Spin Tensor of Electromagnetic Waves

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In the frame of the standard electrodynamics, a torque is calculated, which acts from a circularly polarized electromagnetic beam with a plane phase front on an absorbing surface. And a moment of momentum flux in the same beam is calculated in the frame of the same electrodynamics. It is found that this torque is twice more than the moment of momentum flux. We have inferred that the calculation of the electromagnetic angular momentum flux in the beam is incorrect. Namely, this calculation takes only a moment of momentum into account as an angular momentum, and does not take account of spin. An analysis of the field theory foundations of the electrodynamics confirms this inference. Some changes in the field theory allow obtaining an electrodynamics' spin tensor, which accompanies the Maxwell energy-momentum tensor. Using this spin tensor for the beam yields the equality between the torque and the angular momentum flux. In this way, the electrodynamics is completed by a spin tensor. A criticism of an AOP reviewer and my answer are presented.

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1. Introduction

A circularly polarized light beam carries an angular momentum. It is beyond any doubt. This beam rotated the Beth's birefringent plate [1]. This beam rotates particles trapped in optical tweezers (see, e.g. [2]). However, a troubling question exists: what is the distribution of this angular momentum over the beam section? Can we use a concept of an angular momentum flux density as well as we use an energy flux density or linear momentum flux density? In order to look into this question, in Introduction, we examine an influence of the energy flux and of the momentum flux upon a surface, which absorbs the light beam. The angular momentum flux is considered in the following two Sections. We are convinced that characteristics of mechanical stresses, which are made up by the angular momentum flux, show a location of the absorption of this flux

In Section 4, an analysis of the field theory foundations of the electrodynamics is presented. In Section 5, we arrive at an electrodynamics' spin tensor, which is applied for a calculation of the angular momentum flux in the beam in Section 6. The famous Humblet transformation is considered in Section 7. This transformation is at the heart of an erroneous interpretation of an orbital angular momentum as spin.

If a light beam is absorbed by a material surface, this surface becomes hotter and experiences a pressure. The heat causes a temperature gradient and a heat flow on the surface from the alight zone of the surface to the periphery. The pressure causes a shear stress in the surface, by means of which the pressure force transfers to supports on the periphery

Consider a so-called paraxial circularly polarized beam of radius R with its axis in the z-direction and traveling in this direction [3] (Fig. 1.)

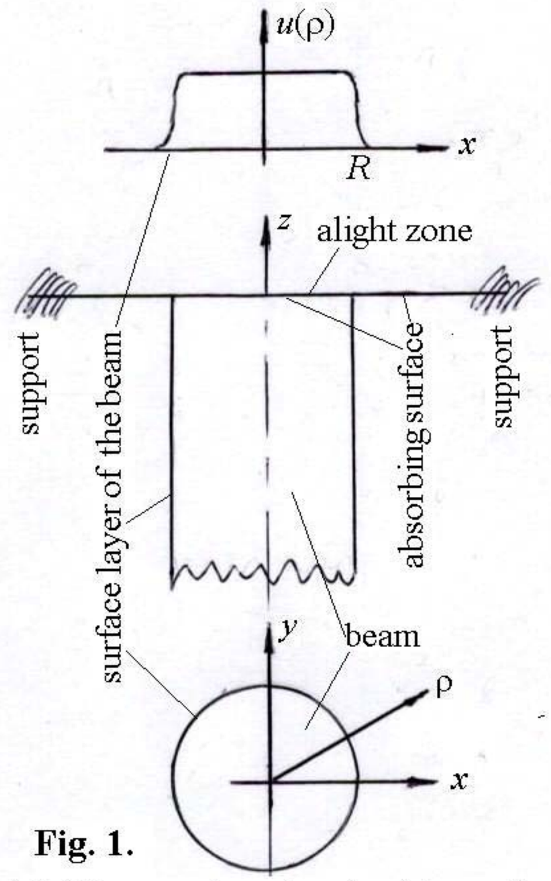

Fig. 1.
A light beam arrives at an absorbing surface
$u(\rho)$ is amplitude of electromagnetic field

$$\breve{\mathbf{E}} = \exp[i(z-t)][\mathbf{x} + i\mathbf{y} + \mathbf{z}(i\partial_x - \partial_y)]u(\rho)\,, \quad \breve{\mathbf{B}} = -i\breve{\mathbf{E}}, \quad \rho^2 = x^2 + y^2\,. \tag{1.1}$$

The symbol 'breve' marks complex vectors and numbers excepting i. $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are the unique coordinate vectors. For short we set $\omega = k = c = 1$, where ω, k, c are the frequency, wave number, and light velocity. $u(\rho)$ is the electric field amplitude. The function $u(\rho)$ is explicitly made constant u_0 over a large central region of the beam. The variation of the function from this constant value to zero is localized within a layer of small thickness, which lies a distance $\rho = R$ from the axis. In the surface layer of the beam, i.e. there where the function $u(\rho)$ decreases, longitudinal components of the electromagnetic fields exist (this components are z-directed). This is because the lines of force are closed, but they cannot transgress the surface of the beam

We set

$$\int u^2 dxdy = \int u^2 2\pi\rho\, d\rho = \int_0^R u^2 2\pi\rho\, d\rho = 1 \tag{1.2}$$

when integrating over the whole of absorbing surface, what is equivalent to integrating over a cross section of the beam (we will ignore the width of the surface layer of the beam when it is admissible).

An energy flux density in the beam is the Poynting vector $\mathbf{E} \times \mathbf{B}$. At first, we consider the z-component of the energy flux density, i.e. T_e^{0z}-component of the Maxwell tensor. Time averaging gives:

$$< T_e^{0z} > = \Re(\breve{E}_x \overline{B}_y - \breve{E}_y \overline{B}_x)/2 = \Re(\breve{E}_x i\overline{E}_y - \breve{E}_y i\overline{E}_x)/2 = u^2\,, \tag{1.3}$$

the dash marks complex conjugating numbers. Thus, the power of our beam, because of (1.2), is

$$W = \int < T_e^{0z} > dxdy = \int u^2 dxdy = \int u^2 2\pi\rho\, d\rho = 1 \tag{1.4}$$

We consider a sufficiently wide beam and neglect the surface layer of the beam here; we set $u(\rho) = u_0 = \mathrm{Const}$ if $\rho < R$. Thus, because of (1.2),

$$u^2(0) = u_0^2 = 1/\pi R^2\,. \tag{1.5}$$

Now one can find a heat flux density in the absorbing surface:

$$Q^i = u_0^2 x^i/2 \;\text{ if }\; \rho < R\,, \;\text{ and }\; Q^i = u_0^2 R^2 x^i / 2\rho^2 \;\text{ if }\; \rho > R \tag{1.6}$$

(index i means $i = x, y$ on the surface). Indeed, a divergence of this flux density equals

$$\partial_i Q^i = u_0^2(\partial_x x + \partial_y y)/2 = u_0^2 \;\text{ if }\; \rho < R\,, \;\text{ and }\; \partial_i Q^i = 0 \;\text{ if }\; \rho > R\,. \tag{1.7}$$

Given heat conductivity, one can calculate the temperature distribution.

The beam pressure on the absorbing surface equals T_e^{zz}-component of the Maxwell tensor. The sense of this component is given by the equality

$$dF^z = T_e^{zz} dxdy\,, \tag{1.8}$$

where dF^z is the force, which acts on an $dxdy$-element of an absorbing surface from an electro-magnetic field. Ignoring the surface layer of the beam, one has a constant pressure in the alight zone of the absorbing surface

$$< T_e^{zz} > = \Re(E_x^2 + E_y^2 + B_x^2 + B_y^2)/4 = u_0^2\,, \tag{1.9}$$

which equals the energy flux density (1.3), as it must be. A mechanical stress in the absorbing surface must balance this pressure. The shear stress is distributed through the thickness of our material surface and is expressed by $T_m^{z\rho}$-component of the stress tensor of the surface. Consider a disk of radius ρ with its center at the axis of the beam, which is chosen from the absorbing surface. A balance conditions for this disk, viz. $u_0^2 \pi\rho^2 = T_m^{z\rho} 2\pi\rho$ for $\rho < R$, and $u_0^2 \pi R^2 = T_m^{z\rho} 2\pi\rho$ for $\rho > R$, give the mechanical stresses in the surface:

$$T_m^{z\rho} = u_0^2 \rho/2 \;\text{ for }\; \rho < R\,, \;\text{ and }\; T_m^{z\rho} = u_0^2 R^2/2\rho \;\text{ for }\; \rho > R\,, \tag{1.10}$$

these expressions are similar to (1.6).

Thus, the heat flux density and mechanical stress in z-direction increase proportionally to the distance ρ from the axis in the alight zone of the absorbing surface. They tend to zero as hyperbole beyond the alight zone.

2. Maxwellian torque

A torque acts on the absorbing surface from the beam, according to the Maxwell electrodynamics, if and only if the surface experiences tangential forces, which are expressed through T_e^{xz}, T_e^{yz}-components of the Maxwell tensor. However, these components equal zero on the surface apart from a boundary of the alight zone of the absorbing surface where the surface layer of the beam is absorbed.

Indeed, the Poynting vector and the momentum density are directed along the direction of propagation, i.e. along z-axis, in the large central region of the beam, as well as in a plane wave. Therefore, the tangential forces act on the absorbing surface only at the boundary of the alight zone, where $\partial_y u^2 \neq 0, \quad \partial_x u^2 \neq 0$,

$$T_e^{xz} = -E_x E_z - B_x B_z, \quad < T_e^{xz} >= -\Re(\breve{E}_x \overline{E}_z + \breve{B}_x \overline{B}_z)/2 = -\Re(\breve{E}_x \overline{E}_z) = \partial_y u^2 / 2, \tag{2.1}$$

$$< T_e^{yz} >= -\partial_x u^2 / 2. \tag{2.2}$$

A disk of radius $\rho < R$ with its center at the axis of the beam, which is chosen from the absorbing surface, does not experience tangential forces and does not experience a torque. Therefore, the alight zone, right up to its boundary, does not contain a mechanical stress, which is caused by a torque.

A torque acts only on the boundary of the alight zone. The torqie equals

$$\tau_2 = \int (x < T_e^{yz} > -y < T_e^{xz} >) dxdy = -\int (x\partial_x u^2 / 2 + y\partial_y u^2 / 2) dxdy = \int u^2 dxdy = 1 \tag{2.3}$$

(index 2 means that this expression is valid in the frame of Section 2). Torque (2.3) must be balanced with a torque, which acts on our surface from supports on the periphery. Therefore the part of the surface for $\rho > R$, which is outside of the alight zone, must contain a mechanical stress which is expressed by $T_{m2}^{\phi\rho}$-component of the surface stress tensor. The sense of this component is given by the equality

$$dF^{\phi} = T_m^{\phi\rho} dl, \tag{2.4}$$

where dF^{ϕ} if the force, which acts on the element dl of a circle and is directed along ϕ-coordinate. A balance condition for a disk of radius $\rho > R$,

$$\tau_2 = \int \rho dF^{\phi} = \int \rho T_{m2}^{\phi\rho} dl = 2\pi\rho^2 T_{m2}^{\phi\rho}, \quad \rho > R, \tag{2.5}$$

gives $T_{m2}^{\phi\rho} = 1/2\pi\rho^2$. As a result, we have, according to the Maxwell electrodynamics, the mechanical stress in the absorbing surface is

$$T_{m2}^{\phi\rho} = 0 \quad \text{for} \quad \rho < R, \qquad T_{m2}^{\phi\rho} = 1/2\pi\rho^2 \quad \text{for} \quad \rho > R. \tag{2.6}$$

The fact, that the moment of momentum relative to the beam axis is contained only in the surface layer of the beam, and, accordingly, the torque acting on the absorbing surface is localized at the boundary of the alight zone, is well known.

For example, Ohanian [4] writes and depicts (see our Fig. 2.)

"In a wave of finite transverse extent, the **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector. Hence the net energy flow is helical. The circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation"

Authors of work [5] present a similar figure (see our Fig. 3) and explain:

"The electric and magnetic fields can have a nonzero z-component only within the skin region of the wave. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only one in which the z-component of angular momentum does not vanish"

All presented here arguments show that, according to the standard electrodynamics, the large central alight zone of the absorbing surface experiences no torque and, accordingly, contains no corresponding mechanical stress. Mechanical stress, causing by torque, arises only in the boundary of the alight zone and extends over the absorbing surface to the periphery, right up to support of the surface. The boundary, and, consequently, supports on the periphery experience the torque from the beam, which equals $\tau_2 = 1$. Because of the power of the beam is $W = 1$ and the frequency $\omega = 1$, one can write down

$$\tau_2 = W / \omega. \tag{2.7}$$

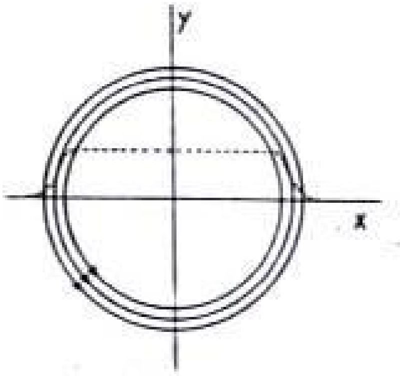

Fig. 1. This pattern of circular flow lines represents the time-average energy flow, or the momentum density, in a circularly polarized electromagnetic wave packet. On a given wave front, say $z = 0$, the fields are assumed to be constant within a circular area and to decrease to zero outside of this area (the dashed line gives the field amplitude as a function of radius). The energy flow has been calculated from an approximate solution of Maxwell's equations. The picture only shows the flow in the transverse directions. The flow in the longitudinal direction is much larger; the net flow is helical.

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Fig. 2.

9.1 ANGULAR MOMENTUM OF THE ELECTROMAGNETIC FIELD 227

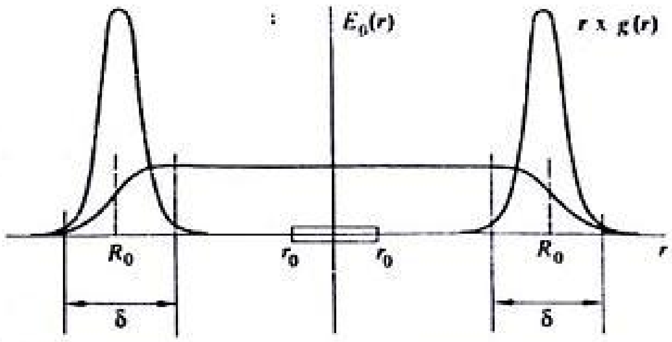

Fig. 9.3. The electric field amplitude and the angular momentum density across a cylindrical beam.

Fig. 3.

However, you must note that the power W is absorbed uniformly by the alight zone, but the moment of momentum, which results in the torque τ_2, is absorbed only by the boundary of the alight zone, i.e. not there where the power is absorbed. Therefore, it is reasonably to suppose that this moment of momentum is not concerned with this energy, and that this energy, which is the energy of a circularly polarized electromagnetic field, is concerned with another angular momentum, which is absorbed uniformly by the alight zone, but is not considered by the standard electrodynamics. On the other hand, the torque τ_2 is caused at the boundary of the alight zone by the longitudinal components of the electromagnetic fields. So, τ_2 cannot have a wave nature and, therefore, cannot be concerned with spin.

The absence of a torque in the large central alight zone of the absorbing surface in the frame of the standard paradigm is confirmed by an interesting reasoning in [6]. The authors cut the beam into two coaxial pieces in their mind: the inner part has radius of $\rho_1 < R$, outer part looks like a thick-wall tube and is located between ρ_1 and R,

$$u(\rho) = u_{in}(\rho) + u_{out}(\rho), \quad (2.8)$$

so $\partial_\rho u_{in}(\rho)|_{\rho_1} = -\partial_\rho u_{out}(\rho)|_{\rho_1}$. The authors rightly affirm that two equal, but opposite torques act on the absorbing surface near the circle of radius ρ_1, which are eliminated mutually.

3. Spin torque

The work [6] was written as a response to a question [7], where it was pointed out that dielectric dipoles of the absorbing surface experience torques from the circularly polarized wave. Because of this, the surface material must experience a volume density of torque, according to [8,9]

$$\tau / V = \mathbf{P} \times \mathbf{E}. \quad (3.1)$$

where $\mathbf{P}$ is the electric polarization

R. Feynman explains the beginning of this torque [10] (see our Fig. 4):

"The electric vector **E** goes in a circle – as drawn in Fig. 17-5(a). Now suppose that such a light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics. We'll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement **r** from its equilibrium position at the origin and goes around with some phase lag with respect to the vector **E.** As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now let's look at the work being done on this electron. The rate that energy is being put into this electron is v, its velocity, times the component of **E** parallel to the velocity:

$$W = eE_t v$$

But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $\tau = eE_t r$ which must be equal to the rate of change of angular momentum dJ_z / dt:

$$dJ_z / dt = \tau = eE_t r.$$

Remembering that $v = \omega r$, we have that

$$\tau = W / \omega\text{»}.$$

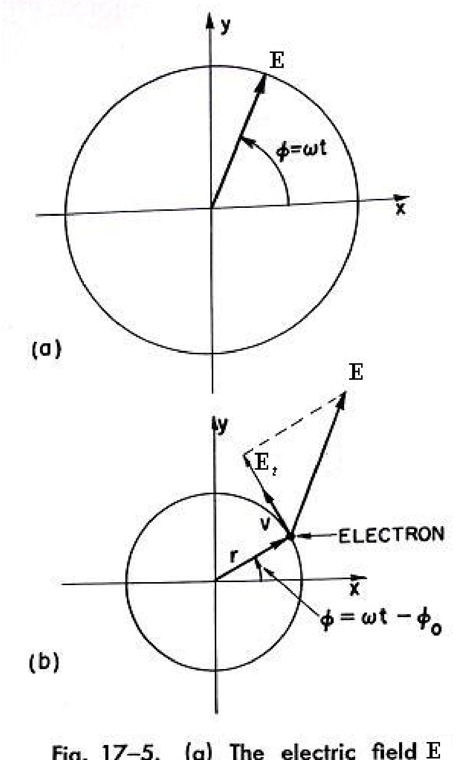

Fig. 17–5. (a) The electric field E in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.

Fig. 4.

Unfortunately, the authors of the work [6] ignored the problems, which arise from taking into account this torque.

Thus, the energy flux density (1.3), which falls on **the** absorbing surface, is accompanied by a torque density, and the energy flux density is in the same relation to the torque density as the whole energy flux (1.4) to the torque τ_2 (2.7), which acts on the boundary of the alight zone, in accordance with the Maxwell theory. However, now the torque density is constant at points of the alight zone and is not expressed in terms of the Maxwell tensor, though the torque undoubtedly cause a mechanical stress, which is expressed in terms of a mechanical stress tensor. We will find this stress by the use of a balance condition, but it is appropriate mention here that the authors of the works [8-10] identify the torque of Section 3 with spin flux of the beam.

Consider a disk of radius $\rho < R$ with its center at the axis of the beam, which is chosen from the absorbing surface. According to (1.3) and (1.5) the disk receives the power $W(\rho) = \pi\rho^2 u_0^2 = \rho^2 / R^2$, and then the disk experiences the torque $\tau_3(\rho) = \rho^2 / R^2$ (index 3 means that this expression is valid in the frame of Section 3). A balance condition for this disk, viz. $\tau = 2\pi\rho^2 T_m^{\phi\rho}$, which is analogous to (2.5), now takes the form of $\rho^2 / R^2 = 2\pi\rho^2 T_{m3}^{\phi\rho}$ for $\rho < R$ and $1 = 2\pi\rho^2 T_{m3}^{\phi\rho}$ for $\rho > R$. It means

$$T_{m3}^{\phi\rho} = 1/2\pi R^2 = \text{Const} \quad \text{for } \rho < R, \qquad T_{m3}^{\phi\rho} = 1/2\pi\rho^2 \quad \text{for } \rho > R. \tag{3.2}$$

Thus, with the regard for the stresses $T_{m2}^{\phi\rho}$ (2.6) and $T_{m3}^{\phi\rho}$ (3.2), we arrive to a double torque at the periphery, i.e. for $\rho > R$,

$$\tau_{\text{tot}} = (T_{m2}^{\phi\rho} + T_{m3}^{\phi\rho})2\pi\rho^2 = 2. \tag{3.3}$$

The result (3.3) was obtained more pronouncedly in [11,12]. The result is evidence that the moment of momentum, which the beam (1.1) brings, according to the standard electrodynamics (see Section 2), is half of the angular momentum, which the absorbing surface receives, according to the same electrodynamics. This means the standard electrodynamics is not complete.

4. Does electrodynamics' spin tensor exist?

We will use the idea mentioned in the works [8-10] about a spin nature of the torque acting on the large central alight zone of the absorbing surface, and will show that the torque is really caused by an absorption of spin flux

As is well known, photons, i.e. electromagnetic waves, carry spin, energy, momentum, and angular momentum that is a moment of the momentum relative to a given point or to a given axis. Energy and momentum of electromagnetic waves are described by the Maxwell energy-momentum tensor (density)

$$T^{\lambda\mu} = -g^{\lambda\alpha}F_{\alpha\nu}F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{4.1}$$

where $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor. An interaction between electromagnetic waves and substance is described by a divergence of the energy-momentum tensor $\partial_\mu T^{\lambda\mu}$, i.e. by the Lorentz force density, viz.,

$$f^{\lambda} = -\partial_\mu T^{\lambda\mu} = F^{\lambda\beta}\partial^\mu F_{\mu\beta} = j_\beta F^{\lambda\beta}. \tag{4.2}$$

The Maxwell equations $\partial_{[\lambda}F_{\mu\nu]} = 0$, $\partial^\mu F_{\mu\beta} = j_\beta$ .are used here.

The angular momentum that is a moment of the momentum can be defined as [13]

$$L^{ij} = \int_V 2x^{[i}T^{j]0}dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV, \tag{4.3}$$

and this construction must be named as an orbital angular momentum in the volume V. However, the modern electrodynamics has no describing of spin, though a concept of classical spin, which differs from the moment of momentum, is contained in the modern theory of fields. Unfortunately, the concept of spin is smothered in the standard electrodynamics as will be shown below.

Realy, the electrodynamics starts from the canonical Lagrangian [14 (4-111)], $\underset{c}{\mathrm{L}} = -F_{\mu\nu}F^{\mu\nu}/4$.

Then, by the Lagrange formalism, the canonical energy-momentum tensor [14 (4-113)]

$$\underset{c}{T}{}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{c}{\mathrm{L}}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\underset{c}{\mathrm{L}} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4 \tag{4.4}$$

and the canonical total angular momentum tensor [14 (4-147)]

$$\underset{c}{J}{}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}{}^{\mu]\nu} + \underset{c}{\Upsilon}{}^{\lambda\mu\nu} \tag{4.5}$$

are obtained. Here

$$\underset{c}{\Upsilon}{}^{\lambda\mu\nu} = -2A^{[\lambda}\delta_\alpha^{\mu]}\frac{\partial \underset{c}{\mathrm{L}}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}, \tag{4.6}$$

is the canonical spin tensor [14 (4-150)]. Its space component is $\mathbf{E}\times\mathbf{A}$:

$$\underset{c}{\Upsilon}{}^{ij0} = \mathbf{E}\times\mathbf{A}, \tag{4.7}$$

The sense of a spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ij0} is a volume density of spin. This means that $dS^{ij} = \Upsilon^{ij0}dV$ is the spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is a flux density of spin flowing in the direction of the x^k axis. For example,

$dS_z / dt = dS^{xy} / dt = d\tau^{xy} = \Upsilon^{xyz} da_z$ is the z-component of spin flux passing through the surface element da_z per unit time, i.e. the torque acting on the element.

The sense of a total angular momentum tensor, $J^{\lambda\mu\nu}$, is that the total angular momentum in an element dV_ν is $dJ^{\lambda\mu} = J^{\lambda\mu\nu} dV_\nu = 2x^{[\lambda} T^{\mu]\nu} dV_\nu + \Upsilon^{\lambda\mu\nu} dV_\nu$. The corresponding integral is

$$J^{\lambda\mu} = L^{\lambda\mu} + S^{\lambda\mu} = \int_V 2x^{[\lambda} T^{\mu]\nu} dV_\nu + \int_V \Upsilon^{\lambda\mu\nu} dV_\nu \,. \tag{4.8}$$

It consists of two terms: the first term involves a moment of momentum and represents an orbital angular momentum; the second term is spin. It must be emphasized that a moment of momentum cannot represent spin. This idea is discussed in the paper [15], which was written in response to [16]

However, the canonical tensors (4.4), (4.5), (4.6) are not electrodynamics tensors. They obviously contradict experiments. For example, consider a uniform electric field:

$$A_0 = -Ex, \;\; A_x = 0, \;\; \partial_\alpha A^\alpha = 0, \;\; F_{x0} = -F^{x0} = \partial_x A_0 = -E \,. \tag{4.9}$$

The canonical energy density (4.4) is negative:

$$\underset{c}{T}^{00} = g^{00} F_{x0} F^{x0} / 2 = -E^2 / 2 \,. \tag{4.10}$$

Another example: consider a circularly polarized plane wave (or a central part of a corresponding light beam),

$$E^x = \cos(z-t), E^y = -\sin(z-t), B^x = \sin(z-t), B^y = \cos(z-t), \;\; A^x = \sin(z-t), A^y = \cos(z-t) \tag{4.11}$$

(for short we set $k = \omega = 1$). A calculation of components of the canonical spin tensor (4.6) yields

$$\underset{c}{\Upsilon}^{xy0} = 1, \;\; \underset{c}{\Upsilon}^{xyz} = 1, \;\; \underset{c}{\Upsilon}^{zxy} = A^x B_x = \sin^2(z-t), \;\; \underset{c}{\Upsilon}^{yzx} = A^y B_y = \cos^2(z-t) \,. \tag{4.12}$$

This result is absurd because, though $\underset{c}{\Upsilon}^{xy0}$ and $\underset{c}{\Upsilon}^{xyz}$ are adequate, the result means that there are spin fluxes in y & x - directions, i.e. in the directions, which are transverse to the direction of the wave propagation.

An opinion exists that a change of the Lagrangian can help to obtain the Maxwell tensor (4.1). A. Barut [17] presented a series of Lagrangians and field equations in Table 1

Table I

Lagrangians and Equations of Motion for the Most Common Fields

Field	Lagrangian	Field Equations
Free Electromagnetic Field	$L_{\mathrm{I}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(E^2 - B^2)$	$F^{\mu\nu}{}_{,\nu} = 0$
	$L_{\mathrm{II}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(A^\mu{}_{,\mu})^2$	$\square^2 A_\mu = 0$
	$L_{\mathrm{III}} = -\frac{1}{2}A^\mu{}_{,\nu}A_\mu{}^{,\nu}$	$\square^2 A_\mu = 0$
	$L_{\mathrm{IV}} = \frac{1}{2}[A_\nu F^{\mu\nu}{}_{,\mu} - A_{\nu,\mu}F^{\mu\nu}] + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$	$\square^2 A_\mu = 0$
Electromagnetic Field with an External Current	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}A_\mu j^\mu$	$F^{\mu\nu}{}_{,\nu} = -\frac{1}{c}j^\mu$

However, A. Barut did not show energy-momentum and spin tensors corresponding to these Lagrangians. So, we add Table 2

Table 2

Electrodynamics' Lagrangians, Energy-Momentum Tensors, and Spin Tensors

Lagrangian	Energy-momentum tensor	Spin tensor
$L_I = \underset{c}{L} = -F_{\mu\nu}F^{\mu\nu}/4$	$T_I^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} = -A_\nu{}^{,\lambda}F^{\mu\nu} + g^{\lambda\mu}F_{\sigma\nu}F^{\sigma\nu}/4$	$\Upsilon_I^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$
$L_{II} = -F_{\mu\nu}F^{\mu\nu}/4 - (A^\mu{}_{,\mu})^2/2$	$T_{II}^{\lambda\mu} = T_I^{\lambda\mu} - A^{\mu,\lambda}A^\sigma{}_{,\sigma} + g^{\lambda\mu}(A^\sigma{}_{,\sigma})^2/2$	$\Upsilon_{II}^{\lambda\mu\nu} = \Upsilon_I^{\lambda\mu\nu} + 2A^{[\lambda}g^{\mu]\nu}A^\sigma{}_{,\sigma}$
$L_{III} = -A^\mu{}_{,\nu}A_\mu{}^{,\nu}/2$	$T_{III}^{\lambda\mu} = -A_\sigma{}^{,\lambda}A^{\sigma,\mu} + g^{\lambda\mu}A_{\sigma,\rho}A^{\sigma,\rho}$	$\Upsilon_{III}^{\lambda\mu\nu} = 2A^{[\lambda}A^{\mu],\nu}$
$L_V = -F_{\mu\nu}F^{\mu\nu}/4 - A_\sigma j^\sigma$	$T_V^{\lambda\mu} = T_I^{\lambda\mu} + g^{\lambda\mu}A_\sigma j^\sigma$	$\Upsilon_V^{\lambda\mu\nu} = \Upsilon_I^{\lambda\mu\nu}$

It is clear, none of these energy-momentum tensors is the Maxwell tensor. And what is more, none of these tensors differs from the Maxwell tensor by a divergence of an antisymmetric quantity. In other words, none of these tensors has true divergence (4.2). A method is unknown to get a tensor with the true divergence in the frame of the standard Lagrange formalism. A desire for such a tensor led Professor Soper to a mistake [18]. He used Lagrangian L_V, but, instead of the tensor $T_V^{\lambda\mu}$, he arrived at a false tensor [18, (8.3.5) – (8.3.9)]

$$\underset{f}{T}^{\lambda\mu} = T_I^{\lambda\mu} + A^\lambda j^\mu\,, \tag{4.13}$$

which differs from the Maxwell tensor by a divergence of an antisymmetric quantity:

$$T^{\lambda\mu} - \underset{f}{T}^{\lambda\mu} = \partial_\alpha A^\lambda F^{\mu\alpha} - A^\lambda j^\mu = \partial_\alpha (A^\lambda F^{\mu\alpha})\,. \tag{4.14}$$

In the frame of the standard procedure, a specific terms,

$$\underset{st}{t}^{\lambda\mu} = -\partial_\nu \tilde{\Upsilon}^{\lambda\mu\nu} / 2 \tag{4.15}$$

and

$$\underset{st}{m}^{\lambda\mu\nu} = -\partial_\kappa (x^{[\lambda} \tilde{\Upsilon}^{\mu]\nu\kappa})\,, \tag{4.16}$$

are added to the canonical tensors (4.4) and (4.5) [19,20] (here $\tilde{\Upsilon}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu}$). This procedure gives a standard energy-momentum tensor $\underset{st}{T}^{\lambda\mu}$ and a standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$,

$$\underset{st}{T}^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu} = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 + \partial_\nu (A^\lambda F^{\mu\nu})\,, \tag{4.17}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + \underset{st}{m}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + \partial_\kappa (x^{[\lambda} A^{\mu]} F^{\nu\kappa})\,. \tag{4.18}$$

Unfortunately, the energy-momentum tensor $\underset{st}{T}^{\lambda\mu}$ (4.17) is obviously invalid, as well as the canonical energy-momentum tensor (4.4). So, the (Belinfante-Rosenfeld) procedure [19,20] is unsuccessful, and the tensors (4.17), (4.18) are never used. But the worst thing is found out when calculating of the standard spin tensor $\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu}$, where the spin addend is

$$\underset{st}{s}^{\lambda\mu\nu} = \underset{st}{m}^{\lambda\mu\nu} - 2x^{[\lambda}\, \underset{st}{t}^{\mu]\nu} = -\partial_\kappa (x^{[\lambda} \tilde{\Upsilon}^{\mu]\nu\kappa}) + x^{[\lambda} \partial_\kappa \tilde{\Upsilon}^{\mu]\nu\kappa} = -\delta_\kappa^{[\lambda} \tilde{\Upsilon}^{\mu]\nu\kappa} = 2\delta_\kappa^{[\lambda} A^{\mu]} F^{\nu\kappa} = -2\delta_\kappa^{[\lambda} A^{\mu]} F^{\kappa\nu} = -2A^{[\mu} F^{\lambda]\nu}$$
$$= 2A^{[\lambda} F^{\mu]\nu} = -\underset{c}{\Upsilon}^{\lambda\mu\nu} \tag{4.19}$$

So, we see the procedure gives a standard spin tensor which equals zero! I.e. the procedure eliminates classical spin at all:

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu} = 0\,. \tag{4.20}$$

That is why a spin term is absent in Eq. (4.22).

Note that the addends $\underset{st}{t}^{\lambda\mu}$ & $\underset{st}{s}^{\lambda\mu\nu}$, though they are unsuitable, satisfy an important equation

$$\partial_\nu\, \underset{st}{s}^{\lambda\mu\nu} = \underset{st}{t}^{[\lambda\mu]}\,. \tag{4.21}$$

In spite of the fact that the standard spin tensor is zero, physicists understand they cannot shut eyes on existence of the classical electrodynamics’ spin. And they proclaim spin is *in* the moment of the momentum (4.3). I.e., the moment of momentum represents the total angular momentum: orbital angular momentum plus spin. I.e., equation (4.3) encompasses both the spin and orbital angular momentum density of a light beam [2-6,9,13,14,16-18,21-23]:

$$J^{ij} = L^{ij} + S^{ij} = \int_V 2x^{[i} T^{j]0} dV = \int_V \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV\,. \tag{4.22}$$

In the end, it is important to point out that an addition of any term to an energy-momentum tensor, including the addition of a divergence-free term like $-\partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu} / 2$ (see, e.g. [23, (3.36)]), changes the energy-momentum distribution and changes the total 4-momentum of the system when the field does not change. Really, it is easy to express the energy-momentum tensor of an uniform ball of radius R in the form of $\partial_\nu \Psi^{\lambda\mu\nu}$ ($\Psi^{\lambda\mu\nu} = -\Psi^{\lambda\nu\mu}$).

$$\Psi^{00i} = -\Psi^{0i0} = \varepsilon x^i / 3 \ \ (r < R), \quad \Psi^{00i} = -\Psi^{0i0} = \varepsilon R^3 x^i / 3r^3 \ \ (r > R) \tag{4.23}$$

give

$$T^{00} = \partial_i \Psi^{00i} = \varepsilon \ \ (r < R), \quad T^{00} = \partial_i \Psi^{00i} = 0 \ \ (r > R)\,. \tag{4.24}$$

5. Electrodynamics' spin tensor exists

Contrary to the Belinfante-Rosenfeld procedure, which eliminates spin, we modify the invalid canonical tensors (4.4) – (4.6) by another way [11,24-29]. In contrast to the procedure [19,20], we use other addends to the canonical energy-momentum and spin tensors. Our addends are

$$t^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu}\,, \tag{5.1}$$

$$s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^\nu\,, \tag{5.2}$$

instead of (4.15), (4.19). $t^{\lambda\mu}$ gives the Maxwell tensor (4.1)

$$T^{\lambda\mu} = \underset{c}{T}{}^{\lambda\mu} + \partial_\nu A^\lambda F^{\mu\nu}\,, \tag{5.3}$$

and $s^{\lambda\mu\nu}$ is obtained from the equation

$$\partial_\nu s^{\lambda\mu\nu} = t^{[\lambda\mu]}\,, \tag{5.4}$$

which is analogous to (4.21). As a result, we arrive at a quantity

$$2A^{[\lambda}\partial^{|\nu|}A^{\mu]} = \underset{c}{\Upsilon}{}^{\lambda\mu\nu} + 2A^{[\lambda}\partial^{\mu]}A^\nu\,, \tag{5.5}$$

instead of the zero, and, at long last, at our spin tensor:

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}\,. \tag{5.6}$$

Here A^λ and Π^λ are magnetic and electric vector potentials which satisfy

$$\partial_\lambda A^\lambda = \partial_\lambda \Pi^\lambda = 0\,, \quad 2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}\,, \quad 2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}\,, \tag{5.7}$$

where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field; $e_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor density. It is evident that the conservation law, $\partial_\nu \Upsilon^{\lambda\mu\nu} = 0$, is held for a free field.

In other words, we introduce a spin tensor $\Upsilon^{\lambda\mu\nu}$ into the modern electrodynamics, i.e. we complete the electrodynamics by introducing the spin tensor, i.e. we claim that the total angular momentum consists of the moment of momentum (4.3) *and* a spin term, that equation (4.22) is incorrect, that the moment of momentum (4.3) does not contain spin at all, that, in reality, the total angular momentum in the volume V equals

$$J^{ij} = L^{ij} + S^{ij} = \int_V (2x^{[i}T^{j]0} + \Upsilon^{ij0})dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV + \int_V \Upsilon^{ij0}dV\,, \tag{5.8}$$

and the angular momentum flux on the area a equals

$$\tau^{ij} = \underset{\text{orb}}{\tau}{}^{ij} + \underset{\text{spin}}{\tau}{}^{ij} = \int_a (2x^{[i}T^{j]k} + \Upsilon^{ijk})da_k = \int_a \mathbf{r}\times(\mathbf{E}\times\mathbf{B})d\mathbf{a} + \int_a \Upsilon^{ijk}da_k\,, \tag{5.9}$$

The difference between our statement (5.8) and the common equation (4.22) is verifiable. The cardinal question concerns the angular momentum flux, i.e. torque $\tau = dJ/dt$, which is carried by the beam (1.1). The common answer, according to (4.22), is

$$\tau = dJ/dt = \mathsf{P}/\omega\,; \tag{5.10}$$

our answer, according to (5.9), is

$$\tau = dJ/dt = 2\mathsf{P}/\omega\,, \tag{5.11}$$

what corresponds to the result (3.3)

6. Spin tensor of a circularly polarized beam

Let us use Eq. (5.9) for proving result (5.11). The first term of (5.9) is already calculated, $\underset{\text{orb}}{\tau} = \tau_2 = 1$ (2.3). This term is independent of the existence of spin tensor. The second term, according to (5.6), uses the vector potentials A^{λ} and Π^{λ} and their derivatives with respect to z. We set the scalar potentials $A^0 = \Pi^0 = 0$, ignore the surface layer of the beam, take into account that $\partial^z = -\partial_z$ because of the signature of the metric $(+---)$. Then we have

$$\breve{\mathbf{A}} = -\int \breve{\mathbf{E}} dt = \exp[i(z-t)](-i\mathbf{x}+\mathbf{y})u_0\,, \quad \breve{\Pi} = \int \breve{\mathbf{B}} dt = -\int i\breve{\mathbf{E}} dt = i\breve{\mathbf{A}}\,, \tag{6.1}$$

$$\partial^z \breve{\mathbf{A}} = \exp[i(z-t)](-\mathbf{x}-i\mathbf{y})u_0\,, \quad \partial^z \breve{\Pi} = i\partial^z \breve{\mathbf{A}}\,, \tag{6.2}$$

$$< \Upsilon^{xyz} >= \Re(\breve{A}^x \partial^z \overline{A}^y - \breve{A}^y \partial^z \overline{A}^x + \breve{\Pi}^x \partial^z \overline{\Pi}^y - \breve{\Pi}^y \partial^z \overline{\Pi}^x)/4 = \Re(\breve{A}^x \partial^z \overline{A}^y - \breve{A}^y \partial^z \overline{A}^x)/2 = u_0^2\,. \tag{6.3}$$

So, the second term of Eq. (5.9), in view of (1.2), equals

$$\underset{\text{spin}}{\tau} = \tau_3 = 1\,. \tag{6.4}$$

Thus, the large central alight zone of the absorbing surface receives spin flux of a constant density over the zone. The corresponding torque density is constant over the zone and causes a specific mechanical stress (3.2). The total angular momentum flux provided by the beam (1.1), accordingly with (5.9), is

$$\tau^{xy} = \underset{\text{orb}}{\tau}{}^{xy} + \underset{\text{spin}}{\tau}{}^{xy} = \int (x < T_e^{yz} > -y < T_e^{xz} >) dxdy + \int < \Upsilon^{xyz} > dxdy = 2\,, \tag{6.5}$$

as it was found in (3.3).

7. The Humblet transformation

The integrand in Eq. (2.3), $x < T_e^{yz} > -y < T_e^{xz} >$, is not zero only at the boundary of the alight zone because T_e^{yz}, T_e^{xz}-components of the Maxwell tensor are not zero only in the surface layer of the beam. It is due to z-component of the moment of momentum is localized in the surface layer of the beam, there where energy flux circulates. However, when calculating the integral (2.3), an integration by parts is applied. As a result, in the end, the integral is calculated by integrating over all alight zone of the absorbing surface. From this mathematical fact, physicists make a strange conclusion that the whole of alight zone, rather than the boundary of the zone, contributes to the torque τ_2 (2.3), and, so, the integral (2.3) can be named spin, which evidently there is within the beam, but is invisible for Maxwell's electrodynamics. Having in view the moment of momentum (4.22), (7.1), Ohanian [4] writes: "The spin receives most of its contribution from the inner region, and the outer region can be neglected".

In reality, this false displacement of the moment of momentum from the surface layer of the beam into the beam is fulfilled by a complicated procedure [4,30,31]. The orbital moment of momentum, localized in the surface layer of the beam, $\int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV$, is expressed through an integral over all interior of the beam, viz,

$$\int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV = \int \mathbf{E} \times \mathbf{A}\, dV\,, \tag{7.1}$$

by means of the following transformation. At first, the integrand takes the form

$$\mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = \mathbf{r} \times [\mathbf{E} \times (\nabla \times \mathbf{A})] = \mathbf{r} \times (E^i \nabla A_i) - \mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}] = -\mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}]\,, \tag{7.2}$$

because $< \mathbf{r} \times (E^i \nabla A_i) >= 0$ for the beam (1.1). Then they write

$$-\mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}] = -\mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}] - \mathbf{E} \times \mathbf{A} + \mathbf{E} \times \mathbf{A} = -\nabla[\mathbf{E}(\mathbf{r} \times \mathbf{A})] + \mathbf{E} \times \mathbf{A}\,, \tag{7.3}$$

and integrating yields Eq. (7.1). This transformation is considered in detail in [11]. Note, a change of an integrating area when calculating, does not entail physical conclusions. For example, consider a solenoid. Let its winding contains electric current of density **j.** Moment of this current, $\int \mathbf{r} \times \mathbf{j}\, dV$, can be easily expressed by an integral over interior of the solenoid [11]:

$$\int \mathbf{r} \times \mathbf{j}\, dV = \int 2\mathbf{B} dV\,. \tag{7.4}$$

However, this does not mean, the interior of solenoid contributes to the moment of the current

8. Supplement

This paper was rejected without a review by Gordon W.F. Drake Editor Physical Review **A**, Manolis Antonoyiannakis Assistant Editor Physical Review Letters, Satoshi Kawata Editor Optics Communications. However, Frank Wilczek, Editor-in-Chief Annals of Physics sent a letter to me:

"Dear Dr. Khrapko: I regret to inform you that the reviewer of your manuscript, referenced above, strongly advised against publication, and we must therefore reject it. The reviewer's comments are included below. Thank you for giving us the opportunity of considering your work. Yours sincerely, (Ms.) Eve Sullivan, Editorial Assistantfor Frank Wilczek, Editor-in-ChiefAnnals of Physics.

Reviewer's comments:

The results stated in this paper are absolutely wrong. The paper contains several fundamental errors, which are fatal and cannot be corrected by any rewriting of the paper. In the following, I will explain these errors in full detail, in the hope of saving the author from wasting any more time on this topic.

SECTIONS 1 and 2. These sections are nearly correct, except for a minor error in the paragraph before Eq. 2.7, where the author erroneously asserts that there is no mechanical stress in the "central alight" zone. Obviously, in this zone there is a stress from the pressure of light (**although no stress that causes a torque**).

SECTION 3. The author quotes Feynman's neat analysis of the absorption of angular momentum from a circularly polarized wave. He tries to use this analysis to deduce that there is a density of spin in this region of the wave, IN ADDITION to the spin carried in the "skin" region, or periphery, of the wave. This is a bad mistake. The absorption of angular momentum a la Feynman can be thought of as **a couple acting on a small area of the absorbing surface**. If there are many such couples, adjacent to each other, the forces acting an adjacent area elements cancel where these areas touch, and the net force acting on the combined area elements is simply the force that acts along the periphery; thus a couple acts on the periphery, but no net couple acts within the interior area. The Feynman picture leads to the same stress (purely peripheral) associated with torque as the calculation presented in Section 2. The Feynman picture merely gives an alternative way of doing the calculation of torque of Section 2, not an ADDITIONAL torque. Therefore the author's claim that "electrodynamics is not complete" is false.

SECTION 4. The author claims that the "standard" energy-momentum tensor given by Soper is "obviously invalid," apparently because Eq. 4.17 seems to contain gauge-noninvariant terms. However, the author's entire treatment deals only with FREE e.m. waves, for which $\partial_\nu F^{\mu\nu} = 0$. Inserting this field equation into 4.17, we immediately find that 4.17 reduces to the usual Maxwell energy-momentum tensor. Thus, the Belinfante-Rosenfeld symmetrization does give us exactly what we want (and, of course, it also gives us exactly what we want if currents are present, because the extra interaction terms in the energy-momentum tensor then cancel out, again leaving the Maxwell tensor; see Soper, Sections 9.4, 9.5).

SECTION 5. The author proposes "addends" for the energy-momentum and the spin tensors. His addend for the former tensor is correct, if the field is a free field (it then coincides with the usual Belinfante-Rosenfeld addend). But his addend for the spin tensor is wrong, even for a free field. The author's fundamental mistake is that the spin term 5.2 CANNOT be postulated by an arbitray ukase; it must be constructed on the basis of the representation of the Lorentz group for the fields contained in the Lagrangian in conjunction with the assumed Belinfante-Rosenfeld addend to the energy-momentum tensor (the construction procedure involves the steps in Eqs. 4.15. 4.16. 4.19; this construction procedure links the spin tensor to the underlying physics, without it, we don't know what this spin tensor really represents). Thus, all of Section 5 beyond Eqs. 5.2 is nonsensical, and Sections 6 and 7 become irrelevant. Eq. 5.11 highlights the author's mistakes, and should have put him on the alert: if this equation were true, photons would have spin 2 instead of spin 1!"

My answer was:

"Dear Frank Wilczek:

I am grateful to you and to your reviewer for quick and detailed comments because they confirm an extreme importance of my work.

The fact of matter is the reviewer agrees that a **couple acts on any small area of the central alight zone**. At the same time he believes that **there is no stress in the zone** (because the forces cancel out). But this belief conflicts with the conservation law: if any motionless area accepts an angular momentum flow, the edge of the area must experience compensative tangential forces from the rest of the surface.

I recommend the reviewer to consider a very simple one-dimensional example. Let a rod experience a distributed torque because of applying a set of couples τ (see Fig. 5).

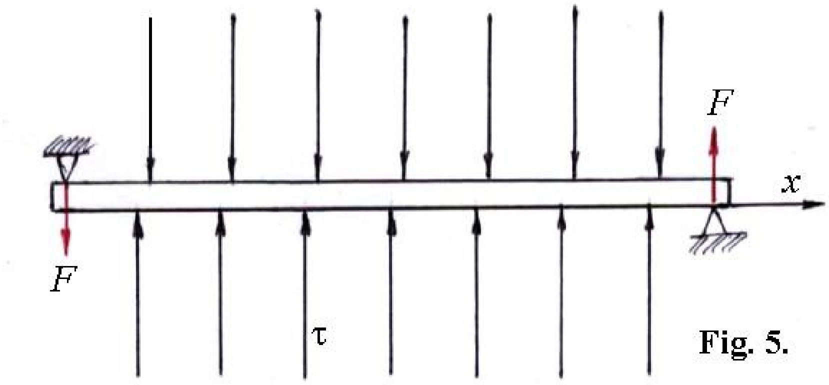

Fig. 5.

The rod experience a distributed torque because of applying a set of couples τ. It is evident that any piece of the rod experiences forces $F = \Delta\tau/\Delta x$ acting on ends of the piece. So a constant shear stress is in the rod

It is evident that any piece of the rod experiences forces $F = \Delta\tau/\Delta x$ acting on ends of the piece. So a constant shear stress is in the rod as well as the constant stress $T_{m3}^{\phi\rho}$ (3.2) is in the central alight zone from my paper (see also [11] or www.sciprint.org, Preprint archives, Search For: author, Text: Khrapko, "Spin_produces_stress"). The stress (3.2) cannot be explained by the Maxwell electrodynamics, so the electrodynamics is not complete.

Dear Frank Wilczek, the reviewer's reasoning is just the common delusion, which I try to expose during ten years.

SECTION 4. The reviewer is mistaken concerning the role of Soper. Soper did not give the standard energy-momentum tensor

$$\underset{st}{T}^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu} = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_\nu (A^\lambda F^{\mu\nu}) . \tag{4.17}$$

Soper used Lagrangian

$$L_V = -F_{\mu\nu}F^{\mu\nu}/4 - A_\sigma j^\sigma .$$

The Lagrangian gives a tensor

$$T_V^{\lambda\mu} = -A_\nu{}^{,\lambda} F^{\mu\nu} + g^{\lambda\mu} F_{\sigma\nu} F^{\sigma\nu}/4 + g^{\lambda\mu} A_\sigma j^\sigma .$$

But Soper mistakenly derived a false tensor

$$\underset{f}{T}^{\lambda\mu} = -A_\nu{}^{,\lambda} F^{\mu\nu} + g^{\lambda\mu} F_{\sigma\nu} F^{\sigma\nu}/4 + A^\lambda j^\sigma$$

from his Lagrangian L_V. So, Soper gave nothing. But it is doesn't matter.

For that matter, the standard energy-momentum tensor (4.17) was derived by the Belinfante-Rosenfeld procedure [19,20] from the canonical energy-momentum tensor (4.4). But the reviewer is mistaken concerning "the Belinfante-Rosenfeld symmetrization." The standard energy-momentum tensor (4.17) is not symmetric as well as the canonical energy-momentum tensor (4.4).

For that matter, the standard energy-momentum tensor (4.17), as well as the canonical energy-momentum tensor (4.4), is obviously invalid if currents are present. And the Lagrange formalism, as well as the Belinfante-Rosenfeld procedure, is incapable of deriving an electrodynamics energy-momentum tensor. The electrodynamics energy-momentum tensor (4.1), i.e. the Maxwell tensor, cannot be obtained by the Lagrange formalism. The coincidence of the standard energy-momentum tensor and the Maxwell tensor when currents is not present is of no importance.

SECTION 5. However, the Belinfante-Rosenfeld procedure is not simply useless, it is extremely harmful because it deprives electrodynamics of spin (4.20). We modify the Belinfante-Rosenfeld procedure. Our procedure (5.1), (5.2) gives the Maxwell tensor and electrodynamics spin tensor, which are valid if currents are present. And the reviewer must not indicate the construction procedure to me. His conjecture about spin 2 indicates that he is confused about a difference between orbital and spin angular momentum. The orbital angular momentum of Section 2 originates from longitudinal components of the electromagnetic fields and does concern neither radiation nor spin."

Conclusions, Notes and Acknowledgements

This paper conveys new physics. We review existing works concerning electrodynamics spin and indicate that existing theory is insufficient to solve spin problems because spin tensor of the modern electrodynamics is zero. Then we show how to resolve the difficulty by introducing a true electrodynamics spin tensor. Our spin tensor doubles a predicted angular momentum of a circularly polarized light beam without an azimuth phase structure. The tensor is needed, in particular, for understanding of essential characteristic features of a rotating dipole radiation [29].

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [7] (was submitted on Oct. 7, 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag).

The expression (5.6) for the spin tensor was submitted to scientific journals (the dates of the submissions are in parentheses): AJP (10 Sep 2001), AO (April 20, 2006), AP (May 5, 2006), APP (28 Jan 2002), CJP (19 Nov 2003), CLEO/QELS Conference (22/11/2006), CMP (May 9, 2006), EJP (June 30, 2005), EPL (15 Oct 2002), FP (May 3, 2002), IJTP (January 25, 2006), JETP (27 Jan 1999), JETP Letters (**14 May 1998**), JMO (Sept 29, 2004), JMP (28 Nov 2002), JOP A (Nov 30, 2003), JOSA A (Apr 7, 2006), JOSA B (Dec 27, 2005), JPA (23 Jun 2002), JPB (Dec 12, 2003), MPEJ (Dec 24, 2004), Nature (Sept 21, 2006), NJP (27 Jun 2003), OC (22 Sept 2002), OL (29 Jul 2003), PLA (22 Jul 2002), PRA (19 Nov 2003), PRD (25 Sep 2001), PRL (Jul 4, 2005), RPJ (18 May 1999), TMP (29 Apr 1999)**,** UFN (25 Feb 1999).

Unfortunately, the expression was rejected 400 times by 35 scientific journals during 10 years since 14 May 1998

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