Quantum Hall effect and Hierarchy of Planck Constants

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Contents

Abstract

I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure. The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal for which second variation of Kähler action vanishes at least for the variations responsible for dynamical symmetries. The interpretation is in terms of quantum criticality. This representation generalizes. One can add imaginary instanton term to the Kähler function and corresponding modified Dirac operator: the hypothesis is that the resulting Dirac determinant equals the exponent of Kähler action and imaginary instanton term.

2. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

3. One important implication is that Poincare and Lorentz invariance are broken inside given CD although they remain exact symmetries at the level of the geometry of world of classical worlds (WCW). The interpretation is as a breaking of symmetries forced by the selection of quantization axis.

4. Anyons would basically correspond to matter at 2-dimensional "partonic" surfaces of macroscopic size surrounding the tip of the light-cone boundary of CD and could be regarded as gigantic elementary particle states with very large quantum numbers and by charge fractionization confined around the tip of CD. Charge fractionization and anyons would be basic characteristic of dark matter (dark only in relative sense). Hence it is not surprising that anyons would have applications going far beyond condensed matter physics. Anyonic dark matter concentrated at 2-dimensional surfaces would play key key role in the the physics of stars and black holes, and also in the formation of planetary system via the condensation of the ordinary matter around dark matter. This assumption was the basic starting point leading to the discovery of the hierarchy of Planck constants. In living matter membrane like structures would represent a key example of anyonic systems as the model of DNA as topological quantum computer indeed assumes.

5. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants.

1 Introduction

Quantum Hall effect [8, 6, 9] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of filling fractions of form $\nu = k/m$ was observed.

The model explaining the QH effect is based on pseudo particles known as anyons [3] , [8] . According to the general argument of [4] anyons have fractional charge νe. Also the TGD based model for fractionization to be discussed later suggests that the anyon charge should be νe quite generally. The braid statistics of anyon is believed to be fractional so that anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For non-Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [3] each of them defining an incompressible hydrodynamical flow. According to [9] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = e/4$ rather than being $Q = \nu e$ [7] . This finding favors non-Abelian models [9] .

Non-Abelian anyons [8, 10] are always created in pairs since they carry a conserved topological charge. In the model of [9] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be show to possess 2^{n-1}-dimensional vacuum degeneracy [11] . When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

Topological quantum computation is perhaps the most promising application of anyons [9, 8, 4, 3, 5, 6] , [1] . I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure [10] . The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results for ordinary integer QHE if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic

transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

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4. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that also the first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants. Also the proposed manifestation of Equivalence Principle at the level of symplectic fusion algebras as a duality between descriptions relying on the symplectic structures of CD and CP_2 [5] forces the hierarchy of Planck constants.

The first sections of the chapter contain summary about theories of quantum Hall effect appearing already in [28] . Second section is a slightly modified version of the description of the generalized imbedding space, which has appeared already in [10, 28, 9] and containing brief description of how to understand QHE in this framework. The third section represents the basic new results about the Kähler structure of generalized imbedding space and represents the resulting model of QHE.

2 About theories of quantum Hall effect

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

2.1 Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [3] , [8] . This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \rightarrow H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i\oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [8] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \rightarrow H$ defines a representation of the braid group. The construction of these representations is discussed in [8] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

2.2 Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A \wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi} Tr\left(A \wedge (dA + \frac{2}{3} A \wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [4, 5] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links [3] .

Witten's article [6] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k \rightarrow \infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3 = \Sigma^2 \times R^1$: in the Coulomb gauge $A_3 = 0$ the action reduces to a sum of $n = dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The configuration space consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [2]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G = SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q = exp(i2\pi/(k+N))$ associated with quantum groups [3] . Note that for $SU(2)$ and $k = 3$, the invariants are expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k+1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [8] .

2.3 Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [6] . Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [3] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow. As will be found, TGD predicts a non-Abelian Chern-Simons term associated with electroweak long range classical fields.

2.4 Topological quantum computation using braids and anyons

By the general mathematical results braids are able to code all quantum logic operations [6] . In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [11] . The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1-1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

   Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [3] . There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [6] . From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [9] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [4] . If the modifications define a closed loop in the space

of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap}dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

3 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

3.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [1] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [22, 18] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [23] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds

in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [26] .

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [1] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and aminoacids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [2, 26] , [2] .

3.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of

CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

(a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

(b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP_2}\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$, $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP_2}/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tedrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

3.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

3.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

3.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [8] .

3.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anticommutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

3.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

3.7.1 1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the spacetime surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \rightarrow CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multivalued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknows. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals

might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

3.7.2 Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multivaluedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi_k^n = \partial L_K/\partial h_n^k$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi_k^n = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π_k^0 and π_k^n. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

   (b) This picture is not quite correct yet. The fixing of π_k^0 and π_k^n should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π_k^0 and π_k^n must be fixed at X^3 and X_l^3 in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X_l^3 as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π_0^k would give n_b branches in CP_2 degrees of freedom and the conditions for π_k^n would split each of these branches to n_a branches.

   (c) The existence of these two kinds of conserved charges (possibly vanishing for π_k^n) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = ng_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

   The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For spacelike 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

3.7.3 Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

4 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [7] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found. The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would

allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

6. The general solution ansatz works for induced Kähler Dirac equation and Chern-Simons Dirac equation and reduces them to ordinary differential equations along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equations along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

4.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the configuration space metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

4.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string

world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} \quad = \quad KJ_{12} \ . \tag{4.1}$$

A more general form of this duality is suggested by the considerations of [13] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} \quad = \quad K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{4.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the configuration space metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m \quad = \quad (1+K)J_{12} \ , \tag{4.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial configuration space metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

4.1.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [1] , [1] read as

$$\begin{aligned}\gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{4.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{4.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned}\frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{4.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned}\alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{4.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

4.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of

the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [19] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{4.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and antifermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{4.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole

4.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced K"ahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{4.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [20]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [27]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

4.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

4.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L \nu_R$. $\nu_L \overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

4.2.2 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four Gaussian Mersennes corresponding to $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [2] .

4.2.3 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [12] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would

be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [15] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [16] .

4.2.4 Should $J + J_1$ appear in Kähler action?

The presence of the S^2 Kähler form J_1 in weak form of electric-magnetic duality was originally suggested by an erratic argument about the reduction to almost topological QFT to be described in the next subsection. In any case this argument raises the question whether one could replace J with $J + J_1$ in the Kähler action. This would not affect the basic non-vacuum extremals but would modify the vacuum degeneracy of the Kähler action. Canonically imbedded M^4 would become a monopole configuration with an infinite magnetic energy and Kähler action due to the monopole singularity at the line connecting tips of the CD. Action and energy can be made small by drilling a small hole around origin. This is however not consistent with the weak form of electro-weak duality. Amusingly, the modified Dirac equation reduces to ordinary massless Dirac equation in M^4.

This extremal can be transformed to a vacuum extremal by assuming that the solution is also a CP_2 magnetic monopole with opposite contribution to the magnetic charge so that $J + J_1 = 0$ holds true. This is achieved if one can regard space-time surface as a map $M^4 \to CP_2$ reducing to a map $(\Theta, \Phi) = (\theta, \pm\phi)$ with the sign chosen by properly projecting the homologically non-trivial $r_M = constant$ spheres of CD to the homologically non-trivial geodesic sphere of CP_2. Symplectic transformations of $S^2 \times CP_2$ produce new vacuum extremals of this kind. Using Darboux coordinates in which one has $J = \sum_{k=1,2} P_k dQ^k$ and assuming that (P_1, Q_1) corresponds to the CP_2 image of S^2, one can take Q_2 to be arbitrary function of P^2 which in turn is an arbitrary function of of M^4 coordinates to obtain even more general vacuum extremals with 3-D CP_2 projection. Therefore the spectrum of vacuum extremals, which is very relevant for the TGD based description of gravitation in long length scales because it allows to satisfy Einstein's equations as an additional condition, looks much richer than for the original option, and it is natural to ask whether this option might make sense.

An objection is that J_1 is a radial monopole field and this breaks Lorentz invariance to $SO(3)$. Lorentz invariance is broken to $SO(3)$ for a given CD also by the presence of the preferred time direction defined by the time-like line connecting the tips of the CD becoming carrying the monopole charge but is compensated since Lorentz boosts of CDs are possible. Could one consider similar compensation also now? Certainly the extremely small breaking of Lorentz invariance and the vanishing of the monopole charge for the vacuum extremals is all that is needed at the space-time level. No new gauge fields would be introduced since only the Kähler field part of photon and Z^0 boson would receive an additional contribution.

The ultimate fate of the modification depends on whether it is consistent with the general relativistic description of gravitation. Since a breaking of spherical symmetry is involved, it is not at all clear whether one can find vacuum extremals which represent small deformations of the Reissner-Nordström metric and Robertson-Walker metric. The argument below shows that this option does not allow the imbedding of small deformations of physically plausible space-time metrics as vacuum extremals.

The basic vacuum extremal whose deformations should give vacuum extremals allowing interpretation as solutions of Einstein's equations is given by a map $M^4 \to CP_2$ projecting the r_M constant spheres S^2 of M^2 to the homologically non-trivial geodesic sphere of CP_2. The winding number of this map is -1 in order to achieve vanishing of the induced Kähler form $J + J_1$. For instance, the following two canonical forms of the map are possible

$$
\begin{array}{rcl}
(\Theta, \Psi) & = & (\theta_M, -\phi_M) \ , \\
(\Theta, \Psi) & = & (\pi - \theta_M, \phi_M) \ .
\end{array}
\tag{4.11}
$$

Here (Θ, Ψ) refers to the geodesic sphere of CP_2 and (θ_M, ϕ_M) to the sphere of M^4.
The resulting space-time surface is not flat and Einstein tensor is non-vanishing. More complex metrics can be constructed from this metric by a deformation making the CP_2 projection 3-dimensional.

Using the expression of the CP_2 line element in Eguchi-Hanson coordinates [4]

$$
\frac{ds^2}{R^2} = \frac{dr^2}{F^2} + \frac{r^2}{F}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + fracr^2 4F sin^2\Theta d\Phi^2)
\tag{4.12}
$$

and s the relationship $r = tan(\Theta)$, one obtains following expression for the CP_2 metric

$$
\frac{ds^2}{R^2} = d\theta_M^2 + sin^2(\theta_M) \left[(d\phi_M + cos(\theta) d\Phi)^2 + \frac{1}{4}(d\theta^2 + sin^2(\theta) d\Phi^2 \right] \ .
\tag{4.13}
$$

The resulting metric is obtained from the metric of S^2 by replacing $d\phi^2$ which 3-D line element. The factor $sin^2(\theta_M)$ implies that the induced metric becomes singular at North and South poles of S^2. In particular, the gravitational potential is proportional to $sin^2(\theta_M)$ so that gravitational force in the radial direction vanishes at equators. It is very difficult to imagine any manner to produce a small deformation of Reissner-Nordstrm metric or Robertson-Walker metric. Hence it seems that the vacuum extremals produce by $J + J_1$ option are not physical.

4.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [4]) so that the Coulombic term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \ . \tag{4.14}$$

   The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{4.15}$$

   This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta}j^K_\beta \partial_\gamma j^K_\delta \quad = \quad 0 \ . \tag{4.16}$$

   j_K is a four-dimensional counterpart of Beltrami field [7] and could be called generalized Beltrami field.

   The integrability conditions follow also from the construction of the extremals of Kähler action [4] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where

$j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha (j^\alpha \phi) \ = \ 0 \ . \tag{4.17}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the imons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant for Chern-Simons Dirac action and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to the Chern-Simons term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

7. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

4.4 Kähler action for Euclidian regions as Kähler function and Kähler action for Minkowskian regions as Morse function?

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked betweeen possibile connections between TGD and Floer homology [31] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

1. All arguments for this have been represented for Minkowskian regions [11] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

   If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. Without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit. Second Chern-Simons term would be however independent of this. For wormhole contacts the two terms could be assigned with opposite wormhole throats and would be identical with their Minkowskian cousins from imaginary unit. This looks a little bit strange.

2. There is however a very delicate issue involved. Quantum classical correspondence requires that the quantum numbers of partonic states must be coded to the space-time geometry, and this is achieved by adding to the action a measurement interaction term which reduces to what is almost a gauge term present only in Chern-Simons-Dirac equation but not at space-time interior [11]. This term would represent a coupling to Poincare quantum numbers at the Minkowskian side and to color and electro-weak quantum numbers at CP_2 side. Therefore the net Chern-Simons contributions and would be different.

3. There is also a very beautiful argument stating that Dirac determinant for Chern-Simons-Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function.

The Minkowskian contribution of Kähler action is imaginary due to the negative of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

Remark: The proportionality of Minkowskian and Euclidian contributions to the same Chern-Simons term implies that the critical points with respect to zero modes appear for both the phase and modulus of vacuum functional. The Kähler function property does not allow extrema for vacuum functional as a function of complex coordinates of WCW since this would mean Kähler metric with non-Euclidian signature. If this were not the case. the stationary values of phase factor and extrema of modulus of the vacuum functional would correspond to different configurations.

4.5 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological field theory. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

4.5.1 Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T \times SO(3) \times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha \left[D_\beta (J^{\alpha\beta} H_A) - j^\alpha_K H^A + T^{\alpha\beta} j^l_A h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{4.18}$$

   The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j^\alpha_K H^A - T^{\alpha\beta} j^k_A h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{4.19}$$

   For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j^\alpha_K J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j^\alpha_K D_\alpha H^A = j^\alpha_K J_\alpha{}^\beta j^A_\beta + T^{\alpha\beta} H^k_{\alpha\beta} j^A_k \ . \tag{4.20}$$

4.5.2 Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J^\alpha_A = j^\alpha_K H^A - T^{\alpha\beta} j^k_A h_{kl} \partial_\beta h^l \tag{4.21}$$

   and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \tag{4.22}$$

   The conservation of J_A gives

$$d * (\Psi_A d\Phi_A) = 0 \ . \tag{4.23}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d * d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \tag{4.24}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{4.25}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d * d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{4.26}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A\Psi_A d\Phi_{G(A)} \ , \ d * (d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{4.27}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [13] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

4.5.3 Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

4.6 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation.

4.6.1 4-dimensional modified Dirac equation and hydrodynamical picture

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \tag{4.28}$$

   The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{4.29}$$

   The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The modified gamma matrices are of form $T^\alpha_k\Gamma^k$, $T^\alpha_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^α_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^\alpha_k\Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi \quad = \quad 0 \ . \qquad (4.30)$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

4.6.2 Generalized eigen modes for the modified Chern-Simons Dirac equation and hydrodynamical picture

Hydrodynamical picture helps to understand also the construction of generalized eigen modes of 3-D Chern-Simons Dirac equation.

The general form of generalized eigenvalue equation for Chern-Simons Dirac action

Consider first the the general form and interpretation of the generalized eigenvalue equation assigned with the modified Dirac equation for Chern-Simons action [6] . This is of course only an approximation since an additional contribution to the modified gamma matrices from the Lagrangian multiplier term guaranteing the weak form of electric-magnetic duality must be included.

1. The modified Dirac equation for Ψ is consistent with that for its conjugate if the coefficient of the instanton term is real and one uses the Dirac action $\overline{\Psi}(D^{\rightarrow} - D^{\leftarrow})\Psi$ giving modified Dirac equation as

$$D_{C-S}\Psi + \frac{1}{2}(D_\alpha\hat{\Gamma}^\alpha_{C-S})\Psi \quad = \quad 0 \ . \qquad (4.31)$$

As noticed, the divergence $D_\alpha\hat{\Gamma}^\alpha_{C-S}$ does not contain second derivatives in the case of Chern-Simons action. In the case of Kähler action they occur unless field equations equivalent with the vanishing of the divergence term are satisfied. The extremals of Chern-Simons action provide a natural manner to define effective 2-dimensionality.

Also the fermionic current is conserved in this case, which conforms with the idea that fermions flow along the light-like 3-surfaces. If one uses the action $\overline{\Psi}D^{\rightarrow}\Psi$, $\overline{\Psi}$ does not satisfy the Dirac equation following from the variational principle and fermion current is not conserved.

2. The generalized eigen modes of D_{C-S} should be such that one obtains the counterpart of Dirac propagator which is purely algebraic and does not therefore depend on the coordinates of the throat. This is satisfied if the generalized eigenvalues are expressible in terms of covariantly constant combinations of gamma matrices and here only M^4 gamma matrices are possible. Therefore the eigenvalue equation would read as

$$D\Psi = \lambda^k\gamma_k\Psi \ , \quad D = D_{C-S} + \frac{1}{2}D_\alpha\hat{\Gamma}^\alpha_{C-S} \ , \quad D_{C-S} = \hat{\Gamma}^\alpha_{C-S}D_\alpha \ . \tag{4.32}$$

Here the covariant derivatives D_α contain the measurement interaction term as an apparent gauge term. For extremals one has

$$D = D_{C-S} \ . \tag{4.33}$$

Covariant constancy allows to take the square of this equation and one has

$$(D^2 + \left[D, \lambda^k\gamma_k\right])\Psi = \lambda^k\lambda_k\Psi \ . \tag{4.34}$$

The commutator term is analogous to magnetic moment interaction.

3. The generalized eigenvalues correspond to $\lambda = \sqrt{\lambda^k\lambda_k}$ and Dirac determinant is defined as a product of the eigenvalues and conjecture to give the exponent of Kähler action reducing to Chern-Simons term. λ is completely analogous to mass. λ_k cannot be however interpreted as ordinary four-momentum: for instance, number theoretic arguments suggest that λ_k must be restricted to the preferred plane $M^2 \subset M^4$ interpreted as a commuting hyper-complex plane of complexified quaternions. For incoming lines this mass would vanish so that all incoming particles irrespective their actual quantum numbers would be massless in this sense and the propagator is indeed that for a massless particle. Note that the eigen-modes define the boundary values for the solutions of $D_K\Psi = 0$ so that the values of λ indeed define the counterpart of the momentum space.

This transmutation of massive particles to effectively massless ones might make possible the application of the twistor formalism as such in TGD framework [29] . $N = 4$ SUSY is one of the very few gauge theory which might be UV finite but it is definitely unphysical due to the masslessness of the basic quanta. Could the resolution of the interpretational problems be that the four-momenta appearing in this theory do not directly correspond to the observed four-momenta?

2. Inclusion of the constraint term

As already noticed one must include also the constraint term due to the weak form of electric-magnetic duality and this changes somewhat the above simple picture.

1. At the 3-dimensional ends of the space-time sheet and at wormhole throats the 3-dimensionality allows to introduce a coordinate varying along the flow lines of Kähler magnetic field $B = *J$. In this case the integrability conditions state that the flow is Beltrami flow. Note that the

value of B^α along the flow line defining magnetic flux appearing in anti-commutation relations is constant. This suggests that the generalized eigenvalue equation for the Chern-Simons action reduces to a collection of ordinary apparently independent differential equations associated with the flow lines beginning from the partonic 2-surface. This indeed happens when the CP_2 projection is 2-dimensional. In this case it however seems that the basis u_n is not of much help.

2. The conclusion is wrong: the variations of Chern-Simons action are subject to the constraint that electric-magnetic duality holds true expressible in terms of Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \ . \tag{4.35}$$

This gives a constraint force to the field equations and also a dependence on the induced 4-metric so that one has only almost topological QFT. This term also guarantees the M^4 part of WCW Kähler metric is non-trivial. The condition that the ends of space-time sheet and wormhole throats are extrema of Chern-Simons action subject to the electric-magnetic duality constraint is strongly suggested by the effective 2-dimensionality. Without the constraint term Chern-Simons action would vanish for its extremals so that Kähler function would be identically zero.

This term implies also an additional contribution to the modified gamma matrices besides the contribution coming from Chern-Simons action so tht the first guess for the modified Dirac operator would not be quite correct. This contribution is of exactly of the same general form as the contribution for any general general coordinate invariant action. The dependence of the induced metric on M^4 degrees of freedom guarantees that also M^4 gamma matrices are present. In the following this term will not be considered.

3. When the contribution of the constraint term to the modifield gamma matrices is neglected, the explicit expression of the modified Dirac operator D_{C-S} associated with the Chern-Simons term is given by

$$\begin{aligned} D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2}D_\mu\hat{\Gamma}^\mu \ , \\ \hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ D_\mu\hat{\Gamma}^\mu &= B^\alpha_K(J_{k\alpha} + \partial_\alpha A_k) \ , \\ B^\alpha_K &= \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl}\partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \tag{4.36}$$

For the extremals of Chern-Simons action one has $D_\alpha\hat{\Gamma}^\alpha = 0$. Analogous condition holds true when the constraing contriabution to the modified gamma matrices is added.

3. Generalized eigenvalue equation for Chern-Simons Dirac action

Consider now the Chern-Simons Dirac equation in more detail assuming that the inclusion of the constraint contribution to the modified gamma matrices does not induce any complications. Assume also extremal property for Chern-Simons action with constraint term and Beltrami flow property.

1. For the extremals the Chern-Simons Dirac operator (constraint term not included) reduces to a one-dimensional Dirac operator

$$D_{C-S} = \hat{\epsilon}^{r\alpha\beta}\left[2J_{k\alpha}A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{4.37}$$

Constraint term implies only a modification of the modified gamma matrices but the form of the operator remains otherwise same when extrema are in question so that one has $D_\alpha\hat{\Gamma^\alpha} = 0$.

2. For the extremals of Chern-Simons action the general solution of the modified Chern-Simons Dirac equation ($\lambda^k = 0$) is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{4.38}$$

The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant (suggested to give rise to the exponent of Kähler function identified as Kähler action). Note that the dependence of these solutions on transversal coordinates of X^3_l is arbitrary which conforms with the hydrodynamic picture. The solutions of Chern-Simons-Dirac are obtained by similar integration procedure also when extremals are not in question.

The formal solution associated with a general eigenvalue λ can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to possibly light-like flow lines of B^α or more general Beltrami field possible induced by the constraint term. There are very strong consistency conditions coming from the conditions that Ψ in the interior is constant along the flow lines of Kähler current and continuous at the ends and throats (call them collectively boundaries), where Ψ has a non-trivial variation along the flow lines of B^α.

1. This makes sense only if the flow lines of the Kähler current are transversal to the boundaries so that the spinor modes at boundaries dictate the modes of the spinor field in the interior. Effective 2-dimensionality means that the spinor modes in the interior can be calculated either by starting from the throats or from the ends so that the data at either upper of lower partonic 2-surfaces dictates everything in accordance with zero energy ontology.

2. This gives an infinite number of commuting diagrams stating that the flow-line time evolution along flow lines along wormhole throats from lower partonic 2-surface to the upper one is equivalent with the flow-line time evolution along the lower end of space-time surface to interior, then along interior to the upper end of the space-time surface and then back to the upper partonic 2-surface. If the space-time surface allows a slicing by partonic 2-surfaces these conditions can be assumed for any pair of partonic 2-surfaces connected by Chern-Simons flow evolution.

3. Since the time evolution along interior keeps the spinor field as constant in the proper gauge and since the flow evolutions at the lower and upper ends are in a reverse direction, there is a strong atemptation to assume that the spinor field at the ends of the of the flow lines of Kähler magnetic field are identical apart from a gauge transformation. This leads to a particle-in-box quantizaton of the values of the pseudo-mass (periodic boundary conditions). These conditions will be assumed in the sequel.

These assumptions lead to the following picture about the generalized eigen modes.

1. By choosing the gauge so that covariant derivative reduces to ordinary derivative and using the constancy of $\hat{\Gamma^r}$, the solution of the generalized eigenvalue equation can be written as

$$\begin{aligned} \Psi &= exp(iL(r)\hat{\Gamma^r}\lambda^k\Gamma_k)\Psi_0 \ , \\ L(r) &= \int_0^r \frac{1}{\sqrt{\hat{g}^{rr}}}dr \ . \end{aligned} \tag{4.39}$$

$L(r)$ can be regarded as the along flux line as defined by the effective metric defined by modified gamma matrices. If λ_k is linear combination of Γ^0 and Γ^{r_M} it anti-commutes with Γ^r which contains only CP_2 gamma matrices so that the pseudo-momentum is a priori arbitrary.

2. When the constraint term taking care of the electric-magnetric duality is included, also M^4 gamma matrices are present. If they are in the orthogonal complement of a preferred plane $M^2 \subset M^4$, anti-commutativity is achieved. This assumption cannot be fully justified yet but conforms with the general physical vision. There is an obvious analogy with the condition that polarizations are in a plane orthogonal to M^2. The condition indeed states that only transversal deformations define quantum fluctuating WCW degrees of freedom contributing to the WCW Kähler metric. In M^8-H duality the preferred plane M^2 is interpreted as a hyper-complex plane belonging to the tangent space of the space-time surface and defines the plane of non-physical polarizations. Also a generalization of this plane to an integrable distribution of planes $M^2(x)$ has been proposed and one must consider also now the possibility of a varying plane $M^2(x)$ for the pseudo-momenta. The scalar function Φ appearing in the general solution ansatz for the field equations satisfies massless d'Alembert equation and its gradient defines a local light-like direction at space-time-level and hence a 2-D plane of the tangent space. Maybe the projection of this plane to M^4 could define the preferred M^2. The minimum condition is that these planes are defined only at the ends of space-time surface and at wormhole throats.

3. If one accepts this hypothesis, one can write

$$\begin{aligned} \Psi &= \left[cos(L(r)\lambda) + isin(\lambda(r))\hat{\Gamma^r}\lambda^k\Gamma_k) \right] \Psi_0 \ , \\ \lambda &= \sqrt{\lambda_k\lambda^k} \ . \end{aligned} \tag{4.40}$$

4. Boundary conditions should fix the spectrum of masses. If the the flow lines of Kähler current coincide with the flow lines of Kähler magnetic field or more general Beltrami current at wormhole throats one ends up with difficulties since the induced spinor fields must be constant along flow lines and only trivial eigenvalues are possible. Hence it seems that the two Beltrami fields must be transversal. This requires that at the partonic 2-surfaces the value of the induced spinor mode in the interior coincides with its value at the throat. Since the induced spinor fields in interior are constant along flow lines, one must have

$$exp(i\lambda L(_{max})) = 1 \ . \tag{4.41}$$

This implies that one has essentially particle in a box with size defined by the effective metric

$$\lambda_n = \frac{n2\pi}{L(r_{max})} \ . \tag{4.42}$$

5. This condition cannot however hold true simultaneously for all points of the partonic 2-surfaces since $L(r_{max})$ depends on the point of the surface. In the most general case one can consider only a subset consisting of the points for which the values of $L(r_{max})$ are rational multiples of the value of $L(r_{max})$ at one of the points -call it L_0. This implies the notion of number theoretical braid. Induced spinor fields are localized to the points of the braid defined by the flow lines of the Kähler magnetic field (or equivalently, any conserved current- this resolves the longstanding issue about the identification of number theoretical braids). The number of the included points depends on measurement resolution characterized somehow by the number rationals which are allowed. Only finite number of harmonics and sub-harmonics of L_0 are possible so that for integer multiples the number of points is finite. If $n_{max}L_0$ and L_0/n_{min} are the largest and smallest lengths involved, one can argue that the rationals n_{max}/n, $n = 1, ..., n_{max}$ and n/n_{min}, $n = 1, ..., n_{min}$ are the natural ones.

6. One can consider also algebraic extensions for which L_0 is scaled from its reference value by an algebraic number so that the mass scale m must be scaled up in similar manner. The spectrum comes also now in integer multiples. p-Adic mass calculations predicts mass scales to the inverses of square roots of prime and this raises the expectation that $\sqrt{n}$ harmonics and sub-harmonics of L_0 might be necessary. Notice however that pseudo-momentum spectrum is in question so that this argument is on shaky grounds.

There is also the question about the allowed values of (λ_0, λ_3) for a given value of λ. This issue will be discussed in the next section devoted to the attempt to calculate the Dirac determinant assignable to this spectrum: suffice it to say that integer valued spectrum is the first guess implying that the pseudo-momenta satisfy $n_0^2 - n_3^2 = n^2$ and therefore correspond to Pythagorean triangles. What is remarkable that the notion of number theoretic braid pops up automatically from the Beltrami flow hypothesis.

5 How to define Dirac determinant?

The basic challenge is to define Dirac determinant hoped to give rise to the exponent of Kähler action associated with the preferred extremal. The reduction to almost topological QFT gives this kind of expression in terms of Chern-Simons action and one might hope of obtaining even more concrete expression from the Chern-Simons Dirac determinant. The calculation of the previous section allowed to calculate the most general spectrum of the modified Dirac operator. If the number of the eigenvalues is infinite as the naive expectation is then Dirac determinant diverges if calculated as the product of the eigenvalues and one must calculate it by using some kind of regularization procedure. Zeta function regularization is the natural manner to do this.

The following arguments however lead to a concrete vision how the regularization could be avoided and a connection with infinite primes. In fact, the manifestly finite option and the option involving zeta function regularization give Kähler functions differing only by a scaling factor and only the manifestly finite option satisfies number theoretical constraints coming from p-adicization. An explicit expression for the Dirac determinant in terms of geometric data of the orbit of the partonic 2-surface emerges.

Arithmetic quantum field theory defined by infinite emerges naturally. The lines of the generalized Feynman graphs are characterized by infinite primes and the selection rules correlating the geometries of the lines of the generalized Feynman graphs corresponds to the conservation of the sum of number theoretic momenta $log(p_i)$ assignable to sub-braids corresponding to different primes p_i assignable to the orbit of parton. This conforms with the vision that infinite primes indeed characterize the geometry of light-like 3-surfaces and therefore also of space-time sheets. The eigenvalues of the modified Dirac operator are proportional $1/\sqrt{p_i}$ where p_i are the primes appearing in the definition of the p-adic prime and the interpretation as analogs of Higgs vacuum expectation values makes sense and is consistent with p-adic length scale hypothesis and p-adic mass calculations. It must be emphasized that all this is essentially due to single basic hypothesis, namely the reduction of quantum TGD to almost topological QFT guaranteed by the Beltrami ansatz for field equations and by the weak form of electric-magnetic duality.

5.1 Dirac determinant when the number of eigenvalues is infinite

At first sight the general spectrum looks the only reasonable possibility but if the eigenvalues correlate with the geometry of the partonic surface as quantum classical correspondence suggests, this conclusion might be wrong. The original hope was the number of eigenvalues would be finite so that also determinant would be finite automatically. There were some justifications for this hope in the definition of Dirac determinant based on the dimensional reduction of D_K as $D_K = D_{K,3} + D_1$ and the identification of the generalized eigenvalues as those assigned to $D_{K,3}$ as analogs of energy eigenvalues assignable to the light-like 3-surface. It will be found that number theoretic input could allow to achieve a manifest finiteness in the case of D_{C-S} and that this option is the only possible one if number theoretic universality is required.

If there are no constraints on the eigenvalue spectrum of D_{C-S} for a given partonic orbit, the naive definition of the determinant gives an infinite result and one must define Dirac determinant using ζ function regularization implying that Kähler function reduces to the derivative of the zeta function $\zeta_D(s)$ -call it Dirac Zeta- associated with the eigenvalue spectrum.

Consider now the situation when the number of eigenvalues is infinite.

1. In this kind of situation zeta function regularization is the standard manner to define the Dirac determinant. What one does is to assign zeta function to the spectrum- let us call it Dirac zeta function and denote by $\zeta_D(s)$- as

$$\zeta_D(s) = \sum_k \lambda_k^{-s} . \tag{5.1}$$

If the eigenvalue λ_k has degeneracy g_k it appears g_k times in the sum. In the case of harmonic oscillator one obtains Riemann zeta for which sum representation converges only for $Re(s) \geq 1$. Riemann zeta can be however analytically continued to the entire complex plane and the idea is that this can be done also in the more general case.

2. By the basic conjecture Kähler function corresponds to the logarithm of the Dirac determinant and equals to the sum of the logarithms of the eigenvalues

$$K = log(\prod \lambda_k) = -\frac{d\zeta_D}{ds}_{|s=0} . \tag{5.2}$$

The expression on the left hand side diverges if taken as such but the expression on the right had side based on the analytical continuation of the zeta function is completely well-defined and finite quantity. Note that the replacement of eigenvalues λ_k by their powers λ_k^n -or equivalently the increase of the degeneracy by a factor n - brings in only a factor n to K: $K \rightarrow nK$.

3. Dirac determinant involves in the minimal situation only the integer multiples of pseudo-mass scale $\lambda = 2\pi/L_{min}$. One can consider also rational and even algebraic multiples $qL_{min} < L_{max}$, $q \geq 1$, of L_{min} so that one would have several integer spectra simultaneously corresponding to different braids. Here L_{min} and L_{max} are the extrema of the braid strand length determined in terms of the effective metric as $L = \int (\hat{g}^{rr})^{-1/2} dr$. The question what multiples are involved will be needed later.

4. Each rational or algebraic multiple of L_{min} gives to the zeta function a contribution which is of same form so that one has

$$\zeta_D = \sum_q \zeta(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ , \ 1 \leq q < \frac{L_{max}}{L_{min}} . \tag{5.3}$$

Kähler function can be expressed as

$$K = \sum_n log(\lambda_n) = -\frac{d\zeta_D(s)}{ds} = -\sum_q log(qx) \frac{d\zeta(s)}{ds}_{|s=0} \ , \ x = \frac{L_{min}}{R} . \tag{5.4}$$

What is remarkable that the number theoretical details of ζ_D determine only the overall scaling factor of Kähler function and thus the value of Kähler coupling strength, which would be purely number theoretically determined if the hypothesis about the role of infinite primes is correct. Also the value of R is irrelevant since it does not affect the Kähler metric.

5. The dependence of Kähler function on WCW degrees of freedom would be coded completely by the dependence of the length scales qL_{min} on the complex coordinates of WCW: note that this dependence is different for each scale. This is reminiscent of the coding of the shape of the drum (or more generally - manifold) by the spectrum of its eigen frequencies. Now Kähler geometry would code for the dependence of the spectrum on the shape of the drum defined by the partonic 2-surface and the 4-D tangent space distribution associated with it.

What happens at the limit of vacuum extremals serves as a test for the identification of Kähler function as Dirac determinant. The weak form of electric magnetic duality implies that all components of the induced Kähler field vanish simultaneously if Kähler magnetic field cancels. In the modified Chern-Simons Dirac equation one obtains $L = \int (\hat{g}^{rr})^{-1/2} dr$. The modified gamma matrix $\hat{\Gamma}^r$ approaches a finite limit when Kähler magnetic field vanishes

$$\hat{\Gamma}^r = \epsilon^{r\beta\gamma}(2J_{\beta k}A_\gamma + J_{\beta\gamma}A_k)\Gamma^k \rightarrow 2\epsilon^{r\beta\gamma}J_{\beta k}\Gamma^k \ . \tag{5.5}$$

The relevant component of the effective metric is $\hat{g}^{rr}$ and is given by

$$\hat{g}^{rr} = (\hat{\Gamma}^r)^2 = 4\epsilon^{r\beta\gamma}\epsilon^{r\mu\nu}J_{\beta k}J_\mu^{\ k}A_\gamma A_\nu \ . \tag{5.6}$$

The limit is non-vanishing in general and therefore the eigenvalues remain finite also at this limit as also the parameter $L_{min} = \int(\hat{g}^{rr})^{-1/2}dr$ defining the minimum of the length of the braid strand defined by Kähler magnetic flux line in the effective metric unless $\hat{g}^{rr}$ goes to zero everywhere inside the partonic surface. Chern-Simons action and Kähler action vanish for vacuum extremals so that in this case one could require that Dirac determinant approaches to unity in a properly chosen gauge. Dirac determinant should approach to unit for vacuum extremals indeed approaches to unity since there are no finite eigenvalues at the limit $\hat{g}^{rr} = 0$.

5.2 Hyper-octonionic primes

Before detailed discussion of the hyper-octonionic option it is good to consider the basic properties of hyper-octonionic primes.

1. Hyper-octonionic primes are of form

$$\Pi_p = (n_0, n_3, n_1, n_2, ..., n_7) \ , \ \Pi_p^2 = n_0^2 - \sum_i n_i^2 = p \ \text{or} \ p^2 \ . \tag{5.7}$$

2. Hyper-octonionic primes have a standard representation as hyper-complex primes. The Minkowski norm squared factorizes into a product as

$$n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3) \ . \tag{5.8}$$

   If one has $n_3 \neq 0$, the prime property implies $n_0 - n_3 = 1$ so that one obtains $n_0 = n_3 + 1$ and $2n_3 + 1 = p$ giving

$$(n_0, n_3) = ((p+1)/2, (p-1)/2) \ . \tag{5.9}$$

   Note that one has $(p+1)/2$ odd for $p \ mod \ 4 = 1$) and $(p+1)/2$ even for $p \ mod \ 4 = 3$). The difference $n_0 - n_3 = 1$ characterizes prime property.

   If n_3 vanishes the prime prime property implies equivalence with ordinary prime and one has $n_3^2 = p^2$. These hyper-octonionic primes represent particles at rest.

3. The action of a discrete subgroup $G(p)$ of the octonionic automorphism group G_2 generates form hyper-complex primes with $n_3 \neq 0$ further hyper-octonionic primes $\Pi(p,k)$ corresponding to the same value of n_0 and p and for these the integer valued projection to M^2 satisfies $n_0^2 - n_3^2 = n > p$. It is also possible to have a state representing the system at rest with $(n_0, n_3) = ((p+1)/2, 0)$ so that the pseudo-mass varies in the range $[\sqrt{p}, (p+1)/2]$. The subgroup $G(n_0, n_3) \subset SU(3)$ leaving invariant the projection (n_0, n_3) generates the hyper-octonionic primes corresponding to the same value of mass for hyper-octonionic primes with same Minkowskian length p and pseudo-mass $\lambda = n \geq \sqrt{p}$.

4. One obtains two kinds of primes corresponding to the lengths of pseudo-momenta equal to p or $\sqrt{p}$. The first kind of particles are always at rest whereas the second kind of particles can be brought at rest only if one interprets the pseudo-momentum as M^2 projection. This brings in mind the secondary p-adic length scales assigned to causal diamonds (CDs) and the primary p-adic lengths scales assigned to particles.

If the M^2 projections of hyper-octonionic primes with length $\sqrt{p}$ characterize the allowed basic momenta, ζ_D is sum of zeta functions associated with various projections which must be in the limits dictated by the geometry of the orbit of the partonic surface giving upper and lower bounds L_{max} and L_{min} on the length L. L_{min} is scaled up to $\sqrt{n_0^2-n_3^2}L_{min}$ for a given projection (n_0,n_3). In general a given M^2 projection (n_0,n_3) corresponds to several hyper-octonionic primes since $SU(3)$ rotations give a new hyper-octonionic prime with the same M^2 projection. This leads to an inconsistency unless one has a good explanation for why some basic momentum can appear several times. One might argue that the spinor mode is degenerate due to the possibility to perform discrete color rotations of the state. For hyper complex representatives there is no such problem and it seems favored. In any case, one can look how the degeneracy factors for given projection can be calculated.

1. To calculate the degeneracy factor $D(n$ associated with given pseudo-mass value $\lambda=n$ one must find all hyper-octonionic primes Π, which can have projection in M^2 with length n and sum up the degeneracy factors $D(n,p)$ associated with them:

$$\begin{aligned}
D(n) &= \sum_p D(n,p) \ , \\
D(n,p) &= \sum_{n_0^2-n_3^2=p} D(p,n_0,n_3) \ , \\
n_0^2-n_3^2 &= n \ , \ \Pi_p^2(n_0,n_3)=n_0^2-n_3^2-\sum_i n_i^2 = n-\sum_i n_i^2 = p \ .
\end{aligned} \tag{5.10}$$

2. The condition $n_0^2-n_3^2=n$ allows only Pythagorean triangles and one must find the discrete subgroup $G(n_0,n_3)\subset SU(3)$ producing hyper-octonions with integer valued components with length p and components (n_0,n_3). The points at the orbit satisfy the condition

$$\sum n_i^2 = p-n \ . \tag{5.11}$$

The degeneracy factor $D(p,n_0,n_3)$ associated with given mass value n is the number of elements of in the coset space $G(n_0,n_3,p)/H(n_0,n_3,p)$, where $H(n_0,n_3,p)$ is the isotropy group of given hyper-octonionic prime obtained in this manner. For $n_0^2-n_3^2=p^2$ $D(n_0,n_3,p)$ obviously equals to unity.

5.3 Three basic options for the pseudo-momentum spectrum

The calculation of the scaling factor of the Kähler function requires the knowledge of the degeneracies of the mass squared eigen values. There are three options to consider.

5.3.1 First option: all pseudo-momenta are allowed

If the degeneracy for pseudo-momenta in M^2 is same for all mass values- and formally characterizable by a number N telling how many 2-D pseudo-momenta reside on mass shell $n_0^2-n_3^2=m^2$. In this case zeta function would be proportional to a sum of Riemann Zetas with scaled arguments corresponding to scalings of the basic mass m to m/q.

$$\zeta_D(s) = N\sum_q \zeta(log(qx)s) \ , \ x=\frac{L_{min}}{R} \ . \tag{5.12}$$

This option provides no idea about the possible values of $1\leq q\leq L_{max}/L_{min}$. The number N is given by the integral of relativistic density of states $\int dk/2\sqrt{k^2+m^2}$ over the hyperbola and is logarithmically divergent so that the normalization factor N of the Kähler function would be infinite.

5.3.2 Second option: All integer valued pseudomomenta are allowed

Second option is inspired by number theoretic vision and assumes integer valued components for the momenta using $m_{max} = 2\pi/L_{min}$ as mass unit. p-Adicization motivates also the assumption that momentum components using m_{max} as mass scale are integers. This would restrict the choice of the number theoretical braids.

Integer valuedness together with masses coming as integer multiples of m_{max} implies $(\lambda_0, \lambda_3) = (n_0, n_3)$ with on mass shell condition $n_0^2 - n_3^2 = n^2$. Note that the condition is invariant under scaling. These integers correspond to Pythagorean triangles plus the degenerate situation with $n_3 = 0$. There exists a finite number of pairs (n_0, n_3) satisfying this condition as one finds by expressing n_0 as $n_0 = n_3 + k$ giving $2n_3k + k^2 = p^2$ giving $n_3 < n^2/2$,$n_0 < n^2/2 + 1$. This would be enough to have a finite degeneracy $D(n) \geq 1$ for a given value of mass squared and ζ_D would be well defined. ζ_D would be a modification of Riemann zeta given by

$$\begin{aligned}
\zeta_D &= \sum_q \zeta_1(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ , \\
\zeta_1(s) &= \sum g_n n^{-s} \ , \ g_n \geq 1 \ . \qquad (5.13)
\end{aligned}$$

For generalized Feynman diagrams this option allows conservation of pseudo-momentum and for loops no divergences are possible since the integral over two-dimensional virtual momenta is replaced with a sum over discrete mass shells containing only a finite number of points. This option looks thus attractive but requires a regularization. On the other hand, the appearance of a zeta function having a strong resemblance with Riemann zeta could explain the finding that Riemann zeta is closely related to the description of critical systems. This point will be discussed later.

5.3.3 Third option: Infinite primes code for the allowed mass scales

According to the proposal of [24] , [3] the hyper-complex parts of hyper-octonionic primes appearing in their infinite counterparts correspond to the M^2 projections of real four-momenta. This hypothesis suggests a very detailed map between infinite primes and standard model quantum numbers and predicts a universal mass spectrum [24] . Since pseudo-momenta are automatically restricted to the plane M^2, one cannot avoid the question whether they could actually correspond to the hyper-octonionic primes defining the infinite prime. These interpretations need not of course exclude each other. This option allows several variants and at this stage it is not possible to exclude any of these options.

1. One must choose between two alternatives for which pseudo-momentum corresponds to hyper-complex prime serving as a canonical representative of a hyper-octonionic prime or a projection of hyper-octonionic prime to M^2.

2. One must decide whether one allows a) only the momenta corresponding to hyper-complex primes, b) also their powers (p-adic fractality), or c) all their integer multiples ("Riemann option").

One must also decide what hyper-octonionic primes are allowed.

1. The first guess is that all hyper-complex/hyper-octonionic primes defining length scale $\sqrt{p}L_{min} \leq L_{max}$ or $pL_{min} \leq L_{max}$ are allowed. p-Adic fractality suggests that also the higher p-adic length scales $p^{n/2}L_{min} < L_{max}$ and $p^nL_{min} < L_{max}$, $n \geq 1$, are possible.

   It can however happen that no primes are allowed by this criterion. This would mean vanishing Kähler function which is of course also possible since Kähler action can vanish (for instance, for massless extremals). It seems therefore safer to allow also the scale corresponding to the trivial prime $(n_0, n_3) = (1, 0)$ (1 is formally prime because it is not divisible by any prime different from 1) so that at least L_{min} is possible. This option also allows only rather small primes unless the partonic 2-surface contains vacuum regions in which case L_{max} is infinite: in this case all primes would be allowed and the exponent of Kähler function would vanish.

2. The hypothesis that only the hyper-complex or hyper-octonionic primes appearing in the infinite hyper-octonionic prime are possible looks more reasonable since large values of p would be possible and could be identified in terms of the p-adic length scale hypothesis. All hyper-octonionic primes appearing in infinite prime would be possible and the geometry of the orbit of the partonic 2-surface would define an infinite prime. This would also give a concrete physical interpretation for the earlier hypothesis that hyper-octonionic primes appearing in the infinite prime characterize partonic 2-surfaces geometrically. One can also identify the fermionic and purely bosonic primes appearing in the infinite prime as braid strands carrying fermion number and purely bosonic quantum numbers. This option will be assumed in the following.

5.4 Expression for the Dirac determinant for various options

The expressions for the Dirac determinant for various options can be deduced in a straightforward manner. Numerically Riemann option and manifestly finite option do not differ much but their number theoretic properties are totally different.

5.4.1 Riemann option

All integer multiples of these basic pseudo-momenta would be allowed for Riemann option so that ζ_D would be sum of Riemann zetas with arguments scaled by the basic pseudo-masses coming as inverses of the basic length scales for braid strands. For the option involving only hyper-complex primes the formula for ζ_D reads as

$$\begin{gathered}
\zeta_D = \zeta(log(x_{min}s)) + \textstyle\sum_{i,n}\zeta(log(x_{i,n}s)) + \sum_{i,n}\zeta(log(y_{i,n}s)) \ \ , \\
x_{i,n} = p_i^{n/2}x_{min} \leq x_{max} \ \ , \ \ p_i \geq 3 \ \ , \ \ y_{i,n} = p_i^n x_{min} \leq x_{max} \ \ . \ \ p_i \geq 2 \ \ ,
\end{gathered} \tag{5.14}$$

L_{max} *resp.* L_{min} is the maximal *resp.* minimal length $L = \int(\hat{g}^{rr})^{-1/2}dr$ for the braid strand defined by the flux line of the Kähler magnetic field in the effective metric. The contributions correspond to the effective hyper-complex prime $p_1 = (1,0)$ and hyper-complex primes with Minkowski lengths $\sqrt{p}$ ($p \geq 3$) and p, $p \geq 2$. If also higher p-adic length scales $L_n = p^{n/2}L_{min} < L_{max}$ and $L_n = p^n L_{min} < L_{max}$, $n > 1$, are allowed there is no further restriction on the summation. For the restricted option only L_n, $n = 0, 2$ is allowed.

The expressions for the Kähler function and its exponent reads as

$$\begin{aligned}
K &= k(log(x_{min}) + \sum_i log(x_i) + \sum_i log(y_i) \ \ , \\
exp(K) &= (\frac{1}{x_{min}})^k \times \prod_i (\frac{1}{x_i})^k \times \prod_i (\frac{1}{y_i})^k \ \ , \\
x_i &\leq x_{max} \ \ , \ \ y_i \leq x_{max} \ \ , \ \ k = -\frac{d\zeta(s)}{ds}_{|s=0} = \frac{1}{2}log(2\pi) \simeq .9184 \ \ .
\end{aligned} \tag{5.15}$$

From the point of view of p-adicization program the appearance of strongly transcendental numbers in the normalization factor of ζ_D is not a well-come property.

If the scaling of the WCW Kähler metric by $1/k$ is a legitimate procedure it would allow to get rid of the transcendental scaling factor k and this scaling would cancel also the transcendental from the exponent of Kähler function. The scaling is not however consistent with the view that Kähler coupling strength determines the normalization of the WCW metric.

This formula generalizes in a rather obvious manner to the cases when one allows M^2 projections of hyper-octonionic primes.

5.4.2 Manifestly finite options

The options for which one does not allow summation over all integer multiples of the basic momenta characterized by the canonical representatives of hyper-complex primes or their projections to M^2 are manifestly finite. They differ from the Riemann option only in that the normalization factor $k =\simeq .9184$ defined by the derivative Riemann Zeta at origin is replaced with $k = 1$. This would mean manifest finiteness of ζ_D. Kähler function and its exponent are given by

$$\begin{aligned} K &= k(log(x_{min}) + \sum_i log(x_i) + \sum_i log(y_i) \ , \ x_i \leq x_{max} \ , \ y_i \leq x_{max} \ , \\ exp(K) &= \frac{1}{x_{min}} \times \prod_i \frac{1}{x_i} \times \prod_i \frac{1}{y_i} \ . \end{aligned} \tag{5.16}$$

Numerically the Kähler functions do not differ much since their ratio is .9184. Number theoretically these functions are however completely different. The resulting dependence involves only square roots of primes and is an algebraic function of the lengths p_i and rational function of x_{min}. p-Adicization program would require rational values of the lengths x_{min} in the intersection of the real and p-adic worlds if one allows algebraic extension containing the square roots of the primes involved. Note that in p-adic context this algebraic extension involves two additional square roots for $p > 2$ if one does not want square root of p. Whether one should allow for R_p also extension based on $\sqrt{p}$ is not quite clear. This would give 8-D extension.

For the more general option allowing all projections of hyper-complex primes to M^2 the general form of Kähler function is same. Instead of pseudo-masses coming as primes and their square roots one has pseudomasses coming as square roots of some integers $n \leq p$ or $n \leq p^2$ for each p. In this case the conservation laws are not so strong.

Note that in the case of vacuum extremals $x_{min} = \infty$ holds true so that there are no primes satisfying the condition and Kähler function vanishes as it indeed should.

5.4.3 More concrete picture about the option based on infinite primes

The identification of pseudo-momenta in terms of infinite primes suggests a rather concrete connection between number theory and physics.

1. One could assign the finite hyper-octonionic primes Π_i making the infinite prime to the sub-braids identified as Kähler magnetic flux lines with the same length L in the effective metric. The primes assigned to the finite part of the infinite prime correspond to single fermion and some number of bosons. The primes assigned to the infinite part correspond to purely bosonic states assignable to the purely bosonic braid strands. Purely bosonic state would correspond to the action of a WCW Hamiltonian to the state.

   This correspondence can be expanded to include all quantum numbers by using the pair of infinite primes corresponding to the "vacuum primes" $X \pm 1$, where X is the product of all finite primes [24] . The only difference with respect to the earlier proposal is that physical momenta would be replaced by pseudo-momenta.

2. Different primes p_i appearing in the infinite prime would correspond to their own sub-braids. For each sub-braid there is a N-fold degeneracy of the generalized eigen modes corresponding to the number N of braid strands so that many particle states are possible as required by the braid picture.

3. The correspondence of infinite primes with the hierarchy of Planck constants could allow to understand the fermion-many boson states and many boson states assigned with a given finite prime in terms of many-particle states assigned to n_a and n_b-sheeted singular covering spaces of CD and CP_2 assignable to the two infinite primes. This interpretation requires that only single p-adic prime p_i is realized as quantum state meaning that quantum measurement always selects a particular p-adic prime p_i (and corresponding sub-braid) characterizing the p-adicity of the quantum state. This selection of number field behind p-adic physics responsible for cognition looks very plausible.

4. The correspondence between pairs of infinite primes and quantum states [24] allows to interpret color quantum numbers in terms of the states associated with the representations of a finite subgroup of $SU(3)$ transforming hyper-octonionic primes to each other and preserving the M^2 pseudo-momentum. Same applies to $SO(3)$. The most natural interpretation is in terms of wave functions in the space of discrete $SU(3)$ and $SO(3)$ transforms of the partonic 2-surface. The dependence of the pseudo-masses on these quantum numbers is natural so that the projection hypothesis finds support from this interpretation.

5. The infinite prime characterizing the orbit of the partonic 2-surface would thus code which multiples of the basic mass $2\pi/L_{min}$ are possible. Either the M^2 projections of hyper-octonionic primes or their hyper-complex canonical representatives would fix the basic M^2 pseudo-momenta for the corresponding number theoretic braid associated. In the reverse direction the knowledge of the light-like 3-surface, the CD and CP_2 coverings, and the number of the allowed discrete $SU(3)$ and $SU(2)$ rotations of the partonic 2-surface would dictate the infinite prime assignable to the orbit of the partonic 2-surface.

One would also like to understand whether there is some kind of conservation laws associated with the pseudo-momenta at vertices. The arithmetic QFT assignable to infinite primes would indeed predict this kind of conservation laws.

1. For the manifestly finite option the ordinary conservation of pseudo-momentum conservation at vertices is not possible since the addition of pseudo-momenta does not respect the condition $n_0 - n_3 = 1$. In fact, this difference in the sum of hyper-complex prime momenta tells how many momenta are present. If one applies the conservation law to the sum of the pseudo-momenta corresponding to different primes and corresponding braids, one can have reactions in which the number of primes involved is conserved. This would give the selection rule $\sum_1^N p_i = \sum_1^N p_f$. These reactions have interpretation in terms of the geometry of the 3-surface representing the line of the generalized Feynman diagram.

2. Infinite primes define an arithmetic quantum field theory in which the total momentum defined as $\sum n_i log(p_i)$ is a conserved quantity. As matter fact, each prime p_i would define a separately conserved momentum so that there would be an infinite number of conservation laws. If the sum $\sum_i log(p_i)$ is conserved in the vertex , the primes p_i associated with the incoming particle are shared with the outgoing particles so that also the total momentum is conserved. This looks the most plausible option and would give very powerful number theoretical selection rules at vertices since the collection of primes associated with incoming line would be union of the collections associated with the outgoing lines and also total pseudo-momentum would be conserved.

3. For the both Riemann zeta option and manifestly finite options the arithmetic QFT associated with infinite primes would be realized at the level of pseudo-momenta meaning very strong selection rules at vertices coding for how the geometries of the partonic lines entering the vertex correlate. WCW integration would reduce for the lines of Feynman diagram to a sum over light-like 3-surfaces characterized by (x_{min}, x_{max}) with a suitable weighting factor and the exponent of Kähler function would give an exponential damping as a function of x_{min}.

5.4.4 Which option to choose?

One should be able to make two choices. One must select between hyper-complex representations and the projections of hyper-octonionic primes and between the manifestly finite options and the one producing Riemann zeta?

Hyper-complex option seems to be slightly favored over the projection option.

1. The appearance of the scales $\sqrt{p_i} x_{min}$ and possibly also their p^n multiples brings in mind p-adic length scales coming as $\sqrt{p}^n$ multiples of CP_2 length scale. The scales $p_i x_{min}$ associated with hyper-complex primes reducing to ordinary primes in turn bring in mind the size scales assignable to CDs. The hierarchy of Planck constants implies also $\hbar/\hbar_0 = \sqrt{n_a n_b}$ multiples of these length scales but mass scales would not depend on n_a and n_b [25] . For large values of p the pseudo-momenta are almost light-like for hyper-complex option whereas the projection option allows also states at rest.

2. Hyper-complex option predicts that only the p-adic pseudo-mass scales appear in the partition function and is thus favored by the p-adic length scale hypothesis. Projection option predicts also the possibility of the mass scales (not all of them) coming as $1/\sqrt{n}$. These mass scales are however not predicted by the hierarchy of Planck constants.

3. The same pseudo-mass scale can appear several times for the projection option. This degeneracy corresponds to the orbit of the hyper-complex prime under the subgroup of $SU(3)$ respecting integer property. Similar statement holds true in the case of $SO(3)$: these groups are assigned to the two infinite primes characterizing parton. The natural assignment of this degeneracy is to the discrete color rotational and rotational degrees associated with the partonic 2-surface itself rather than spinor modes at fixed partonic 2-surface. That the pseudo-mass would depend on color and angular momentum quantum numbers would make sense.

Consider next the arguments in favor of the manifestly finite option.

1. The manifestly finite option is admittedly more elegant than the one based on Riemann zeta and also guarantees that no additional loop summations over pseudo-momenta are present. The strongest support for the manifestly finite option comes from number theoretical universality.

2. One could however argue that the restriction of the pseudo-momenta to a finite number is not consistent with the modified Dirac-Chern-Simons equation. Quantum classical correspondence however implies correlation between the geometry of the partonic orbits and the pseudo-momenta and the summation over all prime valued pseudo-momenta is present but with a weighting factor coming from Kähler function implying exponential suppression.

The Riemann zeta option could be also defended.

1. The numerical difference of the normalization factors of the Kähler function is however only about 8 per cent and quantum field theorists might interpret the replacement the length scales x_i and y_i with x_i^d and y_i^d, $d \simeq .9184$, in terms of an anomalous dimension of these length scales. Could one say that radiative corrections mean the scaling of the original preferred coordinates so that one could still have consistency with number theoretic universality?

2. Riemann zeta with a non-vanishing argument could have also other applications in quantum TGD. Riemann zeta has interpretation as a partition function and the zeros of partition functions have interpretation in terms of phase transitions. The quantum criticality of TGD indeed corresponds to a phase transition point. There is also experimental evidence that the distribution of zeros of zeta corresponds to the distribution of energies of quantum critical systems in the sense that the energies correspond to the imaginary parts of the zeros of zeta [1] .

   The first explanation would be in terms of the analogs of the harmonic oscillator coherent states with integer multiple of the basic momentum taking the role of occupation number of harmonic oscillator and the zeros $s = 1/2 + iy$ of ζ defining the values of the complex coherence parameters. TGD inspired strategy for the proof of Riemann hypothesis indeed leads to the identification of the zeros as coherence parameters rather than energies as in the case of Hilbert-Polya hypothesis [21] and the vanishing of the zeta at zero has interpretation as orthogonality of the state with respect to the state defined by a vanishing coherence parameter interpreted as a tachyon. One should demonstrate that the energies of quantum states can correspond to the imaginary parts of the coherence parameters.

   Second interpretation could be in terms of quantum critical zero energy states for which the "complex square root of density matrix" defines time-like entanglement coefficients of M-matrix. The complex square roots of the probabilities defined by the coefficient of harmonic oscillator states (perhaps identifiable in terms of the multiples of pseudo-momentum) in the coherent state defined by the zero of ζ would define the M-matrix in this situation. Energy would correspond also now to the imaginary part of the coherence parameter. The norm of the state would be completely well-defined.

5.4.5 Representation of configuration Kähler metric in terms of eigenvalues of D_{C-S}

A surprisingly concrete connection of the configuration space metric in terms of generalized eigenvalue spectrum of D_{C-S} results. From the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k \partial_{\bar{l}} K = \frac{\partial_k \partial_{\bar{l}} exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)} \frac{\partial_{\bar{l}} exp(K)}{exp(K)} \ , \tag{5.17}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of D_{C-S} , the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k \partial_{\bar{l}} \lambda_i}{\lambda_i} - \frac{\partial_k \lambda_i}{\lambda_i} \frac{\partial_{\bar{l}} \lambda_i}{\lambda_i} \tag{5.18}$$

for the configuration space metric follows. Here complex coordinates refer to the complex coordinates of configuration space. Hence the knowledge of the eigenvalue spectrum of $D_{C-S}(X^3)$ as function of some complex coordinates of configuration space allows to deduce the metric to arbitrary accuracy. If the above arguments are correct the calculation reduces to the calculation of the derivatives of $log(\sqrt{p} L_{min}/R)$, where L_{min} is the length of the Kähler magnetic flux line between partonic 2-surfaces with respect to the effective metric defined by the anti-commutators of the modified gamma matrices. Note that these length scales have different dependence on WCW coordinates so that one cannot reduce everything to L_{min}. Therefore one would have explicit representation of the basic building brick of WCW Kähler metric in terms of the geometric data associated with the orbit of the partonic 2-surface.

5.4.6 The formula for the Kähler action of CP_2 type vacuum extremals is consistent with the Dirac determinant formula

The first killer test for the formula of Kähler function in terms of the Dirac determinant based on infinite prime hypothesis is provided by the action of CP_2 type vacuum extremals. One of the first attempts to make quantitative predictions in TGD framework was the prediction for the gravitational constant. The argument went as follows.

1. For dimensional reasons gravitational constant must be proportional to p-adic length scale squared, where p characterizes the space-time sheet of the graviton. It must be also proportional to the square of the vacuum function for the graviton representing a line of generalized Feynman diagram and thus to the exponent $exp(-2K)$ of Kähler action for topologically condensed CP_2 type vacuum extremals with very long projection. If topological condensation does not reduce much of the volume of CP_2 type vacuum extremal, the action is just Kähler action for CP_2 itself. This gives

$$\hbar_0 G = L_p^2 exp(2L_K(CP_2) = pR^2 exp(2L_K(CP_2) \ . \tag{5.19}$$

2. Using as input the constraint $\alpha_K \simeq \alpha_{em} \sim 1/137$ for Kähler coupling strengths coming from the comparison of the TGD prediction for the rotation velocity of distant galaxies around galactic nucleus and the p-adic mass calculation for the electron mass, one obtained the result

$$exp(2L_K(CP_2) = \frac{1}{p \times \prod_{p_i \leq 23} p_i} \ . \tag{5.20}$$

The product contains the product of all primes smaller than 24 ($p_i \in \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$). The expression for the Kähler function would be just of the form predicted by the Dirac determinant formula with L_{min} replaced with CP_2 length scale. As a matter fact, this was the first indication that particles are characterized by several p-adic primes but that only one of them is "active". As explained, the number theoretical state function reduction explains this.

3. The same formula for the gravitational constant would result for any prime p but the value of Kähler coupling strength would depend on prime p logarithmically for this option. I indeed proposed that this formula fixes the discrete evolution of the Kähler coupling strength as function of p-adic prime from the condition that gravitational constant is renormalization group invariant quantity but gave up this hypothesis later. It is wisest to keep an agnostic attitude to this issue.

4. I also made numerous brave attempts to deduce an explicit formula for Kähler coupling strength. The general form of the formula is

$$\frac{1}{\alpha_K} = klog(K^2), \quad K^2 = p \times 2 \times 3 \times 5.. \times 23 \ . \tag{5.21}$$

   The problem is the exact value of k cannot be known precisely and the guesses for is value depend on what one means with number theoretical universality. Should Kähler action be a rational number? Or is it Kähler function which is rational number (it is for the Dirac determinant option in this particular case). Is Kähler coupling strength $g_K^2/4\pi$ or g_K^2 a rational number? Some of the guesses were $k = \pi/4$ and $k = 137/107$. The facts that the value of Kähler action for the line of a generalized diagram is not exactly CP_2 action and the value of α_K is not known precisely makes these kind of attempts hopeless in absence of additional ideas.

Also other elementary particles -in particular exchanged bosons- should involve the exponent of Kähler action for CP_2 type vacuum extremal. Since the values of gauge couplings are gigantic as compared to the expression of the gravitational constant the value of Kähler action must be rather small form them. CP_2 type vacuum extremals must be short in the sense that L_{min} in the effective metric is very short. Note however that the p-adic prime characterizing the particle according to p-adic mass calculations would be large also now. One can of course ask whether this p-adic prime characterizes the gravitational space-time sheets associated with the particle and not the particle itself. The assignment of p-adic mass calculations with thermodynamics at gravitational space-time sheets of the particle would be indeed natural. The value of α_K would depend on p in logarithmic manner for this option. The topological condensation of could also eat a lot of CP_2 volume for them.

5.4.7 Eigenvalues of D_{C-S} as vacuum expectations of Higgs field?

Infinite prime hypothesis implies the analog of p-adic length scale hypothesis but since pseudo-momenta are in question, this need not correspond to the p-adic length scale hypothesis for the actual masses justified by p-adic thermodynamics. Note also that L_{min} does not correspond to CP_2 length scale. This is actually not a problem since the effective metric is not M^4 metric and one can quite well consider the possibility that L_{min} corresponds to CP_2 length scale in the the induced metric. The reason is that light-like 3- surface is in question the distance along the Kähler magnetic flux line reduces essentially to a distance along the partonic 2-surface having size scale of order CP_2 length for the partonic 2-surfaces identified as wormhole throats. Therefore infinite prime can code for genuine p-adic length scales associated with the light-like 3-surface and quantum states would correspond by number theoretical state function reduction hypothesis to single ordinary prime.

Support for this identification comes also from the expression of gravitational constant deduced from p-adic length scale hypothesis. The result is that gravitational constant is assumed to be proportional to have the expression $G = L_p^2 exp(-2S_K(CP_2))$, where p characterizes graviton or the space-time sheet mediating gravitational interaction and exponent gives Kähler action for CP_2 type vacuum extremal representing graviton. The argument allows to identify the p-adic prime $p = M_{127}$ associated with electron (largest Mersenne prime which does not correspond to super-astronomical length scale) as the p-adic prime characterizing also graviton. The exponent of Kähler action is proportional to $1/p$ which conforms with the general expression for Kähler function. I have considered several identifications of the numerical factor and one of them has been as product of primes $2 \leq p \leq 23$ assuming that somehow the primes $\{2, ..., 23, p\}$ characterize graviton. This guess is indeed consistent with the prediction of the infinite-prime hypothesis.

The first guess inspired by the p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of D_{C-S} could correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. By the previous argument one would have order of magnitude estimate $h_0 = \sqrt{2\pi/L_{min}}$.

1. The vacuum expectation value of Higgs is only proportional to the scale of λ. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and anti-fermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. For free fermions the vacuum expectation value of Higgs does not seem to be even possible since free fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). If fermion suffers topological condensation as indeed assumed to do in interaction region, a wormhole contact is generated and makes possible the generation of Higgs vacuum expectation value.

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of modified Chern-Simons Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. For the minimal option one has only a finite number of pseudo-mass eigenvalues inversely proportional $\sqrt{p}$ so that the identification as a Higgs vacuum expectation is consistent with the p-adic length scale hypothesis. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. If the coset construction for super-symplectic and super Kac-Moody algebra implying Equivalence Principle is accepted, one encounters what looks like a problem. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign. Therefore it seems that λ_i^2 can define only a deviation of the ground state conformal weight from negative value and is positive.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = -n/2 + \lambda_i^2$ where the negative contribution comes from Super Virasoro representation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

5.4.8 Is there a connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes

The following observations suggest that there might be an intrinsic connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes.

1. p-Adic thermodynamics [14] is based on string mass formula in which mass squared is proportional to conformal weight having values which are integers apart from the contribution of the conformal weight of vacuum which can be non-integer valued. The thermal expectation in p-adic thermodynamics is obtained by replacing the Boltzman weight $exp(-E/T)$ of ordinary thermodynamics with p-adic conformal weight p^{n/T_p}, where n is the value of conformal weight and $1/T_p = m$ is integer values inverse p-adic temperature. Apart from the ground state contribution and scale factor p-adic mass squared is essentially the expectation value

$$\langle n\rangle = \frac{\sum_n g(n) n p^{\frac{n}{T_p}}}{\sum_n g(n) p^{\frac{n}{T_p}}} . \tag{5.22}$$

   $g(n)$ denotes the degeneracy of a state with given conformal weight and depends only on the number of tensor factors in the representations of Virasoro or Super-Virasoro algebra. p-Adic mass squared is mapped to its real counterpart by canonical identification $\sum x_n p^n \to \sum x_n p^{-n}$. The real counterpart of p-adic thermodynamics is obtained by the replacement $p^{-\frac{n}{T_p}}$ and gives under certain additional assumptions in an excellent accuracy the same results as the p-adic thermodynamics.

2. An intriguing observation is that one could interpret p-adic and real thermodynamics for mass squared also in terms of number theoretic thermodynamics for the number theoretic momentum $log(p^n) = nlog(p)$. The expectation value for this differs from the expression for $\langle n\rangle$ only by the factor $log(p)$.

3. In the proposed characterization of the partonic orbits in terms of infinite primes the primes appearing in infinite prime are identified as p-adic primes. For minimal option the p-adic prime characterizes $\sqrt{p}$- or p- multiple of the minimum length L_{min} of braid strand in the effective metric defined by modified Chern-Simons gamma matrice. One can consider also $(\sqrt{p})^n$ and p^n (p-adic fractality)- and even integer multiples of L_{min} if they are below L_{max}. If light-like 3-surface contains vacuum regions arbitrary large p:s are possible since for these one has $L_{min} \to \infty$. Number theoretic state function reduction implies that only single p can be realized -one might say "is active"- for a given quantum state. The powers p_i^n appearing in the infinite prime have interpretation as many particle states with total number theoretic momentum $n_i log(p)_i$. For the finite part of infinite prime one has one fermion and $n_i - 1$ bosons and for the bosonic part n_i bosons. The arithmetic QFT associated with infinite primes - in particular the conservation of the number theoretic momentum $\sum n_i log(p_i)$ - would naturally describe the correlations between the geometries of light-like 3-surfaces representing the incoming lines of the vertex of generalized Feynman diagram. As a matter fact, the momenta associated with different primes are separately conserved so that one has infinite number of conservation laws.

4. One must assign two infinite primes to given partonic two surface so that one has for a given prime p two integers n_+ and n_-. Also the hierarchy of Planck constants assigns to a given page of the Big Book two integers and one has $\hbar = n_a n_b \hbar_0$. If one has $n_a = n_+$ and $n_b = n_-$ then the reactions in which given initial number theoretic momenta $n_{\pm,i} log(p_i)$ is shared between final states would have concrete interpretation in terms of the integers n_a, n_b characterizing the coverings of incoming and outgoing lines.

   Note that one can also consider the possibility that the hierarchy of Planck constants emerges from the basic quantum TGD. Basically due to the vacuum degeneracy of Kähler action the canonical momentum densities correspond to several values of the time derivatives of the imbedding space oordinates so that for a given partonic 2-surface there are several space-time sheets with same conserved quantities defined by isometry currents and Kähler current. This forces the introduction of N-fold covering of $CD \times CP_2$ in order to describe the situation. The splitting of the partonic 2-surface into N pieces implies a charge fractionization during its travel to the upper end of CD. One can also develop an argument suggesting that the coverings factorize to coverings of CD and CP_2 so that the number of the sheets of the covering is $N = n_a n_b$ [13] .

These observations make one wonder whether there could be a connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes.

1. Suppose that one accepts the identification $n_a = n_+$ and $n_b = n_-$. Could one perform a further identification of these integers as non-negative conformal weights characterizing physical states so that conservation of the number theoretic momentum for a given p-adic prime would correspond to the conservation of conformal weight. In p-adic thermodynamics this conformal weight is sum of conformal weights of 5 tensor factors of Super-Virasoro algebra. The number must be indeed five and one could assign them to the factors of the symmetry group. One factor for color symmetries and two factors of electro-weak $SU(2)_L \times U(1)$ are certainly present. The remaining two factors could correspond to transversal degrees of freedom assignable to string like objects but one can imagine also other identifications [14] .

2. If this interpretation is correct, a given conformal weight $n = n_a = n_+$ (say) would correspond to all possible distributions of five conformal weights n_i, $i = 1, ..., 5$ between the n_a sheets of covering of CD satisfying $\sum_{i=1}^{5} n_i = n_a = n_+$. Single sheet of covering would carry only unit conformal weight so that one would have the analog of fractionization also now and a possible interpretation would be in terms of the instability of states with conformal weight $n > 1$. Conformal thermodynamics would also mean thermodynamics in the space of states determined by infinite primes and in the space of coverings.

3. The conformal weight assignable to the CD would naturally correspond to mass squared but there is also the conformal weight assignable to CP_2 and one can wonder what its interpretation might be. Could it correspond to the expectation of pseudo mass squared characterizing the generalized eigenstates of the modified Dirac operator? Note that one should allow in the spectrum also the powers of hyper-complex primes up to some maximum power $p^{n_{max}/2} \leq L_{max}/L_{min}$ so that Dirac determinant would be non-vanishing and Kähler function finite. From the point of conformal invariance this is indeed natural.

6 Quantum Hall effect, charge fractionization, and hierarchy of Planck constants

In this section the most recent view about the relationship between dark matter hierarchy and quantum Hall effect is discussed. This discussion leads to a more realistic view about FQHE allowing to formulate precisely the conditions under which anyons emerge, describes the fractionization of electric and magnetic charges in terms of the delicacies of the Kähler gauge potential of generalized imbedding space, and relates the TGD based model to the original model of Laughlin. The discussion allows also to sharpen the vision about the formulation of quantum TGD itself.

6.1 Quantum Hall effect

Recall first the basic facts. Quantum Hall effect (QHE) [8, 1, 6] is an essentially 2-dimensional phenomenon and occurs at the end of current carrying region for the current flowing transversally along the end of the wire in external magnetic field along the wire. For quantum Hall effect transversal Hall conductance characterizing the 2-dimensional current flow is dimensionless and quantized and given by

$$\sigma_{xy} = 2\nu\alpha_{em} \quad ,$$

ν is so called filling factor telling the number of filled Landau levels in the magnetic field. In the case of integer quantum Hall effect (IQHE) ν is integer valued. For fractional quantum Hall effect (FQHE) ν is rational number. Laughlin introduced his many-electron wave wave function predicting fractional quantum Hall effect for filling fractions $\nu = 1/m$ [6] . The further attempts to understand FQHE led to the notion of anyon by Wilzeck [8] . Anyon has been compared to a vortex like excitation of a dense 2-D electron plasma formed by the current carriers. ν is inversely proportional to the magnetic flux and the fractional filling factor can be also understood in terms of fractional magnetic flux.

The starting point of the quantum field theoretical models is the effective 2-dimensionality of the system implying that the projective representations for the permutation group of n objects are representations of braid group allowing fractional statistics. This is due to the non-trivial first homotopy group of 2-dimensional manifold containing punctures. Quantum field theoretical models allow to assign to the anyon like states also magnetic charge, fractional spin, and fractional electric charge.

Topological quantum computation [28, 9] , [9] , [1] is one of the most fascinating applications of FQHE. It relies on the notion of braids with strands representing the orbits of of anyons. The unitary time evolution operator coding for topological computation is a representation of the element of the element of braid group represented by the time evolution of the braid. It is essential that the group involved is non-Abelian so that the system remembers the order of elementary braiding operations (exchange of neighboring strands). There is experimental evidence that $\nu = 5/2$ anyons possessing fractional charge $Q = e/4$ are non-Abelian [9, 7] .

During last year I have been developing a model for DNA as topological quantum computer [9] . Therefore it is of considerable interest to find whether TGD could provide a first principle description of anyons and related phenomena. The introduction of a hierarchy of Planck constants realized in terms of generalized imbedding space with a book like structure is an excellent candidate in this respect [10] . As a rule the encounters between real world and quantum TGD have led to a more precise quantitative articulation of basic notions of quantum TGD and the same might happen also now.

6.2 A simple model for fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [1] at the level of basic quantum TGD as integer QHE for non-standard value of Planck constant.

The formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [1] .

The model of Laughlin [6] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [5] . Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

Before proposing the TGD based model of FQHE as IQHE with non-standard value of Planck constant, it is good to represent a simple explanation of IQHE effect. Choose the coordinates of the current currying slab so that x varies in the direction of Hall current and y in the direction of the main current. For IQHE the value of Hall conductivity is given by $\sigma = j_y/E_x = n_e ev/vB = n_e e/B = Ne^2/hBS = Ne^2/mh$, were m characterizes the value of magnetized flux and N is the total number of electrons in the current. In the Landau gauge $A_y = xB$ one can assume that energy eigenstates are momentum eigenstates in the direction of current and harmonic oscillator Gaussians in x-direction in which Hall current runs. This gives

$$\Psi \propto exp(iky)H_n(x+kl^2)exp(-\tfrac{(x+kl^2)^2}{2l^2}) \ , \quad l^2 = \tfrac{\hbar}{eB} \ . \tag{6.2}$$

Only the states for which the oscillator Gaussian differs considerably from zero inside slab are important so that the momentum eigenvalues are in good approximation in the range $0 \leq k \leq k_{max} = L_x/l^2$. Using $N = (L_y/2\pi)\int_0^{k_{max}} dk$ one obtains that the total number of momentum eigenstates associated with the given value of n is $N = eBdL_xL_y/h = n$. If ν Landau states are filled, the value of σ is $\sigma = \nu e^2/h$.

The interpretation of FQHE as IQHE with non standard value of Planck constant could explain also the fractionization of charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factor spaces of CP_2 and three of them can lead to the increase or at least fractionization of the Planck constant required by FQHE.

1. The prediction for the filling fraction in FQHE would be

$$\nu = \nu_0 \frac{\hbar_0}{\hbar} \quad , \quad \nu_0 = 1, 2, ... \quad . \tag{6.3}$$

ν_0 denotes the number of filled Landau levels.

2. Let us denote the options as C-C, C-F, F-C, F-F, where the first (second) letter tells whether a singular covering or factor space of CD (CP_2) is in question. The observed filling fractions are consistent with options C-C, C-F, and F-C for which CD or CP_2 or both correspond to a singular covering space. The values of ν in various cases are given by the following table.

Option	$C-C$	$C-F$	$F-C$
ν	$\frac{\nu_0}{n_a n_b}$	$\frac{\nu_0 n_b}{n_a}$	$\frac{\nu_0 n_a}{n_b}$

(6.4)

There is a complete symmetry under the exchange of CD and CP_2 as far as values of ν are considered.

3. All three options are consistent with observations. Charge fractionization allows only the options $C-C$ and $F-C$. If one believes the general arguments stating that also spin is fractionized in FQHE then only the option $C-C$, for which charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively, remains. For $C-C$ option one must allow $\nu_0 > 1$.

4. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [1, 3] . The fractionized charge is believed to be $e/4$ in the latter case [9, 7] . This requires $n_b = 4$ allowing only (C, C) and (F, C) options. $n_i \geq 3$ holds true if coverings and factor spaces are correlates for Jones inclusions and this gives additional constraint. The minimal values of (ν_0, n_a, n_b) are $(2, 1, 4)$ for $\nu = 1/2$ and $(10, 1, 4)$ for $\nu = 5/2$) for both $C-C$ and $F-C$ option. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [5] . $n_b = 2$ would be inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions implying $n_i \geq 3$.

5. A possible problematic aspect of the TGD based model is the experimental absence of even values of m except $m = 2$ (Laughlin's model predicts only odd values of m). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) both n_a and n_b must be odd. This would require that $m = 2$ case differs in some manner from the remaining cases.

6. Large values of m in $\nu = n/m$ emerge as B increases. This can be understood from flux quantization. One has $e \int B dS = n\hbar$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give for (C, C) option $e_F \int B dS = n n_a \hbar_0$. The interpretation is that each of the n_b sheets contributes one unit to the flux for e. Note that the value of magnetic field at given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

7. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires such a low temperature. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of single particle energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

6.3 Description of QHE in terms of hierarchy of Planck constants

The proportionality $\sigma_{xy} \propto \alpha_{em} \propto 1/\hbar$ suggests an explanation of FQHE [8, 1, 6] in terms of the hierarchy of Planck constants. Perhaps filling factors and magnetic fluxes are actually integer valued but the value of Planck constant defining the unit of magnetic flux is changed from its standard value - to its rational multiple in the most general case. The killer test for the hypothesis is to find whether higher order perturbative QED corrections in powers of α_{em} are reduced from those predicted by QED in QHE phase. The proposed general principle governing the transition to large $\hbar$ phase is states that Nature loves lazy theoreticians: if perturbation theory fails to converge, a phase transition increasing Planck constant occurs and guarantees the convergence. Geometrically the phase transition corresponds to the leakage of 3-surface from a given 8-D page to another one in the Big Book having singular coverings and factor spaces of $CD \times CP_2$ as pages. Only cove

The hierarchy of Planck constants strongly suggests the emergence of quantum groups and fractionalization of quantum numbers [3] . The challenge is to figure out the details and see whether this framework is consistent with what is known about FQHE. At least the following questions pop up immediately in the mind of physicist.

1. What the effective 2-dimensionality of the system exhibiting QHE corresponds in TGD framework?

2. What happens in the phase transition leading to the phase exhibiting QHE effect?

3. What are the counterparts anyons? How the fractional electric and magnetic charges emerge at classical and quantum level in the two descriptions?

The TGD inspired description of charge fractionization is based on the weak form of electric-magnetic duality and the reduction of the hierarchy of Planck constants to the basic quantum TGD. Also now one can raise a series of questions.

1. Electric magnetic duality provides a natural description of charge quantization and fractionization. The explanation for the hierarchy of Planck constants predicts that all charges- even Noether charges- are fractionized in the same manner and come as multiples of $1/n_a$ and $1/n_b$. Does this prediction make sense physically?

2. Does the singular gauge part $\Delta A = d\Phi$ of Kähler gauge potential whose exponent is n_a- (n_b-) valued function of appropriate angle coordinates of M^4 and CP_2 provide a description of charge fractionization for a given sheet of the covering associated with a given value of Planck constant? Does this description reduce to the measurement interaction term which is indeed effective gauge part added to the Kähler gauge potential of either space-time surface or of wormhole throats or ends of space-time surface.

3. The Chern-Simons action associated with the induced Kähler gauge potential is Abelian: is this consistent with the non-Abelian character of the braiding matrix?

In the following I try to summarize the basic ideas giving hopes about a coherent description of quantum Hall effect and charge and spin fractionization in TGD framework.

6.3.1 Hierarchy of Planck constants and book like structure of imbedding space

TGD leads to a description for the hierarchy of Planck constants in terms of the generalization of $CD \times CP_2$ to book like structure. To be more precise, the generalization takes place for any region $CD \times CP_2 \subset H$, where CD corresponds to a causal diamond defined as an intersection of future and past directed light-cones of M^4. CDs play key role in the formulation of quantum TGD in zero energy ontology in which the light-like boundaries of CD connected by light-like 3-surfaces can be said to be carriers of positive and negative energy parts of zero energy states. They are also crucial for TGD inspired theory of consciousness, in particular for understanding the relationship between experienced and geometric time [2] .

1. *Should one postulate the hierarchy of Planck constants separately?*

In the most general case both CD and CP_2 are replaced with a book like structure consisting of singular coverings and factor spaces associated with them. A simple geometric argument identifying the square of Planck constant as scaling factor of the covariant metric tensor of M^4 (or actually CD) leads in the most general case to the identification of Planck constant as the ratio $\hbar/\hbar_0 = x_a x_b$, where $x = n$ holds true for a singular covering of X and $x = 1/n$ holds true for a singular factor space. x is the order of the maximal cyclic subgroup of the covering/divisor group $G \subset SO(3)$. The order of G can be thus larger than n. As a consequence, the spectrum of Planck constants is in principle rational-valued. $\hbar_0$ is unique since it corresponds to the unit of rational numbers.

2. Does the hierarchy follow from the basic quantum TGD?

The proposed option is too general if one believes on the argument reducing the hierarchy of Planck constants to the basic quantum TGD. Recall that the argument goes as follows.

1. By the extreme non-linearity of the Kähler action the correspondence between the time derivatives of the imbedding space coordinates and canonical momentum densities is many-to-one. This leads naturally to the introduction of covering spaces of $CD \times CP_2$, which are singular in the sense that the sheets of the covering co-incide at the ends of CD and at wormhole throats. One can say that quantum criticality means also the instability of the 3-surfaces defined by the throats and the ends against the decay to several space-time sheets and consequent charge fractionization. The interpretation is as an instability caused by too strong density of mass and making perturbative description possible since the matter density at various branches is reduced. The nearer the vacuum extremal the system is, the lower the mass density needed to induce the instability is and the larger is the number of sheets resulting in this manner is.

2. The singular regions of the covering are regions in which the integer characterizing the multiple-valuedness of the time derivatives of the imbedding space coordinates as functions of canonical momentum densities is reduced from the maximal value. The reduction to single sheeted covering could (but need not!) take place over any Lagrangian manifold of CP_2 rather than only over a homologically trivial geodesic sphere and would thus directly correspond to the vacuum degeneracy of Kähler action. One can also imagine the reduction of the integer characterizing multivaluedness to a smaller value different from one in non-vacuum regions.

3. In M^4 degrees of freedom branching to a single sheeted covering can occur over any partonic 2-surface which does not enclose the tip of CD. In this case the Kähler gauge potential would contain a singular gauge term having an archetypal form $\Delta A = d\phi/n_a$ at say upper hemisphere so that the magnetic flux would receive a non-vanishing contribution from North pole and give rise to a fractionized Kähler magnetic and therefore also to Kähler electric charge. This term is pure gauge for all partonic 2-surface not containing the tip of CD. Thus one species of anyons would be associated with this kind of partonic 2-surfaces. Second species would correspond to singular gauge transforms about which example would be $\Delta A = d\Psi/n_b$, where Ψ is the angle coordinate associated with a homologically non-trivial geodesic sphere. The modification of the Kähler gauge potential could be interpreted in terms of a measurement interaction term added to the Dirac action and their sum at the ends would give rise to the non-fractional contribution to the measurement interaction term. This kind of term would be also associated with Noether charges such as 4-momentum. Depending on whether one consdiders the end of space-time sheet or at wormhole throat, the measurement interaction term would be given as $1/n_b$ or $1/n_a$ multiple of the measurement interaction term in absence of branching and would be more complex than the simple archetypal forms. The general form of the measurement interaction term is discussed in [11] .

4. Classically the fractional Noether charges would emerge from Chern-Simons representation of Kähler function with the Lagrangian multiplier term realizing the weak form of electric-magnetic duality as a constraint. The latter term would be responsible for the non-vanishing values of four-momentum and angular momentum. The isometry charges in CP_2 degrees of freedom would receive a contribution also from the Chern-Simons term.

5. The situation can be described mathematically either by using effectively only single sheet but an integer multiple of Planck constant or many-sheeted covering and ordinary value of Planck constant. In [10] the argument that this indeed leads to hierarchy of Planck constants including

charge fractionization is developed in detail. The restriction to singular coverings is consistent with the experimental constraints and means that only integer valued Planck constants are possible. A given value of Planck constant corresponds only to a finite number of the pages of the Big Book and that the evolution by quantum jumps is analogous to a diffusion at half-line and tends to increase the value of Planck constant.

6. The following argument would suggest a direct connection between vacuum degeneracy, coverings, and the hierarchy of infinite primes. For vacuum extremal the number of sheets is formally infinite but the sheets are in a well-define sense "passive". On the other hand, by the arguments of [11] the numbers n_a and n_b for sheets correspond to powers p^{n_a} and p^{n_b} for a prime appearing in infinite prime characterizing the partonic 3-surface and having interpretation as particle numbers. The unit infinite primes $X \pm 1$ correspond to the two basic infinite primes having interpretation as fermionic vacua are interpreted as Dirac sea: the numbers of bosons and fermions are vanishing for them. This suggests that the fermions of Dirac sea correspond to the "passive" sheets. This raises the question whether one could characterize the infinite degeneracy associated with vacuum extremals by these two infinite primes and non-vacuum extremals by infinite primes for which boson and fermion numbers are non-vanishing. The two infinite primes would correspond to CD and CP_2 degrees of freedom. They could also correspond to the space-time sheets of Euclidian and Minkowskian signature of the induced metric meeting at the wormhole throat at which the induced 4-metric is degenerate. Bose-Einstein condensate of n_i bosons $(i = a, b)$ or fermion plus $n_i - 1$ bosons would correspond to n_i sheets of covering.

   Arithmetic quantum field theory allows infinite number of conservation laws corresponding to the conservation of the number theoretic momentum $p = \sum_i n_i log(p_i)$ which forces separate conservation of each number theoreticl momentum $n_i log(p_i)$ since the logarithms of primes are linearly independent in the realm of rationals. This conservation law could correlate the partonic lines arriving in the interaction vertices and state that the total number of sheets of the covering is conserved although it can be shared by several partonic space-time sheets in the final state.

The reduction of the hierarchy of Planck constants to basic quantum TGD is of course only an interesting idea and the best strategy to proceed is to develop objections against it.

1. The branching of partonic 2-surfaces at the ends of space-time sheets and wormhole throats is analogous to the branching of the line of Feynman graph. The 3-D lines of generalized Feynman graphs indeed branch at the vertices and this leads to the basic objection against the proposed interpretation of the fractionization. Could one consider the possibility that branching corresponds to what happens in the vertices of Feynman diagrams? This cannot not seem to be the case. The point is that canonical momentum densities are identical so that also the conserved classical Noether and Kähler charges associated with various branches should be the same.

2. The value of gravitational Planck constant is enormous and one would mean enormously manyfold branching of partonic 2-surfaces of astrophysical size. Does this really make sense? Is this simply due the fact that the basic parameter GM_1M_2 characterizing the strength of gravitational interaction is much larger than unity so that perturbation theory in terms of it fails to converge and the splitting to $\hbar_{gr}/\hbar_0$ sheets guarantees that the perturbation theory at each sheet converges.

3. One can also ask whether the fractional charges can be observed directly since it seems that only the partonic 2-surfaces at the ends of the space-time sheet are observable.

4. Perhaps the most serious objection relates to the basic intuition about scaling of quantum lengths by $\hbar$ since this scaling is fundamental for all predictions in the model of quantum biology. It is not obvious why the basic quantum lengths in M^4 degrees of freedom - in particular the size scale of CD - should be scaled up by $n_a n_b$. Could this scaling up result dynamically or can one find some simple kinematic argument forcing the size scale spectrum of CDs? Kinematic argument is more plausible and indeed exists. Suppose that one can speak about plane waves $exp(inEt/\hbar_0)$, where t is proper time coordinate associated with the line connecting the tips of CD. Periodic boundary conditions at $t = T$ imply $E = n\hbar_0/2\pi T$ where T is the proper time

distance between the tips of CD. Suppose that $\hbar_0$ is replaced with its $n_a n_b$ multiple in the plane wave. As a consequence, the plane waves for sheets and for same value of E do not anymore satisfy periodic boundary conditions at $t = T$ anymore. These conditions are however satisfied for $t = n_a n_b T$.

3. Connection with quantum measurement theory

The hierarchy of Planck constants relates closely to quantum measurement theory. The selection of quantization axis implied by the gauge terms ΔA proportional to appropriate angle coordinates has a direct correlate at the level of imbedding space geometry. This means breaking of isometries of H for a given CD with preferred choice time axis (rest frame) and quantization axis of spin. For CP_2 the choice of the quantization axes of color hyper charge and isospin imply symmetry breaking $SU(3) \rightarrow U(2) \rightarrow U(1) \times U(1)$. The "world of classical worlds" (WCW) is union over all Poincare and color translates of given $CD \times CP_2$ so that these symmetries are not lost at the level of WCW although the loss can happen at the level of quantum states.

4. How the different sectors of the generalized imbedding space are glued together?

Intuitively the scaling of Planck constant scales up quantum lengths, in particular the size of CD. This looks trivial but one one must describe precisely what is involved to check internal consistency and also to understand how to model the quantum phase transitions changing Planck constant. The first manner to understand the situation is to consider CD with a fixed range of M^4 coordinates. The scaling up of the covariant Kähler metric of CD by $r^2 = (\hbar/\hbar_0)^2$ scales up the size of CD by r. Another manner to see the situation is by scaling up the linear M^4 coordinates by r for the larger CD so that M^4 metric becomes same for both CDs. The smaller CD is glued to the larger one isometrically together along $(M^2 \cap CD) \subset CD$ anywhere in the interior of the larger CD. What happens is non-trivial for the following reasons.

1. The singular coverings (and possibly also factor spaces) are different and M^4 scaling is not a symmetry of the Kähler action so that the preferred extrema in the two cases do not relate by a simple scaling. The interpretation is in terms of the coding of the radiative corrections in powers of $\hbar$ to the shape of the preferred extremals. This becomes clear from the representation of Kähler action in which M^4 coordinates have the same range for two CDs but M^4 metric differs by r^2 factor.

2. In common M^4 coordinates the M^4 gauge part A_a of CP_2 Kähler potential for the larger CD differs by a factor $1/r$ from that for the smaller CD. This guarantees the invariance of four-momentum assignable to Chern-Simons action in the phase transition changing $\hbar$. The resulting discontinuity of A_a at M^2 is analogous to a static voltage difference between the two CDs and M^2 could be seen as an analog of Josephson junction. In absence of dissipation (expected in quantum criticality) the Kähler voltage could generate oscillatory fermion, em, and Z^0 Josephson currents between the two CDs. Fermion current would flow in opposite directions for fermions and antifermions and also for quarks and leptons since Kähler gauge potential couples to quarks and leptons with opposite signs. In presence of dissipation fermionic currents would be ohmic and could force quarks and leptons and matter and antimatter to different pages of the Big Book. Quarks inside hadrons could have nonstandard value of Planck constant.

6.3.2 Measurement interaction term as gauge transform of Kähler gauge potential and description of charge fractionization in terms of singular gauge transforms

The introduction of a gauge part to the Kähler gauge potential of the imbedding space looks somewhat tricky idea. Can one really assing non-trivial physics to a mere gauge transformation? This is certainly the case if the gauge transformation is singular and induces a fractional Kähler magnetic charge and by electric-magnetic duality also a fractional Kähler electric charge. The introduction of a measurement interaction term as a formal gauge transform of the Kähler gauge potential only in Dirac Kähler action or Kähler Chern-Simons Dirac action but not both provides a second manner to achieve a non-trivial physical effect. It is good to summarize the background in more detail before continuing.

The idea about description of quantum Hall effect in terms of a gauge part of Kähler gauge potential emerged from the idea that Chern-Simons action for Kähler gauge potential (equivalently the for induced classical color gauge field proportional to the Kähler form) could define TGD as an almost topological QFT. It turned out however that Kähler action and the corresponding modified Dirac action containing also Chern-Simons boundary term with the constraint term coming from electric-magnetic duality are the fundamental actions. The general ansatz for the classical field equations based on the proportionality of Kähler current to instanton current reduces TGD to almost topological QFT with action reducing to Chern-Simons term with a Lagrangian multiplier term guaranteing the weak form of electric-magnetic duality. This term is of extreme importance since the extremals of mere Chern-Simons action would give rise to identically vanishing Kähler function and Kähler metric and WCW metric would not have any M^4 part even if one gives up the extremality condition.

The measurement interaction term which corresponds to a gauge part of the Kähler gauge potential and can be added either to the interior part of Kähler Dirac action (and Kähler action) or to the Chern-Simons Dirac action. The measurement interaction term therefore modifies the physics and is visible also in the classical dynamics by the proportionality of Kähler current to instanton current. Note that the modification of Chern-Simons term assigned to the ends of the space-time sheet and to wormhole throats affects the space-time sheet since the Kähler action changes.

For Noether charges the Lagrangian multiplier term guaranteing the weak form of electric magnetic duality in Chern-Simons action gives rise to non-vanishing Noether charges also in M^4 degrees of freedom. The proposed view about the basic process behind the charge fractionization implies that all charges are fractionized in basically the same manner although it seems that M^4 charges are n_b multiples and CP_2 charges n_a multiples of $1/n_a n_b$. Also in this case the additional of a formal gauge term would realize the fractionization at the level of couplings and total anomalous coupling would correspond to a non-singular gauge transformation of A.

One can imagine several kinds of pseudo gauge transformations appearing in the measurement interaction term.

1. The first kind of gauge transformation corresponds to a gauge change for A_μ with no reference to the fact that it is a projection of CP_2 Kähler gauge potential. It is not clear whether measurement interaction could be induced also by this kind of gauge transform. In any case, the proposed form of measurement interaction coan be interpreted in terms of a gauge transform at the level of imbedding space [11]

2. Second kind of gauge transformations are induced by the symplectic transformations of $\delta M^4_\pm \times CP_2$ and in general affect the induced metric and thus the gravitational properties of the system in the case of non-vacuum extremals. Furthermore, there exist no symplectic transformation allowing to eliminate the "gauge part" of A in $M^2 \subset M^4$ or gauge part in $CD\backslash M^2$ or $CP_2\backslash S^2$ if it corresponds to a scalar function which is discontinuous. $\Delta A_\phi = k\phi$, $k \neq n$, where ϕ is an angle variable in M^4 or CP_2 would represent a canonical example of this.

3. Third kind of gauge transform would characterize the pages of the Big Book and give rise to fractional Kähler magnetic charge and by definition would not be reducible to a gauge transform induced by a symplectic transformation. This raises the idea that the gauge parts of A in CD and CP_2 could characterize the pages of the Big Book andthus the charge fractionization. In particular in the case of coverings one might argue that ΔA must be pure gauge in the covering implying $k = m/n_a$ or $k = m/n_b$.

The simplest hypothesis is that the ordinary measurement interaction term for trivial covering is simply scaled down by $1/n_a n_b$ in the interior of the space-time sheet and by $1/n_b$ or $1/n_b$ at its ends and at throats where n_b or n_a sheets co-incide. With this interpretation ΔA would provide a description of physics at a particular sheet of covering and there would be no need to introduce anything new at the level of imbedding space geometry since the coverings of the imbedding space would provide only a formal tool to describe the situation caused by the extreme non-linearity of the Kähler action.

6.4 In what kind of situations do anyons emerge?

Charge fractionization is a fundamental piece of quantum TGD and should be extremely general phenomenon and the basic characteristic of dark matter known to contribute 95 per cent to the matter of Universe.

1. In TGD framework scaling $\hbar = m\hbar_0$ implies the scaling of the unit of angular momentum for m-fold covering of CD only if the many particle state is Z_m singlet. Z_m singletness for many particle states allows of course non-singletness for single particle states. For factor spaces of CD -if present- the scaling $\hbar \rightarrow \hbar/m$ is compensated by the scaling $l \rightarrow ml$ for $L_z = l\hbar$ guaranteing invariance under rotations by multiples of $2\pi/m$. Again one can pose the invariance condition on many-particle states but not to individual particles so that genuine physical effect is in question.

2. There is analogy with Z_3-singletness holding true for many quark states and one cannot completely exclude the possibility that quarks are actually fractionally charged leptons with $m = 3$-covering of CP_2 reducing the value of Planck constant [30, 10] so that quarks would be anyonic dark matter with smaller Planck constant and the impossibility to observe quarks directly would reduce to the impossibility for them to exist at our space-time sheet. Confinement would in this picture relate to the fractionization requiring that the 2-surface associated with quark must surround the tip of CD. Whether this option really works remains an open question. In any case, TGD anyons are quite generally confined around the tip of CD.

3. The model of DNA as topological quantum computer [9] assumes that DNA nucleotides are connected by magnetic flux tubes to the lipids of the cell membrane. In this case, p-adically scaled down u and d quarks and their antiquarks are assumed to be associated with the ends of the flux tubes and provide a representation of DNA nucleotides. Quantum Hall states would be associated with partonic 2-surfaces assignable to the lipid layers of the cell and nuclear membranes and also endoplasmic reticulum filling the cell interior and making it macroscopic quantum system and explaining also its stability. The entire system formed in this manner would be single extremely complex anyonic surface and the coherent behavior of living system would result from the fusion of anyonic 2-surfaces associated with cells to larger anyonic surfaces giving rise to organs and organisms and maybe even larger macroscopically quantum coherent connected systems. An interesting possibility is that the ends of the flux tubes assumed to connect DNA nucleotides to lipids of various membranes carry instead of u, d and their antiquarks fractionally charged electrons and neutrinos and their anti-particles having $n_b = 3$ and large value of n_a.

In astrophysical scales gigantic values of Planck constants would be realized meaning coverings with huge number of sheets. This conforms with the fact that for vacuum extremals the coverings would be formally infinitely many sheeted.

1. Quite generally, one would expect that dark matter and its anyonic forms emerge in situations where the density of plasma like state of matter is very high so that N-fold cover of CD reduces the density of matter by $1/N$ factor at given sheet of covering and thus also the repulsive Coulomb energy. Plasma state resulting in QHE is one example of this. The interiors of neutron stars and black hole like structures are extreme examples of this, and I have proposed that black holes are dark matter with a gigantic value of gravitational Planck constant implying that black hole entropy -which is proportional to $1/\hbar$ - is of the same order of magnitude and even smaller as the entropy assignable to the spin of elementary particle. If the covering results from the basic quantum TGD this entropy would characterize single sheet of the covering only. The fact that there are $n_a n_b$ sheets would mean that the total entropy has just the standard value! Could this mean that entropy is the critical contrple parameter which splits the 3-surface into parallel sheets?

2. The confinement of matter inside black hole could have interpretation in terms of macroscopic anyonic 2-surfaces containing the topologically condensed elementary particles. This conforms with the TGD inspired model for the final state of star [27] inspiring the conjecture that even ordinary stars could possess onion like structure with thin layers with radii given by p-adic length scale hypothesis.

3. The idea about hierarchy of Planck constants was inspired by the finding that planetary orbits can be regarded as Bohr orbits [10] , [1] : the explanation was that visible matter has condensed around dark matter at spherical cells or tubular structures around planetary orbits. This led to the proposal that planetary system has formed through this kind of condensation process around spherical shells or flux tubes surrounding planetary orbits and containing dark matter.

The question why dark matter would concentrate around flux tubes surrounding planetary orbits was not answered. The answer could be that dark matter is anyonic matter at partonic 2-surfaces whose light-like orbits define the basic geometric objects of quantum TGD. These partonic 2-surfaces could contain a central spherical anyonic 2-surface connected by radial flux tubes to flux tubes surrounding the orbits of planets and other massive objects of solar system to form connected anyonic surfaces analogous to elementary particles.

4. If factor spaces appear in M^4 degrees of freedom, they give rise to $Z_n \subset G_a$ symmetries. In astrophysical systems the large value of $\hbar$ necessarily requires a large value of n_a for CD coverings as the considerations of [18] - in particular the model for graviton dark graviton emission and detection - forces to conclude. The same conclusion follows also from the absence of evidence for exact orbifold type symmetries in M^4 degrees of freedom for dark matter in astrophysical scales.

Coverings alone are enough to produce rational number valued spectrum for $\hbar$ consistent with the observed spectrum of ν, and one must keep in mind that the applications of theory do not allow to decide whether singular factor spaces are really needed and that the reduction of the hierarchy of Planck constants to basic quantum TGD for coverings disfavors the factor spaces. The possibility to interpret evolution in terms of the increase of Planck constant also favors coverings-only option.

6.5 What happens in FQHE?

This picture suggest following description for what would happen in QHE in TGD Universe accepting the C-C option implied by the basic quantum TGD.

1. Light-like 3-surfaces - locally random light-like orbits of partonic 2-surfaces- are identifiable as very tiny wormhole throats in the case of elementary particles. This is the case for electrons in particular. Partonic surfaces can be also large, even macroscopic, and the size scales up in the scaling of Planck constant. To avoid confusion, it must be emphasized that light-likeness is with respect to the induced metric and does not imply expansion with light velocity in Minkowski space since the contribution to the induced metric implying light-likeness typically comes from CP_2 degrees of freedom. Strong classical gravitational fields are present near the wormhole throats. Second important point is that regions of space-time surface with Euclidian signature of the induced metric are implied: CP_2 type extremals representing elementary particles and having light-like random curve as CP_2 projection represents basic example of this. Hence rather exotic gravitational physics is predicted to manifest itself in everyday length scales.

2. The simplest identification for what happens in the phase transition to quantum Hall phase is that the end of wire carrying the Hall current corresponds to a partonic 2-surface having a macroscopic size. The electrons in the current correspond to similar 2-surfaces but with size of elementary particle for the ordinary value of Planck constant. As the electrons meet the end of the wire, the tiny wormhole throats of electrons suffer topological condensation to the boundary. One can say that one very large elementary particle having very high electron number is formed.

3. Fractionization occurs for charges in CP_2 degrees of freedom with unit $1/n_a$. If the end of the wire forms part of a spherical surface surrounding the tip of the CD involved fractionization occurs also in CD degrees of freedom so that electrons can become carriers of anomalous electric and magnetic charges. If not then the total spin is n_a multiple of fundamental spin unit.

One of the basic question was whether it is possible to describe non-Abelian FQHE in TGD framework.

1. Chern- Simons action for Kähler gauge potential is Abelian. This raises the question whether the representations of the number theoretical braid group are also Abelian. Since there is evidence for non-Abelian anyons, one might argue that this means a failure of the proposed approach. There are however may reasons to expect that braid group representations are non-Abelian. The action is for induced Kähler form rather than primary Maxwell field, $U(1)$ gauge symmetry is transformed to a dynamical symmetry (symplectic transformations of CP_2 representing isometries of WCW and definitely non-Abelian), and the particles of the theory belong to the

representations of electro-weak and color gauge groups naturally defining the representations of braid group.

2. The finite subgroups of $SU(2)$ defining covering and factor groups are in the general case non-commutative subgroups of $SU(2)$ since the hierarchies of coverings and factors spaces are assumed to correspond to the two hierarchy of Jones inclusions to which one can assign ADE Lie algebras by McKay correspondence. The ADE Lie algebras define effective gauge symmetries having interpretation in terms of finite measurement resolution described in terms of Jones inclusion so that extremely rich structures are expected. The question arises whether the covering option implied by the basic quantum TGD allows coverings defined by finite groups. There seems to be no obvious reason why this could not be the case.

An interesting challenge is to relate concrete models of FQHE to the proposed description. Here only some comments about Laughlin's wave function are made.

1. In the description provided by Lauglin wave function FQHE results from a minimization of Coulomb energy. In TGD framework the tunneling to the page of H with m sheets of covering has the same effect since the density of electrons is reduced by $1/m$ factor.

2. The formula $\nu \propto e^2 N_e/e \int BdS$ with scaling up of magnetic flux by $\hbar/\hbar_0 = m$ implies effective fractional filling factor. The scaling up of magnetic flux results from the presence of m sheets carrying magnetic field with same strength. Since the N_e electrons are shared between m sheets, the filling factor is fractional when one restricts the consideration to single sheet as one indeed does.

3. Laughlin wave function makes sense for $\nu = 1/m$, m odd, and is m:th power of the many electron wave function for IQHE and expressible as the product $\prod_{i<j}(z_i - z_j)^m$, where z represents complex coordinate for the anyonic plane. The relative orbital angular momenta of electrons satisfy $L_z \geq m$ if the value of Planck constant is standard. If Laughlin wave function makes sense also in TGD framework, then m:th power implies that many-electron wave function is singlet with respect to Z_m acting in covering and the value of relative angular momentum indeed satisfies $L_z \geq m\hbar_0$ just as in Laughlin's theory.

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