Mechanical stresses produced by a light beam

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A circularly polarized electromagnetic beam is considered, which is absorbed by a plane, and the mechanical stress produced in the plane by the beam is calculated. It is shown that the central part of the beam produces a torque at the central region of the plane due to the spin of the beam, and the wall of the beam produces an additional torque due to orbital angular momentum of the beam. The total torque acting on the plane equals two power of the beam divided by the frequency. This fact contradicts the standard electrodynamics, which predicts the torque equals power of the beam divided by frequency, and means the electrodynamics is incomplete. An introducing of the spin tensor corrects the electrodynamics.

I. Does electrodynamics' spin tensor exist?

As is well known, photons carry spin, energy, momentum and angular momentum that is a moment of the momentum relative to a given point or to a given axis. Energy and momentum of electromagnetic waves are described by the Maxwell energy-momentum tensor (density)

$$T^{\lambda\mu} = -g^{\lambda\alpha}F_{\alpha\nu}F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{1.1}$$

where $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor. For example, $P^i = \int_V T^{i0}dV$ is the momentum of a waves inside of the volume V, and $dW = \int_a T^{0i}da_i dt$ is the energy that has flowed through the area a in the time dt. The angular momentum that is a moment of the momentum can be defined as[1]

$$L^{ij} = \int_V 2x^{[i}T^{j]0}dV = \int_V \mathbf{r} \times (\mathbf{E} \times \mathbf{B})dV, \tag{1.2}$$

and this construction must be named an orbital angular momentum. However the modern electrodynamics has no describing of spin. Sometimes physicists consider the canonical spin tensor

$$\Upsilon_c^{\lambda\mu\nu} = -2A^{[\lambda}\delta_\alpha^{\mu]}\frac{\partial \mathsf{L}_c}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}, \tag{1.3}$$

where $\mathsf{L}_c = -F_{\mu\nu}F^{\mu\nu}/4$ is the canonical Lagrangian, and A^λ is the magnetic vector potential, $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$. But spin tensor (1.3) is invalid, as well as the canonical energy-momentum tensor independently of any divergence addend. So physicists eliminate the spin tensor by the Belinfante-Rosenfeld procedure.[2,3] As a result, the electrodynamics has no spin tensor, or rather the modern classical electrodynamics spin tensor equals zero.

Nevertheless, physicists understand they cannot shut eyes on existence of the classical electrodynamics spin. And they proclaim spin is *in* the moment of the momentum (1.2). I.e., the moment of momentum represents the total angular momentum, orbital angular momentum plus spin. I.e., equation (1.2) encompasses both the spin and orbital angular momentum density of a light beam:[4-8]

$$J^{ij} = L^{ij} + S^{ij} = \int_V 2x^{[i}T^{j]0}dV = \int_V \mathbf{r} \times (\mathbf{E} \times \mathbf{B})dV, \tag{1.4}$$

Contrary to this paradigm, we introduce a spin tensor $\Upsilon^{\lambda\mu\nu}$ into the modern electrodynamics,[9-14] i.e. we complete the electrodynamics by introducing the spin tensor, i.e. we claim the total angular momentum consists of the moment of momentum (1.2) *and* a spin term, i.e. we claim equation (1.4) is wrong, i.e. we state the moment of momentum does not contain spin at all:

$$J^{ij} = L^{ij} + S^{ij} = \int_V (2x^{[i}T^{j]0} + \Upsilon^{ij0})dV = \int_V \mathbf{r} \times (\mathbf{E} \times \mathbf{B})dV + \int_V \Upsilon^{ij0}dV, \tag{1.5}$$

The sense of the spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ij0} is a volume density of spin. This means that $dS^{ij} = \Upsilon^{ij0}dV$ is the spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is

a flux density of spin flowing in the direction of the x^k axis. For example, $dS_z / dt = dS^{xy} / dt = d\tau^{xy} = Y^{xyz} da_z$ is the z-component of spin flux passing through the surface element da_z per unit time, i.e. the torque acting on the element. The explicit expression for the spin tensor is

$$Y^{\lambda\mu\nu} = (A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]})/2 , \tag{1.6}$$

where A^λ and Π^λ are magnetic and electric vector potentials which satisfy $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$, where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field. A relation between Π and F can be readily obtained in the vector form as follows. If $\text{div}\mathbf{E} = 0$, then $\mathbf{E} = \text{curl}\,\Pi$. And if $\partial\mathbf{E}/\partial t = \text{curl}\mathbf{H}$, then $\partial\Pi/\partial t = \mathbf{H}$. This reasoning is analogous to the common: if $\text{div}\mathbf{B} = 0$, then $\mathbf{B} = \text{curl}\,\mathbf{A}$. And if $\partial\mathbf{B}/\partial t = -\text{curl}\mathbf{E}$, then $\partial\mathbf{A}/\partial t = -\mathbf{E}$.

The difference between our statement (1.5) and the common equation (1.4) is verifiable. The cardinal question is, what angular momentum flux, i.e. torque τ, does a circularly polarized light beam of power P without an azimuth phase structure carry? The common answer, according to (1.4), is

$$\tau = dJ/dt = \mathsf{P}/\omega ; \tag{1.7}$$

our answer, according to (1.5), is

$$\tau = dJ/dt = 2\mathsf{P}/\omega . \tag{1.8}$$

Statements (1.5) & (1.8) are also valid in the case of plane waves or a beam which is much larger than the particle under action if P is the power absorbed by the particle.

Some quantity of theoretical calculations, in particular, the calculation of absorption of a circularly polarized light beam in a dielectric,[7,12] the calculation of a radiation of spin by a rotating electric dipole,[14] as well as numerous experimental works,[8,15-18] confirm our result (1.8). Another manifestation of the spin tensor concerns the mechanical stress that arises in a target absorbing an circular polarization electromagnetic beam. A stress tensor density $T_\wedge^{ij}$ describes this stress. The quantity $T_\wedge^{ij}$ is calculated in Sect. 2 and 3 of the present paper for the cases (1.4) and (1.5), respectively. The stress in the case (1.4) is prove to be in contradiction with the evidence.

In Sect. 4, it is shown that so called decomposition (1.4) of the moment of the Poynting vector in an orbital and spin angular momentums[4,19-22] is false.

II. Absorbing of the moment of Poynting vector flux

According to (1.4), a plane wave traveling in the z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis because $\mathbf{E}\times\mathbf{B}$ is in the z-direction and $[\mathbf{r}\times(\mathbf{E}\times\mathbf{B})]_z = 0$. However, this is no longer the case for a wave with finite extension in the xy-plane. Consider a circularly polarized beam with its axis in the z-direction and traveling in this direction[4]

$$\mathbf{E} = \exp(iz - it)[\mathbf{x} + i\mathbf{y} + \mathbf{z}(i\partial_x - \partial_y)]E_0(x,y) ,$$
$$\mathbf{B} = -i\mathbf{E} . \tag{2.1}$$

Here $E_0(x,y) = E_0(r)$ is the electric field of the beam. Let $E_0(r) = \text{Const}$ inside the beam, and $E_0(r) = 0$ outside the beam. For short we set $\omega = k = c = 1$. At the wall of the beam $r = R_0$, say, we let the amplitude drop to zero. It can be shown that the wall of such a beam gives a finite contribution to J_z because the $\mathbf{E}$ and $\mathbf{B}$ fields have components parallel to the wave vector (the field lines are closed loops) and the energy flow
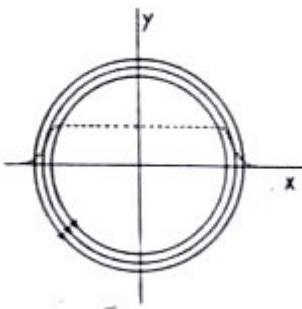

Fig. 1. This pattern of circular flow lines represents the time-average energy flow, or the momentum density, in a circularly polarized electromagnetic wave packet. On a given wave front, say $z = 0$, the fields are assumed to be constant within a circular area and to decrease to zero outside of this area (the dashed line gives the field amplitude as a function of radius). The energy flow has been calculated from an approximate solution of Maxwell's equations. The picture only shows the flow in the transverse directions. The flow in the longitudinal direction is much larger; the net flow is helical.

has component perpendicular to the wave vector. For instance, Fig. 1 from[19] shows the time-average transverse energy flow in the beam. The circulating energy flow in the beam implies the existence of angular momentum, whose direction is along the direction of propagation. Since the fields are identically zero outside the wall and constant inside the wall region, the wall region is the only one in which the z-component of angular momentum does not vanish.

In Fig. 9.3 from[20] an acceptable function $E_0(r)$ is plotted. E_0 is explicitly made constant over a large
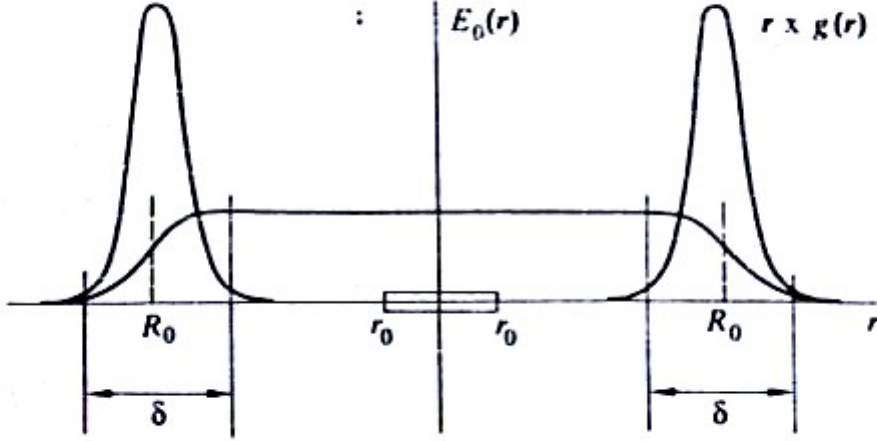
Fig. 9.3. The electric field amplitude and the angular momentum density across a cylindrical beam.

central region of the wave and confined the variation of the function from this constant value to zero to lie within a wall of thickness δ, which lies a distance R_0 from the axis. The profile $E_0(r)$ of the beam may be Gaussian,[23]

$$E_0 = \frac{\sqrt{2/\pi}}{w}\exp\{-\frac{r^2}{w^2}(1-i\frac{z}{z_R}) + i\,\text{arctg}\,\frac{z}{z_R}\}, \quad r^2 = x^2 + y^2, \quad w^2 = \frac{2(z^2 + z_R^2)}{z_R}. \tag{2.2}$$

but it doesn't matter. We set

$$\int_{-\infty}^{+\infty} E_0^2\,dxdy = \int_0^\infty E_0^2\,2\pi r\,dr = 1, \tag{2.3}$$

so, the power of the beam is

$$P = <\int(\mathbf{E}\times\mathbf{B})_z\,dxdy> = \int\Re(E_x\overline{B}_y - E_y\overline{B}_x)dxdy/2 = \int E_0^2\,dxdy = 1 \tag{2.4}$$

(the over lines mark complex conjugate complex numbers).

We need also the p^x, p^y components of momentum density:

$$p^x = <(\mathbf{E}\times\mathbf{B})_x> = \Re(E_y\overline{B}_z - E_z\overline{B}_y)/2 = \partial_y E_0^2/2, \quad p^y = -\partial_x E_0^2/2. \tag{2.5}$$

Let a plane $z = 0$ absorbs the beam. Then the plane, according to (1.7), must experience the torque

$$\tau = P = 1. \tag{2.6}$$

(we ignore the light pressure). We will get the stress tensor density $T_\wedge^{ik}$ of the plane.

We use the cylindrical coordinates r, ϕ, z

$$x = r\cos\phi, \quad y = r\sin\phi, \tag{2.7}$$

with the metric

$$dl^2 = dr^2 + r^2 d\phi^2 + dz^2, \quad g_{\rho\rho} = 1, \quad g_{\phi\phi} = r^2, \quad g_{zz} = 1, \quad \sqrt{g}_\wedge = r, \quad g^{\phi\phi} = 1/r^2. \tag{2.8}$$

Square root of determinant of the metric tensor is a scalar density of weight $+1$. Gothic symbols are usually applied to denote tensor densities.[24] We shall, instead, mark the density with the symbol 'wedge' at the level of bottom indices for a density of weight $+1$ and at the level of top indices for a density of weight

-1. A volume element and a surface element are densities of weight -1, $dV^{\wedge} = dr d\phi dz$, $da^{\wedge} = dr d\phi$ as well as the absolute antisymmetric density $e_{ijk}^{\wedge}$, which equals ± 1, or 0.

ϕ-component p^{ϕ} is obtained by the formulae $p^i = p^a \partial_a^i$, $\partial_a = \partial_a^i \partial_i$, i.e.

$$p^{\phi} = p^x \partial_x^{\phi} + p^y \partial_y^{\phi}, \quad \partial_x = \partial_x^r \partial_r, \quad \partial_y = \partial_y^r \partial_r, \tag{2.9}$$

where the matrix elements ∂_a^i are

$$\partial_x^{\phi} = -y/r^2, \quad \partial_y^{\phi} = x/r^2, \quad \partial_x^r = x/r, \quad \partial_y^r = y/r. \tag{2.10}$$

The physical component of momentum density $p^{\hat{\phi}}$ is equal coordinate component of the tensor density $p_{\wedge}^{\phi}$:

$$p_{\wedge}^{\phi} = p^{\phi} \sqrt{g}_{\wedge} = p^{\hat{\phi}} \sqrt{g^{\phi\phi}} \sqrt{g}_{\wedge} = p^{\hat{\phi}} = -\partial_r E_0^2 / 2. \tag{2.11}$$

So, because $c = 1$, the tensor density $p_{\wedge}^{\phi}$ equals the momentum flux density, i.e. the force tensor density $f_{\wedge}^{\phi}$:

$$f_{\wedge}^{\phi} = -\partial_r E_0^2 / 2. \tag{2.12}$$

This force density $f_{\wedge}^i$ acts on the absorbing plane $z = 0$, thus

$$\nabla_k T_{\wedge}^{ik} = f_{\wedge}^i, \tag{2.13}$$

where $\nabla_k T_{\wedge}^{ik}$ is the covariant divergence of the stress tensor density of the plane. As is known,[24] $\nabla_k T_{\wedge}^{ik} = \partial_k T_{\wedge}^{ik} + \Gamma_{jk}^i T_{\wedge}^{jk}$, where Γ_{jk}^i are the Christoffel symbols. The nonzero symbols are

$$\Gamma_{\phi\phi}^r = -r, \quad \Gamma_{\phi r}^{\phi} = \Gamma_{r\phi}^{\phi} = 1/r, \quad \Gamma_{kr}^k = \Gamma_{rk}^k = 1/r. \tag{2.14}$$

Because $T_{\wedge}^{\phi r} = T_{\wedge}^{r\phi}$, Eq. (2.13) gives,

$$\partial_r T_{\wedge}^{\phi r} + \Gamma_{jk}^{\phi} T_{\wedge}^{jk} \equiv \partial_r T_{\wedge}^{\phi r} + \Gamma_{\phi r}^{\phi} T_{\wedge}^{\phi r} + \Gamma_{r\phi}^{\phi} T_{\wedge}^{r\phi} \equiv \partial_r T_{\wedge}^{\phi r} + 2 T_{\wedge}^{\phi r} / r = f_{\wedge}^{\phi}. \tag{2.15}$$

This equation has a solution

$$T_{\wedge}^{\phi r} = T^{\hat{\phi} r} = C(r)/r^2, \quad \partial_r C / r^2 = -\partial_r E_0^2 / 2. \tag{2.16}$$

It is easy to verify that $T_{\wedge}^{rr} = T_{\wedge}^{\phi\phi} = 0$

Integrating the equation for $C(r)$ yields

$$C(r) = -\int_0^r r^2 dE_0^2 / 2 = -r^2 E_0^2 / 2 + \int_0^r r E_0^2 dr. \tag{2.17}$$

According to (2.17), $C = 0$ and $T_{\wedge}^{\phi r} = 0$ in the central region, where $E_0(r) = E_0(0)$. Thus, there is no mechanical stress in the central region of the target, according to the standard paradigm (1.4). This is depicted in Fig. 2.

$C(r)$ increases up to $C_{\max} = \int_0^{\infty} r E_0^2 dr = 1/2\pi$ in the wall region, in accordance with (2.3).

Correspondingly,

$$T_{\wedge}^{\phi r} = T^{\hat{\phi} r} = 1/2\pi r^2, \quad r > R_0 \tag{2.18}$$

outside the wall region. It is easy to verify that tensor density (2.18) satisfies $\nabla_k T_{\wedge}^{ik} = 0$. A plot of $T^{\hat{\phi} r}(r)$ is in Fig. 2.

By the use of Eq. (2.18), one can get the forces inside the target plane. An element dl of a circumference of radius r experiences the force $dF^{\hat{\phi}} = T^{\hat{\phi} r} dl = C(r) dl / r^2$ and the torque $d\tau = C(r) dl / r$. Thus, the outside part of the plane experiences the torque
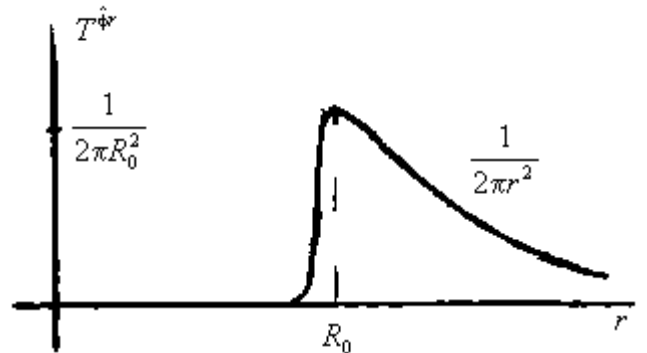

Fig. 2 The component of the stress tensor arisen from the orbital angular momentum

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$$\tau_{\text{orbit}} = \int\limits_{0}^{2\pi r} \frac{C(r)}{r} dl = 2\pi C(r) \tag{2.19}$$

that increases up to the whole torque

$$\tau_{\text{orbit}} = 1 \text{ for } r > R_0, \tag{2.20}$$

in accordance with (1.7), (2.6).

However, a circularly polarized beam, as well as a circularly polarized plane wave, beyond doubt acts on a central region of an absorbing plane by a torque. This torque arises from the fact that the dielectric constant ε is a tensor. Consequently the electric intensity $\mathbf{E}$ is, in general, not parallel to the electric polarization $\mathbf{P}$ in the medium of the plane. The torque per unit volume produced by the action of the electric field on the polarization of the medium is[25]

(a)

(b)

Fig. 17–5. (a) The electric field $\mathcal{E}$ in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.

$$\tau / V = \mathbf{P} \times \mathbf{E}. \tag{2.21}$$

But this torque is not connected with the moment of the Poynting vector (1.2), (1.4)

R. Feynman repeated this explanation.[26] We quote[26] with some abridgements.

"If we have a beam of light containing a large number of photons all circularly polarized the same way, it will carry angular momentum. Now remember what right circularly polarized light is, classically. It's described by an electric field so that the electric vector $\mathbf{E}$ goes in a circle – as drawn in Fig. 17-5(a). Suppose that such a light shines on a plane which is going to absorb it – or at least some of it – and consider an atom in the plane according to the classical physics. We'll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement $\mathbf{r}$ from its equilibrium position at the origin and goes around with some phase lag with respect to the vector $\mathbf{E}$. The relation between $\mathbf{E}$ and $\mathbf{r}$ might be as shown in Fig. 17-5(b). As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $eE_t r$ which must be equal to the rate of change of angular momentum dJ_z / dt:

$$dJ_z / dt = eE_t r". \tag{2.22}$$

This torque is not connected with the moment of the Poynting vector (1.2), (1.4) as well. And the Poynting vector density is zero at the central region of the plane. Thus, the standard paradigm cannot explain this central region torque.

However, Allen and Padgett[27] try to explain the action of a circularly polarized plane wave by a torque on a central region of an absorbing plane in the frame of the standard paradigm. They cut the wave into coaxial pieces in their mind and then claim that every piece produces a torque because the large intensity gradient near the boundary of the piece results in azimuthal components to the momentum density.

I think this is not correct. An intensity gradient near a wall of a beam results in the azimuthal components only in the case of a real beam satisfying the Maxwell equations. There are no azimuthal components in a piece of a wave that is simply cut off from a whole wave. Such a piece cannot be considered at all because it does not satisfy the Maxwell equations. Note, I submitted this reasoning to AJP on 03 Jun 2002 (# 15916).
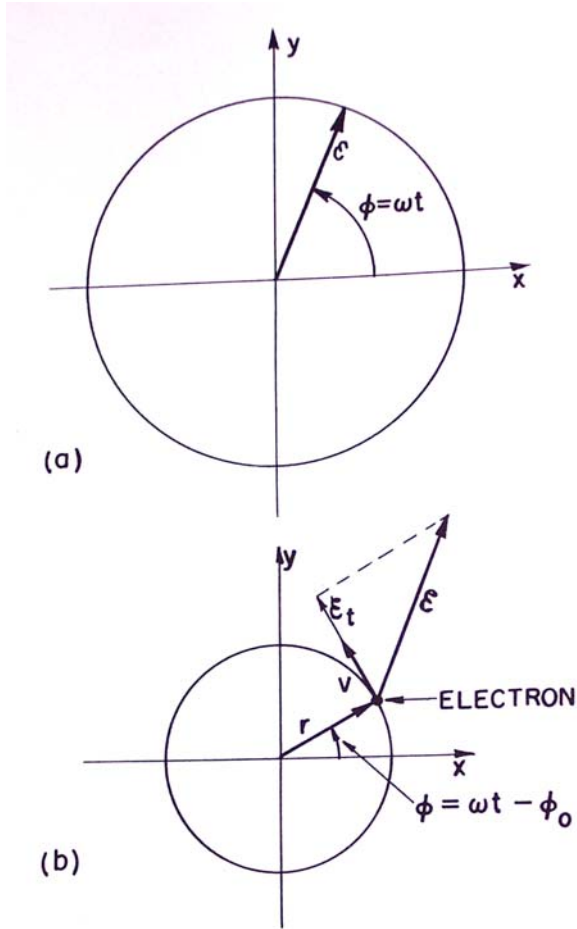
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III. Absorbing of the spin flux density

Now we consider an addition to the tensor density $T_\wedge^{ik}$ of Sect. 2. The tensor density $T_\wedge^{ik}$ of this Sect. is due to absorbing of the angular momentum flux density (1.6), i.e. the spin flux density, by the central region of the target. The field of the central region of the beam (2.1) is

$$\mathbf{E} = \exp(iz - it)(\mathbf{x} + i\mathbf{y})E_0(0), \quad \mathbf{B} = -i\mathbf{E}, \quad x^2 + y^2 < R_0^2. \tag{3.1}$$

The point R_0 (see Fig. 9.3) satisfies the equation

$$\int_0^{R_0} E_0^2(0)2\pi r\, dr = 1, \quad E_0^2(0) = 1/(\pi R_0^2) \tag{3.2}$$

The electromagnetic field (3.1), according to (1.6), corresponds to the component of the spin tensor

$$Y^{xyz}(0) = -\Re(\overline{A}^x\partial^z A^y - \overline{A}^y\partial^z A^x + \overline{\Pi}^x\partial^z\Pi^y - \overline{\Pi}^y\partial^z\Pi^x)/4 = E_0^2(0). \tag{3.3}$$

Here $A^x = -\int E^x dt$, $A^y = -\int E^y dt$, $\Pi^x = \int B^x dt$, $\Pi^y = \int B^y dt$, the over lines mark complex conjugate complex numbers, $\partial^z = -\partial_z$. Now we obtain the cylindrical components of the spin tensor density by the formula $Y_\wedge^{ijk} = \partial_a^i\partial_b^j\partial_c^k Y^{abc}\sqrt{g}_\wedge$ where ∂_a^i are the matrix elements (2.10). For example,

$$Y_\wedge^{r\phi z}(0) = \partial_x^r\partial_y^\phi Y^{xyz}(0)\sqrt{g}_\wedge + \partial_y^r\partial_x^\phi Y^{yxz}(0)\sqrt{g}_\wedge = E_0^2(0). \tag{3.4}$$

As is well known, the local conservation law $\nabla_k T_\wedge^{ik} = 0$ is accompanied by the angular momentum conservation law (see, e.g.,[28] p. 64)

$$\nabla_k Y_\wedge^{ijk} - 2T_\wedge^{[ij]} = 0. \tag{3.5}$$

In our case

$$2T_{3\,\wedge}^{[r\phi]} = \partial_z Y_\wedge^{r\phi z} \quad \text{if} \quad r < R_0 \tag{3.6}$$

where $T_{3\,\wedge}^{[r\phi]}$ is the antisymmetric part of the 3-dimensional stress tensor density in material of the absorbing plane. Using the Eq. (3.6), we arrive to an *antisymmetric* stress tensor, which characterizes medium absorbing angular momentum flux. Integrating Eq. (3.6) over z yields

$$\int_0^\infty 2T_{3\,\wedge}^{[r\phi]}dz = \int_0^\infty \partial_z Y_\wedge^{r\phi z}dz, \quad \text{i.e.,} \quad 2T_\wedge^{[r\phi]} = -Y_\wedge^{r\phi z}(0), \tag{3.7}$$

where $T_\wedge^{r\phi} = \int_0^\infty T_{3\,\wedge}^{r\phi}dz$ stands for 2-tensor density as well as in Sect. 2. Thus,

$$T_\wedge^{\phi r} = -T_\wedge^{r\phi} = T^{\hat\phi r} = -T^{r\hat\phi} = Y_\wedge^{r\phi z}(0)/2 = Y^{r\hat\phi z}(0)/2 = E_0^2(0)/2 = 1/(2\pi R_0^2), \quad \text{and} \quad T_\wedge^{rr} = T_\wedge^{\phi\phi} = 0 \tag{3.8}$$

This means that the edge of a disk of radius r, which is cut off from the plane, i.e. a circle of radius r, acts on the remainder of the plane with a torque. The force acting on an element dl of the edge is $dF^{\hat\phi} = T^{\hat\phi r}dl$, and the torque corresponding to the element is $d\tau = rT^{\hat\phi r}dl$ Integrating yields

$$\tau_{\text{spin}} = \int_0^{2\pi r} rT^{\hat\phi r}dl = r^2/R_0^2, \quad r < R_0. \tag{3.9}$$

Thus, material of the central region transmits the received torque to the periphery by inner tangential stress of (3.6), and the torque increases with r.

The result (3.9) can be obtained in another way. By definition of $Y_\wedge^{r\phi z}$, $d\tau^{r\phi} = Y_\wedge^{r\phi z}da_z^\wedge$, i.e. $d\tau = Y^{r\hat\phi z}(0)2\pi r dr$. So a disk of radius r, which is cut off from the plane, experienses the torque

$$\tau_{\text{spin}} = \int_0^r E_0^2(0)2\pi r dr = E_0^2(0)\pi r^2 = r^2/R_0^2, \quad r < R_0. \tag{3.10}$$

At the edge of the beam, $r = R_0$, Eq. (3.5) is changed to

$$T^{[ij]} = 0, \quad r > R_0. \tag{3.11}$$

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So, for $r > R_0$, $T^{\phi r} = T^{r\phi}$, i.e. the stress tensor is symmetric. But $T^{\phi r}(r)$ must be continuous . Thus, $T^{r\phi}$ changes its sign at $r = R_0$: $T^{r\phi} \rightarrow -T^{r\phi}$.

$$T_{\wedge}^{\phi r} = T_{\wedge}^{r\phi} = 1/2\pi r^2 \quad \text{for } r > R_0, \quad (3.12)$$

as well as in (2.18). A plot of $T^{\hat\phi r}(r)$ is in Fig. 3. So,

$$\tau_{spin} = \int_0^{2\pi r} rT^{\hat\phi r} dl = 1 \quad \text{for } r > R_0. \quad (3.13)$$

We denote the torque (2.19), (2.20) τ_{orbit} because the torque arises from the first term of (1.5), which is an orbital angular momentum, and we denote the torque (3.10), (3.13) τ_{spin} because the torque arises from the second term of (1.5), which is the spin angular momentum. The total torque is

$$\tau = 2\pi C(r) + r^2 / R_0^2 \quad \text{for } r < R_0, \text{ and } \tau = 2 \quad \text{for}$$

Fig. 3 The component of the stress tensor arisen from the spin angular momentum

$$r > R_0, \quad (3.14)$$

according to (1.8).

IV. Vain attempts to find a spin inside the orbital angular momentum

A number of physicists try to decompose the angular momentum of a field (1.4) into the sum of the orbital angular momentum and a spin, and they interpret the circulating flow of a beam (see Fig. 1) as a spin.[29,30] According to the standard procedure of the interpretation, the magnetic vector potential $\mathbf{A}$ is used, $\mathbf{B} = \nabla \times \mathbf{A}$. Then the integrand of (1.4) is written as

$$\mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = \mathbf{r} \times [\mathbf{E} \times (\nabla \times \mathbf{A})] = \mathbf{r} \times (E^i \nabla A_i) - \mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}]. \quad (4.1)$$

However, the first term in the right hand side can be shown to be zero for the beam (2.1). Really, the Lorentz gauge vector potential corresponding to (2.1) is (see, e.g.,[31.])

$$\mathbf{A} = \exp(iz - it)(-i\mathbf{x} + \mathbf{y})E_0. \quad (4.2)$$

So,

$$\Re\{x\overline{E}^x\partial_y A_x + x\overline{E}^y\partial_y A_y - y\overline{E}^x\partial_x A_x - y\overline{E}^y\partial_x A_y) = 0. \quad (4.3)$$

The second therm in the right hand side of (4.1), according to the standard procedure, is augmented by the zero, $0 = -\mathbf{E} \times \mathbf{A} + \mathbf{E} \times \mathbf{A}$. Then the second term is written as

$$-\mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}] = -\mathbf{r} \times [(\mathbf{E} \cdot \nabla)\mathbf{A}] - \mathbf{E} \times \mathbf{A} + \mathbf{E} \times \mathbf{A} = -\nabla[\mathbf{E}(\mathbf{r} \times \mathbf{A})] + \mathbf{E} \times \mathbf{A}, \quad (4.4)$$

and so, the integrand of (1.2) takes the form

$$\mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = -\nabla[\mathbf{E}(\mathbf{r} \times \mathbf{A})] + \mathbf{E} \times \mathbf{A}. \quad (4.5)$$
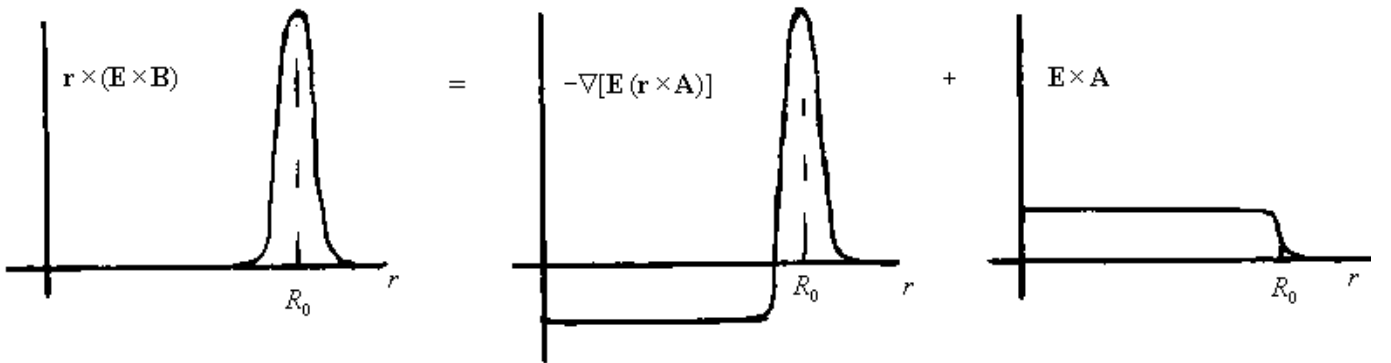

Fig. 4 The term $\mathbf{E} \times \mathbf{A}$ is subtracted from $\mathbf{r} \times (\mathbf{E} \times \mathbf{B})$ and then is added to $\mathbf{r} \times (\mathbf{E} \times \mathbf{B})$

Equation (4.5) is depicted in Fig. 4. When integrating, the first term on the right gives zero and we arrive to the famous equation

$$\int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV = \int \mathbf{E} \times \mathbf{A}\, dV . \tag{4.6}$$

But the equation only expresses a change from integrating over the wall region of the beam to integrating over the bulk. This change does not prove that the torque acts from the bulk of the beam to the central region of the target, and that the moment of the Poynting vector is spin. This change proves nothing. For example, consider an analogous integral $\int \mathbf{r} \times \mathbf{j}\, dV = \int \mathbf{r} \times (\nabla \times \mathbf{H}) dV$ over the surface of a long solenoid where $\mathbf{j}$ is an electric current density of the solenoid. We have

$$\int \mathbf{r} \times (\nabla \times \mathbf{H}) dV = \int (r^i \partial_k H_i - r^i \partial_i H_k) dV = \int [\partial_k (r^i H_i) - H_k - \partial_i (r^i H_k) + \partial_i r^i H_k] dV = \int 2\mathbf{H} dV .$$

This equality between the moment of an electric current and the integral of $\mathbf{H}$ over the solenoid volume proves nothing.

Another transformation of the angular momentum (1.4) is offered by Stewart. He uses the electric vector potential Π (see Sect. 1) instead of the magnetic vector potential $\mathbf{A}$, and he obtains

$$\int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV = \int \Pi \times \mathbf{B} dV \tag{4.7}$$

instead of (4.6) for the beam (2.1). This result can be easily obtained if one rewrites Eq. (4.1) as

$$\mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = -\mathbf{r} \times (\mathbf{B} \times \mathbf{E}) = -\mathbf{r} \times [\mathbf{B} \times (\nabla \times \Pi)] = -\mathbf{r} \times (B^i \nabla \Pi_i) + \mathbf{r} \times [(\mathbf{B} \cdot \nabla)\Pi] \tag{4.1}$$

and then changes $\mathbf{E} \to \mathbf{B}, \quad \mathbf{A} \to \Pi$ in Eq. (4.4).

It must be noted that $\mathbf{E} \times \mathbf{A}$ equals the component $\underset{c}{Y}^{xy0}$ of the invalid canonical spin tensor (1.3), and $\Pi \times \mathbf{B}$ is the magnetic addition to the canonical tensor presented in.[32] For the beam (2.1) $\mathbf{E} \times \mathbf{A} = \Pi \times \mathbf{B}$.

Conclusions, Notes and Acknowledgements

This paper conveys new physics. We review existing works concerning electrodynamics spin and indicate that existing theory is insufficient to solve spin problems because spin tensor of the modern electrodynamics is zero. Then we show how to resolve the difficulty by introducing a true electrodynamics spin tensor. Our spin tensor doubles a predicted angular momentum of a circularly polarized light beam without an azimuth phase structure. The tensor is needed, in particular, for understanding of essential characteristic features of a rotating dipole radiation.

The expression (1.6) for the spin tensor was submitted to scientific journals (the dates of the submissions are in parentheses): **AJP (10 Sep 2001)**, AO (April 20, 2006), AP (May 5, 2006), APP (28 Jan 2002), CJP (19 Nov 2003), CLEO/QELS Conference (22/11/2006), CMP (May 9, 2006), EJP (June 30, 2005), EPL (15 Oct 2002), FP (May 3, 2002), IJTP (January 25, 2006), JETP (27 Jan 1999), JETP Letters (**14 May 1998**), JMO (Sept 29, 2004), JMP (28 Nov 2002), JOP A (Nov 30, 2003), JOSA A (Apr 7, 2006), JOSA B (Dec 27, 2005), JPA (23 Jun 2002), JPB (Dec 12, 2003), MPEJ (Dec 24, 2004), Nature (Sept 21, 2006), NJP (27 Jun 2003), OC (22 Sept 2002), OL (29 Jul 2003), PLA (22 Jul 2002), PRA (19 Nov 2003), PRD (25 Sep 2001), PRL (Jul 4, 2005), RPJ (18 May 1999), TMP (29 Apr 1999), UFN (25 Feb 1999).

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