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Confirmations of Reality of the Electrodynamics' Spin Tensor

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Abstract: Theoretical and experimental results prove that classical electrodynamics' spin described by a nonconventional spin tensor must be taken into account when optically driven micromachines are under consideration. This fact contradicts the standard electrodynamics.

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As is well known, photons carry spin, energy, momentum and angular momentum that is a moment of the momentum relative to a given point or to a given axis. Energy and momentum of electromagnetic waves are described by the Maxwell energy-momentum tensor (density) $T^{\lambda\mu}$. The angular momentum can be defined as

$$L^{ij} = \int_V 2x^{[i}T^{j]0}dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV\,, \tag{1}$$

and this construction must be named as an orbital angular momentum. However, the modern electrodynamics has no describing of spin. The canonical spin tensor

, $\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\dfrac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}$, is invalid, and physicists eliminate it by the Belinfante-Rosenfeld procedure. As a result, the modern classical electrodynamics spin tensor equals zero. Nevertheless, physicists understand they cannot shut eyes on existence of the electrodynamics spin. And they proclaim spin is *in* the moment of the momentum (1). I.e., the moment of momentum represents the total angular momentum, orbital angular momentum plus spin. I.e., equation (1) encompasses both the spin and orbital angular momentum density of a light beam:

$$J^{ij} = L^{ij} + S^{ij} = \int_V 2x^{[i}T^{j]0}dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV\,, \tag{2}$$

Contrary to this paradigm, we introduce a spin tensor $\Upsilon^{\lambda\mu\nu}$ into the modern electrodynamics, [1-7], i.e. we complete the electrodynamics by introducing the spin tensor and claim equation (2) is wrong, i.e. we claim the total angular momentum consists of the moment of momentum (1) *and* a spin term:

$$J^{ij} = L^{ij} + S^{ij} = \int_V (2x^{[i}T^{j]0} + \Upsilon^{ij0})dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV + \int_V \Upsilon^{ij0}dV\,, \tag{3}$$

The explicit expression for the spin tensor is $\Upsilon^{\lambda\mu\nu} = (A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]})/2$, where A^λ and Π^λ are magnetic and electric vector potentials which satisfy $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$, where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field.

The difference between our statement (3) and the common equation (2) is verifiable. According to (2), a circularly polarized light beam of power P without an azimuth phase structure carries angular momentum flux, i.e. torque $\tau = dJ/dt = \mathsf{P}/\omega$. According to (3) the torque is $\tau = dJ/dt = 2\mathsf{P}/\omega$. To verify our statements (3) we use the angular momentum conservation law. We have calculated the torque acting on a dielectric absorbing the beam. We use the standard formula

$$\tau = \int [\mathbf{r}\times(\mathbf{P}\cdot\nabla)\mathbf{E} + \mathbf{r}\times(\mathbf{j}\times\mathbf{B}) + \mathbf{P}\times\mathbf{E}]dV \qquad (4)$$

[see, for example, [8] Eqns. (5.1) & (7.18)]. The point is the accurate calculation gives $\tau = 2\mathsf{P}/\omega$ [4]. Besides, we found that, $\int [\mathbf{r}\times(\mathbf{P}\cdot\nabla)\mathbf{E} + \mathbf{r}\times(\mathbf{j}\times\mathbf{B})]dV = \int \mathbf{P}\times\mathbf{E}\,dV = \mathsf{P}/\omega$. Loudon [8] calculated the torque as well. He used the formula (4) as well, and he obtained $\int [\mathbf{r}\times(\mathbf{P}\cdot\nabla)\mathbf{E} + \mathbf{r}\times(\mathbf{j}\times\mathbf{B})]dV = \mathsf{P}/\omega$ [see his formulae (7.19) – (7.24)]. But he omitted $\mathbf{P}\times\mathbf{E}$ term without explanations, and P/ω was his finish result for the torque. Taking into account the $\mathbf{P}\times\mathbf{E}$ term, he must obtain our result $\tau = 2\mathsf{P}/\omega$.

The work [9] rather confirms our result as well. The authors trapped particles of $r = 10^{-6}$ m by a $\mathrm{LG}_{p=0}^{l=1}$ beam of λ = 1047 nm and power P = 25 mW. When the beam was linearly polarized, it carried an orbital angular momentum flux of $\mathsf{P}/\omega = 1.4\cdot10^{-17}$ J, and the particles were rotated with the rotational rate $\Omega = 13/\sec$. This implies the torque on the particles was $\tau = 8\pi\eta r^3\Omega = 3.3\cdot10^{-19}\,\mathrm{J} = 0.023\mathsf{P}/\omega$ and the authors suggested that the particle absorbed 2.3% of the power, i.e. $\Delta\mathsf{P} = 0.023\mathsf{P}$. However, the point is a Laguerre-Gaussian beam can exert a torque on particles not only when absorbing, but also when being converted into Hermite-Gaussian beams. Because the particles had an irregular form, and because ~98% of the beam passed through the particles in the experiment, it was inevitably that a part of the beam was converted into HG modes. If this part was at least 1.2%, the absorption of $\Delta\mathsf{P} = 0.012\mathsf{P}$ only, instead of 2.3%, could provide the torque $\tau = 3.3\cdot10^{-19}$ J. The main point of the experiment [9] was a cessation of rotating of the particles when the linearly polarized $\mathrm{LG}_{p=0}^{l=1}$ beam became a circularly polarized one if the handedness was opposite to the rotation sense. Thus, we must conclude that the torque associated with the circular polarization equals $2\Delta\mathsf{P}/\omega$ because $\tau = 3.3\cdot10^{-19}\,\mathrm{J} = 2\cdot0.012\mathsf{P}/\omega$.

The work [10] confirms rather our result as well. In this work a linearly polarized $\mathrm{LG}_{p=0}^{l=2}$ beam of λ = 1064 nm and power of P = 20 mW rotates a trapped particle with the rotational rate Ω_1 = 2.5 Hz, and, when circularly polarized, with Ω_2 = 3.0 Hz. The increase, $\Delta\Omega = 2\pi\,0.5$/sec, causes the corresponding increase in the drag torque $\Delta\tau = 12\pi\eta a^3\Delta\Omega = 1.2\cdot10^{-19}$ J. On the other hand, the increase is provided with a change, $\Delta\sigma$, in the degree of the circular polarization σ of the beam as the beam passes through the particle. The quantity σ is determined by signals of two photo-detectors which show powers of right, P_r, and left, P_l, circularly polarized constituents from the formula $\sigma = (\mathsf{P}_r - \mathsf{P}_l)/(\mathsf{P}_r + \mathsf{P}_l)$. Figure 2 of [9] shows $\mathsf{P}_r = 0.999$ units, $\mathsf{P}_l = 0$ without particle, and $\mathsf{P}_r = 0.9982$, $\mathsf{P}_l = 0.0012$ with particle. Thus $\Delta\sigma = 0.003$. These results mean that $\Delta\sigma\mathsf{P}/\omega \cong 0.3\cdot10^{-19}$ J. So, we have, according to [10], $\Delta\tau \cong 4\Delta\sigma\mathsf{P}/\omega$ (we predict $2\Delta\sigma\mathsf{P}/\omega$ against conventional $\Delta\sigma\mathsf{P}/\omega$).

We consider the paper [11] because we are interested in works that show how a trapped particle rotates simultaneously around its own axis (due to spin) and around the beam's axis (due to orbital angular momentum). According to Fig. 1 of the paper, a particle of $r = 1\ \mu\text{m}$ rotates around its own axis with rotational rate $\Omega_{\text{spin}} = 18/\text{sec}$ and around the beam's axis with $\Omega_{\text{orbit}} = 2.4/\text{sec}$ along a circle of radius $R = 2.9\ \mu\text{m}$. The beam is a circularly polarized high-order J_2 Bessel beam ($l = 2$). The azimuthal component of the linear momentum density, $p_\phi = \omega l u^2 / R$, yields the azimuthal force on the particle of $F_\phi = \omega l u^2 \pi r^2 / R$ (we set $\varepsilon_0 = c = 1$). The Stokes's law, $F_\phi = 6\pi\eta r v$, gives $\Omega_{\text{orbit}} = v / R = \omega l u^2 r / 6\eta R^2$. At the same time, formula (3) from [9] gives $\tau = 8\pi\eta r^3 \Omega_{\text{spin}}$. The power impinging on the particle is $\mathsf{P} = \omega^2 u^2 \pi r^2$. We can now obtain $\dfrac{\tau}{\mathsf{P}/\omega} = \dfrac{4l\Omega_{\text{spin}} r^2}{3\Omega_{\text{orbit}} R^2} = 2.3$ that confirms our formula $\tau = 2\mathsf{P}/\omega$. The work [12,13] is also discussed in [7].

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