The Simple Solutions of Four Actual Problems of General Theory of Relativity.

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Abstract: It is quite complicated and difficult generally to solve the problem of the general theory of relativity. We apply the equations of quantitative effect of general theory of relativity and kinematic principle of double-mass which is new-proposed to solve simply four actual problems: gravitational red shift of spectral line, delay of radar echo, perihelion precession of planet, deflection of light in gravitational field, and the results are identical with the formulae that is derived by general method.

Key words: General theory of relativity, Equation of quantitative effect, Kinematic principle of double-mass, Four actual problems

We know that the space-time of general relativity is twisted into Riemann geometric continuum by the matter with mass. So the formulae of problems of general relativity are quite complicated generally and are difficult to solve. Then whether these problems can be simplified by new thinking and way, below we will make a try for it.

1, The equations of quantitative effect of relativity theory and kinematic principle of double-mass

The general relativity considered that the unit length dr or unit time dt or mass m can vary with speed u :

$$dt = \frac{dt_0}{\sqrt{1-u^2/c^2}} \approx \left(1+\frac{u^2}{2c^2}\right)dt_0 \tag{1}$$

$$dr = \sqrt{1-u^2/c^2}\,dr_0 \approx \left(1-\frac{u^2}{2c^2}\right)dr_0 \tag{2}$$

$$m = \frac{m_0}{\sqrt{1-u^2/c^2}} \approx \left(1+\frac{u^2}{2c^2}\right)m_0 \tag{3}$$

dt_0, dr_0 and m_0 are the proper unit time length and mass. The (1), (2) and (3) are the equations of quantitative effect in the special theory of relativity.

The general theory of relativity considers that the unit time and length can vary with gravitational potential, which may be calculate through the principle of equivalence and conservation of energy: in isolated gravitational field of a heavenly body with mass M, let a object with mass m falls free towards the heavenly body from where infinite far, the beginning velocity is zero, and its velocity is u when the object is r away from the heavenly body and where the gravitational potential is φ (it is the zero where infinite far), then

$\frac{1}{2}mu^2 + m\varphi = 0$， namely

$$\varphi = -\frac{1}{2}u^2 \tag{4}$$

Substitute (4) into (1), (2) and (3), we obtain (G is the gravitational constant):

$$dt = \frac{dt_0}{\sqrt{1+2\varphi/c^2}} \approx \left(1-\frac{\varphi}{c^2}\right)dt_0 = \left(1+\frac{GM}{c^2 r}\right)dt_0 \tag{5}$$

$$dr = \sqrt{1+2\varphi/c^2}\,dr_0 \approx \left(1+\frac{\varphi}{c^2}\right)dr_0 = \left(1-\frac{GM}{c^2 r}\right)dr_0 \tag{6}$$

$$m = \frac{m_0}{\sqrt{1+2\varphi/c^2}} \approx (1-\frac{\varphi}{c^2})m_0 = \left(1+\frac{GM}{c^2 r}\right)m_0 \tag{7}$$

The (5), (6) and (7) are the equations of quantitative effect in the general theory of relativity.

The (7) shows that the mass of object can be increased by the addition of gravitational potential. So can say there are two masses: the proper mass m_0 and the effect mass $\frac{\varphi}{c^2}m_0$. The quantitative relation of two masses is decided by (7), they can not interchange as between kinetic energy and potential energy. Now we decompose a motion into proper motion of proper mass and effect motion of effect mass, then what have relations between that two? First the system of proper motion does not be changed by effect motion, while is changed the direction. For example, the system of proper motion of planet is the ellipse, the existence of planet effect mass does not change the form of ellipse, while makes whole ellipse rotated slowly, namely the precession; otherwise the mass is directly proportional to the energy, so the ratio of energy is equal to the ratio of mass for two object. Generally, the ratio of energy is equal to the ratio of speed square; while the movement of heavenly body, which do not include the movement that is caused by the exploder of celestial body, is the movement in the gravitational interaction, the proper and effect motion of a object are the motion in same gravitational interaction, their energy are the ability of making work at this condition, so the ratio of speed or route of two mass motion is equal the ratio of two mass or energy. It is called “kinematic principle of double-masses”.

Kinematic principle of double-masses: The object motion in gravitational field can be decomposed into proper motion of proper mass and effect motion of effect mass; the system of proper motion does not be changed by effect motion, which changes only that direction; the ratio of the velocity (or route) of effect motion to the velocity (or route) of proper motion is equal to the ratio of two masses (or energies), and is approximately $\frac{\varphi}{c^2}$.

Whether the kinematic principle of double-masses can be established, which should ground on whether it can be identical with facts. The following, it is used to calculate the perihelion precession of planet and deflection of light in gravitational field, the results are identical with the approximate formulae that is derived by the general theory of relativity, which shows that it is reasonable.

2, **The derivation of the formula of perihelion precession of planet**

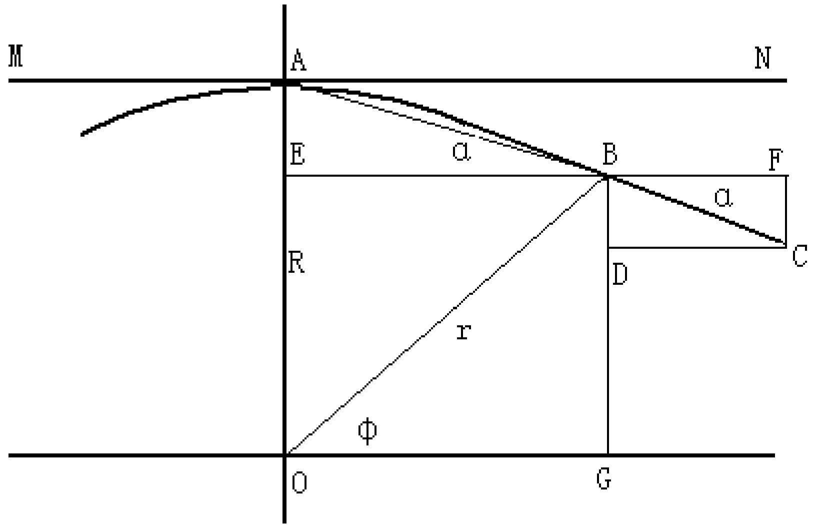
About the perihelion precession of planet, above had pointed out the effect motion has only relevance to precession. Here the effect energy is as extra kinetic energy of angular direction, which makes the angle that the vector radius rotates is not 2π, and is $2\pi+\alpha$ when the planet accomplishes a period's elliptic motion, the α is just the angle of precession. Both of two action are same direction and step between kinetic energy of angular direction of the extra and proper motion, then the precession angle can be derived simple: to derive the ratio of angular direction kinetic energy between precession and proper motion, then the precession angle can be derived in proportion as the ratio when the planet accomplishes a period's elliptic motion.

When the effect motion does not be considered, the angular direction motion is made by angular direction kinetic energy of proper motion. First we derive the proportion of the angular direction kinetic energy to the sum of energy.

For circular orbit, all of kinetic energy are the angular direction kinetic energy, which is half of potential energy, or the angular direction kinetic energy is $\frac{1}{3}$ the sum of energy. For elliptic orbit, part of kinetic energy become the radial direction kinetic energy, which has not relevance to angular direction motion. when the planet lies to the aphelion, its kinetic energy is the least, and is $\frac{GMm}{2(a+c)}$ (G is gravitational constant; M is the solar mass; m is the planet mass; a is the half length of long axis; c is the half of focal length); when the planet lies to the perihelion, its kinetic energy is the largest, and is $\frac{GMm}{2(a-c)}$, so the average kinetic energy of elliptic motion is: $\frac{1}{4}GMm\left(\frac{1}{a-c}+\frac{1}{a+c}\right)=\frac{GMm}{2a\left(1-e^2\right)}$, ($e$ is the eccentricity); while the kinetic energy of circular motion with radius a is $\frac{GMm}{2a}$, which is $1-e^2$ time as much as the average kinetic energy of elliptic motion. Therefore the angular direction kinetic energy of proper motion is approximately $\frac{1-e^2}{3}E$ (E is the sum of energy of proper motion).

According to the kinematic principle of double-masses, the angular direction kinetic energy of planet precession is $\frac{\varphi}{c^2}E$, while the angular direction kinetic energy of proper motion is $\frac{1-e^2}{3}E$, the ratio of them is $\frac{3\varphi}{c^2\left(1-e^2\right)}$, so the angle of precession is $\frac{2\pi\times 3\varphi}{c^2\left(1-e^2\right)}=\frac{6\pi\left(2\pi a/T\right)^2}{c^2\left(1-e^2\right)}=\frac{24\pi^3a^2}{\left(1-e^2\right)c^2T^2}$ (T is the time that the planet goes one period), which is identical with the formulae that is derived by the general theory of relativity.

3, **The derivation of deflection of light in gravitational field**



The sketch map of deflection of
light in gravitational field

Look on the sketch map, if it were without the gravitational field, the photon would move along level line MAN; and actually the photon moves along curve ABC, which is an approximate line and the angle between ABC and MAN is very small actually; the O is the mass centre of heavenly body; R=AO is radius of the heavenly body; MN∥EF∥DC∥OG.

When the photon reach an arbitrary point B on the ABC, ∠FBC=α is the total angle of light deflection. Because the ABC is an approximate line and the light deflection is occurred chiefly nearby the point A, therefore the line AB is in line with the curve BC and its tangent line on point B approximately, when the point B is quite far away the point A. Then ∠ABE=∠FBC=α,

$tg\alpha = \dfrac{AE}{BE} = \dfrac{R - r\sin\phi}{r\cos\phi}$, $r\sin\phi = \dfrac{R}{1 + ctg\phi tg\alpha}$ 。When $\phi = \alpha$, because the angle of deflection of light is very small, the α is equal accurately enough the total angle of light deflection that the photon passes through the whole gravitational field from the point A, at this time,

$$r\sin\phi = \frac{R}{2} \tag{8}$$

The (7) shows that although the photon has not static mass and has only dynamic mass, yet it can produce the effect of general relativity, and may apply the kinematic principle of double-mass. Then the photon's motion can be decomposed into proper motion and effect motion, its proper motion is linear motion with unvaried velocity; its effect motion does not change the speed of proper motion and changes only the direction of that, so it is always perpendicular to the direction of proper motion, the energy ratio between two motions is $\dfrac{GM}{c^2 r}$ (M is the mass of the heavenly body, r is the length of OB), which is a value in a moment. Now we consider another condition: let the mass centre of a heavenly body with same mass and radius BG is in the point G, and the photon passes levelly the point B, in this time, the BF indicates the light velocity; the BD indicates the speed of effect motion, according to the kinematic principle of double-masses, obtain: $\frac{BD}{BF} = \frac{GM}{c^2 BG}$, the deflection angle is just the α. That is to say, the kinematic principle of double-masses can be apply at point B: substitutes level direction for the direction of light; substitutes total deflection angle for the deflection angle in a moment; substitutes $r\sin\phi$ for r. Then using the (8), the total deflection angle that the photon passes trough the whole gravitational field from point A is: $\alpha \approx tg\alpha = \frac{BD}{BF} = \frac{GM}{c^2 r\sin\phi} = \frac{2GM}{c^2 R}$. The moved loci of light before and after point A are symmetric, so the total deflection angle is: $\frac{4GM}{c^2 R}$, which is identical with the approximate formulae that is derived by the general theory of relativity.

4, **Gravitational red shift of spectral-line and gravitational delay of light**

The (5) shows that the time is influenced by gravitational potential. Therefore, the gravitational red shift of spectral-line can be understood: the proper frequency v_0 is invariable, only because the clock in gravitational field goes slower than the clock where is without gravitational field, therefore using clock measures same photon, the frequency of photon in gravitational field is higher than that in without gravitational field. Einstein considers that the frequency of photon is as the number of ticks of the clock per unit time[1], so the frequency of photon $v = \frac{v_0}{\sqrt{1-2\varphi/c^2}} \approx \left(1+\frac{\varphi}{c^2}\right)v_0$. Then using the space-time standard where the photon is located measures the photon which moves towards the direction that the gravitational potential is less, the spectral-line of photon is going red shift.

About the gravitational delay of light, which is analyzed and solved comprehensively in the book Gravitation and Spacetime[2], which points out that the cause of gravitational delay of light is: deflection of light in gravitational field and the light’s velocity is lowered by gravitational field. The route’s addition that is caused by the former is very little, is a two-level amendment, and can be omitted, the major is the latter. In the general theory of relativity, the light velocity is also invariable, but the time-space standard is not united in the gravitational field, if we measure the light velocity of some point in the gravitational field using respective time-space standard, the light velocity will be constant c, while if we measured the light velocity of gravitational field using the space-time standard that is far away the gravitational field, the light velocity would be lowered. In this book, the actual light’s velocity is calculated out by the field equation of general theory of relativity through many steps, the result is: $c_0 = 1 - \frac{2GM}{r}$, (here c =1), then the time of gravitational delay can be derived by the methods of calculus. In fact, applying the (5) and (6) can derive the light’s velocity in the gravitational field, which is to transfer quantitative velocity unit dr/dt into proper velocity unit dr_0/dt_0:

$$dr/dt = \frac{\sqrt{1-2\varphi/c^2}\,dr_0}{dt_0/\sqrt{1-2\varphi/c^2}} = (1-2\varphi/c^2)dr_0/dt_0$$

Then proper light's velocity is: $c_0 = (1-2\varphi/c^2)c = (1-\frac{2GM}{c^2 r})c$. From this, the time of gravitational delay of light can be derived by the methods of calculus.

The equations of quantitative effect in the general theory of relativity are used to solve simply four actual problems of the general theory of relativity on the basis of flat space, which don't mean that the space-time is flat, because when using the general theory of relativity solves these problems, the equation is linearized, which can be considered as that the curved space is flatted. But a probability can not be removed: the space-time should be flat, and the Riemann space is only a mathematical model; the proper physical quantity in relativity theory, which have not relevance to speed and gravitational potential, are Newtonian physical quantity in absolute space-time theory; only the relativity theory has amended quantitatively Newtonian physical quantity, which is reflected by equations of quantitative effect.

References

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