A remark on an ansatz by M.W. Evans and the so-called Einstein-Cartan-Evans unified field theory

Friedrich W. Hehl

Institute for Theoretical Physics,

University at Cologne,

50923 Köln, Germany

and

Department of Physics and Astronomy,

University of Missouri-Columbia,

Columbia, MO 65211, USA

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M.W. Evans tried to relate the electromagnetic field strength to the torsion of a Riemann-Cartan spacetime. We show that this ansatz is untenable for at least two reasons: (i) Geometry: Torsion is related to the (external) translation group and cannot be linked to an internal group, like the $U(1)$ group of electrodynamics. (ii) Electrodynamics: The electromagnetic field strength as a 2-form carries 6 independent components, whereas Evans' electromagnetic construct F^α is a vector-valued 2-form with 24 independent components. This doesn't match. One of these reasons is already enough to disprove the ansatz of Evans.

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I. INTRODUCTION

In 2005, Evans [2] related electromagnetism to the torsion of spacetime. We came across this paper in the context of a refereeing process. We immediately recognized that the ansatz of Evans is shaky; in fact, it will turn out to be incorrect.

As a convenient starting point for our discussion, we can take the viable Einstein-Cartan theory of gravity. There the torsion becomes very transparent from a geometrical as well as from a physical point of view (Sec.II). Then, we come to the Evans ansatz (Sec.III) and show that it represents an overkill for the only 6 components of the electromagnetic field strength.

As a historical note we add that attempts in the direction of the Evans ansatz started in 1925 by Eyraud [3] and in 1926 by Infeld [8] and were shown to lead to nowhere, see Goenner [4] and Tonnelat [10].

II. EINSTEIN-CARTAN THEORY OF GRAVITY (ECT)

It is known from the literature that the Einstein-Cartan theory of gravity, see, for instance, [7, 11], is a *viable* theory of gravitation. Torsion, which geometrically is related to *translations* (it describes closure failures of infinitesimal parallelograms), is, according to the second field equation of the ECT, proportional to the spin angular momentum density of matter. This is not an ansatz, but the result of a variational principle of the Hilbert-Einstein type and of the Noether procedure identifying the right-hand-sides of the field equations as energy-momentum and spin angular momentum, respectively.

Let us shortly sketch the formalism, for details see [5]. We start form the coframe ϑ^α, the metric $g = g_{\alpha\beta}\,\vartheta^\alpha \otimes \vartheta^\beta$, and the connection $\Gamma_\alpha{}^\beta$. Here $\alpha, \beta, ... = 0, 1, 2, 3$ are anholonomic or frame indices. With the connection, we can define an exterior covariant derivative D. In the ECT, one assumes vanishing nonmetricity

$$Q_{\alpha\beta} := -Dg_{\alpha\beta} = 0\,. \tag{1}$$

The torsion T^α and the curvature $R_\alpha{}^\beta$ are defined by

$$T^\alpha \;:=\; D\vartheta^\alpha = d\vartheta^\alpha + \Gamma_\beta{}^\alpha \wedge \vartheta^\beta\,, \tag{2}$$

$$R_\alpha{}^\beta \;:=\; d\Gamma_a{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta\,. \tag{3}$$

These two quantities fulfill the first and the second Bianchi identities:

$$DT^\alpha \;\equiv\; R_\beta{}^\alpha \wedge \vartheta^\beta\,, \tag{4}$$

$$DR_a{}^\beta \;\equiv\; 0\,. \tag{5}$$

Both can be decomposed irreducibly under the local Lorentz group [6]. We define the 1-form $\eta_{\alpha\beta\gamma} := {}^\star(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\gamma)$, where the star denotes the Hodge operator. From (4) we pick the irreducible piece with 6 independent components,

$$DT^\gamma \wedge \eta_{\gamma\alpha\beta} \equiv R_\delta{}^\gamma \wedge \vartheta^\delta \wedge \eta_{\gamma\alpha\beta}\,, \tag{6}$$

and from (5) one with 4 independent components,

$$DR^{\beta\gamma} \wedge \eta_{\beta\gamma\alpha} \equiv 0\,. \tag{7}$$

We will come back to these equations below. The equations (1) to (7) are purely geometrical equations that, at this stage, have no relation to physics.

Studying small loops at some point of the manifold and parallelly transporting vectors, we learn that torsion is related to a translational misfit ($\rightarrow$ dislocations) and curvature to a rotational (or Lorentz) misfit ($\rightarrow$ disclinations), see [9], for example. In this sense, torsion and curvature are related to *external* groups, namely to the translation and to the Lorentz groups. A closer discussion shows that a Riemann-Cartan geometry can be gotten by studying the gauging of the Poincaré group, the semi-direct product of the translation and the Lorentz group. In other words, a Riemann-Cartan geometry is interrelated with the Poincaré group of the tangent Minkowski space. These geometrical facts definitely exclude the possibility to relate torsion to an *internal* group, like the U(1)-phase group of electrodynamics, for example. A proper understanding of geometry excludes such a possibility.

Physics is brought into the Riemann-Cartan spacetime by specifying a gravitational Lagrange 4-form as of the Hilbert-Einstein type according to $L_{\rm grav} = \frac{1}{2\kappa}{}^\star\big(\vartheta^\alpha \wedge \vartheta^\beta\big) \wedge R_{\alpha\beta}$, where κ is Einstein's gravitational constant. Then we find the following two field equations:

$$\frac{1}{2}\,\eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} \;=\; \kappa\Sigma_\alpha\,, \tag{8}$$

$$\frac{1}{2}\,\eta_{\alpha\beta\gamma} \wedge T^{\gamma} \;=\; \kappa\tau_{\alpha\beta}\,. \tag{9}$$

Here Σ_α is the energy-momentum 3-form of matter and $\tau_{\alpha\beta}$ the spin angular momentum 3-form of matter. Upon substitution of (8) and (9) into the irreducible pieces of the Bianchi identities (6) and (7), respectively, we recover the *energy-momentum* and the *angular momentum* laws [6]. In this way we see again the close connection of the ECT to the Noether theorem and the Poincaré group. Clearly, all this couldn't work if torsion (or curvature) would be identified with some electromagnetic field. Energy-momentum and angular momentum are related to translations and Lorentz rotations and under no circumstances to an internal group.

The ECT predicts the existence of a very weak spin-spin-contact interaction proportional to the gravitational constant. It doesn't show up at ordinary laboratory conditions. For vanishing material spin, $\tau_{\alpha\beta} = 0$, one recovers general relativity.

III. THE ANSATZ OF EVANS

Evans takes over the equations (1) to (5) without a proper motivation. As we pointed out above, in the framework of the ECT, the motivation lies in taking the Poincaré group of Minkowski space (which yields the mass-spin classification of elementary particles) as basis and then one gauges this group. This was exactly what Élie Cartan had in mind [1] when he called a Riemann-Cartan space as a space with *Euclidean connection*. In the small, the Riemann-Cartan space is just a Minkowski space. And these small Minkowski "grains" of a Riemann-Cartan space are translated and Lorentz rotated with respect to each other.

Evans instead just formally takes the equations (1) to (5) and brings in his physics by assuming that the coframe, apart from a constant scalar factor $A^{(0)}$, is related to an "electromagnetic potential" by the ad hoc ansatz

$$A^\alpha = A^{(0)}\vartheta^\alpha \,. \tag{10}$$

In coordinate components, we have

$$A_i{}^\alpha = A^{(0)} e_i{}^\alpha \,. \tag{11}$$

One should compare [2], Eq.(12). Evans denotes the components of the coframe by $q^a{}_\mu$. Clearly, the Evans potential A^α has 16 independent components, quite in contrast to the 4 components of the electromagnetic potential A of Maxwell's theory. Of course, the zeroth component of A^α, namely A^0, is *not* covariant under frame transformations and cannot feature as the Maxwellian potential. The same is true for A^1, A^2, or A^3 likewise. According to Evans, we have then for the Evans field strength

$$F^\alpha = DA^\alpha \qquad \text{or} \qquad F^\alpha = A^{(0)}\, D\vartheta^\alpha = A^{(0)}\, T^\alpha \,. \tag{12}$$

(i) Apart from the geometrical arguments which I gave, namely that torsion is related to the translation group (and not to the $U(1)$), (ii) it is also impossible to relate the 6 components of the Maxwell field strength 2-form $F := dA$ (here A and F are the quantities in Maxwell's theory) to the 24 components of the torsion 2-form T^α: They just don't match. And to take only one component of the torsion $T^0 = T_{ij}{}^0\, dx^i \wedge dx^j$ won't help either: It is *not* a covariant relation, in spite of Evan's claim to the contrary (see [2], page 10).

One could think that one decomposes the torsion T^α into its 3 irreducible pieces and attributes the Maxwellian F to one of these pieces. However, this is not possible since these pieces have 16+4+4 independent components, respectively. No irreducible piece has 6 independent components. Hence this possibility is ruled out.

"The Maxwell Heaviside theory is further restricted by the fact that it implicitly suppresses an index a, meaning that only one unwritten scalar components of the tangent bundle spacetime is considered, and then only implicitly" we are told ([2], page 10, last paragraph). It is clear that one cannot *implicitly suppress* the index α in (12) (in Evan's notation the index a) and then somehow get a covariant equation. This is just wishful thinking. With the frame e_α one could build the expression $e_\alpha \rfloor (DT^\alpha)$ and would find 6 independent components. However, we were still left with the 24 components of T^α without being able to reduce them in a covariant way to just 6 independent components.

The ansatz (10) is not only nonsensical from a geometric point of view, it has nothing to do with electrodynamics and with Maxwell's theory either.

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[1] É. Cartan: *On Manifolds with an Affine Connection and the Theory of General Relativity*, English translation of the French original (Bibliopolis, Napoli 1986).

[2] M.W. Evans, *The spinning and curving of spacetime: The electromagnetic and gravitational fields in the Evans unified field theory,* pdf-file supplied by M.W. Evans, dated 6 April 2005, obviously Found. Phys. Lett. 18 (2005) 431-454 (with a changed title).

[3] Eyraud, H., *La Théorie affine asymétrique du champs électromagnétique et gravifique et le rayonnement atomique*, C. R. Acad. Sci. (Paris) **180** (1925) 1245–1248.

[4] H.F.M. Goenner, *On the History of Unified Field Theories*, Living Rev. Relativity **7** (2004) (cited on 01 Dec 2006); http://www.livingreviews.org/lrr-2004-2H.Goenner

[5] F. Gronwald and F.W. Hehl, *On the gauge aspects of gravity,* in *Proc. Int. School of Cosm. & Gravit.* 14$^{\text{th}}$ Course: Quantum Gravity. Held in Erice, Italy. Proceedings, P.G. Bergmann et al. (eds.). World Scientific, Singapore (1996) pp. 148–198; arXiv.org/gr-qc/9602013.

[6] F.W. Hehl and J.D. McCrea, *Bianchi identities and the automatic conservation of energy–momentum and angular momentum in general-relativistic field theories,* Found. Phys. **16** (1986) 267–293.

[7] F.W. Hehl, P. von der Heyde, G.D. Kerlick, and J.M. Nester, *General relativity with spin and torsion: Foundations and prospects*, Rev. Mod. Phys. **48** (1976) 393–416.

[8] L. Infeld, *Zur Feldtheorie von Elektrizität und Gravitation*, Phys. Zeitschr. **29** (1928) 145–147.

[9] M.L. Ruggiero and A. Tartaglia, *Einstein-Cartan theory as a theory of defects in space-time* Am. J. Phys. **71** (2003) 1303–1313.

[10] M.A. Tonnelat, *La théorie du champ unifié d'Einstein et quelques-uns de ses développements,* Gauthier-Villars, Paris (1955).

[11] A. Trautman, *Einstein-Cartan theory,* in *Encyclopedia of Math. Physics*, J.-P. Francoise et al., eds., Elsevier, Oxford (2006) pp. 189–195; arXiv.org/gr-qc/0606062.

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