Doppler shift and aberration for spherical electromagnetic waves

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Spherical wave vs. plane wave approximation to the nature of the electromagnetic waves regarding the Doppler shift and aberration is considered. The first approach is free from the blueshift–redshift transition paradox innate for the second one. It is assumed that for spherical electromagnetic waves, in contrast with the plane ones, not only the magnitude, but also the direction of the light velocity is the same in any inertial frame, which leads to the accepted expression for time dilation. The rest frame of the source of electromagnetic waves is unique among all inertial frames. (In it, the angles of emission and reception always coincide and there is no shift in wavelength in all directions.) The spherical approximation to electromagnetic waves preserves this uniqueness without violating the principle of relativity of uniform motion, while the planar approximation ignores the source completely. Both approximations give the same expression of the Lorentz–FitzGerald contraction. Both spherical and planar approaches give the same Doppler shift in the directions of relative movement of the frames, but in the directions with perpendicular components there may be significant differences. A geometrical picture of the transformation of wavefronts of spherical electromagnetic waves, which differs from the one according to the Lorentz transformation, is suggested.

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Currently accepted mathematical expressions of the Doppler shift and aberration for electromagnetic waves was first derived by A. Einstein using the Lorentz transformation and assuming planar character of those waves [1].

If a source of plane electromagnetic wave is moving with constant velocity v in some frame, then the length of the emitted wave in this frame in the direction making angle θ with the direction of motion of the source (the angle of reception), λ_θ, is connected with the length of the same wave in the rest frame of the source, λ_α, through the equation:

$$\lambda_\theta = \lambda_\alpha \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}\,, \tag{1}$$

$\beta = \frac{v}{c}$, where c is the speed of electromagnetic waves in free space.

The direction of propagation of the same wave in the rest frame of the source, angle α (the angle of emission), is connected with angle θ through the equation of aberration:

$$\cos\alpha = \frac{\cos\theta-\beta}{1-\beta\cos\theta}\,. \tag{2}$$

Angle α in the moving frame is seen as angle α' from the stationary frame. The connection between those angles is:

$$\tan\alpha = \sqrt{1-\beta^2}\tan\alpha'\,.$$

In the rest frame of the source Eqs. (1) and (2) take following forms respectively:

$$\lambda_\alpha = \lambda_\theta \frac{1+\beta\cos\alpha}{\sqrt{1-\beta^2}}\ ; \quad \cos\theta = \frac{\cos\alpha+\beta}{1+\beta\cos\alpha}\,.$$

Those equations make the frames of the source and observer exactly equivalent.

Spherical electromagnetic waves do not fit into the picture of the Lorentz transformation. For instance, the phase of an electromagnetic wave in any point of space (which is closely related with the number of waves passed through that point) must be an invariant. Under the Lorentz transformation, the phase of a plane electromagnetic wave is the same in any inertial frame and that of a spherical one is not.

That the plane wave approximation to the electromagnetic waves is controversial, we can see from the following blueshift-redshift transition paradox, which one has to live with when dealing with those waves.

It is obvious that the blueshift-redshift transition (in respect to the frequency of internal oscillations of the source as seen from an observer's frame) shall happen when the observer receives waves that have traveled the shortest distance between the source and the observer. In any frame moving relative to the source, all waves received by the observer before that instant shall be blueshifted and all waves received after that instant shall be redshifted.

Let us consider this problem in the rest frame of the source (or it may be a frame moving very slowly relative to the source), because that frame is the same for any observer and the events in it do not depend on the kind of transformation theory.

The distance traveled by a wave in the rest frame of the source is the shortest when the wave is issued at angle $\alpha = \frac{\pi}{2}$ to the line of relative velocity. The corresponding angle of reception θ, which is different in different frames, may be found from the equation: $cos\,\theta = \beta$, where β is the relative speed of the source and the observer divided by the speed of light in free space. This result is a consequence of the constancy of the speed of light. It gives blueshift-redshift transition regarding the wavelength in the source's frame, which does not depend on the effect the relative movement of the frames may have on the observed frequency of internal oscillations.

According to Eq. (1), plane waves must have blueshift-redshift transition at the angle of reception, for which $\cos\theta = \frac{1-\sqrt{1-\beta^2}}{\beta}$ (or the angle of emission, for which $\cos\alpha = \frac{\sqrt{1-\beta^2}-1}{\beta}$), but that is inconsistent with the above results (Fig. 1).

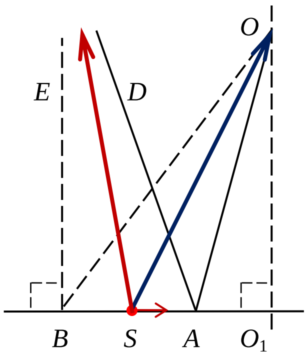

Fig. 1 *Blueshift-redshift transition paradox*. A source of plane electromagnetic waves, S, is moving with constant velocity $v = \beta c$ along the line BO_1 to the right. The constancy of light speed requires that for the observer in point O, the point of emission of electromagnetic waves with no shift in wavelength shall be point B, for which $\alpha = \angle EBO_1 = \frac{\pi}{2}$, and $cos\,\theta = cos\angle OBO_1 = \beta$. On the other hand, Eq. (1) requires that the point of emission for the zero-shift be point A, for which $\cos\alpha = cos\angle DAO_1 = \frac{\sqrt{1-\beta^2}-1}{\beta}$, and $\cos\theta = cos\angle OAO_1 = \frac{1-\sqrt{1-\beta^2}}{\beta}$.

The strange transition from blueshift to redshift (or vice versa) for a given angle of observation by changing only the intensity of the relative velocity of the source and the observer, which cannot be explained in the framework of the currently accepted theory, is considered in [2].

Thus, in a non-controversial equation of the Doppler shift, $\cos\alpha$ (as well as $\cos\alpha'$) must be changing sign when passing the angle for which $\lambda = \lambda_0$ (the zero-shift angle), i.e. $\alpha = \alpha' = \pm\frac{\pi}{2}$; or, which is the same, the angle of observed blueshift-redshift transition $\theta = \pm\,\mathrm{acos}\,\beta$.

We can get the expression of the Doppler shift for spherical electromagnetic waves using some obvious properties of waves.

It is clear that in any frame all waves (wavelengths) emitted by a source of electromagnetic wave in free space are contained between the source and the (outer) wavefront. In any direction the distance between the source and the wavefront of the emitted wave is equal to the sum of wavelengths contained between them.

Let a source of spherical electromagnetic wave, O_1, be moving with a constant velocity $v = \beta c$ from the origin O along the axis X of a stationary frame (Fig. 2).

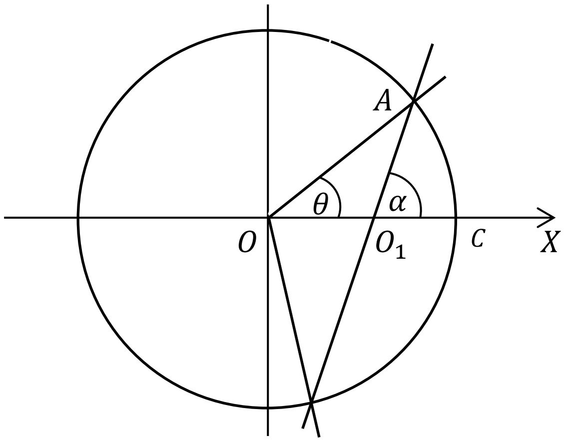

Fig. 2 The wavefront of a spherical electromagnetic wave emitted by a moving source O_1 with the angles of reception and emission: θ and α respectively.

The total number of waves emitted in the period of time t in the stationary frame, n, (which is equal to the number of oscillations within the source of wave), is contained in the space between point O_1 and the sphere with centre in O and radius $R = ct$, which represents the wavefront of those waves in the stationary frame. For some point A of the wavefront in the stationary frame

$$|O_1A| = ct\sqrt{1 - 2\beta\cos\theta + \beta^2}.$$

Assuming that the light speed is the same in the stationary and moving frames,

$$n\tau_\theta = n_0\tau_0\sqrt{1 - 2\beta\cos\theta + \beta^2}$$

and

$$n\lambda_\theta = n_0\lambda_0\sqrt{1 - 2\beta\cos\theta + \beta^2}\,, \tag{3}$$

where n_0 is the number of oscillations within the source of wave during the interval of time t in the rest frame of the source; τ_0 and λ_0 are the period and the length of the wave in the same frame; τ_θ and λ_θ are the period and the length of the wave in direction making $\angle AOO_1 = \theta$ to the direction of velocity of the source in the stationary frame (the angle of reception).

From Eq. (3):

$$\lambda_\theta = \frac{n_0\lambda_0\sqrt{1 - 2\beta\cos\theta + \beta^2}}{n} \tag{4}$$

It is sometimes convenient to use the angle $\angle AO_1C = \alpha$ instead of the angle θ. Angle α is the angle at which, from the point of view of the stationary frame, the ray OA (or the point A of the wavefront) propagates in the moving frame connected with the source. If we assume that the ray OA propagates through an imaginary moving tube of a very small diameter, then angle α

is the angle of slope of the tube relative to the direction of the source's motion. We have no reason to distinguish this angle from the angle of emission in the rest frame of the source.

From ΔAOO_1:

$$\sin\alpha = \frac{\sin\theta}{\sqrt{1-2\beta\cos\theta+\beta^2}}$$

So, instead of Eq. (4) we can use the equation:

$$\lambda_\theta = \frac{n_0\lambda_0\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right)}{n}$$

As it was discussed above, due to the constancy of the speed of electromagnetic waves in free space, when $\lambda_\theta = \lambda_0$, the angle of emission $\alpha = \pm\frac{\pi}{2}$. Then $n = n_0\sqrt{1-\beta^2}$, which gives:

$$\lambda_\theta = \frac{\lambda_0\sqrt{1-2\beta\cos\theta+\beta^2}}{\sqrt{1-\beta^2}},$$

and

$$\lambda_\theta = \frac{\lambda_0\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right)}{\sqrt{1-\beta^2}}.$$

Using vectors we can rewrite the above equations in compact forms:

$$\lambda = \lambda_0\frac{|\vec{c}-\vec{v}|}{\sqrt{c^2-v^2}}, \tag{5}$$

$$\sin\alpha = \frac{c\sin\theta}{|\vec{c}-\vec{v}|}, \tag{6}$$

where λ is the wavelength in the considered direction of propagation of the electromagnetic wave, i.e. in the direction of $\vec{c}$.

At $\theta = 0$ and $\theta = \pm\pi$ Eqs. (1) and (5) coincide absolutely.

In deriving Eq. (5), instant values of the wavelength (and, correspondingly, of the period) of an electromagnetic wave were used. In Fig. 2, at an instant of time t, if $|OA| = \frac{\lambda_0}{\sqrt{1-\beta^2}} = \frac{c\tau_0}{\sqrt{1-\beta^2}}$ (τ_0 is the period of internal oscillations as well as of the wave in the frame of source), then $|O_1A| = \lambda = c\tau$ (λ and τ are the instant values of wavelength and period of the wave in direction θ in the frame of observer). It is obvious that an observer in point A registers the same period ($\tau = \frac{|O_1A|}{c}$), and accordingly, the same wavelength λ. But, will an observer located at some other point on the same line OA register the same values of wave parameters? They would not in classical physics, but considering Planck's and Einstein's hypotheses (assuming that not only the energy of an electromagnetic wave is quantized, but also its period), along with the laws of conservation of momentum and energy, it is to be expected that anyplace on the line OA an observer shall register the same wavelength (and period) of an electromagnetic wave. This anticipation is strengthened by the fact that Eq. (5) gives correct angles for blueshift-redshift transition.

In Fig. 2, ΔOAO_1 represents a space-time quant (photon) of an electromagnetic wave in direction θ, emitted by a source moving with velocity $v = \beta t$ at some instant of time t in the stationary frame. That means, it has no inner parts or points or angles and has to be considered moving as a whole until it gets absorbed by a receiver. For any other θ, β or t we will have another quant that is independent from this one even if they appear to overlap each other in space or time.

Let's consider a simplified model of the photon (quant) consisting of two very short pulses (denoting the beginning and ending points of a photon) with a time period between them equal to that of the real photon. In classical physics those pulses are loosely connected. (The time period between them can expand or contract like a squeezebox.) But, according to Planck's and Einstein's hypotheses, the time connection between those pulses (points A and O_1 in Fig. 2) is rigid: the time period between them ($\tau = \frac{|O_1 A|}{c}$), starting from their emission and ending up with their absorption by a receiver, remains the same in an inertial frame. In other words, the situation that is represented by ΔOAO_1 translates along the line OA with the speed of light. Thus, in contrast to classical physics, both pulses are moving along parallel lines to infinity or until the photon gets absorbed by a receiver.

Fig. 3 shows a comparative picture of the Doppler shifts for the angles of reception θ (curves $1, 2$) and emission α (curves $3, 4$) in the cases of spherical (curves 1,3) and planar (curves $2, 4$) electromagnetic waves when $\beta = 0.9$.

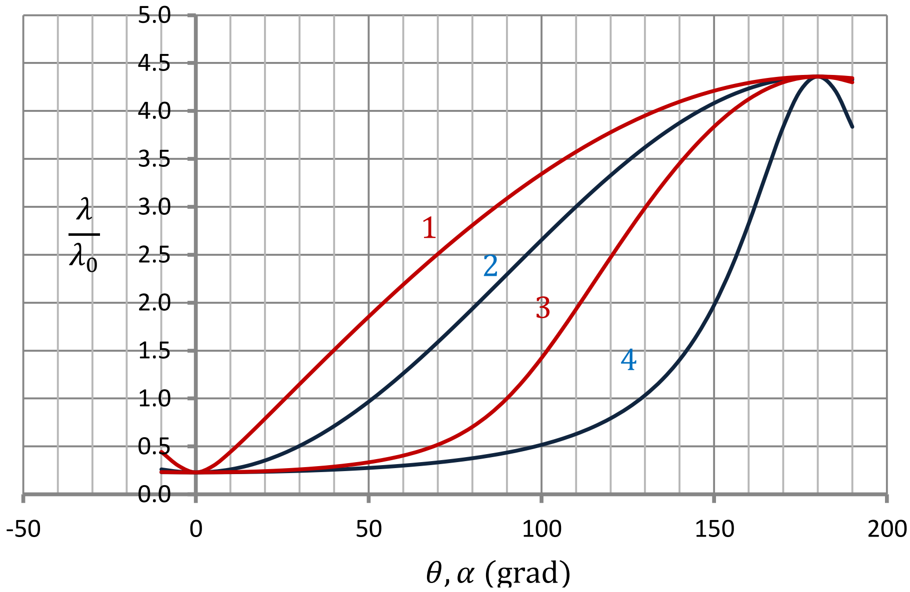

Fig. 3 *Doppler shift*. Dependence of the ratio of the wavelengths of received and emitted waves ($\frac{\lambda}{\lambda_0}$) on the angles of reception θ (curves $1, 2$) and emission α (curves $3, 4$) for the cases of spherical (curves $1, 3$) and planar (curves $2, 4$) electromagnetic waves when $\beta = 0.9$.

For spherical electromagnetic waves the transition to the rest frame of the source is trivial: we have only to alter the sign before the relative velocity in Eqs. (5) and (6) :

$$\lambda = \lambda_0 \frac{|\vec{c} + \vec{v}|}{\sqrt{c^2 - v^2}}, \quad (7)$$

$$\sin \alpha = \frac{c \sin \theta}{|\vec{c} + \vec{v}|}$$

Comparing Eqs. (5) and (7), it can be assumed that vector $\vec{c} = \frac{\overrightarrow{OA}}{t}$ (the vector connecting the points of emission and reception of a pulse or ray of light at the instant of its emission divided by the time of its travel from one end to the other) is the same in the moving and stationary frames, i.e. for spherical electromagnetic waves, the vector of the velocity of an electromagnetic wave is an invariant.

Eq. (6) is the equation of aberration for spherical electromagnetic waves. At the instant when a light pulse arrives at the observer, the angle of the slope of the line connecting the reception point with the source is angle α; at the instant when the pulse is emitted by the source, that angle is θ. For the observer in the rest frame of the source those angles coincide, in any other frame they are different.

For spherical electromagnetic waves, the difference between the angles θ and α is fundamental: angle α is always the direction of a ray in the frame connected with the source (the angle of emission), and θ may be a direction in any frame (the angle of reception), which makes the rest frame of the source unique. That, of course, does not violate the principle of relativity of uniform motion, because the rest frame of the source can always be singled out from the other frames (e.g., in that frame, in contrast with any other frame, the angles of emission and reception are always the same, and there is zero-shift in all directions). In other words, in the case of a spherical electromagnetic wave, the (frame of) source cannot be ignored. In mathematical language this is tantamount to the statement that every sphere has its centre point.

Fig. 4 gives a comparative picture of aberration for spherical and plane waves.

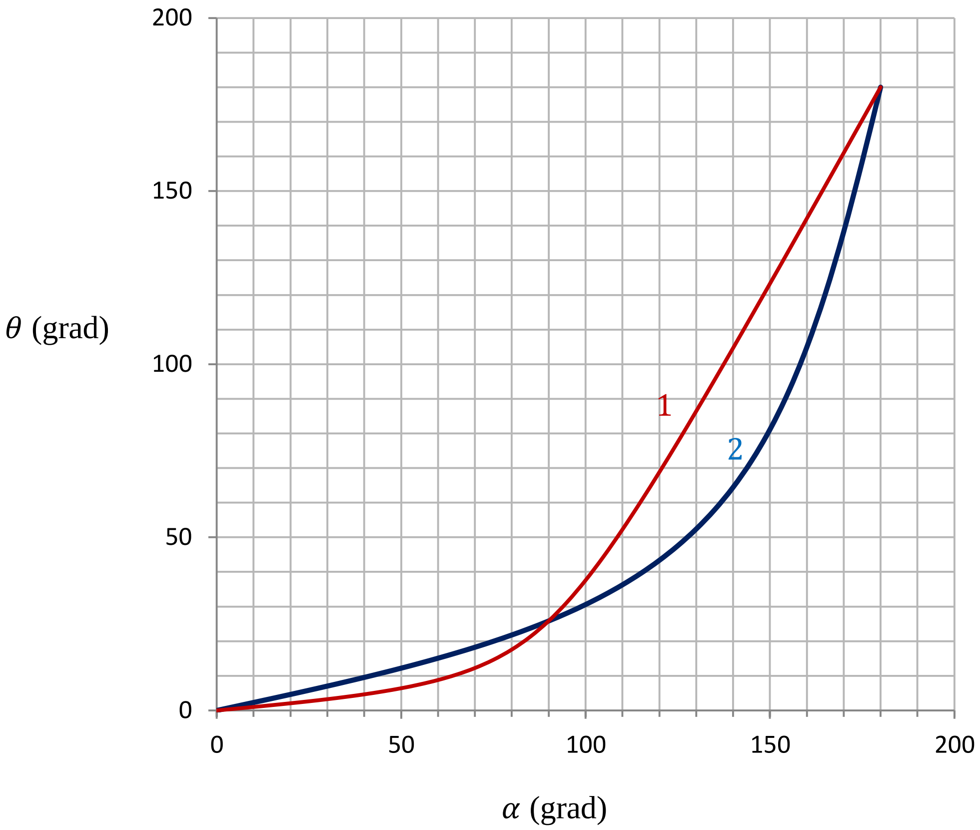

Fig. 4 *Aberration*. Dependence of the angle of reception θ on the angle of emission α for spherical (curve 1) and planar (curve 2) electromagnetic waves when $\beta = 0.9$.

For plane electromagnetic waves the angles θ and α in Eqs. (1) and (2) are equivalent ones, which means we are ignoring the source of wave completely and are considering only a wave in two equivalent inertial frames. That is why when making transition from one frame to the other we have not only to alter the sign before the relative velocity of the frames, but also to swap the angles and wavelengths in those equations. Thus, under the Lorentz transformation the unique character of the rest frame of a source of electromagnetic wave gets lost, which, of course, is also a distortion of the physical reality.

Now let us get the equation of time dilation using as an axiom the constancy of the vector of the light velocity $\vec{c}$, instead of the information about the zero shift angle as above, in order to show that they mean essentially the same thing.

According to Eq. (4), for the case when $\lambda_\theta = \lambda_0$,

$$\frac{n}{n_0} = \frac{|\vec{c} - \vec{v}|}{c}.$$

n_0 and n may be interpreted as the distances between the source and the observer at the moments of emission and reception divided by λ_0 and λ_θ respectively.

In the other frame:

$$\frac{n}{n_0} = \frac{|\vec{c} + \vec{v}|}{c}$$

Since $\frac{n}{n_0}$ is the ratio of the numbers of wavelengths, which are equal to the numbers of *internal oscillations* of the source of waves, its final expression cannot contain any trace of direction ("the source" may be not emitting anything and the direction of the observer's motion cannot affect those numbers).

Besides, while both equations are essentially the same ($|\vec{c} - \vec{v}|$ in one frame is the same as $|\vec{c} + \vec{v}|$ in the other frame), their forms differ in the sign before the relative velocity. The principle of relativity of uniform motion requires that not only the essence of those equations, but also their forms be the same (otherwise an observer would be at a loss which form to use in their frame: in regard to internal oscillations all inertial frames are equivalent). Thus, both forms of those equations must give the same result for both observers, i.e. when $\lambda_\theta = \lambda_0$, then $|\vec{c} - \vec{v}| = |\vec{c} + \vec{v}|$ in both frames, which gives $\frac{n}{n_0} = \sqrt{1 - \beta^2}$ for the case when $(\vec{c} - \vec{v}) = (\vec{c} + \vec{v})$ (and $\vec{v} \neq 0$), or when $\cos\theta = \beta$ in Eq. (4).

As for the other solution: $\frac{n}{n_0} = \sqrt{1 + \beta^2}$, i.e. when $\cos\theta = 0$ in Eq. (4), we discard it because that would mean that a moving observer would be able to count more numbers of waves than emitted by the source.

For each frame n_0 is proportional to the time passed in that frame, and n is proportional to the time passed in the other frame from the moment of starting of emission of the wave. Thus, $\frac{t'}{t} = \frac{n}{n_0} = \sqrt{1 - \beta^2}$.

Or otherwise, since $n\tau' = n_0\tau$ (the same time interval measured in two different units of time), where τ' and τ are the periods of oscillations within the source in the observer's and the source's frames respectively, we get the equation of time dilation: $\frac{\tau}{\tau'} = \sqrt{1 - \beta^2}$.

To get the picture of propagation of the wavefront of a spherical electromagnetic wave in moving and resting frames, let us consider a source of spherical electromagnetic wave in point O' moving from point O to the right with constant velocity $v = \beta c = \frac{|OO'|}{t}$ in the rest frame of the observer (Fig. 5). At the moment of time t in point O, the time in point O' is $t' = t\sqrt{1 - \beta^2}$. The wavefront of the wave emitted by the source in point O is a sphere with the centre in point O and the radius $R = ct = |OA|$ in the rest frame of the observer. In the rest frame of the source the wavefront of the same wave is a sphere with the centre in point O' and the radius $r = ct\sqrt{1 - \beta^2} = |O'A'|$. Because both those spheres are the same, but watched from different frames, there has to be one to one correspondence between their points. Let line $A'B$ be a thin and long enough imaginary tube of arbitrary orientation (angle γ) with the source within it, which is moving together with the source. Propagation of the wave through this tube is watched from both frames. At the instant of time t in the frame of the observer, the wavefront propagating through the tube arrives at points A and B. At the instant of time t' in the frame of the source, the wavefront propagating through the same tube arrives at points A' and B'. That means points A' and B' in the frame of the source are the same points (of the wavefront as well as of the tube) as points A and B respectively in the frame of the observer.

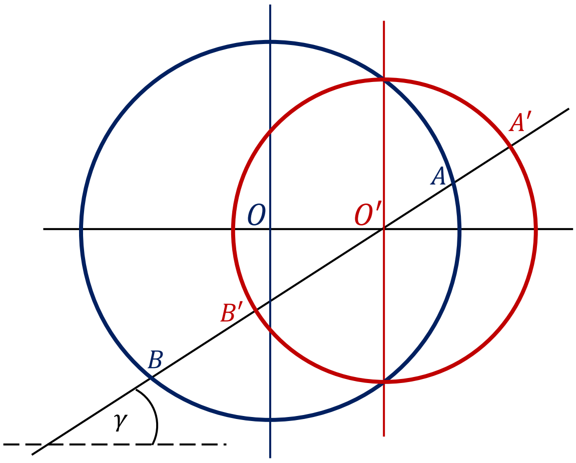

Fig. 5 The wavefront of a spherical electromagnetic wave in the frames of the source and the observer moving relative each other. The origin of the resting frame is in O and the origin of the moving frame is in O'. The ratio $\frac{|A'B'|}{|AB|}$ gives the expression of the Lorentz–FitzGerald contraction.

It is easy to notice that $|A'B'| = |AB|\sqrt{\frac{1-\beta^2}{1-\beta^2 \sin^2\gamma}}$, which is the equation of the Lorentz–FitzGerald contraction [3]. Not going into details here, that means the results of the Michelson-Morley experiment may be interpreted in the terms of spherical waves as well.

It seems obvious that the picture must be the same when the source is resting and the observer and the tube moving. (The observer could be a source of electromagnetic waves as well; what really matters is which one is moving and not which one is the source.)

The Lorentz transformation gives a different picture of the same events. The trouble with the Lorentz transformation seems to be that it allows instant measurement (instant comparison) of lengths in perpendicular to the relative velocity directions, which means that in those direction lengths cannot undergo relativistic changes without violating the principle of relativity of uniform motion. In the directions along the relative velocity, where it is not possible to bring together both ends of compared lengths at the same time (i.e. instant comparison of lengths is impossible), planar and spherical approaches to electromagnetic waves give the same results. But in other directions, where a normal component of length is present, they give different results. There are serious reasons to believe that instant measurement of length is not possible in any directions [4], but again, a more detailed discussion of this question lies beyond the scope of this paper.

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