An Objection to Copenhagen interpretation and an explanation of the two-slit experiment from the viewpoint of waviness.

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Abstract

We suggest that the electron is a wave in the whole process between the electron gun and the sensor. Between the two-slit and the sensor, the following two phenomena happen to the waves: interference and Fraunhofer diffraction. Due to these two phenomena, a considerably sharp shape of wave is finally made in front of the sensor, and a bright spot appears on the sensor. The experiment result that a bright spot appears at random can be explained by the above-mentioned two phenomena and the "fluctuation" of the potential energy that the filament of the biprism makes. All are the wave motion phenomena, and, put simply, the particle called an electron does not exist.

1. Introduction

Richard Feynman once said that the double-slit experiment, which clearly shows both the particle and the wave nature of matter, contained the "heart of quantum mechanics" and was its "only mystery". Tonomura et al[1] actually conducted the two-slit experiment using an electron in an advanced research laboratory at Hitachi Limited, although it had been said that only a thought experiment (Gedanken experiment) was possible. This experiment was explained by the Copenhagen interpretation.

Tonomura et al writes: "According to the interpretation in quantum mechanics, a single electron can pass through both of the slits in a wave from so-called "probability amplitude" when the uncertainty of the electron position in the wall plane covers the two slits, and when no observation is made of the electron at either one of the slits. The electron is then detected as a particle at a point somewhere on the screen according to the probability distribution of the interference pattern."[1] This is the so-called Copenhagen interpretation.

The purpose of our study is to object to the Copenhagen interpretation and to explain this experiment by only waviness. Our analysis differs from the Copenhagen interpretation in that the wave function of Schrodinger equation is not "probability amplitude" but a "real wave".

2. The theory of two-slit experiment in advanced research laboratory, Hitachi Limited; Tonomura et al[1].

The biprism consists of two parallel grounded plates with a fine filament between them, the latter having a positive potential relative to the former. The

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electrostatic potential is given by $V(x, z)$ and the incoming electron wave by $\exp i(k_z z)$, the deflected wave is given by

$$\psi(x, z) = \exp i\left(k_z z - \frac{me}{\hbar^2 k_z}\int_{-\infty}^{z} V(x, z')\, dz'\right), \tag{1}$$

The two waves having passed on each side of the filament can be approximated by $\exp i(k_z z \pm k_x x)$ up to a constant factor, where

$$k_x = -\frac{me}{\hbar^2 k_z}\int_{-\infty}^{\infty}\left(\frac{\partial V(x, z')}{\partial x}\right)_{x=a} dz', \tag{2}$$

and the symmetry $V(x, z) = V(-x, z)$ has been taken into account. This can be interpreted classically also:
$-e\left[\partial V(x, z')/\partial x\right]_{x=a}$ is the x component of force exerted on the electron. Its integral with respect to $dz/v_z = dt$ ($v_z = \hbar k_z/m$) gives the impulse imparted to it, which is the same in absolute value but reversed in sign, depending on which side of the filament the electron passes. If the two waves overlap in the observation plane to give

$$\psi(x, z) = \exp(ik_z z)\left[\exp(-ik_x x) + \exp(ik_x x)\right], \tag{3}$$

then this leads to the interference fringes

$$|\psi(x, z)|^2 = 4\cos^2 k_x x. \tag{4}$$

The Copenhagen interpretation has concluded that $\psi(x, z)$ is the "probability amplitude".

3. Discussion

3.1 Alternative explanation to Copenhagen interpretation: Waviness

The first point to be discussed is, in this experiment, the source of fluctuation is potential energy made by the filament at the center of biprism and the electrode. (For details please refer to 3.2 The reason why the potential energy made by the filament of biprism fluctuates.) In a word, the source of fluctuation is $V(x, z')$ of expression (1)

$$\psi(x, z) = \exp i\left(k_z z - \frac{me}{\hbar^2 k_z}\int_{-\infty}^{z} V(x, z')\, dz'\right)$$

of the Tonomura thesis. Because this fluctuation is a Brownian motion as considered later, it is not symmetric. Because this potential energy fluctuates, k_x of the expression (2)

$$k_x = -\frac{me}{\hbar^2 k_z}\int_{-\infty}^{\infty}\left(\frac{\partial V(x, z')}{\partial x}\right)_{x=a} dz'$$

of the Tonomura thesis is a different value in each wave that has occurred from biprism. Expression (2) means that k_x is determined by the accumulation of the impulse that the electron wave received from potential energy when it went in the biprism. In general, because potential energy fluctuates, the impulse that the first wave received in the biprism and the impulse that the second wave received are different. As a result, the value of k_x is different according to each wave. The interference fringes fluctuate at time because there is k_x also in the expression (4) of the Tonomura thesis

$$|\psi(x, z)|^2 = 4\cos^2 k_x x$$

that shows interference fringes. As shown by the expression (3)

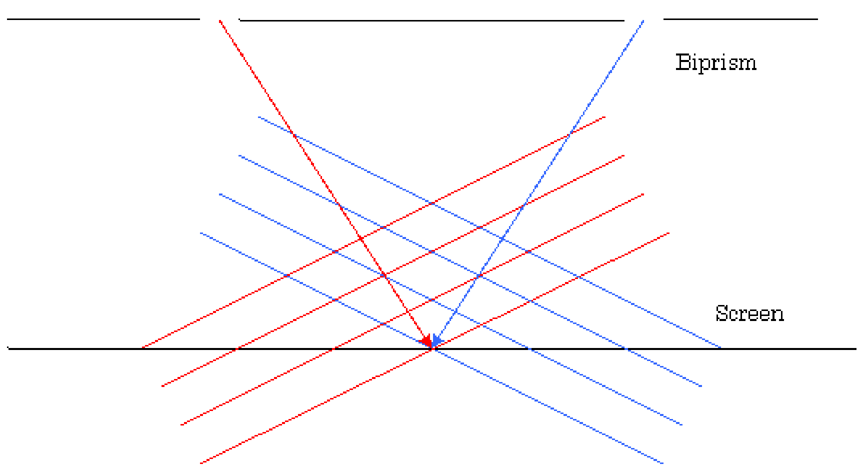
$\psi(x,z) = \exp(ik_z z)\left[\exp(-ik_x x) + \exp(ik_x x)\right]$
of the Tonomura thesis, there is k_x in the wave function that is reaching the screen from right and left biprism. So the phase of wave fluctuates because it receives the influence of fluctuation of potential energy.

The appearance of the interference when wave number k_x fluctuates is seen as follows.

By the expression (3) of the Tonomura thesis, the wave number vector of the wave that comes from the left can be written as $(k_x, 0, k_z)$ and the wave number vector that comes from the right, $(-k_x, 0, k_z)$.

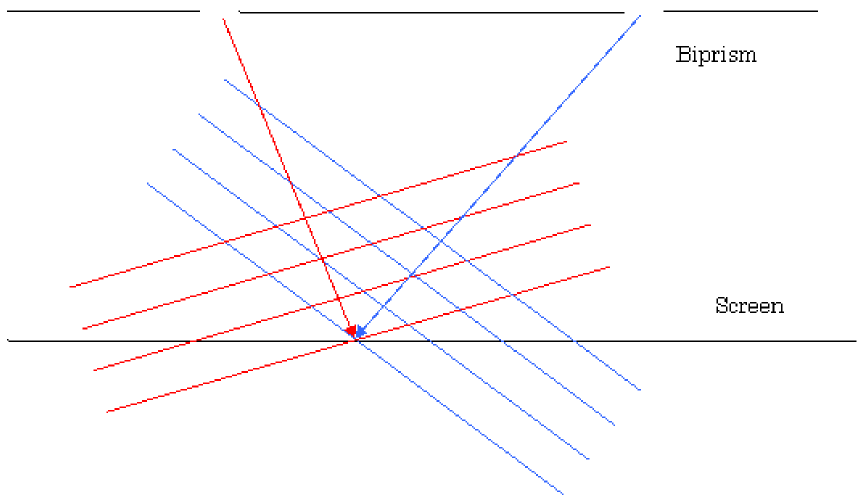
In general, because potential energy $V(x, z')$ doesn't fluctuate symmetrically, neither k_x from the left nor k_x from the right are equal. Then, the wave number vector of the wave that comes from the left is written as $(k_x(L), 0, k_z)$, and the wave number vector of the wave that comes from the right is written as $(-k_x(R), 0, k_z)$. It becomes a different value because of the first wave, the second wave, and the third wave even if paying attention only to $k_x(L)$ (paying attention only to $k_x(R)$).

Figure 1. When the wave number $k_x(L)$ that comes from the left and the wave number $k_x(R)$ that comes from the right are equal:



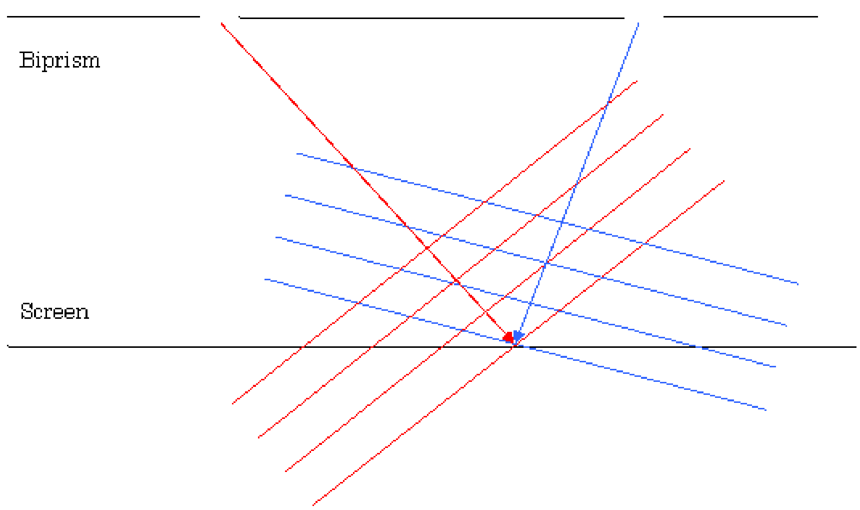
Bright spot appears at the center of the screen.

Figure 2: When the wave number $k_x(R)$ that comes from the right is larger than the Wave number $k_x(L)$ that comes from the left:



Bright spot shifts left.

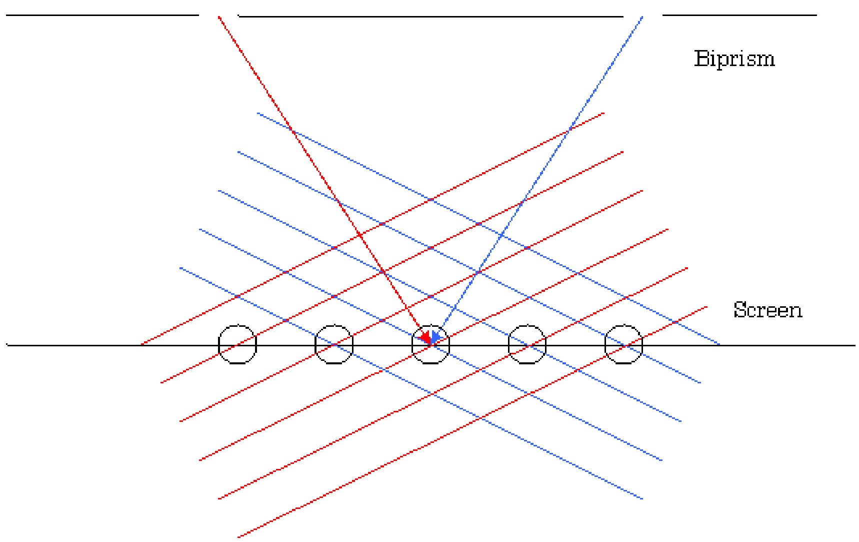
Figure 3: When the wave number $k_x\,(R)$ that comes from the right is smaller than the Wave number $k_x\,(L)$ that comes from the left:



Bright spot shifts right.

We suggest that this is the mechanism by which the bright spot is observed at a random position. Because the potential energy that the filament makes fluctuates, the wave number $k_x\,(L)$ of the wave that passes the left prism sometimes becomes large and at other times it becomes small. The wave number

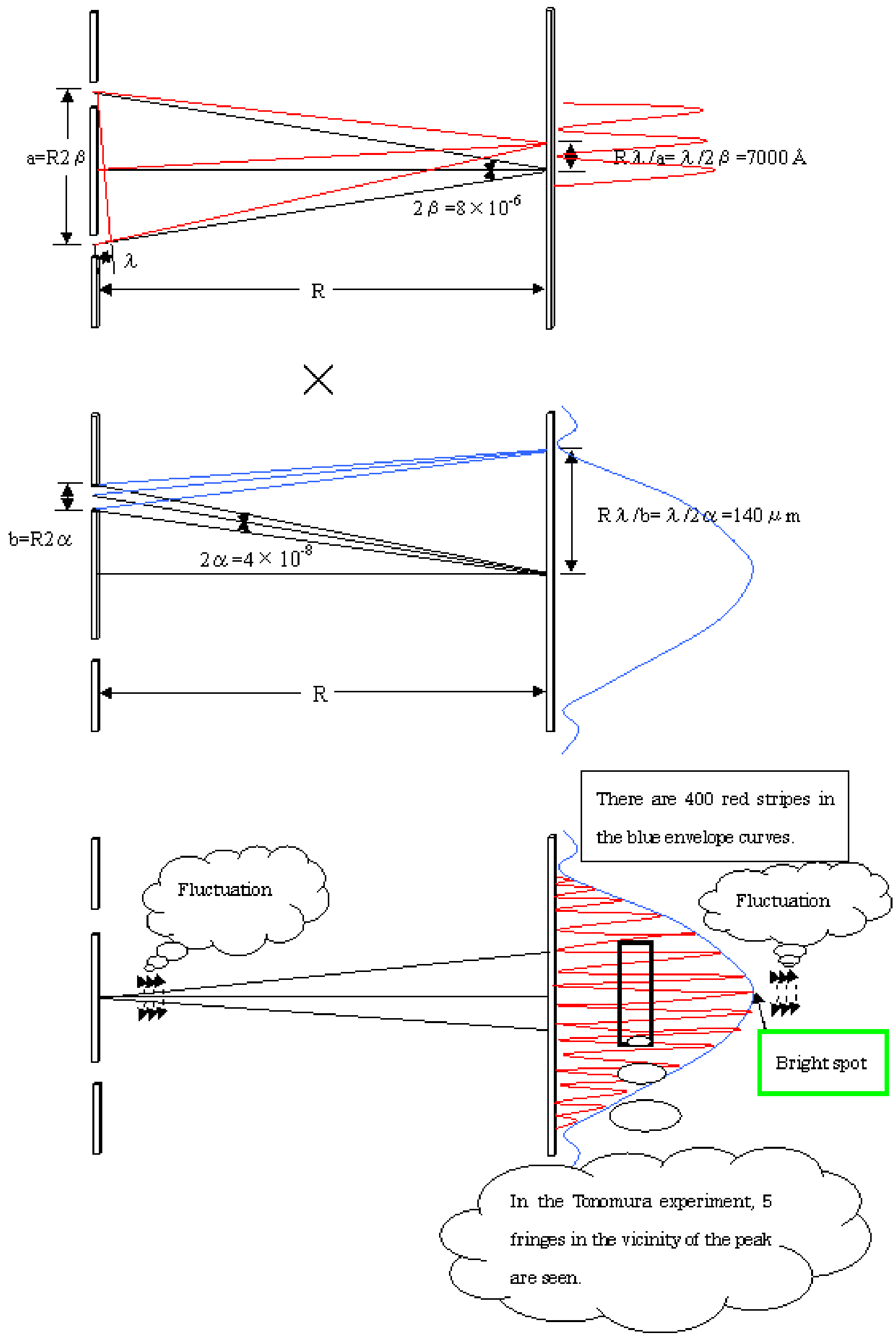
$k_x\,(R)$ of the wave that passes the right prism is also similar. As a result, the position that two waves strengthen each other is different on each occasion as shown in the above figure. In current quantum theory, only the case of Figure 1 is considered, and it is said that the place enclosed in the following figure is a position of the interference fringes.



It follows from this that our interpretation is different from the current interpretation of the quantum theory in that it becomes a very dynamic image like two moving searchlights independently scattering waves of light into the night sky. On the other hand, the image of current quantum theory is very static.

The second point that requires clarification is why it is observed as "a spot" in the experiment when the electron wave is weakened. The reason why it is observed as "a spot" is that the effect of the diffraction (so-called Fraunhofer diffraction) exacerbates the above-mentioned interference because the opening of biprism is not the ideal one like the delta function but has some size in an actual experiment. Therefore, strength of the electron wave on the screen becomes narrowed shape like the interference fringes shown by cos function narrowed by *sinc* function ($\sin x/x$).

(Refer to the figure below. A part of the numerical value is excerpted from the Tonomura thesis.)



Strength of the electron wave on the screen becomes shaped like a sliced mountain. There were an estimated 400 slices in the Tonomura experiment. In addition, only a very narrow area (center part of Airy disk so-called) in the top of a mountain will reflect because the pictures in this experiment were taken with very limited sensitivity. It is concluded that this is the bright spot observed on the screen.

Furthermore, the top of the mountain shakes at random due to the above-mentioned fluctuation. The peak of the distribution of the Fraunhofer diffraction appears at random because the potential energy (electric field) fluctuates and the electron wave fluctuates. Also, because the electron wave discharged from the electron gun is weak, only the part of the peak is taken of a picture.

Up to this point We have explained the two-slit experiment by only waviness.

3.2 The reason why the potential energy made by the filament of biprism fluctuates.

The reason why the potential energy made by the filament fluctuates is as follows. If v_z is deleted from

$dz/v_z = dt$, $v_z = \hbar k_z/m$

that exists between the expression (2) and the expression (3) of the Tonomura thesis, it becomes $\frac{m}{\hbar k_z}dz = dt$. (Refer to Appendix A "Brownian motion of the path of the electron" for an accurate expression.) If this expression is substituted for the expression (1) of the Tonomura thesis, it becomes

$$\psi\left(x,z\right) = \exp i\left(k_z z - \frac{1}{\hbar}\int_{t_0}^{t} eV\left(x,z'\right)dt'\right).$$

And the expression (2) of the Tonomura thesis becomes

$$k_x = -\frac{1}{\hbar}\int_{t_0}^{t} e\left(\frac{\partial V\left(x,z'\right)}{\partial x}\right)_{x=a} dt'.$$

k_x fluctuates as explained in the Appendix A. In the Tonomura thesis, because the kinetic energy of an incidence wave

$$\frac{\hbar^2 k_z^2}{2m} = \frac{p_z^2}{2m} = \frac{1}{2}mv_z^2$$

and the kinetic energy of the scattered wave

$$\frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} = \frac{1}{2}mv^2 \text{ (Here, } v = (v_x, 0, v_z))$$

are omitted, the wave function of Schrodinger equation is originally written

$$\psi\left(x,z\right) = \exp i\left(k_z z - \frac{\hbar k_z^2}{2m}t_1 + \frac{1}{\hbar}\int_{t_1}^{t}\left[\frac{1}{2}mv^2 - eV\left(x,z'\right)\right]dt'\right)$$

$$= \exp i\left(\frac{1}{\hbar}\int_{t_1}^{t}\left[\frac{1}{2}mv^2 - eV\left(x,z'\right)\right]dt'\right)\exp i\left(k_z z - \frac{\hbar k_z^2}{2m}t_1\right).$$

t_1 is time until wave reaches biprism. (Refer to Appendix B "Accurate discussion by Path integral formulation" for an accurate expression that uses the Path integral formulation.)

When reading this expression from the right to the left, this expression can be read that when the plane wave that was emitted from the source reaches the biprism, it is scattered by the potential energy of biprism, and it faces the screen. Because there is

$$\exp i\left(\frac{1}{\hbar}\int_{t_1}^{t}\frac{1}{2}mv^2dt'\right)$$

in this expression, the leveled operation enters the potential energy

$$\Phi=\exp\left(-\frac{i}{\hbar}\int_{t_1}^{t}eV\left(x,z'\right)dt'\right)$$

that fluctuates at random or the wave

$$\psi_1=\exp i\left(-\frac{1}{\hbar}\int_{t_1}^{t}\left[eV\left(x,z'\right)\right]dt'\right)\exp i\left(k_z z-\frac{\hbar k_z^2}{2m}t_1\right)$$

that fluctuates at random by this potential energy. (For details please refer to 3.3 The reason why the kinetic energy exponential part corresponds to the normal distribution function.) In other words, it is

$$\psi\left(x,z\right)=E\left[\psi_1\right]=E\left[\exp i\left(-\frac{1}{\hbar}\int_{t_1}^{t}\left[eV\left(x,z'\right)\right]dt'\right)\exp i\left(k_z z-\frac{\hbar k_z^2}{2m}t_1\right)\right].$$

$E\left[\cdots\right]$ is to mean the expected value is taken.

The solution ψ of Schrodinger equation is the average (expected value) of this random field Φ or the average of the wave ψ_1 that fluctuates at random. The expected value is taken by the normal distribution function in case of non-relativity as understood later. (The normal distribution function is one that the frequency of the random walk (Brownian motion) is infinitely increased.) In other words, fluctuation of Φ can be modeled by the Brownian motion. Φ or potential energy $V\left(x,z'\right)$ fluctuates like this.

Put simply, the wave function ψ that is the solution of Schrodinger equation is an average of the fluctuating wave ψ_1, and the approximate one.

To quote David Bohm: "Fluctuation of the field is not constant but at random. The value of the field described by the quantum theory is the one leveled during a certain time. Fluctuation of the field originates in the more subordinate level than the level that the quantum mechanics targets. This is just the same as Brownian motion of small liquid drop originates in an atomic level that exists in more subordinate position. And, like Newton equation expresses average behavior of liquid drop, Schrodinger wave equation expresses average behavior of the field."[2]

In other words, the two-slit experiment in advanced research laboratory, Hitachi, Ltd is a splendid experiment that visualizes and proves the quantum field fluctuation that David Bohm pointed out.

The quantum field fluctuation is the one that exists in a deeper level than the wave function of Schrodinger equation.

3.3. The reason why the kinetic energy exponential part corresponds to the normal distribution function.

It is because of becoming the normal distribution function when the part of

$$\int Dx\exp\left(\frac{i}{\hbar}\int\frac{1}{2}mv^2dt\right)$$

that corresponds to the kinetic energy is transformed a little. (This is a technique that Feynman often does in "Quantum mechanics and path integrals"[3], etc.)

$$\sqrt{\frac{m}{2\pi\hbar i\Delta t}}\exp\left(-\frac{m}{2\hbar i\Delta t}\left(x_{j+1}-x_j\right)^2\right)$$

shows it. When this and normal distribution function

$$\frac{1}{\sqrt{2\pi\sigma^2 t}}\exp\left(-\frac{x^2}{2\sigma^2 t}\right)$$

are compared, if it is assumed

$$\sigma=\sqrt{\frac{\hbar}{m}}$$

and makes time an imaginary number, these are corresponding. Integrate the kinetic energy with respect to time, put it on the shoulder of exponential, make time an imaginary number, then this exponential part becomes a normal distribution function. It is an interesting point of Feynman's path integrals.

In general, the expected value $E\left[f\left(\xi\right)\right]$ of the function $f\left(\xi\right)$ shown by the random variable ξ that fluctuates by normal distribution is

$$E\left[f\left(\xi\right)\right]=\int\frac{1}{\sqrt{2\pi\sigma^2 t}}\exp\left(-\frac{\xi^2}{2\sigma^2 t}\right)f\left(\xi\right)d\xi.$$

When path integrals of Feynman is used, the solution ψ of Schrodinger equation can be generally solved with

$$\psi=E\left[\exp\left(-\frac{i}{\hbar}\int_{t_0}^{t}V\left(x\left(t'\right)\right)dt'\right)\psi\left(t_0\right)\right].$$

Getting the expected value is due to the fact that the potential energy fluctuates. This is an expression of so-called Feynman-Kac (Feynman-Kac-Nelson) formula. (Please refer to "Techniques and Applications of Path Integration"[4].)

4. Conclusion

In the two-slit experiment, the wave number vector of each wave that occurs from biprism fluctuates by normal distribution. The wave that occurred from biprism is launched in various directions for this fluctuation. This fluctuation is expressed by the probability distribution of normal distribution.

In conclusion, We should note that while in present quantum mechanics it is the wave function that determines the probability distribution of the electron, our observations show that it is not the wave function but this kinetic energy exponential part that determines the probability distribution. As a result, wave motion itself fluctuates. Furthermore, the bright spot observed on the screen is not "an electron" but the peak of the distribution of the Fraunhofer diffraction.

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Appendix A "Brownian motion of the path of the electron"

Because we cannot differentiate the path in Feynman's path integrals (Or, you may say the path of the electron) here and there, we must not express as $dz/v_z = dt$ or $dz = v_z dt$.
It is originally

$$dz = v_z dt + \sqrt{\frac{\hbar}{m}}\sqrt{dt}\xi$$

$\xi \cdots$Standard regular random variable.
So, it is necessary to consider a Brownian motion paragraph. Therefore, it becomes

$$\frac{m}{\hbar k_z}dz = \frac{m}{\hbar k_z}v_z dt + \frac{m}{\hbar k_z}\sqrt{\frac{\hbar}{m}}\sqrt{dt}\xi = dt + \frac{1}{k_z}\sqrt{\frac{m}{\hbar}}\sqrt{dt}\xi.$$

So the impulse that electron receives is not

$$-e\left[\partial V\left(x,z'\right)/\partial x\right]_{x=a}dt$$

but

$$-e\left[\partial V\left(x,z'\right)/\partial x\right]_{x=a}\left(dt + \frac{1}{k_z}\sqrt{\frac{m}{\hbar}}\sqrt{dt}\xi\right).$$

This is the cause of fluctuation. The momentum of electron fluctuates by this fluctuation of impulse, and the wave number vector of electron also fluctuates.

Appendix B "Accurate discussion by Path integral formulation"

$$\psi = \int DxDp\exp\left(\frac{i}{\hbar}\int\left[\dot{p}x - H\left(p,x\right)\right]dt\right)\cdots\text{Hamiltonian Path Integrals}$$

$$\psi = \int Dx\exp\left(\frac{i}{\hbar}\int L\left(x,\dot{x}\right)dt\right)\cdots\text{Lagrangian Path Integrals}$$

$$K\left(b,a\right) = \int_a^b Dx\left(t\right)e^{(i/\hbar)S[b,a]} = \int_a^b Dx\left(t\right)\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}dt\left[\frac{m}{2}\dot{x}\left(t\right)^2 - V\left(x\right)\right]\right)\cdots\text{Propagator}$$

(Wave function)
Wave function that reaches a point b in slit from source a

$$K\left(b,a\right) = \int_a^b DzDp_z\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\left[p_z\dot{z} - \frac{p_z^2}{2m}\right]dt\right) = \int_a^b Dz\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\frac{1}{2}mv_z^2 dt\right)$$

Here, $p_z = \hbar k_z$
Wave function that reaches a point on the screen from a point in slit

$$K\left(c,b\right) = \int_b^c DxDzDp_xDp_z\exp\left(\frac{i}{\hbar}\int_{t_b}^{t_c}\left[p_x\dot{x} + p_z\dot{z} - \frac{p_x^2}{2m} - \frac{p_z^2}{2m} - V\left(x,z\right)\right]dt\right)$$

$$= \int_b^c DxDz\exp\left(\frac{i}{\hbar}\int_{t_b}^{t_c}\left[\frac{1}{2}mv_x^2 + \frac{1}{2}mv_z^2 - V\left(x,z\right)\right]dt\right)$$

Therefore, if the idea of Feynman's path integrals is used, the product of each wave is added by the width of the slit and becomes a wave function that reaches the screen from the source.

$$K\left(c,a\right) = \int_{x_b} dx_b K\left(c,b\right)K\left(b,a\right)$$

$$= \int_{x_b} dx_b\int_b^c DxDz\exp\left(\frac{i}{\hbar}\int_{t_b}^{t_c}\left[\frac{1}{2}mv_x^2 + \frac{1}{2}mv_z^2 - V\left(x,z\right)\right]dt\right)\int_a^b Dz\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\frac{1}{2}mv_z^2 dt\right)$$

$$= \int_{x_b} dx_b\int_b^c DxDz\exp\left(\frac{i}{\hbar}\int_{t_b}^{t_c}\left[\frac{1}{2}mv_x^2 + \frac{1}{2}mv_z^2 - V\left(x,z\right)\right]dt\right)\int_a^b DzDp_z\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\left[p_z\dot{z} - \frac{p_z^2}{2m}\right]dt\right)$$

The phase of the wave function fluctuates when potential energy $V(x, z)$ fluctuates, and, as a result, the position of the top of mountain of the interfered wave fluctuates.

References

[1]A.Tonomura, J.Endo, T.Matsuda, T.Kawasaki, and H.Ezawa, Demonstration of single-electron buildup of an interference pattern, [American Journal of Physics 57, 117-120 (1989)]

[2]David Bohm, Wholeness and the implicate order, 1980

[3]R.P. Feynman/A.R. Hibbs, Quantum mechanics and path integrals, (McGraw-Hill)

[4]L.S. Schulman, Techniques and Applications of Path Integration, (John Wiley & Sons)