A reinterpretation of the law of Hubble

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The law of Hubble is considered as the law of the expansion of the space. However, in this short and very simple paper, we show, using very elemental arguments, that this experimental law can have its origin in the vectorial nature of the escape velocity.

Key words: Light redshift, recession velocity and escape velocity.

1. Introduction

The law of Hubble states that the redshift in the light coming from distant galaxies is proportional to their distances. It is considered the first observational basis for the expanding of the space and the main evidence of theory of the Big Bang.

However, we are going to show, using very elemental arguments, that this experimental law can have its origin in the vectorial nature of the escape velocity.

2. The law of Hubble

This law is stated as

$$v_r = H\ d \tag{2.1}$$

being v_r the velocity of recession (namely, the velocity at which a light source moves away from the observer, due to the expansion of the space between them), H the constant of Hubble and d the distance between the observer and the light source.

Although this is an experimental law, it can be deduced theoretically within the framework of the general relativity theory (GRT) of Einstein, in the context of the model of Friedmann [1] (p. 486).

3. A reinterpretation of the law of Hubble

For an expanding space, in the context of the theory of the Big Bang, and using the method of Milne, the energy E of a test particle in the surface of the expanding universe would be [2]

$$E = T + V = \frac{1}{2} m \left(\frac{dR}{dt} \right)^2 - \frac{GMm}{R} \tag{3.1}$$

being T and V the kinetic and potential energies of the particle, respectively, m its mass, and M and R the mass and the radius of the universe, respectively, and G the gravitational constant of Newton. For a homogeneous and isotropic universe

$$M = \rho \frac{4}{3} \pi R^3 \tag{3.2}$$

being ρ the mass density. And for an expanding space

$$R(t) = s\, a(t) \tag{3.3}$$

being s a constant length and a the scale factor. Substituting (3.2) and (3.3) into (3.1), we obtain that

$$H^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} \tag{3.4}$$

where

$$H = \frac{1}{a} \frac{da}{dt} \tag{3.5}$$

is the constant of Hubble and

$$k = -\frac{2E}{ms^2} \tag{3.6}$$

is the constant of curvature.

For a flat universe, we would have $k = 0$ and $\rho = \rho_c$, being ρ_c the critical mass density, and from (3.4) and (2.1)

$$H = \left(\frac{8\pi G \rho_c}{3} \right)^{\frac{1}{2}} \tag{3.7}$$

$$v_r = \left(\frac{8\pi G \rho_c}{3} \right)^{\frac{1}{2}} d \tag{3.8}$$

But also, from (3.1) and (3.2), we have [2]

$$\frac{1}{2} m \, {v_e}^2 - \frac{GMm}{R} = 0 \tag{3.9}$$

$$v_e = \left(\frac{2GM}{R}\right)^{\frac{1}{2}} = \left(\frac{8\pi G\rho}{3}\right)^{\frac{1}{2}} R \tag{3.10}$$

being v_e the escape velocity of the universe. And from which, we would have that

$$v_d = \left|\vec{v}_{e2} - \vec{v}_{e1}\right| = \left(\frac{8\pi G\rho}{3}\right)^{\frac{1}{2}} \left|\vec{R}_2 - \vec{R}_1\right| = \left(\frac{8\pi G\rho}{3}\right)^{\frac{1}{2}} d \tag{3.11}$$

where $v_d = \left|\vec{v}_{e2} - \vec{v}_{e1}\right|$ represents the variation of $\vec{v}_e = \left(\frac{8\pi G\rho}{3}\right)^{\frac{1}{2}} \vec{R}$ with the direction, and $\vec{R}_1$ and $\vec{R}_2$ are the vector radii of the universe in the points of observation and emission, respectively, and $d = \left|\vec{R}_2 - \vec{R}_1\right|$ is the linear distance between the observer and the light source.

For a flat universe, $\rho(t) = \frac{const.}{t^2}$ and, using (3.3), $R(t) = const.\, t^{\frac{1}{2}}$ (for the radiation) and $R(t) = const.\, t^{\frac{2}{3}}$ (for the matter) [3], and in the limit $t = \infty$, we would have $\rho(\infty) = 0$ and $R(\infty) = \infty$.

Then, for an universe without beginning, $t = \infty$, nor end, flat, $\rho(\infty) = \rho_c$, with $\rho_c = 10^{-29}\, g/cm^3$ [1] (p. 487), homogeneous and isotropic, and infinite, hence, static, we would have, from (3.11) with $\left|\vec{R}_2(\infty) - \vec{R}_1(\infty)\right| = d$, that

$$v_d = \left(\frac{8\pi G\rho_c}{3}\right)^{\frac{1}{2}} d \tag{3.12}$$

which has the form of (3.8). So, we can reinterpret the recession velocity as the variation of the escape velocity of a flat universe with the direction, in a static space.

4. Discussion

Within the framework of the special relativity theory (SRT) of Einstein, the transformation of Lorentz relates the primed coordinates (x', y', z', t') of a moving frame with the unprimed coordinates (x, y, z, t) of a rest frame. The moving frame is moving with a relative constant velocity v with respect to the rest frame. When $v << c$, being c the velocity of the light in the vacuum, the Lorentz transformation

$$x' = (x - vt)\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}},\ y' = y,\ z' = z,\ t' = \left(t - \frac{v}{c^2}x\right)\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{4.1}$$

is converted into the Galileo transformation

$$x' = x - vt,\ y' = y,\ z' = z,\ t' = t \tag{4.2}$$

From which, we have

$$\frac{dx'}{dt'} = \frac{dx'}{dt} = \frac{dx}{dt} - v \tag{4.3}$$

$$v'_x = v_x - v \tag{4.4}$$

$$c' = c - v \tag{4.5}$$

Therefore, we can consider that the light is red shifted because its velocity would be $c - v$, for $v << c$, in a Galilean context. Then, the kinetic energy of the photon, $h\nu_e = h\frac{c}{\lambda}$, decreases until, $h\nu_o = h\frac{c - v}{\lambda}$, being h the constant of Planck, and ν_e and ν_o the frequencies emitted and observed of the photon, respectively, and λ its wavelength. And for a flat universe, we can substitute v by v_r, from (3.8), but only if the space is expanding, and with a velocity $v_r << c$, however, we can always substitute v by v_d, from (3.12), for $v_d << c$, which favours to the case of a static space.

5. Conclusion

We conclude that the experimental law of Hubble can have its origin in the vectorial nature of the escape velocity, namely, in the variation of the escape velocity of a flat universe with the direction, in a static space.

References

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