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Bifurcations and pattern formation in particle physics: an introductory study

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Abstract:

Quantum field theories, regardless of their content, lead to a finite or infinite number of coupled nonlinear field equations. In general, solving these equations in analytic form or managing them through lattice-based computations has been met with limited success. We argue that the theory of nonlinear dynamical systems offers a fresh approach to this challenge. Working from the universal route to chaos in coupled systems of differential equations, we find that: a) particles acquire mass as plane wave solutions of the complex Ginzburg-Landau equation (CGLE), without any reference to the hypothetical Higgs scalar; b) the $U(1)\times SU(2)$ and $SU(3)$ gauge groups, as well as leptons and quarks, are sequentially generated through period-doubling bifurcations of CGLE.

1. Overview and motivation

Quantum Field Theory (QFT) is a mature conceptual framework whose predictive power has been consistently proven in both high-energy physics and condensed matter phenomena [1]. From a historical perspective, QFT represents a successful synthesis of quantum mechanics and special relativity and consists of a broad range of models that have been developed over the years. Among these, the so-called "gauge" theories play a leading role. The Standard Model (SM) is a subset of QFT whose gauge group structure includes the electroweak and strong interactions of all known elementary particles**.** SM is a robust theoretical framework, however, it contains some 20 adjustable parameters whose physical origin is presently unknown and whose numerical values are exclusively fixed by experiments.

QFT consists of several nonlinear field models. Quantizing these types of models is a highly nontrivial effort and raises a series of theoretical challenges [2]. For example, no complete quantum version of classical gravity exists. Quantum chromodynamics (QCD) is considered a reliable field theory at short distances but because its coupling constant becomes large in the infrared sector, standard perturbative techniques do not apply. At present, there is no universal prescription for deriving and handling closed-form solutions of QCD field equations. This is in manifest contrast with quantum electrodynamics (QED) and the electroweak theory, where perturbative methods are applicable and analytic results possible. In general, dealing with closed-form solutions of field theories is seldom a practical alternative. For example, Heisenberg's nonperturbative quantization procedure [3] or Schwinger-Dyson formalism [4] lead to an infinite set of coupled differential equations which connect all orders of Green's functions. This system does not have analytic and uniquely determined solutions. In these instances, conventional wisdom says that one must seek plausible assumptions that simplify the equations *or* employ suitable numerical techniques for approximation.

It is known that Feynman's path integral formulation of QFT reveals its profound connection to equilibrium statistical physics and Boltzmann-Gibbs distribution [1, 5, 11]. As an effective theory, QFT is the correct language whenever one deals with collective phenomena that involve a large number of degrees o freedom [1, 6, 11]. In this context, of special interest is the existence and properties of topological objects in QFT and statistical physics. One may view the topologically stable field configurations (instantons, solitons, monopoles, vortex lines) as a primitive manifestation of *pattern formation*.

Let us further elaborate on this important point. In its traditional form, one frequently cited shortcoming of QFT is its inherent limitation to deal with the effect of highly unstable fluctuations *or* with a dynamics regime that is driven far away from equilibrium [7]. In general, pattern formation is possible in out-of-equilibrium physical systems that are *open* and *nonlinear* [8]. Within a *closed* system patterns may only survive as a transient and die out as a result of the relaxation towards equilibrium. It is for this reason that traditional QFT, with few notable exceptions, is largely unable to properly detect and characterize pattern formation. We now know that pattern formation is relevant to a wide range of applications such as reaction-diffusion processes, nonlinear optics applications, fluid mechanics (Rayleigh-Benard convection and the Taylor-Couette flow), hot plasma, porous and heterogeneous media, arrays of coupled oscillators, lattice quantum field models, traffic models, computational and neural networks and so on [8, 9].

Understanding of non-equilibrium phenomena and pattern formation is still in its infancy. Progress in this field has benefited from tools that have been recently developed for dynamical systems, bifurcation and stability theory [8-10, 19]. Among these we mention new methods for chaotic dynamics, stochastic stability, Liouville-von Newmann formalism [10], new methods in topology, fractional dynamics of systems with long-range interaction and temporal memory, the theory of multifractal sets, non-extensive statistical physics, the dynamics of Levy flows, amplitude equations for spatiotemporal chaos and so on.

Our aim here is to investigate the far from equilibrium sector of classical field theory using some of these newly developed methods. The basic premise is that the theory of coupled dynamical systems provides a superior strategy for dealing with the inherent complexity of field equations. The paper is structured as follows: section 2 introduces the classical equations of motion for a generic model containing a scalar field coupled to a U(1) gauge field; the concept of universality and the emergence of CGLE are discussed in section 3. Section 4 presents the mechanism of mass generation through period-doubling bifurcations of CGLE. The mechanism of dynamic unification for gauge groups and fermions forms the object of section 5. Summary and concluding remarks are detailed in the last section.

We caution from the outset that our contribution is a preliminary research on the topic. As such, it does not claim to be either fully rigorous or comprehensive. The purpose is to convey a new qualitative view rather than an in-depth analysis of phenomena. Future work is required to validate or reject our findings.

2. Classical abelian field theory as a "toy" model

As previously mentioned, field theories amount to a finite or infinite number of coupled nonlinear field equations. In general, handling these equations in closed form or through numerical approximations has been only partially successful. The universal nature of nonlinear dynamics near the threshold of the first instability suggests that one can start from a simple "toy" model and generalize results to more realistic settings.

One of such "toy" models of classical field theory describes an Abelian gauge field $a_\mu(x,t)$ in interaction with a massless scalar field $\varphi(x,t)$. The Lagrangian of this model is given by [11]

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| (\partial_\mu + iea_\mu)\varphi \right|^2 \tag{1}$$

Here, $\mu, \nu = 0,1,2,3$ denote the space-time index, $x = (x_1, x_2, x_3)$ the spatial coordinate, $F^{\mu\nu}$ the field tensor, e the coupling constant and $\partial_\mu \equiv \partial / \partial x^\mu$ the first-order differential operator. Equations of motions derived from (1) are

$$\begin{aligned} D^\mu (D_\mu \varphi) &= 0 \\ \partial^\nu F_{\mu\nu} &= 2e^2 a_\mu \varphi^2 \end{aligned} \tag{2}$$

where $D_\mu \equiv \partial_\mu + iea_\mu$ stands for the covariant derivative operator. Developing (2) yields

$$\begin{aligned} \Box \varphi &= -2iea_\mu \partial^\mu \varphi - ie\varphi \partial^\mu a_\mu + e^2 \varphi a^\mu a_\mu \\ \Box a_\mu &= \partial^\nu \partial_\mu a_\nu - 2e^2 \varphi^2 a_\mu \end{aligned} \tag{3}$$

in which $\Box \equiv \nabla^2 - \partial^2 / \partial t^2$ represents the d'Alembertian operator.

Nonlinear equations of this type are not limited to this particular model. In fact, a *generic* field theoretic model describing dissipative nonlinear oscillators in interaction can be reduced to either one of the following systems of differential equations:

$$\begin{aligned} \partial_t [...] &= \partial_x^2 [...] + f(g, m, ..., \varepsilon, ...[...]) \\ \partial_t^2 [...] &= \partial_x^2 [...] + f(g, m, ..., \varepsilon, ...[...]) \\ \partial_t [...] &= \partial_x [...] + f(g, m, ..., \varepsilon, ...[...]) \end{aligned} \tag{4}$$

Here, ε is an independent control parameter that can be continuously adjusted, g, m are coupling constants and masses, […] are dynamic variables (fields, operators, propagators) and $f(...)$ denote coupling functions. Second order derivatives may be reduced to first order, at the expense of increasing the number of field equations. When dynamic variables depend on ε, the system under study is able to sustain *self-organized pattern formation* [12].

In non-equilibrium models, ε measures the departure from ideal equilibrium conditions. In particular, critical behavior in continuous dimension equates ε with the dimensional parameter of the regularization program $(\varepsilon = 4 - d)$ [13]. This identification enables *fractional dynamics* to become a natural player in non-equilibrium field theory [14-16].

3. Universality and CGLE

Far from equilibrium processes display remarkable universality. Regardless of the specific content of the system, macroscopic patterns that develop near the threshold of a dynamic instability are robust and largely insensitive to microscopic fluctuations [8, 17]. This is the basis for the universal "slaving" mechanism underlying the amplitude equations: fast modes follow the slow modes and can be integrated out. Stated differently, there is a natural "slowing down" of dynamics near the instability and a natural separation of time scales.

CGLE represents a universal amplitude equation for pattern forming systems *or* non-equilibrium spatially extended systems. It describes bifurcating solutions close to the threshold of the first instability [8, 17, 18]. The complex amplitude $A(x,t)$ defines slow modulation in space and time of the underlying spatially periodic pattern. The theory of the reduction to CGLE from generic systems of autonomous nonlinear equations has been developed by several authors. The derivation of CGLE for a 1+1 dimensional system starts from the ansatz

$$\mathbf{u}(x,t) = \mathbf{u}_0 + A(x,\tilde{t})\exp[i(k_c x - \Omega_c t]\mathbf{u}_1 + c.c. \tag{5}$$

where $x, \tilde{t}$ represent slow variables [17, 18]. Replacing in (5) and expanding in power series of the small parameter $\varepsilon = \varepsilon - \varepsilon_c$ leads to CGLE whose standard form is

$$\partial_t A = A + (1 + ic_1)\partial_x^2 A - (1 - ic_3)|A|^2 A \tag{6}$$

Here, the real parameters c_1, c_2 denote the linear and nonlinear dispersion parameters, respectively. The limit $c_1, c_3 \to 0$ corresponds to the real Ginzburg-Landau equation, whereas $c_1^{-1}, c_3^{-1} \to 0$ recovers the nonlinear Schrődinger equation.

4. Higgs-free generation of particle masses

We seek the simplest solutions of CGLE, that is, plane-wave solutions having the form

$$\begin{gathered} a(x,t) = a_0 \exp[-i(qx + mt)] + c.c \\ a_0 = \sqrt{1 - q^2} \end{gathered} \tag{7}$$

where the wave-vector $q \in [-1,1]$ and the frequency m satisfies the dispersion equation

$$m_q = c_1 q^2 - c_3(1 - q^2) \tag{8}$$

The dispersion equation has two complementary limits: $q = \pm 1$ ($a_0 = 0$) and $q = 0$ ($a_0 = \pm 1$). Arguments presented in Appendix A suggest a natural identification of these two modes with the fermion and gauge boson fields of SM.

Despite the fact that we started with a model containing massless fields, both these modes acquire *non-vanishing masses*, namely

$$\begin{gathered} m_\pm = c_1 \\ m_0 = -c_3 \end{gathered} \tag{9}$$

Recall that the plane-wave solution consists of both positive and negative frequencies. Because mass is positive definite, in what follows we are going to only consider **positive** frequencies $c_1 > 0, c_3 > 0$ and take $m_0 = c_3$.

Extensive numerical data [19] show that both parameters of linear and nonlinear dispersion c_1, c_3 are distributed in a geometric progression, that is

$$c_{1,n} = c_{1,\infty} + K_1 \overline{\delta}_1^{-n}$$
$$c_{2,n} = c_{2,\infty} + K_2 \overline{\delta}_2^{-n} \tag{10}$$

Since K_1, K_2, $c_{1,\infty}$ and $c_{2,\infty}$ are independent of the iteration index $n = 1, 2, 3....$, they can be both absorbed into a redefinition of masses. We have, accordingly:

$$m^* = \frac{1}{K_1}(m_\pm - c_{1,\infty})$$
$$M = \frac{1}{K_2}(m_0 - c_{2,\infty}) \tag{11}$$

The ratios of two arbitrary masses consecutively generated through period-doubling bifurcation take the form:

$$\frac{m_j^*}{m_{j+p}^*} = \overline{\delta}^{\,j}$$
$$\frac{M_j}{M_{j+p}} = \overline{\sigma}^{\,j} \tag{12}$$

in which $j = 2^p$, $p = 1, 2, 3....$ and $\overline{\delta}$, $\overline{\sigma}$ are scaling constants. Experimentally, we find the following relationship between $\overline{\delta}$ and $\overline{\sigma}$ [15, 20]:

$$1 - (\frac{M_1}{M_2})^2 = 1 - (\overline{\sigma}^1)^2 \approx \frac{1}{\overline{\delta}} \tag{13}$$

Here $M_1 = M_W, M_2 = M_Z$ are vector boson masses and $3.9 \le \overline{\delta} \le 4.669$, where $\overline{\delta} = 4.669$ represents the Feigenbaum constant for quadratic maps [22]. It is instructive to note that, using the data reported in [19], one finds that the numerical value of the Feigenbaum constant corresponding to CGLE falls in the range $\overline{\delta} \in [1.5,...,10]$.

Tab. 2 and Tab. 3 show a side-by-side comparison between predictions inferred from (12), (13) and experiment for $\overline{\delta} = 4.669$ and $\overline{\delta} = 3.9$, respectively. Actual values of SM parameters, computed at the reference scale given by the mass of the top quark [21], are listed in Tab. 1. Note that the choice of the mass scale is completely arbitrary since (12) involves ratios of consecutive parameters.

Parameter	**Value**	**Units**
m_u	2.12	MeV
m_d	4.22	MeV
m_s	80.90	MeV
m_c	630	MeV
m_b	2847	MeV
m_t	170,800	MeV
M_W	80.46	GeV
M_Z	91.19	GeV
α_{EM}	1/128	-
α_W	0.0338	-
α_{QCD}	0.123	-

Tab. 1: Actual values of selected SM parameters

5. Dynamic unification of fermion fields and gauge groups

The Feigenbaum-Sharkovskii-Magnitskii (FSM) paradigm of universal transition to chaos in nonlinear dissipative systems [19, Appendix B] suggests an intriguing mechanism for the generation of SM particles. The quartet of electroweak fields $(\gamma, W^{+}, W^{-}, Z^{0})$ breaks into the gluon octet and lepton multiplet breaks into the quark multiplet according to the pattern:

$$\boxed{(\gamma, W^{+}, W^{-}, Z^{0}) \Rightarrow (gluon_{1-8})} \tag{14a}$$

$$\boxed{(\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau) + antiparticles \Rightarrow (u, d, c, s, b, t)_{r,g} + antiparticles} \tag{14b}$$

Stated differently, (14a) shows that the dynamical transition $U(1) \times SU(2) \to SU(3)$ means that a stable cycle of period 4 (corresponding to the electroweak quartet) transforms into a stable cycle of period 8. Note that there are 12 distinct leptons and 24 distinct colored quarks in (14b). It follows that transition of leptons to quarks occurs through a bifurcation that generates a stable cycle of period 24 from a stable cycle of period 12.

Two important remarks are in order:

a) color and electrical charge conservation constrains the number of independent (distinct) attractors generated through bifurcations. For example, taking "R" and "G" to represent independent color states, color conservation prohibits formation of distinct attractors of type "B" since $R + G + B = 1$, by definition.

b) there is a natural mixing of cycles prior to their complete separation through bifurcation. As a result of this mixing, transition $U(1) \times SU(2) \to SU(3)$ allows leptons and quarks to couple through electroweak fields, but forbids leptons to couple to gluon fields.

We note that these results are pointing in the same direction with findings reported by the author in an early work [20].

Parameter ratio	Behavior	Actual	Predicted
m_u/m_c	$\bar{\delta}^{-4}$	3.365×10^{-3}	2.104×10^{-3}
m_c/m_t	$\bar{\delta}^{-4}$	3.689×10^{-3}	2.104×10^{-3}
m_d/m_s	$\bar{\delta}^{-2}$	0.052	0.046
m_s/m_b	$\bar{\delta}^{-2}$	0.028	0.046
m_e/m_μ	$\bar{\delta}^{-4}$	4.745×10^{-3}	2.104×10^{-3}
m_μ/m_τ	$\bar{\delta}^{-2}$	0.061	0.046
α_{EM}/α_W	$\bar{\delta}^{-1}$	0.230	0.214
α_{EM}/α_s	$\bar{\delta}^{-2}$	0.0635	0.0459
M_W/M_Z	$(1-\bar{\delta}^{-1})^{1/2}$	0.8823	0.8865

Tab. 2: Actual versus predicted scaling ratios for $\bar{\delta} = 4.669$

Parameter ratio	Behavior	Actual	Predicted
m_u / m_c	$\overline{\delta}^{-4}$	3.365×10^{-3}	4.323×10^{-3}
m_c / m_t	$\overline{\delta}^{-4}$	3.689×10^{-3}	4.323×10^{-3}
m_d / m_s	$\overline{\delta}^{-2}$	0.052	0.066
m_s / m_b	$\overline{\delta}^{-2}$	0.028	0.066
m_e / m_μ	$\overline{\delta}^{-4}$	4.745×10^{-3}	4.323×10^{-3}
m_μ / m_τ	$\overline{\delta}^{-2}$	0.061	0.066
α_{EM} / α_W	$\overline{\delta}^{-1}$	0.230	0.256
α_{EM} / α_s	$\overline{\delta}^{-2}$	0.063	0.066
M_W / M_Z	$(1-\overline{\delta}^{-1})^{1/2}$	0.8823	0.8623

Tab. 3: Actual versus predicted scaling ratios for $\overline{\delta} = 3.9$

6. Summary and conclusions

This brief report has been motivated by recent advances in the theory of nonlinear dynamics and complexity. Through the combined use of CGLE and universal theory of transition to chaos in nonlinear dissipative systems, we have found that:

a) particles acquire mass as plane wave solutions of CGLE, without any reference to the hypothetical Higgs scalar or to a particular symmetry breaking mechanism.

b) there is a natural separation of *non-relativistic modes* ($q = 0$) from *relativistic modes* of maximal group velocity $(q = 1)$. The most straightforward interpretation of this result is that the first group of modes corresponds to gauge bosons and the second group to fermions.

c) the $U(1) \times SU(2)$ and $SU(3)$ gauge groups of SM, as well the currently known fermion generations, are sequentially produced through period-doubling bifurcations of CGLE.

<u>Appendix A</u>

The two dispersion parameters of CGLE are subject to the following dynamic constraints [8, 17]:

a) the Benjamin-Feir-Newell (BFN) criterion states that stability becomes borderline for

$$c_1 c_3 = 1 \tag{A1}$$

b) the group velocity of linear perturbations to the plane wave solutions is given by ( )

$$\mathrm{v}_g = 2q(c_1 + c_3) \tag{A2}$$

Compliance with relativity bounds (A2) to a constant that represents the normalized value of light speed *in vacuo*. It is clear that $q = 0$ represents a *slow mode* (massive gauge boson), while $q = \pm 1$ describes the *fastest mode* (relativistic fermions). Masses associated with these modes are supplied by (9). From the BFN criterion it follows that the borderline value of the normalization constant $Q \equiv \mathrm{v}_g^{\max} / 2$ can be determined from

$$\begin{gathered} c_1 = \frac{Q \pm \sqrt{Q^2 - 4}}{2} \Rightarrow Q \geq 2 \\ c_3 = \frac{1}{c_1} \end{gathered} \tag{A3}$$

(A1) and (A2) imply that, close to the border of BFN instability, gauge boson and fermion masses scale as complementary entities.

<u>Appendix B</u>

For convenience, we briefly outline here the universal FSM paradigm of transition to chaos in nonlinear dissipative systems. The interested reader is referred to [19] for additional details.

Consider the boundary value problem for CGLE in 1+1 space-time dimensions (x,t):

$$\begin{gathered}\partial_t A = A + (1+ic_1)\partial_x^2 A - (1-ic_3)|A|^2 A \\ \partial_x A(0,t) = \partial_x A(L,t) = 0\,,\ A(x,0) = A_0(x)\,,\ 0 \le x \le L\,,\ 0 \le t \le \infty\end{gathered} \tag{B1}$$

This model can be reduced to a three-dimensional system of nonlinear ordinary differential equations with the help of the Galerkin few-mode approximation:

$$A(x,t) \approx \sqrt{\xi(t)}\exp[i\theta_1(t)] + \sqrt{\eta(t)}\exp[i\theta_2(t)]\cos(\frac{\pi}{L}x) \tag{B2}$$

in which

$$\begin{gathered}\partial_t \xi = f_1(\xi,\eta,\theta,c_1,c_3,L) \\ \partial_t \eta = f_2(\xi,\eta,\theta,c_1,c_3,L) \\ \partial_t \theta = f_3(\xi,\eta,\theta,c_1,c_2,L)\end{gathered} \tag{B3}$$

with $\theta(t) \equiv \theta_2(t) - \theta_1(t)$. It can be shown that the transition to chaos in (B3) occurs through a *sequential cascade of bifurcations*. This cascade starts with the Feigenbaum scenario of period-doubling bifurcations of stable cycles, followed by the Sharkovskii subharmonic cascade and ending with the Magnitskii cascade of stable homoclinic cycles.

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