Topological Maxwell Field Theory and Symmetry Breaking.

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Abstract

DRAFT of the first few pages...

1. Topological Field Theory

1.1. Preface

Finally, I have found time to think about, and the incentive to study, how the field theory of Topological thermodynamics, electrodynamics, and hydrodynamics can be compared to field theory concepts that have been developed by Lagrangian methods, for both the classic and quantum mechanical varieties. For more than 30 years I have known that Cartan's topological methods could be applied to dissipative systems; the methods based on diffeomorphic-invariant Lagrangian field theories can not. The incentive came when I realized that the topological methods of Cartan gave dynamical results that can explain "symmetry breaking" and quantization in terms of continuous topological evolution.

1.2. Some recent results

Over the past few months I have spent many, many hours computing examples with Maple, and have discovered a number of interesting results relative to the topological theory of thermodynamics and fields [1] [2] [3] [4] [5]. I will post the Maple programs in pdf format shortly. Some of the most interesting results are:

1. Many examples of Hedgehog **B** fields (but not magnetic monopoles) can be constructed from specific classes of vector-scalar potentials that make up the coefficient functions of the 1-form of Action, A. This 1-form defines a thermodynamic system through its Pfaff Topological dimension, {1, 2, 3, 4} which is in correspondence to {Full-equilibrium, isolated-equilibrium, Closed and Open} thermodynamic (topological) neighborhoods.

2. In certain cases (with examples forthcoming), the Maxwell-Ampere postulate permits the generation of a charge-current 3-form, J, with coefficients that are proportional to the coefficients of the London currents

$$A \Rightarrow F = dA \Rightarrow \text{constitutive relations} \Rightarrow G \Rightarrow J_{Lorentz} = dG = \chi A, \quad (1.1)$$

$$J_{Lorentz} \Rightarrow \chi A = J_{London}. \quad (1.2)$$

   This is not a hypothesis; it is a derived result that does not require modification of the two topological (Maxwell) field theory postulates. $F - dA = 0$, $J - dG = 0$. Bottom line: Superconductivity (vis a vis, The London current) is included in Topological Thermodynamics.

3. From my discovery of the 3-form of topological spin in 1969 [6], $S = A\hat{}G$, it is possible to show that in certain examples (verified by Maple) the divergence of the Spin 3-form dS is zero, and in other cases the divergence, dS, is not zero. When the divergence is zero, there exists a conservation law: The integral - over a bounded neighborhood - of the Spin 3-form is then an evolutionary invariant. When $dS = 0$, the 3-form S is an example of a Noether current. However, there is physical utility and meaning even when the Noether theorem fails; that is, when the symmetry is broken, $dS = 0$.

4. Realize that, in both classical and Quantum Field theory, conservation laws have been associated with some form of a "symmetry". A corner stone of Quantum Field theory is the idea of "spontaneous symmetry breaking", and its relationship to properties of elementary particles. However, the details of the Symmetry breaking *process* (in my opinion) remain somewhat mystical. In topological field theory, the *process* of "symmetry breaking" can be well defined. Symmetry breaking would occur when the divergence of the Spin or Torsion current 3-form changes value from zero (a conservation law) to a non-zero value.

5. The reverse idea represents condensation, and the production of topological coherent collective neighborhoods. What is most remarkable is that a

bounded neighborhood of a 3-form of Torsion or Spin current, where the divergence of the current 3-form is non-zero, can *continuously* evolve to produce an emergent neighborhood (where the divergence of the current 3-form vanishes). The emergent domain is ordered, and can contain topologically quantized values for the Spin or Helicity (Torsion) 3-forms, that involve spin, torsion and charge pairing.

6. For certain example classes of Vector and Scalar potential functions, the coefficients of the Lorentz force, $(i(J)dA)$ induced by the charge-current 3-form, J, is proportional to the coefficients of the Spin current! In those situations where the divergence of the spin is not zero, the non-zero spin energy density, $F\hat{}G - A\hat{}J$, contributes an inertial mass effect (via the Lorentz force). The effect is reminiscent of the Higgs function.!!!!

7. In a symmetry broken mode, where the divergence of the Torsion current is not zero, the value of the divergence represents a "bulk viscosity" coefficient, describing the irreversible expansion (or contraction) of space-time.

8. When the Action 1-form, A, can be represented by a complex function Ψ (or an ordered pair of functions, φ and $\varkappa$) in the Whitney format,

$$A = \Psi^* d\Psi - \Psi d\Psi^* \approx \varphi d\varkappa - x d\varphi, \text{ a Whitney 1-form} \quad (1.3)$$

$$F = dA = -2d\Psi\hat{}d\Psi^* \quad (1.4)$$

It is commonplace (but incorrect) to call the 1-form, A, a "probability" current; a current is a contravariant vector, not a covariant vector. The issue can be resolved by assuming that the 1-form A is "somehow dual" to a 3-form "probability" current. The usual trick is to use the tensor expression and the metric, $\eta_{\mu\nu}$, to construct the contravariant components from the covariant components:

$$\mathbf{J}^{\mu}_{London\_format} = \eta^{\mu\nu}\mathbf{A}_{\nu}, \quad (1.5)$$

$$J_{Prob-current} = i(\mathbf{J}^{\mu})dx\hat{}dy\hat{}dz\hat{}dt. \quad (1.6)$$

The metric dualism generates the format of the London Current: Current proportional to Action. The notion of tensor analysis often confuses the issue between 1-forms and N-1 forms, but in the language of differential forms, it is apparent that the covariant 1-form, A, is not the same as the contravariant current 3-form, J.

9. Never the less, a contribution to the Action, which involves a complex scalar field, $\Psi = u + \sqrt{-1}v$, is a useful extension of how Quantum scalar (but complex) wave functions can enter a topological theory, a theory which is independent from the choice of metric features. Note that if the the two functions are independent, then the 2-form, F, cannot vanish. The exterior derivative of the 1-form becomes:

$$dA = -2d\Psi\hat{}d\Psi^* = -\sqrt{-1}du\hat{}dv. \tag{1.7}$$

The exterior differential of A can not be zero, as the two functions are assumed to be independent. However if the 1-form A is made homogeneous of degree 0, by dividing by a Holder norm,

$$\lambda = \{a \cdot u^p(x,y,z,t) + b \cdot v^p(x,y,z,t)\}^{2/p}, \tag{1.8}$$

or a Buckingham function, homogeneous of degree 2 in the variables, (u,v), then

$$d(A/\lambda) = 0, \text{ or, } div(J_{Prob-current}/\lambda)dx\hat{}dty\hat{}dz\hat{}dt \Rightarrow 0 \tag{1.9}$$

$$\text{but } dx\hat{}dy\hat{}dz\hat{}dt \neq 0. \tag{1.10}$$

Note that result is valid for any constants, (a,b,p), and can be extended t0 Whitney 2-forms and 3-forms. The renormalized Action leads to a closed Probability current. The renormalized Action is of Pfaff Topological dimension 1, while the Whitney form is of Pfaff Topoligical dimension 2. There is an evoutionary integral invariant associated with the renormalized 1-form, which implies a Noether current (the probability current) which is closed (has zero divergence). The unrenormalized Whitney form, represents the state of "broken symmetry", while the renormalized 1-form represents the state of topological coherence. This the Bohm-Aharanov state which can exhibit 1-dimensional period integrals.

I would like to speculate that the divergence of the Spin current represents a shear viscosity, but this idea needs more work. It is a result that the circularly polarized states are due to Topological Torsion. Pairing of the polarized states lead to two CW states (spin 1 or 2), two CCW states (spin -1 or -2), or a pairing of a CW state and a CCW state (Spin 0).

1.3. Remarks

The Topological Theory of Thermodynamics gives to theoretical physics an approach alternate to the ubiquitous Lagrangian variational principle. The topological methods go beyond the methods of Classical and Quantum field theory, yet include thermodynamic arguments that introduce the concepts of Topological Spin and Topological Torsion (and their topological quantization) into a dynamical theory of Continuous Topological Evolution. Continuous Topological Evolution is to be recognized as the dynamical abstraction of the first Law of Thermodynamics [1].

The topological methods yield another viewpoint, an unconventional viewpoint that is useful to the further understanding of the spin-pairing mechanisms in Bose-Einstein Condensation and Superconductivity, the emergence of coherent collective structures, and ordered states in fluids and plasmas at all scales (including stars and galaxies), as well as the effects of bulk and shear viscosity in the theory of hydrodynamic turbulence.

Often, the almost mystical dogmatic concepts of Quantum Field theory can be understood in terms of topological thermodynamics in a non-magical way. For example, the concepts of gauge symmetry breaking can be understood in terms of a continuous transition of local neighborhoods of an Open thermodynamic state of Pfaff Topological Dimension 4 to the emergence of closed neighborhoods of a Closed Thermodynamic state of Pfaff Topological Dimension 3. Bounded structures in the thermodynamic PTD=4 state, can evolve continuously into non-simply connected closed structures of PTD =3.[1] Local neighborhoods of PTD = 3 can evolve continuously into Isolated-Equilibrium neighborhoods of PTD = 2, and these structures can evolve continuously into neighborhoods of Full-Equilibrium States of PTD = 1.

The closed integrals of closed structures can be evaluated to yield rational ratios (deRham cohomology); a result that can be described as topological quantization. (Closed structures are not admissible in the PTD =4 state.) Both the Closed and Open systems are non-equilibrium thermodynamic systems. The Closed PTD = 3 systems will admit exchange of energy(mass) and radiation with its environment, but not particles or spins. The Open PTD = 4 systems will admit exchange of energy, radiation and particles and spins with its environment, which is the entire Open system itself. In essence, the Open thermodynamic

[1] There also exists the possibility of a discontinous transition from a Closed State of Pfaff Topological Dimension 3 to an Open system of Pfaff Topological dimension 4.

state of PTD = 4 could be classified as the Aether, a "vacuum state" that is not an empty void Note that in Quantum Field Theory the meaning of the vacuum is somewhat obscure. It certainly is not a void, but could contain energy, and massless spins.

The topological concepts admit both tensor and spinor features in an unambiguous way. However, the concept of mass, viscosity, radiation impedance and perhaps even dark energy can be related to the lack of a conservation law (which means that the divergence of the Spin Current or Torsion current is not zero) and to the ability of the topological methods to express non-diffeomorphic effects. An evolutionary process is treated as a thermodynamic process that is not restricted by some form of a conservation law (and therefore a symmetry group). The processes are not constrained geometrically to represent Diffeomorphic equivalences.

1.4. Postulates of Topological Field Theory

The basic postulates of the Topological Theory of Thermodynamics (on a 4-dimensional variety) are,

1. There exists a 1-form of Action potentials, A, that encodes a thermodynamic neighborhood in terms of its Pfaff Topological dimension.

2. The exterior differential of the action generates the Limit sets of the Action, encoded in terms of the exact 2-form, $F = dA$, of field intensities ($\mathbf{E}, \mathbf{B}$, using notation of Electromagnetism for convenience).

3. The 2-form of intensities can F can be mapped into a 2-form density G via a set of constitutive relations. For example, $\mathbf{D} = \epsilon\mathbf{E}$, $\mathbf{H} = \mathbf{B}/\mu$, is one possibility.

4. A 3-form of charge-current density can be generated by the Maxwell-Ampere postulate, $J = dG$. The 3-form is closed as $dJ = ddA = 0$, thereby establishing a conservation law, and a "symmetry".

5. Two other 3-form currents can be defined as the Topological Torsion 3-form, $T = i(\mathbf{T}_4)dx\hat{}dy\hat{}dz\hat{}dt = A\hat{}F$, and the Topological Spin3-form, $S = i(\mathbf{S}_4)dx\hat{}dy\hat{}dz\hat{}dt = A\hat{}G$.

6. The symmetry group which preserves T can be broken if the divergence, $dT \neq 0$. It is then possible to demonstrate that the thermodynamic system

is of Pfaff Topological Dimension 4, representing an Open thermodynamic systems. As the divergence is not zero, there is no symmetry. If T is not zero, but $dT = 0$, then the Pfaff Topological dimension is 3, and the neighborhood is a closed thermodynamic system. There exists a conservation law, as but $dT = \not{0}$. The symmetry group is not broken in the PTD=3 state and is broken in the PTD=4 state. The 3-form $A\hat{}F$ represents helicity and/or circular polarization concepts.

7. The symmetry group which preserves S can be broken if the divergence, $dS \neq 0$. If S is not zero, but $dS = 0$, then there exist closed but not exact 3-forms that can serve as the integrands of period integrals, and yield topological quantization. If the divergence of the 3-form is $d(A\hat{}G) \neq 0$, then the symmetry is broken. The divergence establishes an energy density that can be interpreted as mass-energy.

Coming:

2. Dark Energy and Superconductivity

3. Topological Electromagnetism

4. Symmetry breaking and PTD 4 $\Rightarrow$PTD3

5. London current ~ A   Spin current ~ Lorentz Force

6. Torsion and Spin current Pairing

7. Spin Current and viscosity

8. Turbulence to Streamline flow   div(Spin Current) $\Rightarrow$ 0

9. Div Spin current A^G = mass energy

10. Circular Polarization   and A^F not zero

11. Examples: include Yukawa field

12. References

References

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[6] Kiehn, R.M. and Pierce, J. F. (1969), An Intrinsic Transport Theorem, Phys. Fluids, **12**, #9, 1971.