frac{i\hbar}{2m}\,. \tag{4}$$

But the notion of imaginary diffusion is quite difficult to comprehend. Alternatively, one could consider a classical Markov process of diffusion type to consider wave mechanics equation. Consider a continuity equation

$$\frac{\partial\rho}{\partial t} = -\nabla\left(\rho v\right), \tag{5}$$

where $v = v_0 = D\nabla\ln\rho$ (see [1]), which is a Fokker-Planck equation. Then the expectation value for the energy of particle can be written as [1]

*The authors are grateful to Dmitri Rabounski for his valuable comments discussing a case of entangled twin Cats.

†The "coffee-maker" analogue came to mind after a quote: "A mathematician is a device for turning coffee into theorems" — Alfréd Rényi, a Hungarian mathematician, 1921–1970. (As quoted by Christopher J. Mark.)

$$<E> = \int \left(\frac{mv^2}{2} + \frac{D^2 m}{2} D \ln \rho^2 + eV \right) \rho d^3 x \,. \quad (6)$$

Alternatively, it could be shown that there is exact mapping between Schrödinger equation and viscous dissipative Navier-Stokes equations [6], where the square of the wave-function of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6]. This Navier-Stokes interpretation differs appreciably from more standard Euler-Madelung fluid interpretation of Schrödinger equation [1], because in Euler method the fluid is described only in its inviscid limit.

2.5 How neutrosophy could offer solution to Schrödinger's paradox

In this regard, Neutrosophic Logic as recently discussed by one of these authors [22, 23, 24] could offer an interesting application in the context of Schrödinger's cat paradox. It could explain how the "mixed" state could be. It could be shown, that Neutrosophic probability is useful to those events, which involve some degree of indeterminacy (unknown) and more criteria of evaluation — as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events [25]. This new viewpoint for quantum phenomena is required because it is known that Quantum Mechanics is governed by uncertainty, but the meaning of "uncertainty" itself remains uncertain [16].

For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a problem of "alternative worlds theory well represented by the neutrosophic set theory.

In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function ψ, which describes the superposition of possible states, may be simulated by a neutrosophic function, i. e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Now let's return to our cat paradox [25]. Let's consider a Neutrosophic set of a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^-0$ and 1^+. For example: x (0.5, 0.2, 0.3) belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is undecidable); or y (0, 0, 1) belongs to A (which normally means y is not for sure in A); or z (0, 1, 0) belongs to A (which means one does know absolutely nothing about z's affiliation with A). More general, x { (0.2–0.3), (0.40–0.45) ∪ [0.50–0.51], (0.2, 0.24, 0.28) } belongs to the set A, which mean:

- Owning a likelihood in between 20–30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- Owning a probability of 20% or 24% or 28% x is not in A;
- The indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included);
- The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $n\_\sup = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [25], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i. e. we don't know if it is dead or alive), then herein the Neutrosophic Logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic Logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We could expect that someday this proposition based on Neusotrophic Logic could be transformed into a useful guide for experimental verification of quantum paradox [15, 10].

Above results will be expanded into details in our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

References

1. Rosu H. C. arXiv: gr-qc/9411035.
2. Englman R., Yahalom H. arXiv: physics/0406149.
3. Durr D., et al. arXiv: quant-ph/0308039.
4. Hofer W. A. arXiv: physics/9611009; quant-ph/9801044.
5. Hooft G. arXiv: quant-ph/0212095.
6. Kiehn R. M. An interpretation of wavefunction as a measure of vorticity. http://www22.pair.com/csdc/pdf/cologne.pdf.
7. Post E. J. The electromagnetic origin of quantization and the ensuing changes of Copenhagen interpretation. *Annales de la Fondation Louis de Broglie*, 2002, no. 2, 217–240;
8. Aharonov Y., et al. arXiv: quant-ph/0311155.
9. Kron G. Electric circuit model of Schrödinger equation. *Phys. Rev.*, 1945, v. 67, 39–43.
10. Lesovik G., Lebedev A., Blatter G. Appearance of Schrödinger Cat states in measurement process. arXiv: quant-ph/0307044.

11. Buckley P., Peat F. D. A question of physics: conversations in physics and biology. Routledge and Kegan Paul, London and Henley, 1979.

12. Zurek W. *Physics Today*, 1999, v. 44, 36; *Los Alamos Science*, 2002, No. 27; arXiv: quant-ph/0306072; *Rev. Mod. Phys.*, 2003, v. 75, 715; *Complexity, Entropy and the Physics of Information*, Santa Fe Institute Studies, Addison-Wesley, 1990; Haroche S. *Physics Today*, 1998, v. 36.

13. Tegmark M. arXiv: quant-ph/9907009.

14. Vitiello G. arXiv: hep-th/9503135.

15. Schrödinger E. Die gegenwartige Situation in der Quantenmechanik. *Naturwissenschaften*, 1935, Bd. 23; English transl. in: *Quantum Mechanics and Measurement*, ed. J. A. Wheeler and W. Zurek, Princeton UP, 1983.

16. Meglicki Z. Two and three photons: Bell's inequality. http://beige.ucs.indiana.edu/M743/node70.html.

17. Edwards P. M. 75 years of Schrödinger wave equation in 2001. http://ublib.buffalo.edu/libraries/units/sel/exhibits/schrodinger/e_schro.html.

18. Exploring quantum physics. http://xqp.physik.uni-muenchen.de/explore/prog.html.

19. Vaidman L. E. arXiv: hep-th/9310176.

20. Aerts D. *Int. J. Theor. Phys.*, 1998, v. 37, 291–304.

21. Na K., Wyatt R. E. Decoherence demystified: the hydrodynamic viewpoint. arXiv: quant-ph/0201108.

22. Smarandache F. Neutrosophy / neutrosophic probability, set, and logic. American Research Press, Rehoboth, 1998; A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. 3rd ed., American Research Press, 2003.

23. Smarandache F. A unifying field in logics: neutrosophic logic. *Multiple Valued Logic / An International Journal*, 2002, v. 8(3), 385–438.

24. Smarandache F. Definitions derived from neutrosophics. *Multiple Valued Logic / An International Journal*, 2002, v. 8 (5–6), 591–604.

25. Smarandache F. An introduction to the neutrosophic probability applied in quantum physics. *Bull. Pure and Appl. Sciences*, Ser. D (Physics), 2003, v. 22D, No. 1, 13–25.

The Neutrosophic Logic View to Schrödinger's Cat Paradox, Revisited

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The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot. To make this proposition clear, we revisit our previous paper by offering an illustration using modified coin tossing problem, known as Parrondo's game.

1 Introduction

The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. In this article we argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot.

In the preceding article we have discussed how Neutrosophic logic view can offer an alternative method to solve the well-known problem in Quantum Mechanics, i.e. the Schrödinger's cat paradox [1, 2], by introducing indeterminacy of the outcome of the observation.

In other article we also discuss possible re-interpretation of quantum measurement using Unification of Fusion Theories as generalization of Information Fusion [3, 4, 5], which results in proposition that one can expect to neglect the principle of "excluded middle"; therefore Bell's theorem can be considered as merely tautological. [6] This alternative view of Quantum mechanics as Information Fusion has also been proposed by G. Chapline [7]. Furthermore this Information Fusion interpretation is quite consistent with measurement theory of Quantum Mechanics, where the action of measurement implies information exchange [8].

In the first section we will discuss basic propositions of Neutrosophic probability and Neutrosophic logic. Then we discuss solution to Schrödinger's cat paradox. In subsequent section we discuss an illustration using modified coin tossing problem, and discuss its plausible link to quantum game.

While it is known that derivation of Schrödinger's equation is heuristic in the sense that we know the answer to which the algebra and logic leads, but it is interesting that Schrödinger's equation follows logically from de Broglie's *grande loi de la Nature* [9, p.14]. The simplest method to derive Schrödinger's equation is by using simple wave as [9]:

$$\frac{\partial^2}{\partial x^2}\exp(ikx) = -k^2 \cdot \exp(ikx)\,. \tag{1}$$

By deriving twice the wave and defining:

$$k = \frac{2\pi m v}{h} = \frac{mv}{\hbar} = \frac{p_x}{\hbar}\,, \tag{2}$$

where p_x, $\hbar$ represents momentum at x direction, and rationalised Planck constants respectively.

By introducing kinetic energy of the moving particle, T, and wavefunction, as follows [9]:

$$T = \frac{mv^2}{2} = \frac{p_x^2}{2m} = \frac{\hbar^2}{2m}\,k^2, \tag{3}$$

and

$$\psi(x) = \exp(ikx)\,. \tag{4}$$

Then one has the time-independent Schrödinger equation from [1, 3, 4]:

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\,\psi(x) = T \cdot \psi(x)\,. \tag{5}$$

It is interesting to remark here that by convention physicists assert that "the wavefunction is simply the *mathematical function* that describes the wave" [9]. Therefore, unlike the wave equation in electromagnetic fields, one should not consider that equation [5] has any physical meaning. Born suggested that the square of wavefunction represents the probability to observe the electron at given location [9, p.56]. Although Heisenberg rejected this interpretation, apparently Born's interpretation prevails until today.

Nonetheless the founding fathers of Quantum Mechanics (Einstein, De Broglie, Schrödinger himself) were dissatisfied with the theory until the end of their lives. We can summarize the situation by quoting as follows [9, p.13]:

> "The interpretation of Schrödinger's wave function (and of quantum theory generally) remains a matter of continuing concern and controversy among scientists who cling to philosophical belief that the natural world is basically logical and deterministic."

Furthermore, the "pragmatic" view of Bohr asserts that for a given quantum measurement [9, p.42]:

> "A system does not possess objective values of its physical properties until a measurement of one of them is made; the act of measurement is asserted to force the system into an eigenstate of the quantity being measured."

In 1935, Einstein-Podolsky-Rosen argued that the axiomatic basis of Quantum Mechanics is incomplete, and subsequently Schrödinger was inspired to write his well-known cat paradox. We will discuss solution of his cat paradox in subsequent section.

2 Cat paradox and imposition of boundary conditions

As we know, Schrödinger's deep disagreement with the Born interpretation of Quantum Mechanics is represented by his cat paradox, which essentially questioning the "statistical" interpretation of the wavefunction (and by doing so, denying the physical meaning of the wavefunction). The cat paradox has been written elsewhere [1, 2], but the essence seems quite similar to coin tossing problem:

> "Given $p = 0.5$ for each side of coin to pop up, we will never know the state of coin before we open our palm from it; unless we know beforehand the "state" of the coin (under our palm) using ESP-like phenomena. Prop. (1)."

The only difference here is that Schrödinger asserts that the state of the cat is half alive and half dead, whereas in the coin problem above, we can only say that we don't know the state of coin until we open our palm; i.e. the state of coin is *indeterminate* until we open our palm. We will discuss the solution of this problem in subsequent section, but first of all we shall remark here a basic principle in Quantum Mechanics, i.e. [9, p.45]:

> "Quantum Concept: The first derivative of the wavefunction Ψ of Schrödinger's wave equation must be single-valued everywhere. As a consequence, the wavefunction itself must be single-valued everywhere."

The above assertion corresponds to quantum logic, which can be defined as follows [10, p.30; 11]:

$$P \vee Q = P + Q - PQ\,. \tag{6}$$

As we will see, it is easier to resolve this cat paradox by releasing the aforementioned constraint of "single-valuedness" of the wavefunction and its first derivative. In fact, nonlinear fluid interpretation of Schrödinger's equation (using the level set function) also indicates that the physical meaning of wavefunction includes the notion of multivaluedness [12]. In other words, one can say that observation of spin-half electron at location x does not exclude its possibility to pop up somewhere else. This counter-intuitive proposition will be described in subsequent section.

3 Neutrosophic solution of the Schrödinger cat paradox

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set theory*, the definitive and unambiguous assignment of an element of the set {0,1}, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistical theories is the same classical set {0,1}.

For example the Schrödinger's cat paradox says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory. In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function, which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Therefore the question can be summarized as follows [1]:

> "How to describe a particle ζ in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\zeta \in P_1$ and $\zeta \in \neg P_1$ is a true contradiction, with respect to Quantum Concept described above."

Now we will discuss some basic propositions in Neutrosophic logic [1].

3a Non-standard real number and subsets

Let T,I,F be standard or non-standard real subsets $\subseteq]^-0, 1^+[$,

with sup T = t_sup, inf T= t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup + i_sup + f_sup,
n_inf = t_inf + i_inf + f_inf.

Obviously, t_sup, i_sup, f_sup $\leqslant 1^+$; and t_inf, i_inf, f_inf $\geqslant {}^-0$, whereas n_sup $\leqslant 3^+$ and n_inf $\geqslant {}^-0$. The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single element; finite or infinite; union or intersection of various subsets etc. They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case where T, I, F are reduced to points).

For interpretation of this proposition, we can use modal logic [10]. We can use the notion of "world" in modal logic, which is semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is "true in all possible worlds." (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth*, as opposed to possible truth), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a "specific" world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each "world" is dynamic, depending on an ensemble of parameters, we introduce the sub-category "conjuncture" within it to reflect a particular state of the world.

In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the Neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the Neutrosophic indeterminacy value $NL_i(A) = 1$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

3b Neutrosophic probability definition

Neutrosophic probability is defined as: "Is a generalization of the classical probability in which the chance that an event A occurs is t% true — where t varies in the subset T, i% indeterminate — where i varies in the subset I, and f% false — where f varies in the subset F. One notes that NP(A) = (T, I, F)". It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

The universal set, endowed with a Neutrosophic probability defined for each of its subset, forms a Neutrosophic probability space.

3c Solution of the Schrödinger's cat paradox

Let's consider a neutrosophic set a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^-0$ and 1^+. For example: $x\,(0.5, 0.2, 0.3)$ belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is *undecidable*); or $y\,(0, 0, 1)$ belongs to A (which normally means y is not for sure in A); or $z\,(0, 1, 0)$ belongs to A (which means one does know absolutely nothing about z's affiliation with A).

More general, $x\,((0.2–0.3), (0.40–0.45) \cup [0.50–0.51], \{0.2, 0.24, 0.28\})$ belongs to the set A, which means:

- — with a probability in between 20-30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- — with a probability of 20% or 24% or 28% x is not in A;
- — the indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $\text{n\_sup} = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [1, 2], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i.e. we don't know if it is dead or alive), then herein the Neutrosophic logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We have described Neutrosophic solution of the Schrödinger's cat paradox. Alternatively, one may hypothesize four-valued logic to describe Schrödinger's cat paradox, see Rauscher *et al.* [13, 14].

In the subsequent section we will discuss how this Neutrosophic solution involving "possible truth" and "indeterminacy" can be interpreted in terms of coin tossing problem (albeit in modified form), known as Parrondo's game. This approach seems quite consistent with new mathematical formulation of game theory [20].

4 An alternative interpretation using coin toss problem

Apart from the aforementioned pure mathematics-logical approach to Schrödinger's cat paradox, one can use a well-known neat link between Schrödinger's equation and Fokker-Planck equation [18]:

$$D\,\frac{\partial^2 p}{\partial z^2} - \frac{\partial \alpha}{\partial z}\,p - \alpha\,\frac{\partial p}{\partial z} - \frac{\partial p}{\partial t} = 0\,. \tag{7}$$

A quite similar link can be found between relativistic classical field equation and non-relativistic equation, for it is known that the time-independent Helmholtz equation and Schrödinger equation is formally identical [15]. From this reasoning one can argue that it is possible to explain Aharonov effect from pure electromagnetic field theory; and therefore it seems also possible to describe quantum mechanical phenomena without postulating the decisive role of "observer" as Bohr asserted. [16, 17]. In idiomatic form, one can expect that quantum mechanics does not have to mean that "the Moon is not there when nobody looks at".

With respect to the aforementioned neat link between Schrödinger's equation and Fokker-Planck equation, it is interesting to note here that one can introduce "finite difference" approach to Fokker-Planck equation as follows. First, we can define local coordinates, expanded locally about a point $(z_0,\, t_0)$ we can map points between a real space (z, t) and an integer or discrete space (i, j). Therefore we can sample the space using linear relationship [19]:

$$(z, t) = (z_0 + i\lambda, t_0 + j\tau)\,, \tag{8}$$

where λ is the sampling length and τ is the sampling time. Using a set of finite difference approximations for the Fokker-Planck PDE:

$$\frac{\partial p}{\partial z} = A_1 = \frac{p\,(z_0 + \lambda, t_0 - \tau) - p\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,, \tag{9}$$

$$\frac{\partial^2 p}{\partial z^2} = 2A_2 = \\ = \frac{p\,(z_0 - \lambda, t_0 - \tau)\; - 2p\,(z_0, t_0 - \tau) + p\,(z_0 + \lambda, t_0 - \tau)}{\lambda^2}\,, \tag{10}$$

and

$$\frac{\partial p}{\partial t} = B_1 = \frac{p\,(z_0, t_0) - p\,(z_0, t_0 - \tau)}{\tau}\,. \tag{11}$$

We can apply the same procedure to obtain:

$$\frac{\partial \alpha}{\partial z} = A_1 = \frac{\alpha\,(z_0 + \lambda, t_0 - \tau) - \alpha\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,. \tag{12}$$

Equations (9–12) can be substituted into equation (7) to yield the required finite partial differential equation [19]:

$$p\,(z_0, t_0) = a_{-1} \cdot p\,(z_0 - \lambda, t_0 - \tau) - a_0 \cdot p\,(z_0, t_0 - \tau) + \\ + a_{+1} \cdot p\,(z_0 + \lambda, t_0 - \tau)\,. \tag{13}$$

This equation can be written in terms of discrete space by using [8], so we have:

$$p_{i,j} = a_{-1} \cdot p_{i-1,j-1} + a_0 \cdot p_{i,j-1} + a_{+1} \cdot p_{i+1,j-1}\,. \tag{14}$$

Equation (14) is precisely the form required for Parrondo's game. The meaning of Parrondo's game can be described in simplest way as follows [19]. Consider a coin tossing problem with a biased coin:

$$p_{head} = \frac{1}{2} - \varepsilon\,, \tag{15}$$

where ε is an external bias that the game has to "overcome". This bias is typically a small number, for instance 1/200. Now we can express equation (15) in finite difference equation (14) as follows:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon\right) \cdot p_{i-1,j-1} + 0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon\right) \cdot p_{i+1,j-1}\,. \tag{16}$$

Furthermore, the bias parameter can be related to an applied external field.

With respect to the aforementioned Neutrosophic solution to Schrödinger's cat paradox, one can introduce a new "indeterminacy" parameter to represent conditions where the outcome may be affected by other issues (let say, apparatus setting of Geiger counter). Therefore equation (14) can be written as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ + a_0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{17}$$

where unlike the bias parameter (~1/200), the indeterminacy parameter can be quite large depending on the system in question. For instance in the Neutrosophic example given above, we can write that:

$$\eta \sim 0.2 - 0.3 = k\left(\frac{d}{t}\right)^{-1} = k\left(\frac{t}{d}\right) \leqslant 0.50. \tag{18}$$

The only problem here is that in original coin tossing, one cannot assert an "intermediate" outcome (where the outcome is neither A nor B). Therefore one shall introduce modal logic definition of "possibility" into this model. Fortunately, we can introduce this possibility of intermediate outcome into Parrondo's game, so equation (17) shall be rewritten as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ + (2\eta) \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{19}$$

For instance, by setting $\eta \sim 0.25$, then one gets the finite difference equation:

$$p_{i,j} = (0.25 - \varepsilon) \cdot p_{i-1,j-1} + (0.5) \cdot p_{i,j-1} + \\ + (0.25 + \varepsilon) \cdot p_{i+1,j-1}\,, \tag{20}$$

which will yield more or less the same result compared with Neutrosophic method described in the preceding section.

For this reason, we propose to call this equation (19): *Neutrosophic-modified Parrondo's game.* A generalized expression of equation [19] is:

$$p_{i,j} = (p_0 - \varepsilon - \eta) \cdot p_{i-1,j-1} + (z\eta) \cdot p_{i,j-1} + \\ + (p_0 + \varepsilon - \eta) \cdot p_{i+1,j-1}\,, \tag{21}$$

where p_0, z represents the probable outcome in standard coin tossing, and a real number, respectively. For the practical meaning of η, one can think (by analogy) of this indeterminacy parameter as a variable that is inversely proportional to the "thickness ratio" (d/t) of the coin in question. Therefore using equation (18), by assuming $k = 0.2$, coin thickness = 1.0 mm, and coin diameter $d = 50$ mm, then we get $d/t = 50$, or $\eta = 0.2\,(50)^{-1} = 0.004$, which is negligible. But if we use a thick coin (for instance by gluing 100 coins altogether), then by assuming $k = 0.2$, coin thickness = 100 mm, and coin diameter $d = 50\,\text{mm}$, we get $d/t = 0.5$, or $\eta = 0.2(0.5)^{-1} = 0.4$, which indicates that chance to get outcome neither A nor B is quite large. And so forth.

It is worth noting here that in the language of "modal logic" [10, p.54], the "intermediate" outcome described here is given name 'possible true', written $\Diamond A$, meaning that "it is not necessarily true that not-A is true". In other word, given that the cat cannot be found in location x, does not have to mean that it shall be in y.

Using this result (21), we can say that our proposition in the beginning of this paper (Prop. 1) has sufficient reasoning; i.e. it is possible to establish link from Schrödinger wave equation to simple coin toss problem, albeit in modified form. Furthermore, this alternative interpretation, differs appreciably from conventional Copenhagen interpretation.

It is perhaps more interesting to remark here that Heisenberg himself apparently has proposed similar thought on this problem, by introducing "potentia", which means "*a world devoid of single-valued actuality but teeming with unrealized possibility*" [4, p.52]. In Heisenberg's view an atom is certainly real, but its attributes dwell in an existential limbo "halfway between an idea and a fact", a quivering state of attenuated existence. Interestingly, experiments carried out by J. Hutchison seem to support this view, that a piece of metal can come in and out from existence [23].

In this section we discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game. Further observation and theoretical study is recommended to explore more implications of this plausible link.

5 Concluding remarks

In the present paper we revisit the Neutrosophic logic view of Schrödinger's cat paradox. We also discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game.

It is recommended to conduct further experiments in order to verify and explore various implications of this new proposition, including perhaps for the quantum computation theory.

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References

1. Smarandache F. An introduction to the Neutrosophic probability applied in quantum physics. *Bull. Pure and Appl. Sci., Physics*, 2003, v. 22D, no. 1, 13–25.
2. Smarandache F. and Christianto V. The Neutrosophic logic view to Schrödinger's cat paradox. *Progr. in Phys.*, 2006, no. 2.
3. Smarandache F. and Christianto V. A note on geometric and information fusion interpretation of Bell's theorem and quantum measurement. *Progress in Physics*, 2006, no. 4.
4. Smarandache F. and Christianto V. Multivalued logic, neutrosophy and Schrödinger equation. Hexis, Phoenix (AZ), 2006, p.52–54.
5. Smarandache F. and Dezert J. Advances and applications of DSmT for information fusion. American Research Press, Rehoboth (NM), 2004.
6. Kracklauer A. La theorie de Bell, est-elle la plus grande meprise de l'histoire de la physique? *Ann. Fond. Louis de Broglie*, 2000, v. 25, 193.
7. Chapline G. arXiv: adap-org/9906002; quant-ph/9912019; Granik A. and Chapline G. arXiv: quant-ph/0302013.
8. Zurek W. (ed.) Complexity, entropy and the physics of information. Addison-Wesley Publ., 1990, p.378.
9. Hunter G. Quantum chemistry: wave mechanics applied to atoms and molecules. Lecture notes. Chapter 1: Genesis of Quantum Mechanics. 2001, p.14, 26, 42.
10. deVries A. Algebraic hierarchy of logics unifying fuzzy logic and quantum logic. arXiv: math.LO/0707.2161, p.30, 54.
11. Aerts D. Description of many separated physical entities without the paradoxes encountered in Quantum Mechanics. *Found. Phys.*, 1982, v. 12, no. 12, p.1142, 1149–1155.
12. Jin S., Liu H., Osher S., and Tsai R. Computing multivalued physical observables for the semiclassical limits of the Schrödinger equation. *J. Comp. Phys.*, 2005, v. 205, 222–241.
13. Rauscher E.A. and Targ R. The speed of thought: investigation of complex spacetime metric to describe psychic phenomena. *J. Scientific Exploration*, 2001, v. 15, no. 3, 344–354.
14. Rauscher E.A. and Amoroso R. The physical implications of multidimensional geometries and measurement. *Intern. J. Comp. Anticipatory Systems*, D. Dubois (ed.), 2006.
15. Lu J., Greenleaf J., and Recami E. Limited diffraction solutions to Maxwell and Schrödinger equation. arXiv: physics/9610012.
16. Aharonov Y., *et al.* arXiv: quant-ph/0311155.
17. Goldstein S. Quantum theory without observers — part one. *Physics Today*, March 1998, 42–46.
18. Ho C.-L. and Sasaki R. Deformed Fokker Planck equations. arXiv: cond-mat/0612318.
19. Allison A., *et al.* State space visualization and fractal properties of Parrondo's game. arXiv: cond-mat/0205536; cond-mat/0208470.
20. Wu J. A new mathematical representation of game theory. arXiv: quant-ph/0404159.
21. Smarandache F. Unification of fusion theories (UFT). *Intern. J. Appl. Math. and Stat.*, 2004, v. 2, 1–14.
22. Smarandache F. An in-depth look at information fusion rules and unification of fusion theories. Invited speech at NASA Langley Research Center, Hampton, VA, USA, November 5, 2004.
23. Smarandache F., Christianto V., Khrapko R., Yuhua F., Hutchison J. Unfolding labyrinth: open problems in physics, mathematics, astrophysics, and other areas of science. Hexis, Phoenix (AZ), 2006.

A Note on Geometric and Information Fusion Interpretation of Bell's Theorem and Quantum Measurement*

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In this paper we present four possible extensions of Bell's Theorem: Bayesian and Fuzzy Bayesian intrepretation, Information Fusion interpretation, Geometric interpretation, and the viewpoint of photon fluid as medium for quantum interaction.

1 Introduction

It is generally accepted that Bell's theorem [1] is quite exact to describe the linear hidden-variable interpretation of quantum measurement, and hence "quantum reality". Therefore null result of this proposition implies that no hidden-variable theory could provide good explanation of "quantum reality".

Nonetheless, after further thought we can find that Bell's theorem is nothing more than another kind of abstraction of quantum observation based on a set of assumptions and propositions [7]. Therefore, one should be careful before making further generalization on the null result from experiments which are "supposed" to verify Bell's theorem. For example, the most blatant assumption of Bell's theorem is that it takes into consideration only the classical statistical problem of chance of outcome A or outcome B, as result of adoption of Von Neumann's definition of "quantum logic". Another critic will be discussed here, i. e. that Bell's theorem is only a reformulation of statistical definition of correlation; therefore it is merely tautological [5].

Therefore in the present paper we will discuss a few plausible extension of Bell's theorem:

(a) Bayesian and Fuzzy Bayesian interpretation.

(b) Information Fusion interpretation. In particular, we propose a modified version of Bell's theorem, which takes into consideration this multivalued outcome, in particular using the information fusion Dezert-Smarandache Theory (DSmT) [2, 3, 4]. We suppose that in quantum reality the outcome of $P(A \cup B)$ and also $P(A \cap B)$ shall also be taken into consideration. This is where DSmT and Unification of Fusion Theories (UFT) could be found useful [2, 17].

(c) Geometric interpretation, using a known theorem connecting geometry and imaginary plane. In turn, this leads us to 8-dimensional extended-Minkowski metric.

(d) As an alternative to this geometric interpretation, we submit the viewpoint of photon fluid as medium for quantum interaction. This proposition leads us to Gross-Piteavskii equation which is commonly used to describe bose condensation phenomena. In turn we provide a route where Maxwell equations and Schrödinger equation could be deduced from Gross-Pitaevskii equation by using known algebra involving bi-quaternion number. In our opinion, this new proposition provides us a physical mechanism of quantum interaction, beyond conventional "quantum algebra" which hides causal explanation.

By discussing these various approaches, we use an expanded logic beyond "yes" or "no" type logic [3]. In other words, there could be new possibilities to describe quantum interaction: "both can be wrong", or "both can be right", as described in Table 1 below.

In Belnap's four-valued logic there are, besides Truth (T) and Falsehood (F), also Uncertainty (U) and Contradiction (C) but they are inter-related [30]. Belnap's logic is a particular case of Neutrosophic Logic (which considers three components: Truth, Falsehood, and Indeterminacy (I)) when indeterminacy is split into Uncertainty and Contradiction. In our article we have: Yes (Y), No (N), and Indeterminacy (I, which means: neither Yes nor No), but Indeterminacy is split into "both can be wrong" and "both can be right".

It could be expected that a combined interpretation represents multiple-facets of quantum reality. And hopefully it could bring better understanding on the physical mechanism beneath quantum measurement, beyond simple algebraic notions. Further experiments are of course recommended in order to verify or refute this proposition.

2 Bell's theorem. Bayesian and fuzzy Bayesian interpretation

Despite widespread belief of its ability to describe hidden-variables of quantum reality [1], it shall be noted that Bell's theorem starts with a set of assumptions inherent in its formulation. It is assumed that each pair of particles possesses a particular value of λ, and we define quantity $p(\lambda)$ so that probability of a pair being produced between λ and $\lambda + d\lambda$

*Note: The notion "hronir wave" introduced here was inspired from Borges' Tlon, Uqbar, Orbis Tertius.

Alternative	Bell's theorem	Implications	Special relativity
QM is nonlocal	Invalid	Causality breaks down; Observer determines the outcome	Is not always applicable
QM is local with hidden variable	Valid	Causality preserved; The moon is there even without observer	No interaction can exceed the speed of light
Both can be right	Valid, but there is a way to explain QM without violating Special Relativity	QM, special relativity and Maxwell electromagnetic theory can be unified. New worldview shall be used	Can be expanded using 8-dimensional Minkowski metric with imaginary plane
Both can be wrong	Invalid, and so Special Relativity is. We need a new theory	New nonlocal QM theory is required, involving quantum potential	Is not always applicable

Table 1: Going beyond classical logic view of QM

is $p(\lambda)\,d\lambda$. It is also assumed that this is normalized so that:

$$\int p(\lambda)\,d\lambda = 1\,. \tag{1}$$

Further analysis shows that the integral that measures the correlation between two spin components that are at an angle of $(\delta - \phi)$ with each other, is therefore equal to $C''(\delta - \phi)$. We can therefore write:

$$|C''(\phi) - C''(\delta)| - C''(\delta - \phi) \leqslant 1 \tag{2}$$

which is known as Bell's theorem, and it was supposed to represent any local hidden-variable theorem. But it shall be noted that actually this theorem cannot be tested completely because it assumes that all particle pairs have been detected. In other words, we find that a hidden assumption behind Bell's theorem is that it uses classical probability assertion [12], which may or may be not applicable to describe Quantum Measurement.

It is wothnoting here that the standard interpretation of Bell's theorem includes the use of Bayesian posterior probability [13]:

$$P(\alpha\,|\,x) = \frac{p(\alpha)\,p(x\,|\,\alpha)}{\sum_\beta p(\beta)\,p(x\,|\,\beta)}\,. \tag{3}$$

As we know Bayesian method is based on classical two-valued logic. In the meantime, it is known that the restriction of classical propositional calculus to a two-valued logic has created some interesting paradoxes. For example, the Barber of Seville has a rule that all and only those men who do not shave themselves are shaved by the barber. It turns out that the only way for this paradox to work is if the statement is both *true and false simultaneously* [14]. This brings us to *fuzzy Bayesian approach* [14] as an extension of (3):

$$P(s_i|\underline{M}) = \frac{p(\underline{M}\,|\,s_i)\,p(s_i)}{p(\underline{M})}\,, \tag{4}$$

where [14, p. 339]:

$$p(\underline{M}\,|\,s_i) = \sum_{k=1}^{r} p(x_k\,|\,s_i)\,\mu_{\underline{M}}(x_k)\,. \tag{5}$$

Nonetheless, it should also be noted here that there is shortcoming of this Bayesian approach. As Kracklauer points out, Bell's theorem is nothing but a reformulation of statistical definition of correlation [5]:

$$\mathrm{Corr}\,(A,B) = \frac{\langle|AB|\rangle - \langle A\rangle\langle B\rangle}{\sqrt{\langle A^2\rangle\langle B^2\rangle}}\,. \tag{6}$$

When $\langle A\rangle$ or $\langle B\rangle$ equals to zero and $\langle A^2\rangle\langle B^2\rangle = 1$ then equation (6) reduces to Bell's theorem. Therefore as such it could be considered as merely tautological [5].

3 Information fusion interpretation of Bell's theorem. DSmT modification

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set theory*, the definitive and unambiguous assignment of an element of the set $\{0,1\}$, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistal theories is the same classical set $\{0,1\}$.

Not only that, there is also criticism over the use of Bayesian approach, i. e.: [13]

(a) In real world, neither class probabilities nor class densities are precisely known;

(b) This implies that one should adopt a parametric model for the class probabilities and class densities, and then use empirical data.

(c) Therefore, in the context where multiple sensors can be used, information fusion approach could be a better alternative to Bayes approach.

In other words, we should find an extension to standard proposition in statistical theory [8, p. 388]:

$$P(AB|C) = P(A|BC)P(B|C) \tag{7}$$
$$= P(B|AC)P(A|C) \tag{8}$$
$$P(A|B) + P(\bar{A}|B) = 1\,. \tag{9}$$

Such an extension is already known in the area of information fusion [2], known as Dempster-Shafer theory:

$$m(A) + m(B) + m(A \cup B) = 1\,. \tag{10}$$

Interestingly, Chapline [13] noted that neither Bayesian theory nor Dempster-Shafer could offer insight on how to minimize overall energy usage in the network. In the meantime, Dezert-Smarandache (DSmT) [2] introduced further improvement of Dempster-Shafer theory by taking into consideration chance to observe intersection between A and B:

$$m(A) + m(B) + m(A \cup B) + m(A \cap B) = 1\,. \tag{11}$$

Therefore, introducing this extension from equation (11) into equation (2), one finds a modified version of Bell's theorem in the form:

$$\begin{gathered} |C''(\phi) - C''(\delta)| - \\ - C''(\delta - \phi) + C''(\delta \cup \phi) + C''(\delta \cap \phi) \leqslant 1\,, \end{gathered} \tag{12}$$

which could be called as modified Bell's theorem according to Dezert-Smarandache (DSmT) theory [2]. Its direct implications suggest that it could be useful to include more sensors in order to capture various possibilities beyond simple $\{0, 1\}$ result, which is typical in Bell's theorem.

Further generalization of DSmT theory (11) is known as Unification of Fusion Theories [15, 16, 17]:

$$\begin{gathered} m(A) + m(B) + m(A \cup B) + m(A \cap B) + \\ + m(\bar{A}) + m(\bar{B}) + m(\bar{A} \cup \bar{B}) + m(\bar{A} \cap \bar{B}) = 1\,, \end{gathered} \tag{13}$$

where $\bar{A}$ is the complement of A and $\bar{B}$ is the complement of B (if we consider the set theory).

(But if we consider the logical theory then $\bar{A}$ is the negation of A and $\bar{B}$ is the negation of B. The set theory and logical theory in this example are equivalent, hence doesn't matter which one we use from them.) In equation (13) above we have a complement/negation for A. We might define the $\bar{A}$ as the entangle of particle A. Hence we could expect to further extend Bell's inequality considering UFT; nonetheless we leave this further generalization for the reader.

Of course, new experimental design is recommended in order to verify and to find various implications of this new proposition.

4 An alternative geometric interpretation of Bell-type measurement. Gross-Pitaevskii equation and the "hronir wave"

Apart from the aforementioned Bayesian interpretation of Bell's theorem, we can consider the problem from purely geometric viewpoint. As we know, there is linkage between geometry and algebra with imaginary plane [18]:

$$x + iy = \rho e^{i\phi}. \tag{14}$$

Therefore one could expect to come up with geometrical explanation of quantum interaction, provided we could generalize the metric using imaginary plane:

$$X + i\,X' = \rho e^{i\phi}. \tag{15}$$

Interestingly, Amoroso and Rauscher [19] have proposed exactly the same idea, i. e. generalizing Minkowski metric to become 8-dimensional metric which can be represented as:

$$Z^{\mu} = X^{\mu}_{re} + i\,X^{\mu}_{im} = \rho e^{i\phi}. \tag{16}$$

A characteristic result of this 8-dimensional metric is that "space separation" vanishes, and quantum-type interaction could happen in no time.

Another viewpoint could be introduced in this regard, i. e. that the wave nature of photon arises from "photon fluid" medium, which serves to enable photon-photon interaction. It has been argued that this photon-fluid medium could be described using Gross-Pitaevskii equation [20]. In turns, we could expect to "derive" Schrödinger wave equation from the Gross-Pitaevskii equation.

It will be shown, that we could derive Schrödinger wave equation from Gross-Pitaevskii equation. Interestingly, a new term similar to equation (14) arises here, which then we propose to call it "hronir wave". Therefore one could expect that this "hronir wave" plays the role of "invisible light" as postulated by Maxwell long-time ago.

Consider the well-known Gross-Pitaevskii equation in the context of superfluidity or superconductivity [21]:

$$i\hbar\,\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + \left(V(x) - \gamma\,|\Psi|^{p-1}\right)\Psi, \tag{17}$$

where $p < 2N/(N-2)$ if $N \geqslant 3$. In physical problems, the equation for $p = 3$ is known as Gross-Pitaevskii equation. This equation (17) has standing wave solution quite similar to Schrödinger equation, in the form:

$$\Psi(x,t) = e^{-iEt/\hbar} \cdot u(x)\,. \tag{18}$$

Substituting equation (18) into equation (17) yields:

$$-\frac{\hbar^2}{2m}\,\Delta u + \left(V(x) - E\right)u = |u|^{p-1}\,u\,, \tag{19}$$

which is nothing but time-independent linear form of Schrödinger equation, except for term $|u|^{p-1}$ [21]. In case the right-hand side of this equation is negligible, equation (19) reduces to standard Schrödinger equation. Using Maclaurin series expansion, we get for (18):

$$\Psi(x,t) = \left(1 - \frac{iEt}{\hbar} + \frac{\left(\frac{iEt}{\hbar}\right)^2}{2!} + \frac{\left(-\frac{iEt}{\hbar}\right)^3}{3!} + \ldots\right) \cdot u(x)\,. \tag{20}$$

Therefore we can say that standing wave solution of Gross-Pitaevskii equation (18) is similar to standing wave solution of Schrödinger equation (u), except for nonlinear term which comes from Maclaurin series expansion (20). By neglecting third and other higher order terms of equation (20), one gets an approximation:

$$\Psi(x,t) = \left[1 - iEt/\hbar\right] \cdot u(x)\,. \tag{21}$$

Note that this equation (21) is very near to hyperbolic form $z = x + iy$ [18]. Therefore one could conclude that standing wave solution of Gross-Pitaevskii equation is merely an extension from ordinary solution of Schrödinger equation into Cauchy (imaginary) plane. In other words, there shall be "hronir wave" part of Schrödinger equation in order to describe Gross-Pitaevskii equation. We will use this result in the subsequent section, but first we consider how to derive bi-quaternion from Schrödinger equation.

It is known that solutions of Riccati equation are logarithmic derivatives of solutions of Schrödinger equation, and *vice versa* [22]:

$$u'' + vu = 0\,. \tag{22}$$

Bi-quaternion of differentiable function of $x = (x_1, x_2, x_3)$ is defined as [22]:

$$Dq = -\mathrm{div}\,(q) + \mathrm{grad}\,(q_0) + \mathrm{rot}\,(q)\,. \tag{23}$$

By using alternative representation of Schrödinger equation [22]:

$$\left[-\Delta + u\right] f = 0\,, \tag{24}$$

where f is twice differentiable, and introducing quaternion equation:

$$Dq + q^2 = -u\,. \tag{25}$$

Then we could find q, where q is purely vectorial differentiable bi-quaternion valued function [22].

We note that solutions of (24) are related to (25) as follows [22]:

- For any nonvanishing solution f of (24), its logarithmic derivative:

$$q = \frac{Df}{f}\,, \tag{26}$$

  is a solution of equation (25), and *vice versa* [22].

Furthermore, we also note that for an arbitrary scalar twice differentiable function f, the following equality is permitted [22]:

$$\left[-\Delta + u\right] f = \left[D + M^h\right]\left[D - M^h\right] f\,, \tag{27}$$

provided h is solution of equation (25).

Therefore we can summarize that given a particular solution of Schrödinger equation (24), the general solution reduces to the first order equation [22, p. 9]:

$$\left[D + M^h\right] F = 0\,, \tag{28}$$

where

$$h = \frac{D\sqrt{\varepsilon}}{\varepsilon}\,. \tag{29}$$

Interestingly, equation (28) is equivalent to **Maxwell equations**. [22] Now we can generalize our result from the preceding section, in the form of the following conjecture:

Conjecture 1 *Given a particular solution of Schrödinger equation (24), then the approximate solution of Gross-Pitaevskii equation (17) reduces to the first order equation:*

$$\left[1 - iEt/\hbar\right]\left[D + M^h\right] F = 0\,. \tag{30}$$

Therefore we can conclude here that there is neat linkage between Schrödinger equation, Maxwell equation, Riccati equation via biquaternion expression [22, 23, 24]. And approximate solution of Gross-Pitaevskii equation is similar to solution of Schrödinger equation, except that it exhibits a new term called here "*the hronir wave*" (30).

Our proposition is that considering equation (30) has imaginary plane wave, therefore it could be expected to provided "physical mechanism" of quantum interaction, in the same sense of equation (14). Further experiments are of course recommended in order to verify or refute this

5 Some astrophysical implications of Gross-Pitaevskii description

Interestingly, Moffat [25, p. 9] has also used Gross-Pitaevskii in his "phion condensate fluid" to describe CMB spectrum. Therefore we could expect that this equation will also yield interesting results in cosmological scale.

Furthermore, it is well-known that Gross-Pitaevskii equation could exhibit *topologically* non-trivial vortex solutions [26, 27], which can be expressed as quantized vortices:

$$\oint p \bullet dr = N_v 2\pi\hbar\,. \tag{31}$$

Therefore an implication of Gross-Pitaevskii equation [25] is that topologically quantized vortex could exhibit in astrophysical scale. In this context we submit the viewpoint that this proposition indeed has been observed in the form of Tifft's quantization [28, 29]. The following description supports this assertion of topological quantized vortices in astrophysical scale.

We start with standard definition of Hubble law [28]:

$$z = \frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{32}$$

or

$$r = \frac{c}{H}\,z\,. \tag{33}$$

Now we suppose that the major parts of redshift data could be explained via Doppler shift effect, therefore [28]:

$$z = \frac{\delta\lambda}{\lambda} = \frac{v}{c}\,. \tag{34}$$

In order to interpret Tifft's observation of quantized redshift corresponding to quantized velocity 36.6 km/sec and

72.2 km/sec, then we could write from equation (34):

$$\frac{\delta v}{c} = \delta z = \delta\left(\frac{\delta\lambda}{\lambda}\right). \tag{35}$$

Or from equation (33) we get:

$$\delta r = \frac{c}{H}\,\delta z\,. \tag{36}$$

In other words, we submit the viewpoint that Tifft's observation of quantized redshift implies a quantized distance between galaxies [28], which could be expressed in the form:

$$r_n = r_0 + n\,(\delta r)\,. \tag{35a}$$

It is proposed here that this equation of quantized distance (5) is resulted from topological quantized vortices (31), and agrees with Gross-Pitaevskii (quantum phion condensate) description of CMB spectrum [25]. Nonetheless, further observation is recommended in order to verify the above proposition.

Concluding remarks

In the present paper we review a few extension of Bell's theorem which could take into consideration chance to observe outcome beyond classical statistical theory, in particular using the information fusion theory. A new geometrical interpretation of quantum interaction has been considered, using Gross-Pitaevskii equation. Interestingly, Moffat [25] also considered this equation in the context of cosmology.

It is recommended to conduct further experiments in order to verify and also to explore various implications of this new proposition, including perhaps for the quantum computation theory [8, 13].

Acknowledgment

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References

1. Shimony A. http://plato.stanford.edu/entries/bell-theorem/ ; http://plato.stanford.edu/entries/kochen-specker/.
2. Smarandache F. and Dezert J. Advances and applications of of DSmT for information fusion. American Research Press, Rehoboth, 2004.
3. Smarandache F. *Bulletin of Pure and Applied Sciences*, Ser. Physics, 2003, v. 22D, No. 1, 13–25.
4. Smarandache F. and Christianto V. Multivalued logic, neutrosophy and Schrödinger equation. Hexis, Phoenix, 2005.
5. Kracklauer A. La theorie de Bell, est-elle la plus grande meprise de l'histoire de la physique? *Annales de la Fondation Louis de Broglie*, v. 25, 2000, 193.
6. Aharonov Y. *et al.* arXiv: quant-ph/0311155.
7. Rosu H. C. arXiv: gr-qc/9411035.
8. Zurek W. (ed.), Complexity, entropy and the physics of information. Santa Fe Inst. Studies, Addison-Wesley, 1990.
9. Schrieffer J. R. Macroscopic quantum phenomena from pairing in superconductors. Lecture, December 11, 1972.
10. Anandan J. S. In: *Quantum Coherence and Reality*. Columbia SC, World Sci., 1994; arXiv: gr-qc/9504002.
11. Goldstein S. Quantum theory without observers — part one. *Physics Today*, March 1998, 42–46.
12. Pitowski I. arXiv: quant-ph/0510095.
13. Chapline G. arXiv: adap-org/9906002; quant-ph/9912019; Granik A. and Chapline G. arXiv: quant-ph/0302013; http://www.whatsnextnetwork.com/technology/index.php/2006/03/.
14. Ross T. J. fuzzy logic with engineering applications. McGraw-Hill, 1995, 196–197, 334–341.
15. Smarandache F. Unification of Fusion Theories (UFT). *Intern. J. of Applied Math. & Statistics*, 2004, v. 2, 1–14.
16. Smarandache F. An in-depth look at information fusion rules and unification of fusion theories. Invited speech at NASA Langley Research Center, Hampton, VA, USA, Nov 5, 2004.
17. Smarandache F. Unification of the fusion theory (UFT). Invited speech at NATO Advance Study Institute, Albena, Bulgaria, May 16–27, 2005.
18. Gogberashvili M. arXiv: hep-th/0212251.
19. Rauscher E. A. and Amoroso R. *Intern. J. of Comp. Anticipatory Systems*, 2006.
20. Chiao R. *et al.*. arXiv: physics/0309065.
21. Dinu T. L. arXiv: math.AP/0511184.
22. Kravchenko V. arXiv: math.AP/0408172.
23. Lipavsky P. *et al*. arXiv: cond-mat/0111214.
24. De Haas E. P. *Proc. of the Intern. Conf. PIRT-2005*, Moscow, MGTU Publ., 2005.
25. Moffat J. arXiv: astro-ph/0602607.
26. Smarandache F. and Christianto V. *Progress in Physics*, 2006, v. 2, 63–67.
27. Fischer U. arXiv: cond-mat/9907457; cond-mat/ 0004339.
28. Russell Humphreys D. Our galaxy is the centre of the universe, "quantized" red shifts show. *TJ Archive*, 2002, v. 16(2), 95–104; http://answersingenesis.org/tj/v16/i2/galaxy.asp.
29. Setterfield B. http://www.journaloftheoretics.com.
30. Belnap N. A useful four-valued logic, modern uses of multiple-valued logic. (D. Reidel, editor), 8–37, 1977.

Extension of Inagaki General Weighted Operators

and

A New Fusion Rule Class of Proportional Redistribution of Intersection Masses

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Abstract.
In this paper we extend Inagaki Weighted Operators fusion rule (WO) in information fusion by doing redistribution of not only the conflicting mass, but also of masses of non-empty intersections, that we call Double Weighted Operators (DWO).
Then we propose a new fusion rule Class of Proportional Redistribution of Intersection Masses (CPRIM), which generates many interesting particular fusion rules in information fusion.
Both formulas are presented for 2 and for $n \geq 3$ sources.

1. Introduction.

Let $\theta = \{\theta_1, \theta_2, ..., \theta_n\}$, for $n \geq 2$, be the frame of discernment, and $S^\theta = (\theta, \cup, \cap, \tau)$ its super-power set, where $\tau(x)$ means complement of x with respect to the total ignorance.

Let I_t = total ignorance = $\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n$.

$S^\theta = 2$ ^θ_{refined} = 2^(2^θ) = $D^{\theta \cup \theta c}$, when refinement is possible, where $\theta_c = \{\tau(\theta_1), \tau(\theta_2), \ldots, \tau(\theta_n)\}$.

We consider the general case when the domain is S^θ, but S^θ can be replaced by $D^\theta = (\theta, \cup, \cap)$ or by $2^\theta = (\theta, \cup)$ in all formulas from below.

Let $m_1(\cdot)$ and $m_2(\cdot)$ be two normalized masses defined from S^θ to $[0,1]$.

We use the conjunction rule to first combine $m_1(\cdot)$ with $m_2(\cdot)$ and then we redistribute the mass of $m(X \cap Y) \neq 0$, when $X \cap Y = \Phi$.

Let's denote $m_{2\cap}(A) = (m_1 \oplus m_2)(A) = \sum_{\substack{X,Y \in S^\theta \\ (X \cap Y) = A}} m_1(X) m_2(Y)$ using the conjunction rule.

Let's note the set of intersections by:

$$S_{\cap} = \left\{ \begin{array}{l} X \in S^{\theta} \mid X = y \cap z, \text{ where } y, z \in S^{\theta} \setminus \{\Phi\}, \\ X \text{ is in a canonical form, and} \\ X \text{ contains at least an } \cap \text{ symbol in its formula} \end{array} \right\}. \quad (1)$$

In conclusion, $S_{\cap}$ is a set of formulas formed with singletons (elements from the frame of discernment), such that each formula contains at least an intersection symbol $\cap$, and each formula is in a canonical form (easiest form).

For example: $A \cap A \notin S_{\cap}$ since $A \cap A$ is not a canonical form, and $A \cap A = A$. Also, $(A \cap B) \cap B$ is not in a canonical form but $(A \cap B) \cap B = A \cap B \in S_{\cap}$.

Let

$S_{\cap}^{\Phi}$ = the set of all empty intersections from $S_{\cap}$,

and

$S_{\cap,r}^{non\Phi}$ = {the set of all non-empty intersections from $S_{\cap}^{non\Phi}$ whose masses are redistributed to other sets, which actually depends on the sub-model of each application}.

2. Extension of Inagaki General Weighted Operators (WO).

Inagaki general weighted operator (WO) is defined for two sources as:

$$\forall\, A \in 2^{\theta} \setminus \{\Phi\},\ m_{(WO)}(A) = \sum_{\substack{X, Y \in 2^{\theta} \\ (X \cap Y) = A}} m_1(X) m_2(Y) + W_m(A) \cdot m_{2\cap}(\Phi), \quad (2)$$

where

$$\sum_{X \in 2^{\theta}} W_m(X) = 1 \text{ and all } W_m(\cdot) \in [0,1]. \quad (3)$$

So, the conflicting mass is redistributed to non-empty sets according to these weights $W_m(\cdot)$.

In the extension of this WO, which we call the Double Weighted Operator (DWO), we redistribute not only the conflicting mass $m_{2\cap}(\Phi)$ but also the mass of some (or all) non-empty intersections, i.e. those from the set $S_{\cap,r}^{non\Phi}$, to non-empty sets from S^{θ} according to some weights $W_m(\cdot)$ for the conflicting mass (as in WO), and respectively according to the weights $V_m(.)$ for the non-conflicting mass of the elements from the set $S_{\cap,r}^{non\Phi}$:

$$\forall\, A \in \left(S^{\theta} \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi\},\ m_{DWO}(A) = \sum_{\substack{X, Y \in S^{\theta} \\ (X \cap Y) = A}} m_1(X) m_2(Y) + W_m(A) \cdot m_{2\cap}(\Phi) + V_m(A) \cdot \sum_{z \in S_{\cap,r}^{non\Phi}} m_{2\cap}(z), \quad (4)$$

where

$$\sum_{X \in S^{\theta}} W_m(X) = 1 \text{ and all } W_m(\cdot) \in [0,1], \text{ as in (3)}$$

and

$$\sum_{z \in S_{\cap,r}^{non\Phi}} V_m(z) = 1 \text{ and all } V_m(\cdot) \in [0,1]. \tag{5}$$

In the free and hybrid modes, if no non-empty intersection is redistributed, i.e. $S_{\cap,r}^{non\Phi}$ contains no elements, DWO coincides with WO.

In the Shafer's model, always DWO coincides with WO.

For $s \geq 2$ sources, we have a similar formula:

$$\forall A \in \left(S^\theta \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi\},\ m_{DWO}(A) = \sum_{\substack{X_1, X_2, \ldots, X_n \in S^\theta \\ \bigcap_{i=1}^{s} X_i = A}} \prod_{i=1}^{s} m_i(X_i) + W_m(A) \cdot m_{s\cap}(\Phi) + V_m(A) \cdot \sum_{z \in S_{\cap,r}^{non\Phi}} m_{s\cap}(z) \tag{6}$$

with the same restrictions on $W_m(\cdot)$ and $V_m(\cdot)$.

3. A Fusion Rule Class of Proportional Redistribution of Intersection Masses

For $A \in \left(S^\theta \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi, I_t\}$ for two sources we have:

$$m_{CPR\backslash M}(A) = m_{2\cap}(A) + f(A) \cdot \sum_{\substack{X,Y \in S^\theta \\ \{\Phi = X \cap Y \text{ and } A \subseteq M\} \\ \text{or } \left\{\Phi \neq X \cap Y \in S_{\cap,r}^{non\Phi} \text{ and } A \subseteq N\right\}}} \frac{m_1(X) m_2(Y)}{\sum_{z \subseteq M} f(z)}, \tag{7}$$

where $f(X)$ is a function directly proportional to X, $f : S^\theta \to [0, \infty]$. (8)

For example, $f(X) = m_{2\cap}(X)$, or (9)

$f(X) = card(X)$, or

$f(X) = \dfrac{card(X)}{card(M)}$ (ratio of cardinals), or

$f(X) = m_{2\cap}(X) + card(X)$, etc.;

and M is a subset of S^θ, for example: (10)

$M = \tau(X \cup Y)$, or

$M = (X \cup Y)$, or

M is a subset of $X \cup Y$, etc.,

where N is a subset of S^θ, for example: (11)

$N = X \cup Y$, or

N is a subset of $X \cup Y$, etc.

And

$$m_{CPR\backslash M}(I_t) = m_{2\cap}(I_t) + \sum_{\substack{X,Y \in S^\theta \\ \left\{X \cap Y = \Phi \text{ and } (M = \Phi \text{ or } \sum_{z \subseteq M} f(z) = 0)\right\}}} m_1(X) m_2(Y). \tag{12}$$

These formulas are easily extended for any $s \geq 2$ sources $m_1(\cdot), m_2(\cdot), ..., m_s(\cdot)$.
Let's denote, using the conjunctive rule:

$$m_{s\cap}(A) = (m_1 \oplus m_2 \oplus ... \oplus m_s)(A) = \sum_{\substack{X_1, X_2, ..., X_s \in S^\wedge\Theta \\ \bigcap_{i=1}^{s} X_i = A}} \prod_{i=1}^{s} m_i(x_i) \tag{13}$$

$$m_{s\cap}(A) = m_{s\cap}(A) + \text{f(A)} \cdot \sum_{\substack{X_1, X_2, ..., X_n \in S^\theta \\ \left\{\Phi = \bigcap_{i=1}^{s} X_i \text{ and } A \subseteq M\right\} \\ \text{or } \left\{\Phi \neq \bigcap_{i=1}^{s} X_i \in S_{\cap,r}^{non\Phi} \text{ and } A \subseteq N\right\}}} \frac{\prod_{i=1}^{s} m_i(X_i)}{\sum_{z \subseteq M} f(z) \neq 0} \tag{14}$$

where $f(\cdot)$, M, and N are similar to the above where instead of $X \cup Y$ (for two sources) we take $X_1 \cup X_2 \cup ... \cup X_s$ (for s sources), and instead of $m_{2\cap}(X)$ for two sources we take $m_{s\cap}(X)$ for s sources.

This new fusion rule **Class of Proportional Redistribution of Intersection Masses (CPRIM)** generates many interesting particular fusion rules in information fusion.

References:

[1] T. Inagaki, Independence Between Safety-Control Policy and Multiple-Sensors Schemes via Dempster-Shafer Theory, IEEE Transaction on Reliability, 40, 182-188, 1991.

[2] E. Lefèbvre, O. Colot, P. Vannoorenberghe, Belief Function Combination and Conflict Management, Information Fusion 3, 149-162, 2002.

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n-ary Fuzzy Logic and Neutrosophic Logic Operators

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Abstract.

We extend Knuth's 16 Boolean binary logic operators to fuzzy logic and neutrosophic logic binary operators. Then we generalize them to n-ary fuzzy logic and neutrosophic logic operators using the smarandache codification of the Venn diagram and a defined vector neutrosophic law. In such way, new operators in neutrosophic logic/set/probability are built.

Keywords: binary/trinary/n-ary fuzzy logic operators, T-norm, T-conorm, binary/trinary/n-ary neutrosophic logic operators, N-norm, N-conorm

Introduction.

For the beginning let's consider the Venn Diagram of two variables x and y, for each possible operator, as in Knuth's table, but we adjust this table to the Fuzzy Logic (FL).

Let's denote the fuzzy logic values of these variables as

$$FL(x) = (t_1, f_1)$$

where

t_1 = truth value of variable x,

f_1 = falsehood value of variable x,

with $0 \le t_1, f_1 \le 1$ and $t_1 + f_1 = 1$;

and similarly for y:

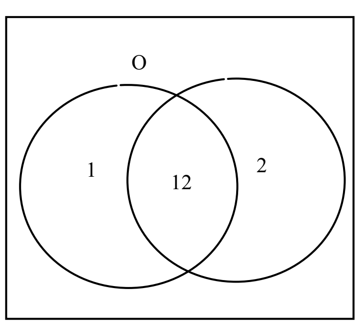
$$FL(y) = (t_2, f_2)$$

with the same $0 \le t_2, f_2 \le 1$ and $t_2 + f_2 = 1$.

We can define all 16 Fuzzy Logical Operators with respect to two FL operators: FL conjunction (FLC) and FL negation (FLN).

Since in FL the falsehood value is equal to 1- truth value, we can deal with only one component: the truth value.

The Venn Diagram for two sets X and Y



has $2^2 = 4$ disjoint parts:

0 = the part that does not belong to any set (the complement or negation)
1 = the part that belongs to 1st set only;
2 = the part that belongs to 2nd set only;
12 = the part that belongs to 1st and 2nd set only;
{called Smarandache's codification [1]}.

Shading none, one, two, three, or four parts in all possible combinations will make $2^4 = 2^{2^2} = 16$ possible binary operators.

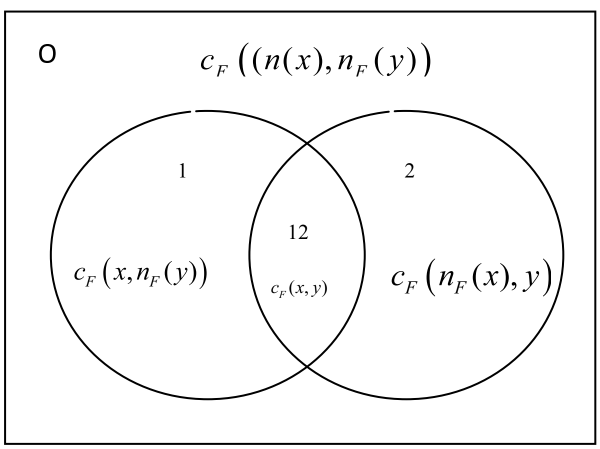
We can start using a $T-norm$ and the negation operator.

Let's take the binary conjunction or intersection (which is a $T-norm$) denoted as $c_F(x,y)$:

$$c_F : \left(\left[0,1\right]\times\left[0,1\right]\right)^2 \to \left[0,1\right]\times\left[0,1\right]$$

and unary negation operator denoted as $n_F(x)$, with:

$$n_F : \left[0,1\right]\times\left[0,1\right] \to \left[0,1\right]\times\left[0,1\right]$$



The fuzzy logic value of each part is:

$P12 = part12 =$ intersection of x and y; so $FL(P12) = c_F(x,y)$.

$P1 = part1 =$ intersection of x and negation of y; $FL(P1) = c_F(x, n_F(y))$.

$P2 = part2 =$ intersection of negation of x and y; $FL(P2) = c_F(n_F(x), y)$.

$P0 = part0 =$ intersection of negation of x and the negation of y; $FL(P0) = c_F(n_F(x), n_F(y))$, and for normalization we set the condition:

$$c_F(x,y) + c_F\left((n(x), y\right) + c_F\left(x, n_F(y)\right) + c_F\left(n_F(x), n_F(y)\right) = (1,0).$$

Then consider a binary $T-conorm$ (disjunction or union), denoted by $d_F(x,y)$:

$$d_F:\left([0,1]\times[0,1]\right)^2 \to [0,1]\times[0,1]$$

$$d_F(x,y)=\left(t_1+t_2, f_1+f_2-1\right)$$

if x and y are disjoint and $t_1+t_2\le 1$.

This fuzzy disjunction operator of disjoint variables allows us to add the fuzzy truth-values of disjoint parts of a shaded area in the below table. When the truth-value increases, the false value decreases. More general, $d_F^k\left(x_1,x_2,...,x_k\right)$, as a k-ary disjunction (or union), for $k\ge 2$, is defined as:

$$d_F^k:\left([0,1]\times[0,1]\right)^k \to [0,1]\times[0,1]$$

$$d_F^k\left(x_1,x_2,...,x_k\right)=\left(t_1+t_2+...+t_k, f_1+f_2+...+f_k-k+1\right)$$

if all x_i are disjoint two by two and $t_1+t_2+...+t_k\le 1$.

As a particular case let's take as a binary fuzzy conjunction:

$$c_F(x,y)=\left(t_1t_2, f_1+f_2-f_1f_2\right)$$

and as unary fuzzy negation:

$$n_F(x)=\left(1-t_1,1-f_1\right)=\left(f_1,t_1\right),$$

where

$FL(x)=(t_1,f_1)$, with $t_1+f_1=1$, and $0\le t_1, f_1\le 1$;

$FL(y)=(t_2,f_2)$, with $t_2+f_2=1$, and $0\le t_2, f_2\le 1$.

whence:

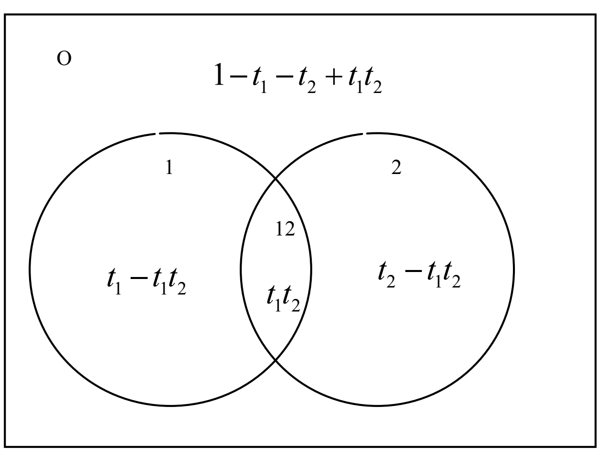
$$FL(P12)=\left(t_1t_2, f_1+f_2-f_1f_2\right)$$

$$FL(P1)=\left(t_1f_2, f_1+t_2-f_1t_2\right)$$

$$FL(P2)=\left(f_1t_2, t_1+f_2-t_1f_2\right)$$

$$FL(P0)=\left(f_1f_2, t_1+t_2-t_1t_2\right)$$

The Venn Diagram for $n=2$ and considering only the truth values, becomes:



since

$$t_1 f_2 = t_1(1-t_2) = t_1 - t_1 t_2$$
$$f_1 t_2 = (1-t_1)t_2 = t_2 - t_1 t_2$$
$$f_1 f_2 = (1-t_1)(1-t_2) = 1 - t_1 - t_2 + t_1 t_2 .$$

We now use:

$$d_F^k\left(P12, P1, P2, P0\right) = \left(\left(t_1 t_2\right) + \left(t_1 - t_1 t_2\right) + \left(t_2 - t_1 t_2\right) + \left(1 - t_1 - t_2 + t_1 t_2\right),\right.$$
$$\left.\left(f_1 + f_2 - f_1 f_2\right) + \left(f_1 + t_2 - f_1 t_2\right) + \left(t_1 + f_2 - t_1 f_2\right) + \left(t_1 + t_2 - t_1 t_2\right) - 3\right) = \left(1, 0\right).$$

So, the whole fuzzy space is normalized under $FL(\cdot)$.

For the neurosophic logic, we consider

$$NL(x) = (T_1, I_1, F_1), \text{ with } 0 \le T_1, I_1, F_1 \le 1;$$
$$NL(y) = (T_2, I_2, F_2), \text{ with } 0 \le T_2, I_2, F_2 \le 1;$$

if the sum of components is 1 as in Atanassov's intuitionist fuzzy logic, i.e. $T_i + I_i + F_i = 1$, they are considered *normalized*; otherwise *non-normalized*, i.e. the sum of the components is <1 (*sub-normalized*) or >1 (*over-normalized*).

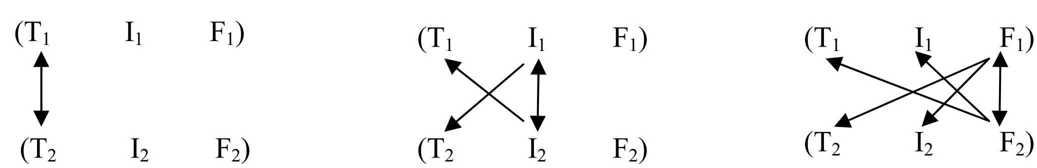
We define a binary neutrosophic conjunction (intersection) operator, which is a particular case of an N-norm (neutrosophic norm, a generalization of the fuzzy t-norm):

$$c_N : \left(\left[0,1\right] \times \left[0,1\right] \times \left[0,1\right]\right)^2 \to \left[0,1\right] \times \left[0,1\right] \times \left[0,1\right]$$
$$c_N(x, y) = \left(T_1 T_2, I_1 I_2 + I_1 T_2 + T_1 I_2, F_1 F_2 + F_1 I_2 + F_1 T_2 + F_2 T_1 + F_2 I_1\right).$$

The neutrosophic conjunction (intersection) operator $x \wedge_N y$ component truth, indeterminacy, and falsehood values result from the multiplication

$$\left(T_1 + I_1 + F_1\right) \cdot \left(T_2 + I_2 + F_2\right)$$

since we consider in a prudent way $T \prec I \prec F$, where "$\prec$" means "weaker", i.e. the products $T_i I_j$ will go to I, $T_i F_j$ will go to F, and $I_i F_j$ will go to F (or reciprocally we can say that F prevails in front of I and of T,



So, the truth value is $T_1 T_2$, the indeterminacy value is $I_1 I_2 + I_1 T_2 + T_1 I_2$ and the false value is $F_1 F_2 + F_1 I_2 + F_1 T_2 + F_2 T_1 + F_2 I_1$. The norm of $x \wedge_N y$ is $\left(T_1 + I_1 + F_1\right) \cdot \left(T_2 + I_2 + F_2\right)$. Thus, if x and y are normalized, then $x \wedge_N y$ is also normalized. Of course, the reader can redefine the neutrosophic conjunction operator, depending on application, in a different way, for example in a more optimistic way, i.e. $I \prec T \prec F$ or T prevails with respect to I, then we get:

$$c_N^{ITF}(x, y) = \left(T_1 T_2 + T_1 I_2 + T_2 I_1, I_1 I_2, F_1 F_2 + F_1 I_2 + F_1 T_2 + F_2 T_1 + F_2 I_1\right).$$

Or, the reader can consider the order $T \prec F \prec I$, etc.

Let's also define the unary neutrosophic negation operator:

$$n_N : \left[0,1\right] \times \left[0,1\right] \times \left[0,1\right] \to \left[0,1\right] \times \left[0,1\right] \times \left[0,1\right]$$

$$n_N\left(T,I,F\right)=\left(F,I,T\right)$$

by interchanging the truth T and falsehood F vector components.
Then:

$$NL(P12)=\left(T_1T_2,I_1I_2+I_1T_2+I_2T_1,F_1F_2+F_1I_2+F_1T_2+F_2T_1+F_2I_1\right)$$

$$NL(P1)=\left(T_1F_2,I_1I_2+I_1F_2+I_2T_1,F_1T_2+F_1I_2+F_1F_2+T_2T_1+T_2I_1\right)$$

$$NL(P2)=\left(F_1T_2,I_1I_2+I_1T_2+I_2F_1,T_1F_2+T_1I_2+T_1T_2+F_2F_1+F_2I_1\right)$$

$$NL(P0)=\left(F_1F_2,I_1I_2+I_1F_2+I_2F_1,T_1T_2+T_1I_2+T_1F_2+T_2F_1+T_2I_1\right)$$

Similarly as in our above fuzzy logic work, we now define a binary $N-conorm$ (disjunction or union), i.e. neutrosophic conform.

$$d_N:\left([0,1]\times[0,1]\times[0,1]\right)^2\to[0,1]\times[0,1]\times[0,1]$$

$$d_N(x,y)=\left(T_1+T_2,\left(I_1+I_2\right)\cdot\frac{\tau-T_1-T_2}{I_1+I_2+F_1+F_2},\left(F_1+F\right)\cdot\frac{\tau-T_1-T_2}{I_1+I_2+F_1+F_2}\right)$$

if x and y are disjoint, and $T_1+T_2\le 1$ where τ is the neutrosophic norm of $x\vee_N y$, i.e.

$$\tau=\left(T_1+I_1+F_1\right)\cdot\left(T_2+I_2+F_2\right).$$

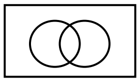
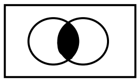
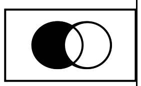
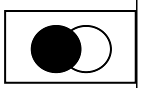
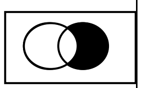
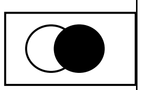
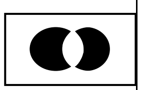
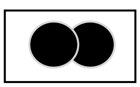
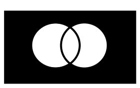
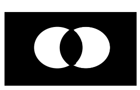
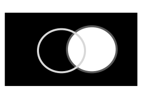
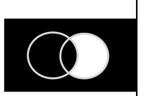
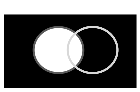
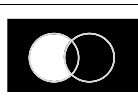
We consider as neutrosophic norm of x, where $NL(x)=T_1+I_1+F_1$, the sum of its components: $T_1+I_1+F_1$, which in many cases is 1, but can also be positive <1 or >1.

When the truth value increases $\left(T_1+T_2\right)$ is the above definition, the indeterminacy and falsehood values decrease proportionally with respect to their sums I_1+I_2 and respectively F_1+F_2.

This neutrosophic disjunction operator of disjoint variables allows us to add neutrosophic truth values of disjoint parts of a shaded area in a Venn Diagram.

Now, we complete Donald E. Knuth’s Table of the Sixteen Logical Operators on two variables with Fuzzy Logical operators on two variables with Fuzzy Logic truth values, and Neutrosophic Logic truth/indeterminacy/false values (for the case $T\prec I\prec F$).

Table 1

Fuzzy Logic Truth Values	**Venn Diagram**	**Notations**	**Operator symbol**	**Name(s)**
0		0	$\perp$	Contradiction, falsehood; constant 0
t_1t_2		$xy,\ x\wedge y,\ x\,\&\,y$	$\wedge$	Conjunction; and
$t_1-t_1t_2$		$x\wedge\overline{y}, x\not\supset y, [x>y], x-y$	$\overline{\supset}$	Nonimplication; difference, but not
t_1		x	L	Left projection
$t_2-t_1t_2$		$\overline{x}\wedge y, x\not\subset y, [x<y], y-x$	$\overline{\subset}$	Converse nonimplication; not…but
t_2		y	R	Right projection
$t_1+t_2-2t_1t_2$		$x\oplus y, x\not\equiv y, x^\wedge y$	$\oplus$	Exclusive disjunction; nonequivalence; “xor”
$t_1+t_2-t_1t_2$		$x\vee y, x\mid y$	$\vee$	(Inclusive) disjunction; or; and/or
$1-t_1-t_2+t_1t_2$		$\overline{x}\wedge\overline{y}, \overline{x\vee y}, x\,\overline{\vee}\,y, x\uparrow y$	$\overline{\vee}$	Nondisjunction, joint denial, neither…nor
$1-t_1-t_2+2t_1t_2$		$x\equiv y, x\leftrightarrow y, x\Leftrightarrow y$	$\equiv$	Equivalence; if and only if
$1-t_2$		$\overline{y}, \neg y, !y, \sim y$	$\overline{R}$	Right complementation
$1-t_2+t_1t_2$		$x\vee\overline{y}, x\subset y, x\Leftarrow y,$ $[x\geq y], x^y$	$\subset$	Converse implication if
$1-t_1$		$\overline{x}, \neg x, !x, \sim x$	$\overline{L}$	Left complementation
$1-t_1+t_1t_2$		$\overline{x}\vee y, x\supset y, x\Rightarrow y,$ $[x\leq y], y^x$	$\supset$	Implication; only if; if..then

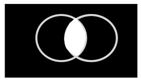
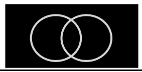
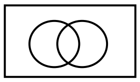
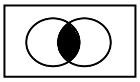
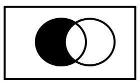
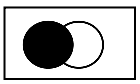
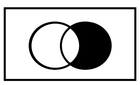
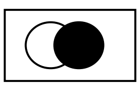
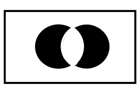
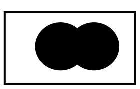
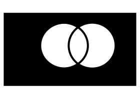
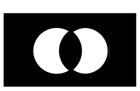
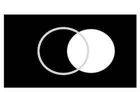
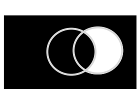
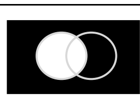
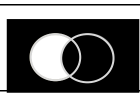
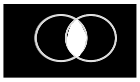
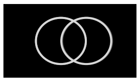
$1-t_1t_2$		$\overline{x}\vee\overline{y}, \overline{x\wedge y}, x\,\overline{\wedge}\,y, x \mid y$	$\overline{\wedge}$	Nonconjunction, not both…and; “nand”
1		1	T	Affirmation; validity; tautology; constant 1

Table 2

Venn Diagram	**Neutrosophic Logic Values**
	$(0,0,1)$
	$(T_1T_2, I_1I_2 + IT, F_1F_2 + FI + FT)$, where $IT = I_1T_2 + I_2T_1$ similarly FI, FT ;
	$\left(T_1F_2, \underbrace{I_1I_2 + IT_{\overline{y}}}_{I_{P1}}, \underbrace{F_{\overline{y}}F_{\overline{y}} + F_{\overline{y}}I + F_{\overline{y}}T}_{F_{P2}}\right)$
	(T_1, I_1, F_1)
	$\left(F_1T_2, \underbrace{I_1I_2 + IT_{\overline{x}}}_{I_{P2}}, \underbrace{F_{\overline{x}}F_{\overline{x}} + F_{\overline{x}}I + F_{\overline{x}}T}_{F_{P2}}\right)$
	(T_2, I_2, F_2)
	$\left(TF, (I_{P1} + I_{P2}) \cdot \frac{\tau - TF}{I_{P1} + I_{P2} + F_{P1} + F_{P2}}, (F_{P1} + F_{P2}) \cdot \frac{\tau - TF}{I_{P1} + I_{P2} + F_{P1} + F_{P2}}\right)$ Where $\tau = (T_1 + I_1 + F_1) \cdot (T_2 + I_2 + F_2)$ which is the neutrosophic norm
	$(T_1T_2 + TI + TF, I_1I_2 + IF, F_1F_2)$
	$(F_1F_2, I_1I_2 + IF, T_1T_2 + TI + TF)$
	$\left((F_{P1} + F_{P2}) \cdot \frac{\tau - TF}{I_{P1} + I_{P2} + F_{P1} + F_{P2}}, (I_{P1} + I_{P2}) \cdot \frac{\tau - TF}{I_{P1} + I_{P2} + F_{P1} + F_{P2}}, TF\right)$
	(F_2, I_2, T_2)
	$(F_{\overline{x}}F_{\overline{x}} + F_{\overline{x}}I + F_{\overline{x}}T, I_1I_2 + IT_{\overline{x}}, F_1T_2)$
	(F_1, I_1, T_1)
	$(F_{\overline{y}}F_{\overline{y}} + F_{\overline{y}}I + F_{\overline{y}}T, I_1I_2 + IT_{\overline{y}}, T_1F_2)$

	$\left(F_1F_2 + FI + FT, I_1I_2 + IT, T_1T_2\right)$
	$(1,0,0)$

These 16 neutrosophic binary operators are approximated, since the binary N-conorm gives an approximation because of 'indeterminacy' component.

Tri-nary Fuzzy Logic and Neutrosophic Logic Operators

In a more general way, for $k \geq 2$:

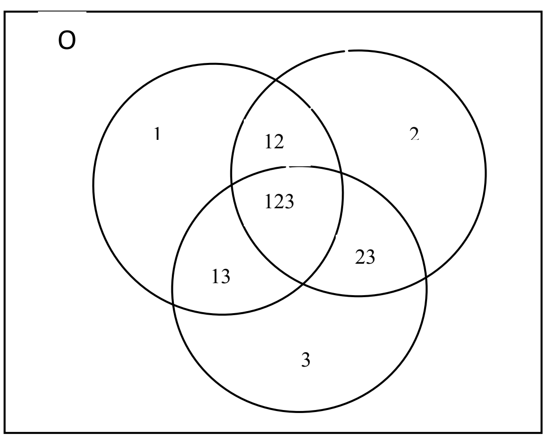
$$d_N^k : \left([0,1]\times[0,1]\times[0,1]\right)^k \to [0,1]\times[0,1]\times[0,1],$$

$$d_N^k\left(x_1, x_2, ..., x_k\right) = \left(\sum_{i=1}^{k} T_i, \left(\sum_{i=1}^{k} I_i\right)\cdot\frac{\tau_k - \sum_{i=1}^{k} T_i}{\sum_{i=1}^{k}\left(I_i + F_i\right)}, \left(\sum_{i=1}^{k} F_i\right)\cdot\frac{\tau_k - \sum_{i=1}^{k} T_i}{\sum_{i=1}^{k}\left(I_i + F_i\right)}\right)$$

if all x_i are disjoint two by two, and $\sum_{i=1}^{k} T_i \leq 1$.

We can extend Knuth's Table from binary operators to tri-nary operators (and we get $2^{2^3} = 256$ tri-nary operators) and in general to n-ary operators (and we get 2^{2^n} n-ary operators).

Let's present the tri-nary Venn Diagram, with 3 variables x, y, z



using the name Smarandache codification.

This has $2^3 = 8$ disjoint parts, and if we shade none, one, two, …, or eight of them and consider all possible combinations we get $2^8 = 256$ tri-nary operators in the above tri-nary Venn Diagram.

For n=3 we have:

$$P123 = c_F(x, y, z)$$
$$P12 = c_F\left(x, y, n_F(z)\right)$$
$$P13 = c_F\left(x, n_F(y), z\right)$$
$$P23 = c_F\left(n_F(x), y, z\right)$$
$$P1 = c_F\left(x, n_F(y), n_F(z)\right)$$
$$P2 = c_F\left(n_F(x), y, n_F(z)\right)$$
$$P3 = c_F\left(n_F(x), n_F(y), z\right)$$
$$P0 = c_F\left(n_F(x), n_F(y), n_F(z)\right)$$

Let

$FL(x) = (t_1, f_1)$, with $t_1 + f_1 = 1, \ \ 0 \le t_1, f_1 \le 1$,
$FL(y) = (t_2, f_2)$, with $t_2 + f_2 = 1, \ \ 0 \le t_2, f_2 \le 1$,
$FL(z) = (t_3, f_3)$, with $t_3 + f_3 = 1, \ \ 0 \le t_3, f_3 \le 1$.

We consider the particular case defined by tri-nary conjunction fuzzy operator:

$$c_F : \left([0,1]\times[0,1]\right)^3 \to [0,1]\times[0,1]$$
$$c_F(x, y, z) = \left(t_1t_2t_3, f_1 + f_2 + f_3 - f_1f_2 - f_2f_3 - f_3f_1 + f_1f_2f_3\right)$$

because

$$\left((t_1, f_1)\wedge_F (t_2, f_2)\right)\wedge_F (t_3, f_3) = \left(t_1t_2, f_1 + f_2 - f_1f_2\right)\wedge_F (t_3, f_3) =$$
$$= \left(t_1t_2t_3, f_1 + f_2 + f_3 - f_1f_2 - f_2f_3 - f_3f_1 + f_1f_2f_3\right)$$

and the unary negation operator:

$$n_F : \left([0,1]\times[0,1]\right) \to [0,1]\times[0,1]$$
$$n_F(x) = (1 - t_1, 1 - f_1) = (f_1, t_1).$$

We define the function:

$$L_1 : [0,1]\times[0,1]\times[0,1] \to [0,1]$$
$$L_1(\alpha, \beta, \gamma) = \alpha\cdot\beta\cdot\gamma$$

and the function

$$L_2 : [0,1]\times[0,1]\times[0,1] \to [0,1]$$
$$L_2(\alpha, \beta, \gamma) = \alpha + \beta + \gamma - \alpha\beta - \beta\gamma - \gamma\alpha + \alpha\beta\gamma$$

then:

$$FL(P123) = \left(L_1(t_1, t_2, t_3), L_2(f_1, f_2, f_3)\right)$$
$$FL(P12) = \left(L_1(t_1, t_2, f_3), L_2(f_1, f_2, t_3)\right)$$
$$FL(P13) = \left(L_1(t_1, f_2, t_3), L_2(f_1, t_2, f_3)\right)$$
$$FL(P23) = \left(L_1(f_1, t_2, t_3), L_2(t_1, f_2, f_3)\right)$$
$$FL(P1) = \left(L_1(t_1, f_2, f_3), L_2(f_1, t_2, t_3)\right)$$
$$FL(P2) = \left(L_1(f_1, t_2, f_3), L_2(t_1, f_2, t_3)\right)$$
$$FL(P3) = \left(L_1(f_1, f_2, t_3), L_2(t_1, t_2, f_3)\right)$$
$$FL(P0) = \left(L_1(f_1, f_2, f_3), L_2(t_1, t_2, t_3)\right)$$

We thus get the fuzzy truth-values as follows:

$$FL_t(P123) = t_1 t_2 t_3$$
$$FL_t(P12) = t_1 t_2 (1 - t_3) = t_1 t_2 - t_1 t_2 t_3$$
$$FL_t(P13) = t_1 (1 - t_2) t_3 = t_1 t_3 - t_1 t_2 t_3$$
$$FL_t(P23) = (1 - t_1) t_2 t_3 = t_2 t_3 - t_1 t_2 t_3$$
$$FL_t(P1) = t_1 (1 - t_2)(1 - t_3) = t_1 - t_1 t_2 - t_1 t_3 + t_1 t_2 t_3$$
$$FL_t(P2) = (1 - t_1) t_2 (1 - t_3) = t_2 - t_1 t_2 - t_2 t_3 + t_1 t_2 t_3$$
$$FL_t(P3) = (1 - t_1)(1 - t_2) t_3 = t_3 - t_1 t_3 - t_2 t_3 + t_1 t_2 t_3$$
$$FL_t(P0) = (1 - t_1)(1 - t_2)(1 - t_3) = 1 - t_1 - t_2 - t_3 + t_1 t_2 + t_1 t_3 + t_2 t_3 - t_1 t_2 t_3.$$

We, then, consider the same disjunction or union operator $d_F(x, y) = t_1 + t_2, f_1 + f_2 - 1$, if x and y are disjoint, and $t_1 + t_2 \leq 1$ allowing us to add the fuzzy truth values of each part of a shaded area.

Neutrophic Composition Law

Let's consider $k \geq 2$ neutrophic variables, $x_i\left(T_i, I_i, F_i\right)$, for all $i \in \{1, 2, ..., k\}$. Let denote

$$T = \left(T_1, ..., T_k\right)$$
$$I = \left(I_1, ..., I_k\right)$$
$$F = \left(F_1, ..., F_k\right).$$

We now define a neutrosophic composition law o_N in the following way:

$$o_N : \{T, I, F\} \to [0,1]$$

If $z \in \{T, I, F\}$ then $z_{o_N} z = \prod_{i=1}^{k} z_i$.

If $z, w \in \{T, I, F\}$ then

$$z_{o_N} w = w_{o_N} z \quad = \sum_{\substack{r=1 \\ \{i_1,...,i_r,j_{r+1},...,j_k\}\equiv\{1,2,...,k\} \\ (i_1,...,i_r)\in C^r(1,2,...,k) \\ (j_{r+1},...,j_k)\in C^{k-r}(1,2,...,k)}}^{k-1} z_{i_1}..z_{i_r} w_{j_{r+1}}...w_{j_k}$$

where $C^r\left(1,2,...,k\right)$ means the set of combinations of the elements $\left\{1,2,...,k\right\}$ taken by r. [Similarly for $C^{k-r}\left(1,2,...,k\right)$].

In other words, $z_{o_N} w$ is the sum of all possible products of the components of vectors z and w, such that each product has at least a z_i factor and at least w_j factor, and each product has exactly k factors where each factor is a different vector component of z or of w. Similarly if we multiply three vectors:

$$T_{o_N} I_{o_N} F = \sum_{\substack{u,v,k-u-v=1 \\ \{i_1,...,i_u,j_{u+1},...,j_{u+v},l_{u+v+1},...,l_k\}\equiv\{1,2,...,k\} \\ (i_1,...,i_u)\in C^u(1,2,...,k),(j_{u+1},...,j_{u+v})\in \\ \in C^v(1,2,...,k),(l_{u+v+1},...,l_k)\in C^{k-u-v}(1,2,...,k)}}^{k-2} T_{i_1...i_u} I_{j_{u+1}...j_{u+v}} F_{l_{u+v+1}}...F_{l_k}$$

Let's see an example for $k=3$.

$$x_1\left(T_1, I_1, F_1\right)$$
$$x_2\left(T_2, I_2, F_2\right)$$
$$x_3\left(T_3, I_3, F_3\right)$$

$$T_{o_N} T = T_1 T_2 T_3,\; I_{o_N} I = I_1 I_2 I_3, \quad \mathrm{F}_{o_N} F = F_1 F_2 F_3$$
$$T_{o_N} I = T_1 I_2 I_3 + I_1 T_2 I_3 + I_1 I_2 T_3 + T_1 T_2 I_3 + T_1 I_2 T_3 + I_1 T_2 T_3$$
$$T_{o_N} F = T_1 F_2 F_3 + F_1 T_2 F_3 + F_1 F_2 T_3 + T_1 T_2 F_3 + T_1 F_2 T_3 + F_1 T_2 T_3$$
$$I_{o_N} F = I_1 F_2 F_3 + F_1 I_2 F_3 + F_1 F_2 I_3 + I_1 I_2 F_3 + I_1 F_2 I_3 + F_1 I_2 I_3$$
$$T_{o_N} I_{o_N} F = T_1 I_2 F_3 + T_1 F_2 I_3 + I_1 T_2 F_3 + I_1 F_2 T_3 + F_1 I_2 T_3 + F_1 T_2 I_3$$

For the case when indeterminacy I is not decomposed in subcomponents {as for example $I = P \cup U$ where P=paradox (true and false simultaneously) and U=uncertainty (true or false, not sure which one)}, the previous formulas can be easily written using only three components as:

$$T_{o_N} I_{o_N} F = \sum_{i,j,r\in\mathcal{P}(1,2,3)} T_i I_j F_r$$

where $\mathcal{P}(1,2,3)$ means the set of permutations of $(1,2,3)$ i.e.

$$\left\{(1,2,3),(1,3,2),(2,1,3),(2,3,1,),(3,1,2),(3,2,1)\right\}$$

$$z_{o_N} w = \sum_{\substack{\mathrm{i}=1 \\ (i,j,r)\equiv(1,2,3) \\ (j,r)\in\mathcal{P}^2(1,2,3)}}^{3} z_i w_j w_{j_r} + w_i z_j z_r$$

This neurotrophic law is associative and commutative.

Neutrophic Logic Operators

Let's consider the neutrophic logic cricy values of variables x, y, z (so, for $n=3$) $NL(x)=(T_1, I_1, F_1)$ with $0 \le T_1, I_1, F_1 \le 1$

$$NL(y)=(T_2, I_2, F_2) \text{ with } 0 \le T_2, I_2, F_2 \le 1$$

$$NL(z)=(T_3, I_3, F_3) \text{ with } 0 \le T_3, I_3, F_3 \le 1$$

In neutrosophic logic it is not necessary to have the sum of components equals to 1, as in intuitionist fuzzy logic, i.e. $T_k + I_k + F_k$ is not necessary 1, for $1 \le k \le 3$

As a particular case, we define the tri-nary conjunction neutrosophic operator:

$$c_N : ([0,1]\times[0,1]\times[0,1])^3 \to [0,1]\times[0,1]\times[0,1]$$

$$c_N(x,y) = \left(T_{o_N} T, I_{o_N} I + I_{o_N} T, F_{o_N} F + F_{o_N} I + F_{o_N} T\right)$$

If x or y are normalized, then $c_N(x,y)$ is also normalized.

If x or y are non-normalized then $|c_N(x,y)| = |x| \cdot |y|$ where $|\cdot|$ means norm.

c_N is an N-norm (neutrosophic norm, i.e. generalization of the fuzzy t-norm).

Again, as a particular case, we define the unary negation neutrosophic operator:

$$n_N : [0,1]\times[0,1]\times[0,1] \to [0,1]\times[0,1]\times[0,1]$$

$$n_N(x) = n_N(T_1, I_1, F_1) = (F_1, I_1, T_1).$$

We take the same Venn Diagram for $n=3$.

So,

$$NL(x)=(T_1, I_1, F_1)$$

$$NL(y)=(T_2, I_2, F_2)$$

$$NL(z)=(T_3, I_3, F_3).$$

Vectors

$$\mathrm{T}=\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}, \quad \mathrm{I}=\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \text{ and } \mathrm{F}=\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}.$$

We note $\mathrm{T}_{\bar{x}}=\begin{pmatrix} F_1 \\ T_2 \\ T_3 \end{pmatrix}$, $\mathrm{T}_{\bar{y}}=\begin{pmatrix} T_1 \\ F_2 \\ T_3 \end{pmatrix}$, $\mathrm{T}_{\bar{z}}=\begin{pmatrix} T_1 \\ T_2 \\ F_3 \end{pmatrix}$, $\mathrm{T}_{\overline{xy}}=\begin{pmatrix} F_1 \\ F_2 \\ T_3 \end{pmatrix}$, etc.

and similarly

$$\mathrm{F}_{\bar{x}}=\begin{pmatrix} T_1 \\ F_2 \\ F_3 \end{pmatrix}, \quad F_{\bar{y}}=\begin{pmatrix} F_1 \\ T_2 \\ F_3 \end{pmatrix}, \quad \mathrm{F}_{\overline{xz}}=\begin{pmatrix} T_1 \\ F_2 \\ T_3 \end{pmatrix}, \text{ etc.}$$

For shorter and easier notations let's denote $z_{o_N} w = zw$ and respectively $z_{o_N} w_{o_N} v = zwv$ for the vector neutrosophic law defined previously.

Then

$$NL(P123) = c_N(x,y) = (TT, II + IT, FF + FI + FT + FIT) =$$

$$
\begin{aligned}
&= \big(T_1 T_2 T_3, I_1 I_2 I_3 + I_1 I_2 T_3 + I_1 T_2 I_3 + T_1 I_2 I_3 + I_1 T_2 T_3 + T_1 I_2 T_3 + T_1 T_2 I_3, \\
&F_1 F_2 F_3 + F_1 F_2 I_3 + F_1 I_2 F_3 + I_1 F_2 F_3 + F_1 I_2 I_3 + I_1 F_2 I_3 + I_1 I_2 F_3 + \\
&+ F_1 F_2 T_3 + F_1 T_2 F_3 + T_1 F_2 F_3 + F_1 T_2 T_3 + T_1 F_2 T_3 + T_1 T_2 F_3 + \\
&+ T_1 I_2 F_3 + T_1 F_2 I_3 + I_1 F_2 T_3 + I_1 T_2 F_3 + F_1 I_2 T_3 + F_1 T_2 I_3\big)
\end{aligned}
$$

$$NL(P12) = c_N(x, y, n_N(z)) = \left(T_{\bar{z}} T_{\bar{z}}, II + IT_{\bar{z}}, F_{\bar{z}} F_{\bar{z}} + F_{\bar{z}} I + F_{\bar{z}} T_{\bar{z}} + F_{\bar{z}} IT_{\bar{z}}\right)$$

$$NL(P13) = c_N(x, n_N(y), z) = \left(T_{\bar{y}} T_{\bar{y}}, II + IT_{\bar{y}}, F_{\bar{y}} F_{\bar{y}} + F_{\bar{y}} I + F_{\bar{y}} T_{\bar{y}} + F_{\bar{y}} IT_{\bar{y}}\right)$$

$$NL(P23) = c_N(n_N(x), y, z) = \left(T_{\bar{x}} T_{\bar{x}}, II + IT_{\bar{x}}, F_{\bar{x}} F_{\bar{x}} + F_{\bar{x}} I + F_{\bar{x}} T_{\bar{x}} + F_{\bar{x}} IT_{\bar{y}}\right)$$

$$NL(P1) = c_N(x, n_N(y), n_N(z)) = \left(T_{\overline{yz}} T_{\overline{yz}}, II + IT_{\overline{yz}}, F_{\overline{yz}} F_{\overline{yz}} + F_{\overline{yz}} I + F_{\overline{yz}} T_{\overline{yz}} + F_{\overline{yz}} IT_{\overline{yz}}\right)$$

$$NL(P2) = c_N(n_N(x), y, n_N(z)) = \left(T_{\overline{xz}} T_{\overline{xz}}, II + IT_{\overline{xz}}, F_{\overline{xz}} F_{\overline{xz}} + F_{\overline{xz}} I + F_{\overline{xz}} T_{\overline{xz}} + F_{\overline{xz}} IT_{\overline{xz}}\right)$$

$$
\begin{aligned}
NL(P0) &= c_N(n_N(x), n_N(y), n_N(z)) = \left(T_{\overline{xyz}} T_{\overline{xyz}}, II + IT_{\overline{xyz}}, F_{\overline{xyz}} F_{\overline{xyz}} + F_{\overline{xyz}} I + F_{\overline{xyz}} T_{\overline{xyz}} + F_{\overline{xyz}} IT_{\overline{xyz}}\right) = \\
&= (FF, II + IF, TT + TI + TF + TIF).
\end{aligned}
$$

n-ary Fuzzy Logic and Neutrosophic Logic Operators

We can generalize for any integer $n \geq 2$.

The Venn Diagram has 2^{2^n} disjoint parts. Each part has the form $Pi_1 ... i_k j_{k+1} ... j_n$, where $0 \leq k \leq n$, and of course $0 \leq n-k \leq n$; $\{i_1, ..., i_k\}$ is a combination of k elements of the set $\{1, 2, ..., n\}$, while $\{j_{k+1}, ..., j_n\}$ the $n-k$ elements left, i.e. $\{j_{k+1}, ..., j_k\} = \{1, 2, ..., n\} \setminus \{i_1, ..., i_k\}$. $\{i_1, ..., i_k\}$ are replaced by the corresponding numbers from $\{1, 2, ..., n\}$, while $\{j_{k+1}, ..., j_n\}$ are replaced by blanks.

For example, when $n = 3$,

$$Pi_1 i_2 j_3 = P13 \text{ if } \{i_1, i_2\} = \{1, 3\},$$

$$Pi_1 j_2 j_3 = P1 \text{ if } \{i_1\} = \{1\}.$$

Hence, for fuzzy logic we have:

$$Pi_1 ... i_k j_{k+1} ... j_n = c_F\left(x_{i_1}, ..., x_{i_k}, n_F\left(x_{j_{k+1}}\right), ..., n_F\left(x_{j_n}\right)\right)$$

whence

$$FL(Pi_1 ... i_k j_{k+1} ... j_n) = \left(\left(\prod_{r=1}^{k} t_{i_r}\right)\left(\prod_{s=k+1}^{n} \left(1 - t_{j_s}\right)\right), \varphi(f_1 f_2, ..., f_n)\right)$$

where $\varphi : [0,1]^n \to [0,1]$,

$$\varphi(\alpha_1, \alpha_2, ..., \alpha_n) = S_1 - S_2 + S_3 + ... + (-1)^{n+1} S_n = \sum_{l=1}^{n} (-1)^{l+1} S_l$$

where

$$S_1 = \sum_{i=1}^{n} \alpha_i$$

$$S_2 = \sum_{1 \le i < j \le n} \alpha_i \alpha_j$$

....

$$S_l = \sum_{1 \le i_1 < i_2 < \ldots < i_l \le n} \alpha_{i_1} \alpha_{i_2} .. \alpha_{i_l}$$

....

$$S_n = \alpha_1 \cdot \alpha_2 \cdot \ldots \cdot \alpha_n$$

And for neutrosophic logic we have:

$$Pi_1 ... i_k j_{k+1} ... j_n = c_N \left(x_{i_1}, ..., x_{i_k}, n_N \left(x_{j_{k+1}} \right), ..., n_N \left(x_{j_n} \right) \right)$$

whence:

$$NL\left(Pi_1 ... i_k j_{k+1} ... j_n \right) = \left(T_{12...n}, I_{12...n}, F_{12...n} \right),$$

where

$$T_{12...n} = T_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} T_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} = \left(\prod_{r=1}^{k} T_{i_r} \right) \cdot \prod_{s=k+1}^{n} F_{j_s} .$$

$$I_{12...n} = II + IT_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} ,$$

$$F_{12...n} = F_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} F_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} + F_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} I + F_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} T_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} + F_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}} IT_{\bar{x}_{j_{k+1}} ... \bar{x}_{j_n}}$$

Conclusion:

A generalization of Knuth's Boolean binary operations is presented in this paper, i.e. we present n-ary Fuzzy Logic Operators and Neutrosophic Logic Operators based on Smarandache's codification of the Venn Diagram and on a defined vector neutrosophic law which helps in calculating fuzzy and neutrosophic operators.

Better neutrosophic operators than in [2] are proposed herein.

References:

[1] F. Smarandache & J. Dezert, Advances and Applications of DSmt for Information Fusion, Am. Res. Press, 2004.

[2] F. Smarandache, A unifying field in logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, 1998, 2001, 2003.

[3] Knuth, Art of Computer Programming, The, Volumes 1-3 Boxed Set (2nd Edition), Addison-Wesley Professional, 1998.

[4] Zadeh, Fuzzy Sets, Information and Control, Vol. 8, 338-353, 1965.

[5] Driankov, Dimiter; Hellendoorn, Hans; and Reinfrank, Michael: *An Introduction to Fuzzy Control.* Springer, Berlin/Heidelberg, 1993.

[6] Atanassov, K., Stoyanova, D., Remarks on the intuitionistic fuzzy sets. II, Notes on Intuitionistic Fuzzy Sets, Vol. 1, No. 2, 85 – 86, 1995.

Schrödinger Equation and the Quantization of Celestial Systems

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In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems. While this hypothesis has been described by some authors, including Nottale, here we argue that such a macroquantization was formed by topological superfluid vortice. We also provide derivation of Schrödinger equation from Gross-Pitaevskii-Ginzburg equation, which supports this superfluid dynamics interpretation.

1 Introduction

In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems, based on logarithmic nature of Schrödinger equation, and also its exact mapping to Navier-Stokes equations [1].

While this notion of macro-quantization is not widely accepted yet, as we will see the logarithmic nature of Schrödinger equation could be viewed as a support of its applicability to larger systems. After all, the use of Schrödinger equation has proved itself to help in finding new objects known as extrasolar planets [2, 3]. And we could be sure that new extrasolar planets are to be found in the near future. As an alternative, we will also discuss an outline for how to derive Schrödinger equation from simplification of Ginzburg-Landau equation. It is known that Ginzburg-Landau equation exhibits fractal character, which implies that quantization could happen at *any scale*, supporting topological interpretation of quantized vortices [4].

First, let us rewrite Schrödinger equation in its common form [5]

$$\left[i\,\frac{\partial}{\partial t} + \frac{\bar{\nabla}^2}{2m} - U(x) \right] \psi = 0 \tag{1}$$

or

$$i\,\frac{\partial \psi}{\partial t} = H\,\psi\,. \tag{2}$$

Now, it is worth noting here that Englman and Yahalom [5] argues that this equation exhibits logarithmic character

$$\ln \psi(x,t) = \ln\big(|\psi(x,t)|\big) + i \arg\big(\psi(x,t)\big)\,. \tag{3}$$

Schrödinger already knew this expression in 1926, which then he used it to propose his equation called “eigentliche Wellengleichung” [5]. Therefore equation (1) can be re-written as follows

$$2m\,\frac{\partial\big(\ln|\psi|\big)}{\partial t} + 2\bar{\nabla}\ln|\psi|\,\bar{\nabla}\arg\big[\psi\big] + \bar{\nabla}\bar{\nabla}\arg\big[\psi\big] = 0\,. \tag{4}$$

Interestingly, Nottale’s scale-relativistic method [2, 3] was also based on generalization of Schrödinger equation to describe quantization of celestial systems. It is known that Nottale-Schumacher’s method [6] could predict new exoplanets in good agreement with observed data. Nottale’s scale-relativistic method is essentially based on the use of first-order scale-differentiation method defined as follows [2]

$$\frac{\partial V}{\partial(\ln \delta t)} = \beta(V) = a + b\,V + \dots\ . \tag{5}$$

Now it seems clear that the natural-logarithmic derivation, which is essential in Nottale’s scale-relativity approach, also has been described properly in Schrödinger’s original equation [5]. In other words, its logarithmic form ensures applicability of Schrödinger equation to describe macro-quantization of celestial systems. [7, 8]

2 Quantization of celestial systems and topological quantized vortices

In order to emphasize this assertion of the possibility to describe quantization of celestial systems, let us quote Fischer’s description [4] of relativistic momentum from superfluid dynamics. Fischer [4] argues that the circulation is in the relativistic dense superfluid, defined as the integral of the momentum

$$\gamma_s = \oint p_\mu\, dx^\mu = 2\pi N_v\,\hbar\,, \tag{6}$$

and is quantized into multiples of Planck’s quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of γ_s. And then Fischer [4] concludes that the Maxwell equations of ordinary electromagnetism can be written in the form of conservation equations of relativistic perfect fluid hydrodynamics [9]. Furthermore, the topological character of equation (6) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [25]. For the plausible linkage between superfluid dynamics and cosmological phenomena, see [16–24].

It is worth noting here, because vortices could be defined as elementary objects in the form of stable topological excitations [4], then equation (6) could be interpreted as Bohr-Sommerfeld-type quantization from *topological quantized vortices*. Fischer [4] also remarks that equation (6) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization for celestial systems is known in literature [7, 8], which here in the context of Fischer's arguments it has special meaning, i. e. it suggests that *quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale* [4]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [16–24].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider the problem of quantization of celestial orbits in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [26, 27]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity [28]; it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s \, dl = 2\pi n\hbar/m$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [28]

$$\oint_{\Gamma} p \, dx = 2\pi n\hbar \tag{6a}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [28]

$$\int_0^T v^2 d\tau = \omega^2 T = 2\pi\omega \,, \tag{7}$$

where $T = 2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion [28]

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \,. \tag{8}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum, a new constant g was introduced [28]

$$mvr = \frac{ng}{2\pi} \,. \tag{9}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [28]

$$r = \frac{n^2 g^2}{4\pi^2 GMm^2} \,, \tag{10}$$

which can be rewritten in the known form of gravitational Bohr-type radius [2, 7, 8]

$$r = \frac{n^2 GM}{v_0^2} \,, \tag{11}$$

where r, n, G, M, v_0 represents orbit radii, quantum number ($n = 1, 2, 3, \dots$), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (11), we denote [28]

$$v_0 = \frac{2\pi}{g} \, GMm \,. \tag{12}$$

The value of m is an adjustable parameter (similar to g) [7, 8]. In accordance with Nottale, we assert that the specific velocity v_0 is 144 km/sec for planetary systems. By noting that m is meant to be mass of celestial body in question, then we could find g parameter (see also [28] and references cited therein).

Using this equation (11), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and use M in terms of reduced mass $\mu = \frac{(M_1 + M_2)}{M_1 M_2}$. From this viewpoint the result is shown in Table 1 below [28].

For comparison purpose, we also include some recent observation by Brown-Trujillo team from Caltech [29–32]. It is known that Brown et al. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52 AU), 2005FY9 (at 52 AU), 2003VB12 (at 76 AU, dubbed as Sedna). And recently Brown-Trujillo team reported a new planetoid finding, called 2003UB31 (97 AU). This is not to include their previous finding, Quaoar (42 AU), which has orbit distance more or less near Pluto (39.5 AU), therefore this object is excluded from our discussion. It is interesting to remark here that all of those new "planetoids" are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 1). While this prediction is not so precise compared to the observed data, one could argue that the 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until $n = 9$ of Jovian planets (outer solar system), it seems that there are sufficient reasons to suppose that more planetoids in the Oort Cloud will be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we include in Table 1 new expected orbits based

Object	No.	Titius	Nottale	CSV	Observ.	Δ, %
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	-6.95
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit) [28].

on the same quantization procedure we outlined before. For Jovian planets corresponding to quantum number $n = 10$ and $n = 11$, our method suggests that it is likely to find new orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new planetoids around these predicted orbits.

As an interesting alternative method supporting this proposition of quantization from superfluid-quantized vortices (6), it is worth noting here that Kiehn has argued in favor of re-interpreting the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [1]. From this viewpoint, Kiehn suggests that there is *exact mapping* from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [1]. Interestingly, de Andrade and Sivaram [33] also suggest that there exists formal analogy between Schrödinger equation and the Navier-Stokes viscous dissipation equation:

$$\frac{\partial V}{\partial t} = \nu \nabla^2 V \,, \tag{13}$$

where ν is the kinematic viscosity. Their argument was based on propagation torsion model for quantized vortices [23]. While Kiehn's argument was intended for ordinary fluid, nonetheless the neat linkage between Navier-Stokes equation and superfluid turbulence is known in literature [34, 24].

At this point, it seems worth noting that some criticism arises concerning the use of quantization method for describing the motion of celestial systems. These criticism proponents usually argue that quantization method (wave mechanics) is oversimplifying the problem, and therefore cannot explain other phenomena, for instance planetary migration etc. While we recognize that there are phenomena which do not correspond to quantum mechanical process, at least we can argue further as follows:

1. Using quantization method like Nottale-Schumacher did, one can expect to predict new exoplanets (extra-solar planets) with remarkable result [2, 3];
2. The "conventional" theories explaining planetary migration normally use fluid theory involving *diffusion* process;
3. Alternatively, it has been shown by Gibson et al. [35] that these migration phenomena could be described via Navier-Stokes approach;
4. As we have shown above, Kiehn's argument was based on *exact-mapping* between Schrödinger equation and Navier-Stokes equations [1];
5. Based on Kiehn's vorticity interpretation one these authors published prediction of some new planets in 2004 [28]; which seems to be in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt;
6. To conclude: while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction;
7. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8 AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud);
8. There are of course other theories which have been developed to explain planetoids and exoplanets [36]. Therefore quantization method could be seen as merely a "plausible" theory between others.

All in all, what we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But at least it can be used to predict something quantitatively, i. e. measurable (exoplanets, and new planetoids in the outer solar system etc.).

In the meantime, it seems also interesting here to consider a plausible generalization of Schrödinger equation in particular in the context of viscous dissipation method [1]. First, we could write Schrödinger equation for a charged particle interacting with an external electromagnetic field [1] in the form of Ulrych's unified wave equation [14]

$$\begin{aligned}\left[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu \psi\right] = \\ = \left[-i2m\,\frac{\partial}{\partial t} + 2mU(x)\right]\psi\,.\end{aligned} \tag{14}$$

In the presence of electromagnetic potential, one could include another term into the LHS of equation (14)

$$\begin{aligned}\left[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu + eA_0\right]\psi = \\ = 2m\left[-i\,\frac{\partial}{\partial t} + U(x)\right]\psi\,.\end{aligned} \tag{15}$$

This equation has the physical meaning of Schrödinger equation for a charged particle interacting with an external electromagnetic field, which takes into consideration Aharonov effect [37]. Topological phase shift becomes its immediate implication, as already considered by Kiehn [1].

As described above, one could also derived equation (11) from scale-relativistic Schrödinger equation [2, 3]. It should be noted here, however, that Nottale's method [2, 3] differs appreciably from the viscous dissipative Navier-Stokes approach of Kiehn [1], because Nottale only considers his equation in the Euler-Newton limit [3]. Nonetheless, it shall be noted here that in his recent papers (2004 and up), Nottale has managed to show that his scale relativistic approach has linkage with Navier-Stokes equations.

3 Schrödinger equation derived from Ginzburg-Landau equation

Alternatively, in the context of the aforementioned superfluid dynamics interpretation [4], one could also derive Schrödinger equation from simplification of Ginzburg-Landau equation. This method will be discussed subsequently. It is known that Ginzburg-Landau equation can be used to explain various aspects of superfluid dynamics [16, 17]. For alternative approach to describe superfluid dynamics from Schrödinger-type equation, see [38, 39].

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m^* can be described as [40]

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \kappa\,|\psi|^2\,\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{16}$$

For some conditions, it is possible to replace the potential energy term in equation (16) with Hulthen potential. This substitution yields

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + V_{\text{Hulthen}}\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{17}$$

where

$$V_{\text{Hulthen}} = -Ze^2\,\frac{\delta\,e^{-\delta r}}{1 - e^{-\delta r}}\,. \tag{18}$$

This equation (18) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (18) approximates the effective Coulomb potential, in particular for large radius

$$V^{\text{eff}}_{\text{Coulomb}} = -\frac{e^2}{r} + \frac{\ell(\ell+1)\,\hbar^2}{2mr^2}\,. \tag{19}$$

By inserting (19), equation (17) could be rewritten as

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \left[-\frac{e^2}{r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2}\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}. \tag{20}$$

For large radii, second term in the square bracket of LHS of equation (20) reduces to zero [41],

$$\frac{\ell(\ell+1)\,\hbar^2}{2mr^2} \to 0\,, \tag{21}$$

so we can write equation (20) as

$$\left[-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2 + U(x)\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{22}$$

where Coulomb potential can be written as

$$U(x) = -\frac{e^2}{r}\,. \tag{22a}$$

This equation (22) is nothing but Schrödinger equation (1), except for the mass term now we get mass of Cooper pairs. In other words, we conclude that it is possible to re-derive Schrödinger equation from simplification of (Gross-Pitaevskii) Ginzburg-Landau equation for superfluid dynamics [40], in the limit of small screening parameter, δ. Calculation shows that introducing this Hulthen effect (18) into equation (17) will yield essentially similar result to (1), in particular for small screening parameter. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (20) is *essentially* the same with the result derived from equation (1). Now, to derive gravitational Bohr-type radius equation (11) from Schrödinger equation, one could use Nottale's scale-relativistic method [2, 3].

4 Concluding remarks

What we would emphasize here is that this derivation of Schrödinger equation from (Gross-Pitaevskii) Ginzburg-Landau equation is in good agreement with our previous conjecture that equation (6) implies macroquantization corresponding to superfluid-quantized vortices. This conclusion is the main result of this paper. Furthermore, because Ginzburg-Landau equation represents superfluid dynamics at low-temperature [40], the fact that we can derive quantization of celestial systems from this equation seems to support the idea of Bose-Einstein condensate cosmology [42, 43]. Nonetheless, this hypothesis of Bose-Einstein condensate cosmology deserves discussion in another paper.

Above results are part of our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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References

1. Kiehn R. M. An interpretation of wavefunction as a measure of vorticity. http://www22.pair.com/csdc/pdf/cologne.pdf.
2. Nottale L., et al., *Astron. Astrophys.*, 1997, v. 322, 1018.
3. Nottale L. *Astron. Astrophys.*, 1997, v. 327, 867-889.
4. Fischer U. W. arXiv: cond-mat/9907457 (1999).
5. Englman R. and Yahalom H. arXiv: physics/0406149 (2004).
6. Nottale L., Schumacher G., Levefre E. T. *Astron. Astrophys.*, 2000, v. 361, 379–387.
7. Rubcic A. and Rubcic J. The quantization of solar-like gravitational systems. *Fizika B*, 1998, v. 7(1), 1–13.
8. Agnese A. G. and Festa R. Discretization of the cosmic scale inspired from the Old Quantum Mechanics. *Proc. Workshop on Modern Modified Theories of Grav. and Cosmology*, 1997; arXiv: astro-ph/9807186.
9. Fischer U. W. arXiv: cond-mat/9907457.
10. Aharonov Y., et al. arXiv: quant-ph/0311155.
11. Hofer W. A. arXiv: physics/9611009; quant-ph/9801044.
12. Hooft G. arXiv: quant-ph/0212095.
13. Blasone M., et al. arXiv: quant-ph/0301031.
14. Rosu H. C. arXiv: gr-qr-qc/9411035.
15. Oudet X. The quantum state and the doublets. *Annales de la Fondation Louis de Broglie*, 2000, v. 25(1).
16. Zurek W. Cosmological experiments in superfluids and superconductors. *Proc. Euroconference Formation and Interaction of Topological Defects*, ed. A. Davis and R. Brandenberger, Plenum, 1995; arXiv: cond-mat/9502119.
17. Volovik G. Superfluid analogies of cosmological phenomena. arXiv: gr-qc/0005091.
18. Volovik G. Links between gravity and dynamics of quantum liquids. *Int. Conf. Cosmology. Relativ. Astrophysics. Cosmoparticle Physics* (COSMION-99); arXiv: gr-qc/0004049.
19. Volovik G. arXiv: gr-qc/0104046.
20. Nozieres P. and Pines D. The theory of quantum liquids: Superfluid Bose Liquid. Addison-Wesley, 1990, 116–124.
21. Winterberg F. *Z. Naturforsch.*, 2002, v. 57a, 202–204; presented at *9th Canadian Conf. on General Relativity and Relativ. Astrophysics*, Edmonton, May 24–26, 2001.
22. Winterberg F. Maxwell's aether, the Planck aether hypothesis, and Sommerfeld's fine structure constant. http://www.znaturforsch.com/56a/56a0681.pdf.
23. Kaivarainen A. arXiv: physics/020702.
24. Kaivarainen A. Hierarchic models of turbulence, superfluidity and superconductivity. arXiv: physics/0003108.
25. Wiegmann P. arXiv: cond-mat/9808004.
26. Schrieffer J. R. Macroscopic quantum phenomena from pairing in superconductors. Lecture, December 11th, 1972.
27. Coles P. arXiv: astro-ph/0209576.
28. Christianto V. *Apeiron*, 2004, v. 11(3).
29. NASA News Release (Jul 2005), http://www.nasa.gov/vision/universe/solarsystem/newplanet-072905.html.
30. *BBC News* (Oct 2004), http://news.bbc.co.uk/1/hi/sci/tech/4730061.stm.
31. Brown M., et al. *ApJ. Letters*, Aug. 2004; arXiv: astro-ph/0404456; *ApJ.*, forthcoming issue (2005); astro-ph/0504280.
32. Brown M. (July 2005), http://www.gps.caltech.edu/∼mbrown/planetlila/.
33. de Andrade L. G. and Sivaram C. arXiv: hep-th/9811067.
34. Godfrey S. P., et al. A new interpretation of oscillating flow experiments in superfluid Helium II, *J. Low Temp. Physics*, Nos. 112, Oct 2001.
35. Gibson C. and Schild R. arXiv: astro-ph/0306467.
36. Griv E. and Gedalin M. The formation of the Solar System by Gravitational Instability. arXiv: astro-ph/0403376.
37. Anandan J. S. *Quantum Coherence and Reality, Proc. Conf. Fundamental Aspects of Quantum Theory*, Columbia SC., ed. by J. S. Anandan and J. L. Safko, World Scientific, 1994; arXiv: gr-qc/9504002.
38. Varma C. M. arXiv: cond-mat/0109049.
39. Lipavsky P., et al. arXiv: cond-mat/0111214.
40. Infeld E., et al. arXiv: cond-mat/0104073.
41. Pitkänen M. http://www.physics.helsinki.fi/∼matpitka/articles/nottale.pdf.
42. Trucks M. arXiv: gr-qc/9811043.
43. Castro C., et al. arXiv: hep-th/0004152.

Plausible Explanation of Quantization of Intrinsic Redshift from Hall Effect and Weyl Quantization

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Using phion condensate model as described by Moffat [1], we consider a plausible explanation of (Tifft) intrinsic redshift quantization as described by Bell [6] as result of Hall effect in rotating frame. We also discuss another alternative to explain redshift quantization from the viewpoint of Weyl quantization, which could yield Bohr-Sommerfeld quantization.

1 Introduction

In a recent paper by Moffat [1] it is shown that quantum phion condensate model with Gross-Pitaevskii equation yields an approximate fit to data corresponding to CMB spectrum, and it also yields a modified Newtonian acceleration law which is in good agreement with galaxy rotation curve data. It seems therefore interesting to extend further this hypothesis to explain quantization of redshift, as shown by Tifft *et al.* [2, 6, 7]. We also argue in other paper that this redshift quantization could be explained as signature of topological quantized vortices, which also agrees with Gross-Pitaevskiian description [3, 5].

Nonetheless, there is remaining question in this quantized vortices interpretation, i. e. how to provide explanation of "intrinsic redshift" argument by Bell [6]. In the present paper, we argue that it sounds reasonable to interpret the intrinsic redshift data from the viewpoint of rotating Hall effect, i. e. rotational motion of clusters of galaxies exhibit quantum Hall effect which can be observed in the form of "intrinsic redshift". While this hypothesis is very new, it could be expected that we can draw some prediction, including possibility to observe small "blue-shift" effect generated by antivortex part of the Hall effect [5a].

Another possibility is to explain redshift quantization from the viewpoint of Weyl-Moyal quantization theory [25]. It is shown that Schrödinger equation can be derived from Weyl approach [8], therefore quantization in this sense comes from "graph"-type quantization. In large scale phenomena like galaxy redshift quantization one could then ask whether there is possibility of "super-graph" quantization.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

2 Interpreting quantized redshift from Hall effect. Cosmic String

In a recent paper, Moffat [1, p. 9] has used Gross-Pitaevskii in conjunction with his *phion condensate fluid* model to describe CMB spectrum data. Therefore we could expect that this equation will also yield interesting results in galaxies scale. See also [1b, 1c, 13] for other implications of low-energy phion fluid model.

Interestingly, it could be shown, that we could derive (approximately) Schrödinger wave equation from Gross-Pitaevskii equation. We consider the well-known Gross-Pitaevskii equation in the context of superfluidity or superconductivity [14]:

$$i\hbar\,\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + \big(V(x) - \gamma\,|\Psi|^{p-1}\big)\,\Psi, \qquad (1)$$

where $p < 2N/(N-2)$ if $N \geqslant 3$. In physical problems, the equation for $p = 3$ is known as Gross-Pitaevskii equation. This equation (1) has standing wave solution quite similar to solution of Schrödinger equation, in the form:

$$\Psi(x,t) = e^{-iEt/\hbar} \cdot u(x) \qquad (2)$$

Substituting equation (2) into equation (1) yields:

$$-\frac{\hbar^2}{2m}\Delta u + \big(V(x) - E\big)\,u = |u|^{p-1}\,u\,, \qquad (3)$$

which is nothing but a time-independent linear form of Schrödinger equation, except for term $|u|^{p-1}$ [14]. If the right-hand side of this equation is negligible, equation (3) reduces to standard Schrödinger equation.

Now it is worth noting here that from Nottale *et al.* we can derive a gravitational equivalent of Bohr radius from generalized Schrödinger equation [4]. Therefore we could also expect a slight deviation of this gravitational Bohr radius in we consider Gross-Pitaevskii equation instead of generalized Schrödinger equation.

According to Moffat, the phion condensate model implies a modification of Newtonian acceleration law to become [1, p. 11]:

$$a(r) = -\frac{G_\infty\,M}{r^2} + K\,\frac{\exp\,(-\mu_\phi r)}{r^2}\,(1 + \mu_\phi r)\,, \qquad (4)$$

where

$$G_\infty = G\left[1+\sqrt{\frac{M_0}{M}}\right]. \tag{5}$$

Therefore we can conclude that the use of phion condensate model implies a modification of Newton gravitational constant, G, to become (5). Plugging in this new equation (5) into a Nottale's gravitational Bohr radius equation [4] yields:

$$r_n \approx n^2\,\frac{GM}{v_0^2}\left[1+\sqrt{\frac{M_0}{M}}\right] \approx \chi\cdot n^2\,\frac{GM}{v_0^2}\,, \tag{6}$$

where n is integer (1,2,3 ...) and:

$$\chi = \left[1+\sqrt{\frac{M_0}{M}}\right]. \tag{7}$$

Therefore we conclude that — provided the higher order Yukawa term of equation (4) could be neglected — one has a *modified* gravitational Bohr-radius in the form of (6). It can be shown (elsewhere) that using similar argument one could expect to explain a puzzling phenomenon of *receding Moon* at a constant rate of $\pm 1.5''$ per year. And from this observed fact one could get an estimate of this χ factor. It is more interesting to note here, that a number of *coral reef* data also seems to support the same idea of modification factor in equation (5), but discussion of this subject deserves another paper.

A somewhat similar idea has been put forward by Masreliez [18] using the metric:

$$ds^2 = e^{\alpha\beta}\left[dx^2+dy^2+dz^2-(icdt)^2\right]. \tag{8}$$

Another alternative of this metric has been proposed by Socoloff and Starobinski [19] using multi-connected hypersurface metric:

$$ds^2 = dx^2 + e^{-2x}\left(dy^2+dz^2\right) \tag{9}$$

with boundaries: $e^{-x} = \Lambda$.

Therefore one can conclude that the use of phion condensate model has led us to a form of expanding metric, which has been discussed by a few authors.

Furthermore, it is well-known that Gross-Pitaevskii equation could exhibit *topologically* non-trivial vortex solutions [4, 5], which also corresponds to quantized vortices:

$$\oint p\cdot dr = N_v\,2\pi\hbar\,. \tag{10}$$

Therefore an implication of Gross-Pitaevskii equation [1] is that topologically quantized vortex could exhibit in astrophysical scale. In this context we submit the viewpoint that this proposition indeed has been observed in the form of Tifft's redshift quantization [2, 6]:

$$\delta r = \frac{c}{H}\,\delta z\,. \tag{11}$$

In other words, we submit the viewpoint that Tifft's observation of quantized redshift implies a quantized distance between galaxies [2, 5], which could be expressed in the form:

$$r_n = r_0 + n(\delta r)\,, \tag{12}$$

where n is integer (1,2,3, ...) similar to quantum number. Because it can be shown using standard definition of Hubble law that redshift quantization implies quantized distance between galaxies in the same cluster, then one could say that this equation of quantized distance (11) is a result of *topological* quantized vortices (9) in astrophysical scale [5]; and it agrees with Gross-Pitaevskii (quantum phion condensate) description of CMB spectrum [1]. It is perhaps more interesting if we note here, that from (11) then we also get an equivalent expression of (12):

$$\frac{c}{H}\,z_n = \frac{c}{H}\,z_0 + n\left(\frac{c}{H}\,\delta z\right) \tag{13}$$

or

$$z_n = z_0 + n(\delta z) \tag{14}$$

or

$$z_n = z_0\left[1+n\left(\frac{\delta z}{z_0}\right)\right]. \tag{15}$$

Nonetheless, there is a problem here, i. e. how to explain intrinsic redshift related to Tifft quantization as observed in Fundamental Plane clusters and also from various quasars data [6, 6a]:

$$z_{iQ} = z_f\left[N - 0.1 M_N\right] \tag{16}$$

where $z_f = 0.62$ is assumed to be a fundamental redshift constant, and $N(=1,2,3\ \ldots)$, and M is function of N [6a]. Meanwhile, it is interesting to note here similarity between equation (15) and (16). Here, the number M seems to play a rôle similar to second quantum number in quantum physics [7].

Now we will put forward an argument that intrinsic redshift quantization (16) could come from rotating quantum Hall effect [5a].

It is argued by Fischer [5a] that "Hall quantization is of necessity derivable from a *topological quantum number* related to this (quantum) coherence". He used total particle momentum [5a]:

$$p = mv + m\Omega\times r + qA\,. \tag{17}$$

The uniqueness condition of the collective phase represented in (9) then leads, if we take a path in the bulk of electron liquid, for which the integral of mv can be neglected, to the quantization of the sum of a *Sagnac flux*, and the magnetic flux [5a]:

$$\begin{aligned}\Phi = q\oint A\cdot dr + m\oint \Omega\times r\cdot dr &= \\ = \iint B\cdot dS &= N_v\,2\pi\hbar\,.\end{aligned} \tag{18}$$

This flux quantization rule corresponds to the fact that a *vortex* is fundamentally characterised by the winding number N alone [5a]. In this regard the vortex could take the form of cosmic string [22]. Now it is clear from (15) that quantized vortices could be formed by different source of flux.

After a few more reasonable assumptions one could obtain a generalised Faraday law, which in rotating frame will give in a non-dissipative Hall state the quantization of Hall conductivity [5a].

Therefore one could observe that it is quite natural to interpret the quantized distance between galaxies (11) as an implication of quantum Hall effect in rotating frame (15). While this proposition requires further observation, one could think of it in particular using known analogy between condensed matter physics and cosmology phenomena [10, 22]. If this proposition corresponds to the facts, then one could think that redshift quantization is an imprint of generalized quantization in various scales from microphysics to macrophysics, just as Tifft once put it [2]:

> "The redshift has imprinted on it a pattern that appears to have its origin in microscopic quantum physics, yet it carries this imprint across cosmological boundaries".

In the present paper, Tifft's remark represents natural implication of topological quantization, which could be formed at any scale [5]. We will explore further this proposition in the subsequent section, using Weyl quantization.

Furthermore, while this hypothesis is new, it could be expected that we can draw some new prediction, for instance, like possibility to observe small "blue-shift" effect generated by the Hall effect from antivortex-galaxies [23]. Of course, in order to observe such a "blue-shift" one shall first exclude other anomalous effects of redshift phenomena [6]. (For instance: one could argue that perhaps Pioneer spacecraft anomaly's blue-shifting of Doppler frequency may originate from the same effect as described herein.)

One could expect that further observation in particular in the area of low-energy neutrino will shed some light on this issue [20]. In this regard, one could view that the Sun is merely a remnant of a neutron star in the past, therefore it could be expected that it also emits neutrino similar to neutron star [21].

3 An alternative interpretation of astrophysical quantization from Weyl quantization. Graph and quantization

An alternative way to interpret the above proposition concerning topological quantum number and topological quantization [5a], is by using Weyl quantization.

In this regards, Castro [8, p. 5] has shown recently that one could derive Schrödinger equation from Weyl geometry using continuity equation:

$$\frac{\partial\rho}{\partial t} + \frac{1}{\sqrt{g}}\,\partial_i\big(\sqrt{g}\rho v^i\big) \tag{19}$$

and Weyl metric:

$$R_{\text{Weyl}} = (d-1)(d-2)\left(A_k A^k\right) - 2(d-1)\,\partial_k A^k\,. \tag{20}$$

Therefore one could expect to explain astrophysical quantization using Weyl method in lieu of using generalised Schrödinger equation as Nottale did [4]. To our knowledge this possibility has never been explored before elsewhere.

For instance, it can be shown that one can obtain Bohr-Sommerfeld type quantization rule from Weyl approach [24, p. 12], which for kinetic plus potential energy will take the form:

$$2\pi N\hbar = \sum_{j=0}^{\infty} \hbar^j S_j(E)\,, \tag{21}$$

which can be solved by expressing $E = \sum \hbar^k E_k$ as power series in $\hbar$ [24]. Now equation (10) could be rewritten as follows:

$$\oint p\cdot dr = N_v\, 2\pi\hbar = \sum_{j=0}^{\infty} \hbar^j S_j(E)\,. \tag{22}$$

Or if we consider quantum Hall effect, then equation (18) can be used instead of equation (10), which yields:

$$\begin{aligned}\Phi = q\oint A\cdot dr + m\oint \Omega\times r\cdot dr \;=\\ = \iint B\cdot dS = \sum_{j=0}^{\infty}\hbar^j S_j(E)\,.\end{aligned} \tag{23}$$

The above method is known as "graph kinematic" [25] or Weyl-Moyal's quantization [26]. We could also expect to find Hall effect quantization from this deformation quantization method.

Consider a harmonic oscillator, which equation can be expressed in the form of deformation quantization instead of Schrödinger equation [26]:

$$\left(\left(x + \frac{i\hbar}{2}\,\partial_p\right)^2 + \left(p - \frac{i\hbar}{2}\,\partial_x\right)^2 - 2E\right) f(x,p) = 0\,. \tag{24}$$

This equation could be separated to become two simple PDEs. For imaginary part one gets [26]:

$$\left(x\,\partial_p - p\,\partial_x\right) f = 0\,. \tag{25}$$

Now, considering Hall effect, one can introduce our definition of total particle momentum (17), therefore equation (25) may be written:

$$\big(x\,\partial_p - (mv + m\Omega\times r + qA)\,\partial_x\big) f = 0\,. \tag{26}$$

Our proposition here is that in the context of deformation quantization it is possible to find quantization solution of harmonic oscillator without Schrödinger equation. And because it corresponds to graph kinematic [25], generalized Bohr-Sommerfeld quantization rule for quantized vortices (22) in astrophysical scale could be viewed as signature of "*super-graph*"*quantization*.

This proposition, however, deserves further theoretical considerations. Further experiments are also recommended in order to verify and explore further this proposition.

Concluding remarks

In a recent paper, Moffat [1] has used Gross-Pitaevskii in his "phion condensate fluid" to describe CMB spectrum data. We extend this proposition to explain Tifft redshift quantization from the viewpoint of topological quantized vortices. In effect we consider that the intrinsic redshift quantization could be interpreted as result of Hall effect in rotating frame.

Another alternative to explain redshift quantization is to consider quantized vortices from the viewpoint of Weyl quantization (which could yield Bohr-Sommerfeld quantization).

It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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References

1. Moffat J. arXiv: astro-ph/0602607; [1a] Consoli M. arXiv: hep-ph/0109215; [1b] Consoli M. *et al.* arXiv: physics/0306094.
2. Russell Humphreys D. Our galaxy is the centre of the universe, "quantized" red shifts show. *TJ Archive*, 2002, v. 16(2), 95–104; http://answersingenesis.org/tj/v16/i2/galaxy.asp.
3. Smarandache F. and Christianto V. A note on geometric and information fusion interpretation of Bell theorem and quantum measurement. *Progress in Physics*, 2006, v. 4, 27–31.
4. Smarandache F. and Christianto V. Schrödinger equation and the quantization of celestial systems. *Progress in Physics*, 2006, v. 2, 63–67.
5. Fischer U. arXiv: cond-mat/9907457; [5a] arXiv: cond-mat/0004339.
6. Bell M. B. arXiv: astro-ph/0111123; [6a] arXiv: astro-ph/0305112; [6b] arXiv: astro-ph/0305060.
7. Setterfield B. http://www.journaloftheoretics.com; http://www.setterfield.org.
8. Castro C. and Mahecha J. On nonlinear Quantum Mechanics, Brownian motion, Weyl geometry, and Fisher information. *Progress in Physics*, 2006, v. 1, 38–45.
9. Schrieffer J. R. Macroscopic quantum phenomena from pairing in superconductors. Lecture, December 11, 1972.
10. Zurek W. H. Cosmological experiments in superfluids and superconductors. In: *Proc. Euroconference in Formation and Interaction of Topological Defects*, Plenum Press, 1995; arXiv: cond-mat/9502119.
11. Anandan J. S. In: *Quantum Coherence and Reality, Proc. Conf. Fundamental Aspects of Quantum Theory*, Columbia SC., edited by J. S. Anandan and J. L. Safko, World Scientific, 1994; arXiv: gr-qc/9504002.
12. Rauscher E. A. and Amoroso R. The physical implications of multidimensional geometries and measurement. *Intern. J. of Comp. Anticipatory Systems*, 2006.
13. Chiao R. *et al.* arXiv: physics/0309065.
14. Dinu T. L. arXiv: math.AP/0511184.
15. Kravchenko V. arXiv: math.AP/0408172.
16. Lipavsky P. *et al.* arxiv: cond-mat/0111214.
17. De Haas E. P. *Proc. of the Intern. Conf. PIRT-2005*, Moscow, MGTU Publ., 2005.
18. Masreliez J. *Apeiron*, 2005, v. 12.
19. Marc L.-R. and Luminet J.-P. arXiv: hep-th/9605010.
20. Lanou R. arXiv: hep-ex/9808033.
21. Yakovlev D. *et al.* arXiv: astro-ph/0012122.
22. Volovik G. arXiv: cond-mat/0507454.
23. Balents L. *et al.* arXiv: cond-mat/9903294.
24. Gracia-Saz A. *Ann. Inst. Fourier*, Grenoble, 2005, v. 55(5), 1001–1008.
25. Asribekov V. L. arXiv: physics/0110026.
26. Zachos C. arXiv: hep-th/0110114.

Numerical Solution of Time-Dependent Gravitational Schrödinger Equation

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In recent years, there are attempts to describe quantization of planetary distance based on time-independent gravitational Schrödinger equation, including Rubcic & Rubcic's method and also Nottale's Scale Relativity method. Nonetheless, there is no solution yet for time-dependent gravitational Schrödinger equation (TDGSE). In the present paper, a numerical solution of time-dependent gravitational Schrödinger equation is presented, apparently for the first time. These numerical solutions lead to gravitational Bohr-radius, as expected. In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat. Alternatively one can consider this condensate from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the past few years, there have been some hypotheses suggesting that quantization of planetary distance can be derived from a gravitational Schrödinger equation, such as Rubcic & Rubcic and also Nottale's scale relativity method [1, 3]. Interestingly, the gravitational Bohr radius derived from this gravitational Schrödinger equation yields prediction of new type of astronomical observation in recent years, i.e. extrasolar planets, with unprecedented precision [2].

Furthermore, as we discuss in preceding paper [4], using similar assumption based on gravitational Bohr radius, one could predict new planetoids in the outer orbits of Pluto which are apparently in good agreement with recent observational finding.. Therefore one could induce from this observation that the gravitational Schrödinger equation (and gravitational Bohr radius) deserves further consideration.

In the meantime, it is known that all present theories discussing gravitational Schrödinger equation only take its time-independent limit. Therefore it seems worth to find out the solution and implication of time-dependent gravitational Schrödinger equation (TDGSE). This is what we will discuss in the present paper.

First we will find out numerical solution of time-independent gravitational Schrödinger equation which shall yield gravitational Bohr radius as expected [1, 2, 3]. Then we extend our discussion to the problem of time-dependent gravitational Schrödinger equation.

In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat [5]. Alternatively one can consider this phion condensate model from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation. To our knowledge this proposition of coupled time-independent gravitational Schrödinger equation has never been considered before elsewhere.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

All numerical computation was performed using Maple. Please note that in all conditions considered here, we use only gravitational Schrödinger equation as described in Rubcic & Rubcic [3], therefore we neglect the scale relativistic effect for clarity.

2 Numerical solution of time-independent gravitational Schrödinger equation and time-dependent gravitational Schrödinger equation

First we write down the time-independent gravitational Schrödinger radial wave equation in accordance with Rubcic & Rubcic [3]:

$$\begin{aligned} \frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2 E'}{H^2}R + \\ + \frac{2}{r}\frac{4\pi^2 GMm^2}{H^2}R - \frac{\ell(\ell+1)}{r^2}R = 0\,. \end{aligned} \tag{1}$$

When H, V, E' represents gravitational Planck constant, Newtonian potential, and the energy per unit mass of the orbiting body, respectively, and [3]:

$$H = h\left(2\pi f \frac{Mm_n}{m_0^2}\right), \tag{2}$$

$$V(r) = -\frac{GMm}{r}, \tag{3}$$

$$E' = \frac{E}{m}. \tag{4}$$

By assuming that R takes the form:

$$R = e^{-\alpha r} \tag{5}$$

and substituting it into equation (1), and using simplified terms only of equation (1), one gets:

$$\Psi = \alpha^2 e^{-\alpha r} - \frac{2\alpha e^{-\alpha r}}{r} + \frac{8\pi GMm^2 e^{-\alpha r}}{r H^2}. \tag{6}$$

After factoring this equation (7) and solving it by equating the factor with zero, yields:

$$RR = -\frac{2\left(4\pi GMm^2 - H^2\alpha\right)}{\alpha^2 H^2} = 0, \tag{7}$$

or

$$RR = 4\pi GMm^2 - H^2\alpha = 0, \tag{8}$$

and solving for α, one gets:

$$a = \frac{4\pi^2 GMm^2}{H^2}. \tag{9}$$

Gravitational Bohr radius is defined as inverse of this solution of α, then one finds (in accordance with Rubcic & Rubcic [3]):

$$r_1 = \frac{H^2}{4\pi^2 GMm^2}, \tag{10}$$

and by substituting back equation (2) into (10), one gets [3]:

$$r_1 = \left(\frac{2\pi f}{\alpha c}\right)^2 GM. \tag{11}$$

Equation (11) can be rewritten as follows:

$$r_1 = \frac{GM}{\nu_0^2}, \tag{11a}$$

where the "specific velocity" for the system in question can be defined as:

$$\nu_0 = \left(\frac{2\pi f}{\alpha c}\right)^{-1} = \alpha_g c. \tag{11b}$$

The equations (11a)-(11b) are equivalent with Nottale's result [1, 2], especially when we introduce the quantization number: $r_n = r_1 n^2$ [3]. For complete Maple session of these all steps, see Appendix 1. Furthermore, equation (11a) may be generalised further to include multiple nuclei, by rewriting it to become: $r_1 = (GM)/v^2 \Rightarrow r_1 = (G\,\Sigma M)/v^2$, where ΣM represents the sum of central masses.

Solution of time-dependent gravitational Schrödinger equation is more or less similar with the above steps, except that we shall take into consideration the right hand side of Schrödinger equation and also assuming time dependent form of r:

$$R = e^{-\alpha r(t)}. \tag{12}$$

Therefore the gravitational Schrödinger equation now reads:

$$\begin{aligned} &\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2 E'}{H^2}R + \\ &+\frac{2}{r}\frac{4\pi^2 GMm^2}{H^2}R - \frac{\ell(\ell+1)}{r^2}R = H\frac{dR}{dt}, \end{aligned} \tag{13}$$

or by using Leibniz chain rule, we can rewrite equation (15) as:

$$\begin{aligned} -H\frac{dR}{dr(t)}\frac{dr(t)}{dt} + \frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2 E'}{H^2}R + \\ + \frac{2}{r}\frac{4\pi^2 GMm^2}{H^2}R - \frac{\ell(\ell+1)}{r^2}R = 0. \end{aligned} \tag{14}$$

The remaining steps are similar with the aforementioned procedures for time-independent case, except that now one gets an additional term for RR:

$$\begin{aligned} RR' = H^3\alpha\left(\frac{d}{dt}r(t)\right)r(t) - \alpha^2 r(t)H^2 + \\ + 8\pi GMm^2 - 2H^2\alpha = 0. \end{aligned} \tag{15}$$

At this point one shall assign a value for $\frac{d}{dt}r(t)$ term, because otherwise the equation cannot be solved. We choose $\frac{d}{dt}r(t) = 1$ for simplicity, then equation (15) can be rewritten as follows:

$$RR' := \frac{rH^3\alpha}{2} + \frac{rH^2\alpha^2}{2} + 4\pi^2 GMm^2 - H^2\alpha = 0. \tag{16}$$

The roots of this equation (16) can be found as follows:

$$\begin{aligned} a1 &:= \frac{-r^2H + 2H + \sqrt{r^4H^4 - 4H^3r + 4H^2 - 32rGMm^2\pi^2}}{2rH}, \\ a2 &:= \frac{-r^2H + 2H - \sqrt{r^4H^4 - 4H^3r + 4H^2 - 32rGMm^2\pi^2}}{2rH}. \end{aligned} \tag{17}$$

Therefore one can conclude that there is time-dependent modification factor to conventional gravitational Bohr radius (10). For complete Maple session of these steps, see Appendix 2.

3 Gross-Pitaevskii effect. Bogoliubov-deGennes approximation and coupled time-independent gravitational Schrödinger equation

At this point it seems worthwhile to take into consideration a proposition by Moffat, regarding modification of Newtonian acceleration law due to phion condensate medium, to include Yukawa type potential [5, 6]:

$$a(r) = -\frac{G_\infty M}{r^2} + K\frac{\exp(-\mu_\phi r)}{r^2}(1 + \mu_\phi r). \tag{18}$$

Therefore equation (1) can be rewritten to become:

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{8\pi m^2E'}{H^2}R+ \\ +\frac{2}{r}\frac{4\pi^2\big(GM-K\exp(-\mu_\phi r)(1+\mu_\phi r)\big)m^2}{H^2}R- \\ -\frac{\ell(\ell+1)}{r^2}R=0\,, \tag{19}$$

or by assuming $\mu = 2\mu_0 = \mu_0 r$ for the exponential term, equation (19) can be rewritten as:

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{8\pi m^2E'}{H^2}R+ \\ +\frac{2}{r}\frac{4\pi^2\big(GM-Ke^{-2\mu_0}(1+\mu_0 r)\big)m^2}{H^2}R-\frac{\ell(\ell+1)}{r^2}R=0\,. \tag{20}$$

Then instead of equation (8), one gets:

$$RR''=8\pi GMm^2-2H^2\alpha-8\pi^2m^2Ke^{-\mu_0}(1+\mu)=0\,. \tag{21}$$

Solving this equation will yield a modified gravitational Bohr radius which includes Yukawa effect:

$$r_1=\frac{H^2}{4\pi^2(GM-Ke^{-2\mu_0})m^2} \tag{22}$$

and the modification factor can be expressed as ratio between equation (22) and (10):

$$\chi=\frac{GM}{(GM-Ke^{-2\mu_0})}\,. \tag{23}$$

(For complete Maple session of these steps, see Appendix 3.)

A careful reader may note that this "Yukawa potential effect" as shown in equation (20) could be used to explain the small discrepancy (around $\pm 8\%$) between the "observed distance" and the computed distance based on gravitational Bohr radius [4, 6a]. Nonetheless, in our opinion such an interpretation remains an open question, therefore it may be worth to explore further.

There is, however, an alternative way to consider phion condensate medium i.e. by introducing coupled Schrödinger equation, which is known as Bogoliubov-deGennes theory [7]. This method can be interpreted also as generalisation of assumption by Rubcic-Rubcic [3] of subquantum structure composed of positive-negative Planck mass. Therefore, taking this proposition seriously, then one comes to hypothesis that there shall be coupled Newtonian potential, instead of only equation (3).

To simplify Bogoliubov-deGennes equation, we neglect the time-dependent case, therefore the wave equation can be written in matrix form [7, p.4]:

$$[A]\,[\Psi]=0\,, \tag{24}$$

where $[A]$ is $2{\times}2$ matrix and $[\Psi]$ is $2{\times}1$ matrix, respectively, which can be represented as follows (using similar notation with equation 1):

$$[A]=\begin{pmatrix} \frac{8\pi GMm^2e^{-\alpha r}}{rH^2} & \alpha^2e^{-\alpha r}-\frac{2\alpha e^{-\alpha r}}{r} \\ \alpha^2e^{-\alpha r}-\frac{2\alpha e^{-\alpha r}}{r} & -\frac{8\pi GMm^2e^{-\alpha r}}{rH^2} \end{pmatrix} \tag{25}$$

and

$$[\Psi]=\begin{pmatrix} f(r) \\ g(r) \end{pmatrix}. \tag{26}$$

Numerical solution of this matrix differential equation can be found in the same way with the previous methods, however we leave this problem as an exercise for the readers.

It is clear here, however, that Bogoliubov-deGennes approximation of gravitational Schrödinger equation, taking into consideration phion condensate medium will yield nonlinear effect, because it requires solution of matrix differential equation* (21) rather than standard ODE in conventional Schrödinger equation (or time-dependent PDE in 3D-condition). This perhaps may explain complicated structures beyond Jovian Planets, such as Kuiper Belt, inner and outer Oort Cloud etc. which of course these structures cannot be predicted by simple gravitational Schrödinger equation. In turn, from the solution of (21) one could expect that there are numerous undiscovered celestial objects in the Oort Cloud.

Further observation is also recommended in order to verify and explore further this proposition.

4 Concluding remarks

In the present paper, a numerical solution of time-dependent gravitational Schrödinger equation is presented, apparently for the first time. This numerical solution leads to gravitational Bohr-radius, as expected.

In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat. Alternatively one can consider this condensate from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation.

It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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*For recent articles discussing analytical solution of matrix differential equations, the reader is referred to Electronic Journal of Differential Equations (free access online on many mirrors as http://ejde.math.txstate.edu, http://ejde.math.unt.edu, http://www.emis.de/journals/EJDE etc.).

References

1. Nottale L. *et al. Astron. & Astrophys.*, 1997, v. 322, 1018.
2. Nottale L., Schumacher G. and Levefre E. T. *Astron. & Astrophys.*, 2000, v. 361, 379–389; accessed online on http://daec.obspm.fr/users/nottale.
3. Rubcic A. and Rubcic J. The quantization of solar like gravitational systems. *Fizika*, B-7, 1998, v. 1, No. 1–13.
4. Smarandache F. and Christianto V. *Progress in Physics*, 2006, v. 2, 63–67.
5. Moffat J. arXiv: astro-ph/0602607.
6. Smarandache F. and Christianto V. *Progress in Physics*, 2006, v. 4, 37–40; [6a] Christianto V. *EJTP*, 2006, v. 3, No. 12, 117–144; accessed onle on http://www.ejtp.com.
7. Lundin N. I. Microwave induced enhancement of the Josephson DC. *Chalmers University of Technology & Gotterborg University Report*, p. 4–7.
8. Griffin M. arXiv: cond-mat/9911419.
9. Tuszynski J. *et al. Physica A*, 2003, v. 325, 455–476; accessed onle on http://sciencedirect.com.
10. Toussaint M. arXiv: cs.SC/0105033.
11. Fischer U. arXiv: cond-mat/9907457; [8a] arXiv: cond-mat/0004339.
12. Zurek W. (ed.) In: *Proc. Euroconference in Formation and Interaction of Topological Defects*, Plenum Press, 1995; accessed online: arXiv: cond-mat/9502119.
13. Volovik G. arXiv: cond-mat/0507454.

Appendix 1 Time-independent gravitational Schrödinger equation

\> restart;
\> with (linalg);
\> R: = exp (−(alpha*r));

$$R := e^{-\alpha\, r}$$

\> D1R:=diff (R,r); D2R:=diff (D1R,r);

$$D1R := -\alpha\, e^{-\alpha\, r}$$

$$D2R := -\alpha^2\, e^{-\alpha\, r}$$

\> SCHEQ1:=D2R+D1R*2/r+8*pi^2*m*E*R/h^2+8*pi^2*G*M*m^2*R/(r*h^2)−l*(l+1)*R/r^2=0;
\> XX1:=factor (SCHEQ1);
\> #Using simplified terms only from equation (A*8, of Rubcic & Rubcic, 1998)
\> ODESCHEQ:=D2R+D1R*2/r+8*pi^2*G*M*m^2*R/(r*h^2)=0;

$$ODESCHEQ := \alpha^2\, e^{-\alpha\, r} - \frac{2\alpha\, e^{-\alpha\, r}}{r} + \frac{8\pi^2 G M m^2 e^{-\alpha\, r}}{r H^2} = 0$$

\> XX2:=factor (SCHEQ2);

$$XX2 := \frac{e^{-\alpha\, r}\left(\alpha^2 r H^2 - 2H^2\alpha + 8\pi^2 G M m^2\right)}{r H^2} = 0$$

\> RR:= solve (XX2, r);

$$RR := -\frac{2(4\pi^2 G M m^2 - H^2\alpha)}{\alpha^2 H^2}$$

\> #Then solving for RR=0, yields:

\> SCHEQ3:= 4*pi^2*G*M*m^2−h^2*alpha=0;

$$SCHEQ3 := 4\pi^2 G M m^2 - H^2\alpha = 0$$

\> a:= solve (SCHEQ3, alpha);

$$a := \frac{4\pi^2 G M m^2}{H^2}$$

\> #Gravitational Bohr radius is defined as inverse of alpha:
\> gravBohrradius:=1/a;

$$r_{gravBohr} := \frac{H^2}{4\pi^2 G M m^2}$$

Appendix 2 Time-dependent gravitational Schrödinger equation

\> #Solution of gravitational Schrodinger equation (Rubcic, Fizika 1998);
\> restart;
\> #with time evolution (Hagendorn's paper);
\> S:=r(t); R:=exp(−(alpha*S)); R1:=exp(−(alpha*r));

$$S := r(t)$$

$$R := e^{-\alpha\, r}$$

\> D4R:=diff(S,t); D1R:=−alpha*exp(−(alpha*S)); D2R:=−alpha^2*exp(−(alpha*S)); D5R:=D1R*D4R;

$$D4R := \frac{d}{dt} r(t)$$

$$D1R := -\alpha\, e^{-\alpha\, r(t)}$$

$$D2R := -\alpha^2\, e^{-\alpha\, r(t)}$$

$$D1R := -\alpha\, e^{-\alpha\, r(t)}\, \frac{d}{dt} r(t)$$

\> #Using simplified terms only from equation (A*8)
\> SCHEQ3:=−h*D5R+D2R+D1R*2/S+8*pi^2*G*M*m^2*R/(S*h^2);
\> XX2:=factor(SCHEQ3);

$$XX2 := \frac{e^{-\alpha\, r(t)}\left(H^3\alpha \frac{dr(t)}{dt} r(t) - \alpha^2 r(t) H^2 - 2H^2\alpha + 8\pi^2 G M m^2\right)}{r(t) H^2} = 0$$

\> #From standard solution of gravitational Schrodinger equation, we know (Rubcic, Fizika 1998):
\> SCHEQ4:=4*pi^2*G*M*m^2−h^2*alpha;

$$SCHEQ4 := 4\pi^2 G M m^2 - H^2\alpha$$

\> #Therefore time-dependent solution of Schrodinger equation may introduce new term to this gravitational Bohr radius.
\> SCHEQ5:=(XX2*(S*h^2)/(exp(−(alpha*S))))−2*SCHEQ4;

$$ODESCHEQ5 := H^3\alpha \frac{dr(t)}{dt} r(t) - \alpha^2 r(t) H^2$$

\> #Then we shall assume for simplicity by assigning value to d[r(t)]/dt:
\> D4R:=1;
\> Therefore SCHEQ5 can be rewritten as:
\> SCHEQ5:= H^3*alpha*r/2+alpha^2*r*H^2/2−4*pi^2*G*M*m^2−H^2*alpha=0;

$$SCHEQ5 := \frac{r H^3\alpha}{2} + \frac{r H^2\alpha^2}{2} + 4\pi^2 G M m^2 - H^2\alpha = 0$$

\> Then we can solve again SCHEQ5 similar to solution of SCHEQ4:
\> a1:=solve(SCHEQ5,alpha);

$$a1 := \frac{-r^2 H + 2H + \sqrt{r^4 H^4 - 4H^3 r + 4H^2 - 32 r G M m^2 \pi^2}}{2rH}$$

$$a2 := \frac{-r^2 H + 2H - \sqrt{r^4 H^4 - 4H^3 r + 4H^2 - 32 r G M m^2 \pi^2}}{2rH}$$

\> #Therefore one could expect that there is time-dependent change of gravitational Bohr radius.

Appendix 3 Time-independent gravitational Schrödinger equation with Yukawa potential [5]

> #Extension of gravitational Schrodinger equation (Rubcic, Fizika 1998);
> restart;
> #departure from Newton potential;
> R:=exp(−(alpha*r));

$$R := e^{-\alpha r}$$

> D1R:=diff(R,r); D2R:=diff(D1R,r);

$$D1R := -\alpha e^{-\alpha r}$$

$$D2R := -\alpha^2 e^{-\alpha r}$$

> SCHEQ2:=D2R+D1R*2/r+8*piˆ2*(G*M−K*exp(−2*mu)*(1+mu*r))*mˆ2*R/(r*hˆ2)=0;

$$ODESCHEQ := \alpha^2 e^{-\alpha r} - \frac{2\alpha e^{-\alpha r}}{r} + + \frac{8\pi^2(GM - Ke^{-2\mu}(1+\mu r))m^2 e^{-\alpha r}}{rH^2} = 0$$

> XX2:=factor(SCHEQ2);
> RR1:=solve(XX2,r);

$$RR1 := -\frac{2(-H^2\alpha + 4\pi^2 GMm^2 - 4\pi^2 m^2 Ke^{-2\mu})}{-\alpha^2 H^2 + 8\pi^2 m^2 Ke^{-2\mu}}$$

> #from standard gravitational Schrodinger equation we know:
> SCHEQ3:=4*piˆ2*G*M*mˆ2−hˆ2*alpha=0;
> a:=solve(SCHEQ3, alpha);
> #Gravitational Bohr radius is defined as inverse of alpha:
> gravBohrradius:=1/a;

$$r_{gravBohr} := \frac{H^2}{4\pi^2 GMm^2}$$

> #Therefore we conclude that the new terms of RR shall yield new terms (YY) into this gravitational Bohr radius:
> PI:=(RR*(alphaˆ2*hˆ2)−(−8*piˆ2*G*M*mˆ2+2*hˆ2*alpha));
> #This new term induced by pion condensation via Gross-Pitaevskii equation may be observed in the form of long-range potential effect. (see Moffat J., arXiv: astro-ph/0602607, 2006; also Smarandache F. and Christianto V. Progress in Physics, v. 2, 2006, & v. 1, 2007, www.ptep-online.com)
> #We can also solve directly:
> SCHEQ5:=RR*(alphaˆ2*hˆ2)/2;

$$SCHEQ5 := \frac{\alpha^2 H^2(-H^2\alpha + 4\pi^2 GMm^2 - 4\pi^2 m^2 Ke^{-2\mu})}{-\alpha^2 H^2 + 8\pi^2 m^2 Ke^{-2\mu}}$$

> a1:=solve(SCHEQ5, alpha);

$$a1 := 0, 0, \frac{4\pi^2 m^2(GM - Ke^{-2\mu})}{H^2}$$

> #Then one finds modified gravitational Bohr radius in the form:
> modifgravBohrradius:=1/(4*piˆ2*(G*M−K*exp(−2*mu))*mˆ2/hˆ2);

$$r_{modified.gravBohr} := \frac{H^2}{4\pi^2 m^2(GM - Ke^{-2\mu})}$$

> #This modification can be expressed in chi-factor:
> chi:=modifgravBohrradius/gravBohrradius;

$$\chi := \frac{GM}{GM - Ke^{-2\mu}}$$

Kaluza-Klein-Carmeli Metric from Quaternion-Clifford Space, Lorentz' Force, and Some Observables

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It was known for quite long time that a quaternion space can be generalized to a Clifford space, and vice versa; but how to find its neat link with more convenient metric form in the General Relativity theory, has not been explored extensively. We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there obtains Carmeli metric, which can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric). Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4, 5, 6]. In subsequent section we explain Podkletnov's rotating disc experiment. We also note possible implications to quantum gravity. Further observations are of course recommended in order to refute or verify this proposition.

1 Introduction

It was known for quite long time that a quaternion space can be generalized to a Clifford space, and *vice versa*; but how to find its neat link to more convenient metric form in the General Relativity theory, has not been explored extensively [2].

First it is worth to remark here that it is possible to find a flat space representation of quaternion group, using its algebraic isomorphism with the ring division algebra [3, p.3]:

$$E_i E_j = -\delta_{ij} + f_{ijk} E_k \,. \tag{1}$$

Working for R^{dim}, we get the following metric [3]:

$$ds^2 = dx_\mu dx^\mu , \tag{2}$$

imposing the condition:

$$x_\mu x^\mu = R^2 . \tag{3}$$

This rather elementary definition is noted here because it was based on the choice to use the square of the radius to represent the distance (x_μ), meanwhile as Riemann argued long-time ago it can also been represented otherwise as the square of the square of the radius [3a].

Starting with the complex $n = 1$, then we get [3]:

$$q = x_0 + x_1 E_1 + x_2 E_2 + x_3 E_3 \,. \tag{4}$$

With this special choice of x_μ we can introduce the special metric [3]:

$$ds^2 = R^2 (\delta_{ij} \partial \Phi_i \partial \Phi_j) . \tag{5}$$

This is apparently most direct link to describe a flat metric from the ring division algebra. In the meantime, it seems very interesting to note that Trifonov has shown that the geometry of the group of nonzero quaternions belongs to closed FLRW metric. [1] As we will show in the subsequent Section, this approach is more rigorous than (5) in order to describe neat link between quaternion space and FLRW metric.

We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there we argue that one can obtain Carmeli 5D metric [4] from this group with non-zero quaternions. The resulting metric can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric).

Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4–7]. Possible implications to the Earth geochronometrics and possible link to coral growth data are discussed. In the subsequent Section we explain Podkletnov's rotating disc experiment. We also note a possible near link between Kaluza-Klein-Carmeli and Yefremov's Q-Relativity, and also possible implications to quantum gravity.

The reasons to consider this Carmeli metric instead of the conventional FLRW are as follows:

- One of the most remarkable discovery from WMAP is that it reveals that our Universe seems to obey Euclidean metric (see Carroll's article in *Nature*, 2003);
- In this regards, to explain this observed fact, most arguments (based on General Relativity) seem to agree that in the edge of Universe, the metric will follow Euclidean, because the matter density tends to approaching zero. But such a proposition is of course in contradiction with the basic "assumption" in GTR itself, i.e. that the Universe is *homogenous isotropic* everywhere, meaning that the matter density should be the same too in the edge of the universe. In other words, we need a new metric to describe the *inhomogeneous isotropic* spacetime.

$$g_{\alpha\beta} = \begin{pmatrix} \tau(\eta)(\frac{\dot{R}}{R})^2 & 0 & 0 & 0 \\ 0 & -\tau(\eta) & 0 & 0 \\ 0 & 0 & -\tau(\eta)\sin^2(\chi) & 0 \\ 0 & 0 & 0 & -\tau(\eta)\sin^2(\chi)\sin^2(\vartheta) \end{pmatrix}. \tag{6}$$

- Furthermore, from astrophysics one knows that spiral galaxies do not follow Newtonian potential exactly. Some people have invoked MOND or modified (Post-) Newton potential to describe that deviation from Newtonian potential [8, 9]. Carmeli metric is another possible choice [4], and it agrees with spiral galaxies, and also with the redshift data [5–7].
- Meanwhile it is known, that General Relativity is strictly related to Newtonian potential (Poisson's equation). All of this seems to indicate that General Relativity is only applicable for some limited conditions, but it may not be able to represent the *rotational aspects of gravitational phenomena*. Of course, there were already extensive research in this area of the generalized gravitation theory, for instance by introducing a torsion term, which vanishes in GTR [10].

Therefore, in order to explain spiral galaxies' rotation curve and corresponding "dark matter", one can come up with a different route instead of invoking a kind of strange matter. In this regards, one can consider dark matter as a property of the metric of the spacetime, just like the precession of the first planet is a property of the spacetime in General Relativity.

Of course, there are other methods to describe the *inhomogeneous* spacetime, see [15, 16], for instance in [16] a new differential operator was introduced: $\frac{\delta}{\delta\tau} = \frac{1}{H_o}\frac{1}{c}\frac{\delta}{\delta t}$, which seems at first glance as quite similar to Carmeli method. But to our present knowledge Carmeli metric is the most consistent metric corresponding to generalized FLRW (derived from a quaternion group).

Further observations are of course recommended in order to refute or verify this proposition.

2 FLRW metric associated to the group of non-zero quaternions

The quaternion algebra is one of the most important and well-studied objects in mathematics and physics; and it has natural Hermitian form which induces Euclidean metric [1]. Meanwhile, Hermitian symmetry has been considered as a method to generalize the gravitation theory (GTR), see Einstein paper in *Ann. Math.* (1945).

In this regards, Trifonov has obtained that a natural extension of the structure tensors using nonzero quaternion bases will yield formula (6). (See [1, p.4].)

Interestingly, by *assuming* that [1]:

$$\tau(\eta)\left(\frac{\dot{R}}{R}\right)^2 = 1\,, \tag{7}$$

then equation (6) reduces to closed FLRW metric [1, p.5]. Therefore one can say that closed FLRW metric is neatly associated to the group of nonzero quaternions.

Now consider equation (7), which can be rewritten as:

$$\tau(\eta)(\dot{R})^2 = R^2. \tag{8}$$

Since we choose (8), then the radial distance can be expressed as:

$$dR^2 = dz^2 + dy^2 + dx^2. \tag{9}$$

Therefore we can rewrite equation (8) in terms of (9):

$$\tau(\eta)(d\dot{R})^2 = (dR)^2 = dz^2 + dy^2 + dx^2, \tag{10}$$

and by defining

$$\tau(\eta) = \tau^2 = \frac{1}{H_0^2(\eta)} = \frac{1}{\alpha(H_0^2)^n}\,. \tag{11}$$

Then we can rewrite equation (10) in the form:

$$\tau(\eta)(d\dot{R})^2 = \tau^2(dv)^2 = dz^2 + dy^2 + dx^2, \tag{12}$$

or

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 = 0\,, \tag{13}$$

which is nothing but an original Carmeli metric [4, p.3, equation (4)] and [6, p.1], where H_0 represents Hubble constant (by setting $\alpha = n = 1$, while in [12] it is supposed that $\alpha = 1.2$, $n = 1$). Further extension is obviously possible, where equation (13) can be generalized to include the $(i\,cdt)$ component in the conventional Minkowski metric, to become (Kaluza-Klein)-Carmeli 5D metric [5, p.1]:

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 + (icdt)^2 = 0\,. \tag{14}$$

Or if we introduce equation (13) in the general relativistic setting [4, 6], then one obtains:

$$ds^2 = \tau^2(dv)^2 - e^{\xi}\cdot dr^2 - R^2\cdot(d\vartheta^2 + \sin^2\vartheta\cdot d\phi^2). \tag{15}$$

The solution for (15) is given by [6, p.3]:

$$\frac{dr}{dv} = \tau\cdot\exp\left(-\frac{\xi}{2}\right)\,, \tag{16}$$

which can be written as:

$$\frac{d\dot{r}}{dr} = \frac{dv}{dr} = \tau^{-1}\cdot\exp\left(\frac{\xi}{2}\right). \tag{17}$$

This result implies that there shall be a metric deformation, which may be associated with astrophysics observation, such as the possible AU differences [11, 12].

Furthermore, this proposition seems to correspond neatly to the Expanding Earth hypothesis, because [13]:

"In order for expansion to occur, the moment of inertia constraints must be overcome. An expanding Earth would necessarily rotate more slowly than a smaller diameter planet so that angular momentum would be conserved." (Q.1)

We will discuss these effects in the subsequent Sections.

We note however, that in the original Carmeli metric, equation (14) can be generalized to include the potentials to be determined, to become [5, p.1]:

$$ds^2 = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \left(1 + \frac{\Phi}{c^2}\right) c^2 dt^2, \quad (18)$$

where

$$dr^2 = dz^2 + dy^2 + dx^2. \quad (19)$$

The line element represents a spherically symmetric *inhomogeneous* isotropic universe, and the expansion is a result of the spacevelocity component. In this regards, metric (18) describes *funfbein* ("five-legs") similar to the standard Kaluza-Klein metric, for this reason we propose the name Kaluza-Klein-Carmeli for all possible metrics which can be derived or extended from equations (8) and (10).

To observe the expansion at a definite time, the $(i\,cdt)$ term in equation (14) has been ignored; therefore the metric becomes "*phase-space*" Minkowskian. [5, p.1]. (A similar phase-space Minkowskian has been considered in various places, see for instance [16] and [19].) Therefore the metric in (18) reduces to (by taking into consideration the isotropic condition):

$$dr^2 + \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 = 0\,. \quad (20)$$

Alternatively, one can suppose that in reality this assumption may be reasonable by setting $c \to 0$, such as by considering the metric for the phonon speed c_s instead of the light speed c; see Volovik, etc. Therefore (18) can be rewritten as:

$$\begin{aligned} ds^2_{phonon} = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \\ + \left(1 + \frac{\Phi}{c_s^2}\right) c_s^2\, dt^2. \end{aligned} \quad (21)$$

To summarize, in this Section we find out that not only closed FLRW metric is associated to the group of nonzero quaternions [1], but also the same group yields Carmeli metric. In the following Section we discuss some plausible implications of this proposition.

3 Observable A: the Earth geochronometry

One straightforward implication derived from equation (8) is that the ratio between the velocity and the radius is directly proportional, regardless of the scale of the system in question:

$$\left(\frac{\dot{R}}{R}\right)^2 = \tau(\eta)^{-1}\,, \quad (22)$$

or

$$\left(\frac{R_1}{\dot{R}_1}\right) = \left(\frac{R_2}{\dot{R}_2}\right) = \sqrt{\tau(\eta)}\,. \quad (23)$$

Therefore, one can say that there is a direct proportionality between the *spacevelocity* expansion of, let say, Virgo galaxy and the Earth geochronometry. Table 1 displays the calculation of the Earth's radial expansion using the formula represented above [17]:

Therefore, the Earth's radius increases at the order of ~ 0.166 cm/year, which may correspond to the decreasing angular velocity (Q.1). This number, albeit very minute, may also correspond to the Continental Drift hypothesis of A. Wegener [13, 17]. Nonetheless the reader may note that our calculation was based on Kaluza-Klein-Carmeli's phase-space *spacevelocity* metric.

Interestingly, there is a quite extensive literature suggesting that our Earth experiences a continuous deceleration rate. For instance, J. Wells [14] described a increasing day-length of the Earth [14]:

"It thus appears that the length of the day has been increasing throughout geological time and that the number of days in the year has been decreasing. At the beginning of the Cambrian the length of the day would have been 21^{h}." (Q.2)

Similar remarks have been made, for instance by G. Smoot [13]:

"In order for this to happen, the lunar tides would have to slow down, which would affect the length of the lunar month. ... an Earth year of 447 days at 1.9 Ga decreasing to an Earth year of 383 days at 290 Ma to 365 days at this time. However, the Devonian coral rings show that the *day is increasing by 24 seconds every million years,* which would allow for an expansion rate of about 0.5% for the past 4.5 Ga, all other factors being equal." (Q.3)

Therefore, one may compare this result (Table 1) with the increasing day-length reported by J. Wells [13].

4 Observable B: the Receding Moon from the Earth

It is known that the Moon is receding from the Earth at a constant rate of $\sim$4cm/year [17, 18].

Using known values: $G = 6.6724 \times 10^{-8}$ cm^2/(g $\cdot$ sec^2) and $\rho = 5.5 \times 10^6$ g/m^3, and the Moon's velocity $\sim$7.9 km/sec, then one can calculate using known formulas:

$$\mathrm{Vol} = \frac{4}{3}\,\pi \cdot (R + \Delta R)^3, \quad (24)$$

$$M + \Delta M = \mathrm{Vol} \cdot \rho\,, \quad (25)$$

$$r + \Delta r = \frac{G \cdot (M + \Delta M)}{v^2}\,, \quad (26)$$

where r, v, M each represents the distance from the Moon to the Earth, the Moon's orbital velocity, and the Earth's mass,

Nebula	Radial velocity (mile/s)	Distance (10^3 kly)	Ratio (10^{-5} cm/yr)	the Earth dist. (R, km)	Predicted the Earth exp. (ΔR, cm/year)
Virgo	750	39	2.617	6371	0.16678
Ursa Mayor	9300	485	2.610	6371	0.166299
Hydra	38000	2000	2.586	6371	0.164779
Bootes 2	86000	4500	2.601	6371	0.165742
Average			2.604		0.1659

Table 1: Calculation of the radial expansion from the Galaxy velocity/distance ratio. Source: [17].

respectively. Using this formula we obtain a prediction of the Receding Moon at the rate of 0.00497 m/year. This value is around 10% compared to the observed value 4 cm/year.

Therefore one can say that this calculation shall take into consideration other aspects. While perhaps we can use other reasoning to explain this discrepancy between calculation and prediction, for instance using the "*conformal brane*" method by Pervushin [20], to our best knowledge this effect has neat link with the known paradox in astrophysics, i.e. the observed matter only contributes around $\sim$1–10% of all matter that is supposed to be "there" in the Universe.

An alternative way to explain this discrepancy is that there is another type of force different from the known Newtonian potential, i.e. by taking into consideration the expansion of the "surrounding medium" too. Such a hypothesis was proposed recently in [21]. But we will use here a simple argument long-time ago discussed in [22], i.e. if there is a force other than the gravitational force acting on a body with mass, then it can be determined by this equation [22, p.1054]:

$$\frac{d(mv_0)}{dt} = F + F_{gr}, \tag{27}$$

where v_0 is the velocity of the particle relative to the absolute space [22a]. The gravitational force can be defined as before:

$$F_{gr} = -\, m\, \nabla V\,, \tag{28}$$

where the function V is solution of Poisson's equation:

$$\nabla^2 V = 4\pi K \mu\,, \tag{29}$$

and K represents Newtonian gravitational constant. For system which does not obey Poisson's equation, see [15].

It can be shown, that the apparent gravitational force that is produced by an aether flow is [22]:

$$F_{gr} = m\,\frac{\partial v}{\partial t} + m\,\nabla\left(\frac{v^2}{2}\right) - mv_0 \times \nabla \times v + v\,\frac{dm}{dt}\,, \tag{30}$$

which is an extended form of Newton law:

$$\vec{F} = \frac{d}{dt}\left(\vec{m}\vec{v}\right) = m\left(\frac{d\vec{v}}{dt}\right) + v\left(\frac{d\vec{m}}{dt}\right). \tag{31}$$

If the surrounding medium be equivalent to Newton's theory, this expression shall reduce to that given in (27). Supposing the aether be irrotational relative to the particular system of the coordinates, and $m = \text{const}$, then (29) reduces [22]:

$$F_{gr} = -m\left(-\frac{\partial v}{\partial t} - \nabla\left(\frac{v^2}{2}\right)\right), \tag{32}$$

which will be equivalent to equation (27) only if:

$$\nabla V = \frac{\partial v}{\partial t} + \nabla\left(\frac{v^2}{2}\right). \tag{33}$$

Further analysis of this effect to describe the Receding Moon from the Earth will be discussed elsewhere. In this Section, we discuss how the calculated expanding radius can describe (at least partially) the Receding Moon from the Earth. Another possible effect, in particular the deformation of the surrounding medium, shall also be considered.

5 Observable C: Podkletnov's rotation disc experiment

It has been discussed how gravitational force shall take into consideration the full description of Newton's law. In this Section, we put forth the known equivalence between Newton's law (31) and Lorentz' force [23], which can be written (supposing m to be constant) as follows:

$$\vec{F} = \frac{d}{dt}\left(\gamma\vec{m}\vec{v}\right) = \gamma m\left(\frac{d\vec{v}}{dt}\right) = q\left(\vec{E} + \frac{1}{c}\,\vec{v}\times\vec{B}\right), \tag{34}$$

where the relativistic factor is defined as:

$$\gamma = \pm\sqrt{\frac{1}{1-\beta^2}}\,. \tag{35}$$

while we can expand this equation in the cylindrical coordinates [23], we retain the simplest form in this analysis. In accordance with Spohn, we define [24]:

$$E = -\nabla A\,. \tag{36}$$

$$B = \nabla \times A\,. \tag{37}$$

For Podkletnov's experiment [26–28], it is known that there in a superconductor $E = 0$ [25], and by using the mass m in lieu of the charge ratio $\frac{e}{c}$ in the right hand term of (34) called the "gravitational Lorentz force", we get:

$$m\left(\frac{d\vec{v}}{dt}\right) = \frac{m}{\gamma}\left(\vec{v}\times\vec{B}\right) = \frac{1}{\gamma}\left(\vec{p}\times\vec{B}\right). \tag{38}$$

Let us suppose we conduct an experiment with the weight $w = 700$ g, the radius $r = 0.2$ m, and it rotates at $f = 2$ cps (cycle per second), then we get the velocity at the edge of the disc as:

$$v = 2\pi \cdot f\, r = 2.51 \text{ m/sec}, \tag{39}$$

and with known values for $G = 6.67\times10^{-11}$, $c \simeq 3\times10^8$m/sec, $M_{earth} = 5.98\times10^{24}$ kg, $r_{earth} = 3\times10^6$ m, then we get:

$$F_{gr} = \frac{G}{c^2 r}\, Mv \approx 3.71\times10^{-9} \text{ newton/kgm sec}. \tag{40}$$

Because $B = F/\text{meter}$, then from (39), the force on the disc is given by:

$$F_{disc} = \vec{B}_{earth} \cdot \vec{p}_{disc} \approx B_{earth} \cdot \left(m\, \frac{c}{\gamma} \right). \tag{41}$$

High-precision muon experiment suggests that its speed can reach around $\sim 0.99\, c$. Let us suppose in our disc, the particles inside have the speed $0.982\, c$, then $\gamma^{-1} = 0.1889$. Now inserting this value into (40), yields:

$$\begin{aligned} F_{disc} = (3.71\times10^{-9}) \cdot (0.7) \cdot (3\times10^8) \cdot 0.189 \ = \\ = \ 0.147 \text{ newton} = 14.7 \text{ gr}. \end{aligned} \tag{42}$$

Therefore, from the viewpoint of a static observer, the disc will get a mass reduction as large as $\frac{14.7}{700} = 2.13\%$, which seems quite near with Podkletnov's result, i.e. the disc can obtain a mass reduction up to 2% of the static mass.

We remark here that we use a simplified analysis using Lorentz' force, considering the fact that superconductivity may be considered as a relativistic form of the ordinary electromagnetic field [25].

Interestingly, some authors have used different methods to explain this apparently bizarre result. For instance, using Tajmar and deMatos' [29] equation: $\gamma_0 = \frac{a\Omega}{2} = \frac{0.2\cdot 2}{2} = 0.2$. In other words, it predicts a mass reduction around $\sim \frac{0.2}{9.8} = 2\%$, which is quite similar to Podkletnov's result.

Another way to describe those rotating disc experiments is by using simple Newton law [33]. From equation (31) one has (by setting $F = 0$ and because $g = \frac{dv}{dt}$):

$$\frac{dm}{dt} = -\frac{m}{v}\, g = -\frac{m}{\omega R}\, g\,, \tag{43}$$

Therefore one can expect a mass reduction given by an angular velocity (but we're not very how Podkletnov's experiment can be explained using this equation).

We end this section by noting that we describe the rotating disc experiment by using Lorentz' force in a rotating system. Further extension of this method in particular in the context of the (extended) Q-relativity theory, will be discussed in the subsequent Section.

6 Possible link with Q-Relativity. Extended 9D metric

In the preceding Section, we have discussed how closed FLRW metric is associated to the group with nonzero quaternions, and that Carmeli metric belongs to the group. The only problem with this description is that it neglects the directions of the velocity other than against the x line.

Therefore, one can generalize further the metric to become [1, p.5]:

$$-\tau^2 (dv_R)^2 + dz^2 + dy^2 + dx^2 = 0\,, \tag{44}$$

or by considering each component of the velocity vector [23]:

$$\begin{aligned} (i\,\tau\, dv_X)^2 + (i\,\tau\, dv_Y)^2 + (i\,\tau\, dv_Z)^2 + \\ + dz^2 + dy^2 + dx^2 = 0\,. \end{aligned} \tag{45}$$

From this viewpoint one may consider it as a generalization of Minkowski's metric into biquaternion form, using the modified Q-relativity space [30, 31, 32], to become:

$$ds = (dx_k + i\,\tau\, dv_k)\, q_k. \tag{46}$$

Please note here that we keep using definition of Yefremov's quaternion relativity (Q-relativity) physics [30], albeit we introduce dv instead of dt in the right term. We propose to call this metric *quaternionic Kaluza-Klein-Carmeli metric*.

One possible further step for the generalization this equation, is by keep using the standard Q-relativistic dt term, to become:

$$ds = (dx_k + i\, c\, dt_k + i\,\tau\, dv_k)\, q_k\,, \tag{47}$$

which yields 9-Dimensional extension to the above quaternionic Kaluza-Klein-Carmeli metric. In other words, this generalized 9D KK-Carmeli metric is seemingly capable to bring the most salient features in both the standard Carmeli metric and also Q-relativity metric. Its prediction includes plausible time-evolution of some known celestial motion in the solar system, including but not limited to the Earth-based satellites (albeit very minute). It can be compared for instance using Arbab's calculation, that the Earth accelerates at rate 3.05 arcsec/cy^2, and Mars at 1.6 arcsec/cy^2 [12]. Detailed calculation will be discussed elsewhere.

We note here that there is quaternionic multiplication rule which acquires the compact form [30–32]:

$$1 q_k = q_k 1 = q_k\,, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n\,, \tag{48}$$

where δ_{kn} and ε_{jkn} represent 3-dimensional symbols of Kronecker and Levi-Civita, respectively [30]. It may also be worth noting here that in 3D space Q-connectivity has clear geometrical and physical treatment as movable Q-basis with behavior of Cartan 3-frame [30].

In accordance with the standard Q-relativity [30, 31], it is also possible to write the dynamics equations of Classical Mechanics for an inertial observer in the constant Q-basis, as follows:

$$m\, \frac{d^2}{dt^2}\, (x_k q_k) = F_k q_k\,. \tag{49}$$

Because of the antisymmetry of the connection (the generalized angular velocity), the dynamics equations can be written in vector components, by the conventional vector notation [30, 32]:

$$m\left(\vec{a}+2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right)=\vec{F}\,, \qquad (50)$$

which represents known types of classical acceleration, i.e. the linear, the Coriolis, the angular, and the centripetal acceleation, respectively.

Interestingly, as before we can use the equivalence between the inertial force and Lorentz' force (34), therefore equation (50) becomes:

$$m\left(\frac{d\vec{v}}{dt}+2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right)= = q_{\otimes}\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right), \qquad (51)$$

or

$$\left(\frac{d\vec{v}}{dt}\right)=\frac{q_{\otimes}}{m}\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right)- -\frac{2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})}{m}\,. \qquad (52)$$

Please note that the variable q here denotes electric charge, not quaternion number.

Therefore, it is likely that one can expect a new effects other than Podkletnov's rotating disc experiment as discussed in the preceding Section.

Further interesting things may be expected, by using (34):

$$\vec{F}=m\left(\frac{d\vec{v}}{dt}\right)=q\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right)\Rightarrow \Rightarrow m\,(d\vec{v})=q\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right)dt\,. \qquad (53)$$

Therefore, by introducing this Lorentz' force instead of the velocity into (44), one gets directly a plausible extension of Q-relativity:

$$ds=\left[dx_k+i\,\tau\,\frac{q}{m}\left(\vec{E}_k+\frac{1}{c}\,\vec{v}_k\times\vec{B}_k\right)dt_k\right]q_k\,. \qquad (54)$$

This equation seems to indicate how a magnetic wormhole can be induced in 6D Q-relativity setting [16, 19]. The reason to introduce this proposition is because there is known link between magnetic field and rotation [34]. Nonetheless further experiments are recommended in order to refute or verify this proposition.

7 Possible link with quantum gravity

In this Section, we remark that the above procedure to derive the closed FLRW-Carmeli metric from the group with nonzero quaternions has an obvious advantage, i.e. one can find Quantum Mechanics directly from the quaternion framework [35]. In other words, one can expect to put the gravitational metrical (FLRW) setting and the Quantum Mechanics setting in equal footing. After all, this may be just a goal sought in "quantum gravity" theories. See [4a] for discussion on the plausible quantization of a gravitational field, which may have observable effects for instance in the search of extrasolar planets [35a].

Furthermore, considering the "phonon metric" described in (20), provided that it corresponds to the observed facts, in particular with regards to the "surrounding medium" vortices described by (26–29), one can say that the "surrounding medium" is comprised of the phonon medium. This proposition may also be related to the superfluid-interior of the Sun, which may affect the Earth climatic changes [35b]. Therefore one can hypothesize that the signatures of quantum gravity, in the sense of the quantization in gravitational large-scale phenomena, are possible because the presence of the phonon medium. Nonetheless, further theoretical works and observations are recommended to explore this new proposition.

8 Concluding remarks

In the present paper we begun with a representation of a group with non-zero quaternions to derive closed FLRW metric [1], and we obtained Carmeli 5D metric [4] from this group. The resulting metric can be extended further to become 5D and 6D metric (called by us *Kaluza-Klein-Carmeli metric*).

Thereafter we discussed some plausible implications of this metric. Possible implications to the Earth geochronometrics and possible link to the coral growth data were discussed. In subsequent Section we explained Podkletnov's rotating disc experiment. We also noted possible neat link between Kaluza-Klein-Carmeli metric and Yefremov's Q-Relativity, in particular we proposed a further extension of Q-relativity to become 9D metric. Possible implications to quantum gravity, i.e. possible observation of the quantization effects in gravitation phenomena was also noted.

Nonetheless we do not pretend to have the last word on some issues, including quantum gravity, the structure of the aether (phonon) medium, and other calculations which remain open. There are also different methods to describe the Receding Moon or Podkletnov's experiments. What this paper attempts to do is to derive some known gravitational phenomena, including Hubble's constant, in a simplest way as possible, without invoking a strange form of matter. Furthermore, the Earth geochronometry data may enable us to verify the cosmological theories with unprecedented precision.

Therefore, it is recommended to conduct further observations in order to verify and also to explore the implications of our propositions as described herein.

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References

1. Trifonov V. Geometry of the group of nonzero quaternions. arXiv: physics/0301052; [1a] arXiv: math-ph/0606007.
2. Carrion H.L., et al. Quaternionic and octonionic spinors. A classification. arXiv: hep-th/0302113.
3. Abdel-Khalek K. The ring division self duality. arXiv: hep-th/9710177; [3a] Riemann B. In which line-element may be expressed as the fourth root of a quartic differential expression. *Nature*, v. VIII, nos. 183 and 184, 14–17, 36, and 37. Translated by William K. Clifford.
4. Carmeli M. Is galaxy dark matter a property of spacetime? arXiv: astro-ph/9607142; [4a] Carmeli M., et al. The SL(2,c) gauge theory of gravitation and the quantization of gravitational field. arXiv: gr-qc/9907078.
5. Hartnett J.G. Carmeli's accelerating universe is spatially flat without dark matter. arXiv:gr-qc/0407083.
6. Hartnett J.G. Extending the redshift-distance relation in Cosmological General Relativity to higher redshifts. arXiv: physics.gen-ph/0705.3097.
7. Hartnett J.G. The distance modulus determined from Carmeli's cosmology. arXiv: astro-ph/0501526.
8. Fujii Y. Cosmological constant, quintessence and Scalar-Tensor theories of gravity. arXiv: gr-qc/0001051.
9. Fujii Y. Quintessence, Scalar-Tensor theories, and non-Newtonian gravity. arXiv:gr-qc/9911064.
10. Hehl F.W. and Obukhov Y. arXiv: gr-qc/0711.1535, p.1–2; [10a] Rapoport D. In: *Quantization in Astrophysics, Brownian motion, and Supersymmetry*, F. Smarandache and V. Christianto (eds), Chenai, Tamil Nadu, 2007.
11. Noerdlinger P. arXiv: astro-ph/0801.3807.
12. Arbab I.A. On the planetary acceleration and the rotation of the Earth. arXiv: astro-ph/0708.0666, p. 5–7.
13. Smoot N.C. Earth geodynamic hypotheses updated. *Journal of Scientific Exploration*, 2001, v. 15, no. 3, 465–494; http://www.scientificexploration.org/jse/
14. Wells J.W. Coral growth and geochronometry. *Nature*, 9 March 1963; http://freepages.genealogy.rootsweb.com/∼springport/geology/coral_growth.html
15. Christodoulou D. and Kazanas D. Exact solutions of the isothermal Lane-Emdeen equations with rotation and implications for the formation of planets and satellites. arXiv: astro-ph/0706.3205.
16. Sussman R. A dynamical system approach to inhomogeneous dust solutions. arXiv: gr-qc/0709.1005.
17. Sollanych M.J. Continental drift and the expansion of the universe. http://www3.bc.sympathico.ca/moon/index.html
18. O'Brien R. In: *2002 Yearbook of Astronomy*, P. Moore (ed.), Macmillan Publ. Ltd., London, 2002, 214–223, 262–263.
19. Friedman J.L., et al. Reduced phase space formalism for spherically symmetric geometry with a massive dust shell. arXiv: gr-qc/970605, p.19
20. Pervushin V.N., et al. Astrophysics in relative units as the theory of conformal brane. arXiv: hep-th/0209070.
21. Rughede O.L. On the theory and physics of aether. *Progress in Physics*, 2006, v. 2.
22. Kirkwood R.L. Gravitational field equations. *Phys. Rev.*, 1954, v. 95, no. 4, 1051–1056; [22a] Cahill R. Dynamical fractal 3-space and the generalized Schrödinger equation: equivalence principle and vorticity effects. *Progress in Physics*, 2006, v. 2.
23. Murad P.A. Relativistic fluid dynamics and light speed travel, *AIAA*, 1999.
24. Spohn H. Semiclassical limit of the Dirac equation and spin precession. arXiv: quant-ph/9911065.
25. Cui H.Y. Relativistic mechanism of superconductivity. arXiv: physics/0212059.
26. Podkletnov E. and Nieminen R. *Physica C*, 1992, v. 203, 441.
27. Podkletnov E. Weak gravitation shielding properties of composite bulk $YBa_2Cu_3O_{7-x}$ superconductor below 70 K under e.m. field. arXiv: cond-mat/9701074.
28. Podkletnov E. and Modanese G. Impulse gravity generator based on charged $YBa_2Cu_3O_{7-y}$ superconductor with composite crystal structure. arXiv: physics/0108005.
29. Tajmar M. and deMatos C. Induction and amplification of non-Newtonian gravitational fields. arXiv: gr-qc/0107012; [29a] deMatos C. and Tajmar M. Gravitational pointing vector and gravitational Larmor theorem in rotating bodies with angular acceleration. arXiv: gr-qc/0107014.
30. Yefremov A. Quaternions: algebra, geometry and physical theories. *Hypercomplex Numbers in Geometry and Physics*, 2004, v. 1(1), 105.
31. Yefremov A. In: *Quantization in Astrophysics, Brownian motion, and Supersymmetry*, F. Smarandache and V. Christianto (eds), Chenai, Tamil Nadu, 2007.
32. Smarandache F. and Christianto V. Less mundane explanation of Pioneer anomaly from Q-relativity. *Progress in Physics*, 2007, v. 3.
33. Shipov G. Shipov on torsion. http://www.shipov.com/200506_news03.pdf
34. Dzhunushaliev V. Duality between magnetic field and rotation. arXiv: gr-qc/0406078.
35. Trifonov V. Geometrical modification of quaternionic quantum mechanics. arXiv: math-ph/0702095; [35a] Bower G.C., et al. Radio astrometric detection and characterization of extra-Solar planets: a white paper submitted to the NSF ExoPlanet Task Force. arXiv: astro-ph/0704.0238; [35b] Manuel O.K., et al. Superfluidity in the Solar interior: implications for Solar eruptions and climate. *J. Fusion Energy*, 2002, v. 21, 192–198; arXiv: astro-ph/0501441.

A Note of Extended Proca Equations and Superconductivity

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It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations. The implications of introducing Proca equations include an alternative description of superconductivity, via extending London equations. In the light of another paper suggesting that Maxwell equations can be written using quaternion numbers, then we discuss a plausible extension of Proca equation using biquaternion number. Further implications and experiments are recommended.

1 Introduction

It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations, to become electrodynamics with finite photon mass [11]. The implications of introducing Proca equations include description of superconductivity, by extending London equations [18]. In the light of another paper suggesting that Maxwell equations can be generalized using quaternion numbers [3, 7], then we discuss a plausible extension of Proca equations using biquaternion number. It seems interesting to remark here that the proposed extension of Proca equations by including quaternion differential operator is merely the next logical step considering already published suggestion concerning the use of quaternion differential operator in electromagnetic field [7, 8]. This is called Moisil-Theodoresco operator (see also Appendix A).

2 Maxwell equations and Proca equations

In a series of papers, Lehnert argued that the Maxwell picture of electrodynamics shall be extended further to include a more "realistic" model of the *non-empty vacuum*. In the presence of electric space charges, he suggests a general form of the Proca-type equation [11]:

$$\left(\frac{1}{c^2}\frac{\partial}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu, \qquad \mu = 1,2,3,4. \quad (1)$$

Here $A_\mu = (A, i\phi/c)$, where A and ϕ are the magnetic vector potential and the electrostatic potential in three-space, and:

$$J_\mu = (j,\ i c\bar{\phi}) \,. \quad (2)$$

However, in Lehnert [11], the right-hand terms of equations (1) and (2) are now given a new interpretation, where $\bar{\phi}$ is the nonzero electric charge density in the vacuum, and j stands for an associated three-space current-density.

The background argument of Proca equations can be summarized as follows [6]. It was based on known definition of derivatives [6, p. 3]:

$$\left.\begin{aligned} \partial^\mu &= \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}; \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \left(\partial^0; -\nabla\right) \\ \partial_\mu &= \frac{\partial}{\partial x^\mu} = \left(\partial^0; \nabla\right) \end{aligned}\right\}, \quad (3)$$

$$\partial_\mu a^\mu = \frac{\partial a^0}{\partial t} + \nabla \vec{a} \,, \quad (4)$$

$$\partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \partial_0^2 - \nabla^2 = \partial^\mu \partial_\mu \,, \quad (5)$$

where ∇^2 is Laplacian and $\partial_\mu \partial^\mu$ is d'Alembertian operator. For a massive vector boson (spin-1) field, the Proca equation can be written in the above notation [6, p. 7]:

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) + m^2 A^\nu = j^\nu. \quad (6)$$

Interestingly, there is also a neat link between Maxwell equations and quaternion numbers, in particular via the *Moisil-Theodoresco D* operator [7, p. 570]:

$$D = i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + i_3 \frac{\partial}{\partial x_3} \,. \quad (7)$$

There are also known links between Maxwell equations and Einstein-Mayer equations [8]. Therefore, it seems plausible to extend further the Maxwell-Proca equations to biquaternion form too; see also [9, 10] for links between Proca equation and Klein-Gordon equation. For further theoretical description on the links between biquaternion numbers, Maxwell equations, and unified wave equation, see Appendix A.

3 Proca equations and superconductivity

In this regards, it has been shown by Sternberg [18], that the classical *London equations* for superconductors can be written in differential form notation and in relativistic form, where they yield the Proca equations. In particular, the field itself acts as its own charge carrier [18].

Similarly in this regards, in a recent paper Tajmar has shown that superconductor equations can be rewritten in terms of Proca equations [19]. The basic idea of Tajmar appears similar to Lehnert's extended Maxwell theory, i.e. to include *finite photon mass* in order to explain superconductivity phenomena. As Tajmar puts forth [19]:

> "In quantum field theory, superconductivity is explained by a *massive photon*, which acquired mass due to gauge symmetry breaking and the Higgs mechanism. The wavelength of the photon is interpreted as the London penetration depth. With a nonzero photon mass, the usual Maxwell equations transform into the so-called Proca equations which will form the basis for our assessment in superconductors and are only valid for the superconducting electrons."

Therefore the basic Proca equations for superconductor will be [19, p. 3]:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}\,, \tag{8}$$

and

$$\nabla \times B = \mu_0 \bar{j} + \frac{1}{c^2}\frac{\partial \bar{E}}{\partial t} - \frac{1}{\lambda^2}\bar{A}\,. \tag{9}$$

The Meissner effect is obtained by taking curl of equation (9). For non-stationary superconductors, the same equation (9) above will yield second term, called London moment.

Another effects are recognized from the finite Photon mass, i.e. the photon wavelength is then interpreted as the London penetration depth and leads to a photon mass about 1/1000 of the electron mass. This furthermore yields the Meissner-Ochsenfeld effect (shielding of electromagnetic fields entering the superconductor) [20].

Nonetheless, the use of Proca equations have some known problems, i.e. it predicts that a charge density rotating at angular velocity should produce huge magnetic fields, which is *not observed* [20]. One solution of this problem is to recognize that the value of photon mass containing charge density is different from the one in free space.

4 Biquaternion extension of Proca equations

Using the method we introduced for Klein-Gordon equation [2], then it is possible to generalize further Proca equations (1) using biquaternion differential operator, as follows:

$$(\diamondsuit\bar{\diamondsuit})A_\mu - \mu_0 J_\mu = 0\,, \qquad \mu = 1,2,3,4, \tag{10}$$

where (see also Appendix A):

$$\diamondsuit = \nabla^q + i\nabla^q = \left(-i\,\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + \\ + i\left(-i\,\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right). \tag{11}$$

Another way to generalize Proca equations is by using its standard expression. From d'Alembert wave equation we get [6]:

$$\left(\frac{1}{c^2}\frac{\partial}{\partial t^2} - \nabla^2\right)A_\mu = \mu_0 J_\mu\,, \qquad \mu = 1,2,3,4, \tag{12}$$

where the solution is Liennard-Wiechert potential. Then the Proca equations are [6]:

$$\left[\left(\frac{1}{c^2}\frac{\partial}{\partial t^2} - \nabla^2\right) + \left(\frac{m_p c}{\hbar}\right)^2\right]A_\mu = 0\,, \quad \mu = 1,2,3,4, \tag{13}$$

where m is the photon mass, c is the speed of light, and $\hbar$ is the reduced Planck constant. Equation (13) and (12) imply that photon mass can be understood as charge density:

$$J_\mu = \frac{1}{\mu_0}\left(\frac{m_p c}{\hbar}\right)^2. \tag{14}$$

Therefore the "biquaternionic" extended Proca equations (13) become:

$$\left[\diamondsuit\bar{\diamondsuit} + \left(\frac{m_p c}{\hbar}\right)^2\right]A_\mu = 0\,, \qquad \mu = 1,2,3,4. \tag{15}$$

The solution of equations (10) and (12) can be found using the same computational method as described in [2].

Similarly, the generalized structure of the wave equation in electrodynamics — *without* neglecting the finite photon mass (Lehnert-Vigier) — can be written as follows (instead of eq. 7.24 in [6]):

$$\left[\diamondsuit\bar{\diamondsuit} + \left(\frac{m_p c}{\hbar}\right)^2\right]A^a_\mu = RA^a_\mu\,, \quad \mu = 1,2,3,4. \tag{16}$$

It seems worth to remark here that the method as described in equation (15)-(16) or ref. [6] is not the only possible way towards generalizing Maxwell equations. Other methods are available in literature, for instance by using topological geometrical approach [14, 15].

Nonetheless further experiments are recommended in order to verify this proposition [23, 24]. One particular implication resulted from the introduction of biquaternion differential operator into the Proca equations, is that it may be related to the notion of "active time" introduced by Paine & Pensinger sometime ago [13]; the only difference here is that now the time-evolution becomes nonlinear because of the use of 8-dimensional differential operator.

5 Plausible new gravitomagnetic effects from extended Proca equations

While from Proca equations one can expect to observe gravitational London moment [4, 22] or other peculiar gravitational shielding effect unable to predict from the framework of General Relativity [5, 16, 22], one can expect to derive new gravitomagnetic effects from the proposed extended Proca equations using the biquaternion number as described above.

Furthermore, another recent paper [1] has shown that given the finite photon mass, it would imply that if m is due to a Higgs effect, then the *Universe is effectively similar to a Superconductor.* This may support De Matos's idea of dark energy arising from superconductor, in particular via *Einstein-Proca* description [1, 5, 16].

It is perhaps worth to mention here that there are some indirect observations [1] relying on the effect of Proca energy (assumed) on the galactic plasma, which implies the limit:

$$m_A = 3{\times}10^{-27}\ \text{eV}. \tag{17}$$

Interestingly, in the context of cosmology, it can be shown that Einstein field equations with cosmological constant are approximated to the second order in the perturbation to a flat background metric [5]. Nonetheless, further experiments are recommended in order to verify or refute this proposition.

6 Some implications in superconductivity research

We would like to mention the Proca equation in the following context. Recently it was hypothesized that the creation of superconductivity at room temperature may be achieved by a resonance-like interaction between an everywhere present background field and a special material having the appropriate crystal structure and chemical composition [12]. According to Global Scaling, a new knowledge and holistic approach in science, the everywhere present background field is given by oscillations (standing waves) in the universe or physical vacuum [12].

The just mentioned hypothesis how superconductivity at room temperature may come about, namely by a resonance-like interaction between an everywhere present background field and a special material having the appropriate crystal structure and chemical composition, seems to be supported by a statement from the so-called ECE Theory which is possibly related to this hypothesis [12]:

> "... One of the important practical consequences is that a material can become a superconductor by absorption of the inhomogeneous and homogeneous currents of ECE space-time ..." [6].

This is a quotation from a paper with the title "ECE Generalizations of the d'Alembert, Proca and Superconductivity Wave Equations ..." [6]. In that paper the Proca equation is derived as a special case of the ECE field equations.

These considerations raises the interesting question about the relationship between (a possibly new type of) superconductivity, space-time, an everywhere-present background field, and the description of superconductivity in terms of the Proca equation, i.e. by a massive photon which acquired mass by symmetry breaking. Of course, how far these suggestions are related to the physical reality will be decided by further experimental and theoretical studies.

7 Concluding remarks

In this paper we argue that it is possible to extend further Proca equations for electrodynamics of superconductivity to biquaternion form. It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations, to become electrodynamics with finite photon mass. The implications of introducing Proca equations include description of superconductivity, by extending London equations. Nonetheless, further experiments are recommended in order to verify or refute this proposition.

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Appendix A: Biquaternion, Maxwell equations and unified wave equation [3]

In this section we're going to discuss Ulrych's method to describe unified wave equation [3], which argues that it is possible to define a unified wave equation in the form [3]:

$$D\phi(x) = m_\phi^2 \cdot \phi(x), \tag{A.1}$$

where unified (wave) differential operator D is defined as:

$$D = \left[(P - qA)_\mu \left(\bar{P} - qA\right)^\mu\right]. \tag{A.2}$$

To derive Maxwell equations from this unified wave equation, he uses free photon expression [3]:

$$DA(x) = 0, \tag{A.3}$$

where potential A(x) is given by:

$$A(x) = A^0(x) + jA^1(x), \tag{A.4}$$

and with electromagnetic fields:

$$E^i(x) = -\partial^0 A^i(x) - \partial^i A^0(x), \tag{A.5}$$

$$B^i(x) = \in^{ijk} \partial_j A_k(x). \tag{A.6}$$

Inserting these equations (A.4)-(A.6) into (A.3), one finds Maxwell electromagnetic equation [3]:

$$\begin{aligned} &-\nabla \bullet E(x) - \partial^0 C(x) + ij\nabla \bullet B(x) - \\ &- j(\nabla x B(x) - \partial^0 E(x) - \nabla C(x)) - \\ &- i(\nabla x E(x) + \partial^0 B(x)) = 0. \end{aligned} \tag{A.7}$$

For quaternion differential operator, we define quaternion Nabla operator:

$$\begin{aligned} \nabla^q \equiv c^{-1}\frac{\partial}{\partial t} + \left(\frac{\partial}{\partial x}\right) i + \left(\frac{\partial}{\partial y}\right) j + \left(\frac{\partial}{\partial z}\right) k = \\ = c^{-1}\frac{\partial}{\partial t} + \vec{i} \cdot \vec{\nabla}. \end{aligned} \tag{A.8}$$

And for biquaternion differential operator, we may define a diamond operator with its conjugate [3]:

$$\diamond\bar{\diamond} \equiv \left(c^{-1}\frac{\partial}{\partial t} + c^{-1}\, i\,\frac{\partial}{\partial t}\right) + \{\vec{\nabla}\}* \tag{A.9}$$

where Nabla-star-bracket operator is defined as:

$$\begin{aligned}\{\vec{\nabla}\}* \equiv &\left(\frac{\partial}{\partial x} + i\,\frac{\partial}{\partial X}\right) i + \\ &+ \left(\frac{\partial}{\partial y} + i\,\frac{\partial}{\partial Y}\right) j + \left(\frac{\partial}{\partial z} + i\,\frac{\partial}{\partial Z}\right) k\,.\end{aligned} \tag{A.10}$$

In other words, equation (A.9) can be rewritten as follows:

$$\begin{aligned}\diamond\bar{\diamond} \equiv &\left(c^{-1}\frac{\partial}{\partial t} + c^{-1}\, i\,\frac{\partial}{\partial T}\right) + \left(\frac{\partial}{\partial x} + i\,\frac{\partial}{\partial X}\right) i + \\ &+ \left(\frac{\partial}{\partial y} + i\,\frac{\partial}{\partial Y}\right) j + \left(\frac{\partial}{\partial z} + i\,\frac{\partial}{\partial Z}\right) k\,.\end{aligned} \tag{A.11}$$

From this definition, it shall be clear that there is neat link between equation (A.11) and the Moisil-Theodoresco D operator, i.e. [7, p. 570]:

$$\begin{aligned}\diamond\bar{\diamond} &\equiv \left(c^{-1}\frac{\partial}{\partial t} + c^{-1}\, i\,\frac{\partial}{\partial t}\right) + (D_{xi} + i\,D_{Xi}) = \\ &= \left(c^{-1}\frac{\partial}{\partial t} + c^{-1} i\frac{\partial}{\partial T}\right) + \left[i_1\frac{\partial}{\partial x_1} + i_2\frac{\partial}{\partial x_2} + i_3\frac{\partial}{\partial x_3}\right] + \\ &+ i\left[i_1\frac{\partial}{\partial X_1} + i_2\frac{\partial}{\partial X_2} + i_3\frac{\partial}{\partial X_3}\right].\end{aligned} \tag{A.12}$$

In order to define biquaternionic representation of Maxwell equations, we could extend Ulrych's definition of *unified differential operator* [3,17,21] to its biquaternion counterpart, by using equation (A.2) and (A.10), to become:

$$\{D\}* \equiv \left[\left(\{P\}* - q\{A\}*\right)_{\mu}\left(\{\bar{P}\}* - q\{A\}*\right)^{\mu}\right], \tag{A.13}$$

or by definition $P = -i\hbar\nabla$, equation (A.13) could be written as:

$$\{D\}* \equiv \left[\left(-\hbar\{\vec{\nabla}\}* - q\{A\}*\right)_{\mu}\left(-\hbar\{\vec{\nabla}\}* - q\{A\}*\right)^{\mu}\right], \tag{A.14}$$

where each component is now defined in term of biquaternionic representation. Therefore the biquaternionic form of the *unified wave equation* [3] takes the form:

$$\{D\} * \phi(x) = m_{\phi}^2 \cdot \phi(x)\,, \tag{A.15}$$

which is a wave equation for massive electrodynamics, quite similar to Proca representation.

Now, biquaternionic representation of free photon fields could be written as follows:

$$\{D\} * A(x) = 0\,. \tag{A.16}$$

References

1. Adelberger E., Dvali G., and Gruzinov A. Photon-mass bound destroyed by vortices. *Phys. Rev. Lett.*, 2007, v. 98, 010402.
2. Christianto V. and Smarandache F. Numerical solution of radial biquaternion Klein-Gordon equation. *Progress in Physics*, 2008, v.1.
3. Christianto V. *Electronic J. Theor. Physics*, 2006, v. 3, no. 12.
4. De Matos C. J. arXiv: gr-qc/0607004; Gravio-photon, superconductor and hyperdrives. http://members.tripod.com/da_theoretical1/warptohyperdrives.html
5. De Matos C.J. arXiv: gr-qc/0609116.
6. Evans M.W. ECE generalization of the d'Alembert, Proca and superconductivity wave equations: electric power from ECE space-time. §7.2; http://aias.us/documents/uft/a51stpaper.pdf
7. Kravchenko V.V. and Oviedo H. On quaternionic formulation of Maxwell's equations for chiral media and its applications. *J. for Analysis and its Applications*, 2003, v. 22, no. 3, 570.
8. Kravchenko V.G. and Kravchenko V.V. arXiv: math-ph/0511092.
9. Jakubsky V. and Smejkal J. A positive definite scalar product for free Proca particle. arXiv: hep-th/0610290.
10. Jakubsky V. *Acta Polytechnica*, 2007, v. 47, no. 2–3.
11. Lehnert B. Photon physics of revised electromagnetics. *Progress in Physics*, 2006, v. 2.
12. Lichtenberg F. Presentation of an intended research project: searching for room temperature superconductors. August, 2008, http://www.sciprint.org; http://podtime.net/sciprint/fm/uploads/files/1218979173Searching_for_Room_Temperature_Superconductors.pdf
13. Paine D.A. and Pensinger W.L. *Int. J. Quantum Chem.*, 1979, v.15, 3; http://www.geocities.com/moonhoabinh/ithapapers/hydrothermo.html
14. Olkhov O.A. Geometrization of classical wave fields. arXiv: 0801.3746.
15. Olkhov O.A. *Zh. Fiz. Khim.*, 2002, v. 21, 49; arXiv: hep-th/0201020.
16. Poenaru D. A. Proca (1897–1955). arXiv: physics/0508195; http://th.physik.uni-frankfurt.de/~poenaru/PROCA/Proca.pdf
17. Ulrych S. arXiv: physics/0009079.
18. Sternberg S. On the London equations. *PNAS*, 1992, v. 89, no. 22, 10673–10675.
19. Tajmar M. Electrodynamics in superconductors explained by Proca equations. arXiv: 0803.3080.
20. Tajmar M. and De Matos C.J. arXiv: gr-qc/0603032.
21. Yefremov A., Smarandache F. and Christianto V. Yang-Mills field from quaternion space geometry, and its Klein-Gordon representation. *Progress in Physics*, 2007, v. 3.
22. Gravitational properties of superconductors. http://functional-materials.at/rd/rd_spa_gravitationalproperties_de.html
23. Magnetism and superconductivity observed to exist in harmony. Aug. 28, 2008, http://www.physorg.com/news139159195.html
24. Room temperature superconductivity. Jul. 8, 2008, http://www.physorg.com/news134828104.html

Less Mundane Explanation of Pioneer Anomaly from Q-Relativity

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There have been various explanations of Pioneer blueshift anomaly in the past few years; nonetheless no explanation has been offered from the viewpoint of Q-relativity physics. In the present paper it is argued that Pioneer anomalous blueshift may be caused by Pioneer spacecraft experiencing angular shift induced by similar Q-relativity effect which may also affect Jupiter satellites. By taking into consideration "aether drift" effect, the proposed method as described herein could explain Pioneer blueshift anomaly within ∼0.26% error range, which speaks for itself. Another new proposition of redshift quantization is also proposed from gravitational Bohr-radius which is consistent with Bohr-Sommerfeld quantization. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the past few years, it is becoming well-known that Pioneer spacecraft has exhibited an anomalous Doppler frequency blueshifting phenomenon which cannot be explained from conventional theories, including General Relativity [1, 4]. Despite the nature of such anomalous blueshift remains unknown, some people began to argue that a post-einsteinian gravitation theory may be in sight, which may be considered as further generalisation of pseudo-Riemannian metric of general relativity theory.

Nonetheless, at this point one may ask: Why do we require a generalization of pseudo-Riemannian tensor, instead of using "patch-work" as usual to modify general relativity theory? A possible answer is: sometimes too much path-work doesn't add up. For instance, let us begin with a thought-experiment which forms the theoretical motivation behind General Relativity, an elevator was put in free-falling motion [8a]. The passenger inside the elevator will not feel any gravitational pull, which then it is interpreted as formal analogue that "inertial acceleration equals to gravitational acceleration" (Equivalence Principle). More recent experiments (after Eötvös) suggest, however, that this principle is only applicable at certain conditions.

Further problem may arise if we ask: what if the elevator also experiences lateral rotation around its vertical axis? Does it mean that the inertial acceleration will be slightly higher or lower than gravitational pull? Similarly we observe that a disc rotating at high speed will exert out-of-plane field resemble an acceleration field. All of this seems to indicate that the thought-experiment which forms the basis of General Relativity is only applicable for some limited conditions, in particular the $F = m\frac{dv}{dt}$ part (because General Relativity is strictly related to Newtonian potential), but it may not be able to represent the rotational aspects of gravitational phenomena. Einstein himself apparently recognizes this limitation [8a, p.61]:

> ". . . all bodies of reference K' should be given preference in this sense, and they should be exactly equivalent to K for the formation of natural laws, provided that they are in a state of *uniform rectilinear and non-rotary motion* with respect to K." (Italic by Einstein).

Therefore, it shall be clear that the restriction of *non-rotary motion* remains a limitation for all considerations by relativity theory, albeit the *uniform rectilinear* part has been relaxed by general relativity theory.

After further thought, it becomes apparent that it is required to consider a new kind of metric which may be able to represent the rotational aspects of gravitation phenomena, and by doing so extends the domain of validity of general relativity theory.

In this regard, the present paper will discuss the aforementioned Pioneer blueshift anomaly from the viewpoint of Q-relativity physics, which has been proposed by Yefremov [2] in order to bring into application the quaternion number. Despite the use of quaternion number in physical theories is very scarce in recent years — apart of Pauli matrix — it has been argued elsewhere that using quaternion number one could expect to unify all known equations in Quantum Mechanics into the same framework, in particular via the known isomorphism between Dirac equation and Maxwell equations [5].

Another problem that was often neglected in most treatises on Pioneer spacecraft anomaly is the plausible role of aether drift effect [6]. Here it can be shown that taking this effect into consideration along with the aforementioned Q-relativity satellite's apparent shift could yield numerical prediction of Pioneer blueshift within ∼0.26% error range, which speaks for itself.

We also suggest a new kind of Doppler frequency shift which can be predicted using Nottale-type gravitational Bohr-radius, by taking into consideration varying G parameter as described by Moffat [7]. To our knowledge this proposition of new type of redshift corresponding to gravitational Bohr-radius has never been considered before elsewhere.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

2 Some novel aspects of Q-relativity physics. Pioneer blueshift anomaly

In this section, first we will review some basic concepts of quaternion number and then discuss its implications to quaternion relativity (Q-relativity) physics [2]. Then we discuss Yefremov's calculation of satellite time-shift which may be observed by precise measurement [3]. We however introduce a new interpretation here that such a satellite Q-timeshift is already observed in the form of Pioneer spacecraft blueshift anomaly.

Quaternion number belongs to the group of "very good" algebras: of real, complex, quaternion, and octonion [2]. While Cayley also proposed new terms such as quantic, it is less known than the above group. Quaternion number can be viewed as an extension of Cauchy imaginary plane to become [2]:

$$Q \equiv a + bi + cj + dk\,, \tag{1}$$

where a, b, c, d are real numbers, and i, j, k are imaginary quaternion units. These Q-units can be represented either via 2×2 matrices or 4×4 matrices [2].

It is interesting to note here that there is quaternionic multiplication rule which acquires compact form:

$$1q_k = q_k 1 = q_k\,, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n\,, \tag{2}$$

where δ_{kn} and ε_{jkn} represent 3-dimensional symbols of Kronecker and Levi-Civita, respectively [2]. Therefore it could be expected that Q-algebra may have neat link with pseudo-Riemannian metric used by General Relativity. Interestingly, it has been argued in this regard that such Q-units can be generalised to become Finsler geometry, in particular with Berwald-Moor metric. It also can be shown that Finsler-Berwald-Moor metric is equivalent with pseudo-Riemannian metric, and an expression of Newtonian potential can be found for this metric [2a].

It may also be worth noting here that in 3D space Q-connectivity has clear geometrical and physical treatment as movable Q-basis with behaviour of Cartan 3-frame [2].

It is also possible to write the dynamics equations of Classical Mechanics for an inertial observer in constant Q-basis. $SO(3,R)$-invariance of two vectors allow to represent these dynamics equations in Q-vector form [2]:

$$m\frac{d^2}{dt^2}\left(x_k q_k\right) = F_k q_k\,. \tag{3}$$

Because of antisymmetry of the connection (generalised angular velocity) the dynamics equations can be written in vector components, by conventional vector notation [2]:

$$m\left(\vec{a} + 2\vec{\Omega}\times\vec{v} + \vec{\Omega}\times\vec{r} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right)\right) = \vec{F}\,. \tag{4}$$

Therefore, from equation (4) one recognizes known types of classical acceleration, i.e. linear, coriolis, angular, centripetal. Meanwhile it is known that General Relativity introduces Newton potential as *rigid* requirement [2a, 6b]. In other words, we can expect — using Q-relativity — to predict new effects that cannot be explained with General Relativity.

From this viewpoint one may consider a generalisation of Minkowski metric into biquaternion form [2]:

$$dz = \left(dx_k + i dt_k\right) q_k\,, \tag{5}$$

with some novel properties, i.e.:

- temporal interval is defined by imaginary vector;
- space-time of the model appears to have six dimensions (6D);
- vector of the displacement of the particle and vector of corresponding time change must always be normal to each other, or:

$$dx_k dt_k = 0\,. \tag{6}$$

It is perhaps quite interesting to note here that Einstein himself apparently once considered similar approach, by proposing tensors with Riemannian metric with Hermitian symmetry [8]. Nonetheless, there is difference with Q-relativity described above, because in Einstein's generalised Riemannian metric it has 8-dimensions, rather than 3d-space and 3d-imaginary time.

One particularly interesting feature of this new Q-relativity (or rotational relativity) is that there is universal character of motion of the bodies (including non-inertial motions), which can be described in unified manner (Hestenes also considers Classical Mechanics from similar spinor language). For instance advanced perihelion of planets can be described in term of such rotational precession [2].

Inspired by this new Q-relativity physics, it can be argued that there should be anomalous effect in planets' satellite motion. In this regard, Yefremov argues that there should be a deviation of the planetary satellite position, due to discrepancy between calculated and observed from the Earth motion magnitudes characterizing cyclic processes on this planet or near it. He proposes [2]:

$$\Delta\varphi \approx \frac{\omega V_e V_p}{c^2}\,t\,, \tag{7}$$

or

$$\Delta\varphi' \approx -\frac{\omega V_e V_p}{c^2}\,t'. \tag{8}$$

Therefore, given a satellite orbit radius r, its position shift is found in units of length $\Delta l = r\,\Delta\varphi$. His calculation

Satellites	Cycle frequency ω, 1/s	Angular shift $\Delta\varphi$, $''$/100 yrs	Linear shift Δl, km/100 yrs	Linear size a, km
Phobos (Mars)	0.00023	18.2	54	20
Deimos (Mars)	0.00006	4.6	34	12
Metis (Jupiter)	0.00025	**10.6**	431	40
Adrastea (Jupiter)	0.00024	**10.5**	429	20
Amalthea (Jupiter)	0.00015	6.3	361	189

Table 1: The following table gives values of the effect for five fast satellites of Mars and Jupiter. Orbital linear velocities are: of the Earth $V_E = 29.8$ km/s, of Mars $V_P = 24.1$ km/s, of Jupiter $V_P = 13.1$ km/s; the value of the light velocity is $c = 299\,793$ km/s; observation period is chosen 100 years. Courtesy of A. Yefremov, 2006 [3].

for satellites of Mars and Jupiter is given in Table 1. Nonetheless he gave no indication as to how to observe this anomalous effect.

In this regard, we introduce here an alternative interpretation of the aforementioned Q-satellite time-shift effect by Yefremov, i.e. this effect actually has similar effect with Pioneer spacecraft blueshift anomaly. It is known that Pioneer spacecraft exhibits this anomalous Doppler frequency while entering Jupiter orbit [1, 4], therefore one may argue that this effect is caused by Jupiter planetary gravitational effect, which also may cause similar effect to its satellites.

Despite the apparent contradiction with Yefremov's own intention, one could find that the aforementioned Q-satellite time-shift could yield a natural explanation of Pioneer spacecraft blueshift anomaly. In this regard, Taylor [9] argues that there is possibility of a mundane explanation of anomalous blueshift of Pioneer anomaly (5.99×10^{-9} Hz/sec). The all-angle formulae for relativistic Doppler shift is given by [9a, p.34]:

$$v' = v_0 \gamma \frac{(1 - \beta \cos\phi)}{\sqrt{1-\beta^2}}\,, \tag{9}$$

where $\beta = v/c$. By neglecting the $\sqrt{1-\beta^2}$ term because of low velocity, one gets the standard expression:

$$v' = v_0 \gamma (1 - \beta \cos\phi)\,. \tag{9a}$$

The derivative with respect to ϕ is:

$$\frac{dv'}{d\phi} = v_0 \gamma \beta \sin\phi\,, \tag{10}$$

where $\frac{dv'}{d\phi} = 5.99\times10^{-9}$ Hz/sec, i.e. the observed Pioneer anomaly. Introducing this value into equation (10), one gets requirement of an effect to explain Pioneer anomaly:

$$d\phi = \frac{\arcsin\left(5.99\times10^{-9}\,\text{Hz}\right)}{v_0 \gamma \beta} = 1.4\times10^{-12}\ \text{deg/sec}. \tag{11}$$

Therefore, we can conclude that to explain 5.99×10^{-9} Hz/sec blueshift anomaly, it is required to find a shift of emission angle at the order 1.4×10^{-12} degree/sec only (or around $15.894''$ per 100 years).

Interestingly this angular shift can be explained with the same order of magnitude from the viewpoint of Q-satellite angular shift (see Table 1), in particular for Jupiter's Adrastea ($10.5''$ per 100 years). There is however, a large discrepancy at the order of 50% from the expected angular shift.

It is proposed here that such discrepancy between Q-satellite angular shift and expected angular shift required to explain Pioneer anomaly can be reduced if we take into consideration the "aether drift" effect [6]. Interestingly we can use experimental result of Thorndike [6, p.9], saying that the aether drift effect implies a residual apparent Earth velocity is $v_{obs} = 15 \pm 4$ km/sec. Therefore the effective V_e in equation (8) becomes:

$$V_{e.eff} = v_{obs} + V_e = 44.8\ \text{km/sec}. \tag{12}$$

Using this improved value for Earth velocity in equation (8), one will get larger values than Table 1, which for Adrastea satellite yields:

$$\Delta\varphi_{obs} = \frac{\omega V_{e.eff} V_p}{c^2}\, t = \frac{V_{e.eff}}{V_e}\, \Delta\varphi = 15.935''/100\ \text{yrs}. \tag{13}$$

Using this improved prediction, the discrepancy with required angular shift only ($15.894''$ per 100 years) becomes $\sim 0.26\%$, which speaks for itself. Therefore one may conclude that this less mundane explanation of Pioneer blueshift anomaly with Q-relativity may deserve further consideration.

3 A new type of redshift from gravitational Bohr radius. Possible observation in solar system.

In preceding paper [10, 11] we argued in favour of an alternative interpretation of Tifft redshift quantization from the viewpoint of quantized distance between galaxies. A method can be proposed as further test of this proposition both at solar system scale or galaxies scale, by using the known quantized Tifft redshift [14, 15, 16]:

$$\delta r \approx \frac{c}{H}\, \delta z\,. \tag{14}$$

In this regards, we use gravitational Bohr radius equation:

$$r_n = n^2 \frac{GM}{v_0^2}\,. \tag{15}$$

Inserting equation (15) into (14), then one gets quantized redshift expected from gravitational Bohr radius:

$$z_n = \frac{H}{c} n^2 \frac{GM}{v_0^2} \tag{16}$$

which can be observed either in solar system scale or galaxies scale. To our present knowledge, this effect has never been described elsewhere before.

Therefore, it is recommended to observe such an accelerated Doppler-freequency shift, which for big jovian planets this effect may be detected. It is also worth noting here that according to equation (16), this new Doppler shift is quantized.

At this point one may also take into consideration a proposition by Moffat, regarding modification of Newtonian acceleration law to become [7]:

$$a(r) = -\frac{G_\infty M}{r^2} + K \frac{\exp(-\mu_\phi r)}{r^2} (1 + \mu_\phi r) \tag{17}$$

where

$$G_\infty = G\left[1 + \sqrt{\frac{M_0}{M}}\right]. \tag{17a}$$

Therefore equation (16) may be rewritten to become:

$$z_n \approx \frac{H}{c} n^2 \frac{GM}{v_0^2} \left[1 + \sqrt{\frac{M_0}{M}}\right] \approx \chi \frac{H}{c} n^2 \frac{GM}{v_0^2} \tag{18}$$

where n is integer (1, 2, 3, …) and:

$$\chi = \left[1 + \sqrt{\frac{M_0}{M}}\right]. \tag{18a}$$

To use the above equations, one may start by using Bell's suggestion that there is fundamental redshift $z = 0.62$ which is typical for various galaxies and quasars [14]. Assuming we can use equation (16), then by setting $n = 1$, we can expect to predict the mass of quasar centre or galaxy centre. Then the result can be used to compute back how time-variation parameter affects redshift pattern in equation (18). In solar system scale, time-varying radius may be observed in the form of changing Astronomical Unit [4].

This proposition, however, deserves further theoretical considerations. Further observation is also recommended in order to verify and explore further this proposition.

4 Concluding remarks

In the present paper it is argued that Pioneer anomalous blueshift may be caused by Pioneer spacecraft experiencing angular shift induced by similar Q-relativity effect which may also affect Jupiter satellites. By taking into consideration aether drift effect, the proposed method as described herein could predict Pioneer blueshift within ∼0.26% error range, which speaks for itself. Further observation is of course recommended in order to refute or verify this proposition.

Another new proposition of redshift quantization is also proposed from gravitational Bohr-radius which is consistent with Bohr-Sommerfeld quantization. It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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References

1. Anderson J.D., Campbell J.K., & Nieto M.M. arXiv: astro-ph/ 0608087; [1a] Nieto M.M. & Anderson J.D. arXiv: gr-qc/0507052.
2. Yefremov A. *Hypercomplex Numbers in Geometry and Physics*, 2004, v. 1(1), 105; [2a] Pavlov D. G. arXiv: math-ph/ 0609009.
3. Yefremov A. Private communication, October 2006. Email: a.yefremov@rudn.ru.
4. Laemmerzahl C. & Dittus H. Clocks and gravity, from quantum to cosmos. UCLA, 2006, http://www.physics.ucla.edu/ quantum_to_cosmos/q2c06/Laemmerzahl.pdf
5. Christianto V. *EJTP*, 2006, v. 3, No. 12, http://www.ejtp.com.
6. Consoli M. arXiv: physics/0306094, p. 9; [6a] Consoli M. *et al.* arXiv: gr-qc/0306105; [6b] arXiv: hep-ph/0109215.
7. Moffat J. arXiv: astro-ph/0602607.
8. Einstein A. *Ann. Math.*, 1945, v. 46; [8a] Einstein A. Relativity: the special and general theory. Crown Trade Paperback, New York, 1951, pp. 61, 66–70.
9. Taylor S. arXiv: physics/0603074; [9a] Engelhardt W. *Apeiron*, 2003, v. 10, No. 4, 34.
10. Smarandache F. & Christianto V. *Progress in Physics*, 2006, v. 4, 27–31.
11. Smarandache F. & Christianto V. *Progress in Physics*, 2006, v. 4, 37–40.
12. Smarandache F. & Christianto V. *Progress in Physics*, 2006, v. 2, 63–67.
13. Fischer U. arXiv: cond-mat/9907457; [13a] arXiv: cond-mat/ 0004339.
14. Bell M.B. arXiv: astro-ph/0111123; [14a] Bell M.B. arXiv: astro-ph/0305112; [14b] Bell M.B. arXiv: astro-ph/0305060.
15. Humphreys R. *TJ Archive*, v. 16, http://answersingenesis.org.
16. Múnera H. *Apeiron*, 1998, v. 5, No. 3–4.
17. Zurek W. (ed.) *Proc. Euroconference in Formation and Interaction of Topological Defects*, Plenum, 1995; arXiv: cond-mat/9502119.
18. Volovik G. arXiv: cond-mat/0507454.

Notes on Pioneer Anomaly Explanation by Sattellite-Shift Formula of Quaternion Relativity: Remarks on "Less Mundane Explanation of Pioneer Anomaly from Q-Relativity"

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Use of satellite shift formula emerging in Quaternion (Q-) model of relativity theory for explanation of Pioneer anomaly [1] is critically discussed. A cinematic scheme more suitable for the case is constructed with the help of Q-model methods. An appropriate formula for apparent deceleration resulting from existence of observer-object relative velocity is derived. Preliminary quantitative assessments made on the base of Pioneer 10/11 data demonstrate closure of the assumed "relativistic deceleration" and observed "Doppler deceleration" values.

1 Introduction. Limits of satellite-shift formula

Recently [1] there was an attempt to give an explanation of Pioneer anomaly essentially using formula for relativistic shift of planet's fast satellites observed from the Earth. This formula was derived within framework of Q-method developed to calculate relativistic effects using $\mathrm{SO}(1,2)$ form-invariant quaternion square root from space-time interval rather than the interval itself [2]; in particular this advantageously permits to describe relativistic motions of any non-inertial frames. The last option was used to find mentioned formula that describes cinematic situation comprising three Solar System objects: the Earth (with observer on it), a planet, and its satellite revolving with comparatively large angular velocity. Due to existence of Earth-planet relative velocity, not great though and variable but permanent, the cycle frequency of satellite rotation (observed from the Earth) is apparently less that in realty, i.e. the "planet's clock" is slowing down, and calculation shows that the gap is growing linearly with time. Visually it looks that the satellite position on its orbit is apparently behind an expected place. For very fast satellites (like Jupiter's Metis and Adrastea) and for sufficiently long period of time the effect can probably be experimentally detected. Same effect exists of course for Mars's satellites and it is computed that monthly apparent shift on its orbit of e.g. Phobos is about 50 meters (that is by the way can be important and taken into account when planning expedition of spacecraft closely approaching the moon).

In paper of F. Smarandache and V. Christianto [1] the discussed formula was used to describe famous Pioneer effect, implying that the last great acceleration the space probe received when approached very close to Jupiter; in particular data concerning Adrastea, whose location was as close to Jupiter as the space probe, were cited in [1]. Combined with ether drift effect the formula gives good coincidence (up to 0.26%) with value of emission angle shift required to explain observation data of Pioneer's signal Doppler residuals [3].

This surprisingly exact result nevertheless should not lead to understanding that obtained by Q-method mathematical description of a specific mechanical model can bear universal character and fit to arbitrary relativistic situation. One needs to recognize that Pioneer cinematic scheme essentially differs from that of the Earth-planet-satellite model; but if one tries to explain the Pioneer effect using the same relativistic idea as for satellite shift then an adequate cinematic scheme should be elaborated. Happily the Q-method readily offers compact and clear algorithm for construction and description of any relativistic models. In Section 2 a model referring observed frequency shift of Pioneer spacecraft signals to purely relativistic reasons is regarded; some quantitative assessments are made as well as conclusions on ability of the model to explain the anomaly. In Section 3 a short discussion is offered.

2 Earth-Pioneer Q-model and signal frequency shift

Paper [3] enumerates a number of factors attracted to analyze radio data received from Pioneer 10/11 spacecraft, among them gravitational planetary perturbations, radiation pressure, interplanetary media, General Relativity*, the Earth's precession and nutation. It is worth noting here that one significant factor, time delay caused by relative probe-observer motion, is not distinguished in [3]. The fact is understandable: relative motion of spacecraft and observer on the Earth is utterly non-inertial one; Special Relativity is not at all able to cope with the case while General Relativity methods involving specific metric and geodesic lines construction

*Unfortunately paper [3] does not indicate to what depth General Relativity is taken into account: whether only Newtonian gravity is modified by Schwarzschild, Kerr (or other) metrics, or cinematic effects are regarded too.

(with all curvature tensor components zero) or additional vector transport postulates are mathematically difficult. Contrary to this the Q-relativity method easily allows building of any non-inertial relativistic scheme; an example describing a spacecraft (probe) and an Earth's observer is given below.

Assume that Pioneer anomaly is a purely relativistic effect caused by existence of Earth-Pioneer relative velocity, variable but permanent. Construct respective model using the Q-method algorithm. Choose Q-frames. Let $\Sigma = (\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$ be the Earth's frame whose Cartesian directing vectors are given by quaternion "imaginary" units $\mathbf{q}_k$ obeying the multiplication rule*

$$1\,\mathbf{q}_k = \mathbf{q}_k\,1 = \mathbf{q}_k\,, \qquad \mathbf{q}_k\,\mathbf{q}_l = -\delta_{kl} + \varepsilon_{klj}\,\mathbf{q}_j\,. \quad (1)$$

Let Q-frame $\Sigma' = \{\mathbf{q}_{k'}\}$ belong to a probe. Suppose for simplicity that vectors $\mathbf{q}_2$, $\mathbf{q}_3$ are in the ecliptic plane as well as (approximately) the probe's trajectory. Assume that vector $\mathbf{q}_2$ of Σ is always parallel to Earth-probe relative velocity V. Now one is able to write rotational equation, main relation of Q-relativity, which ties two frames

$$\Sigma' = O_1^{-i\psi}\,\Sigma\,, \quad (2)$$

here $O_1^{-i\psi}$ is 3×3 orthogonal matrix of rotation about axis No. 1 at imaginary angle $-i\psi$

$$O_1^{-i\psi} = \begin{pmatrix} \cos(i\psi) & -\sin(i\psi) & 0 \\ \sin(-i\psi) & \cos(i\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cosh\psi & -i\sinh\psi & 0 \\ i\sinh\psi & \cosh\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

thus "converting" frame Σ into Σ'. The first row in the matrix equation (2)

$$\mathbf{q}_{1'} = \mathbf{q}_1\cosh\psi - \mathbf{q}_2\,i\sinh\psi$$

after straightforward algebra

$$\mathbf{q}_{1'} = \cosh\psi\,(\mathbf{q}_1 - \mathbf{q}_2\,i\tanh\psi) \;\Rightarrow \mathbf{q}_{1'} = \frac{dt}{dt'}(\mathbf{q}_1 - \mathbf{q}_2\,iV\psi)$$

with usual relativistic relations

$$V = \tanh\psi, \qquad dt = dt'\cosh\psi \quad (3)$$

acquires the form of basic cinematic space-time object of Q-relativity

$$idt'\,\mathbf{q}_{1'} = idt\,\mathbf{q}_1 + dr\,\mathbf{q}_2\,,$$

a specific quaternion square root from space-time interval of Special Relativity

$$(idt'\mathbf{q}_{1'})(idt'\mathbf{q}_{1'}) = (idt\mathbf{q}_1 + dr\mathbf{q}_2)(idt\mathbf{q}_1 + dr\mathbf{q}_2) \;\Rightarrow$$
$$\Rightarrow\; dt'^2 = dt^2 - dr^2,$$

dt' being proper time segment of the probe. Eq. (3) yields ratio for probe-Earth signal period (small compared to time of observation) $T = T'\cosh\psi$, i.e. observed from Earth the period is apparently longer than it really is. Vice versa, observed frequency $f = 1/T$ is smaller than the real one f'

$$f = \frac{1}{T} = \frac{1}{T\cosh\psi} = \frac{f'}{\cosh\psi} = f'\sqrt{1-(V/c)^2}, \quad (4)$$

or for small relative velocity

$$f \cong f'\left(1 - \frac{V^2}{2c^2}\right).$$

This means that there exists certain purely apparent relativistic shift of the probe's signal detected by the Earth observer

$$\Delta f = f' - f = f'\,\frac{V^2}{2c^2}\,, \quad \text{or} \quad \frac{\Delta f}{f'} = \frac{V^2}{2c^2} = \frac{\varepsilon}{c^2}\,, \quad (5)$$

ε being the probe's kinetic energy per unit mass computed in a chosen frame. Contrary to pure Doppler effect the shift given by Eq. (5) does not depend on the direction of relative velocity of involved objects since in fact it is just another manifestation of relativistic delay of time. Light coming to observer from any relatively (and arbitrary) moving body is universally "more red" than originally emitted signal; as well all other frequencies attributed to observed moving bodies are smaller then original ones, and namely this idea was explored for derivation of satellite shift formula.

Experimental observation of the frequency change (5) must lead to conclusion that there exists respective "Doppler velocity" V_D entering formula well known from Special Relativity

$$f = \frac{f'}{\sqrt{1-(V_D/c)^2}}\left(1 - \frac{V_D}{c}\cos\beta\right), \quad (6)$$

β being angle between velocity vector and wave vector of emitted signal. If $\beta = 0$ and smaller relativistic correction are neglected then Eq. (6) can be rewritten in the form similar to Eq. (5)

$$\frac{\Delta f}{f'} \cong \frac{V_D}{c^2}\,; \quad (7)$$

comparison of Eqs. (7) and (5) yields very simple formula for calculated (and allegedly existent) "Doppler velocity" corresponding to observed relativistic frequency change

$$V_D \cong \frac{\varepsilon}{c}\,. \quad (8)$$

Estimation of the value of V_D can be done using picture of Pioneer 10/11 trajectories (Fig. 1) projected upon ecliptic plane (provided in NASA report [4]); other spacecraft traces are also shown, the Earth's orbit radius too small to be indicated.

Schematically the cinematic situation for Pioneer 10 is shown at Fig. 2 where the trajectory looks as a straight line inclined at constant angle λ to axis $\mathbf{q}_2$, while the Earth's position on its orbit is determined by angle $\alpha = \Omega t$, $\Omega = 3.98\times10^{-7}\,s^{-1}$ being the Earth's orbital angular velocity. Vectors of the probe's and Earth's velocities in Solar Ecliptic

*Latin indices are 3-dimensional (3D), δ_{kl} is 3D Kroneker symbol, ε_{jkl} is 3D Levi-Civita symbol; summation convention is assumed.

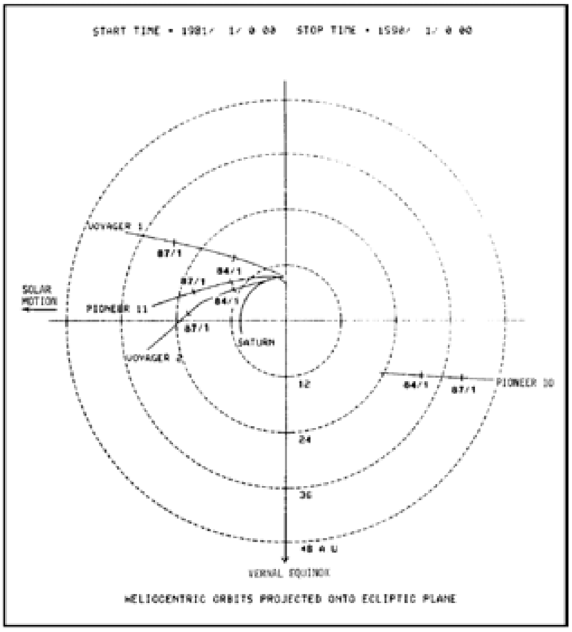

Fig. 1: Spacecraft trajectories on the ecliptic plane. (After NASA original data [4]. Used by permission.)

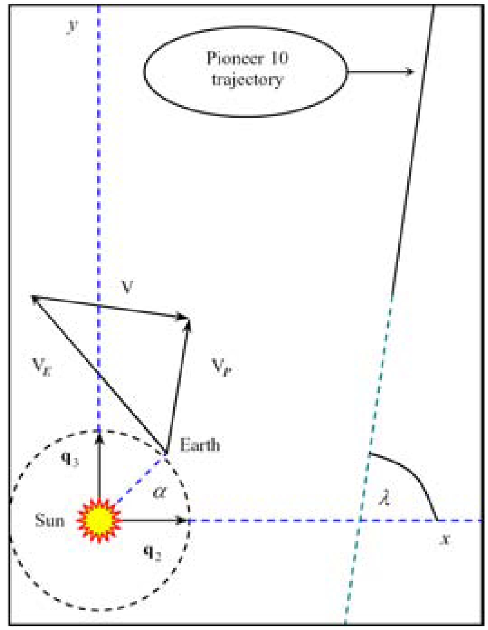

Fig. 2: Earth-Pioneer 10 cinematic scheme, where the trajectory looks as a straight line inclined at constant angle λ to axis $\mathbf{q}_2$.

(SE) coordinate system* are respectively denoted as $\mathbf{V}_P$ and $\mathbf{V}_E$; their vector subtraction gives relative Earth-probe velocity $\mathbf{V} = \mathbf{V}_P - \mathbf{V}_E$ so that

$$V_D(t) = \frac{V^2}{2c} = \frac{V_P^2 + V_E^2 - 2V_P V_E \cos(\Omega t - \lambda)}{2c}, \tag{9}$$

and respective "Doppler acceleration" is

$$a_D = \dot{V}_D(t) = \\ = \frac{V_P \dot{V}_P - \dot{V}_P V_E \cos(\Omega t - \lambda) + \Omega V_P V_E \sin(\Omega t - \lambda)}{c}. \tag{10}$$

In Eq. (10) the first term in the numerator claims existence of secular deceleration, since escaping from the Sun's and Jupiter's gravity the probe is permanently decelerated, $\dot{V}_p < 0$; the result is that the frequency gap shrinks giving rise to pure relativistic blue shift. Other sign-changing terms in right-hand-side of Eq. (10) are periodic (annual) ones; they may cause blue shift as well as red shift. Thus Eq. (10) shows that, although relative probe-Earth velocity incorporates into difference between real and observed frequency, nevertheless secular change of the difference is to be related only to relative probe-Sun velocity. Distinguish this term temporary ignoring the annual modulations; then the secular deceleration formula is reduced as

$$a_{SD} \cong \frac{\dot{V}_P V_P}{c}. \tag{11}$$

*The SE is a heliocentric coordinate system with the z-axis normal to and northward from the ecliptic plane. The x-axis extends toward the first point of Aries (Vernal Equinox, i.e. to the Sun from Earth in the first day of Spring). The y-axis completes the right handed set.

Below only radial components of the probe's velocity and acceleration in Newtonian gravity are taken into account in Eq. (11); it is quite a rough assessment but it allows to conceive order of values. The probe's acceleration caused by the Sun's Newtonian gravity is

$$\dot{V}_P = -\frac{GM_\odot}{R^2}, \tag{12}$$

$G = 6.67\times10^{-11}$ m^3/kg$\times$s^2, $M_\odot = 1.99\times10^{30}$ kg are respectively gravitational constant and mass of the Sun. NASA data [5] show that in the very middle part (1983–1990) of the whole observational period of Pioneer 10 its radial distance from the Sun changes from $R \cong 28.8$ AU $= 4.31\times10^{12}$ m to $R \cong 48.1$ AU $= 7.2\times10^{12}$ m, while year-mean radial velocity varies from $V_P = 15.18\times10^3$ m/s to $V_P = 12.81\times10^3$ m/s. Respective values of the secular "relativistic deceleration" values for this period computed with the help of Eqs. (11), (12) vary from $a_{SD} = -3.63\times10^{-10}$ m/s^2 to $a_{SD} = -1.23\times10^{-10}$ m/s^2. It is interesting (and surprising as well) that these results are very close in order to anomalous "Doppler deceleration" of the probe $a_P = -(8\pm3)\times10^{-10}$ m/s^2 cited in [3].

Analogous computations for Pioneer 11, as checking point, show the following. Full time of observation of Pioneer 11 is shorter so observational period is taken from 1984 to 1989, with observational data from the same source [5]. Radial distances for beginning and end of the period are $R \cong 15.1$ AU $= 2.26\times10^{12}$ m, $R \cong 25.2$ AU $= 3.77\times10^{12}$ m; respective year-mean radial velocities are $V_P = 11.86\times10^3$ m/s, $V_P = 12.80\times10^3$ m/s. Computed "relativistic deceleration" values for this period are then $a_{SD} = -10.03\times10^{-10}$ m/s^2,

$a_{SD} = -5.02\times10^{-10}$ m/s^2: this is even in much better correlation (within limits of the cited error) with experimental value of a_P.

3 Discussion

Quantitative estimations presented above allow to conclude: additional blue shift, experimentally registered in Pioneer 10 and 11 signals, and interpreted as Sun-directed acceleration of the spacecraft to some extent, support the assumption of pure relativistic nature of the anomaly. Of course one notes that while Pioneer 11 case shows good coincidence of observed and calculated values of deceleration, values of a_{SD} for Pioneer 10 constitute only (45–15)% of observed Doppler residual; moreover generally in this approach "relativistic deceleration" is a steadily decreasing function, while experimentally (though not directly) detected deceleration a_P is claimed nearly constant. These defects could find explanation first of all in the fact that a primitive "Newtonian radial model" was used for assessments. Preliminary but more attentive reference to NASA data allows noticing that observed angular acceleration of the probes too could significantly incorporate to values of "relativistic deceleration". This problem remains to be regarded elsewhere together with analysis of the angular acceleration itself.

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References

1. Smarandache F. and Christianto V. *Progr. in Phys.*, 2007, v. 1, 42–45.
2. Yefremov A. *Grav. & Cosmol.*, 1996, v. 2, No. 4, 335–341.
3. Anderson J. D. et al. arXiv: gr-qc/9808081.
4. http://cohoweb.gsfc.nasa.gov/helios/book1/b1_62.html
5. http://cohoweb.gsfc.nasa.gov/helios/book2/b2_03.html

Reply to "Notes on Pioneer Anomaly Explanation by Satellite-Shift Formula of Quaternion Relativity"

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In the present article we would like to make a few comments on a recent paper by A. Yefremov in this journal [1]. It is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration. We argue that perhaps it will be necessary to consider extension of Lorentz transformation to Finsler-Berwald metric, as discussed by a number of authors in the past few years. In this regard, it would be interesting to see if the use of extended Lorentz transformation could also elucidate the long-lasting problem known as Ehrenfest paradox. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

We are delighted to read A. Yefremov's comments on our preceding paper [3], based on his own analysis of Pioneer anomalous "apparent acceleration" [1]. His analysis made use of a method called Quaternion Relativity, which essentially is based on $\mathrm{SO}(1,2)$ form invariant quaternion square root from space-time interval rather than the interval itself [1, 2]. Nonetheless it is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration [1].

In this regard, we would like to emphasize that our preceding paper [3] was based on initial "conjecture" that in order to explain Pioneer anomaly, it would be necessary to generalize pseudo-Riemann metric of General Relativity theory into broader context, which may include Yefremov's Quaternion Relativity for instance. It is interesting to note here, however, that Yefremov's analytical method keeps use standard Lorentz transformation in the form Doppler shift effect (Eq. 6):

$$f = \frac{f'}{\sqrt{1-\left(\frac{v_D}{c}\right)^2}}\left(1-\frac{v_D}{c}\cos\beta\right). \tag{1}$$

While his method using relativistic Doppler shift a la Special Relativity is all right for such a preliminary analysis, in our opinion this method has a drawback that it uses "standard definition of Lorentz transformation" based on 2-dimensional problem of *rod-on-rail* as explained in numerous expositions of relativity theory [5]. While this method of rod-on-rail seems sufficient to elucidate why "simultaneity" is ambiguous term in physical sense, it does not take into consideration 3-angle problem in more general problem. This is why we pointed out in our preceding paper that apparently General Relativity inherits the same drawback from Special Relativity [3].

Another problem of special relativistic definition of Lorentz transformation is known as "reciprocity postulate", because in Special Relativity it is assumed that: $x \leftrightarrow x'$, $t \leftrightarrow t'$, $v \leftrightarrow -v'$ [6]. This is why Doppler shift can be derived without assuming reciprocity postulate (which may be regarded as the "third postulate" of Special Relativity) and without special relativistic argument, see [7]. Nonetheless, in our opinion, Yefremov's Quaternion Relativity is free from this "reciprocity" drawback because in his method there is difference between moving-observer and static-observer [2].

An example of implications of this drawback of 1-angle problem of Lorentz transformation is known as Ehrenfest paradox, which can be summarized as follows: "According to Special Relativity, a moving rod will exhibit apparent length-reduction. This is usually understood to be an observational effect, but if it is instead considered to be a real effect, then there is a paradox. According to Ehrenfest, the perimeter of a rotating disk is like a sequence of rods. So does the rotating disk shatter at the rim?" Similarly, after some thought Klauber concludes that "*The second relativity postulate does not appear to hold for rotating systems*" [8].

While it is not yet clear whether Quaternion-Relativity is free from this Ehrenfest paradox, we would like to point out that an alternative metric which is known to be nearest to Riemann metric is available in literature, and known as Finsler-Berwald metric. This metric has been discussed adequately by Pavlov, Asanov, Vacaru and others [9–12].

2 Extended Lorentz-transformation in Finsler-Berwald metric

It is known that Finsler-Berwald metric is subset of Finslerian metrics which is nearest to Riemannian metric [12], therefore it is possible to construct pseudo-Riemann metric based on Berwald-Moor geometry, as already shown by Pavlov [4]. The neat link between Berwald-Moor metric and Quaternion Relativity of Yefremov may also be expected because Berwald-Moor metric is also based on analytical functions of the H4 variable [4].

More interestingly, there was an attempt in recent years to extend 2d-Lorentz transformation in more general framework on H4 of Finsler-Berwald metric, which in limiting cases will yield standard Lorentz transformation [9, 10]. In this letter we will use extension of Lorentz transformation derived by Pavlov [9]. For the case when all components but one of the velocity of the new frame in the old frame coordinates along the three special directions are equal to zero, then the transition to the frame moving with velocity V_1 in the old coordinates can be expressed by the new frame as [9, p.13]:

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} [F] & [0] \\ [0] & [F] \end{bmatrix} = \begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} \tag{2}$$

where the transformation matrix for Finsler-Berwald metric is written as follows [9, p.13]:

$$[F] = \begin{pmatrix} \frac{1}{\sqrt{1-V_1^2}} & \frac{V_1}{\sqrt{1-V_1^2}} \\ \frac{V_1}{\sqrt{1-V_1^2}} & \frac{1}{\sqrt{1-V_1^2}} \end{pmatrix} \tag{3}$$

and

$$[0] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{4}$$

Or

$$x_0 = \frac{x'_0 + Vx'_1}{\sqrt{1-V_1^2}}\, x_1 = \frac{Vx'_0 + x'_1}{\sqrt{1-V_1^2}}\,, \tag{5}$$

and

$$x_2 = \frac{x'_2 + Vx'_3}{\sqrt{1-V_1^2}}\, x_3 = \frac{Vx'_2 + x'_3}{\sqrt{1-V_1^2}}. \tag{6}$$

It shall be clear that equation (5) $(x'_0, x'_1) \leftrightarrow (x_0, x_1)$ coincides with the corresponding transformation of Special Relativity, while the transformation in equation (6) differs from the corresponding transformation of Special Relativity where $x_2 = x'_2$, $x_3 = x'_3$ [9].

While we are not yet sure whether the above extension of Lorentz transformation could explain Pioneer anomaly better than recent analysis by A. Yefremov [1], at least it can be expected to see whether Finsler-Berwald metric could shed some light on the problem of Ehrenfest paradox. This proposition, however, deserves further theoretical considerations.

In order to provide an illustration on how the transformation keeps the Finslerian metric invariant, we can use Maple algorithm presented by Asanov [10, p.29]:

```
> c1:=cos(tau);c2:=cos(psi);c3:=cos(phi);
> u1:=sin(tau);u2:=sin(psi);u3:=sin(phi);
> l1:=c2*c3−c1*u2*u3;l2:=−c2*u3−c1*u2*c3;l3:=u1*u2;
> m1:=u2*c3+c1*c2*u3;m2:=−u2*u3+c1*c2*c3;m3:=−u1*c2;
> n1:=u1*u3; u1*c3; c1;
> F1:=(e1)ˆ((l1+m1+n1+l2+m2+n2+l3+m3+n3+1)/4)*
(e2)ˆ((−l1−m1−n1+l2+m2+n2−l3−m3−n3+1)/4)*
(e3)ˆ((l1+m1+n1−l2−m2−n2−l3−m3−n3+1)/4)*
(e4)ˆ((−l1−m1−n1−l2−m2−n2+l3+m3+n3+1)/4):
> F2:=(e1)ˆ((−l1+m1−n1−l2+m2−n2−l3+m3−n3+1)/4)*
(e2)ˆ((l1−m1+n1−l2+m2−n2+l3−m3+n3+1)/4)*
(e3)ˆ((−l1+m1−n1+l2−m2+n2+l3−m3+n3+1)/4)*
(e4)ˆ((l1−m1+n1+l2−m2+n2−l3+m3−n3+1)/4):
> F3:=(e1)ˆ((l1−m1−n1+l2−m2−n2+l3−m3−n3+1)/4)*
(e2)ˆ((−l1+m1+n1+l2−m2−n2−l3+m3+n3+1)/4)*
(e3)ˆ((l1−m1−n1−l2+m2+n2−l3+m3+n3+1)/4)*
(e4)ˆ((−l1+m1+n1−l2+m2+n2+l3−m3−n3+1)/4):
> F4:=(e1)ˆ((−l1−m1+n1−l2−m2+n2−l3−m3+n3+1)/4)*
(e2)ˆ((l1+m1−n1−l2−m2+n2+l3+m3−n3+1)/4)*
(e3)ˆ((−l1−m1+n1+l2+m2−n2+l3+m3−n3+1)/4)*
(e4)ˆ((l1+m1−n1+l2+m2−n2−l3−m3+n3+1)/4):
> a:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
a[i,j]:=diff(F||i,e||j);
end do:
end do:
> b:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
b[i,j]:=simplify(add(1/F||k*diff(a[k,i],e||j),k=1..4),symbolic);
end do:
end do:
> print(b);
```

The result is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

This result showing that all the entries of the matrix are zeroes support the argument that the metricity condition is true [10].

3 Concluding remarks

In the present paper we noted that it is possible to generalise standard Lorentz transformation into H4 framework of Finsler-Berwald metric. It could be expected that this extended Lorentz transformation could shed some light not only to Pioneer anomaly, but perhaps also to the long-lasting problem of Ehrenfest paradox which is also problematic in General Relativity theory, or by quoting Einstein himself:

> "... Thus all our previous conclusions based on general relativity would appear to be called in question. In reality we must make a subtle detour in order to be able to apply the postulate of general relativity exactly" [5].

This reply is not intended to say that Yefremov's preliminary analysis is not in the right direction, instead we only highlight a possible way to improve his results (via extending Lorentz transformation). Furthermore, it also does not mean to say that Finsler-Berwald metric could predict better than Quaternion Relativity. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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References

1. Yefremov A. Notes on Pioneer anomaly explanation by satellite-shift formula of Quaternion Relativity. *Progress in Physics*, 2007, v. 3, 93–96.
2. Yefremov A. Quaternions: algebra, geometry and physical theories. *Hypercomplex Numbers in Geometry and Physics*, 2004, v. 1(1), 105.
3. Smarandache F. and Christianto V. Less mundane explanation of Pioneer anomaly from Q-relativity. *Progress in Physics*, 2007, v. 1, 42–45.
4. Pavlov D. G. Construction of the pseudo Riemannian geometry on the base of the Berwald-Moor geometry. arXiv: math-ph/0609009.
5. Einstein A. Relativity: the special and general theory. Crown Trade Paperback, New York, 1951, 66–70 (quoted from the ebook version, p. 97: http://www.ibiblio.org/ebooks/Einstein/Einstein_Relativity.pdf); [5a] Janssen M. http://www.tc.umn.edu/∼janss011/pdf%20files/appendix-SR.pdf; [5b] http://www.ph.utexas.edu/∼gleeson/NotesChapter11.pdf.
6. De Lange O. L. Comment on Space-time exchange invariance: Special relativity as a symmetry principle. *Am. J. Phys.*, 2002, v. 70, 78–79.
7. Engelhardt W. Relativistic Doppler effect and the principle of relativity. *Apeiron*, 2003, v. 10, No. 4, 29–49.
8. Klauber R. Toward a consistent theory of relativistic rotation. In: *Relativity in Rotating Frames*, Kluwer Academic, 2004; arXiv: physics/0404027.
9. Pavlov D. *et al.* he notions of distance and velocity modulus in the linear Finsler spaces. *Hypercomplex Numbers in Geometry and Physics*, 2005, v. 1(3), 13.
10. Asanov G. S. Finslerian metric function of totally anisotropic type. Relativistic aspects. arXiv: math-ph/0510007; [10a] Asanov G. S. arXiv: math-ph/0510049.
11. Vacaru S., Stavrinos P., Gaburov E. and Gonta D. Clifford and Riemann-Finsler structures in geometric mechanics and gravity. Geometry Balkan Press, 2005; arXiv: gr-qc/0508023, p. 39.
12. Szabo Z. Berwald metrics constructed by Chevalley's polynomials. arXiv: math.DG/0601522.
13. Weber T. Measurements on a rotating frame in relativity. *Am. J. Phys.*, 1997, v. 65, 946–953.

Yang-Mills Field from Quaternion Space Geometry, and Its Klein-Gordon Representation

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Analysis of covariant derivatives of vectors in quaternion (Q-) spaces performed using Q-unit spinor-splitting technique and use of SL(2C)-invariance of quaternion multiplication reveals close connexion of Q-geometry objects and Yang-Mills (YM) field principle characteristics. In particular, it is shown that Q-connexion (with quaternion non-metricity) and related curvature of 4 dimensional (4D) space-times with 3D Q-space sections are formally equivalent to respectively YM-field potential and strength, traditionally emerging from the minimal action assumption. Plausible links between YM field equation and Klein-Gordon equation, in particular via its known isomorphism with Duffin-Kemmer equation, are also discussed.

1 Introduction

Traditionally YM field is treated as a gauge, "*auxiliary*", field involved to compensate local transformations of a 'main' (e.g. spinor) field to keep invariance of respective action functional. Anyway there are a number of works where YM-field features are found related to some geometric properties of space-times of different types, mainly in connexion with contemporary gravity theories.

Thus in paper [1] violation of $SO(3,1)$-covariance in gauge gravitation theory caused by distinguishing time direction from normal space-like hyper-surfaces is regarded as spontaneous symmetry violation analogous to introduction of mass in YM theory. Paper [2] shows a generic approach to formulation of a physical field evolution based on description of differential manifold and its mapping onto "model" spaces defined by characteristic groups; the group choice leads to gravity or YM theory equations. Furthermore it can be shown [2b] that it is possible to describe altogether gravitation in a space with torsion, and electroweak interactions on 4D real spacetime C^2, so we have in usual spacetime with torsion a unified theory (modulo the non treatment of the strong forces).

Somewhat different approach is suggested in paper [3] where gauge potentials and tensions are related respectively to connexion and curvature of principle bundle, whose base and gauge group choice allows arriving either to YM or to gravitation theory. Paper [4] dealing with gravity in Riemann-Cartan space and Lagrangian quadratic in connexion and curvature shows possibility to interpret connexion as a mediator of YM interaction.

In paper [5] a unified theory of gravity and electroweak forces is built with Lagrangian as a scalar curvature of space-time with torsion; if trace and axial part of the torsion vanish the Lagrangian is shown to separate into Gilbert and YM parts. Regardless of somehow artificial character of used models, these observations nonetheless hint that there may exist a deep link between supposedly really physical object, YM field and pure math constructions. A surprising analogy between main characteristics of YM field and mathematical objects is found hidden within geometry induced by quaternion (Q-) numbers.

In this regard, the role played by Yang-Mills field cannot be overemphasized, in particular from the viewpoint of the Standard Model of elementary particles. While there are a number of attempts for describing the Standard Model of hadrons and leptons from the viewpoint of classical electromagnetic Maxwell equations [6, 7], nonetheless this question remains an open problem. An alternative route toward achieving this goal is by using quaternion number, as described in the present paper. In fact, in Ref. [7] a somewhat similar approach with ours has been described, i.e. the generalized Cauchy-Riemann equations contain 2-spinor and C-gauge structures, and their integrability conditions take the form of Maxwell and Yang-Mills equations.

It is long ago noticed that Q-math (algebra, calculus and related geometry) naturally comprise many features attributed to physical systems and laws. It is known that quaternions describe three "imaginary" Q-units as unit vectors directing axes of a Cartesian system of coordinates (it was initially developed to represent subsequent telescope motions in astronomical observation). Maxwell used the fact to write his equations in the most convenient Q-form. Decades later Fueter discovered a formidable coincidence: a pure math Cauchy-Riemann type condition endowing functions of Q-variable with analytical properties turned out to be identical in shape to vacuum equations of electrodynamics [9].

Later on other surprising Q-math — physics coincidences were found. Among them: "automatic" appearance of Pauli magnetic field-spin term with Bohr magneton as a coefficient when Hamiltonian for charged quantum mechanical particle was built with the help of Q-based metric [10]; possibility to endow "imaginary" vector Q-units with properties of not only stationary but movable triad of Cartan type and use it for a very simple description of Newtonian mechanics in rotating frame of reference [11]; discovery of inherited in Q-math variant of relativity theory permitting to describe motion of non-inertial frames [12]. Preliminary study shows that YM field components are also formally present in Q-math.

In Section 2 notion of Q-space is given in necessary detail. Section 3 discussed neat analogy between Q-geometric objects and YM field potential and strength. In Section 4 YM field and Klein-Gordon correspondence is discussed. Concluding remarks can be found in Section 5.

Part of our motivation for writing this paper was to explicate the hidden electromagnetic field origin of YM fields. It is known that the Standard Model of elementary particles lack systematic description for the mechanism of quark charges. (Let alone the question of whether quarks do exist or they are mere algebraic tools, as Heisenberg once puts forth: *If quarks exist, then we have redefined the word "exist".*) On the other side, as described above, Maxwell described his theory in quaternionic language, therefore it seems natural to ask whether it is possible to find neat link between quaternion language and YM-fields, and by doing so provide one step toward describing mechanism behind quark charges.

Further experimental observation is of course recommended in order to verify or refute our propositions as described herein.

2 Quaternion spaces

Detailed description of Q-space is given in [13]; shortly but with necessary strictness its notion can be presented as following.

Let U_N be a manifold, a geometric object consisting of points $M \in U_N$ each reciprocally and uniquely corresponding to a set of N numbers-coordinates $\{y^A\}: M \leftrightarrow \{y^A\}$, $(A = 1, 2 \ldots N)$. Also let the sets of coordinates be transformed so that the map becomes a homeomorphism of a class C_k. It is known that U_N may be endowed with a proper tangent manifold T_N described by sets of orthogonal unite vectors $\mathbf{e}_{(A)}$ generating in T_N families of coordinate lines $M \to \{X^{(A)}\}$, indices in brackets being numbers of frames' vectors. Differentials of coordinates in U_N and T_N are tied as $dX^{(A)} = g^{(A)}_B dy^B$, with Lamé coefficients $g^{(A)}_B$, functions of y^A, so that $X^{(A)}$ are generally non-holonomic. Irrespectively of properties of U_N each its point may be attached to the origin of a frame, in particular presented by "imaginary" Q-units $\mathbf{q}_k$, this attachment accompanied by a rule tying values of coordinates of this point with the triad orientation $M \leftrightarrow \{y^A, \Phi_\xi\}$. All triads $\{\mathbf{q}_k\}$ so defined on U_N form a sort of "tangent" manifold $T(U, \mathbf{q})$, (really tangent only for the base U_3). Due to presence of frame vectors $\mathbf{q}_k(y)$ existence of metric and at least proper (quaternionic) connexion $\omega_{jkn} = -\omega_{jnk}$, $\partial_j \mathbf{q}_k = \omega_{jkn} \mathbf{q}_n$, is implied, hence one can tell of $T(U, \mathbf{q})$ as of a Q-tangent space on the base U_N. Coordinates x_k defined along triad vectors $\mathbf{q}_k$ in $T(U, \mathbf{q})$ are tied with non-holonomic coordinates $X^{(A)}$ in proper tangent space T_N by the transformation $dx_k \equiv h_{k(A)} dX^{(A)}$ with $h_{k(A)}$ being locally depending matrices (and generally not square) of relative $\mathbf{e}_{(A)} \leftrightarrow \mathbf{q}_k$ rotation. Consider a special case of unification $U \oplus T(U, \mathbf{q})$ with 3-dimensional base space $U = \mathbf{U}_3$. Moreover, let quaternion specificity of T_3 reflects property of the base itself, i.e. metric structure of $\mathbf{U}_3$ inevitably requires involvement of Q-triads to initiate Cartesian coordinates in its tangent space. Such 3-dimensional space generating sets of tangent quaternionic frames in each its point is named here "quaternion space" (or simply Q-space). Main distinguishing feature of a Q-space is non-symmetric form of its metric tensor* $\mathbf{g}_{kn} \equiv \mathbf{q}_k \mathbf{q}_n = -\delta_{kn} + \varepsilon_{knj} \mathbf{q}_j$ being in fact multiplication rule of "imaginary" Q-units. It is easy to understand that all tangent spaces constructed on arbitrary bases as designed above are Q-spaces themselves. In most general case a Q-space can be treated as a space of affine connexion $\Omega_{jkn} = \Gamma_{jkn} + Q_{jkn} + S_{jkn} + \omega_{jnk} + \sigma_{jkn}$ comprising respectively Riemann connexion Γ_{jkn}, Cartan contorsion Q_{jkn}, segmentary curvature (or ordinary non-metricity) S_{jkn}, Q-connexion ω_{jnk}, and Q-non-metricity σ_{jkn}; curvature tensor is given by standard expression $R_{knij} = \partial_i \Omega_{j\,kn} - \partial_j \Omega_{i\,kn} + \Omega_{i\,km} \Omega_{j\,mn} - \Omega_{j\,nm} \Omega_{i\,mk}$. Presence or vanishing of different parts of connexion or curvature results in multiple variants of Q-spaces classification [13]. Further on only Q-spaces with pure quaternionic characteristics (Q-connexion and Q-non-metricity) will be considered.

3 Yang-Mills field from Q-space geometry

Usually Yang-Mills field $A_{B\mu}$ is introduced as a gauge field in procedure of localized transformations of certain field, e.g. spinor field [14, 15]

$$\psi_a \to U(y^\beta)\, \psi_a\,. \tag{1}$$

If in the Lagrangian of the field partial derivative of ψ_a is changed to "covariant" one

$$\partial_\beta \to D_\beta \equiv \partial_\beta - g A_\beta\,, \tag{2}$$

*Latin indices are 3D, Greek indices are 4D; δ_{kn}, ε_{knj} are Kronecker and Levi-Civita symbols; summation convention is valid.

$$A_\beta \equiv i A_{C\,\beta}\,\mathbf{T}_C\,, \tag{3}$$

where g is a real constant (parameter of the model), $\mathbf{T}_C$ are traceless matrices (Lie-group generators) commuting as

$$[\mathbf{T}_B,\,\mathbf{T}_C] = i f_{BCD}\,\mathbf{T}_D \tag{4}$$

with structure constants f_{BCD}, then

$$D_\beta U \equiv (\partial_\beta - g A_\beta)\,U = 0\,, \tag{5}$$

and the Lagrangian keeps invariant under the transformations (1). The theory becomes "self consistent" if the gauge field terms are added to Lagrangian

$$L_{YM} \sim F^{\alpha\beta} F_{\alpha\beta}\,, \tag{6}$$

$$F_{\alpha\beta} \equiv F_{C\,\alpha\beta}\,\mathbf{T}_C\,. \tag{7}$$

The gauge field intensity $F^{\mu\nu}_B$ expressed through potentials $A_{B\mu}$ and structure constants as

$$F_{C\,\alpha\beta} = \partial_\alpha A_{C\,\beta} - \partial_\beta A_{C\,\alpha} + f_{CDE}\,A_{D\,\alpha} A_{E\,\beta}\,. \tag{8}$$

Vacuum equations of the gauge field

$$\partial_\alpha F^{\alpha\beta} + \left[A_\alpha, F^{\alpha\beta}\right] = 0 \tag{9}$$

are result of variation procedure of action built from Lagrangian (6).

Group Lie, e.g. SU(2) generators in particular can be represented by "imaginary" quaternion units given by e.g. traceless 2×2-matrices in special representation (Pauli-type) $i\mathbf{T}_B \to \mathbf{q}_{\tilde{k}} = -i\sigma_k$ (σ_k are Pauli matrices),

Then the structure constants are Levi-Civita tensor components $f_{BCD} \to \varepsilon_{knm}$, and expressions for potential and intensity (strength) of the gauge field are written as:

$$A_\beta = g\,\frac{1}{2}\,A_{\tilde{k}\,\beta}\,\mathbf{q}_{\tilde{k}}\,, \tag{10}$$

$$F_{k\,\alpha\beta} = \partial_\alpha A_{k\,\beta} - \partial_\beta A_{k\,\alpha} + \varepsilon_{kmn}\,A_{m\,\alpha} A_{n\,\beta}\,. \tag{11}$$

It is worthnoting that this conventional method of introduction of a Yang-Mills field type essentially exploits *heuristic base* of theoretical physics, first of all the postulate of minimal action and formalism of Lagrangian functions construction. But since description of the field optionally uses quaternion units one can assume that some of the above relations are appropriate for Q-spaces theory and may have geometric analogues. To verify this assumption we will use an example of 4D space-time model with 3D spatial quaternion section.

Begin with the problem of 4D space-time with 3D spatial section in the form of Q-space containing only one geometric object: proper quaternion connexion. Q-covariant derivative of the basic (frame) vectors $\mathbf{q}_m$ identically vanish in this space:

$$\tilde{D}_\alpha\,\mathbf{q}_k \equiv (\delta_{mk}\,\partial_\alpha + \omega_{\alpha\,mk})\,\mathbf{q}_m = 0\,. \tag{12}$$

This equation is in fact equivalent to definition of the proper connexion $\omega_{\alpha\,mk}$. If a transformation of Q-units is given by spinor group (leaving quaternion multiplication rule invariant)

$$\mathbf{q}_k = U(y)\,\mathbf{q}_{\tilde{k}}\,U^{-1}(y) \tag{13}$$

($\mathbf{q}_{\tilde{k}}$ are constants here) then Eq. (12) yields

$$\partial_\alpha U\,\mathbf{q}_{\tilde{k}}\,U^{-1} + U\,\mathbf{q}_{\tilde{k}}\,\partial_\alpha U^{-1} = \omega_{\alpha k n}\,U\,\mathbf{q}_{\tilde{n}}\,U^{-1}. \tag{14}$$

But one can easily verify that each "imaginary" Q-unit $\mathbf{q}_{\tilde{k}}$ can be always represented in the form of tensor product of its eigen-functions (EF) $\psi_{(\tilde{k})}$, $\varphi_{(\tilde{k})}$ (no summation convention for indices in brackets):

$$\mathbf{q}_{\tilde{k}}\,\psi_{(\tilde{k})} = \pm i\psi_{(\tilde{k})}, \qquad \varphi_{(\tilde{k})}\mathbf{q}_{\tilde{k}} = \pm i\varphi_{(\tilde{k})} \tag{15}$$

having spinor structure (here only EF with positive parity (with sign +) are shown)

$$\mathbf{q}_{\tilde{k}} = i\,(2\psi_{(\tilde{k})}\varphi_{(\tilde{k})} - 1); \tag{16}$$

this means that left-hand-side (lhs) of Eq. (14) can be equivalently rewritten in the form

$$\begin{aligned} &\frac{1}{2}\,(\partial_\alpha U\,\mathbf{q}_{\tilde{k}}\,U^{-1} + U\,\mathbf{q}_{\tilde{k}}\,\partial_\alpha U^{-1}) = \\ &\quad = (\partial_\alpha U\,\psi_{(\tilde{k})})\,\varphi_{(\tilde{k})} U^{-1} + U\,\psi_{(\tilde{k})}\,(\varphi_{(\tilde{k})}\partial_\alpha U^{-1}) \end{aligned} \tag{17}$$

which strongly resembles use of Eq. (1) for transformations of spinor functions.

Here we for the first time underline a remarkable fact: *form-invariance of multiplication rule of Q-units under their spinor transformations gives expressions similar to those conventionally used to initiate introduction of gauge fields of Yang-Mills type.*

Now in order to determine mathematical analogues of these "physical fields", we will analyze in more details Eq. (14). Its multiplication (from the right) by combination $U\,\mathbf{q}_{\tilde{k}}$ with contraction by index $\tilde{k}$ leads to the expression

$$-3\,\partial_\alpha U + U\,\mathbf{q}_{\tilde{k}}\,\partial_\alpha U^{-1} U \mathbf{q}_{\tilde{k}} = \omega_{\alpha k n}\,U\,\mathbf{q}_{\tilde{n}}\,\mathbf{q}_{\tilde{k}}\,. \tag{18}$$

This matrix equation can be simplified with the help of the always possible development of transformation matrices

$$U \equiv a + b_k\,\mathbf{q}_{\tilde{k}}\,, \qquad U^{-1} = a - b_k\,\mathbf{q}_{\tilde{k}}\,, \tag{19}$$

$$UU^{-1} = a^2 + b_k\,b_k = 1\,, \tag{20}$$

where a, b_k are real scalar and 3D-vector functions, $\mathbf{q}_{\tilde{k}}$ are Q-units in special (Pauli-type) representation. Using Eqs. (19), the second term in lhs of Eq. (18) after some algebra is reduced to remarkably simple expression

$$\begin{aligned} &U\,\mathbf{q}_{\tilde{k}}\,\partial_\alpha U^{-1} U \mathbf{q}_{\tilde{k}} = \\ &= (a + b_n \mathbf{q}_{\tilde{n}})\,\mathbf{q}_{\tilde{k}}\,(\partial_\alpha a - \partial_\alpha b_m \mathbf{q}_{\tilde{m}})\,(a + b_l\,\mathbf{q}_{\tilde{l}})\,\mathbf{q}_{\tilde{k}} = \\ &= \partial_\alpha (a + b_n \mathbf{q}_{\tilde{n}}) = -\partial_\alpha U \end{aligned} \tag{21}$$

so that altogether lhs of Eq. (18) comprises $-4\,\partial_\alpha U$ while right-hand-side (rhs) is

$$\omega_{\alpha kn}\, U\,\mathbf{q}_{\tilde{n}}\mathbf{q}_{\tilde{k}} = -\,\varepsilon_{knm}\omega_{\alpha kn}\, U\,\mathbf{q}_{\tilde{m}}\,; \tag{22}$$

then Eq. (18) yields

$$\partial_\alpha U - \frac{1}{4}\,\varepsilon_{knm}\omega_{\alpha kn} U\,\mathbf{q}_{\tilde{m}} = 0\,. \tag{23}$$

If now one makes the following notations

$$A_{k\,\alpha} \equiv \frac{1}{2}\,\varepsilon_{knm}\,\omega_{\alpha kn}\,, \tag{24}$$

$$A_{\,\alpha} \equiv \frac{1}{2}\,A_n\mathbf{q}_{\tilde{n}}\,, \tag{25}$$

then notation (25) exactly coincides with the definition (10) (provided $g = 1$), and Eq. (23) turns out equivalent to Eq. (5)

$$U\overleftarrow{D}_\alpha \equiv U(\overleftarrow{\partial}_\alpha - A_\alpha) = 0\,. \tag{26}$$

Expression for "covariant derivative" of inverse matrix follows from the identity:

$$\partial_\alpha U\, U^{-1} = -\,U\partial_\alpha U^{-1}. \tag{27}$$

Using Eq. (23) one easily computes

$$-\,\partial_\alpha U^{-1} - \frac{1}{4}\,\varepsilon_{knm}\omega_{\alpha kn}\,\mathbf{q}_{\tilde{m}} U^{-1} = 0 \tag{28}$$

or

$$D_\alpha U^{-1} \equiv (\partial_\alpha + A_\alpha)\,U^{-1} = 0\,. \tag{29}$$

Direction of action of the derivative operator is not essential here, since the substitution $U^{-1} \to U$ и $U \to U^{-1}$ is always possible, and then Eq. (29) exactly coincides with Eq. (5).

Now let us summarize first results. We have a remarkable fact: form-invariance of Q-multiplication has as a corollary "covariant constancy" of matrices of spinor transformations of vector Q-units; moreover one notes that proper Q-connexion (contracted in skew indices by Levi-Civita tensor) plays the role of "gauge potential" of some Yang-Mills-type field. By the way the Q-connexion is easily expressed from Eq. (24)

$$\omega_{\alpha kn} = \varepsilon_{mkn} A_{m\,\alpha}\,. \tag{30}$$

Using Eq. (25) one finds expression for the gauge field intensity (11) (contracted by Levi-Civita tensor for convenience) through Q-connexion

$$\begin{aligned}
&\varepsilon_{kmn}F_{k\,\alpha\beta} = \\
&= \varepsilon_{kmn}(\partial_\alpha A_{k\,\beta} - \partial_\beta A_{k\,\alpha}) + \varepsilon_{kmn}\varepsilon_{mlj}A_{l\,\alpha}A_{j\,\beta} = \\
&= \partial_\alpha\omega_{\beta\,mn} - \partial_\beta\omega_{\,\alpha\,mn} + A_{m\,\alpha}A_{n\,\beta} - A_{m\beta}A_{n\alpha}\,.
\end{aligned} \tag{31}$$

If identically vanishing sum

$$-\delta_{mn}A_{j\,\alpha}A_{j\,\beta} + \delta_{mn}A_{j\beta}A_{j\alpha} = 0 \tag{32}$$

is added to rhs of (31) then all quadratic terms in the right hand side can be given in the form

$$\begin{aligned}
&A_{m\,\alpha}A_{n\,\beta} - A_{m\beta}A_{n\,\alpha} - \delta_{mn}A_{j\,\alpha}A_{j\,\beta} + \delta_{mn}A_{j\beta}A_{j\alpha} = \\
&= (\delta_{mp}\delta_{qn} - \delta_{mn}\delta_{qp})(A_{p\,\alpha}A_{q\,\beta} - A_{p\beta}A_{q\,\alpha}) = \\
&= \varepsilon_{kmq}\varepsilon_{kpn}(A_{p\,\alpha}A_{q\,\beta} - A_{p\beta}A_{q\,\alpha}) = \\
&= -\,\omega_{\alpha\,kn}\,\omega_{\beta\,km} + \omega_{\beta\,kn}A_{\alpha\,km}\,.
\end{aligned}$$

Substitution of the last expression into Eq. (31) accompanied with new notation

$$R_{mn\,\alpha\beta} \equiv \varepsilon_{kmn}F_{k\,\alpha\beta} \tag{33}$$

leads to well-known formula:

$$\begin{aligned}
R_{mn\,\alpha\beta} = \partial_\alpha\omega_{\beta\,mn} - \partial_\beta\omega_{\,\alpha\,mn} &+ \\
+\,\omega_{\alpha\,nk}\,\omega_{\beta\,km} &- \omega_{\beta\,nk}\,\omega_{\alpha\,km}\,.
\end{aligned} \tag{34}$$

This is nothing else but curvature tensor of Q-space built out of proper Q-connexion components (in their turn being functions of 4D coordinates). By other words, Yang-Mills field strength is mathematically (geometrically) identical to quaternion space curvature tensor. But in the considered case of Q-space comprising only proper Q-connexion, all components of the curvature tensor are identically zero. So Yang-Mills field in this case has potential but no intensity.

The picture absolutely changes for the case of quaternion space with Q-connexion containing a proper part $\omega_{\beta\,kn}$ and also Q-non-metricity $\sigma_{\beta\,kn}$

$$\Omega_{\beta\,kn}(y^\alpha) = \omega_{\beta\,kn} + \sigma_{\beta\,kn} \tag{35}$$

so that Q-covariant derivative of a unite Q-vector with connexion (35) does not vanish, its result is namely the Q-non-metricity

$$\hat{D}_\alpha\mathbf{q}_k \equiv (\delta_{mk}\partial_\alpha + \Omega_{\alpha\,mk})\,\mathbf{q}_m = \sigma_{\alpha\,mk}\,\mathbf{q}_k\,. \tag{36}$$

For this case "covariant derivatives" of transformation spinor matrices may be defined analogously to previous case definitions (26) and (29)

$$U\hat{\overleftarrow{D}}_\alpha \equiv \hat{U}(\overleftarrow{\partial}_\alpha - \hat{A}_\alpha), \quad \hat{D}_\alpha U^{-1} \equiv (\partial_\alpha + \hat{A}_\alpha)U\,. \tag{37}$$

But here the "gauge field" is built from Q-connexion (35)

$$\hat{A}_{k\,\alpha} \equiv \frac{1}{2}\,\varepsilon_{knm}\Omega_{\alpha kn}, \qquad \hat{A}_{\,\alpha} \equiv \frac{1}{2}\,\hat{A}_n\mathbf{q}_{\tilde{n}}\,. \tag{38}$$

It is not difficult to verify whether the definitions (37) are consistent with non-metricity condition (36). Action of the "covariant derivatives" (37) onto a spinor-transformed unite Q-vector

$$\begin{aligned}
&\hat{D}_\alpha\,\mathbf{q}_k \to (\hat{D}_\alpha U)\,\mathbf{q}_{\tilde{k}}\,\partial_\alpha U^{-1} + U\,\mathbf{q}_{\tilde{k}}\,(\hat{D}_\alpha U^{-1}) = \\
&= \left(U\overleftarrow{D}_\alpha - \frac{1}{4}\,\varepsilon_{jnm}\Omega_{\alpha\,nm}U\mathbf{q}_{\tilde{j}}\,\mathbf{q}_{\tilde{k}}\right)U^{-1} + \\
&+ U\,\mathbf{q}_{\tilde{k}}\left(D_\alpha U^{-1} + \frac{1}{4}\,\varepsilon_{jnm}\Omega_{\alpha\,nm}\,\mathbf{q}_{\tilde{j}}\,U^{-1}\right)
\end{aligned}$$

together with Eqs. (26) and (29) demand:

$$U\overleftarrow{D}_{\alpha} = D_{\alpha}U^{-1} = 0 \tag{39}$$

leads to the expected results

$$\hat{D}_{\alpha}\mathbf{q}_{k} \rightarrow \frac{1}{2}\,\varepsilon_{jnm}\sigma_{\alpha\,nm}U\varepsilon_{jkl}\,\mathbf{q}_{\tilde{l}}\,U^{-1} = \\ = \sigma_{\alpha\,kl}\,U\,\mathbf{q}_{\tilde{l}}\,U^{-1} = \sigma_{\alpha\,kl}\,\mathbf{q}_{l}$$

i.e. "gauge covariant" derivative of any Q-unit results in Q-non-metricity in full accordance with Eq. (36).

Now find curvature tensor components in this Q-space; it is more convenient to calculate them using differential forms. Given Q-connexion 1-form

$$\Omega_{\,kn} = \Omega_{\beta\,kn}dy^{\beta} \tag{40}$$

from the second equation of structure

$$\frac{1}{2}\,\hat{R}_{kn\alpha\beta}\,dy^{\alpha}\wedge dy^{\beta} = d\Omega_{kn} + \Omega_{km}\wedge\Omega_{mn} \tag{41}$$

one gets the curvature tensor component

$$\hat{R}_{kn\alpha\beta} = \partial_{\alpha}\Omega_{\beta\,kn} - \partial_{\beta}\Omega_{\alpha\,kn} + \\ + \Omega_{\alpha\,km}\Omega_{\beta\,mn} - \Omega_{\alpha\,nm}\Omega_{\beta\,mk} \tag{42}$$

quite analogously to Eq. (34). Skew-symmetry in 3D indices allows representing the curvature part of 3D Q-section as 3D axial vector

$$\hat{F}_{m\,\alpha\beta} \equiv \frac{1}{2}\,\varepsilon_{knm}\hat{R}_{kn\alpha\beta} \tag{43}$$

and using Eq. (38) one readily rewrites definition (43) in the form

$$\hat{F}_{m\,\alpha\beta} = \partial_{\alpha}\hat{A}_{m\,\beta} - \partial_{\beta}\hat{A}_{m\,\alpha} + \varepsilon_{knm}\hat{A}_{k\,\alpha}\hat{A}_{n\,\beta} \tag{44}$$

which exactly coincides with conventional definition (11). QED.

4 Klein-Gordon representation of Yang-Mills field

In the meantime, it is perhaps more interesting to note here that such a neat linkage between Yang-Mills field and quaternion numbers is already known, in particular using Klein-Gordon representation [16]. In turn, this neat correspondence between Yang-Mills field and Klein-Gordon representation can be expected, because both can be described in terms of SU(2) theory [17]. In this regards, quaternion decomposition of SU(2) Yang-Mills field has been discussed in [17], albeit it implies a different metric from what is described herein:

$$ds^{2} = d\alpha_{1}^{2} + \sin^{2}\alpha_{1}\,d\beta_{1}^{2} + d\alpha_{2}^{2} + \sin^{2}\alpha_{2}\,d\beta_{2}^{2}\,. \tag{45}$$

However, the O(3) non-linear sigma model appearing in the decomposition [17] looks quite similar (or related) to the Quaternion relativity theory (as described in the Introduction, there could be neat link between Q-relativity and SO(3, 1)).

Furthermore, sometime ago it has been shown that four-dimensional coordinates may be combined into a quaternion, and this could be useful in describing supersymmetric extension of Yang-Mills field [18]. This plausible neat link between Klein-Gordon equation, Duffin-Kemmer equation and Yang-Mills field via quaternion number may be found useful, because both Duffin-Kemmer equation and Yang-Mills field play some kind of significant role in description of standard model of particles [16].

In this regards, it has been argued recently that one can derive standard model using Klein-Gordon equation, in particular using Yukawa method, without having to introduce a Higgs mass [19, 20]. Considering a notorious fact that Higgs particle has not been observed despite more than three decades of extensive experiments, it seems to suggest that an alternative route to standard model of particles using (quaternion) Klein-Gordon deserves further consideration.

In this section we will discuss a number of approaches by different authors to describe the (quaternion) extension of Klein-Gordon equation and its implications. First we will review quaternion quantum mechanics of Adler. And then we discuss how Klein-Gordon equation leads to hypothetical imaginary mass. Thereafter we discuss an alternative route for quaternionic modification of Klein-Gordon equation, and implications to meson physics.

4.1 Quaternion Quantum Mechanics

Adler's method of quaternionizing Quantum Mechanics grew out of his interest in the Harari-Shupe's rishon model for composite quarks and leptons [21]. In a preceding paper [22] he describes that in quaternionic quantum mechanics (QQM), the Dirac transition amplitudes are quaternion valued, i.e. they have the form

$$q = r_0 + r_1 i + r_2 j + r_3 k \tag{46}$$

where r_0, r_1, r_2, r_3 are real numbers, and i, j, k are quaternion imaginary units obeying

$$i^2 = j^2 = k^2 = -1, \quad ij = -ji = k, \\ jk = -kj = i, \qquad ki = -ik = j\,. \tag{47}$$

Using this QQM method, he described composite fermion states identified with the quaternion real components [23].

4.2 Hypothetical imaginary mass problem in Klein-Gordon equation

It is argued that dynamical origin of Higgs mass implies that the mass of W must always be pure imaginary [19, 20]. Therefore one may conclude that a real description for (composite) quarks and leptons shall avoid this problem, i.e. by not including the problematic Higgs mass.

Nonetheless, in this section we can reveal that perhaps the problem of imaginary mass in Klein-Gordon equation is not completely avoidable. First we will describe an elementary derivation of Klein-Gordon from electromagnetic wave equation, and then by using Bakhoum's assertion of total energy we derive alternative expression of Klein-Gordon implying the imaginary mass.

We can start with 1D-classical wave equation as derived from Maxwell equations [24, p.4]:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0\,. \tag{48}$$

This equation has plane wave solutions:

$$E(x,t) = E_0 e^{i(kx-\omega t)} \tag{49}$$

which yields the relativistic total energy:

$$\varepsilon^2 = p^2c^2 + m^2c^4. \tag{50}$$

Therefore we can rewrite (48) for non-zero mass particles as follows [24]:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right)\Psi e^{\frac{i}{\hbar}(px-Et)} = 0\,. \tag{51}$$

Rearranging this equation (51) we get the Klein-Gordon equation for a free particle in 3-dimensional condition:

$$\left(\nabla - \frac{m^2c^2}{\hbar^2}\right)\Psi = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}\,. \tag{52}$$

It seems worthnoting here that it is more proper to use total energy definition according to Noether's theorem in lieu of standard definition of relativistic total energy. According to Noether's theorem [25], the total energy of the system corresponding to the time translation invariance is given by:

$$E = mc^2 + \frac{cw}{2}\int_0^\infty \left(\gamma^2\, 4\pi r^2\, dr\right) = k\mu c^2 \tag{53}$$

where k is *dimensionless* function. It could be shown, that for low-energy state the total energy could be far less than $E = mc^2$. Interestingly Bakhoum [25] has also argued in favor of using $E = mv^2$ for expression of total energy, which expression could be traced back to Leibniz. Therefore it seems possible to argue that expression $E = mv^2$ is more generalized than the standard expression of special relativity, in particular because the total energy now depends on actual velocity [25].

From this new expression, it is possible to rederive Klein-Gordon equation. We start with Bakhoum's assertion that it is more appropriate to use $E = mv^2$, instead of more convenient form $E = mc^2$. This assertion would imply [25]:

$$H^2 = p^2c^2 - m_0^2c^2v^2. \tag{54}$$

A bit remark concerning Bakhoum's expression, it does not mean to imply or to interpret $E = mv^2$as an assertion that it implies zero energy for a rest mass. Actually the problem comes from "mixed" interpretation of what we mean with "velocity". In original Einstein's paper (1905) it is defined as "kinetic velocity", which can be measured when standard "steel rod" has velocity approximates the speed of light. But in quantum mechanics, we are accustomed to make use it deliberately to express "photon speed" $= c$. Therefore, in special relativity 1905 paper, it should be better to interpret it as "speed of free electron", which approximates c. For hydrogen atom with 1 electron, the electron occupies the first excitation (quantum number $n = 1$), which implies that their speed also approximate c, which then it is quite safe to assume $E \sim mc^2$. But for atoms with large number of electrons occupying large quantum numbers, as Bakhoum showed that electron speed could be far less than c, therefore it will be more exact to use $E = mv^2$, where here v should be defined as "average electron speed" [25].

In the first approximation of relativistic wave equation, we could derive Klein-Gordon-type relativistic equation from equation (54), as follows. By introducing a new parameter:

$$\zeta = i\,\frac{v}{c}\,, \tag{55}$$

then we can use equation (55) in the known procedure to derive Klein-Gordon equation:

$$E^2 = p^2c^2 + \zeta^2 m_0^2 c^4, \tag{56}$$

where $E = mv^2$. By using known substitution:

$$E = i\hbar\,\frac{\partial}{\partial t}\,, \qquad p = \frac{\hbar}{i}\,\nabla\,, \tag{57}$$

and dividing by $(\hbar c)^2$, we get Klein-Gordon-type relativistic equation [25]:

$$-c^{-2}\,\frac{\partial\Psi}{\partial t} + \nabla^2\Psi = k_0'^2\,\Psi\,, \tag{58}$$

where

$$k_0' = \frac{\zeta m_0 c}{\hbar}\,. \tag{59}$$

Therefore we can conclude that imaginary mass term appears in the definition of coefficient k_0' of this new Klein-Gordon equation.

4.3 Modified Klein-Gordon equation and meson observation

As described before, quaternionic Klein-Gordon equation has neat link with Yang-Mills field. Therefore it seems worth to discuss here how to quaternionize Klein-Gordon equation. It can be shown that the resulting modified Klein-Gordon equation also exhibits imaginary mass term.

Equation (52) is normally rewritten in simpler form (by asserting $c = 1$):

$$\left(\nabla - \frac{\partial^2}{\partial t^2}\right)\Psi = \frac{m^2}{\hbar^2}\,. \tag{60}$$

Interestingly, one can write the Nabla-operator above in quaternionic form, as follows:

A. Define quaternion-Nabla-operator as analog to quaternion number definition above (46), as follows [25]:

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\,, \tag{61}$$

where e_1, e_2, e_3 are quaternion imaginary units. Note that equation (61) has included partial time-differentiation.

B. Its quaternion conjugate is defined as follows:

$$\bar{\nabla}^q = -i\frac{\partial}{\partial t} - e_1\frac{\partial}{\partial x} - e_2\frac{\partial}{\partial y} - e_3\frac{\partial}{\partial z}\,. \tag{62}$$

C. Quaternion multiplication rule yields:

$$\nabla^q\bar{\nabla}^q = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z}\,. \tag{63}$$

D. Then equation (63) permits us to rewrite equation (60) in quaternionic form as follows:

$$\nabla^q\bar{\nabla}^q\Psi = \frac{m^2}{\hbar^2}\,. \tag{64}$$

Alternatively, one used to assign standard value $c=1$ and also $\hbar=1$, therefore equation (60) may be written as:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\varphi(x,t) = 0\,, \tag{65}$$

where the first two terms are often written in the form of square Nabla operator. One simplest version of this equation [26]:

$$-\left(\frac{\partial S_0}{\partial t}\right)^2 + m^2 = 0 \tag{66}$$

yields the known solution [26]:

$$S_0 = \pm mt + \text{constant}\,. \tag{67}$$

The equation (66) yields wave equation which describes a particle at rest with positive energy (lower sign) or with negative energy (upper sign). Radial solution of equation (66) yields Yukawa potential which predicts meson as observables.

It is interesting to note here, however, that numerical 1-D solution of equation (65), (66) and (67) each yields slightly different result, as follows. (All numerical computation was performed using Mathematica [28].)

- For equation (65) we get:

(−D[#,x,x]+mˆ2+D[#,t,t])&[y[x,t]]==

$$m^2 + y^{(0,2)}[x,t] - y^{(2,0)}[x,t] = 0$$

DSolve[%,y[x,t],{x,t}]

$$\left\{\left\{y[x,t] \to \frac{m^2x^2}{2} + C[1][t-x] + C[2][t+x]\right\}\right\}$$

- For equation (66) we get:

(mˆ2−D[#,t,t])&[y[x,t]]==

$$m^2 + y^{(0,2)}[x,t] = 0$$

DSolve[%,y[x,t],{x,t}]

$$\left\{\left\{y[x,t] \to \frac{m^2t^2}{2} + C[1][x] + tC[2][x]\right\}\right\}$$

One may note that this numerical solution is in quadratic form $\frac{m^2t^2}{2}$ + constant, therefore it is rather different from equation (67) in [26].

In the context of possible supersymetrization of Klein-Gordon equation (and also PT-symmetric extension of Klein-Gordon equation [27, 29]), one can make use biquaternion number instead of quaternion number in order to generalize further the differential operator in equation (61):

E. Define a new "diamond operator" to extend quaternion-Nabla-operator to its biquaternion counterpart, according to the study [25]:

$$\begin{aligned}\diamondsuit = \nabla^q + i\nabla^q = \left(-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\right)+ \\ +\, i\left(-i\frac{\partial}{\partial T}+e_1\frac{\partial}{\partial X}+e_2\frac{\partial}{\partial Y}+e_3\frac{\partial}{\partial Z}\right),\end{aligned} \tag{68}$$

where e_1, e_2, e_3 are quaternion imaginary units. Its conjugate can be defined in the same way as before.

To generalize Klein-Gordon equation, one can generalize its differential operator to become:

$$\left[\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)+i\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\right]\varphi(x,t) = -m^2\varphi(x,t), \tag{69}$$

or by using our definition in (68), one can rewrite equation (69) in compact form:

$$\left(\diamondsuit\bar{\diamondsuit} + m^2\right)\varphi(x,t) = 0, \tag{70}$$

and in lieu of equation (66), now we get:

$$\left[\left(\frac{\partial S_0}{\partial t}\right)^2 + i\left(\frac{\partial S_0}{\partial t}\right)^2\right] = m^2. \tag{71}$$

Numerical solutions for these equations were obtained in similar way with the previous equations:

- For equation (70) we get:

(−D[#,x,x]+D[#,t,t]−I*D[#,x,x]+I*D[#,t,t]+mˆ2)
&[y[x,t]]==

$$m^2 + (1+i)\,y^{(0,2)}[x,t] - (1+i)\,y^{(2,0)}[x,t] = 0$$

DSolve[%,y[x,t],{x,t}

$$\left\{\left\{y[x,t] \to \left(\frac{1}{4}-\frac{i}{4}\right)m^2x^2 + C[1][t-x] + C[2][t+x]\right\}\right\}$$

- For equation (71) we get:

$$(-\text{m}\hat{}2+\text{D}[\#,\text{t},\text{t}]+\text{I*D}[\#,\text{t},\text{t}])\&[\text{y}[\text{x},\text{t}]]==$$

$$m^2 + (1+i)\, y^{(0,2)}[x,t] = 0$$

$$\text{DSolve}[\%,\text{y}[\text{x},\text{t}],\{\text{x},\text{t}\}]$$

$$\left\{\left\{y[x,t] \to \left(\frac{1}{4} - \frac{i}{4}\right) m^2 x^2 + C[1][x] + t C[2][x]\right\}\right\}$$

Therefore, we may conclude that introducing biquaternion differential operator (in terms of "diamond operator") yield quite different solutions compared to known standard solution of Klein-Gordon equation [26]:

$$y(x,t) = \left(\frac{1}{4} - \frac{i}{4}\right) m^2 t^2 + \text{constant}\,. \qquad (72)$$

In other word: we can infer hat $t = \pm \frac{1}{m} \sqrt{y/\left(\frac{1}{4} - \frac{i}{4}\right)}$, therefore it is likely that there is imaginary part of time dimension, which supports a basic hypothesis of the aforementioned BQ-metric in Q-relativity.

Since the potential corresponding to this biquaternionic KGE is neither Coulomb, Yukawa, nor Hulthen potential, then one can expect to observe a new type of matter, which may be called "*supersymmetric-meson*". If this new type of particles can be observed in near future, then it can be regarded as early verification of the new hypothesis of PT-symmetric QM and CT-symmetric QM as considered in some recent reports [27, 29]. In our opinion, its presence may be expected in particular in the process of breaking of Coulomb barrier in low energy schemes.

Nonetheless, further observation is recommended in order to support or refute this proposition.

5 Concluding remarks

If 4D space-time has for its 3D spatial section a Q-space with Q-connexion $\Omega_{\beta\, kn}$ containing Q-non-metricity $\sigma_{\beta\, kn}$, then the Q-connexion, geometric object, is algebraically identical to Yang-Mills potential

$$\hat{A}_{k\alpha} \equiv \frac{1}{2}\, \varepsilon_{knm} \Omega_{\alpha kn}\,,$$

while respective curvature tensor $\hat{R}_{kn\alpha\beta}$, also a geometric object, is algebraically identical to Yang-Mills "physical field" strength

$$\hat{F}_{m\alpha\beta} \equiv \frac{1}{2}\, \varepsilon_{knm} \hat{R}_{kn\alpha\beta}\,.$$

Thus Yang-Mills gauge field Lagrangian

$$L_{YM} \sim \hat{F}^{\alpha\beta}_{k} \hat{F}_{k\alpha\beta} = \frac{1}{4}\, \varepsilon_{kmn} \varepsilon_{kjl}\, \hat{R}^{\alpha\beta}_{mn} \hat{R}_{jl\alpha\beta} = \frac{1}{2}\, \hat{R}^{\alpha\beta}_{mn} \hat{R}_{mn\alpha\beta}$$

can be geometrically interpreted as a Lagrangian of "non-linear" or "quadratic" gravitational theory, since it contains quadratic invariant of curvature Riemann-type tensor contracted by all indices. Hence Yang-Mills theory can be regarded as a theory of pure geometric objects: Q-connexion and Q-curvature with Lagrangian quadratic in curvature (as: Einstein's theory of gravitation is a theory of geometrical objects: Christoffel symbols and Riemann tensor, but with linear Lagrangian made of scalar curvature).

Presence of Q-non-metricity is essential. If Q-non-metricity vanishes, the Yang-Mills potential may still exist, then it includes only proper Q-connexion (in particular, components of Q-connexion physically manifest themselves as "forces of inertia" acting onto non-inertially moving observer); but in this case all Yang-Mills intensity components, being in fact components of curvature tensor, identically are equal to zero.

The above analysis of Yang-Mills field from Quaternion Space geometry may be found useful in particular if we consider its plausible neat link with Klein-Gordon equation and Duffin-Kemmer equation. We discuss in particular a biquaternionic-modification of Klein-Gordon equation. Since the potential corresponding to this biquaternionic KGE is neither Coulomb, Yukawa, nor Hulthen potential, then one can expect to observe a new type of matter. Further observation is recommended in order to support or refute this proposition.

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References

1. Antonowitcz M. and Szczirba W. Geometry of canonical variables in gravity theories. *Lett. Math. Phys.*, 1985, v. 9, 43–49.
2. Rapoport D. and Sternberg S. On the interactions of spin with torsion. *Annals of Physics*, 1984, v. 158, no. 11, 447–475. MR 86e:58028; [2a] Rapoport D. and Sternberg S. Classical Mechanics without lagrangians nor hamiltoneans. *Nuovo Cimento A*, 1984, v. 80, 371–383, MR 86c:58055; [2b] Rapoport D. and Tilli M. Scale Fields as a simplicity principle. *Proceedings of the Third International Workshop on Hadronic Mechanics and Nonpotential Interactions*, Dept. of Physics, Patras Univ., Greece, A. Jannussis (ed.), in *Hadronic J. Suppl.*, 1986, v. 2, no. 2, 682–778. MR 88i:81180.
3. Chan W. K. and Shin F. G. Infinitesimal gauge transformations and current conservation in Yang theory of gravity. *Meet. Frontier Phys.*, Singapore, 1978, v. 2.
4. Hehl F. M. and Sijacki D. To unify theory of gravity and strong interactions? *Gen. Relat. and Grav.*, 1980, v. 12 (1), 83.
5. Batakis N. A. Effect predicted by unified theory of gravitational and electroweak fields. *Phys. Lett. B*, 1985, v. 154 (5–6), 382–392.
6. Kyriakos A. Electromagnetic structure of hadrons. arXiv: hep-th/0206059.

7. Kassandrov V. V. Biquaternion electrodynamics and Weyl-Cartan geometry of spacetime. *Grav. and Cosmology*, 1995, v. 1, no. 3, 216–222; arXiv: gr-qc/0007027.
8. Smarandache F. and Christianto V. Less mundane explanation of Pioneer anomaly from Q-relativity. *Progress in Physics*, 2007, v. 1, 42–45.
9. Fueter R. *Comm. Math. Helv.*, 1934–1935, v. B7, 307–330.
10. Yefremov A. P. *Lett. Nuovo. Cim.*, 1983, v. 37(8), 315–316.
11. Yefremov A. P. *Grav. and Cosmology*, 1996, v. 2(1), 77–83.
12. Yefremov A. P. *Acta Phys. Hung.*, Series — Heavy Ions, 2000, v. 11(1–2), 147–153.
13. Yefremov A. P. *Gravitation and Cosmology*, 2003, v. 9(4), 319–324. [13a] Yefremov A. P. Quaternions and biquaternions: algebra, geometry, and physical theories. arXiv: math-ph/0501055.
14. Ramond P. Field theory, a modern primer. The Benjamin/Cumming Publishing Co., ABPR Massachussetts, 1981.
15. Huang K. Quarks, leptons and gauge fields. World Scientific Publishing Co., 1982.
16. Fainberg V. and Pimentel B. M. Duffin-Kemmer-Petiau and Klein-Gordon-Fock equations for electromagnetic, Yang-Mills and external gravitational field interactions: proof of equivalence. arXiv: hep-th/0003283, p. 12.
17. Marsh D. The Grassmannian sigma model in SU(2) Yang-Mills theory. arXiv: hep-th/07021342.
18. Devchand Ch. and Ogievetsky V. Four dimensional integrable theories. arXiv: hep-th/9410147, p. 3.
19. Nishikawa M. Alternative to Higgs and unification. arXiv: hep-th/0207063, p. 18.
20. Nishikawa M. A derivation of the electro-weak unified and quantum gravity theory without assuming a Higgs particle. arXiv: hep-th/0407057, p. 22.
21. Adler S. L. Adventures in theoretical physics. arXiv: hep-ph/0505177, p. 107.
22. Adler S. L. Quaternionic quantum mechanics and Noncommutative dynamics. arXiv: hep-th/9607008.
23. Adler S. L. Composite leptons and quarks constructed as triply occupied quasiparticles in quaternionic quantum mechanics. arXiv: hep-th/9404134.
24. Ward D. and Volkmer S. How to derive the Schrödinger equation. arXiv: physics/0610121.
25. Christianto V. A new wave quantum relativistic equation from quaternionic representation of Maxwell-Dirac equation as an alternative to Barut-Dirac equation. *Electronic Journal of Theoretical Physics*, 2006, v. 3, no. 12.
26. Kiefer C. The semiclassical approximation of quantum gravity. arXiv: gr-qc/9312015.
27. Znojil M. PT-symmetry, supersymmetry, and Klein-Gordon equation. arXiv: hep-th/0408081, p. 7–8; [27a] arXiv: math-ph/0002017.
28. Toussaint M. Lectures on reduce and maple at UAM-I, Mexico. arXiv: cs.SC/0105033.
29. Castro C. The Riemann hypothesis is a consequence of CT-invariant Quantum Mechanics. Submitted to *JMPA*, Feb. 12, 2007.

Numerical Solution of Radial Biquaternion Klein-Gordon Equation

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In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we present numerical /computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the preceding article [1] we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we presented here for the first time a numerical/computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential.

This biquaternionic effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [2]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Radial biquaternionic KGE (radial BQKGE)

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)+i\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\right]\varphi(x,t)= \\ =-m^2\,\varphi(x,t), \tag{1}$$

or this equation can be rewritten as:

$$\left(\diamondsuit\bar{\diamondsuit}+m^2\right)\varphi(x,t)=0, \tag{2}$$

provided we use this definition:

$$\diamondsuit=\nabla^q+i\,\nabla^q=\left(-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\right)+ \\ +i\left(-i\frac{\partial}{\partial T}+e_1\frac{\partial}{\partial X}+e_2\frac{\partial}{\partial Y}+e_3\frac{\partial}{\partial Z}\right), \tag{3}$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1=i$, $e_2=j$, $e_3=k$):

$$\begin{aligned} i^2=j^2=k^2=-1\,, \quad & ij=-ji=k\,, \\ jk=-kj=i\,, \quad & ki=-ik=j\,. \end{aligned} \tag{4}$$

and quaternion *Nabla operator* is defined as [1]:

$$\nabla^q=-i\,\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\,. \tag{5}$$

(Note that (3) and (5) included partial time-differentiation.)

In the meantime, the standard Klein-Gordon equation usually reads [3, 4]:

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\varphi(x,t)=-m^2\varphi(x,t)\,. \tag{6}$$

Now we can introduce polar coordinates by using the following transformation:

$$\nabla=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-\frac{\ell^2}{r^2}\,. \tag{7}$$

Therefore, by substituting (7) into (6), the radial Klein-Gordon equation reads — by neglecting partial-time differentiation — as follows [3, 5]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-\frac{\ell(\ell+1)}{r^2}+m^2\right)\varphi(x,t)=0\,, \tag{8}$$

and for $\ell=0$, then we get [5]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+m^2\right)\varphi(x,t)=0\,. \tag{9}$$

The same method can be applied to equation (2) for radial biquaternionic KGE (BQKGE), which for the 1-dimensional situation, one gets instead of (8):

$$\left(\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right)-i\,\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right)+m^2\right)\varphi(x,t)=0\,. \tag{10}$$

In the next Section we will discuss numerical/computer solution of equation (10) and compare it with standard solution of equation (9) using Maxima software package [6]. It can be shown that equation (10) yields potential which differs appreciably from standard Yukawa potential. For clarity, all solutions were computed in 1-D only.

3 Numerical solution of radial biquaternionic Klein-Gordon equation

Numerical solution of the standard radial Klein-Gordon equation (9) is given by:

(%i1) diff(y,t,2)-'diff(y,r,2)+m^2*y;

(%o1) $m^2 \cdot y - \frac{d^2}{d^2x} y$

(%i2) ode2 (%o1, y , r);

(%o2) $y = \%k_1 \cdot \% \exp(mr) + \%k_2 \cdot \% \exp(-mr)$ (11)

In the meantime, numerical solution of equation (10) for radial biquaternionic KGE (BQKGE), is given by:

(%i3) diff(y,t,2)- (%i+1)*'diff(y,r,2)+m^2*y;

(%o3) $m^2 \cdot y - (i+1)\, \frac{d^2}{d^2r} y$

(%i4) ode2 (%o3, y , r);

(%o4) $y = \%k_1 \cdot \sin\left(\frac{|m|r}{\sqrt{-\%i-1}}\right) + \%k_2 \cdot \cos\left(\frac{|m|r}{\sqrt{-\%i-1}}\right)$ (12)

Therefore, we conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different result compared to the solution of standard Klein-Gordon equation; and it *differs appreciably* from the well-known Yukawa potential [3, 7]:

$$u(r) = -\frac{g^2}{r}\, e^{-mr}. \qquad (13)$$

Meanwhile, Comay puts forth argument that the Yukawa lagrangian density has theoretical inconsistency within itself [3].

Interestingly one can find argument that biquaternion Klein-Gordon equation is nothing more than quadratic form of (modified) Dirac equation [8], therefore BQKGE described herein, i.e. equation (12), can be considered as a plausible solution to the problem described in [3]. For other numerical solutions to KGE, see for instance [4].

Nonetheless, we recommend further observation [9] in order to refute or verify this proposition of new type of potential derived from biquaternion Klein-Gordon equation.

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VC would like to dedicate this article for RFF.

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References

1. Yefremov A., Smarandache F. and Christianto V. Yang-Mills field from quaternion space geometry, and its Klein-Gordon representation. *Progress in Physics*, 2007, v. 3, 42–50.
2. Storms E. http://www.lenr-canr.org
3. Comay E. *Apeiron*, 2007, v. 14, no. 1; arXiv: quant-ph/0603325.
4. Li Yang. Numerical studies of the Klein-Gordon-Schrödinger equations. MSc thesis submitted to NUS, Singapore, 2006, p. 9 (http://www.math.nus.edu.sg/∼bao/thesis/Yang-li.pdf).
5. Nishikawa M. A derivation of electroweak unified and quantum gravity theory without assuming Higgs particle. arXiv: hep-th/0407057, p. 15.
6. Maxima from http://maxima.sourceforge.net (using GNU Common Lisp).
7. http://en.wikipedia.org/wiki/Yukawa_potential
8. Christianto V. A new wave quantum relativistic equation from quaternionic representation of Maxwell-Dirac equation as an alternative to Barut-Dirac equation. *Electronic Journal of Theoretical Physics*, 2006, v. 3, no. 12.
9. Gyulassy M. Searching for the next Yukawa phase of QCD. arXiv: nucl-th/0004064.

On PT-Symmetric Periodic Potential, Quark Confinement, and Other Impossible Pursuits

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As we know, it has been quite common nowadays for particle physicists to think of six impossible things before breakfast, just like what their cosmology fellows used to do. In the present paper, we discuss a number of those impossible things, including PT-symmetric periodic potential, its link with condensed matter nuclear science, and possible neat link with Quark confinement theory. In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists. We begin with a review of some results from a preceding paper discussing derivation of PT-symmetric periodic potential from biquaternion Klein-Gordon equation and proceed further with the remaining issues. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

As we know, it has been quite common nowadays for particle physicists to think of six impossible things before breakfast [1], just like what their cosmology fellows used to do. In the present paper, we discuss a number of those impossible things, including PT-symmetric periodic potential, its link with condensed matter nuclear science, and possible neat link with Quark Confinement theory.

In this regards, it is worth to remark here that there were some attempts in literature to generalise the notion of symmetries in Quantum Mechanics, for instance by introducing CPT symmetry, chiral symmetry etc. In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists [2, 3]. It is expected that the discussions presented here would shed some light on these issues.

We begin with a review of results from our preceding papers discussing derivation of PT-symmetric periodic potential from biquaternion Klein-Gordon equation [4–6]. Thereafter we discuss how this can be related with both Gribov's theory of Quark Confinement, and also with EQPET/TSC model for condensed matter nuclear science (aka low-energy reaction or "cold fusion") [7]. We also highlight its plausible implication to the calculation of Gamow integral for the (periodic) non-Coulomb potential.

In other words, we would like to discuss in this paper, whether there is PT symmetric potential which can be observed in Nature, in particular in the context of condensed matter nuclear science (CMNS) and Quark confinement theory.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 PT-symmetric periodic potential

It has been argued elsewhere that it is plausible to derive a new PT-symmetric Quantum Mechanics (PT-QM; sometimes it is called pseudo-Hermitian Quantum Mechanics [3, 9]) which is characterized by a PT-symmetric potential [2]

$$V(x) = V(-x)\,. \tag{1}$$

One particular example of such PT-symmetric potential can be found in sinusoidal-form potential

$$V = \sin\varphi\,. \tag{2}$$

PT-symmetric harmonic oscillator can be written accordingly [3]. Znojil has argued too [2] that condition (1) will yield Hulthen potential

$$V(\xi) = \frac{A}{\left(1-e^{2i\xi}\right)^2} + \frac{B}{\left(1-e^{2i\xi}\right)}\,. \tag{3}$$

Interestingly, a similar periodic potential has been known for quite a long time as Posch-Teller potential [9], although it is not always related to PT-Symmetry considerations. The Posch-Teller system has a unique potential in the form [9]

$$U(x) = -\lambda\cosh^{-2}x\,. \tag{4}$$

It appears worth to note here that Posch-Teller periodic potential can be derived from conformal D'Alembert equations [10, p.27]. It is also known as the second Posch-Teller potential

$$V_\mu(\xi) = \frac{\mu\,(\mu-1)}{\sinh^2\xi} + \frac{\ell\,(\ell+1)}{\cosh^2\xi}. \tag{5}$$

The next Section will discuss biquaternion Klein-Gordon equation [4, 5] and how its radial version will yield a sinusoidal form potential which appears to be related to equation (2).

3 Solution of radial biquaternion Klein-Gordon equation and a new sinusoidal form potential

In our preceding paper [4], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = \\ = -m^2\varphi(x,t)\,, \qquad (6)$$

or this equation can be rewritten as

$$\left(\diamondsuit\bar{\diamondsuit} + m^2\right)\varphi(x,t) = 0 \qquad (7)$$

provided we use this definition

$$\diamondsuit = \nabla^q + i\nabla^q = \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + \\ + i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right), \qquad (8)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols $e_1 = i$, $e_2 = j$, $e_3 = k$):

$$i^2 = j^2 = k^2 = -1\,, \quad ij = -ji = k\,, \qquad (9)$$

$$jk = -kj = i\,, \quad ki = -ik = j\,, \qquad (10)$$

and quaternion *Nabla operator* is defined as [4]

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\,. \qquad (11)$$

Note that equation (11) already included partial time-differentiation.

Thereafter one can expect to find solution of *radial* biquaternion Klein-Gordon Equation [5, 6].

First, the standard Klein-Gordon equation reads

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\varphi(x,t) = -m^2\varphi(x,t)\,. \qquad (12)$$

At this point we can introduce polar coordinate by using the following transformation

$$\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\ell^2}{r^2}\,. \qquad (13)$$

Therefore by introducing this transformation (13) into (12) one gets (setting $\ell = 0$)

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \qquad (14)$$

By using the same method, and then one gets radial expression of BQKGE (6) for 1-dimensional condition as follows [5, 6]

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - i\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \quad (15)$$

Using Maxima computer package we find solution of equation (15) as a new potential taking the form of sinusoidal potential

$$y = k_1 \sin\left(\frac{|m|\,r}{\sqrt{-i-1}}\right) + k_2\cos\left(\frac{|m|\,r}{\sqrt{-i-1}}\right), \qquad (16)$$

where k_1 and k_2 are parameters to be determined. It appears very interesting to remark here, when k_2 is set to 0, then equation (16) can be written in the form of equation (2)

$$V = k_1 \sin\varphi\,, \qquad (17)$$

by using definition

$$\varphi = \sin\left(\frac{|m|\,r}{\sqrt{-i-1}}\right). \qquad (18)$$

In retrospect, the same procedure which has been traditionally used to derive the Yukawa potential, by using radial biquaternion Klein-Gordon potential, yields a PT-symmetric periodic potential which takes the form of equation (1).

4 Plausible link with Gribov's theory of Quark Confinement

Interestingly, and quite oddly enough, we find the solution (17) may have deep link with Gribov's theory of Quark confinement [8, 11]. In his Third Orsay Lectures he described a periodic potential in the form [8, p.12]

$$\ddot{\psi} - 3\sin\psi = 0\,. \qquad (19)$$

By using Maxima package, the solution of equation (19) is given by

$$\left.\begin{aligned} x_1 &= k_2 - \frac{\int \frac{1}{\sqrt{k_1-\cos(y)}}\,dy}{\sqrt{6}} \\ x_2 &= k_2 + \frac{\int \frac{1}{\sqrt{k_1-\cos(y)}}\,dy}{\sqrt{6}} \end{aligned}\right\}, \qquad (20)$$

while Gribov argues that actually the equation shall be like nonlinear oscillation with damping, the equation (19) indicates close similarity with equation (2).

Therefore one may think that PT-symmetric periodic potential in the form of (2) and also (17) may have neat link with the Quark Confinement processes, at least in the context of Gribov's theory. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

5 Implication to condensed matter nuclear science. Comparing to EQPET/TSC model. Gamow integral

In accordance with a recent paper [6], we interpret and compare this result from the viewpoint of EQPET/TSC model which has been suggested by Prof. Takahashi in order to explain some phenomena related to Condensed matter nuclear Science (CMNS).

Takahashi [7] has discussed key experimental results in condensed matter nuclear effects in the light of his EQPET/TSC model. We argue here that his potential model with inverse barrier reversal (STTBA) may be comparable to the periodic potential described above (17).

In [7] Takahashi reported some findings from condensed matter nuclear experiments, including intense production of helium-4, ^{4}He atoms, by electrolysis and laser irradiation experiments. Furthermore he [7] analyzed those experimental results using EQPET (Electronic Quasi-Particle Expansion Theory). Formation of TSC (tetrahedral symmetric condensate) were modeled with numerical estimations by STTBA (Sudden Tall Thin Barrier Approximation). This STTBA model includes strong interaction with negative potential near the center.

One can think that apparently to understand the physics behind Quark Confinement, it requires fusion of different fields in physics, perhaps just like what Langland program wants to fuse different branches in mathematics.

Interestingly, Takahashi also described the Gamow integral of his STTBA model as follows [7]

$$\Gamma_n = 0.218 \left(\mu^{1/2}\right) \int_{r_0}^{b} (V_b - E_d)^{1/2} dr \,. \tag{21}$$

Using $b = 5.6$ fm and $r = 5$ fm, he obtained [7]

$$P_{4D} = 0.77, \tag{22}$$

and

$$V_B = 0.257\,\text{MeV}, \tag{23}$$

which gave significant underestimate for 4D fusion rate when rigid constraint of motion in 3D space attained. Nonetheless by introducing different values for λ_{4D} the estimate result can be improved. Therefore we may conclude that Takahashi's STTBA potential offers a good approximation (just what the name implies, STTBA) of the fusion rate in condensed matter nuclear experiments.

It shall be noted, however, that his STTBA lacks sufficient theoretical basis, therefore one can expect that a sinusoidal periodic potential such as equation (17) may offer better result.

All of these seem to suggest that the cluster deuterium may yield a different inverse barrier reversal which cannot be predicted using the D-D process as in standard fusion theory. In other words, the standard procedure to derive Gamow factor should also be revised [12]. Nonetheless, it would need further research to determine the precise Gamow energy and Gamow factor for the cluster deuterium with the periodic potential defined by equation (17); see for instance [13].

In turn, one can expect that Takahashi's EQPET/TSC model along with the proposed PT-symmetric periodic potential (17) may offer new clues to understand both the CMNS processes and also the physics behind Quark confinement.

6 Concluding remarks

In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists.

In the present paper, it has been shown that one can find a new type of PT-symmetric periodic potential from solution of the radial biquaternion Klein-Gordon Equation. We also have discussed its plausible link with Gribov's theory of Quark Confinement and also with Takahashi's EQPET/TSC model for condensed matter nuclear science. All of which seems to suggest that the Gribov's Quark Confinement theory may indicate similarity, or perhaps a hidden link, with the Condensed Matter Nuclear Science (CMNS). It could also be expected that thorough understanding of the processes behind CMNS may also require revision of the Gamow factor to take into consideration the cluster deuterium interactions and also PT-symmetric periodic potential as discussed herein.

Further theoretical and experiments are therefore recommended to verify or refute the proposed new PT symmetric potential in Nature.

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References

1. http://www-groups.dcs.st-and.ac.uk/∼history/Quotations/Dodgson.html
2. Znojil M. arXiv: math-ph/0002017.
3. Znojil M. arXiv: math-ph/0104012, math-ph/0501058.
4. Yefremov A.F., Smarandache F., and Christianto V. *Progress in Physics*, 2007, v. 3, 42; also in: *Hadron Models and Related New Energy Issues*, InfoLearnQuest, USA, 2008.
5. Christianto V. and Smarandache F. *Progress in Physics*, 2008, v. 1, 40.
6. Christianto V. *EJTP*, 2006, v. 3, no. 12.
7. Takahashi A. In: *Siena Workshop on Anomalies in Metal-D/H Systems*, Siena, May 2005; also in: *J. Condensed Mat. Nucl. Sci.*, 2007, v. 1, 129.
8. Gribov V.N. arXiv: hep-ph/9905285.
9. Correa F., Jakubsky V., Plyushckay M. arXiv: 0809.2854.
10. De Oliveira E.C. and da Rocha R. *EJTP*, 2008, v. 5, no. 18, 27.
11. Gribov V.N. arXiv: hep-ph/9512352.
12. Fernandez-Garcia N. and Rosas-Ortiz O. arXiv: 0810.5597.
13. Chugunov A.I., DeWitt H.E., and Yakovlev D.G. arXiv: astro-ph/0707.3500.

FLORENTIN SMARANDACHE & V. CHRISTIANTO

Meet in person at International Airport, Jakarta, Indonesia, May 2006

Seminar of Neutrosophic Logic and its applications in Central Java, Indonesia, May 2006

With Dr. Peter Handoko (first from left), Dr. S. Trihandaru (second from right), May 2006

Seminar of Neutrosophic Logic and its applications in Central Java, Indonesia, May 2006

Discussion with Dr Harsono (lecturer of Fuzzy Logic), May 2006

Seminar of Neutrosophic Logic and its applications in Central Java, Indonesia, May 2006

A few audience of the seminar (including Prof. Theo Van Beusekom fourth from right and Prof .L Wilardjo second from right), May 2006

Seminar of Neutrosophic Logic and its applications in Central Java, Indonesia, May 2006

Audience of the seminar (mostly students from Electronics Dept.)

FLORENTIN SMARANDACHE

Visiting Blanco Museum, Bali Island, Indonesia, May 2006

NEUTROSOPHIC LOGIC, WAVE MECHANICS AND OTHER STORIES: SELECTED WORKS 2005-2008

There is beginning for anything; we used to hear that phrase. The same wisdom word applies to us too. What began in 2005 as a short email on some ideas related to interpretation of the Wave Mechanics results in a number of papers and books up to now. Some of these papers can be found in *Progress in Physics* or elsewhere.

Our purpose here is to present a selection of those papers in a compilation which enable the readers to find some coherent ideas which appeared in those articles. For this reason, the ordering of the papers here is based on categories of ideas.

While some of these articles have been published in book format elsewhere, we hope that reading this book will give the readers impression of progression of our thoughts.

We wish to extend our sincere gratitude to plenty of colleagues, friends and relatives all around the world for sharing their ideas, insightful discussions etc. Special thanks to D. Rabounski, S. Crothers, L. Borissova for their great service in *Progress in Physics* journal.

FS & VC
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