A note on quaternionic Maxwell-Dirac isomorphism, Klein-Gordon equation, Unified wave equation from relativistic fluid, and Gravitation from Aharonov effect

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ABSTRACT. While nowadays it is almost trivial to prove explicitly that there is exact correspondence (isomorphism) between Dirac equation and Maxwell electromagnetic equations via biquaternionic representation, nonetheless their physical meaning remains open for discussion. In the present note we submit the viewpoint that it would be more conceivable if we interpret the *vierbein* in terms of superfluid velocity. Furthermore using the notion of Hodge bracket operator, we could find a neat linkage between Dirac equation and Klein-Gordon equation. From this viewpoint it seems possible to suggest a generalised *unified wave equation* from relativistic fluid dynamics, which is thus far never proposed. Furthermore, the present note argues that it is possible to derive an alternative description of gravitational phenomena from Aharonov effect in relativistic spacetime, which then could be used to explain anomalous gravitational phenomenon known as Podkletnov's experiment. Further experimental observation to verify or refute this proposition is recommended. For clarity, each new equation in the present note, which never appears before elsewhere, is denoted by (#) notation.

1 Introduction

While nowadays it is almost trivial to prove explicitly that there is exact correspondence (isomorphism) between Dirac equation and Maxwell electromagnetic equations via biquaternionic representation [1][5][6], nonetheless their physical meaning remains open for discussion. A plausible reasoning for this problem, as noted by Simulik & Krivsky [4] is because the standard Maxwell electrodynamics cannot be applied to intraatomic region, and *its equations are not mathematically equivalent to any of quantum* *mechanical equations* for electron (Schrödinger equation, Dirac equation, etc).

However, there is another route, which is less known thus far, i.e. to find electromagnetic representation of Quantum Mechanics. It has been shown by Aharonov *et al.* [8] that there is classical analog of non-local quantum interference. This seems to support argument by Hofer [9] that a precise electromagnetic interpretation of Quantum Mechanics could lead us to Maxwell equations. From this viewpoint one could expect to reconcile Maxwell electrodynamics and various relativistic wave equations in a coherent way. With regard to Gsponer's remark [2] on fundamental distinction between Maxwell equations and Dirac equation, one could argue that this distinction could be reconciled because it is possible to find analytic continuation from Lorentzian spacetime to Euclidean spacetime via Cauchy spacetime (with imaginary number) [56]:

$$diag\{-1,1,1,1\} \rightarrow diag\{e^{-ij},1,1,1\} \rightarrow diag\{1,1,1,1\} \tag{1}$$

Therefore, in the present note, using biquaternion representation we argue that there is a unified wave equation, which reduces to Dirac equation, Klein-Gordon equation, and Maxwell electrodynamics equations as its subsets. This conclusion is not new, however, because a unified equation has been considered by Moisil-Theodoresco around seven decades ago [5]:

$$Df + f.\vec{a} = 0 \tag{2}$$

where

$$Df = \sum_{k=1}^{3} e_k \partial_k f \tag{2a}$$

It is also worthnoting here that a somewhat different approach has been made by Einstein-Mayer [2], albeit in the same direction, i.e. to find unified representation, which reduces to Maxwell electromagnetic as well as Quantum Mechanics results. All of these are nothing new to us, as plenty of discussion is available in literature.

What we would argue in the present note, instead, is that one could expect to extend further this form of unified wave equation, in particular using Ulrych's representation [7]. Furthermore in subsequent section, we prove that Ulrych's wave equation could be given relativistic fluid interpretation [10][11], which leads us to suggest that it is conceivable to interpret the vierbein formalism in terms of superfluid velocity [12][13]. While such an attempt to interpret *vierbein* of Dirac equation has been made by de Broglie (in terms of 'Dirac fluid' [57]), it seems that to find its exact representation in terms of superfluid velocity has never been made before. Furthermore, because gravitational Lorentz force could be derived from biquaternionic

Maxwell equations, therefore we also argue in favor of description of gravitational phenomena from Aharonov effect. An introduction to this issue will be discussed in last section, using Freitas metric. This approach will enable us to offer an alternative explanation of an anomalous gravitational phenomenon, known as Podkletnov's experiment.

Provided this proposition of unified wave equation in terms of superfluid velocity *vierbein* corresponds to the observed facts, then it could be used to predict some new observations, for instance to describe elementary particle [54][55]. Further experiment to support or refute this proposition is therefore recommended.

2 Biquaternion, Imaginary algebra, Unified relativistic wave equation

Before we discuss biquaternionic Maxwell equations from unified wave equation, first we should review Ulrych's method [7] by defining imaginary number representation as follows [7]:

$$x = x_0 + j.x_1, \qquad j^2 = -1 \tag{3}$$

This leads to the multiplication and addition (or substraction) rules for any number which is composed of real part and imaginary number:

$$(x \pm y) = (x_0 \pm y_0) + j.(x_1 \pm y_1), \tag{4}$$

$$(xy) = (x_0 y_0 + x_1 y_1) + j.(x_0 y_1 + x_1 y_0). \tag{5}$$

From these basic imaginary numbers, Ulrych [7] argues that it is possible to find a new *relativistic algebra*, which could be regarded as modified form of standard quaternion representation [46]. Alternatively, one could extend this imaginary number to Clifford algebra [3a][3][6][33].

Once we define this imaginary number, it is possible to define further some relations as follows [14]. Given $w = x_0 + j.x_1$, then its D-conjugate of w could be written as:

$$\overline{w} = x_0 - j.x_1 \tag{6}$$

Also for any given two imaginary numbers $w_1, w_2 \in D$, we get the following relations [14]:

$$\overline{w_1 + w_2} = \overline{w}_1 + \overline{w}_2 \tag{7}$$

$$\overline{w_1 \bullet w_2} = \overline{w_1} \bullet \overline{w_2} \tag{8}$$

$$|w|^2 = \overline{w} \bullet w = x_0^{\ 2} - x_1^{\ 2} \tag{9}$$

$$|w_1 \bullet w_2|^2 = |w_1|^2 \bullet |w_2|^2 \quad (10)$$

Again, all of these provide us nothing new. For extension of these imaginary numbers in Quantum Mechanics, see [76]. Now we will review a few elementary definitions of biquaternion, which are useful in subsequent discussions.

It is known that biquaternion could describe Maxwell equations in its original form, and some of the use of biquaternion have been discussed in [1][5][50][51][52][57].

Quaternion number, Q is defined as:

$$Q = a + b.i + c.j + d.k\,. \quad (11)$$

Biquaternion is an extension of this quaternion number, and it is described here using Hodge-bracket operator, in lieu of known Hodge operator ($**=-1$) [53]:

$$\{Q\}* = (a + iA) + (b + iB).i + (c + iC).j + (d + iD).k \quad (12)$$

For differential operator, we could define Nabla-Hodge-bracket operator:

$$\{\nabla\}* = c^{-1}.\partial/\partial t + \vec{i}.\vec{\nabla} \quad (13)$$

It is worthnoting here that equations (4)-(10) also applicable for biquaternion number. While equations (3)-(13) are known in the existing literature, and sometimes called 'hyper-complex' or 'biparavector' (Baylis), we prefer to call it 'imaginary algebra' with emphasize on the use of Hodge-bracket operator.

Now we are ready to discuss Ulrych's method to describe unified wave equation [7], who argues that it is possible to define a unified wave equation in the form [7]:

$$Df(x) = m_f^{\,2}.f(x)\,, \quad (14)$$

where unified (wave) differential operator D is defined as:

$$D = \left[(P - qA)_m (P - qA)^m\right]. \quad (15)$$

Note here that equation (14)-(15) has similar form with equation (2)-(2a) from Moisil-Theodoresco [5]. The distinction with Ulrych's representation is that Ulrych's method could be generalised to describe relativistic fluid, as we will discuss in subsequent section. To derive Maxwell equations from this unified wave equation, he uses free photon fields expression [7]:

$$DA(x) = 0\,, \quad (16)$$

where potential A(x) is given by:

$$A(x) = A^0(x) + jA^1(x)\,, \quad (17)$$

and with electromagnetic fields:

$$E^i(x) = -\partial^0 A^i(x) - \partial^i A^0(x)\,, \tag{18}$$

$$B^i(x) = \in^{ijk} \partial_j A_k(x)\,. \tag{19}$$

Inserting these equations (17)-(19) into (16), one finds Maxwell electromagnetic equation [7]:

$$\begin{aligned} & -\nabla \bullet E(x) - \partial^0 C(x) \\ & + ij\nabla \bullet B(x) \\ & - j(\nabla x B(x) - \partial^0 E(x) - \nabla C(x)) \\ & - i(\nabla x E(x) + \partial^0 B(x)) = 0 \end{aligned} \tag{20}$$

The gauge transformation of the vector potential A(x) is given according to [7]:

$$A'(x) = A(x) + \nabla \Lambda(x)/e\,, \tag{21}$$

where $\Lambda(x)$ is a scalar field. As equations (17)-(18) only use simple definitions of imaginary numbers (3)-(5), then an extension from (20) and (21) to biquaternionic representation of Maxwell equations is possible [1][5].

In order to define a biquaternionic representation of Maxwell equations, we could extend Ulrych's definition of unified differential operator to its biquaternion counterpart, by using equation (12), to become:

$$\{D\}* = \left[(\{P\}* - q\{A\}*)_m (\{P\}* - q\{A\}*)^m \right], \tag{22\#}$$

or by definition $P = -i\hbar\nabla$ and (13), equation (22#) could be written as:

$$\{D\}* = \left[(-\hbar\{\nabla\}* - q\{A\}*)_m (-\hbar\{\nabla\}* - q\{A\}*)^m \right], \tag{22a\#}$$

where each component is now defined in its biquaternionic representation. Therefore the biquaternionic form of unified wave equation takes the form:

$$\{D\}*f(x) = m_f{}^2.f(x)\,, \tag{23\#}$$

if we assume the wavefunction is not biquaternionic, and

$$\{D\}*\{f(x)\}* = m_f{}^2.\{f(x)\}*\,. \tag{24\#}$$

if we suppose that the wavefunction also takes the same biquaternionic form.

We note here the similarity between equation (23#) and (2). Now, for free photon fields, its biquaternionic representation could be written as:

$$\{D\}*A(x) = 0 \tag{25\#}$$

We will not explore here complete solution of this biquaternion equation, as it has been discussed in various literatures aforementioned above, including [51].

However, immediate implications of this biquaternion extension of unified wave equation can be described as follows.

Ulrych's fermion wave equation in the presence of electromagnetic field reads [7]:

$$\left[(P - qA_m)(\overline{P} - qA^m)y\right] = -m^2.y \ , \qquad (26)$$

which asserts c=1 (conventionally used to write wave equations), and be using definition of momentum operator:

$$P = -i\hbar\nabla \ . \qquad (27)$$

So we get three-dimensional relativistic wave equation [7]:

$$\left[(-i\hbar\nabla_m - qA_m)(-i\hbar\nabla^m - qA^m)y\right] = -m^2.c^2.y \ . \qquad (28)$$

which is Klein-Gordon equation. A plausible extension to (28) using biquaternion differential operator defined above (22#) yields:

$$\left[(-\hbar\{\nabla_m\}*-q\{A_m\}*)(-\hbar\{\nabla^m\}*-q\{A^m\}*)y\right] = -m^2.c^2.y \ , \qquad (29\#)$$

which could be called as 'biquaternionic' representation of Klein-Gordon equation.

Therefore we conclude that there is neat correspondence between Ulrych's fermion wave equation and Klein Gordon equation, in particular via biquaternionic representation. At this time it is also worthnoting that it could be shown that Schrodinger equation could be derived from Klein-Gordon equation [11], and Klein-Gordon equation also neatly corresponds to Duffin-Kemmer-Petiau equation [44]. Furthermore it could be proved that modified (quaternion) Klein-Gordon equation could be related to Dirac equation [45]. All of these linkages seem to support argument by Gursey and Hestenes who find plenty of interesting features using quaternionic Dirac equation [45][46][47].

It is worthnoting here that Meessen has proposed a method to describe elementary particle from Klein-Gordon equation [48]. Therefore it becomes interesting to find whether equation (28) could lead us to charge description as described by Meessen [48].

By assigning imaginary numbers to each component [7], equation (26) could be rewritten as follows:

$$\left[(P - qA)_m (P - qA)^m - eE^i ijs_i - eB^i s_i + m^2\right]y = 0 \ , \qquad (30)$$

where Pauli matrices s_i are written explicitly. Now it is possible to rewrite equation (30) in complete tensor formalism [7], if Pauli matrices and electromagnetic fields are expressed with antisymmetric tensor, so we get:

$$\left[(P-qA)_m (P-qA)^m - e s_{mn} F^{mn} + m^2\right] y = 0 \,, \tag{31}$$

where

$$F_{mn} = (\partial_m A_n - \partial_n A_m) \,. \tag{32}$$

Note that equation (31) is formal identical to *quadratic form* of Dirac equation [7], which supports argument suggesting that modified (quaternion) Klein-Gordon equation could be related to Dirac equation [45]. Ulrych [7a] has proposed an outline of solution for (31):

$$m^2 \approx 1 - z^2 . a^2 \tag{33}$$

Alternatively, it seems possible to find solution of (31) by using its similarity with standard Klein-Gordon equation (28), by rewritting:

$$\left[(P-qA)_m (P-qA)^m\right] y = \left[e s_{mn} F^{mn} - m^2\right] y \;, \tag{34\#}$$

or

$$\left[(P-qA)_m (P-qA)^m\right] y = V^2 . m^2 . y \;, \tag{35\#}$$

where

$$V = \sqrt{\;}\,(e s_{mn} F^{mn} / m^2 - 1) \tag{36\#}$$

Therefore the solution of conventional Klein-Gordon equation as describe by Meessen [48] with mass m will be displaced according to equation (36#). This perhaps could be used to explain why the observed quark charge is not exactly the same with predicted values, i.e. multiple of 1/3. To our present knowledge, equation (36#) is *never* dicussed yet in the particle physics literature. It seems like the solutions of the relativistic wave equation should be somewhat displaced, in order to get the observed value [49]:

Table 1. Predicted quark charge and its observed value [49]

Quark	Nonrelativistic prediction	Observed value
Δu	4/3	0.84±0.04
Δs	-1/3	-0.42±0.04
Δd	0	-0.09±0.04
$\Delta\Sigma$	1	0.33+0.08

Of course, to how extent this proposition of charge displacement (36#) could explain the observed quark values remains to be explored.

We will discuss here implications of biquaternionic Maxwell equations to Lorentz force, which will be useful in subsequent section. From definition of quaternionic force [51]:

$$F = P/c + i.F_1 + j.F_2 + k.F_3 \quad (37)$$

then

$$F_q = v.P_q = q\left[(E + vxB) - v(c^{-2}.\partial/\partial t + \nabla A) + i(cB - vxE/c)\right] \quad (38)$$

which represents the quaternionic form of Lorentz force (QLF). By imposing Lorentz gauge condition we use only the first term of (38):

$$F_q = q(E + vxB) \quad (39a)$$

This equation is exactly similar to Spohn's BRST equation [31].

Furthermore, using similar method we can also find gravitational Lorentz force. As discussed in [51], gravitational Lorentz force takes the same form with Lorentz force, and then it could be written as:

$$F_q = m(G + vxT) \quad (39b)$$

Because gravitational Lorentz force can be given vierbein formalism using teleparallel gravity, then it seems worth to define teleparallel equation in terms of biquaternion. The teleparallel equation equivalent to general relativity can be written in the form [29a]:

$$\left(\partial_m x^a + (m_g/m_i)B^a_m\right)du_a/ds = (m_g/m_i)F^a{}_{mr}u_a u^r \quad (40a)$$

Note here that in teleparallel representation, gravitational mass (m_g) does not have to be the same with inertial mass (m_i), which indicates that teleparallel equation remains valid even in case weak equivalence principle is broken. Because gravitational Aharonov effect could be described using teleparallel equation [29], then it seems that it is possible to define gravitation from Aharonov effect.

Then, the biquaternion representation of teleparallel equation becomes:

$$\left(\{\partial_m\} * x^a + (m_g/m_i)\{B^a_m\} *\right)\{du_a/ds\} * = (m_g/m_i)\{F^a{}_{mr}\} * u_a u^r \quad (40b)$$

In subsequent section, we will use this result to argue in favor of gravitation phenomena from Aharonov effect.

3 Vierbein from superfluid velocity and Unified wave equation

It is known that in the literature, fine structure and electronic spin of hydrogen could be derived using various approaches. In this regard, it is worthnoting that Simulik & Krivsky [4] have derived Bohr's quantization rule in hadronic scale using a slightly extended Maxwell electromagnetic theory. However, very few attempts have been made to describe relativistic wavefunction or electronic spin in terms of four-velocity (vierbein) of relativistic continuum [10], except perhaps in [54]. Therefore in the present article we offer an alternative version of relativistic wavefunction for hydrogen based on Cui's method [11], where the notion of electronic spin could be given a *new interpretation*. An advantage of the proposed method outlined here is that it could be used to find direct observables for the four-velocity, in particular using superfluid experiments [12][13]. Furthermore, Bohr-Sommerfeld quantization also emerges from *topological* vortice dynamics, which seems to support Simulik & Krivsky's argument [4] and also Oudet argument to reconsider Sommerfeld's quantization method [15]. This topological nature of electromagnetic and quantization also seems to support arguments by Post [4c].

In his article discussing relativistic wave mechanics, Bakhoum [22a] argues that the fine structure of hydrogen could be derived directly from the expression of potential energy for a bound electron:

$$\left(\pm v \sum_r \vec{p}_r \left[b_r\right]\right)^2 - \left(-e^2/R\right)^2 = 0\,, \qquad (41)$$

by using Dirac's analysis procedure. And because $-e^2/R = -mv^2$ [22a], then equation (41) could be expressed in terms of relativistic four-velocity:

$$\left(\pm v \sum_r \vec{p}_r \left[b_r\right]\right)^2 - \left[-m.(u_m u_m)(v/c)^2\right]^2 = 0\,. \qquad (42)$$

This equation (42) suggests that the fine structure of hydrogen could be derived alternatively from four-velocity of relativistic continuum [10]. Interestingly, Cui [11] argues that the motion of particle of an ideal fluid could be represented in terms of the relativistic Newton's law:

$$m.du_m / dt = q.F_{mn}.u_n\,, \qquad (43)$$

$$u_m.u_m = -c^2\,, \qquad (44)$$

where equation (44) is the relativistic energy-momentum relation *when multiplying it by squared mass*. Note that we use c^2 instead of v^2 in the right hand side of equation (44), in accordance with Bakhoum's argument [22a] that for fine structure analysis, we consider electron in its lowest possible theoretical position, therefore its velocity is c. Therefore term (v/c) in equation (42) becomes 1.

Substituting the following momentum-wavefunction relation by introducing vector potential of electromagnetic field [10]:

$$mu_m = y^{-1}.\left(-i\hbar\partial_m - qA_m\right)y \tag{45}$$

into equation (26), then we get a new representation of quantum wave function:

$$\left[\left(-i\hbar\partial_m - qA_m\right)y\right].\left[\left(-i\hbar\partial_m - qA_m\right)y\right] = -m^2.c^2.y^2 \tag{46}$$

This equation is identical to one-dimensional form of equation (28), provided we multiply both sides of equation (28) with y, therefore we could consider this equation as comparable to one-dimensional Klein-Gordon equation.

A noted characteristic of equation (46) is that the fine structure of hydrogen energy could be calculated directly, without introducing multicomponent wavefunction in Dirac equation [11]. However, its disadvantage is that the meaning of assertion in equation (44) is not quite clear, in particular how to find experimental observables of four-velocity. Therefore we will discuss here how this relativistic wavefunction could be improved.

To find observational meaning of equation (44), it is conjectured here that we could introduce a slightly modification by using the definition that equation (44) is the relativistic energy-momentum relation *when multiplying it by squared mass*. In other words, we submit the viewpoint that the four-velocity field (*vierbein*) in Cui's equation could be interpreted as 'superfluid velocity [55]. Therefore we could use instead of equation (44) an alternative assertion proposed by Carter & Langlois sometime ago [12]:

$$m_r.m^r = -c^2.m^2 \tag{47}$$

by replacing m with the effective mass variable m. This equation has the meaning of cylindrically symmetric superfluid with known metric [12]:

$$g_{rs}.dx^r.dx^s = -c^2.dt^2 + dz^2 + r^2.df^2 + dr^2 \tag{48}$$

Furthermore, equation (47) could be made similar to equation (44), by dividing with quadratic of the effective mass:

$$m_r.m^r / m^2 = -c^2 \tag{49}$$

Introducing this term directly to define equation (44), then we get an alternative relativistic wavefunction instead of equation (46):

$$\left[\left(-i\hbar\partial_r - qA_r\right)y\right].\left[\left(-i\hbar\partial^r - qA^r\right)y\right] = -m^2.c^2.y^2 \qquad (50\#)$$

An interesting characteristic here compared with equation (46), is that in the strong equilibrium conditions, we could define energy, momentum, and angular momentum per particle [12]:

$$k^r.m_r = -E \qquad (51)$$

$$\ell^r.m_r = L \qquad (52)$$

$$m^r.m_r = M \qquad (53)$$

and then we could also write [12]:

$$c^2.m^2 = E^2/c^2 - M^2/r^2 - L^2 \qquad (54)$$

Therefore, in this condition, equation (50) could be rewritten as:

$$\left[\left(-i\hbar\partial_r - qA_r\right)y\right].\left[\left(-i\hbar\partial^r - qA^r\right)y\right] = -\left[E^2/c^2 - M^2/r^2 - L^2\right].y^2 \qquad (55\#)$$

Now it seems possible to find out some physical observables, in particular in the context of rotating superfluid experiments.

Further extension of equation (47) is possible, as discussed by Fischer [13], where the effective mass variable term also appears in the LHS of velocity equation, by defining momentum of the continuum as:

$$p_a = m.u_a\,. \qquad (56)$$

Therefore equation (47) now becomes:

$$m^2.u_a.u^a = -c^2.m^2\,, \qquad (57)$$

where the effective mass variable now acquires the meaning of chemical potential [13]:

$$m = \partial \in /\partial r\,, \qquad (58)$$

and

$$r.p_a/m = \left(K/\hbar^2\right)p_a = j_a\,, \qquad (59)$$

$$K = \hbar^2\left(r/m\right). \qquad (60)$$

The quantity K is defined as the stiffness coefficient against variations of the order parameter phase. Now the sound speed is related to the equations above, for a barotropic fluid [13], as:

$$c_s = d\left(\ln m\right)/d\left(\ln r\right) = \left(K/\hbar^2\right)d^2 \in /dr^2\,. \qquad (61\#)$$

Using this definition, then equation (61) could be rewritten as follows:

$$p_a = j_a\left(\hbar^2/K\right) = (j_a/c_s).d^2 \in /dr^2\,, \qquad (62\#)$$

Introducing this result (62) into equation (50) via our definition (56) and (57), we get:

$$\left[\left(-i\hbar\partial_r - qA_r\right)y\right].\left[\left(-i\hbar\partial^r - qA^r\right)y\right] = -\left((j_a / c_s).d^2 \in / dr^2\right)^2.y^2 \quad (63\#)$$

which is an alternative expression of relativistic wavefunction in terms of superfluid sound speed, c_s. Note that this equation could appear only if we interpret Cui's equation (44) in terms of superfluid four-velocity [55]. While equation (46) is known, to our present knowledge, equation (63#) has never been proposed before elsewhere, so we propose to call it Cui-Carter-Langlois-Fischer's (CCLF) wave equation.

Now, using similarity between equation (46) and (28), it is more convenient to write equation (63) in 3-dimensional Klein-Gordon form (28):

$$\left[\left(-i\hbar\nabla_m - qA_m\right)\left(-i\hbar\nabla^m - qA^m\right)y\right] = -\left((j_a / c_s).d^2 \in / dr^2\right)^2.y \quad (64\#)$$

Therefore this equation is Klein-Gordon equation, where vierbein is defined in terms of superfluid velocity. Because equation (64#) also has never been proposed before elsewhere, so we propose to call it Ulrych-Carter-Langlois-Fischer's (UCLF) wave equation. Alternatively, in condition without electromagnetic charge, then we can rewrite equation (64#) in the known form of standard Klein-Gordon equation [72]:

$$\left[D_m D^m y\right] = -\left((j_a / c_s).d^2 \in / dr^2\right)^2.y \quad (65\#)$$

In this way, then this alternative representation of Klein-Gordon equation has the physical meaning of relativistic wave equation for superfluid phonon [73][74].

A plausible extension of (64#) is also possible using our definition of biquaternionic differential operator (22):

$\{D\} * y = -\left((j_a / c_s).d^2 \in / dr^2\right)^2 y$	(66#)

which is an alternative expression from Ulrych's [7] of biquaternionic unified relativistic wave equation, where the vierbein is defined in terms of superfluid sound speed, c_s. This is the main result of this section. As alternative, equation (66) could be written in compact form:

$$[\{D\} * + \Gamma]\Psi = 0 \quad (66a\#)$$

where the operator Γ is defined according to the quadratic of equation (62):

$$\Gamma = \left((j_a / c_s).d^2 \in / dr^2\right)^2 \quad (66b\#)$$

We note here that equation (66a) takes the same form with Moisil-Theodoresco equation (2), with differential operator has biquaternionic form (22).

Furthermore, Fischer [13] argues that the circulation leading to equation (56)-(57) is in the relativistic dense superfluid, defined as the integral of the momentum:

$$g_s = \oint p_m dx^m = 2p.N_v\hbar \,, \qquad (67)$$

and is quantized into multiples of Planck's quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of g_s. And then Fischer [13] concludes that the Maxwell equations of ordinary electromagnetism can be cast into the form of conservation equations of relativistic perfect fluid hydrodynamics [10]. Furthermore, the topological character of equation (67) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [43].

It is worthnoting here, because here vortices are defined as elementary objects in the form of stable topological excitations, then equation (67) could be interpreted as signatures of Bohr-Sommerfeld quantization of topological quantized vortices. Fischer [13] also remarks that equation (67) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization to celestial systems is known in literature [61][62], which here in the context of Fischer's arguments it seems to suggest that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale [60]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [16]-[20].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider an illustration of quantization of celestial orbit in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [37]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity, it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2p.n\hbar / m_4$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [59]:

$$\oint_\Gamma p.dx = 2p.n\hbar \qquad (67a)$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [59]

$$\int_0^T v^2.dt = w^2.T = 2p.w \tag{68}$$

where T=2π/ω is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $w = n\hbar$. [59] Then we can write the force balance relation of Newton's equation of motion [60]:

$$GMm/r^2 = mv^2/r \tag{69}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (67a), a new constant g was introduced [60]:

$$mvr = ng/2p \tag{70}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = n^2.g^2/(4p^2.GM.m^2) \tag{71}$$

which can be rewritten in the known form [61][62]:

$$r = n^2.GM/v_o^2 \tag{72}$$

where r, n, G, M, v_o represents orbit radii, quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (72), we denote:

$$v_o = (2p/g).GMm \tag{73}$$

The value of m is an adjustable parameter (similar to g). [60]

It is worthnoting here that Nottale [63][64][65] also derived the same result (72) using gravitational-Schrödinger equation by arguing that the equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation. See also [66] for a review of Nottale's method.

Using this equation (72), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and define M in terms of reduced mass $m = (M_1 + M_2)/(M_1.M_2)$. From this viewpoint the result is shown in Table 2 below [67]:

Table 2. Comparison of prediction and observed orbit distance of planets in Solar system (in 10xAU) [67]

Object	No.	Titius	Nottale	CSV	Observed	Δ (%)
	1		0.4	0.428		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	-6.95
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96

For comparison purpose, we also include some recent observation by M. Brown *et al.* from Caltech [68][69][70][71]. It is known that Brown *et al.* have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52AU), 2005FY9 (at 52AU), 2003VB12 (at 76AU, dubbed as *Sedna*. And recently Brown and his team report a new planetoid finding, called 2003UB31 (97AU). This is not to include *Quaoar* (42AU), which has orbit distance more or less near Pluto (39.5AU), therefore this object is excluded from our discussion. It is therefore interesting to remark here that all of those new 'planetoids' are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 2). While this prediction has not been made in so high precision, one could argue that this 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until n=9 of Jovian planets (outer solar system), it seems that there are reasons to suppose that more planetoids are to be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values.

4 Introduction to Gravitation from Aharonov effect. Podkletnov experiment

In this section, we discuss an alternative route to describe gravitational phenomena from Aharonov effect. As to the question of why should we attempt to describe gravitation phenomena using other approach, in lieu of using conventional methods, there are some reasons to do this. Among other things, we note here that Einstein himself seemed to anticipate that a new development such as quantum theory would have to change not only "Maxwellian electrodynamics, but also (his) the new theory of gravitation." [72]. However, we have discussed in the preceding section, that historical development of quantum theory takes continuation along this path: De Broglie wave → Schrödinger equation → Klein-Gordon equation → Maxwell/Dirac equation → Unified wave equation. Therefore it seems also quite expected that a modified version of standard notions of gravitation is required. Another principle is that continuation of development requires that gravitation should be derivable from the Unified wave equation.

Moreoever, as we have shown that the new Unified wave equation implies that vierbein could be related to superfluid velocity, and then it seems conceivable to expect that gravitational phenomena should related to superfluidity. In this regard, it is known that gravitational effects in superfluidity correspond to (gravitational) Aharonov effect [27][28]. Therefore, it becomes apparent that the corresponding logical development is to assert that Aharonov effect could exhibit various gravitational phenomena.

While this proposition of alternative description of gravitation phenomena via Aharonov effect could be derived from teleparallel equation (40) [29], we will describe it via Freitas' metric [21]. The use of Freitas's metric enables us to find the meaning of 'charge-like' metric.

In order to do so, first we will define distance in relativistic spacetime in terms of energy. It is known that the theory of special relativity basically asserts that we could define distance ($\Delta x = \sqrt{x_2^2 - x_1^2}$) in terms of time-elapsed, and *vice versa.*

In the same way, for decades it has been customary for astronomers to use lightyears (ly) to represent stellar distance, while in particle physics practically we could transform distance unit to time unit via c, and time scale to energy scale by $\hbar$, and angular frequency could also be represented in energy unit. This would mean that distance could always be expressed in terms of energy, and vice-versa [20]. This seems to support Feynman's remark that spacetime itself is composed of *intense potential* [41]:

> "We may think of E(x, y, z, t) and B(x, y, z, t) as giving the forces that would be experienced at the time t by a charge located at (x, y, z), with the condition that placing the charge there did not disturb the positions or motion of all the other charges responsible for the fields."

Another plausible logical reasoning to express distance in terms of energy is as follows: Using Gibbs-Ehrenfest theorem, it is argued that time could be related to entropy flow [39], therefore we could say that time is also another form of energy (entropy). Because in STR distance is equivalent to time, therefore we could also say that distance also has similar energy (entropy) meaning. However, this Gibbs-Ehrenfest argument has disadvantage because it implies that time is irreversible. While this notion could be useful (albeit arguable) to describe time arrow, it is physically meaningless to argue that distance is also irreversible. Therefore we don't use this argument here. For further discussion on the meaning of time irreversibility in the light of Quantum Mechanics, see Aharonov [26] and Zurek [40].

Therefore in this section we argue in favor of Freitas [21] and Bakhoum's argument of modified special relativity, STR [22], in order to define distance in terms of energy for relativistic spacetime. This notion will be proved useful when we put gravitational potential to our definition. Bakhoum's argument is used here because it could naturally reconcile STR and Quantum Mechanics, while Freitas' method is used to reduce the number of geometrical dimension required to unify electromagnetic and gravitation theories.

We start by clarifying the meaning of charge-momentum conservation in spacetime metric.

Historically, the idea of deriving relativistic wave (Dirac) equation from Kaluza-Klein's 5D spacetime has been discussed by various authors [23] with various results. Alternatively, Freitas [21] discusses an interesting extension of standard STR as an alternative of Kaluza-Klein metric in order to describe charge conservation in terms of momentum conservation. This result implies that charge may be defined as another form of momentum in the fourth direction. Now we will extend his argument to describe charge-momentum relation, but using Bakhoum's argument [22] $E=m.v^2$ instead of $E=m.c^2$.

Starting from Bakhoum's expression [22], we write:

$$H^2 = p^2.c^2 - m_o{}^2.c^2.v^2 \tag{74}$$

By using our definition p=m.v and H=m.v², equation (74) could be written in the form:

$$m^2.c^2.(v/c)^2.v^2 = m^2.c^2.(v^2 - m_o{}^2/m^2.v^2) \qquad (75)$$

At this point, it is worthnoting here that equation (74)-(75) could be used only in the context that kinetic energy equals to relativistic energy.

Now, if the particle has the Cartesian three-space coordinates x,y,z then its velocity could be written as [21]:

$$v^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 . \qquad (76)$$

Inserting this equation (76) into (77) and dividing both sides by $(m.c)^2$ yields:

$$h^2.v^2 = (1-x^2).\left[(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2\right], \qquad (77\#)$$

where:

$$h = v/c , \qquad (78)$$

$$x = m_o/m . \qquad (79\#)$$

We note that this equation (78) is slightly different from Freitas' equation [21]. Now, by using the known relativistic time expression of STR [21]:

$$t_o = t.\sqrt{1-h^2} , \qquad (80)$$

or

$$dt_o/dt = \sqrt{1-h^2} . \qquad (81)$$

Then we could derive the following expression [21]:

$$c^2.(dt_o/dt)^2 = c^2 - v^2 , \qquad (82)$$

or

$$v^2 = c^2 - c^2.(dt_o/dt)^2 . \qquad (83)$$

Inserting equation (83) into LHS of equation (77) and rearranging yields:

$$c^2 = (1-x^2)/h^2.\left[(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 + c^2.(dt_o/dt)^2.h^2/(1-x^2)\right] \qquad (84\#)$$

Therefore, by using velocity expression, we find a new *chargelike* component of the velocity in the form:

$$(dw/dt)^2 = \left[c^2.(dt_o/dt)^2.h^2/(1-x^2)\right], \qquad (85\#)$$

which is a bit different from Freitas' definition [21], because we use here Bakhoum's argument E=m.v² instead of E=m.c² [22].

Therefore, by multiplying both sides of (84) with dt², and inserting (85) into the right hand side of equation (84), then the expression of flat spacetime metric could be rewritten in terms of this new *chargelike-metric*:

$$dw^2 = \left[c^2.dt_o{}^2.h^2/(1-x)^2 - dx^2 - dy^2 - dz^2\right]. \qquad (86\#)$$

By assuming wave nature of matter, then we get special relativistic waveequation. Interestingly, it could be shown [21] that Klein-Gordon-type equation could be derived from these equations (84).

Now it seems possible to define distance in terms of energy from equation (86), again by using Bakhoum's argument of E=m.v^2, which could be written in the form:

$$\Delta x^2 = \Delta E.\Delta t^2/m. \qquad (87\#)$$

Therefore, equation (87) could be rewritten in terms of:

$$dw^2 = dE_w.dt^2/m = \left[dt_{t,o}{}^2.h^2/(1-x^2) - dt_x{}^2 - dt_y{}^2 - dt_z{}^2\right]dE/m. \quad (88)$$

Alternatively, equation (89) could be written in terms of energy:

$$dw^2 = dt^2/m.\sqrt{\left[dE_{t,o}{}^2.h^2/(1-x^2) - dE_x{}^2 - dE_y{}^2 - dE_z{}^2\right]}. \quad (89\#)$$

While it seems quite simple at first glance, to this author's knowledge equation (88) and (89) have never been derived before. It could be noted that this equation for expressing relativistic spacetime in terms of energy is also different from Smolin's method [25].

This equation (88) becomes interesting once we consider gravitational interaction energy definition, which is known since Nordström's era [24]:

$$E = \left(\int d^3x.rf\right)/2 = -G\sum_{a<b} m_1.m_2/\left|r_1 - r_2\right|. \qquad (90)$$

Inserting this definition, now equation (88) becomes:

$$dw^2 = \left[dt_{t,o}{}^2.h^2/(1-x^2) - dt_x{}^2 - dt_y{}^2 - dt_z{}^2\right].\left(-G.m_1.m_2/\left|r_1 - r_2\right|m\right). \quad (91\#)$$

Now it becomes interesting to note here that gravitational interaction energy could be represented in terms of *charge-like* component of the relativistic spacetime metric of STR. Similarly, replacing interaction energy (90) with Coulomb interaction will result in electromagnetic charge-like metric [24]. In other words, this result supports the aforementioned conjecture that the equivalence between distance-energy-charge characterizes special theory of relativity. This is the main result of this section. Now we use this result to describe gravitation phenomena from Aharonov effect.

Interestingly, Bakhoum [22] has also remarked that his proposition E=m.v^2 instead of E=m.c^2 could affect the description of motion of a charged particle in a magnetic field. In the presence of a magnetic field, the change in the particle's momentum that occurs as a result of the interaction with the field is given by [22]:

$$\Delta p = (e/c).A, \tag{92}$$

where e is the particle's charge and A is the magnitude of the vector of magnetic potential. By using his definition $E=m.v^2$ he gets this equation [22]

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / 2m = mv^2 / 2 + \hbar |e| \left(\sum_r \left\| \vec{M}[b_r] \right\| \right) / 2mc \tag{93}$$

It is interesting here to note that when $\vec{M} = 0$, that is when particle is away from the magnetic field lines, and then equation (93) becomes:

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / 2m = mv^2 / 2, \tag{94\#}$$

which is a direct confirmation of Aharonov effect [22]. This proves that the components p_r of the momentum will be altered while the kinetic energy remains constant. By dividing both sides by 2, equation (94) becomes:

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / m = E. \tag{95\#}$$

Inserting this result to equation (88) yields:

$$dw^2 = \left[dt_{t,o}{}^2 .h^2 / (1 - x^2) - dt_x{}^2 - dt_y{}^2 - dt_z{}^2 \right] . d\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / m^2 \tag{96}$$

which is a *charge-like* component of Aharonov effect.

Things become more interesting if we introduce equation (87) into the right hand side of equation (95), then we get:

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / m = m.\Delta x^2 / \Delta t^2 . \tag{97\#}$$

By dividing both sides with Δx, then we get:

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r / c)^2 \right) / (m.\Delta x) = d(m\vec{v}) / dt \approx m.g , \tag{98\#}$$

which is a gravitational description from Aharonov effect in relativistic spacetime of STR. This is the main result of this section. It is recommended, therefore to verify this proposition in particular using superfluid experiment where the notion of gravitational Aharonov is known [27][28]. A somewhat different approach is discussed in [29], where gravitational Aharonov could be derived from gauge theory.

At this point, it is worthnoting here that some authors have derived a kind of 'inertial' motion from Lorentz force [30][31][32]. While Hestenes' method is more recognizable because it has been discussed in numerous articles [33], we will use here only Spohn's method [31], which is more consistent with the previous discussion based on biquaternion. But most of these methods are essentially the same; for instance Hestenes [30] obtained the following relative equation of motion in terms of "Lorentz form":

$$dv/dt = E + vxB, \tag{99}$$

which is equivalent to Spohn's result [31], except with charge e, and mass, m introduced into the LHS of equation (90):

$$g.d[m.u_t]/dt = e\left(E(q_t) + u_t xB(q_t)/c\right), \tag{100}$$

where

$$E = -\nabla f\ , \tag{101}$$

$$B = \nabla xA. \tag{102}$$

Spohn's result appears interesting here because it could be shown that electron follows the *classical orbit* with high precision. This verifies our aforementioned conjecture (91) that there is no essential difference between gravitational potential and electromagnetic potential, except different energy interaction form. Spohn also argues that this equation in semiclassical limit is equivalent to Schrödinger equation [31]:

$$i\hbar\partial\Psi/\partial t = \left(-\hbar^2\Delta/2m + V(ex)\right)\Psi \tag{103}$$

Now, assuming equation (95) defines intrinsic energy of the system, and inserting equation (100) into (98), yields:

$$\left(\sum_r (\vec{p}_r + e.\vec{A}_r/c)^2\right)/(m.\Delta x) = d[m.u_t]/dt = e\left(E + u_t xB/gc\right). \tag{104\#}$$

In order to verify this equation, let us note that inside superconductive system it is known that E=0, therefore equation (100) becomes:

$$d[m.u_t]/dt = \left(eu_t xB(q_t)/gc\right), \tag{105}$$

or if we use mass m in the right hand side of equation (105) in lieu of charge ratio e/c, then we get (corresponding to gravitational Lorentz force (39b)):

$$d[m.u_t]/dt = \left(m.u_t xB(q_t)\right)/g = \left(p_t xB(q_t)\right)/g\ . \tag{106\#}$$

Interestingly, Williams [75] has also discussed similar gravitomagnetic field produced by a rotating mass on a test particle, in the form $b = w.e_\Phi$. This seems to support our aforementioned conjecture (105)-(106).

Let suppose we conduct an experiment with superconducting disc with weight w=700 grams, radius r=0.2 m, and it rotates at f=2 cps (cycle per second). We get velocity at the edge of the disc is:

$$v = 2p . f .r = 2p .(0.2).(2) = 2.51 m / \sec , \quad (107)$$

and with the known G=6.67x10^{-11}, c~3x10^{8}, r_e=3x10^{6}, M_e=5.98x10^{24}, then the induced gravitomagnetic field from Earth is given by:

$$F_{earth} = (G / c^2 r).Mv \approx 3.71x10^{-9} newtons / kgm / \sec \quad (108)$$

Because B=F/meter, then from equation (106), the force on disc is given by:

$$F_{disc} = B_{earth} .p_{disc} = B_{earth} .(mv) \approx B.mc / g \quad (109)$$

Now we should discuss what is the proper value for v. As discussed above, Bakhoum argues that momentum-energy equivalence could happen at speed less than c [22]. However, high precision experiment for muon suggests that its speed could reach 0.9994c [31]. Therefore, for superconducting disc in our discussion, we suppose its velocity is somewhere between this range, let say 0.982c (yields $g^{-1} = 0.18889$). We will use these values as lower bound and upper bound limit of our prediction. Inserting this value into equation (100), then we get an estimate for reduced weight:

$$F = (3.71x10^{-9})(0.7)(3x10^{8}) x0.189 = 0.147 newton = 14.7 gr \quad (109a)$$

Therefore, from the viewpoint of static observer, the disc will get a mass reduction as large as 14.7gr/700gr=2.103%. This is very near to what is observed in Podkletnov's experiment, where he obtained up to 2% mass reduction, depending on the speed of rotation [34]. This slight deviation may come from our simplifying assumption on the exact value of v. Interestingly, he also noted [35]:

> "As soon as the main solenoids were switched on and the disk began to rotate in the vapors of liquid helium, the shielding effect increased."

It is perhaps also worthnoting that some authors have attempted to derive similar result, for example Tajmar & deMatos [36]. Tajmar's equation $g = a.\Omega / 2$ =(0.2)(2)/2=0.2 predicts mass reduction of 0.2/9.8~ 2% which is near to Podkletnov's recent result [35].

For experimental prediction purpose, we include here Table 3, Table 4, and Chart 1, plotting relationship (for lower bound and upper bound limit) between rotation speed and expected weight reduction ratio for the same condition discussed above. Provided this proposition corresponds to the facts, and then it seems to suggest that our reasoning leading to equation (106) is good enough as *approximation*.

Table 3. Prediction of rotating speed (cps) effect to weight reduction (%), upper bound limit (v=0.982c)

Rotation speed (cps)	Velocity at the disc edge	F exerted to disc (gr)	Weight reduct. (%)	Podkletnov's exp. [34][35]
2	2.51	14.72	2.10%	~2.0%
10	12.6	73.60	10.51%	
30	37.7	220.79	31.54%	
50	62.8	367.99	52.57%	
85	106.76	625.57	89.36%	

Table 4. Prediction of rotating speed (cps) effect to weight reduction (%), lower bound limit (v=0.9994c)

Rotation speed (cps)	Velocity at the disc edge	F exerted to disc (gr)	Weight reduct. (%)	Podkletnov's exp. [34][35]
2	2.51	2.699	0.39%	~2.0%
10	12.6	13.49	1.93%	
30	37.7	40.487	5.78%	
50	62.8	67.479	9.64%	
85	106.76	114.71	16.39%	

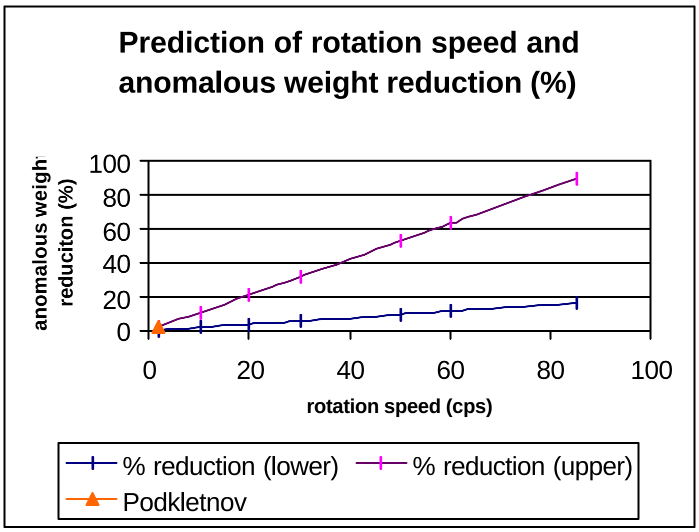

Chart 1. Prediction of rotating speed (cps) effect to weight reduction (%).

It should be clear from Chart I above, that equation (105) is very sensitive to the notion of electron speed. Here we define that the electron speed corresponds to the average speed of electron in the atom. Therefore atoms having large amount of electrons will have *average speed less* than atoms having only one electron (because more electrons are in highly excited state, corresponding to large quantum number, n). This large difference between lower bound and upper bound limit perhaps could also explain why some experiments failed to produce the same result as expected. We supposed that the problem could come from using (simple) molecule with small amount of electrons, instead of using superconducting material exactly the same with Podkletnov's experiment. Alternatively, one could argue that the scattered experimental result comes from other components of biquaternion gravitational Lorentz force (38), which for some conditions become unavoidable. It is recommended therefore to ensure that these components are kept to minimum.

This ends our introduction to describing gravitational phenomena from Aharonov effect. Further step from this introduction could include deriving similar results using teleparallel equation [29][42]. Other implications of this proposition in astrophysics and other fields remain to be explored.

5 Concluding remarks

In the present article, we attempt to find plausible linkage between Quantum Mechanics and Maxwell's classical electrodynamics. It could be shown that using biquaternionic representation, there is exact correspondence between Klein-Gordon equation, Dirac equation, and Maxwell equations. We also derive an alternative description of gravitation phenomenon from Aharonov effect in relativistic spacetime. This proposition enables us to derive some interesting results, including explanation of Podkletnov's superconducting disc experiment, also alternative description of unified wave equation in terms of superfluid velocity (vierbein).

After all, the present article was not intended to rule out the existing methods in the literature to find linkage between Quantum Mechanics and classical electrodynamics, but instead to argue that perhaps there is coherent way to describe these systems provided we suppose wavemechanics description for electrodynamics interaction, and *vice-versa*.

If all of the abovementioned propositions correspond to the observed facts, and then in principle it seems to support argument suggesting that correspondence between physical theories seem indicate that the sought after unification of physical theories could be achieved. Our argument here is merely to show that chance to find such unification of physical theories seems promising if we use quaternion or biquaternion representation. Alternatively, one could expect to find such unification of all physical theories via Clifford-space representation, which also corresponds to quaternion representation [33]. Nonetheless, other implications to cosmology, astrophysics, and also particle physics remain to be seen.

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