Basics of Deuteron-Cluster Dynamics by Langevin Equation

Akito Takahashi

Technova Inc., Uchisaiwaicho 1-1-1, Chiyodaku, Tokyo, 100-001 Japan

This paper gives the follow-up study on the basics for our previous papers[2)], Study on 4D/Tetrahedral Symmetric Condensate Condensation Motion by Non-Linear Langevin Equation, Symposium 998-Low Energy Nuclear Reactions Source Book, ACS, published on August 2008 from Oxford University Press.

Pertaining to the quantum mechanics, the basics of new approach using the stochastic differential equation (Langevin equation) is written for quantifying dynamic motion of known molecules as D_2^+, D_2 and D_3^+ as well as D-atom state. The role of the Platonic symmetry in these known molecules are discussed for deducing simple one-dimensional (R_{dd} dependent; here R_{dd} is distance between nearest d-d pair) Langevin equation and making quantum-mechanical ensemble averaging to obtain equation for expectation value.

The methodology is applied for more complicated D-clusters as 4D/TSC and 6D/OSC which would keep the Platonic symmetry, by introducing the force fluctuation deviating from the ideal Platonic symmetry. Time-dependent TSC and OSC trapping potentials which take balance to getting back to the Platonic symmetry from the distorted states were defined and used for numerical solution of Langevin equation. Finally, time-dependent fusion rate formula for simultaneous 4D interaction was obtained based on the Fermi's golden rule and one-pion exchange potential of strong interaction. The 4D fusion is regarded to

cause radiation-less excess heat and ^{4}He ash in metal-deuterium systems under dynamic conditions.

Introduction

This paper gives the follow-up basics for our previous paper[2)], Study on 4D/Tetrahedral Symmetric Condensate Condensation Motion by Non-Linear Langevin Equation, Symposium 998-Low Energy Nuclear Reactions Source Book, ACS, published on August 2008 from Oxford University Press.

The reports of apparent hard-radiation-less excess heat with helium-4 ash are of key results in CMNS (condensed matter nuclear science) experiments, especially in dynamic PdDx systems. We have done a long series of study for modeling D-cluster (or multi-body deuteron) fusion reaction mechanisms to have reached at our latest theory based on Langevin equations[1,2)].

This paper describes the quantum mechanical basics of formulation of Langevin equations (stochastic differential equations) for D-cluster dynamics, especially for D-atom, D_2 molecule, D_2^+ ion, D_3^+ ion, 4D/TSC (tetrahedral symmetric condensate) and $6D^{2-}$/OSC (octahedral symmetric condensate). This paper describes our extended explanation to our previous paper[14)] to the Catania Workshop.

First one-dimensional Langevin equations for D-clusters with the R_{dd} d-d distance are formulated under the Platonic symmtery[1,3)] conditions of multi-particle D-cluster systems with deuterons and quantum-mechanical electron centers. The known D (or H) molecular systems can be regarded as three-dimensional symmetric geometry keeping the Platonic symmetry[14)]. Under the orthogonally coupled Platonic symmetry for a Platonic deuteron-system and a Platonic electron system, a simple one-dimensional Langevin equation for the inter-nuclear d-d distance R_{dd} can be formulated[14)], as we will show in this paper with further explanations, for the known deuteron-molecules and clusters.

The ensemble averaging of one-dimensional Langevin equation with the weight of quantum mechanical wave-functions for electrons and deuterons can be done, by assuming the Born-Oppenheimer treatment for the separability of electron and deuteron wave functions. Usually, the electron wave length is much larger than the deuteron wave length, so that the Born-Oppenheimer treatment is appropriate. We can further derive a time dependent equation for expectation value $<R_{dd}>$, which is nonlinear, but can be solved by the Verlet time step method[2)]. We show in this paper that only 4D(or H)/TSC can condense ultimately to be finally very small charge neutral entity with about 10-20 fm radius, among considered cluster states.

At the final stage of 4D/TSC condensation in about $2x10^{-20}$ s, 4D fusion with 2 ^{4}He products takes place with almost 100% probability, according to our HMEQPET calculation for barrier factors and fusion rate formula by the Fermi's first golden rule[2)].

We show in Chapter 2 the derivation of Langevin equations for known systems as D-atom, D_2 molecule, D_2^+ ion, and D_3^+ ion. This procedure gives the basics of methodology for formulating Langevin equations of complex D-cluster systems as 4D/TSC and 6D/OSC in Chapter 3. In analogy, we apply the methodology and derive Langevin equations for 4D/TSC and 6D/OSC condensation motions. In the following chapters, estimation of fusion rates for the simultaneous 4D fusion induced by the very rapid 4D/TSC condensation is given. A speculative discussion on the final state interaction of 4D fusion is given lastly.

Langevin Equations for Known D(H)-Atom and Molecules

Langevin Equation in General

The Langevin equation is useful to treat dynamic motion of particles under the existence of friction (or constraint) and random fluctuation of force-field. This is a kind of stochastic differential equation in mathematical view. The generally known form is:

$$m\frac{d^2R}{dt^2} = -F_c - \varsigma\frac{dR}{dt} + f'(t) \qquad (1)$$

Here m is the particle mass, R is particle position, F_c is the Coulombic force, ς is the coefficient for friction (or constraint) and f'(t) is the randomly fluctuated force term (white noise usually but we need modification for complex systems), for our deuterons plus electrons systems. The Langevin equation takes the balance of force with the fluctuation force source term f'(t), into which we include the quantum mechanical effect. This equation is complimentary to the quantum mechanical Schroedinger equation which takes balance of the system total energy (Hamiltonian) with the Heisenberg Uncertainty Principle (HUP) in operator forms.

Langevin Equation for D-atom

In **Fig.1**, a simple quantum mechanical image of D-atom is drawn. In the view of Platonic symmetry, D-atom is the orthogonal coupling of central point (deuteron) and sphere (electron-wave) to form symmetry in the three-dimensional space. The Langevin equation is given as the balance of the centripetal force of Coulombic attraction between plus-charged deuteron and minus-charged electron and the centrifugal force of electron rotation motion around the center-of-mass point (actually very close to the center position of deuteron wave);

$$m_e \frac{d^2 R_{de}}{dt^2} = -\frac{e^2}{[R_{de}]^2} + \frac{m_e v_e^2}{R_{de}} + f(t) \qquad (2)$$

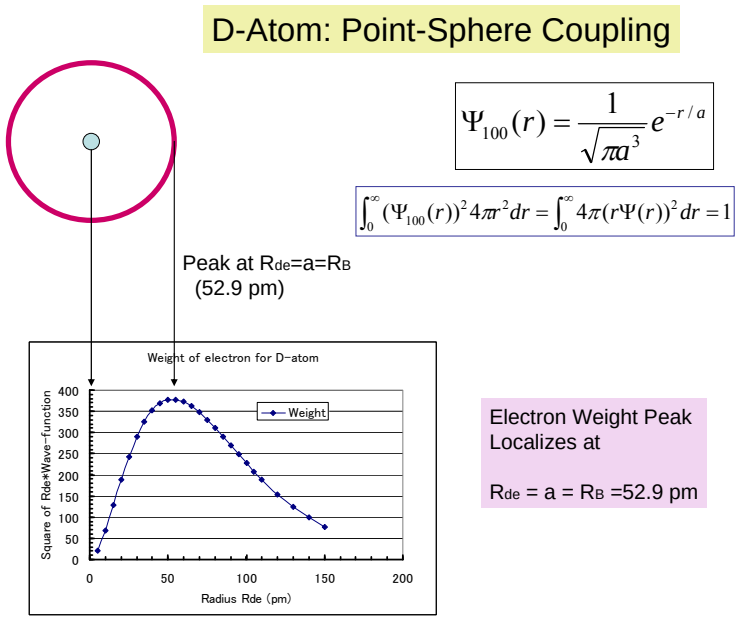


Fig. 1: Quantum mechanical image of D-atom

Here, m_e is the electron mass, R_{de} is the d-e distance, e is the unit charge and v_e is the electron velocity.

Basic quantum mechanical data for D(H)-atom are:

$$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \quad \text{; 1-S electron wave function} \tag{2-2}$$

$$\langle \Psi_{100} | E_{C-D} | \Psi_{100} \rangle = \int_0^\infty (-e^2/r) \Psi_{100}{}^2 4\pi r^2 dr = -1.44/r \quad \text{; Coulomb energy} \tag{2-3}$$

Using r=R_B=52.9 pm, we get <Coulomb Energy>= -27.2eV.
Using the ground state energy of the system;

$$\langle H \rangle = \langle \Psi_{100} | H | \Psi_{100} \rangle = \langle \Psi_{100} | -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{r} | \Psi_{100} \rangle = E_0 = -13.6eV \tag{2-4}$$

We obtain <Mean Electron Kinetic Energy>=13.6eV.
The corresponding de Broglie wave length $\lambda = h/(mv)$ to this mean electron kinetic energy is 332 pm, which satisfies the condition that $2\pi R_B = 332pm$, namely the Bohr's condition or the continuity of wave function for one turn around the center of mass point (very close to the deuteron or proton position in this case). For λ> 332 pm, the centrifugal force of orbital electron, $(mv)^2/R_{de}$ is smaller than the centripetal Coulombic attractive force, $e^2/R_{de}{}^2$. Forλ< 332 pm, the centrifugal force is greater than the centripetal force.
Therefore the λ= 332 pm state (corresponding to the mean electron kinetic energy 13.6 eV) is the only one steady state, namely the ground state for the d-e or p-e system. No other electron kinetic energy states than 332 pm can satisfy the Heisenberg uncertainty principle (HUP) and the exact balancing of the centrifugal force and the centripetal force. Thus the electron motion around deuteron (or proton) with different kinetic energy from 13.6 eV should draw spiral motion to getting back to the 332 pm ground state.

We have no friction in this case for formulating Langevin equation. By taking ensemble average of Eq.(2) with the weight of squared 1-S wave function $\Psi_{100}{}^2$, we obtain,

$$m_e \frac{d^2 \langle R_{de} \rangle}{dt^2} = -\left\langle \frac{e^2}{R_{de}^2} \right\rangle + \left\langle \frac{m_e v_e^2}{R_{de}} \right\rangle = 0 \tag{3}$$

The right side becomes zero, because of the average kinetic energy $\langle E_{KE} \rangle = \frac{1}{2} m_e \langle v_e^2 \rangle = \frac{e^2}{2R_B} = 13.6eV$ and the average Coulomb energy

$\langle E_C \rangle = -\frac{e^2}{R_B} = -27.2eV$ as well known for hydrogen (or deuterium) atom.

However the exact mathematical proof is needed, as we regarded the factor $1/R_{de}$ separable in the ensemble averaging.
We can integrate Eq.(3) over time to get,

$$m_e \frac{d\langle R_{de} \rangle}{dt} = F(T) = \int_0^T f(t)dt = \langle f(t) \rangle = 0 \tag{4}$$

The time-average (integral) of random fluctuation f(t) is equal to the ensemble average <f(t)> due to the ergodic process. We integrate Eq.(4) to obtain the well known result as,

$$\langle R_{de} \rangle(t) = R_0 = R_B = 52.9\,pm \tag{5}$$

Namely, expectation value $<R_{de}>$ of radial electron orbit is constant to be Bohr radius R_B = 52.9 pm. Dynamic motion of electron starting at arbitrary position should converge to this ground state after spiral motion.

Quantum Mechanical Ensemble Average for D-Cluster

Since we can regard that both positions of electrons and deuterons fluctuate quantum mechanically for D-cluster systems, we need to average with both weights of wave functions for electrons and deuterons. Applying the Born-Oppenheimer (adiabatic) approximation for total wave function, we can make step-wise averaging for electron-waves and then for deuteron-waves. The adiabatic wave function for D_2 molecule is;

$$\Psi(R_{dd}; r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \Psi_{2D} \cdot X(R_{dd}) \tag{6}$$

The electron wave function of D_2 molecule is given[4] by,

$$\Psi_{2D} = \frac{1}{\sqrt{(2+2\Delta)}} \left[\Psi_{100}(r_{A1})\Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\Psi_{100}(r_{B1}) \right] X_s(S1, S2) \tag{7}$$

And the wave function for a d-d pair is approximated by the Gaussian wave function as, rewriting X with Ψ and putting $R_{dd} = R$,

$$\Psi(R, R') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(R'-R)^2/(2\sigma^2)) \tag{8}$$

Quantum-mechanical ensemble average of observable G is given for every step,

$$\left\langle G\right\rangle_{ensemble} = \left\langle \Psi \middle| G \middle| \Psi \right\rangle \tag{9}$$

Langevin Equation for D_2 Molecule

Electron localization (weight distribution) of D_2 molecule is roughly understood by the normalization equation of electron wave function as,

$$(4\pi)^4 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \Psi_{2D}^2(r_{A1}, r_{A2}, r_{B1}, r_{B2}) r_{A1}^2 r_{A2}^2 r_{B1}^2 r_{B2}^2 dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1 \tag{10}$$

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1 \tag{11}$$

$$\rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \frac{(4\pi)^4}{2+2\Delta} \left\{ \begin{array}{l} \left[r_{A1}^2 \Psi_{100}^2(r_{A1}) r_{B2}^2 \Psi_{100}^2(r_{B2}) \right] r_{A2}^2 r_{B1}^2 \\ + 2\left[r_{A1} \Psi_{100}(r_{A1}) r_{A2} \Psi_{100}(r_{A2}) r_{B1} \Psi_{100}(r_{B1}) r_{B2} \Psi_{100}(r_{B2}) \right] r_{A1} r_{A2} r_{B1} r_{B2} \\ + \left[r_{A2}^2 \Psi_{100}^2(r_{A2}) r_{B1}^2 \Psi_{100}^2(r_{B1}) \right] r_{A1}^2 r_{B2}^2 \end{array} \right\} \tag{12}$$

Since $\left| r\Psi \right|^2$-type term is the element of particle density localization function, localized peaks appear at $r_{A1}=r_{A2}=r_{B1}=r_{B2}=a=R_B$ (52.9 pm); namely the drawn sphere with Bohr radius is a good measure of electron localization.

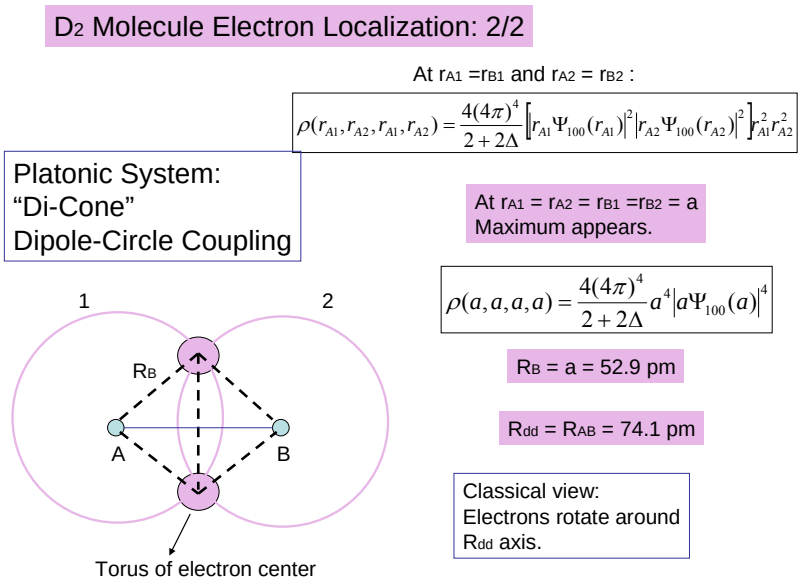


Fig.2: Localization of electron wave and semi-classical image of D_2 molecule

In the view of Platonic symmetry, D_2 molecule is an orthogonal coupling of the d-d line (dipole) and the circle (torus of electron center) to form a dicone (see the left-bottom figure in Fig.2). Freedom of electron motion is constrained by the existence of counterpart deuteron and two electrons to form the torus of electron centers, but averaged centrifugal force exists as the rotation of electrons around R_{dd} axis.

Major quantum mechanical parameters for D_2 molecule are;

$$\left\langle \Psi_{2D}\middle|H\middle|\Psi_{2D}\right\rangle = -35.1eV \quad \text{; total energy of the } D_2 \text{ system} \qquad (12\text{-}2)$$

$$\left\langle E_{C-2D}\right\rangle = 4(-\frac{e^2}{a_B}) + 2(\frac{e^2}{\sqrt{2}a_B}) = -70.3eV \quad \text{; Coulomb energy} \qquad (12\text{-}3)$$

$$\left\langle E_{ke-2D}\right\rangle = \left\langle H_{2D}\right\rangle - \left\langle E_{C-2D}\right\rangle = 70.3eV - 35.1eV = 35.2eV$$
$$\text{; mean kinetic energy of two electrons} \qquad (12\text{-}4)$$

We now know that the mean kinetic energy of one electron in the D_2 system is 17.6 eV. The de Broglie wave length of 17.6 eV electron is 230 nm, which satisfies the condition $2\pi R_e = 230nm$ with $R_e = R_{ee}/2 = R_B/\sqrt{2}$. This feature

provides us a semi-classical image that two quantum-mechanical electron-centers are rotating around the center-of-mass point of the system and draws a torus (circle).

The Langevin equation for D_2 molecule thus becomes as,

$$m_d \frac{d^2 R_{dd}}{dt^2} = -(4\sqrt{2}-2)\frac{e^2}{R_{dd}^2} + \frac{2m_e v_e^2}{(R_{ee}/2)} - \frac{\partial V_{s2}(R_{dd};1,1)}{\partial R_{dd}} + f(t) \qquad (13)$$

Here the Coulomb force term under Platonic symmetry is derived by derivative (minus sign) of Coulomb energy,

$$E_C = -4\frac{e^2}{R_{de}} + \frac{e^2}{R_{dd}} + \frac{e^2}{R_{ee}} \qquad (14)$$

$$E_C \approx -4\sqrt{2}\frac{e^2}{R_{dd}} + 2\frac{e^2}{R_{dd}} \qquad (15)$$

By taking QM-ensemble average with weight of squared electron wave function,

$$m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\left\langle \frac{5.26}{R_{dd}^2} \right\rangle + 4\left\langle \frac{m_e v_e^2}{R_{ee}} \right\rangle - \frac{\partial V_{s2}(R_{dd};1,1)}{\partial \langle R_{dd}\rangle} + \langle f(t)\rangle \qquad (16)$$

The first and second term of Eq.(16) right side cancels each other[4], and we obtain,

$$m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{\partial V_{s2}(R_{dd};1,1)}{\partial \langle R_{dd}\rangle} + \langle f(t)\rangle \qquad (17)$$

By taking ensemble average with the Gaussian wave function of d-d pair, the second term of Eq.(17) <f(t)> becomes zero, since we have no distortion in the d-d dipole line by the QM fluctuation which causes deviation from the Platonic symmetry. Thus, Eq.(17) becomes the well known Newtonian mechanical equation, with constraint by molecular trapping potential V_{s2} (R_{dd};1,1) which we must however calculate by quantum mechanics. Mathematical formulas for trapping (shielded) potentials of D_2 and D_2^+ systems are given in our previous papers[2, 5]. Figure of plotted data for two potentials are shown in **Fig.3**.

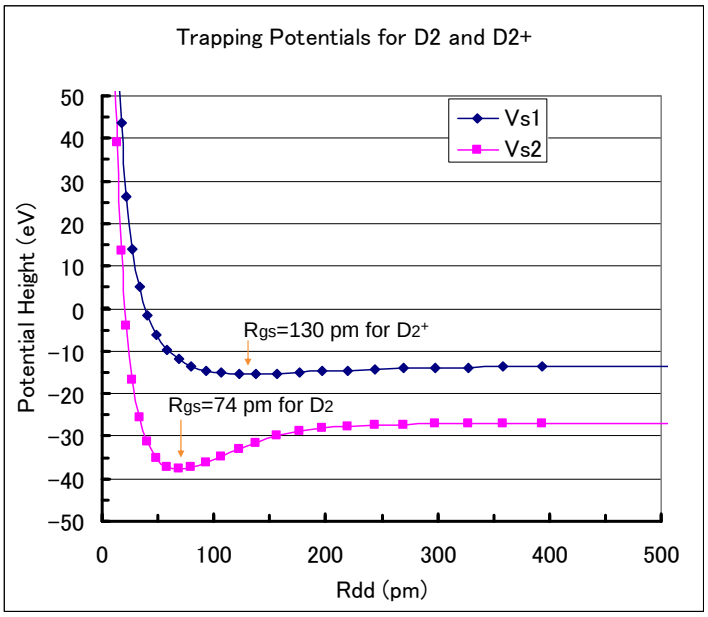


Fig.3: Trapping potential of d-d pair for D_2 molecule and D_2^+ ion

As understood by potential shape for D_2 molecule given in Fig.3, the Langevin equation for expectation value $<R_{dd}>$ gives always convergence to R_{gs} =74 pm after time-dependent spiral motion starting from arbitrary position R_{dd} (t=0). If $R_{dd} > R_{gs}$, we have acceleration force. If $R_{dd} < R_{gs}$, we have deceleration force. The feature is drawn in Fig.3-2.

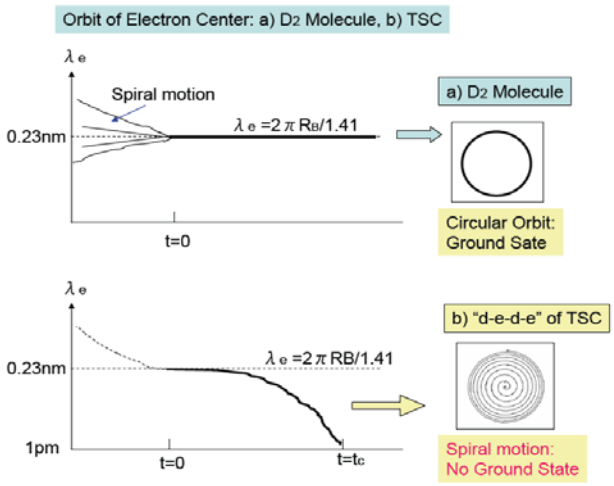


Fig.3-2: Feature of spiral motion for D_2 system; a) stable D_2 molecule, b) a face of TSC under condensation.[12)]

Langevin Equation of D_2^+ Ion

The D_2^+ ion molecule is the three body system of d-e-d. The electron wave function is given by the normalized linear combination of 1-S wave functions from two deuteron positions. The energy eigen-value of electron as a function of d-d distance R_{dd} is given by the variational method to be $V_s(R_{dd};1,1)$, curve of which is given in Fig.3.

The electron center (averaged position with weighting of squared wave function) makes a torus around the center-of-mass of the d-e-d system (very close to the mid-point of R_{dd}). In the view of the Platonic symmetry, D_2^+ molecule is the elongated dicone.

In analogy to the D_2 molecule, Langevin equation for D_2^+ ion (stable in vacuum) is thus given by,

$$m_d \frac{d^2 R_{dd}}{dt^2} = -2\frac{e^2}{R_{de}^2} + \frac{e^2}{R_{dd}^2} + \frac{m_e v_e^2}{R_e} - \frac{\partial V_s(R_{dd};1,1)}{\partial R_{dd}} + f(t) \qquad (18)$$

By taking QM-ensemble average with electron wave function and Gaussian d-d wave function, we obtain,

$$-e^2\left\langle \frac{2}{R_{de}^2}-\frac{1}{R_{dd}^2}\right\rangle+\left\langle \frac{m_e v_e^2}{R_e}\right\rangle=0 \tag{19}$$

$$m_d\frac{d^2\langle R_{dd}\rangle}{dt^2}=-\frac{\partial V_s(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} \tag{20}$$

Potential curve is shown in Fig.3. In the view of Platonic symmetry, D_2^+ ion molecule is an elongated dicone with R_{dd}=130 pm (approximate value for the ground state), with rotating triangle of "d-e-d" type face around the R_{dd} axis. Dynamic motion of deuteron by Eq.(20) gives convergence to $R_{dd}=R_{gs}$=138 pm after spiral motion. The eigen-value of thre system Hamiltonian is calculated by GW2 code[2] to be -15.1eV. The system Coulomb energy is -28.7 eV. We obtain the electron mean kinetic energy to be 13.6 eV. The radius of torus for the rotation of quantum-mechanical electron center around the center-of-mass (very close to the mid point of the d-d dipole) is calculated to be 52.9 pm (equal to the Bohr radius). The d-d distance of ground state is 138 pm. These are the conditions to make the system energy of d-e-d three-body minimum.

In the view of classical mechanics, the straight line-up of d-e-d sytem shall give the minimum system energy, and it would converge to the mid point to form a "very dimished size dde system" which might cause strong interaction, namely d-d fusion. However to fulfill this requirement, sizes of deuterons and electrons must be zero (just points); This is the consequence of Newtonian mechanics and the charge-density and the mass density should diverge to infinity at the points. This is the great contradiction agaist nature for the classical mechanics. To avoid the divergence, particles should have finite sizes reflecting the HUP (Heisenberg Uncertainty Principle).

In the quantum mechanical view, the weight of charge density at the center-of-mass point (very close to the middle line of d-d dipole) is just zero dute to HUP. Namely the much larger de Broglie wave length of electron than that of deuterons should make the effective quantum-mechanically averaged charge center existing at the rotaing torus around the center-of-mass point. Consequently, the d-e-d system draws a "rotating triangle" around the d-d dipole line, in the semi-classical view. This is the enrgy-minimized sate of the d-e-d three-body system in the quantum mechanics.

In the following for complex D-cluster systems, Eq.(17) and Eq.(20) with those potentials will provide intrinsic components of friction (constraint) by QM electron waves with D-cluster condensation.

Langevin Equations of Expectation Values for Complex D-Clusters

In complex D-cluster systems under the Platonic symmetry, averaged rotation motion of particles over whole system is prohibited by the constraints of many particle arrangements. This *form*[3)] of self-organization makes simpler treatment to derive one-dimensional Langevin equation possible. The term *form* is meta-physical concept.

The QM-ensemble average on electron wave function can be subdivided as multiple constraint functions of either the "d-e-d" type or the "d-e-d-e" type potential derivative as,

$$\left\langle Constra\,\mathrm{int}\right\rangle_{electron-wave} = -N_f \frac{\partial V_{si}(R_{dd};1,1)}{\partial R_{dd}} \tag{21}$$

Here N_f is the number of faces of the Platonic polyhedron for a D-cluster, and i=1 for the "d-e-d" type (D_2^+ type) face and i=2 for the "d-e-d-e" type (D_2 type) face.

The Langevin equation for a D-cluster under the Platonic symmetry with N_e number of d-d edges and N_f number of faces is written as,

$$N_e m_d \frac{d^2R}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + f(t) \tag{22}$$

Here k is constant (11.8 for 4D/TSC, 29.3 for $6D^{--}$/OSC, 6.13 for $3D^+$, and zero for D_2 and D_2^+).

The QM-ensemble average with the d-d wave function (assuming Gaussian form) is given by

$$\begin{aligned} N_e m_d \left\langle \Psi(R,R') \left| \frac{d^2R}{dt^2} \right| \Psi(R,R') \right\rangle &= -\left\langle \Psi(R,R') \left| \frac{k}{R^2} \right| \Psi(R,R') \right\rangle \\ - N_f \left\langle \Psi(R,R') \left| \frac{\partial V_s}{\partial R} \right| \Psi(R,R') \right\rangle &+ \left\langle \Psi(R,R') \left| f(t) \right| \Psi(R,R') \right\rangle \end{aligned} \tag{23}$$

with a Gaussian wave function for d-d pair of D-cluster,

$$\Psi(R,R') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(R'-R)^2/(2\sigma^2)) \tag{24}$$

We drive a Langevin equation for expectation value $<R_{dd}> = <R>$ as,

$$N_e m_d \frac{d^2 < R >}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + < f(t) > \tag{25}$$

For complex D-cluster, <f(t)> value does not always zero because of the deviation of D-cluster system from the ideal Platonic symmetry, due to the distortion of the system by the quantum mechanical fluctuation of d-positions. The perturbed force component by this QM distortion is approximately given by the next formula, which is the change of system Coulomb energy derivative, as,

$$\begin{aligned} -\frac{\partial \Delta E_C}{\partial R} &\approx -k\frac{1}{2}\left(\frac{1}{(R+\Delta R)^2} + \frac{1}{(R-\Delta R)^2}\right) + k\frac{1}{R^2} \\ &\approx -\frac{k}{2R^2}\left(1 - \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R}\right)^2 + 1 + \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R}\right)^2\right) + \frac{k}{R^2} \\ &= \frac{k}{R^2}\left(\frac{\Delta R}{R}\right)^2 \end{aligned} \tag{26}$$

By using a Gaussian squared wave function for d-d pair fluctuation, we write,

$$(\Delta R)^2 = (\sigma R)^2 \tag{27}$$

The change of Coulomb force by distortion is given by the following simple formula,

$$<\text{Change of Coulombic Force}> = \frac{k\sigma^2}{(R_{dd})^2} \tag{28}$$

Langevin Equation of D_3^+ Ion Molecule

It is well known that tri-atomic hydrogen molecular ion D_3^+ (or H_3^+) is generated in ion source and glow discharged plasma and very stable in vacuum. However, quantum molecular physics for the system is of difficult problem to solve by many-body Schroedinger equations, and studies are being continued in astrophysics needs. The D_3^+ molecule is the d-e-d-e-d five-body system.

The analogy of the present methodology for D (or H)-cluster under the orthogonal coupling of Platonic symmetries for electrons and deuterons (or protons) can provide rather simple way of modeling its dynamics. Applying Eq.(25), we obtain the Langevin equation of D_3^+ molecule, for the expectation value $<R_{dd}>$,

$$3m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{6.13}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle \qquad (29)$$

Here the force is given with unit of [keV/pm]. Image of D_3^+ ion is given in **Fig.4**.

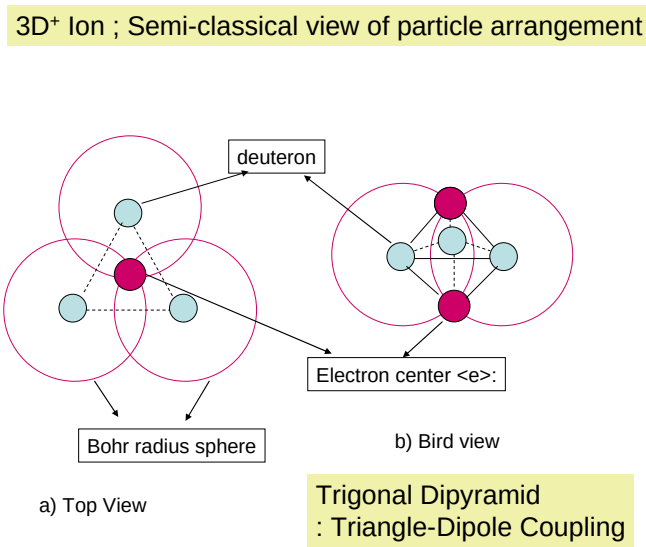


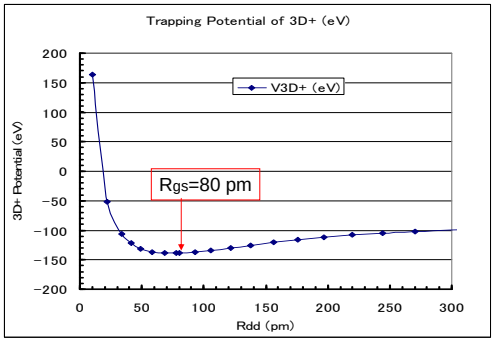
Fig.4: Tri-atomic hydrogen (deuterium) molecular ion and its Platonic arrangement

The system Coulomb energy and its derivative can be calculated by simple geometry exercise for the Platonic symmetry system of trigonal di-pyramid which is the orthogonal coupling of the 3d regular triangle and the <e> - <e> line (dipole). Here two electron centers (or electron balls) appear in the system, and system-averaged rotation of electrons is prohibited (no averaged centrifugal force). However, there remains partial electron rotation motion around a d-d line as a face of the system.

By distortion of the ideal Platonic symmetry due to the QM fluctuations of deuteron positions, we have positive <f(t)> bias. As we have 3 d-d edges in the system, 3 times of Eq.(28) becomes the bias (about 30 % of main Coulomb acceleration force). Therefore the total potential of the system becomes in expected value equation, as,

$$V_{3D+main}(R_{dd}) = -\frac{6.13}{R_{dd}} + 6V_s(R_{dd};1,1) + (\langle f(t)\rangle \text{ component}) \tag{30}$$

The calculated curve of this potential is shown in **Fig.5**.



$$3m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{6.13}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle$$

$$\langle f(t)\rangle = \left\langle -\frac{\partial \Delta E_C}{\partial R'} \mathrm{X}^2(R_{dd};\sigma,t)\right\rangle$$

:Distortion of Coulomb force from 3D Regular triangle arrangement (about 30%)

Fig.5: Trapping potential of D_3^+ ion molecule with Langevin equation for expectation value of d-d distance of 3d regular triangle

The tri-atomic hydrogen ion is thus stable and has its ground state at R_{gs}= 80 pm. As a reference, Helm et al (Freiburg University, 2003; google triatomic hydrogen ion and Helm) gave about R_{gs} = 85 pm[8] which agrees considerably well with our result taking into account that appropriate sigma-value of wave function[2] is about 30 % of R_{dd}.

We can conclude that our approach with one-dimensional Langevin equations for D-cluster systems look successful. We are ready to apply for more complex systems.

Langevin Equation for 4D/TSC and Numerical Solution

Double Platonic Symmetry

In **Fig.6**, we show the feature of electron cloud for 4D/TSC (t=0), compared with those of D-atom and D_2 molecule.

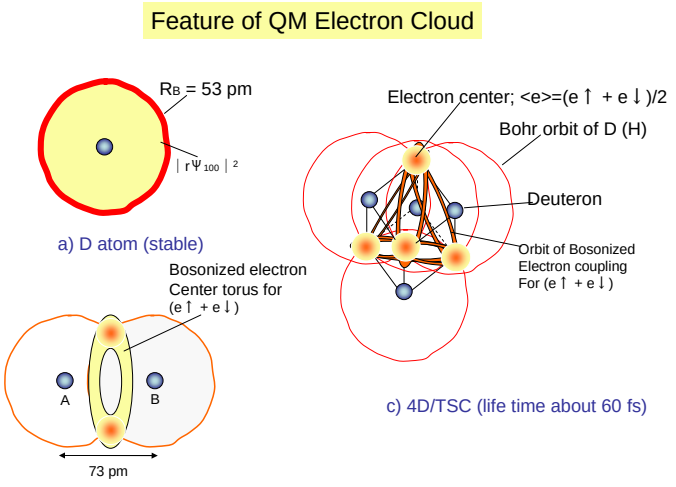


Fig.6: Feature of QM electron clouds for 4D/TSC (t=0), compared with those of D-atom and D_2 molecule

The *form*[3] of 4D/TSC (t=0) wave function is given[1,3,4] as,

Wave Function for 4D/TSC (t=0)

- $\Psi_{4D} \sim$ **a1** $[\Psi_{100}(r_{A1})\ \Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\ \Psi_{100}(r_{B1})]X_s(S1,S2)$
  +a2 $[\Psi_{100}(r_{A1})\ \Psi_{100}(r_{D4}) + \Psi_{100}(r_{A4})\ \Psi_{100}(r_{D1})]X_s(S1,S4)$
  +a3 $[\Psi_{100}(r_{A2})\ \Psi_{100}(r_{C4}) + \Psi_{100}(r_{A4})\ \Psi_{100}(r_{C2})]X_s(S2,S4)$
  +a4 $[\Psi_{100}(r_{B1})\ \Psi_{100}(r_{D3}) + \Psi_{100}(r_{B3})\ \Psi_{100}(r_{D1})]X_s(S1,S3)$
  +a5 $[\Psi_{100}(r_{B2})\ \Psi_{100}(r_{C3}) + \Psi_{100}(r_{B3})\ \Psi_{100}(r_{C2})]X_s(S2,S3)$
  +a6 $[\Psi_{100}(r_{C3})\ \Psi_{100}(r_{D4}) + \Psi_{100}(r_{C4})\ \Psi_{100}(r_{D3})]X_s(S3,S4)$

6-Bonds of "Bosonozed" electron-pairs (e ↑ + e ↓), which forms **Regular Tetrahedron (PA)**
4-Electron-Centers at Vertexes of **Regular Tetrahedron (PA)**

$u_{1s1}(r) = \Psi_{100}(r) = (1/\pi)^{1/2}(1/a_B)^{3/2}\exp(-r/a_B)$

Top equation (31)

This TSC system has double symmetry of regular tetrahedrons for deuterons and electron-centers, namely the *double Platonic symmetry* which is the most ideal system in 3-dimensional condensation squeezing into the central focal point (Center-of-Mass; CMS).

Langevin Equation for 4D/TSC and Solution

The system Coulomb energy and its derivative are given in our previous work[1,2]. We write here resulting final Langevin equation for Monte-Carlo calculation.

$$6m_d \frac{d^2R_{dd}(t)}{dt^2} = -\frac{11.85}{[R_{dd}(t)]^2} - 6\frac{\partial V_{s2}(R_{dd}(t);1,1)}{\partial R_{dd}(t)} + \langle f(t)\rangle + f'(t) \qquad (32)$$

with

$$f'(t) = f(t) - \langle f(t)\rangle \qquad (33)$$

$$f(t) = [-\frac{\partial \Delta E_c(R_{dd})}{\partial R_{dd}}] \mathrm{mod}[X^2(R'_{dd}; R_{dd}(t))] \tag{34}$$

$$\mathrm{X}^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd} - R_{dd}(t))^2 / (2\sigma^2)] \tag{35}$$

For QM-ensemble averaged equation, we obtaied[2],

$$6m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{11.85}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(\langle R_{dd}\rangle; m, Z)}{\partial \langle R_{dd}\rangle} + 6.6\left\langle \frac{(R' - R_{dd})^2}{R_{dd}{}^4} \right\rangle \tag{36}$$

The time-dependent TSC trapping potential for this equation was given[2] as,

$$V_{tsc}(R': R_{dd}(t)) = -\frac{11.85}{R_{dd}(t)} + 6V_s(R_{dd}(t); m, Z) + 2.2\frac{|R' - R_{dd}(t)|^3}{[R_{dd}(t)]^4} \tag{37}$$

Here we fixed m=2 and Z=2 for V_s (dde*(2,2) potential) for numerical calculation.
The third term of right side of Eq.(36) gives about 15% positive bias to main Coulomb force (first term), and was merged[2] in the first term by multiplying factor 0.85 in the numerical calculation by the Verlet method.

In **Fig.7**, we show the calculated trapping potential of 4D/TSC, compared with that of $6D^{2-}$/OSC (shown later). 4D(or H)/TSC keeps in average the always accelerating force in its condensation motion, hence it can condense ultimately until when TSC-min state (about 10-20 fm radius) comes, as illustrated in **Fig.8**. On the contrary, $6D^{2-}$/OSC converges to R_{dd}=40 pm on the way of condensation (we derive equation later). *Within the presently studied 5 kinds of D-clusters, only 4D/TSC can condense ultimately to very small charge neutral entity.*

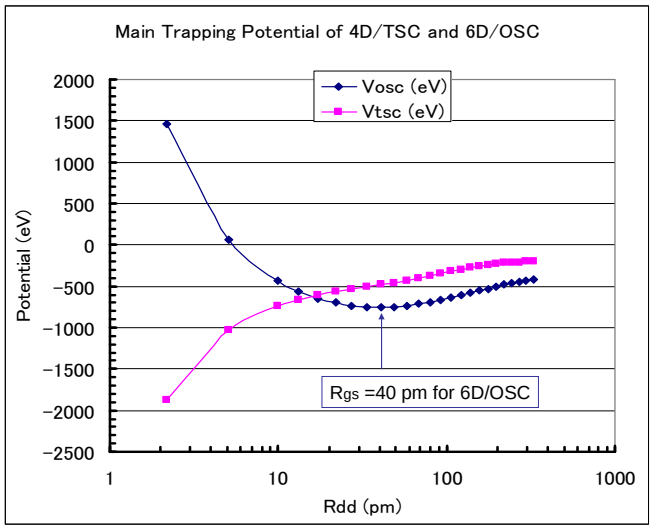


Fig.7: Central trapping potential (at R=R') of 4D(or H)/TSC, always attractive, compared with $6D^{2-}$/OSC potential which has converging point (about 40 pm) at its ground state

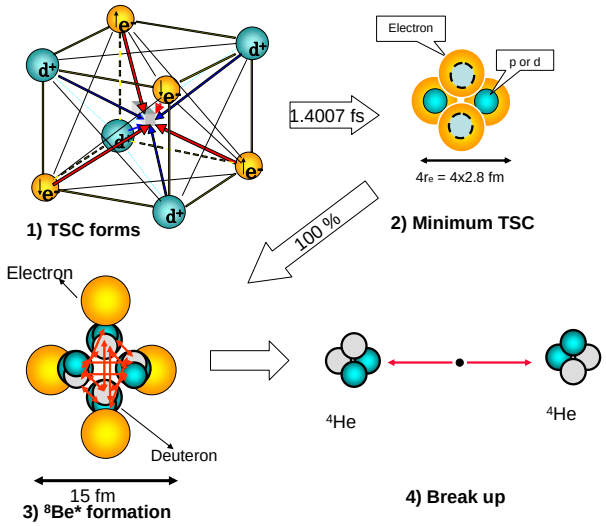


Fig.8 Condensation of 4D/TSC and 4D-fusion to two ^{4}He-particles break-up

Numerical solution of Eq.(36) was obtained by a computer code based on the Verlet time-step method[2], a standard result is shown in **Fig.9**.

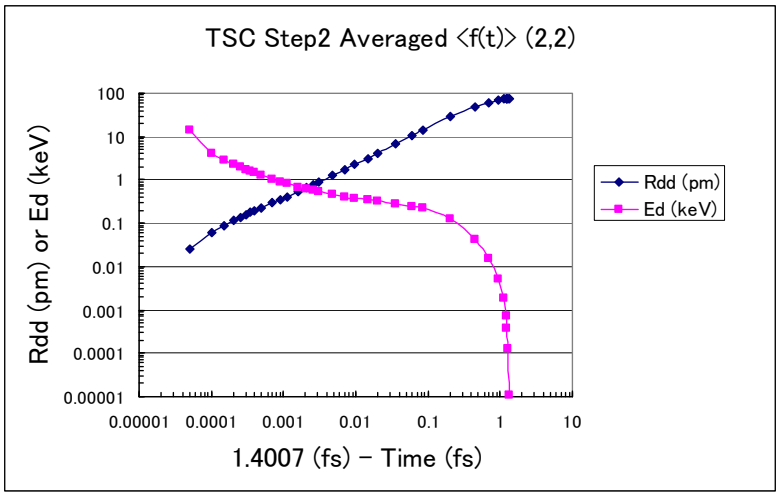


Fig.9: Numerical results of 4D/TSC condensation motion; time-variation of $<R_{dd}>$ and mean deuteron kinetic energy $<E_d>$

The condensation time of 4D/TSC is very fast as 1.4007 fs. As we show in the next section, 4D fusion reaction takes place with almost 100% probability in the final stage of condensation within the time interval of about $2x10^{-20}$ s. For other details of time-dependent behavior of TSC dynamics with different conditions, see our previous paper[2]. Since the effective time interval $2x10^{-20}$s with averaged effective deuteron energy about 10 keV are of our surprise, we can say that the cluster deuteron fusion by the TSC condensation mechanism is not "cold fusion". The relative kinetic energy about 10 keV for the 4D simultaneous fusion reaction seems comparable to the "target" d-t plasma temperature 10 keV of the ITER device (hot fusion), by chance.

We can consider also that, to make visible fusion rate in the condensed matter environment at room temperature, the deuteron cluster (or a d-d pair) should be confined within very deep trapping potential (time-dependent, in this case of TSC) to keep elevated kinetic energy of deuterons due to the requirement of HUP. However, the event happens in very much in short time as $2x10^{-20}$ s.

To trigger the TSC condensation, we need the initial sate interaction in dynamics of solid-state-physics level to form TSC (t=0) sate. For example, the TSC (t=0) state was speculated to form by the optical phonon excitation in the PdD regular lattice[1].

As the potential depth of PdD Bloch potential trapping deuterons in Pd lattice is about 0.2 eV, we need 0.2 eV at most for the incubation of the TSC(t=0) sate.

As we got about 100% 4D fusion yield per TSC generation in our previous study[2] and we know that $2x10^{11}$ 4D-fusion/s corresponds to one watt. The necessary "input energy" is $0.2eVx1.6x10^{-19}x2x10^{11} = 6.4x10^{-9}$ joule, for generating one joule energy by the TSC induced 4D fusions, if we have the full D-loaded Pd, namely PdD, on near surface of PdDx sample. Namely the nominal gain of this reaction is about 10^{8}. In other words, if the TSC generation probability is larger than 10^{-8} for the dynamic condition of PdD lattice, we will have more out-put energy than the input energy.

As the production of $2x10^{11}$ TSC/s corresponds to about the portion of 10^{-11} of PdD density (on the order of 10^{22} per cm^3) in cubic centimeters, the transient phenomena causing TSC generation is regarded to be in the very "dissipative-structure", much less than "ppm and ppb" levels, of PdD lattice dynamics or surface dynamics which the usual solid state or surface physics have never treated.

As discussed in our past works[5,7,9,11,12], 4D/TSC(t=0) sate may be formed in the PdD dynamics by the optical phonon excitation (see Fig.10).

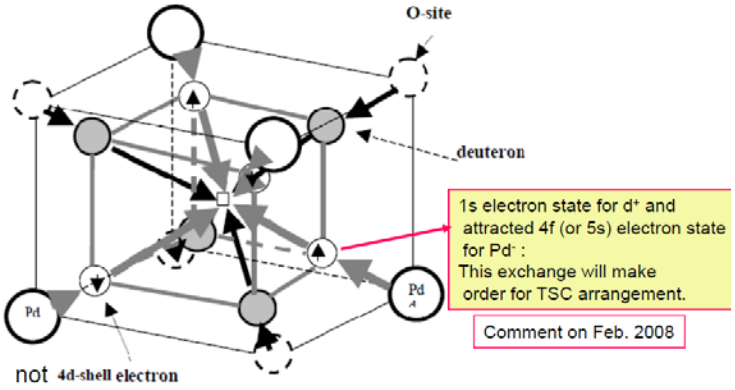


Fig.10: Incubation of 4D/TSC (t=0) in the PdD dynamics under D-phonon excitation. The exchange of 1s electron for the d+ sate and the 5s (or 4f) electron of Pd- state may arrange the TSC configuration.

We made a crude estimation of the 4D cluster formation probability of more than 10^{-6} in the past work[5,7,9]. We consider now that we need much less probability as 10^{-11} to be studied in the dissipative dynamics.

The PdD (PdDx: x=1) lattice maybe formed in the thin near surface zone of Pd lattice which absorbes deuterons in octahedral sites. However the full loaded PdD state would block deuteron diffusion into much deeper zone. Therefore, we

need wide surface area as much as possible in Pd zone. This may be the main reason that we need nano-modificated Pd metal samples to cause the CMNS effect with visible amount of reaction rates.

Of course for practical R&D of power generating device, we need to add additional inputs. However, we have a good principle to realize "clean radiation-less nuclear energy device" if the rapid condensation mechanism of 4D/TSC works well in real devices.

Langevin Equation for $6D^{2-}$/OSC

To fulfill the orthogonally coupled Platonic symmetry for 6D-cluster, 8 electron centers should appear on the center lines of 8 regular triangle faces of 6d octahedron; see **Fig.11**. Therefore to keep the geometrical symmtery, the Platonic OSC should be with 2- negative ion state.

The Langevin equation for $6D^{2-}$/OSC becomes as,

$$12m_d \frac{d^2 R_{dd}(t)}{dt^2} = -\frac{29.3}{[R_{dd}(t)]^2} - 24\frac{\partial V_s(R_{dd}(t);1,1)}{\partial R_{dd}(t)} + \langle f(t)\rangle + f'(t) \qquad (38)$$

The effective trapping potential of this system was already given in Fig.7, which tells us that $6D^{2-}$/OSC does not make ultimate condensation. However, in transient condensation process, we may have small probability that d-d distance would approach in shorter d-d distances than 40 pm of its ground state and 6D fusion rate may be somehow enhanced. We need numerical study for this.

We need a different study on if there exists a condensing system of neutral 6D-cluster (face-centered dodecahedron[11)] by coupling of two octahedrons (one of 6 deuterons and the other of 6 electron-centers)).

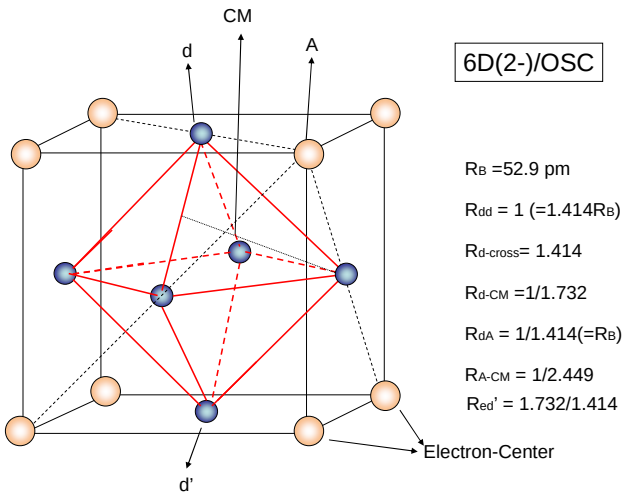


Fig.11: Geometrical view of $6D^{--}$/OSC structure

As we know the proposed 6D/OSC in the condensed matter should be charge-neutral (rapid charge neutralization should happen), we might consider the 8 QM electron centers with "6/8 charge weight" at each point to keep the charge neutrality of 6D system and its Platonic symmetry. We need further study to validate such idea if 6D/OSC (neutral) would make ultimate condensation as 4D/TSC (neutral).

HMEQPET Method for Fusion Rate Quantification

A slice of time-dependent TSC trapping potential at the final stage (TSC-min) of condensation is shown in **Fig.13**.

Depth of the trapping potential is -130.4keV. Calculated relative kinetic energy of d-d pair is 13.68keV. In approximate view, this potential can be regarded as an adiabatic potential having the d-d pair "quasi-ground state" with E_{gs}=13.68

keV trapped in 130.4keV deep potential for very short time-interval of 10^{-20} s. In every time step of numerical calculation (by the Verlet method[2)]), we can draw approximate adiabatic potential which changes continuously with the change of time.

Kinetic energy of particle is given by $E = \frac{1}{2}mv^2$. The de Brpglie wave length is given by $\lambda\!\!\!^{-} = \hbar/(mv)$.

Mean electron kinetic energy increases in a "d-e-d-e" face during condensation motion as shown in Fig.12. As electron wave length with increased kinetic energy (or momentum) can be replaced with heavy mass fermion to keep the same "d-e-d-e" size. The HMEQPET method is based on this idea.

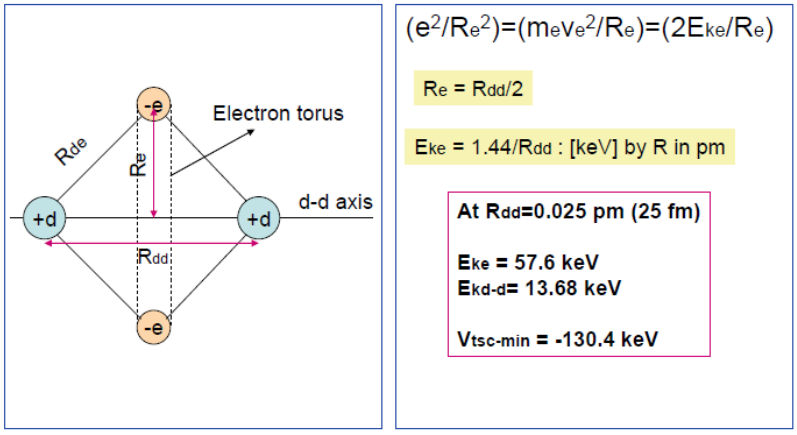


Fig.12: Model of a "d-e-d-e" face of TSC 6 faces; time-dependent continuous diminishment of size keeping a seudo-molecular system with higher electron mean kinetic energy is assumed here.

From Fig.12, mean kinetic energy of electron in a "d-e-d-e" face is 57.6keV at R_{dd}=25fm. As TSC at t=0 has mean electron kinetic energy about 18eV (17.6 eV in exact number), the equivalent mass of "heavy" fermion is estimated to be $\sqrt{57.6x1000/18} = 56.57$ times the electron mass. Depth of dde*(56.57,2) potential is about -4.8keV and comparable to the trapping potential of muonic d-d molecule (See Table-1 and Table-2).

Table-1: Typical-point-wise HMEQPET potentials to be replaced with time-dependent TSC trapping potential

Molecule	b0 (pm)	Rmin (pm)	Vs-min (keV)	Ed-d (keV)	**Rgs (pm)**	Egs (keV)
D_2	22	70	-0.03782	0.00268	**76.69**	-0.03514
dde*(2,2)	4.5	19.3	-0.1804	0.01013	**21.82**	-0.17027
dde*(5,2)	1.9	7.6	-0.4509	0.0208	**8.72**	-0.43007
dde*(10,2)	0.90	3.8	-0.9019	0.0418	**4.36**	-0.86012
dde*(20,2)	0.45	1.9	-1.8039	0.0837	**2.18**	-1.7202
dde*(50,2)	0.18	0.76	-4.5097	0.2094	**0.873**	-4.3003
dde*(100,2)	0.09	0.38	-9.0194	0.4196	**0.436**	-8.5998
dde*(200,2)	0.045	0.19	-18.039	0.843	**0.218**	-17.196
dde*(500,2)	0.018	0.076	-45.097	2.135	**0.0873**	-42.968
dde*(1000,2)	0.009	0.038	-90.194	4.336	**0.0436**	-85.858
dde*(2000,2)	0.0045	0.019	-180.39	8.984	**0.0218**	-171.406

Since the depth of 4D/TSC trapping potential at R_{dd}=25fm was -130.4keV, we must assume much heavier fermion to quantitatively approximate the TSC trapping potential by EQPET dde*(m,Z) potential[2].

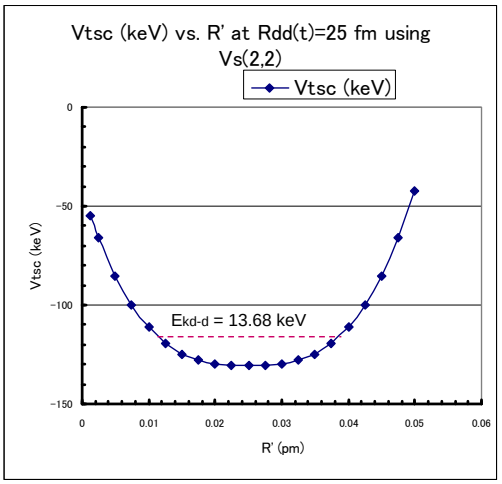


Fig.13: TSC trapping potential at the final stage (TSC-min) of condensation motion by Langevin equation

We used Gaussian wave function for d-d pair in Langevin equations. As discussed[2], we cannot use Gaussian wave function for the estimation of Coulomb barrier penetration probability (barrier factor), because the tail-distribution of Gaussian function is not accurate enough. Instead, we can use trapping potentials of dde*(m,2) EQPET molecule and Gamow integrals. The assumed quasi-particle state is heavy Cooper pair e*(m,2) of two "heavy" electrons in a "d-e-d-e" system.

If there exists the one-to-one relation between m and $<R_{dd}>(t)$, we can replace all time-dependent TSC trapping potentials with $V_s(R_{dd}(t);m,2)$ potentials of HMEQPET, continuously by adopting real number of m.
Typical parameters of calculated $V_s(R_{dd}(t);m,2)$ potentials are shown in **Table-1**. From this table, we eventually found the following empirical laws.

$$b_0(m,2) = 0.206 R_{gs}(m,2) \tag{39}$$

$$m = 9000/b_0(m,2) \tag{40}$$

Here b_0 and R_{gs} values are given in fm unit.
The muonic dd-molecule has R_{gs}=805 fm, and the sate of m=54m_e of HMEQPET potential corresponds to it. The final stage of TSC potential corresponds to m=2000m_e.
Calculated barrier factors by HMEQPET method are already given in Table-2.

Table-2: Calculated time-dependent barrier factors under TSC condensation

Elapsed Time (fs)	R_{dd} (pm)	P_{2d}: 2D barrier facotor	P_{4d}: 4D barrier factor
0	74.1 (D_2 molecule)	1.00E-85	1.00E-170
1.259	21.8 (dde*(2.2); Cooper pair	1.30E-46	1.69E-92
1.342	10.3	2.16E-32	4.67E-64
1.3805	4.12	9.38E-21	8.79E-41
1.3920	2.06	6.89E-15	4.75E-29
1.3970	1.03	9.69E-11	9.40E-21
1.39805	0.805 (muon-dd molecule)	1.00E-9	1.00E-18
1.39960	0.412	9.40E-7	2.16E-13
1.40027	0.206	3.35E-5	1.12E-9
1.40047	0.103	1.43E-3	2.05E-6
1.40062	0.0412	1.05E-2	1.12E-4
1.40070	0.0206 (TSC-min)	4.44E-2	1.98E-3

Finally, we copy again the illustration of 4D/TSC condensation motion in **Fig.14**.

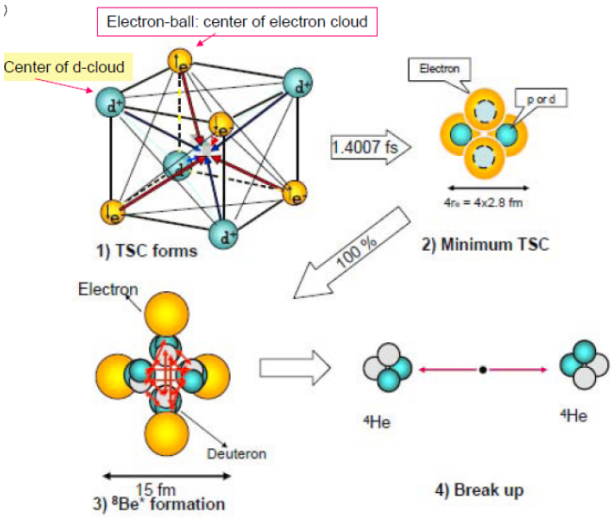


Fig.14: Four steps of 4D/TSC condensation; 1) TSC is just formed (t=0), 2) TSC condensed in 1.4 fs to TSC-min state, 3) ^{8}Be* state is born by strong nuclear interaction (charged pion exchange between nucleons), 4) final state interaction for break up to two alpha particles

Estimation of Fusion Rates

Fusion Rate Formula and Calculated Fusion rates for Steady Clusters

The Gaussian wave function was quite useful for making QM-ensemble averaging of Langevin equation, but unfortunately the accuracy in its tails (for very small R_{dd} values) is not high enough to apply for the estimation of barrier factor of d-d pair and 4d cluster, as we illustrate the relation between trapping

(shielded) Coulomb potential, wave function and the very short (within r_0 = about 5 fm) range of nuclear strong interaction, in **Fig.15**.

To obtain usable accuracy in barrier factor, we introduced the Heavy Mass Electronic Quasi-Particle Expansion Method (HMEQPET) in the previous chapter to provide equivalent time-dependent potentials for the squeezing 4D/TSC system. Detail description is given in our previous paper[2, 12].

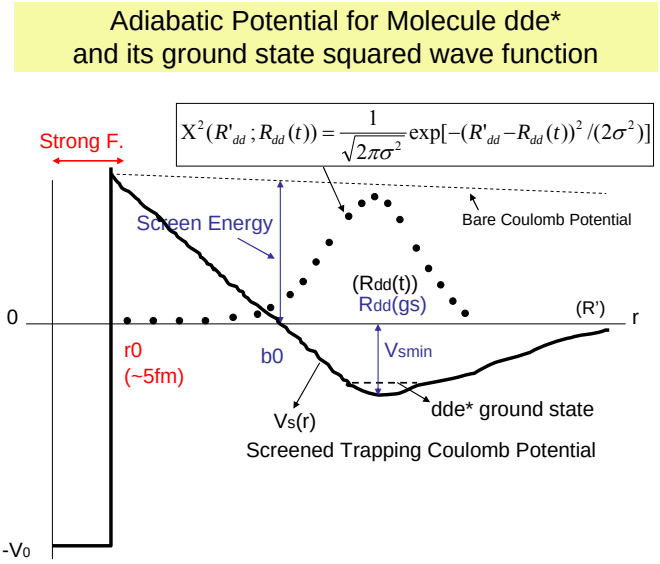


Fig.15: Relation between d-d trapping (shielded) Coulomb potential, wave function and the very short (within about 5 fm) range of nuclear strong interaction. By calculating Gamow integral from R_{gs} to r_0, we obtain barrier factor for fusion rate estimation.

Time-dependent or equivalently R_{dd}-dependent barrier factors were calculated as we copy from our previous paper[2] as given in **Table-2**. TSC starts at R_{dd}=74 pm and condenses very rapidly to reach at the final TSC-min state with R_{dd} = 0.0206 pm (20.6 fm) in 1.4007 fs in this case. On the way of condensation, TSC passes the equivalent state with that of dd-muon molecule for which we have reference data[6] to show good agreement with our calculation. Barrier factors for 2d and 4d fusion were calculated using the WKB approximation with Gamow integral1[1,2,3].

Fusion rates for steady molecules were then calculated based on the Fermi's first golden rule[3],

$$\lambda_{nd} = \frac{2}{\hbar}\langle W\rangle P_{nd}(r_0) = 3.04\times 10^{21} P_{nd}(r_0)\langle W\rangle \tag{41}$$

Here P_{nd} is barrier factor for nD-cluster and <W> is the averaged value of imaginary part of nuclear optical potential[3]. The extrapolation of <W> value to 4d fusion was made[2,12] by using the scaling law $\langle W\rangle \propto (PEF)^5$ with PEF-value which is given in unit of derivative of one pion exchange potential (OPEP) (simple case of Hamada-Johnston potential[10] for pion exchange model) given by

$$\langle OnePEF\rangle = -\frac{\partial\langle V_{OPEP}(x)\rangle_{\tau,\sigma}}{\partial r} = -\frac{1}{1.43}\frac{\partial\langle V_{OPEP}(x)\rangle_{\tau,\sigma}}{\partial x} \tag{42}$$

$$V_{OPEP}(x) = v_0\cdot(\vec{\tau}_1\cdot\vec{\tau}_2)\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2 + (1+\frac{3}{x}+\frac{3}{x^2})S_{12}\right\}\frac{\exp(-x)}{x} \tag{43}$$

Here $x = \frac{m_\pi c}{\hbar} r = \frac{r}{1.43}[fm]$

and S_{12} is the tensor operator $S_{12} = 3\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2$

and $v_0 = \frac{1}{3}\frac{f^2 m_\pi c^2}{\hbar c} = 3.65[MeV]$.

And $\vec{\tau}$ is the isospin operator for n-p charged pion exchange and $\vec{\sigma}$ is the spin operator for nucleon state. The table of <W> values is given in our previous paper[2]. Actual force strength of d-d interaction is shown in Fig.16.

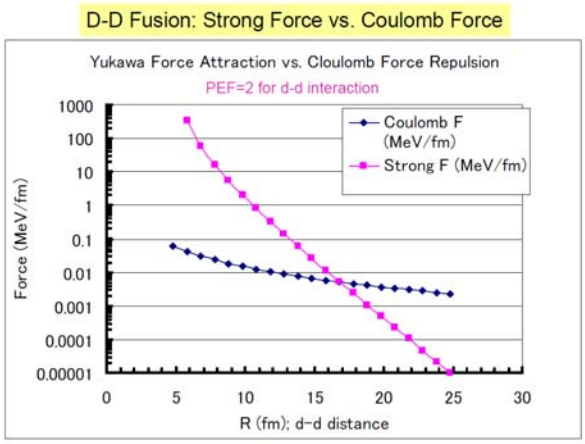


Fig.16: Strength of d-d strong nuclear calculated based on the one-pion exchange potential, compared with Coulombic repulsive force between two deuterons.

Calculated fusion rates for "steady state molecules and clusters" are given in Table-3.

Table-3: Fusion rates by Fermi's golden rule for steady molecules

Molecule	$R_{dd}=R_{gs}$ (pm)	P_{nd} ; B-Factor	<W> (MeV)	λ_{2d} (f/s)	λ_{4d} (f/s)
D_2	74.1	1.0E-85	0.008	2.4E-66	
dde*(2,2)	21.8	1.3E-46	0.008	3.16E-27	
μ dd	0.805	1.0E-9	0.008	2.4E+10	
4D/TSC-min	0.021	1.98E-3	62		3.7E+20

Time-Dependent Fusion Rates

Time-integrated fusion yield per TSC generation was given[2] by the following formulas,

$$\eta_{4d} = 1 - \exp(-\int_0^{t_c} \lambda_{4d}(t)dt) \tag{44}$$

$$\lambda_{4d}(t) = 3.04\times10^{21}\langle W\rangle P_{4d}(r_0;R_{dd}(t)) = 1.88\times10^{23} P_{4d}(r_0;R_{dd}(t)) \tag{45}$$

$$\int_0^{t_c} \lambda_{4d}(t)dt = 1.88\times10^{23}\int_0^{t_c} P_{4d}(r_0;R_{dd}(t))dt \tag{46}$$

$$\int_0^{t_c} P_{4d}(r_0 : R_{dd}(t)dt = 2.31\times10^{-22} \tag{47}$$

$$\eta_{4d} \cong 1.0 \tag{48}$$

Macroscopic fusion rate is given by

$$Y_{4d} = Q_{tsc}\eta_{4d} \tag{49}$$

We have obtained that 4D fusion may take place with almost 100 % yield per a TSC generation, so that macroscopic 4d fusion yield is given by simply with TSC generation rate Q_{tsc} in the experimental conditions of CMNS. However, when we consider that one deuteron has spin-parity 1+ and combination of 4d has total spin state 4, 3, 2, 1 and 0, the 4d fusion with out-going channel to two ^{4}He (0+:gs) particles is forbidden, by spin-parity conservation (for S-wave in/out channels), except for the 0+ spin-parity state (T=0) of 4d combination, to be explained detail analysis including P-wave and D-wave states with isospin elsewhere (some discussions are given in the next section.

The ultimate condensation is possible only when the double Platonic symmetry of 4D/TSC is kept in its dynamic motion. The sufficient increase (super screening) of barrier factor is also only possible as far as the Platonic symmetric 4D/TSC system is kept. Therefore, there should be always 4 deuterons in barrier penetration and fusion process, so that 4d simultaneous fusion should take place predominantly. The portion of 2D (usual) fusion rate is considered to be negligible[2].

Major nuclear products of 4D fusion are speculated to be two 23.8 MeV α-particles[5,7]. We discuss in the next section about the possible out-going channels and particle spectra.

4H/TSC should condense in the same way until when TSC-min state with classical electron radius (2.8 fm) comes, but no strong interaction exists among protons and will make 1p to 4p capture transmutations with host metal nuclei when 4H/TSC has sufficient drift (CMS) momentum.

Some Considerations on Final State Interaction

We have already discussed about the final state interaction and break-up products in our earlier work[5, 13].

We do not know exactly about the ^{8}Be* state just formed. There are several possibilities of initial spin-parity state combinations as shown in Fig.17.

Fig.18 shows possible break-up products for the S-wave states.

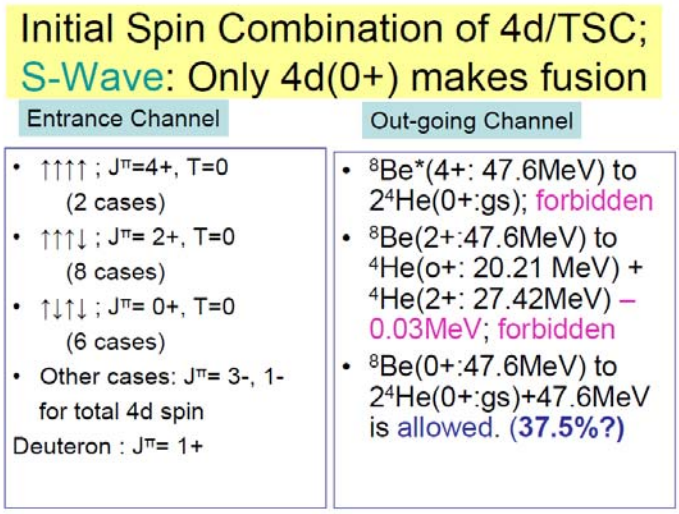


Fig.17: Spin-parity and iso-spin states for initial state interaction to form ^{8}Be*; The S-wave inlet channel selects the S-wave (hence decay to two alpha-particles) out going channels. P-wave inlet and P-wave out going channels are also possible.

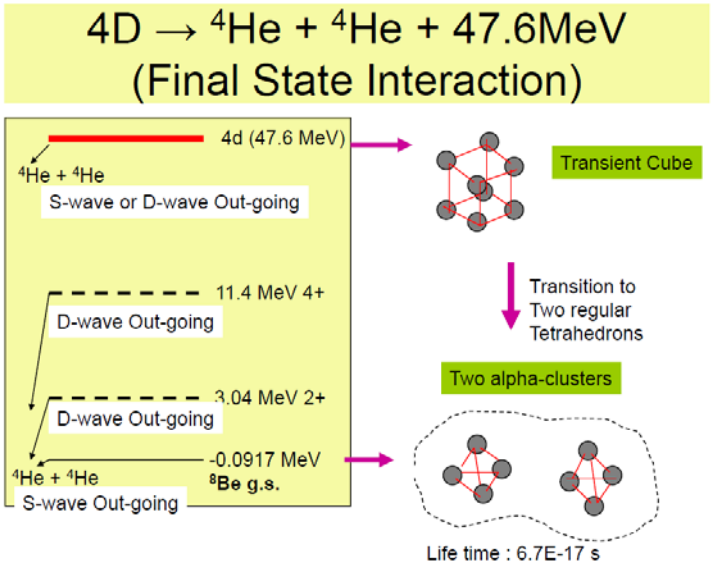


Fig.18: Speculated break-up channels for the S-wave states

The nuclear excited energy of ^{8}Be* by the simultaneous 4D fusion is 47.6MeV. This corresponds to 5.95 MeV per nucleon. The nuclear binding energy of ^{4}He nucleus (α particle) is 28.0 MeV, which corresponds to 7.0 MeV per nucleon. The α-cluster in nucleus has therefore a hard entity in the excited state compound nucleus ^{8}Be*. Since total binding energy of two α-clusters is 56 MeV which is larger than the nuclear excited energy 47.6 MeV of ^{8}Be* by the 4D fusion, the excited state of ^{8}Be* is speculated to be an tandem oscillation mode of two α-clusters as shown in Fig.18. The tandem oscillation mode will be formed in very short time after the simultaneous 4D fusion, as drawn in Fig.18, and will shortly decay directly to two α-particles with 23.8MeV kinetic energy per ^{4}He nucleus (in the case of S-wave transition) or will first make electro-magnetic transition (in the case of P-wave transition) to lower excited states with even-spin and even-parity and secondly break up to two α-particles with lower kinetic energy as shown in the left figure in Fig.18.

If the P-wave states (as the 3- state discussed in our past work[13)]) dominate in the ^{8}Be*, we have possibility of the QED-photon energy transfer to PdDx lattice vibration (phonons) as discussed in Ref.13 in 1994. In this case, kinetic energy of emitted α-particle becomes very small as 46 keV and no secondary X-rays will be observed.

Possible minor decay products are shown in Fig.19. Because of very hard cores of α-clusters which are supposed to form in the excited state of ^{8}Be*, decay products as proton, neutron, triton and ^{3}He are speculated to be minor products, even for the highly excited energy of 47.6 MeV which exceeds threshold energies of these "minor" channels.

Final Sate Interaction of ^{8}Be*(Ex=47.6MeV)

$$
\begin{aligned}
^{8}\mathrm{Be(g.s.)} &\rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He} + 91.86\ \mathrm{keV},\\
&\rightarrow {}^{3}\mathrm{He} + {}^{5}\mathrm{He}\,(\mathrm{n} + {}^{4}\mathrm{He}) - 11.13\ \mathrm{MeV},\\
&\rightarrow \mathrm{t} + {}^{5}\mathrm{Li}\,(\mathrm{p} + {}^{4}\mathrm{He}) - 21.68\ \mathrm{MeV},\\
&\rightarrow \mathrm{d} + {}^{6}\mathrm{Li} - 22.28\ \mathrm{MeV},\\
&\rightarrow \mathrm{p} + {}^{7}\mathrm{Li} - 17.26\ \mathrm{MeV},\\
&\rightarrow \mathrm{n} + {}^{7}\mathrm{Be} - 18.90\ \mathrm{MeV}.
\end{aligned}
$$

Fig.19: Possible break-up products from the ^{8}Be* (47.6MeV) state; In the case of S-wave transition, the spin-parity conservation prohibits the out-going channels emitting neutron, proton, triton and ^{3}He.

We need further study with experimental observations of emitted charged particle spectra to make definite conclusions.

Conclusions

1) The quantum-mechanical Langevin equation (stochastic differential equation) is complimentary to the Schroedinger equation of standard quantum mechanics.
2) The Platonic symmetry appears in D-atom, D_2, D_2^+,D_3^+ molecule, and 4D/TSC, and we could introduce the dynamics approach based on Langevin equations.
3) Platonic symmetry appears in the initial sate interactions in the CMNS of 4D/TSC for both of the Coulombic interaction and the strong interaction.
4) Dynamic Platonic symmetry is of key for the super-screening of Coulomb repulsion and the 4D simultaneous cluster fusion.
5) We have obtained good solutions of molecular dynamics with Langevin Equations, for Platonic symmetric systems as, D, D_2, D_3^+ and 4D(or H)/TSC.
6) It was shown that about 100% 4D-fusion per TSC generation is possible, by the present work, in the condensed matter nuclear effects.
7) Only 4D (or H)/TSC can condense ultimately to a very small charge neutral entity with 10-20 fm radius size, as far as 5 kinds of D(or H)-clusters studied in this work.
8) Bosonized e(spin=1/2)+e(spin=-1/2) coupling for the "d-e-d-e" system makes D_2 type faces of 4D(or H)/TSC to help its ultimate condensation.
9) $6D^{2-}$/OSC converges its condensation at about R_{dd}=40 pm, but closer d-d distance in transient may appear with small probability.
10) Single quantum-mechanical <e- >-center states for the "d-e-d" (D_2^+) type faces of the D_3^+ ion molecule and the $6D^{2-}$/OSC cluster should enhance constraint (friction) for their condensation motion.
11) The HMEQPET method is useful tool to approximate the time-dependent TSC trapping potential and to calculate sufficiently accurate time-dependent barrier penetration probabilities for estimating fusion reaction rates.
12) Fusion rate formulas for time-dependent D-cluster condensation were given.
13) We need further efforts to study the solid state/surface-physics conditions to efficiently incubate TSC(t=0) transient clusters in metal-deuterium and

metal-hydrogen systems.

14) The final state interaction of $^{8}Be^{*}$ is complicated, due to unknown spin-parity and isospin states, to study further.

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