About the Nature of Time

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October 10, 2008

Abstract

The identification of the experienced time t_e and geometric time t_g involves well-known problems. Physicist is troubled by the reversibility of t_g contra irreversibility of t_e, by the conflict between determinism of Schrödinger equation and the non-determinism of state function reduction, and by the poorly understood the origin of the arrow of t_g. In biology the second law of thermodynamics might be violated in its standard form for short time intervals. Neuroscientist knows that the moment of sensory experience has a finite duration, does not understand what memories really are, and is bothered by the Libet's puzzling finding that neural activity seems to precede conscious decision. These problems are discussed in the framework of Topological Geometrodynamics (TGD) and TGD inspired theory of consciousness constructed as a generalization of quantum measurement theory. In TGD space-times are regarded as 4-dimensional surfaces of 8-dimensional space-time $H = M^4 \times CP_2$ and obey classical field equations. The basic notions of consciousness theory are quantum jump and self. Subjective time is identified as a sequence of quantum jumps. Self has as a geometric correlate a fixed volume of H- "causal diamond"-defining the perceptive field of self. Quantum states are regarded as quantum superpositions of space-time surfaces of H and by quantum classical correspondence assumed to shift towards the geometric past of H quantum jump by quantum jump. This creates the illusion that perceiver moves to the direction of the geometric future. Self is curious about the geometric future and induces the shift bringing it to its perceptive field. Macroscopic quantum coherence and the identification of space-times as surfaces in H play a crucial role in this picture allowing to understand also other problematic aspects in the relationship between experienced and geometric time.

1 Introduction

The notion of time remains one of the most problematic concepts of physics. In classical physics the different properties of the time of Newton's equations and thermodynamical time are puzzling. In special relativity and general relativity the notion of simultaneity becomes a problematic concept and challenges the naive Newtonian view about time flow as a motion of 3-D time=constant snapshot of 4-D space-time. The replacement of time=constant 3-surface with past directed light-cone assignable to the world-line of observer resolves this problem. In general relativity the problem is that past light-cones need make sense only locally. In quantum measurement theory the localization of the state function reduction process into a finite space-time volume is in conflict with the determinism of Schrödinger equation. In biology the presence of self-organization processes like self assembly challenge second law of thermodynamics in short time scales. In neuroscience the finding of Libet suggesting that neural activity seems to precede conscious decision forces to give up the notion of free will or the naive identification of experienced and geometrical time.

In this essay I will consider a new view about time based on Topological Geometrodynamics [13], which can be regarded as an attempt to unify fundamental interactions assuming that space-times are representable as 4-dimensional surfaces of certain higher-dimensional space-time $H = M^4 \times CP_2$ (M^4 denotes 4-D Minkowski space and CP_2 complex projective space of 2 complex dimensions) fixed by

the requirement that the theory explains standard model symmetries and provides a geometrization of classical gauge fields and gravitational fields.

The construction of quantum TGD leads to a radical revision of space-time concept (many-sheeted space-time and topological field quantization), and forces also to generalize the original view about imbedding space. p-Adic physics as physics of cognition and intentionality is part of TGD inspired theory of consciousness and the need to fuse real and p-adic physics to single coherent whole forces to revise the notions of number and space-time. It has been also necessary to replace the standard positive energy ontology with what I call zero energy ontology. These generalizations are of special importance in TGD inspired theory of consciousness and of quantum biology.

There are several first principle approaches to quantum TGD and following gives only a very concise summary of them.

1. Generalization of Einstein's program of geometrizing classical physics so that quantum theory can be seen as a theory of *classical* spinor fields in the world of classical worlds (WCW) consisting of light-like 3-surfaces and possessing Kähler geometry [4, 5, 6]. By general coordinate classical physics becomes an exact part of quantum theory in a well-defined sense. A geometrization of Fermi statistics is obtained, and the Clifford algebra associated with the spinors of WCW can be regarded as a direct sum of von Neumann algebras known as hyper-finite factors of type II_1 (HFFs) closely related to quantum groups and non-commutative geometry;

2. Quantum TGD as almost topological field theory (TQFT) with fundamental objects identified as light-like 3-surfaces and having generalized super-conformal symmetries as symmetries [7, 8]: the notion of braid is the basic building block of this approach;

3. There are two kinds of conformal symmetries corresponding to the boundary of light-cone of Minkowski space and light-like 3-surfaces, and these symmetries alone dictate to high degree the physics. Quite recently it turned that also a symplectic analog of conformal field theory emerges naturally in TGD framework (super-canonical symmetries) and this led to a concrete proposal for how to construct n-point functions needed to calculate M-matrix [13];

4. Physics as a generalized number theory involving three different threads corresponding to need fuse real and various p-adic physics to single coherent whole by using a generalization of number concept obtained by gluing reals and various p-adic number fields and their extensions together along rationals and common algebraics [10]; the observation that standard model symmetries and dynamics of quantum and classical TGD are to high degree dictated by classical number fields [11]; and the ideas inspired by the notion of infinite prime [12];

5. The identification of WCW Clifford algebra elements as hyper-octonion (subspace of complexified octonions spanned by real unit and octonionic imaginary units multiplied by the commuting additional imaginary unit) valued conformal fields having values in HFF provides a justification for the concept of number theoretic braid needed both in the fusion of real and p-adic physics and in TGD as almost TQFT approach;

6. The hierarchy of Planck constants realizing quantum criticality [9] forces a generalization of the notion of imbedding space by replacing it with a book like structure having as its pages singular coverings and factor spaces of H and allowing to realize geometric correlates for the choice of quantization axis in quantum measurement: the particles at different pages of this book are "relatively dark" since they do not possess local interaction vertices which means a radically new manner to interpret dark matter;

7. Zero energy ontology and the notion of finite measurement resolution formulated in terms of inclusions of HFFs fix quantum dynamics highly in terms of Connes tensor product allowing to interpret quantum theory as a square root of thermodynamics [13, 8]: finite measurement resolution has number theoretic braid as its space-time correlate so that various approaches to TGD are closely related;

8. Quantum theory of consciousness as a generalization of quantum measurement theory to include observer to the theory [14].

The notion of number theoretic braid is a common denominator of various approaches to quantum TGD and leads to effective discretization of the imbedding space which is however due to the finite measurement resolution and number theoretic constraints rather than being something fundamental so that there are no problems with standard conservation laws.

The article series about TGD and its applications to biology and consciousness [13, 14, 15, 16, 17, 18, 19, 20] gives an overall view about quantum TGD. In the following I will concentrate only on the aspects of quantum TGD relevant for the notion of time. I will first describe zero energy ontology and p-adicization program and after that consider the problem of time.

2 The most recent vision about zero energy ontology and p-adicization

The generalization of the number concept obtained by fusing real and p-adics along rationals and common algebraics is the basic philosophy behind p-adicization. One must be able to speak about rational points common to real and various p-adic variants of H. The basic objection is the necessity to fix some special coordinates in turn implying the loss of a manifest general coordinate invariance. The isometries of the imbedding space could save the situation provided one can identify some special coordinate system in which isometry group reduces to its discrete subgroup. The loss of the full isometry group could be compensated by assuming that WCW is union over sub-WCWs obtained by applying isometries on basic sub-WCW with discrete subgroup of isometries.

The combination of zero energy ontology realized in terms of a hierarchy of causal diamonds (CDs) and hierarchy of Planck constants providing a description of dark matter and leading to a generalization of the notion of imbedding space suggests that it is possible to realize this dream. The article [13] provides a brief summary about recent state of quantum TGD helping to understand the big picture behind the following considerations.

2.1 Zero energy ontology briefly

1. The basic construct in the zero energy ontology is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4. In zero energy ontology physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.

2. CDs form a fractal hierarchy and one can glue smaller CDs within larger CDs. This construction recipe when combined with TGD inspired theory of consciousness allows to understand the asymmetry between positive and negative energies and why the arrow of experienced time corresponds to the arrow of geometric time and why the contents of sensory experience is located to so narrow interval of geometric time. One can imagine evolution to occur as quantum leaps in which the size of the largest CD in the hierarchy of personal CDs increases in such a manner that it becomes sub-CD of a larger CD. p-Adic length scale hypothesis [13] follows if the values of temporal distance T between tips of CD come in powers of 2^n: $T = 2^n T_0$. All conserved quantum numbers for zero energy states have vanishing net values. The interpretation of zero energy states in the framework of positive energy ontology is as physical events, say scattering events with positive and negative energy parts of the state interpreted as initial and final states of the event.

3. In the realization of the hierarchy of Planck constants $CD \times CP_2$ is replaced with a Cartesian product of book like structures formed by almost copies of CDs and CP_2s defined by singular coverings and factors spaces of CD and CP_2 with singularities corresponding to intersection $M^2 \cap CD$ and homologically trivial geodesic sphere S^2 of CP_2 for which the induced Kähler form vanishes. The

coverings and factor spaces of CDs are glued together along common $M^2 \cap CD$. The coverings and factors spaces of CP_2 are glued together along common homologically non-trivial geodesic sphere S^2. The choice of preferred M^2 as subspace of tangent space of X^4 at all its points and interpreted as space of non-physical polarizations, brings M^2 into the theory also in different manner. S^2 in turn defines a subspace of the much larger space of vacuum extremals as surfaces inside $M^4 \times S^2$.

4. Configuration space (the world of classical worlds, WCW) decomposes into a union of sub-WCWs corresponding to different choices of M^2 and S^2 and also to different choices of the quantization axes of spin and energy, color isospin and hyper-charge for each choice of this kind. This means breaking down of the isometries to a subgroup. This can be compensated by the fact that the union can be taken over the different choices of this subgroup.

5. p-Adicization requires a further breakdown to discrete subgroups of the resulting sub-groups of the isometry groups but again a union over sub-WCWs corresponding to different choices of the discrete subgroup can be assumed. Discretization relates also naturally to the notion of number theoretic braid.

Is it really possible to speak about zero energy states for a given sector defined by generalized imbedding space with fixed M^2 and S^2? Classically this is possible and conserved quantities are well defined. In quantal situation the presence of the light-cone boundaries breaks full Poincare invariance although the infinitesimal version of this invariance is preserved. Note that the basic dynamical objects are 3-D light-like "legs" of the generalized Feynman diagrams glued together along their ends at generalized vertices.

2.2 Definition of energy in zero energy ontology

The approach relying on the two super conformal structures of quantum TGD gives hopes of defining the notion of energy for positive and negative energy parts of the state.

1. Since light-like 3-surfaces assignable to incoming and outgoing legs of the generalized Feynman diagrams are the basic objects, one can hope of having enough translational invariance to define the notion of energy. If translations are restricted to time-like translations acting in the direction of the future (past) then one has local translation invariance of dynamics for classical field equations inside $\delta M_{\pm}^4$ as a kind of semigroup. Also the M^4 translations leading to interior of X^4 from the light-like 2-surfaces surfaces act as translations. Classically these restrictions correspond to non-tachyonic momenta defining the allowed directions of translations realizable as particle motions. These two kinds of translations have been assigned to super-canonical conformal symmetries at $\delta M_{\pm}^4 \times CP_2$ and and super Kac-Moody type conformal symmetries at light-like 3-surfaces. Equivalence Principle in TGD framework states that these two conformal symmetries define a structure completely analogous to a coset representation of conformal algebras so that the four-momenta associated with the two representations are identical [7].

2. The condition selecting preferred extremals of Kähler action is induced by a global selection of $M^2 \subset M^4$ as a plane belonging to the tangent space of X^4 at all its points [7] and interpreted as a plane of nonphysical polarizations so that direct connection with number theory and gauge symmetries emerges. The M^4 translations of X^4 as a whole in general respect the form of this condition in the interior. Furthermore, if M^4 translations are restricted to M^2, also the condition itself - rather than only its general form - is respected. This observation, the earlier experience with p-adic mass calculations, and also the treatment of quarks and gluons in QCD encourage to consider the possibility that translational invariance should be restricted to M^2 translations so that mass squared, longitudinal momentum and transversal mass squared would be well defined quantum numbers. This would be enough to realize zero energy ontology. Encouragingly, M^2 appears also in the generalization of the causal diamond to a book-like structure forced by the realization of the hierarchy of Planck constant at the level of the imbedding space.

4

3. That the cm degrees of freedom for CD would be gauge like degrees of freedom sounds strange. The paradoxical feeling disappears as one realizes that this is not the case for sub-CDs, which indeed can have non-trivial correlation functions with either upper or lower tip of the CD playing a role analogous to that of an argument of n-point function in QFT description. One can also say that largest CD in the hierarchy defines infrared cutoff.

2.3   p-Adic variants of the imbedding space

The need to fuse p-adic physics with TGD emerged originally from the discovery that p-adic mass calculations based on p-adic thermodynamics give excellent predictions for elementary particle masses if one assumes p-adic length scale hypothesis stating that primes near integer powers of 2 are physically favored [13]. Later came the interpretation of p-adic physics as cognition cognition and intentionality. The following somewhat technical construction of p-adic variants of the imbedding space provides new insights concerning the understanding of the arrow of geometric time.

1. Rational values of p-adic coordinates are non-negative so that light-cone proper time $a_{4,+} = \sqrt{t^2 - z^2 - x^2 - y^2}$ is the unique Lorentz invariant choice for the p-adic time coordinate near the lower tip of CD. For the upper tip the identification of a_4 would be $a_{4,-} = \sqrt{(t - T)^2 - z^2 - x^2 - y^2}$. In the p-adic context the simultaneous existence of both square roots poses additional conditions on T. For 2-adic numbers $T = 2^n T_0$, $n \geq 0$ (or more generally $T = \sum_{k \geq n_0} b_k 2^k$), would allow to satisfy these conditions, which would be one additional reason for $T = 2^n T_0$ implying p-adic length scale hypothesis. The remaining coordinates of CD are naturally (hyperbolic) cosines and sines of the spherical coordinates θ and ϕ (hyperbolic angle $\eta_{\pm,4}$).

2. The existence of the preferred plane M^2 of un-physical polarizations would suggest that 2-D light-cone proper times $a_{2,+} = \sqrt{t^2 - z^2}$ $a_{2,-} = \sqrt{(t - T)^2 - z^2}$ can be also considered. The remaining coordinates would be naturally $\eta_{\pm,2}$ and cylindrical coordinates (ρ, ϕ).

3. The p-adically transcendental values of a_4 and a_2 are literally infinite as real numbers and could be visualized as points in infinitely distant geometric future so that the arrow of time might be said to emerge number theoretically.

4. The selection of the preferred quantization axes of energy and angular momentum unique apart from a Lorentz transformation of M^2 would have purely number theoretic meaning in both cases. One must allow a union over sub-WCWs labeled by points of $SO(1,1)$. This suggests a deep connection between number theory, quantum theory, quantum measurement theory, and even quantum theory of mathematical consciousness.

5. In the case of CP_2 there are three real coordinate patches involved [21]. The compactness of CP_2 allows to use cosines and sines of the preferred angle variable for a given coordinate patch.

$$
\begin{aligned}
\xi^1 &= tan(u)exp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\
\xi^2 &= tan(u)exp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ .
\end{aligned}
\tag{1}
$$

The ranges of the variables u, Θ, Φ, Ψ are $[0, \pi/2], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively. Note that u has naturally only positive values in the allowed range. S^2 corresponds to the values $\Phi = \Psi = 0$ of the angle coordinates.

6. The rational values of the (hyperbolic) cosine and sine correspond to Pythagorean triangles having sides of integer length and thus satisfying $m^2 = n^2 + r^2$ ($m^2 = n^2 - r^2$). These conditions are equivalent and allow the well-known explicit solution [22]. One can construct a p-adic completion for the set of Pythagorean triangles by allowing p-adic integers which are infinite as real integers

5

as solutions of the conditions $m^2 = r^2 \pm s^2$. These angles correspond to genuinely p-adic directions having no real counterpart. Hence one obtains p-adic continuum also in the angle degrees of freedom. Algebraic extensions of the p-adic numbers bringing in cosines and sines of the angles π/n lead to a hierarchy increasingly refined algebraic extensions of generalized imbedding space. Since the different sectors of WCW directly serve as correlates of selves, this means a direct correlation with the evolution of the mathematical consciousness. Trigonometric identities allow to construct points which in the real context correspond to sums and differences of angles.

7. Negative rational values of the cosines and sines correspond as p-adic integers to infinite real numbers and it seems that one use several coordinate patches obtained as copies of the octant ($x \geq 0, y \geq 0, z \geq 0,$). An analogous picture applies in CP_2 degrees of freedom.

3 Zero energy ontology, self hierarchy, and the notion of time

Consider now the formulation of TGD inspired quantum theory of consciousness [14] and quantum biology [15] in terms of zero energy ontology.

One should understand the asymmetry between positive and negative energies and between two directions of geometric time at the level of conscious experience, the correspondence between experienced and geometric time, and the emergence of the arrow of time. One should explain why human sensory experience is about a rather narrow time interval of about .1 seconds and why memories are about the interior of much larger CD with time scale of order life time. One should have a vision about the evolution of consciousness: how quantum leaps leading to an expansion of consciousness occur.

Negative energy signals to geometric past - about which phase conjugate laser light represents an example - provide an attractive tool to realize intentional action as a signal inducing neural activities in the geometric past (this would explain Libet's classical findings), a mechanism of remote metabolism, and the mechanism of declarative memory as communications with geometric past. One should understand how these signals are realized in zero energy ontology and why their occurrence is so rare.

In the following I try to demonstrate that TGD inspired theory of consciousness and quantum TGD proper indeed are in tune.

3.1 Causal diamonds as correlates for selves

Quantum jump as a moment of consciousness, self as a sequence of quantum jumps integrating to self, and self hierarchy with sub-selves experienced as mental images, are the basic notions of TGD inspired theory of consciousness. In the most ambitious vision self hierarchy reduces to a fractal hierarchy of quantum jumps within quantum jumps.

It is natural to interpret CDs as correlates of selves. CDs can be interpreted either as subsets of the generalized imbedding space or as sectors of WCW. Accordingly, selves correspond to CDs of the generalized imbedding space or sectors of WCW, literally separate interacting quantum Universes. The spiritually oriented reader might speak of Gods. Sub-selves correspond to sub-CDs geometrically. The contents of consciousness of self is about the interior of the corresponding CD at the level of imbedding space. For sub-selves the wave function for the position of tip of CD brings in the delocalization of sub-WCW.

The fractal hierarchy of CDs within CDs is the geometric counterpart for the hierarchy of selves: the quantization of the time scale of planned action and memory as $T(k) = 2^k T_0$ suggest an interpretation for the fact that we experience octaves as equivalent in music experience.

3.2 Why sensory experience is about so short time interval?

CD picture implies automatically the 4-D character of conscious experience and memories form part of conscious experience even at elementary particle level. Amazingly, the secondary p-adic time scale of electron is $T = 0.1$ seconds defining a fundamental time scale in living matter. The problem is to understand why the sensory experience is about a short time interval of geometric time rather than

about the entire personal CD with temporal size of order life-time. The explanation would be that sensory input corresponds to subselves (mental images) with $T \simeq .1$ s at the upper light-like boundary of CD in question. This requires a strong asymmetry between upper and lower light-like boundaries of CDs.

The localization of the contents of the sensory experience to the upper light-cone boundary and local arrow of time could emerge as a consequence of self-organization process involving conscious intentional action. Sub-CDs would be in the interior of CD and self-organization process would lead to a distribution of CDs concentrated near the upper or lower boundary of CD. The local arrow of geometric time would depend on CD and even differ for CD and sub-CDs.

1. The localization of contents of sensory experience to a narrow time interval would be due to the concentration of sub-CDs representing mental images near the either boundary of CD representing self.

2. Phase conjugate signals identifiable as negative energy signals to geometric past are important when the arrow of time differs from the standard one in some time scale. If the arrow of time establishes itself as a phase transition, this kind of situations are rare. Negative energy signals as a basic mechanism of intentional action and transfer of metabolic energy would explain why living matter is so special.

3. Geometric memories would correspond to the regions near "lower" boundaries of CD. Since the density of sub-CDs is small there geometric memories would be rare and not sharp. A temporal sequence of mental images, say the sequence of digits of a phone number, would correspond to a temporal sequence of sub-CDs.

4. Sharing of mental images corresponds to a fusion of sub-selves/mental images to single sub-self by quantum entanglement: the space-time correlate could be flux tubes connecting space-time sheets associated with sub-selves represented also by space-time sheets inside their CDs.

3.3 Arrow of time

TGD forces a new view about the relationship between experienced and geometric time. Although the basic paradox of quantum measurement theory disappears the question about the arrow of geometric time remains.

1. Selves correspond to CDs. The CDs and their projections to the imbedding space do not move anywhere. Therefore the standard explanation for the arrow of geometric time cannot work.

2. The only plausible interpretation at classical level relies on quantum classical correspondence and the fact that space-times are 4-surfaces of the imbedding space. If quantum jump corresponds to a shift for a quantum superposition of space-time sheets towards geometric past in the first approximation (as quantum classical correspondence suggests), one can understand the arrow of time. Space-time surfaces simply shift backwards with respect to the geometric time of the imbedding space and therefore to the 8-D perceptive field defined by the CD. This creates in the materialistic mind a temporal variant of train illusion. Space-time as 4-surface and macroscopic and macro-temporal quantum coherence are absolutely essential for this interpretation to make sense.

Why this shifting should always take place to the direction of geometric past of the imbedding space? Does it so always? The proposed mechanism for the localization of sensory experience to a short time interval suggests an explanation in terms of intentional action.

1. CD defines the perceptive field for self. Selves are curious about the space-time sheets outside their perceptive field and perform quantum jumps tending to shift the superposition of the space-time sheets so that unknown regions of space-time sheets emerge to the perceptive field. Either the upper or lower boundary of CD wins in the competition and the arrow of time results as a spontaneous

symmetry breaking. The arrow of time can depend on CD but tends to be the same for CD and its sub-CDs. Global arrow of time could establish itself by a phase transitions establishing the same arrow of time globally by a mechanism analogous to percolation phase transition.

2. Since the news come from the upper boundary of CD, self concentrates its attention to this region and improves the resolution of sensory experience. The sub-CDs generated in this manner correspond to mental images with contents about this region. Hence the contents of conscious experience, in particular sensory experience, tends to be about the region near the upper boundary.

3.4 Can selves interact and evolve?

Interesting questions relate to how dynamical selves are.

1. Is self doomed to live inside the same sub-WCW eternally as a lonely god? This question has been already answered: there are interactions between sub-CDs of given CD, and one can think of selves as quantum superposition of states in CDs with wave function having as its argument the tips of CD, or rather only the second one since T is assumed to be quantized.

2. Is there largest CD in the personal CD hierarchy of self in an absolute sense? Or is the largest CD present only in the sense that the contribution to the contents of consciousness coming from very large CDs is negligible? Long time scales T correspond to low frequencies and thermal noise might mask these contributions. Here however the hierarchy of Planck constants and generalization of the imbedding space could come in rescue by allowing dark EEG photons to have energies above thermal energy.

3. Can selves evolve in the sense that the size of CD increases in quantum leaps so that the corresponding time scale $T = 2^k T_0$ of memory and planned action increases? Geometrically this kind of leap would mean that CD becomes a sub-CD of a larger CD - either at the level of conscious experience or in absolute sense. The leap can occur in two senses: as an increase of the largest p-adic time scale in the personal hierarchy of space-time sheets or as increase of the largest value of Planck constants in the personal dark matter hierarchy. At the level of individual organism this would mean emergence of new lower frequencies of generalized EEG and levels of personal dark matter hierarchy with larger value of Planck constant.

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