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Experimental Determination of Angular Momentum Flux Density in a Circularly Polarized Light Beam

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Abstract: An interference experiment is proposed to answer a troubling question, how is angular momentum distributed over a circularly polarized light beam with plane phase front.

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A circularly polarized light beam carries an angular momentum [1,2]. A torque acts on a body, which absorbs the beam or/and changes the polarization of the beam. However, a troubling question exists: what is the distribution of this angular momentum over the beam section? Can we use a concept of an angular momentum flux density as well as we use an energy flux density or linear momentum flux density?

According to a common opinion [3,4], the angular momentum flux is localized on the surface of the beam where the **E** and **B** fields have components parallel to the wave vector (the field lines are closed loops) and where the energy flow has components perpendicular to the wave vector. So, in the surface layer of the beam, the net energy flow is helical, and the circulating energy flow implies the existence of angular momentum, whose direction is along the direction of propagation (*z*-direction). Accordingly, T^{xz}, T^{yz}, or $T^{\phi z}$-components of the Maxwell stress tensor are nonzero only at the edge of the alight zone of the body, and the central part of the beam carries no angular momentum as well as a plane wave. If the body is fastened by its periphery, the torque must be balanced with a torque, which acts on the body's periphery from supports. Thus, the outer part of the body experiences shear stress, and the inner part has no stress (besides that from light pressure).

Contrary to this concept, R. Feynman explains the beginning of a torque acting on the body in another manner [5]. He write, "The electric vector **E** goes in a circle – as drawn in Fig. 17-5(a). Now suppose that such a light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics. We'll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement **r** from its equilibrium position at the origin and goes around with some phase lag with respect to the vector **E.** As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now look, there is angular momentum being poured into this electron, because there is always a torque about the origin." So, according to Feynman, the central part of the body receives a distribution of a density of the angular momentum. This inference is confirmed by the concept of photons. Photons are absorbed by the central part uniformly and bring energy $\hbar\omega$, momentum $\hbar\omega/c$, and angular momentum $\hbar$ per photon. This entails a uniform shear stress in the central part of the body [6,7], and the origin of the stress cannot be expressed in terms of Maxwell stress tensor.

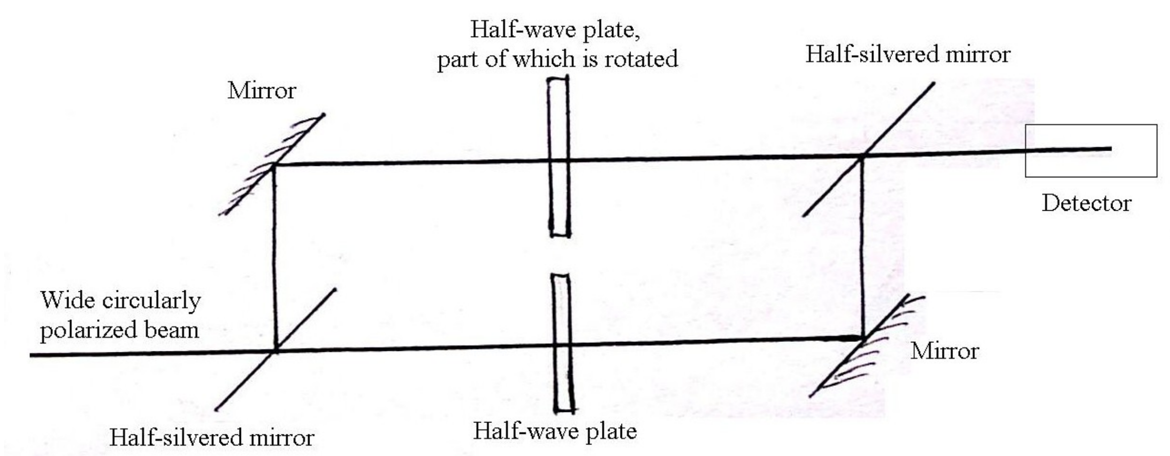
This ambiguity gives rise to a question [8]. Suppose that our body is divided concentrically into outer and inner parts. Will the inner part perceive a torque (and rotate)?

Allen et al. answered this question [9]. They divide the beam into coaxial pieces in their mind: into an inner cilindric part and outer part, which looks like a thick-wall tube. The authors affirm that two equal, but opposite, torques act on the body near the boundary of the division. I think this is not correct. Such pieces of a beam cannot be considered at all because they do not satisfy the Maxwell equations [6,7]. Now we propose an experimental method to measure the torque acting on the central part of the alight zone.

Let the beam pass through a half-wave plate, which reverses the handedness of the circular polarization so that the plate experiences a torque. If the plate rotates in its own plane, work will be done. This (positive or negative) amount of work must reappear as an alteration in the energy of the photons, i.e., in the frequency of the light, which will result in moving interference fringes in any suitable interference experiment [10].

We propose to place two half-wave plates in the paths of the beams in an interferometer (Mach-Zehnder, for example), but for all that, one of the plates is divided into an inner disc and a closely fitting outer annulus so that one can rotate the inner disc of the plate independently of the annulus. It is expedient to use a thin slab mode of operation of the interferometer. In this case, one can see moving interference rings at the alight zone of the half-wave plate.

Let $N = T^{0z}$ denotes the energy flux density, i.e. the component of Poynting vector, and μ denotes the angular momentum flux density, i.e., density of torque acting on points of the half-wave plate. Because energy and spin of a quantum are $\hbar\omega$ and $\hbar$, we have $\mu = 2N/\omega$. Thus a rotation of a part of the half-wave plate with the angular frequency Ω yields the alteration $\Delta N = \mu\Omega = 2N\Omega/\omega$ and the light angular frequency alteration $\Delta\omega = \Delta N\,\omega/N = 2\Omega$. Coresponding phase shift in time t is $\varphi = 2\Omega t$. The phase shift per revolution ($\Omega t = 2\pi$) is 4π, and the fringes shift is two per revolution.



A positive result of the experiment will mean the standard electrodynamics is not complete because there is no expression for a spin flux density in the frame of the electrodynamics. Such an expression, i.e. the electrodynamics' spin tensor

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]},$$

is offered in [6,7,11-13] as an addition to the electrodynamics, instead of the invalid canonical spin tensor, which is eliminated by the Belinfante-Rosenfeld procedure in the frame of the standard Lagrange formalism [6,7]. Here A^λ and Π^λ are the magnetic and electric vector potentials which satisfy

$$\partial_\lambda A^\lambda = \partial_\lambda \Pi^\lambda = 0\,, \quad 2\partial_{[\mu} A_{\nu]} = F_{\mu\nu}\,, \quad 2\partial_{[\mu} \Pi_{\nu]} = -e_{\mu\nu\alpha\beta} F^{\alpha\beta}\,,$$

where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field; $e_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor density. It is evident that the conservation law, $\partial_\nu \Upsilon^{\lambda\mu\nu} = 0$, is held for a free field. Note, the expression for spin tensor gives a physical sense to the vector potentials of Lorentz gauge.

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