Calculation of absorbed spin contradicts electrodynamics and an experiment

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Absorption of a circularly polarized light beam without an azimuth phase structure in a dielectric is calculated in the frame of the standard electrodynamics. We calculate a transfer of energy, momentum, and angular momentum to the dielectric. The calculation shows that the force acting on the dielectric does not have a surface term and the angular momentum flux in the beam equals to two power of the beam divided by frequency. This results contradict an experiment of A. Bishop et al. (PRL, 92, 198104), a paper of R. Loudon (PRA, 68, 013806), and another part of standard electrodynamics, which predicts the flux equals to power of the beam divided by frequency. At the same time the last part of the electrodynamics contradicts to the classical Beth's experiment (PR, 50, 115). To correct this part of the standard electrodynamics we introduce a spin tensor. The corrected electrodynamics is in accordance with our calculation and with the Beth's experiment. But we cannot help the Bishop's and Loudon's results.

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A remarkable work of A. Bishop et al. [1] is devoted to a measurement of viscous properties on a micronic scale. They used polarized light beams, which passed through objects in a single direction. Their results were in agreement with the established value for the viscosity if they considered an angular momentum flux in a circularly polarized beam was

$$\tau = \mathsf{P}/\omega \tag{1}$$

where P is the power and ω is the angular frequency of the beam. So, [1] confirms (1). Eq. (1) is also supported by an enormous number of authors. It is a corollary of standard expressions for a total angular momentum **J** and energy U of a circularly polarized beam [2, 3]

$$\mathbf{J} = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV, \quad J_z = U/\omega \tag{2}$$

(V is the volume of a piece of the beam). Nevertheless we have reasons to doubt Eq. (1). To verify the statements (1), (2) we consider absorption of the beam by semi-infinite dielectric. We find that the dielectric absorbs twice as much as (1), (2), i.e.

$$\tau = 2\mathsf{P}/\omega, \quad J_z = 2U/\omega \tag{3}$$

Here is our calculation.

We use the Jackson's expression [3] for the beam,

$$\breve{\mathbf{E}} = \exp[i(\breve{k}z - t)][\mathbf{x} + i\mathbf{y} + \mathbf{z}\frac{1}{\breve{k}}(i\partial_x - \partial_y)]u(\rho), \quad \breve{\mathbf{B}} = -i\breve{k}\breve{\mathbf{E}}, \quad \rho^2 = x^2 + y^2, \quad \breve{k}^2 = \breve{\varepsilon}, \quad \breve{k} = \eta + i\kappa, \quad |\breve{k}| = k. \tag{4}$$

The symbol 'breve' marks complex vectors and numbers excepting i. $\breve{k}$ is the complex wave number, and $\breve{\varepsilon}$ is the permittivity. For short, we set speed of light in vacuum, $c = 1$, and the frequency, $\omega = 1$. Note that our fields are increased by a factor of $\sqrt{2}$ relative to the fields of Allen, Loudon and others [4, 5]. They write (the over lines mark complex conjugate complex numbers)

$$\breve{\mathbf{E}}_A = \exp[i(\breve{k}z - t)][i(\breve{\alpha}\mathbf{x} + \breve{\beta}\mathbf{y}) - \mathbf{z}\frac{1}{\breve{k}}(\breve{\alpha}\partial_x + \breve{\beta}\partial_y)]u(\rho), \quad \sigma = i(\breve{\alpha}\overline{\breve{\beta}} - \overline{\breve{\alpha}}\breve{\beta}) = \pm 1. \tag{5}$$

A profile of the beam (4) may be Gaussian [6],

$$u = \exp\{-\rho^2/w^2\}\sqrt{2/\pi}/w, \quad kw >> 1, \tag{6}$$

but it doesn't matter. We use the cylindrical coordinates ρ, ϕ, z

$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad \rho = \sqrt{x^2 + y^2} \tag{7}$$

with the metric

$$dl^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2, \quad g_{\rho\rho} = 1, \quad g_{\phi\phi} = \rho^2, \quad g_{zz} = 1, \quad \sqrt{g}_{\wedge} = \rho, \quad g^{\phi\phi} = 1/\rho^2 \tag{8}$$

Square root of determinant of the metric tensor is a scalar density of weight $+1$. Gothic symbols are usually applied to denote tensor densities. We shall, instead, mark the density with the symbol ‘wedge’ at the level of bottom indices for a density of weight $+1$ and at the level of top indices for a density of weight -1. A volume element and a surface element are densities of weight -1, $dV^{\wedge} = d\rho d\phi dz,\ da^{\wedge} = d\rho d\phi,$ as well as the absolute antisymmetric density $e^{\wedge}_{ijk}$, which equals to ± 1, or 0.

The coordinate transformation for the *covariant* components of the vectors **E**, **B** in (4) gives [7]

$$\underset{\to}{\breve{E}} = \exp[i(\breve{k}z - t + \phi)](\underset{\to}{\rho} + i\rho\underset{\to}{\phi} + \underset{\to}{z}\frac{i}{\breve{k}}\partial_\rho)u(\rho), \qquad \underset{\to}{\breve{B}} = -i\breve{k}\,\underset{\to}{\breve{E}}. \tag{9}$$

The arrow placed under a symbol means a covariant vector, or a covariant coordinate vector.

If a beam of the type (9) with $\breve{k} = 1$ for $z < 0$,

$$\underset{\to}{\breve{E}}_1 = \exp[i(z - t + \phi)](\underset{\to}{\rho} + i\rho\underset{\to}{\phi} + \underset{\to}{z}\,i\partial_\rho)u(\rho), \qquad \underset{\to}{\breve{B}}_1 = -i\,\underset{\to}{\breve{E}}_1. \tag{10}$$

impinges normally on a surface of a dielectric which is characterized by $\breve{k}$, the beam divides into a reflected part (for $z < 0$)

$$\underset{\to}{\breve{E}}_2 = \frac{1-\breve{k}}{1+\breve{k}}\exp[i(-z - t + \phi)](\underset{\to}{\rho} + i\rho\underset{\to}{\phi} - \underset{\to}{z}\,i\partial_\rho)u(\rho), \qquad \underset{\to}{\breve{B}}_2 = i\,\underset{\to}{\breve{E}}_2 \tag{11}$$

and a transmitted part (for $z > 0$)

$$\underset{\to}{\breve{E}}_3 = \frac{2}{1+\breve{k}}\exp[i(\breve{k}z - t + \phi)](\underset{\to}{\rho} + i\rho\underset{\to}{\phi} + \underset{\to}{z}\frac{i}{\breve{k}}\partial_\rho)u(\rho), \qquad \underset{\to}{\breve{B}}_3 = -i\breve{k}\,\underset{\to}{\breve{E}}_3 \tag{12}$$

in accordance with the reflected and the transmission coefficients $\breve{R} = \dfrac{1-\breve{k}}{1+\breve{k}},\ \breve{T} = \dfrac{2}{1+\breve{k}}$. We set

$$\int u^2 2\pi\rho\, d\rho = 1, \tag{13}$$

so an average power that enters the dielectric is

$$\mathsf{P} = 1 - R^2 = \eta T^2 = \frac{4\eta}{1 + 2\eta + k^2}, \quad R = |\breve{R}|, \quad T = |\breve{T}|. \tag{14}$$

It coincides with eqns. (6.10) and (6.14) from [5] in spite of the field (5) is less than ours. This is because the authors use wrong expression of the Poynting vector.

A pressure acting on the dielectric from vacuum can be readily obtained as a component of the Maxwell tensor (density), $T^{zz}_{\wedge}$, by the use of the sums

$$\underset{\to}{\breve{E}}_V = \underset{\to}{\breve{E}}_1 + \underset{\to}{\breve{E}}_2, \quad \underset{\to}{\breve{B}}_V = \underset{\to}{\breve{B}}_1 + \underset{\to}{\breve{B}}_2: \tag{15}$$

$$< T^{zz}_{\wedge} >|_{z<0} = \sqrt{g}_{\wedge}(E^2_{V\rho} + E^2_{V\varphi} - E^2_{Vz} + B^2_{V\rho} + B^2_{V\varphi} - B^2_{Vz})/4 = \rho[u^2 - (\partial_\rho u)^2/2](1 + R^2). \tag{16}$$

Integration over the complete cross section with use of the waist w from (6) gives:

$$F\,|_{z<0} = \int \rho[u^2 - (\partial_\rho u)^2/2](1 + R^2)d\rho d\phi = (1 - 2/w^2)(1 + R^2) = (1 - 2/w^2)\frac{2(\eta^2 + 1 + \kappa^2)}{1 + 2\eta + k^2}. \tag{17}$$

For a wide beam, $w \to \infty$, this expression reduces to that for a plane wave and coincides with eqn. (7.1) from [5] in spite of eqn. (7.1) is obtained for a common beam. The force (17) is less than that for a plane wave because z-components of the **E** and **B** fields are near the wall of a beam (the field lines are closed loops) [8]. These components give rise to a negative pressure.

A pressure acting on the dielectric under its surface can be readily obtained as a component of the Maxwell tensor by the use of (12) instead of (15):

$$< T^{zz}_{\wedge} >|_{z>0} = \sqrt{g}_{\wedge}(E^2_{3\rho} + E^2_{3\varphi} - E^2_{3z} + B^2_{3\rho} + B^2_{3\varphi} - B^2_{3z})\exp(-2\kappa z)/4$$
$$= \rho[u^2 - (\partial_\rho u)^2/2k^2](1+R^2)\exp(-2\kappa z)\,. \qquad (18)$$

It is seen that the flux density of momentum (18) decreases with z. Accordingly, a force density acts on the dielectric,

$$f^z_{\wedge} = -\partial_z < T^{zz}_{\wedge} >|_{z>0} = 2\kappa\rho[u^2 - (\partial_\rho u)^2/2k^2]\frac{2(\eta^2+1+\kappa^2)}{1+2\eta+k^2}\exp(-2\kappa z)\,. \qquad (19)$$

A resulting force

$$F|_{z>0} = \int\rho[u^2 - (\partial_\rho u)^2/2k^2](1+R^2)d\rho d\phi = (1-2/k^2w^2)(1+R^2) = (1-2/k^2w^2)\frac{2(\eta^2+1+\kappa^2)}{1+2\eta+k^2} \qquad (20)$$

acting on the dielectric at $z>0$ exceeds the force (17) acting from vacuum by a small surface force

$$F|_{z>0} - F|_{z<0} = \frac{k^2-1}{k^2w^2}2(1+R^2) = \frac{4(\eta^2-1+\kappa^2)(\eta^2+1+\kappa^2)}{k^2w^2(1+2\eta+k^2)} \qquad (21)$$

directed against the z-direction and acting near the beam wall. If ignoring the wall effect, the surface force is zero because of continuity of **E** and **B** field at $z=0$.

The surface force (21) can be readily obtained by calculating a surface charge. The charge density is

$$\breve{\sigma} = [\breve{E}_{3z} - \breve{E}_{1z} - \breve{E}_{2z}]_{z=0} = \frac{2i(1-\breve{\varepsilon})}{(1+\breve{k})\breve{k}}\exp[i(-t+\phi)]\partial_\rho u(\rho)\,. \qquad (22)$$

The *z*-component of the time averaged surface force density $\underset{a}{F}_{\wedge}$ acts on this charge and directs against the z-direction:

$$\underset{a}{F}_{\wedge} = \sqrt{g}_{\wedge}(\varepsilon^2-1)E^2_{3z}/4 = \rho(k^4-1)T^2(\partial_\rho u)^2/4k^2\,. \qquad (23)$$

Integration over the complete cross section with use of $(1+k^2)T^2 = 2(1+R^2)$ gives eqn. (21). Thus the force (17) acting on the dielectric from vacuum is divided into the small surface part (21) and the bulk part (17):

$$F = (1-2/w^2)\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2} = \overbrace{-\frac{4(\eta^2-1+\kappa^2)(\eta^2+1+\kappa^2)}{k^2w^2[(\eta+1)^2+\kappa^2]}}^{surface} + \overbrace{(1-2/k^2w^2)\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2}}^{bulk}\,. \qquad (24)$$

This division is not coincided with eqns (7.10), (8.1) from [5] even if $w\to\infty$:

$$\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2} = \overbrace{\frac{2(\eta^2-1+\kappa^2)}{(\eta+1)^2+\kappa^2}}^{surface} + \overbrace{\frac{4}{(\eta+1)^2+\kappa^2}}^{bulk}\,, \qquad (7.10)$$

When an electromagnetic wave passes through a dielectric, the electric field polarizes the dielectric. The polarization and time derivative of the polarization, i.e. the displacement current, are

$$\breve{\mathbf{P}} = (\breve{\varepsilon}-1)\breve{\mathbf{E}}, \quad \breve{\mathbf{j}} = \partial_t\breve{\mathbf{P}}\,. \qquad (25)$$

These allow us to calculate the torque on the absorbing dielectric using a formula suggested by a referee of PRL (see also [9]).

$$\tau = \Re\{\int[\mathbf{r}\times(\breve{\mathbf{j}}\times\overline{\mathbf{B}}) + \mathbf{r}\times(\breve{\mathbf{P}}\cdot\nabla)\overline{\mathbf{E}} + \breve{\mathbf{P}}\times\overline{\mathbf{E}}]dV\}/2\,. \qquad (26)$$

We interpret the z-component of the vector product, $[\mathbf{r}\times(\mathbf{j}\times\mathbf{B})]_z$, as a *wall* torque $\underset{w}{\tau}_z$ because the **E** and **B** fields have a component parallel to the wave vector near the wall of a beam. The torque equals to the integral of

$$d\underset{w}{\tau}_z = \Re\{\rho(\breve{j}_{3z}\overline{B}_{3\rho} - \breve{j}_{3\rho}\overline{B}_{3z})\sqrt{g}_{\wedge}\}dV^{\wedge}/2 \qquad (27)$$

over the dielectric $(z>0)$. Substituting $\breve{E}_{3\rho}$, $\breve{E}_{3z}$, $\overline{B}_{3\rho}=i\overline{k}\overline{E}_{3\rho}$, $\overline{B}_{3z}=i\overline{k}\overline{E}_{3z}$ from (12) into (27) and integrating with respect to ϕ yields

$$\underset{w}{\tau}{}_z=\pi\int\rho^2\Re\{(\breve{\varepsilon}-1)i\overline{k}(\partial_t\breve{E}_{3z}\overline{E}_{3\rho}-\partial_t\breve{E}_{3\rho}\overline{E}_{3z})\}d\rho dz=\frac{4\pi\eta\Re\{(\breve{\varepsilon}-1)i\overline{k}\}}{k^2(1+2\eta+k^2)}\int\rho^2\partial_\rho(u^2)\exp(-2\kappa z)\,d\rho dz\,. \quad (28)$$

Integrating with respect to *z* and by parts with respect to ρ yields [7]

$$\underset{w}{\tau}{}_z=\frac{2\eta(k^2+1)}{k^2(1+2\eta+k^2)} \quad (29)$$

The second term of eqn. (26) is written as a volume integral. But in reality volume electric forces do not act on polarized dielectric because it does not have volume charge. I have explained it [10]. The second term torque acts on the surface of the dielectric thanks to the charge (22). We interpret this term as a *surface* torque $\underset{\sigma}{\tau}$, and I repeat here the explanation.

$$\underset{\sigma}{\tau}{}_z=\int\rho(\varepsilon-1)E^i\partial_iE^\phi dV=\int(\varepsilon-1)\partial_i(\rho E^iE^\phi)dV=\oint(\varepsilon-1)\rho E^iE^\phi da_i=\int\rho\sigma E^\phi da_z \quad (30)$$

Substituting (22) and $E_{3\phi}$ from (12) into (30) yields

$$\underset{\sigma}{\tau}{}_z=\int_{z=0}\rho\Re\{\breve{\sigma}\overline{E}_{3\phi}\}d\rho d\phi/2=\frac{2\pi\Re\{(1-\breve{\varepsilon})\overline{k}\}}{k^2(1+2\eta+k^2)}\int\rho^2\partial_\rho(u^2)\,d\rho=\frac{2\eta(k^2-1\}}{k^2(1+2\eta+k^2)} \quad (31)$$

The third term of (26) is a really volume term (see also [11]). We name it a bulk torque $\underset{b}{\tau}$. We have

$$\underset{b}{\tau}{}^z=\int\Re(\breve{P}_\rho\overline{E}_{3\phi}-\breve{P}_\phi\overline{E}_{3\rho})e^{\rho\phi z}_{\wedge}(d\rho d\phi dz)^\wedge/2=\frac{4\eta}{1+2\eta+k^2} \quad (32)$$

It is remarkable that the sum of the wall part (29) and the surface part (31) of the total torque τ equals to the bulk part (32) of the torque

$$\underset{w}{\tau}+\underset{\sigma}{\tau}=\frac{2\eta(k^2+1)}{k^2(1+2\eta+k^2)}+\frac{2\eta(k^2-1)}{k^2(1+2\eta+k^2)}=\frac{4\eta}{1+2\eta+k^2}=\underset{b}{\tau} \quad (33)$$

So, the total torque that is experienced by our dielectric equals to the double quantity

$$\tau=\underset{w}{\tau}+\underset{\sigma}{\tau}+\underset{b}{\tau}=\frac{8\eta}{1+2\eta+k^2}\,, \quad (34)$$

and eqn. (14) gives $\tau=2\mathsf{P}$, i.e. we arrive to eqn. (3), instead of (1), (2),

$$\tau=2\mathsf{P}/\omega\,. \quad (3)$$

Our results (33), (34) do not coincide with the division of the torque in [5].

Hereby we show that eqns. (1), (2), which are a part of the standard electrodynamics, contradict the calculation of angular momentum in the frame of the standard electrodynamics.

In view of this result, we undertake another verification of the statements (1), (2). We apply Eq. (2) to the classical Beth's experiment [11] and immediately find that the statement (2) predicts zero result of the experiment [12, 13]. The point is the circularly polarized beam, which exerts a torque on a doubly refracting plate in the Beth's experiment, passes through the plate there and back. Therefore the Poynting vector $\mathbf{E}\times\mathbf{B}$ is obviously zero in the experiment because the passed beam is added with the reflected one. So, Eq. (2) yields zero.

This result means that the torque arises due to another factor, and the factor must be added to the expressions (2) for doubling the angular momentum of a circularly polarized light beam. This factor is a spin tensor $\Upsilon_{\alpha\beta\gamma}$ [12, 13]

$$\Upsilon_{\alpha\beta\gamma}=A_{[\alpha}\partial_{|\gamma|}A_{\beta]}+\Pi_{[\alpha}\partial_{|\gamma|}\Pi_{\beta]},\qquad 2\partial_{[\alpha}A_{\beta]}=F_{\alpha\beta},\quad 2\partial_{[\alpha}\Pi_{\beta]}=-F^*_{\alpha\beta} \quad (35)$$

where A_α, Π_α are magnetic and electric vector potentials, $F_{\alpha\beta}$, $\overset{*}{F}_{\alpha\beta} = e_{\alpha\beta\mu\nu}F^{\mu\nu}$ are the field strength tensor and the dual field strength tensor. Thus, eqn. (2) must be replaced by

$$J_z = \int (2r^{[x}T^{y]0} + \Upsilon^{xy0})dV \qquad (36)$$

Eq. (36) explains the Beth's experiment, but we cannot help the Bishop's and Loudon's results.

This result, (35), (36), was submitted to "JETP" on Jan. 27, 1999. It was rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (70), AJP (14), EJP (4), EPL (5), PRA (2), PRD (4), PRE (2), APP (5), FP (6), PLA (9), OC (2), JPA (4), JPB (1), JMP (4), JOPA (1), JMO (2), CJP (1), OL (1), NJP (2), MREJ (3), arXiv (70). In particular, this paper was submitted to JMO on September 29, 2004 5:03 PM; to PRD on December 9, 2004 1:47 PM; to PRL on Jul 4 12:51:47 2005

The use of the spin tensor (35) is presented at web sites http://www.mai.ru/projects/mai_works/.

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