APRI-TH-PHY-2007-02

Einstein versus the Physical Review on Gravitational Waves and the Principle of Causality

C. Y. Lo

Applied and Pure Research Institute

17 Newcastle Drive, Nashua, NH 03060

March 2007

Abstract

Einstein & Rosen discovered the non-existence of wave solutions. However, the Physical Review found that the singularities they discovered are removable. Since the editorial accepted wave solutions with unbounded amplitude, Einstein's requirement on weak gravity was implicitly rejected in disagreement with general relativity. On the other hand, from the Hulse-Taylor binary pulsars experiment, it has been found that the nonexistence of gravitational wave solution is due to a violation of the principle of causality. It will be shown that such a violation can be seen directly from the cylindrical "waves" of Einstein and Rosen. The root of such an error lasting more than half a century is due to that the editorial of Physical Review and others did not understand Einstein's equivalence principle and incorrectly regarded it as essentially the same as Pauli's version. Consequently, they do not understand a basic principle in science, the principle of causality adequately. Moreover, the rejection of Einstein's requirement on weak gravity was based on the invalid interim "covariance principle". Currently, misinterpretations of Einstein's equivalence principle and the acceptance of the invalid "covariance principle", in effect, have developed into efforts conspired to give deceptive predictions.

04.20.-q, 04.20.Cv, 04.30.-w

Key Words: Einstein's equivalence principle, Euclidean-like structure, principle of causality, plane-wave.

1. Introduction

A mistake among theorists [1] including Einstein [2], Feynman [3], Landau & Lifshitz [4], etc., was assuming the existence of dynamic solutions for the Einstein equation of 1915. This question of dynamic solutions was raised by Gullstrand [5] in his 1921 report to the Nobel Committee. Due to conceptual errors such as ambiguity of coordinates as pointed out by Whitehead [6], Fock [7], and Zhou [8], many cannot reconcile the non-existence of dynamic solutions with the three accurate predictions. It was not until 1995 that the nonexistence of dynamic solution is proven [9, 10] and related issues are addressed in 2000 [1] and 2002 [11, 12]. Meanwhile, unphysical solutions were accepted [13] because Einstein's equivalence principle (of 1921) was not well understood [1] (see also Appendix A).

In fact, all the existing "wave" solutions are unbounded in amplitude, but physicists had to accept this as if a new feature since physical principles could not be used. Because coordinates were ambiguous [11, 12], the physical meaning of Einstein's equivalence principle and other physical principles were not clear. Recently, however, it is proven that a physical space must have a frame of reference that has the Euclidean-like structure.

To explain the Hulse-Taylor binary pulsar experiment, it is necessary to modify Einstein equation with an added source term, the gravitation energy-stress tensor (with an antigravity coupling), to accommodate the waves [9, 10] and to have a valid linearized equation. Since the gravitational wave whose sources are energy-stress tensors, carries energy-momentum, a gravitational wave should carry a source along. Then, it becomes clear that the non-existence of gravitational wave is due to a violation of the principle of causality [1)] [9].

Historically, Einstein & Rosen [14] could be considered as the first to discover the non-existence of wave solution, but the Physical Review found that the singularities they discovered are removable [15]. Her editorial implicitly rejected Einstein's requirement on weak gravity. Moreover, the editorials of the Physical Review and other journals failed to identify the violation of physical principles such as the principle of causality (see Section 4) because Einstein's equivalence principle of 1921 was not understood adequately (see Section 2).

In this paper, it will be shown that the violation of the principle of causality can be identified directly from the cylindrical "wave" solution of Einstein-Rosen [14]. Thus, the Physical Review was correct only at a point in mathematics. Recently, based on Einstein's equivalence principle, the physical meaning of coordinates has been clarified [11, 12]. Then, it is possible to justify Einstein's requirement on weak gravity in terms of the principle of causality. As the editorial of the Royal Society pointed out, Einstein's requirement for weak gravity was rejected because of the "covariance principle" (see Appendix B), which is due to Einstein's failure [16, 17] to identify the physical meaning of coordinates. It is hoped that this paper would awaken theorists to study the problem of gravitational waves anew with physical considerations included.

2. Einstein's Equivalence Principle and its Misinterpretation

Einstein claimed that his equivalence principle implies the Einstein-Minkowski condition (see Appendix A) that a free fall results in a co-moving local Minkowski space from which the time dilation and space contractions are obtained [16, 17]. However, in spite of his objection [18], his equivalence principle was regarded the same as Pauli's [19]:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

Thus, Pauli regards the equivalence principle as merely the mathematical existence of locally constant spaces. Moreover, Pauli's version would make that arbitrary coordinate systems have to be used (see section 4).

Apparently, Pauli, Misner, Thorne, & Wheeler [20], Straumann [21], and Will [22], overlooked (or disagreed with) Einstein's remark [17, p.144], "For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of coordinates…"

Moreover, John L. Friedman, Divisional Associate Editor of *Physical Review Letters*, officially claimed [23],

> "The experimentally well-tested theory of general relativity relies not on the author's version of the equivalence principle, but on a well-defined theory in which the existence of local Minkowski space has replaced the equivalence principle that initially motivated it."

However, the fact is that the theory based on Pauli's version is not well-defined because of the ambiguity in coordinates.

In addition, Synge [24] and Fock [7], who incorrectly claimed that Einstein's equivalence principle is nonessential, have misled many theorists. Such a view is echoed by Erick J. Weinberg, Editor of *Physical Review D*, who claimed [23], "These considerations do not, however, lead to experimentally testable consequences that are different from those obtained by the generally accepted application of general relativity."

Recently, it has been shown that Einstein's equivalence principle is crucial in the theory of measurement in general relativity [25]. In particular, Einstein's equivalence principle shows that Einstein's theory of measurement is actually invalid, and this is supported by experiments such as the observed bending of light [16, 25] since the velocities are defined without the metric.

Of course, the editorial of the Physical Review is not alone in such errors. For example, the editorial of the Royal Society implicitly rejected Einstein's requirement on weak gravity (see Appendix C). Some scientists perceived as experts for instance, Misner, Thorne, & Wheeler [20] incorrectly claimed that Einstein's equivalence principle is as follows:

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitestimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any other local Lorentz frame in the same or any other region. "

However, the Einstein-Minkowski condition [16, 17] is missing. Notably Misner et al. [20] claimed that gravitational energy-Momentum could not be located due to Einstein's equivalence principle. However, such a misinterpretation is in disagreement with the fact that the field energy would be exchanged with the energy of a particle [9].

A misinterpretation is not free. In their eq. (40.14) [20], they got an incorrect conclusion on the local time of the earth in the solar system since they did not understand [23] Einstein's equivalence principle and related theorems in Riemannian space [24]. Moreover, Ohanian & Ruffini [26] also ignored the Einstein-Minkowski condition and had the same problems in their eq. (50). (In fact, Ohanian, Ruffini, and Wheeler [26] proclaimed themselves as non-believers of Einstein's principles.) However, Straumann [21], Wald [27], and Weinberg [28] did not make the same mistake.

Misunderstanding of Einstein's equivalence principle keeps the coordinates to be ambiguous [11, 12]. Then, the so-called "covariance principle" appeared to be necessary, and led to the rejection of Einstein's requirement on weak gravity (Appendix C). Zhou [8] pointed out, "The concept that coordinates don't matter in the interpretation of Einstein's theory … necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory." The ambiguity of coordinates was a major difficulty in applying the principle of causality. However, as shown in next section, Penrose ignored the principle of causality, even when a precise physical meaning of coordinates is not needed.

Unexpectedly, a key to the existence of the Euclidean-like structure is Einstein's equivalence principle [11, 12]. One may wonder why this clearly illustrated fact in Einstein's solution was not recognized. In Einstein's theory of measurement, [2)] he overlooked that his measurements for a physical Riemannian space is not static but dynamics [25]

3. Gravitational Plane-Waves and Physical Requirements

The principle of causality requires the metric of weak gravity must be bounded in amplitude. The Maxwell-Newton Approximation (MNA) supports such a conclusion [29]. Nevertheless, all "plane waves" solutions are unbounded [1].

A metric solution that convinced many the existence of gravitational wave solution was obtained by Bondi, Pirani, & Robinson [30], who claimed that a wave from a distant source, is

$$ds^2 = \exp(2\phi)(d\tau^2 - d\xi^2) - u^2[\mathrm{ch}2\beta\,(d\eta^2 + d\zeta^2) + \mathrm{sh}2\beta\,\cos 2\theta\,(d\eta^2 - d\zeta^2) - 2\mathrm{sh}2\beta\,\sin 2\theta\, d\eta d\zeta], \qquad (1)$$

where ϕ, β, θ are functions of u (= $\tau - \xi$). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u(\beta'^2 + \theta'^2\,\mathrm{sh}^2 2\beta). \qquad (2)$$

However, metric (1) is not bounded because of the factor u^2 that grows anomaly large as time τ goes by.

According to the Maxwell-Newton Approximation, as shown by Einstein [31], a plane-wave with a finite source is always bounded. Thus, metric (1) is not valid in physics. Note that a violation of the principle of causality is independent of a physical coordinate system, and thus it is not necessary to show this for all admissible systems.

Another well known "plane wave" is the metric considered by Misner, Thorne & Wheeler [15] as follows:

$$ds^2 = dt'^2 - dx'^2 - L^2(e^{2\beta}dy^2 + e^{-2\beta}dz^2), \qquad (3)$$

where $L = L(u')$, $\beta = \beta\,(u')$, $u' = t' - x'$. Then, the Einstein's field equation becomes

$$\frac{d^2L}{du'^2} + L\left(\frac{d\beta}{du'}\right)^2 = 0\,. \qquad (4)$$

However, this equation has no bounded plane-wave solution [3)] since L'' cannot be always negative for a weak solution.

Moreover, a "plane wave", which is clearly unphysical, is accepted by Penrose [32] as follows:

$$ds^2 = du\,dv + Hdu^2 - dx_i\,dx_i, \qquad \text{where} \qquad H = h_{ij}(u)\,x_i\,x_j \tag{5}$$

where $u = ct - z$, $v = ct + z$, $x = x_1$ and $y = x_2$, $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$.

This metric satisfies the harmonic gauge, and its cause can be an electromagnetic plane wave. Metric (5) satisfies

$$\eta^{\alpha\beta}\,\partial_\alpha\partial_\beta\,\gamma_{tt} = -2\{h_{xx}(u) + h_{yy}(u)\}, \qquad \text{where} \qquad \gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}. \tag{6}$$

However, causality is not satisfied. For the force is related to $\Gamma^z{}_{tt} = (1/2)(1 + H)\partial H/\partial t$; and there are arbitrary non-physical parameters (the choice of origin) that are unrelated to the cause (a plane wave).

Now, it is clear that Einstein's requirement on weak gravity was rejected. A common problem in the theory of gravitational wave is that physical considerations are being neglected (see also Appendix C) [13].

4. The Principle of Causality and the Cylindrical Symmetric Metrics of Einstein and Rosen

Since Einstein's equivalence principle has not been understood adequately [23, 25], physical principles are often neglected [12, 29]. Among the existing so-called wave solutions, not only Einstein's equivalence principle, but also others such as the principle of causality and the principle of relativistic causality [11] are not satisfied.

Let us examine their cylindrical "waves" again. In coordinates, ρ, φ, and z, the solution of Einstein & Rosen [14] is

$$ds^2 = \exp(2\gamma - 2\Psi)(dT^2 - d\rho^2) - \rho^2\exp(-2\Psi)d\varphi^2 - \exp(2\Psi)dz^2 \tag{7}$$

where T is the product of the velocity of light and the time coordinate, and γ and ψ are functions of ρ and T. They satisfy

$$\Psi_{\rho\rho} + (1/\rho)\Psi_\rho - \Psi_{TT} = 0, \qquad \gamma_\rho = \rho[\Psi_\rho{}^2 + \Psi_T{}^2], \qquad \text{and} \qquad \gamma_T = 2\rho\Psi_\rho\Psi_T. \tag{8}$$

Rosen [35] consider the energy-stress tensor $T_{\mu\nu}$ and Einstein's gravitational pseudotensor $t_{\mu\nu}$. He found that

$$T_4{}^4 + t_4{}^4 = 0, \qquad \text{and} \qquad T_4{}^l + t_4{}^l = 0, \tag{9}$$

where $t_4{}^l$ is momentum in the ρ-direction. In other words, the total energy density and momentum in ρ-direction are zero.

However, Weber & Wheeler [33] argued that these results are meaningless since $t_{\mu\nu}$ is not a tensor. Moreover, they claimed, "We concluded that many of the otherwise apparently paradoxical properties of this cylindrical wave can be understood by taking into account the analogy between gravitational waves and electromagnetic waves, and the special demands of the equivalence principle, which rules out a special role for any particular frame of reference." [4)] They failed to see that due to Pauli version of equivalence, arbitrary coordinate systems had to used [34].

Weber and Wheeler did not supply any evidence for their claimed analogy between gravitational waves and electromagnetic waves. Although the first equation in (8) is a Maxwell-type, the Maxwell-Newton Approximation, a full analogy, is not satisfied since metric (7) does not satisfy the linearized gauge harmonic conditions. In fact, because of the other two equations in (8), the function γ satisfies a non-linear equation $\gamma_{\rho\rho} + \gamma_\rho/\rho - \gamma_{TT} = 2\psi_T{}^2$ (see also Appendix D)

The relation (9) reflects that the cylindrical metric (7) cannot really have a radiation propagating in the ρ-direction as seen from the following solution [33].

$$\Psi = AJ_0(\omega\rho)\cos\omega t + BN_0(\omega\rho)\sin\omega t, \tag{10a}$$

where A and B are constants. The second function γ in the special case B = A, reduces to

$$\begin{aligned}\gamma = \frac{1}{2}A^2\omega\rho\{&J_0(\omega\rho)J_0'(\omega\rho) + N_0(\omega\rho)N_0'(\omega\rho) + \omega\rho[(J_0(\omega\rho))^2 + (J_0'(\omega\rho))^2 + (N_0(\omega\rho))^2 + (N_0'(\omega\rho))^2] \\ &+ [J_0(\omega\rho)J_0'(\omega\rho) - N_0(\omega\rho)N_0'(\omega\rho)]\cos 2\omega T + [J_0(\omega\rho)N_0'(\omega\rho) + N_0(\omega\rho)J_0'(\omega\rho)]\sin 2\omega T\} - \frac{2}{\pi}A^2\omega T\end{aligned} \tag{10b}$$

(10b) implies that $\exp(2\gamma)$ would become very small. (Note that T and $\omega\rho$ should have been respectively t and $\omega\rho$/c.)

According (10b) the function $\exp(2\gamma)$ gets very large as T goes to negatively large. Thus, 't Hooft's [35] claim of boundedness is a mistake. Since $\exp(2\gamma)$ approaches to zero as T gets positively large (thus $g_{tt} = g_{\rho\rho} \cong 0$), the condition for weak gravity ($1 >> |\gamma_{\mu\nu}|$) would fail, independent of parameters such as the amplitude and the frequency.

For instance, one can prove that metric (7) cannot satisfy coordinate relativistic causality. Weber and Wheeler agreed with Fierz's analysis, based on (8), that γ is a strictly positive where $\Psi(\rho, T) \approx 0$ for large ρ [33]. On the other hand, for a cylindrical coordinate system, time dilation and space contractions would mean

$$- g_{\rho\rho} \geq 1 \geq g_{tt} , \qquad - g_{\varphi\varphi}/\rho^2 \geq 1 , \quad \text{and} \qquad - g_{zz} \geq 1 . \tag{11a}$$

These would imply coordinate relativistic causality [11]. Thus, from metric (7) one has $\exp(2\gamma) \leq 1$ and $\exp(2\gamma) \leq \exp(2\Psi)$. It thus follows that $\gamma \leq 0$. Hence, the condition $\gamma > 0$ cannot be met. In fact, it follows (11a) directly that

$$(2\gamma - 2\Psi) \geq 0 \geq (2\gamma - 2\Psi), \qquad -2\Psi \geq 0 , \qquad \text{and} \qquad 2\Psi \geq 0. \tag{11b}$$

Thus $\Psi = 0$ and $\gamma = 0$. This shows again that there is no physical wave solution for $G_{\mu\nu} = 0$.

Thus, invalidity of the "wave" solutions, raised by Rosen [33], has been verified after the physical meaning of space-time coordinates is clarified [11, 12].

5. Discussions and Conclusions

Weber & Wheeler [33] and 't Hooft [35] draw physical interpretations almost exclusively from Ψ, which satisfies a Maxwell-type equation, although γ is different (see also Appendix D). Moreover, they were unaware violations of the principle of causality. They invalidly ignored how the waves are generated and that gravity cannot be screened. A criterion for a valid plane-wave solution is whether it is compatible with the Maxwell-Newton Approximation with a source [1, 29, 36].

Whitehead [6] and Fock [7] objected general relativity because the physical meaning of coordinated was ambiguous [16, 17]. Zhou [37] pointed out that coordinates must have physical meaning since all of physical quantities have to be expressed in terms of coordinates. In addition, Liu & Zhou [36] are among the earliest, who recognized that physical requirements must be considered for a solution of plan-waves. An irony of Einstein's theory of measurement is that its method cannot be executed for measuring an extended object [38].

In contrast, Hawking and Ellis [39] seem to forget the physical conditions from which mathematical expressions were obtained.[5)] Ellis [40] and 't Hooft [41].claimed there is no reason to reject an unbounded plane-wave since the Schwarzschild solution, which is not bounded at the event of horizon, is accepted. Apparently, he failed to see that such a Schwarzschild solution is reducibly unbounded because the internal solution is different, whereas the plane-waves are irreducibly unbounded. (see Appendix C). They are currently good examples to illustrate that that Penrose is not the only well known theorist who does not understand the principle of causality adequately.

A common error [6)] was assuming the existence of dynamic solutions for the Einstein equation of 1915 [1]. For dynamic situations, the linearized equation is unrelated to Einstein equation [3, 9]. However, a conclusion on the existence of dynamic solutions was almost impossible unless Einstein's equivalence principle is understood so that the physical meaning of coordinates is clarified. Many wrongly regarded Einstein's equivalence principle of 1921 the same as the 1911 assumption of equivalence between Newtonian gravity and an accelerated frame [42]. Moreover, the formula of Landau & Lifshitz [4] for space contractions was rejected because of inadequate understanding of Einstein's equivalence principle [25].

One may ask what would be responsible to the mistake on the question of gravitational waves that has last more than half a century. A problem is that Einstein's equivalence principle has not been well understood. Consequently, this field is dominated by applied mathematicians who care little about physical requirements. Because the large volume of papers must be handled in a timely manner, the practical criterion is often just an opinion of the perceived expert(s), many of those actually do not understand relativity, and the Physical Review D becomes a victim of this traditional pressure.

Now, it is clear that the cylindrical symmetric "wave" solution of Einstein and Rosen is not valid. Who could have imagined that the a basic reason for failure of some well known theorists in general relativity is due to inadequate understanding of a basic principle of science, the principle of causality? In physics, the fundamental concepts are often difficult to grasp. A well-known example is that Schrödinger for a long time did not understand the solutions of his equation [43]. Einstein seems not an exception of such a problem. He failed to see that his equivalence principle is actually inconsistent with his theory of measurements [12, 25]. Thus, he believed incorrectly [44] the solutions of with different gauges as equally valid.

Currently, misinterpretations of Einstein's equivalence principle and the acceptance of the invalid "covariance principle", in effect, have developed into efforts conspired to give deceptive predictions. For example, Will [45], the Chair of NASA's Science Advisory Committee for Gravity Probe B, in a paper published in Physical Review D, claimed that there is only one covariant formula for the de Sitter precession. However, Will has a record of being dishonest in science [46]. Detailed calculations show that there is no covariant formula, but different formulas for the de Sitter precession [34]. Thus, the Physical Review D has become a tool for Will's game of deception.

The Physical Review is a major journal that serves the physics community well. However, like any human institutes, she is not perfect; especially in the handling of gravity are not as good as other branches of physics. In this paper, it has been shown that a metric satisfying the principle of causality is intimately related to boundedness of its magnitude. It is hoped that this paper would fairly settle the historical account of the events between Einstein and the Physical Review. More important, the editorials would rectify the errors and give an added impetus for solving the severe problems in theories of gravitational waves.

Acknowledgments

The author is grateful for stimulating discussions with S. L. Cao, G. F. R. Ellis, G. ‘t Hooft and Liu Liao on plane-waves and the analysis of Weber and Wheeler on the cylindrical symmetric metric of Einstein and Rosen. This publication is supported in part by Innotec Design, Inc., U.S.A.

Appendix A: The Einstein-Minkowski Condition, and the “Covariance Principle”.

Einstein’s equivalence principle is explained very clearly in page 57 of “The Meaning of Relativity” [16]. A consequence is the Einstein-Minkowski condition [17, p. 161], which has its foundation from theorems [24] as follows:

Theorem 1. Given any point P in any Lorentz manifold (whose metric signature is the same as a Minkowski space) there always exist coordinate systems (x^μ) in which $\partial g_{\mu\nu}/\partial x^\lambda = 0$ at P.

Theorem 2. Given any time-like geodesic curve Γ there always exists a coordinate system (so-called Fermi coordinates) (x^μ) in which $\partial g_{\mu\nu}/\partial x^\lambda = 0$ along Γ.

In these theorems, the local space of a particle is locally constant.

However, after some algebra, a local Minkowski metric exists at any given point. For metric,

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2, \quad \text{(A1)}$$

the geodesic is a straight line, and the local metric of a particle in free fall is not locally Minkowski.

What Einstein added to theses theorems is the requirement that *for a physically valid space*, such a locally constant metric must be locally Minkowski, and thus special relativity is a special case [16]. From metric (A1), it is clear that Einstein’s equivalence principle is inconsistent with his “covariance principle”.[7)]

In calculations of the bending light [16, 17], Einstein assumed, but did not prove validity of the Einstein-Minkowski condition. Moreover, such metrics are not equivalent because an unphysical metric can produce the correct result [44]. Thus, the correct physical gauge remains to be determined.

Appendix B: Invalidity of the “Covariance Principle”

The so-called “covariance principle” is a favorite among applied mathematicians, who often over-looked physical requirements. In fact, the creation of such a principle is due to Einstein’s failure to identify adequately the physical meaning of the coordinates. Einstein called it the “principle of covariance” [17], "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." In view of that the “principle of covariance” is an interim assumption due to Einstein’s certain ignorance of the space-time coordinates [17], it is only natural that such an invalid “principle” is a source of theoretical inconsistence in Einstein’s theory [23]. However, this “principle” is an extension of covariance in special relativity, and thus would be accepted by those who do not deliberate the physical requirements carefully.

For instance, in special relativity, there is no exchange of the time coordinate system and a space coordinate system. Later Einstein [16] realized this problem and remarked “As in the special theory of relativity, we have to discriminate between time-like and space-like line elements in the four dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds.”

This “principle” leads to the notion of Lorentz manifolds [27] that cannot be one-one corresponding to a four-dimensional Minkowski space. Then, for such a manifold, Einstein’s requirement for weak gravity may not be applicable since a mathematical coordinate system may not relate to a physical frame of reference. The crucial point of the covariance principle is the validity of any Gaussian coordinate system as a space-time coordinate system in physics. For this, Einstein’s supporting arguments [17] are as follows:

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

Einstein's arguments, though highly deceptive, are actually false. Note that the meaning of measurements is crucially omitted. First, his arguments are incompatible with his earlier argument for defining time relating to local clocks [17]. Moreover, in order to predict events, one must be able to relate events of different locations in a definite manner [34]. Moreover, Zhou correctly pointed out that coordinates must have physical meaning [8].

Zhou argued [8], "When we come to solve the field equation of moving matter, we must first define the geometrical configuration of matter, the symmetry of the configuration, its density distribution, pressure, and velocity of motion in space-time. All of them have to be expressed in terms of coordinates." Note that all physical predictions, including Einstein's own three tests, must be understood in terms of the physical meaning of coordinates [34].

Thus, the "covariance principle" is clearly not just a mathematical covariance as Wald explained [27]. For instance, in support of this principle Einstein [16] mentioned that the bending of light can be derived from different metrics with the same frame of reference. However, these metrics give different formulas for the de Sitter precession [34], and thus become good counter examples. Since the root of such covariance is due to Einstein's failure in laying down coordinates in a definite manner [17], to resolve this problem, one must identify the physical meaning of space-time coordinates [12].

Appendix C: Einstein's Requirement on Weak Gravity and the Principle of Causality

For a space-time metric $g_{\mu\nu}$, Einstein's requirement of weak gravity is

$$\gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}\,, \qquad \text{and} \quad 1 \quad >> \quad |\gamma_{\mu\nu}| \tag{C1}$$

where $\eta_{\mu\nu}$ is flat metric in a Minkowski space. The foundation of this requirement is the principle of causality that requires a weaker source would produce weaker gravity [29]. Thus, the plane-waves and the cylindrical metric of Einstein and Rosen [33] should be reducible to the case of a metric with bounded amplitudes.

Nevertheless, as shown by the metric of Bondi, Pirani, & Robinson [31], editorials of the Royal Society rejected Einstein's requirement on weak gravity and claimed [47] it as meaningless in physics because of being coordinate-dependent. Their justification is Einstein's "covariance principle" that allows to assume the physical space as a manifold. (Thus, Misner et al [20] disagrees with the editorial of the Royal Society.) This confirms further that Einstein's theory is not self-consistent [34].

It has been shown in Appendix A that the "covariance principle" is inconsistent with Einstein's equivalence principle. This illustrates that understanding Einstein's equivalence principle crucial in general relativity.

The equivalence principle leads to the conclusion that a physical space must have a frame of reference with the Euclidean-like structure [11, 12]. Moreover, since such a structure is defined by measurements, there is only one such structure for a given frame of reference. Then, Einstein's requirement on weak gravity can be defined in term of the reduced metric elements in which the factors from the line elements of the Euclidean-like structure are removed [29]. Thus, his requirement (C1) on weak gravity is generally applicable. It thus follows that the irreducibly unbounded plane-waves are not valid in physics.

Appendix D: The Cylindrical Symmetry "Wave" Solution by G. 't Hooft

Professor 't Hooft produced a cylindrical solution that satisfied the condition having a weak limit as follows:

1. Explicit bounded solution

Equations:

$$\Psi_{rr} + \frac{1}{r}\Psi_r - \Psi_{tt} = 0\,, \qquad \gamma_r = r(\Psi_r^2 + \Psi_t^2)\,, \qquad \gamma_t = 2\Psi_r\Psi_t\,. \tag{1.1}$$

One of the many bounded solutions:

$$\Psi = A\int_0^{2\pi} d\varphi e^{-\alpha(t - r\cos\varphi)^2}$$

where A and α are free parameters. For simplicity, take them to be one. Any linear superposition of such expressions of course also qualifies. $|\psi|$ is everywhere bounded by 2π (A =1). At large value for t and r, we see that the stationary points of the cosine dominate, so that there are peaks at r = |t|. This is a wave packet coming from r = ∞ at t = - ∞ bouncing against the origin at t ≅ 0 (always obeying the correct boundary condition there), and moving to r = ∞ again at t = - ∞ , bouncing against the origin at t ≅ 0 (always obeying the correct boundary condition there), and moving to r = ∞ again at t →∞. At |t| >> r, the function ψ drops off exponentially, and at r >> |t| as a power:

$$\Psi \to 2\sqrt{\frac{\pi}{r^2 - t^2}}$$

(easily derived by looking at the value that contribute most).

The other function γ is found by integrating one of the two other equations (they of course yield the same value). γ drops off exponentially at | t | >> r and for r >> | t | it goes asymptotically to :

$$\gamma \to \frac{-2\pi r^2}{(r^2 - t^2)} + Cst.$$

Note the t ↔ -t symmetry throughout.

2. Green function

Given arbitrary values for $\Psi(r, 0)$ and $\Psi_t(r, 0)$, the solution obeys

$$\Psi(x,t) = \int_0^\infty dr (G(x,r,t)\Psi_t(r,0) + G_t(x,r,t)\Psi(r,0)) , \qquad t > 0 \qquad (1.2)$$

where G is

$$G(x,r,t) = \frac{r}{2\pi} \oint_{\sqrt{+}} d\varphi \frac{1}{\sqrt{2xr\cos\varphi + t^2 - x^2 - r^2}} , \qquad (1.3)$$

where the symbol √+ indicates that the integral should only be taken over the values of φ for which the entry of the square root is positive. If t < |x - r| there are no real φ values obeying this, so G vanishes there.

Derivation: take action functional S in 3 + 1 dimensions:

$$S = \int d^3\vec{x}\, dt \left(-\frac{1}{2}(\vec{\partial}\Psi)^2 + \frac{1}{2}\Psi_t^2 + J(\vec{x},t)\Psi\right) , \qquad (1.4)$$

leading to the field equation

$$\Psi_{tt} - \vec{\partial}^2\Psi = J(\vec{x},t) . \qquad (1.5)$$

General solution:

$$\Psi(\vec{x},t) = \int d^3\vec{x} \int_0^\infty d\tau\, G(\vec{x} - \vec{x}', \tau) J(\vec{x}, t - \tau) \qquad (1.6)$$

$$G(\vec{x},t) = \frac{1}{4\pi r}\delta(r - \tau)\theta(\tau) , \qquad r^2 = \vec{x}^2 . \qquad (1.7)$$

G is the so-called Green function of the system. Now take the cylindrically symmetric case,

$$J = J(r, t) , \qquad \psi = \psi(r, t) , \qquad (1.8)$$

where

$$\frac{S}{2\pi} = \int_0^\infty dr \int dt \left(-\frac{1}{2} r\Psi_r^2 + \frac{1}{2} r\Psi_t^2 + rJ\Psi\right) , \qquad (1.9)$$

in which case the field equation becomes ours with source term inserted:

$$\Psi_{tt} - \Psi_{rr} - \frac{1}{r}\Psi_r = J(r,t) \qquad (1.10)$$

Using (1.6) and (1.7), we find

$$\Psi(r_1,t) = \int dr \oint d\varphi \int dz \int d\tau \frac{r}{4\pi\tau}\delta(\rho-\tau)J(r,t-\tau)\,, \tag{1.11}$$

where $\rho \equiv \sqrt{z^2 + r^2 + r_1^2 - 2rr_1\cos\varphi}$. (1.12)

The delta function can be written as

$$\delta(\rho-\tau) = \frac{\tau}{z}\delta\left(z - \sqrt{\tau^2 + 2rr_1\cos\varphi - r^2 - r_1^2}\right). \tag{1.13}$$

If the square root exists there are two solutions for z, so

$$\Psi(r_1,t) = \int_0^\infty dr \oint_{\sqrt{+}} d\varphi \int_0^\infty dt G(r_1,r,t)J(r,t-\tau)\,, \tag{1.14}$$

where G is as given in (1.3). The equation (1.2) is obtained by choosing $\Psi(r, t) = 0$ if $t < 0$ and a source

$$J(r, t) = \delta(t)\Phi_2(r) + \delta'(t)\Phi_1(r)\,, \tag{1.15}$$ [8)]

So that the field equation (1.10) forces

$$\Psi(r, t) \to \Phi_1(r)\,, \qquad \psi_t(r, t) \to \Phi_2(r)\,, \tag{1.16}$$

$$\text{as} \quad t \downarrow 0 \tag{1.17}$$

To see that (1.16) indeed follows from (1.15) and (1.10), evaluate G for $t << r, r_1$:

$$G(x,r,t) \to \frac{1}{2}(\theta(r-x+t) - \theta(r-x-\tau))\,. \tag{1.18}$$

To see that (1.3) indeed satisfies the field equation

$$G_{tt} - G_{xx} - (1/x)\, G_x = \delta(x - r)\delta(t) \tag{1.19}$$

is an elementary exercise. It follows from the construction, and the direct argument is as follows: the integrand,

$$g(x,t,\varphi) = \frac{1}{\sqrt{2xr\cos\varphi + t^2 - x^2 - r^2}}$$

obeys the partial differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} - \frac{\partial^2}{\partial t^2} + \frac{1}{x^2}\frac{\partial^2}{\partial\varphi^2}\right)g(x,t,\varphi) = 0\,, \tag{1.20}$$

so that integration of Eq. (1.19) over φ only gives boundary terms. To see the contribution of these boundary terms, reexpress the integral in terms of a closed contour integral in the complex plane. One finds that G can also be written as

$$G(x,r,t) = \frac{r}{2\pi}\int_{\sqrt{+}} d\varphi \frac{1}{\sqrt{2xr\cosh\varphi + x^2 + r^2 - t^2}}\,, \tag{1.21}$$

where the integral runs from $-\infty$ to ∞ or over the domain where the square root exists. The field equation then leads to an integral over φ of a function that is a pure derivation of a function of φ with appropriate boundary conditions. Either in Eq. (1.3) or in Eq. (1.12), we see that the boundaries vanish, unless $r = x$ and $t = 0$. The delta functions in (1.19) are deduced by inspecting the points where G is singular. Expression (1.3) ensures that $G = 0$ as soon as $|x - r| > t$.

This completes the proof that these solutions are completely causal. It is an elementary exercise in mathematical physics.

ENDNOTES

1) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality [1, 11, 29]. Parameters unrelated to any physical cause is not allowed, and a dynamic solution must be related to a dynamic source that generates the wave. Einstein and others have used this principle implicitly on symmetry considerations [16, 28] such as for a circle in a uniformly rotating disk and the metric for a spherical symmetric mass distribution. A physi-

cal solution due to sources must have weak limit that satisfies Einstein's condition for weak gravity. Moreover, a violation of any physical principle is also a violation of the principle of causality.

2) Einstein's theory of measurement is incorrect because theoretically it is in conflict with his equivalence principle [25], and it also led to speeds of light that result in disagreement with the observed light bending [23].

3) Misner et al [20] believe that their equation has a plane-wave solution with bounded amplitude.

4) Here, Weber & Wheeler [33] have mistaken the "Covariance principle" as Einstein's equivalence principle.

5) In the singularity theorems of Hawking and Penrose, a crucial assumption is the unique sign of all coupling constants. Such an assumption is based on misinterpreting the famous formula $E = mc^2$ as unconditional equivalence between mass and any type of energy. However, such an assumption has been proven invalid theoretically [48-51] and experimentally by the Hulse and Taylor observations on binary pulsars [1, 9]. Nevertheless, Hawking in his recently (June 2006) visit to China, still misleadingly told his audience that his theory was based on general relativity only.

6) It was believed [2, 3] that the two-body problem could be solved in Einstein's equation. However, as suspected by Gullstrand [5] and conjectured by Hogarth [52], the opposite is correct.

7) Einstein supported his "covariance principle" with that the Schwarzschild solution and the isotropic solution give the same deflection of light. Thus, this principle is different from the pure mathematical covariance, which allows only scalars as invariant. However, it has been shown that these metrics give different formulas for the de Sitter precession [34].

8) Since a normal source of gravity cannot be a delta function of time, this is a violation of causality.

REFERENCES

1. C. Y. Lo, Phys. Essays, **13** (4), 527-539 (December, 2000).
2. A. Einstein, L. Infeld, & B. Hoffmann, Annals of Math. **39** (1), 65-100 (Jan. 1938).
3. R. P. Feynman, ***The Feynman Lectures on Gravitation*** (Addison-Wesley, New York, 1996).
4. L. D. Landau & E. M. Lifshitz, ***Classical Theory of Fields*** (Addison-Wesley, Reading Mass. 1962).
5. A. Gullstrand, Ark. Mat. Astr. Fys. **16**, No. 8 (1921); A. Gullstrand, Ark. Mat. Astr. Fys. **17**, No. 3 (1922).
6. A. N. Whitehead, ***The Principle of Relativity*** (Cambridge Univ. Press, Cambridge, 1922).
7. V. A. Fock, ***The Theory of Space Time and Gravitation***, translated by N. Kemmer (Pergamon Press, 1964).
8. Zhou Peiyuan, "On Coordinates and Coordinate Transformation in Einstein's Theory of Gravitation" in ***Proc. of the Third Marcel Grossmann Meetings on Gen. Relativ.*** ed. Hu Ning, Sci. Press & North Holland. (1983), 1-20.
9. C. Y. Lo, Astrophys. J. **455**: 421-428 (Dec. 20, 1994).
10. C. Y. Lo, Phys. Essays, **12** (2), 226-241 (June, 1999).
11. C. Y. Lo, Phys. Essays, **15** (3), 303-321 (September 2002).
12. C. Y. Lo, Chinese J. of Phys., **41** (4), 332-342 (August 2003).
13. D. Kramer, H. Stephani, E. Herlt, & M. MacCallum, **Exact Solutions of Einstein's Field Equations**, ed. E. Schmutzer (Cambridge Univ. Press, Cambridge, 1980).
14. A. Einstein & N. Rosen, J. Franklin Inst. **223**, 43 (1937).
15. Daniel Kennefick, "Einstein versus the Physical Review," Physics Today (September, 2004).
16. A. Einstein, ***The Meaning of Relativity*** (Princeton Univ. Press, 1954).
17. A. Einstein, H. A. Lorentz, H. Minkowski, H. Weyl, ***The Principle of Relativity*** (Dover, New York, 1923).
18. J. Norton, "What was Einstein's Principle of Equivalence?" in Einstein's Studies Vol. 1: ***Einstein and the History of General Relativity,*** eds. D. Howard & J. Stachel (Birkhäuser, 1989).
19. W. Pauli, ***Theory of Relativity*** (Pergamon Press, London, 1958).
20. C. W. Misner, K. S. Thorne, & J. A. Wheeler, ***Gravitation*** (W. H. Freeman, San Francisco, 1973).

21. N. Straumann, ***General Relativity and Relativistic Astrophysics*** (Springer, New York, 1984).

22. C. M. Will, ***Theory and Experiment in Gravitational Physics*** (Cambridge University Press, 1981).

23. C. Y. Lo, Phys. Essays, **16** (1), 84-100 (March 2003).

24. J. L. Synge, Relativity; The General Theory (North-Holland, Amsterdam, 1971), pp. IX–X.

25. C. Y. Lo, Phys. Essays, **18** (4) (Dec. 2004).

26. H. C. Ohanian & R. Ruffini, ***Gravitation and Spacetime*** (Norton, New York, 1994).

27. R. M. Wald, ***General Relativity*** (The Univ. of Chicago Press, Chicago, 1984).

28. S. Weinberg, ***Gravitation and Cosmology*** (John Wiley, New York, 1972), p. 273.

29. C. Y. Lo, Phys. Essays, **12** (2), 508-526 (September, 1999).

30. H. Bondi, F. A. E. Pirani, & I. Robinson, Proc. R. Soc. London A **251**, 519-533 (1959).

31. A. Einstein, Sitzungsberi, Preuss, Acad. Wis. 1918, 1: 154.

32. R. Penrose, Rev. Mod. Phys. **37** (1), 215-220 (1964).

33. J. Weber & J. A. Wheeler, Rev. Modern Phys. **29** (3) 509 (1957).

34. C. Y. Lo, Phys. Essays 18 (1), 112-124 (March 2005).

35. G. 't Hooft claimed that the Einstein-Rosen solutions [14] are bounded in amplitude (September 23, 2004). Later, he regarded these metrics as physically valid "wave" solutions (Private communications, August 8, 2004).

36. H. Y. Liu & P.-Y. Zhou, Scientia Sincia (Series A) 1985, XXVIII (6) 628-637.

37. Zhou (Chou) Pei-Yuan, Scientia Sinica (Series A), 1982 XXV (6): 628-643.

38. C. Y. Lo, Phys. Essays 18 (4), 539 (Dec. 2005).

39. S. W. Hawking & G. F. R. Ellis, ***The large scale structure of space-time*** (Cambridge University press, 1979).

40. G. F. R. Ellis, Ph.D. Editor-in-Chief, General Relativity and Gravitation, supports the validity of unbounded plane-waves (Feb. 22, 2007). Thus, just like Penrose, Ellis also disagrees with Einstein, who obtained his physical requirement on weak gravity [16, 17] by using the principle of causality [1, 11].

41. G. t' Hooft also supports the validity of unbounded plane-waves and wants his view to be known (February 23, 2007).

42. Tai-Pei Cheng, ***Relativity, Gravitation, and Cosmology*** – a basic introduction (Oxford University Press, 2005).

43. L. Motz & J. H. Weaver, ***The Story of Physics*** (Avon, New York, 1989).

44. C. Y. Lo, Chinese Physics, **13** (2), 159-167 (February 2004).

45. C. M. Will, Phys. Rev. D, 67, 062003-1 (2003)

46. C. Y. Lo, Astrophys. J. **477,** 700-704 (1997).

47. Fiona Pring, The Royal Society, on Board Member's Comments, "The author wishes to require that the components of the metric be bounded. This is a coordinate-dependent condition, and is physically meaningless." (Jan. 8, 2007).

48. Einstein A, '$E = mc^2$' (1946) in ***Ideas and Opinions*** (Dover, 1982).

49. C. Y. Lo, 2006, Progress in Phys. Vol. 4, 14-18.

50. C. Y. Lo & C. Wong, Bull. Pure and Applied Sci., Vol. 25D, No.2, 109-117 (2006).

51. C. Y. Lo, 2007, Chin. Phys. (Beijing), **16** (3) 635-639 (March).

52. J. E. Hogarth**, *"Particles, Fields, and Rigid Bodies in the Formulation of Relativity Theories*"**, Ph. D. Thesis 1953, Dept. of Math., Royal Holloway College, University of London (1953), p. 6.