Quantization in Dynamic Smarandache Multi-Space

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Abstract: Discussing the applications of Dynamic Smarandache Multi-Space (DSMS) Theory. Supposing for the n different dynamic spaces (n is a dynamic positive integer and the function of time) the different equations have been established, as these n different dynamic spaces synthesize the DSMS, and they are mutually affected, some new coupled equations need to establish in the DSMS to replace some equations in the original dynamic spaces, as well as supply other equations to process the contact, boundary conditions and so on. For the unified processing of all equations in the DSMS, this paper proposes to run the quantization processing to all the variables and all the equations and establish the unified variational principle of quantization with the collocation method based on the method of weighted residuals, and simultaneously solve all the equations in the DSMS with the optimization method. Thus by using the unified variational principle of quantization in the DSMS and the fractal quantization method, will pave the way for the unified processing of the theory of relativity and the quantum mechanics, and the unified processing of the four foundational interactions. Finally the coupled solution for the problem of relativity and quantum mechanics is discussed.

Key words: Dynamic Smarandache Multi-Space (DSMS), coupled equation, collocation method, unified variational principle of quantization, fractal quantization, four foundational interactions, unified processing

Introduction

In this paper we will propose the structure of Dynamic Smarandache Multi-Space (DSMS) firstly, then discuss the problems of the new coupled equations in the DSMS, unified processing all the equations in the DSMS, running the quantization processing to all the variables and equations with the collocation method based on the method of weighted residuals, variable quantization method, fractal quantization method, the unified variational principle of quantization, the coupled solution for the problem of relativity and quantum mechanics, and so on.

1 Smarandache Multi-Space and Dynamic Smarandache Multi-Space

The notion of Smarandache Multi-Space was proposed by Smarandache in 1969[1-3].

A Smarandache multi-space is a union of n sets or spaces $M_1, M_2, \ldots M_n$.

$$M = M_1 \cup M_2 \cup \ldots \cup M_n$$

where n is an integer, and n≥1.

For the reason that n is a constant, so the Smarandache Multi-Space is a static Multi-Space.

But in many practical problems, for the continuous change of the number and structure of the sets or spaces, the dynamic multi-space should be considered.

We define that a Dynamic Smarandache Multi-Space (DSMS) is a union of n(t) sets or spaces $M_1(t), M_2(t), \ldots M_n(t)$.

$M(t) = M_1(t) \cup M_2(t) \cup \ldots \cup M_n(t)$

where n(t) is integers, and the function of time t n(t)≥1.

2 Unified variational principle in single space

In the reference [4], a unified variational principle of fluid mechanics was presented for the single fluid space.

2.1 Optimization method of weighted residuals

Optimization method of weighted residuals (OMWR) has been used for solving some problems in mechanics, physics and astronomy[5], it can be stated briefly as follows.

Supposing that we have the operator equation

$F = 0$ in domain V (1)

Boundary condition

$B = 0$ on boundary S (2)

Then the functional Π defined by OMWR reads

$$\Pi = \int_V A^+(F)dV + W\int_S A^+(B)dS = \min_0 \quad (3)$$

where $A^+(F)$ is a non-negative functional about F its value being

$$A^+(F) \begin{matrix} = 0 & \text{if } F = 0 \\ > 0 & \text{if } F \neq 0 \end{matrix} \quad (4)$$

$\min_0$ denotes minimum and its value should be equal to zero. It was introduced in the reference [6]. W is positive weighted number, and in many cases it can be taken as $W = 1$. The way for choosing W in general cases is shown in the reference [7].

In Eq.(4), if $A^+(F) = F^2$, then it gives the Least Square Method. In the reference [7], the cases of $A^+(F) = |F|$ F^4 $\sqrt{|F|}$ $|e^F - 1|$ $|F|_{\max}$ and the like have also been discussed.

Obviously the condition of zero minimum value of functional Π in Eq.(3) is equivalent to the solution of Eqs.(1) and (2), the proof is shown in the reference [4].

It should be noted that, if the domain V consists of n solitary points $P_1 \sim P_n$, the boundary S consists of m solitary points $Q_1 \sim Q_m$ (such as the quantization problems we will discuss in this paper), then the integral in Eq.(3) should be replaced by the sum

$$\Pi = \sum_1^n W_i A^+(F(P_i)) + W\sum_1^m W_j A^+(B(Q_j)) = \min_0 \quad (3')$$

In addition, the establishment of functional Π in OMWR is very easy, and the minimum value of Π is known in advance (equal to zero).

As for finding the minimum value of Π, two methods can be used. One is solving the

equations $\Pi' = 0$ given by the stationary condition. Another is using the optimization methods used in the references [4-8] such as steepest descent method, searching method.

2.2 A unified variational principle of fluid mechanics (UVPFM)

Basic equations of fluid mechanics and boundary condition read

Continuity equation $F = 0$ in domain V (5)

Equation of motion $G = 0$ in domain V (6)

Energy equation $H = 0$ in domain V (7)

Constitutive equation $I = 0$ in domain V (8)

Equation of state $J = 0$ in domain V (9)

Boundary condition $B = 0$ on boundary S (10)

According to OMWR, the UVPFM in the following form can be obtained

$$\Pi = W_1 \int_V A^+(F)dV + W_2 \int_V A^+(G)dV + W_3 \int_V A^+(H)dV +$$

$$W_4 \int_V A^+(I)dV + W_5 \int_V A^+(J)dV + W_6 \int_S A^+(B)dS = \min_0 \qquad (11)$$

where W_i is suitable positive weighted numbers, $A^+(F)$ $A^+(G)$ and the like are non-negative functionals defined by Eq.(4).

By using this method, in the reference [8] unified processing various water gravity wave theories, in the reference [4] according to the OMWR for solitary domain or solitary point, the solution of hydrodynamics equation for the solitary domain or for the solitary point (point solution) is developed. In the solving process, the compatibility with other domain or other point does not need to be considered.

3 Unified variational principle in the DSMS

All the equations should be considered in the DSMS are as follows

The equations that are established in the original n(t) sets or spaces, and still correct in the DSMS

$$F_i = 0 \qquad \text{in domain } V_i \quad i = 1 \sim n(t) \qquad (12)$$

The boundary conditions that are established in the original n(t) sets or spaces, and still correct in the DSMS

$$B_i = 0 \qquad \text{on boundary } S_i \quad i = 1 \sim n(t) \qquad (13)$$

As the n(t) different dynamic sets or spaces synthesize the DSMS, and they are mutually affected, some new coupled equations need to establish in the DSMS to replace some equations in the original dynamic sets or spaces, as well as supply other equations to process the contact, boundary conditions and so on. Here the contact condition denotes the condition should be satisfied in the case that two sets or spaces have partial common elements.

As establishing the new coupled equations in the DSMS, the existing coupled equations in physics can be referred, such as the pressure-velocity coupled equation,

temperature-stress coupled equation, and so on.

For the sake of convenience, all the new coupled equations, the contact and boundary conditions and so on will be written in the unified form as follows (in which V_j may be domain or boundary)

$$C_j = 0 \qquad \text{for } V_j \quad j = 1 \sim m(t) \qquad (14)$$

According to OMWR, the unified variational principle in the DSMS is as follows

$$\Pi = \sum_1^{n(t)} W_{1i} \int_{V_i} A^+(F_i) dV_i + \sum_1^{n(t)} W_{2i} \int_{S_i} A^+(B_i) dS_i +$$

$$\sum_1^{m(t)} W_{3j} \int_{V_j} A^+(C_j) dV_j = \min_0 \qquad (15)$$

It also should be noted that, if the domain V_i consists of $n'(t)$ solitary points $P_{i1} \sim P_{in'}$, the boundary S_i consists of $m'(t)$ solitary points $Q_{i1} \sim Q_{im'}$, the domain V_j consists of $k(t)$ solitary points $P_{j1} \sim P_{jk}$, then the integral in Eq.(15) should be replaced by the sum, and the unified variational principle of quantization may be obtained

$$\Pi = \sum_1^{n(t)} W_{1i} \sum_1^{n'(t)} W_{i'} A^+(F_i(P_{ii'})) + \sum_1^{n(t)} W_{2i} \sum_1^{m'(t)} W_{i''} A^+(B_i(Q_{ii''})) +$$

$$\sum_1^{m(t)} W_{3j} \sum_1^{k(t)} W_{j'} A^+(C_j(P_{jj'})) = \min_0 \qquad (15')$$

4 Variable quantization and equation quantization

The variable quantization consists two methods: average value method and representative value method.

The average value method means that the value of a whole interval is represented by the average value of this interval.

For example, a spaceship navigates along a straight line. Its speed originally is continual. But we take the continual five line segments, and take the average speed of each line segment, moreover the speed of entire line segment is represented by the average speed. Then the speed has not been continual, we may think that the speed has been carried on the quantization processing (similarly, other parameters such as energy, temperature and so on, may be carried on the quantization processing).

The representative value method means that the value of a whole interval is represented by the value of a representative point that is chosen suitably in this interval.

For the equation quantization, the representative value method can be used only, because the solution of equation is not known in advance.

The finite difference method and the finite element method are all the typical

representative value method for the equation quantization.

Obviously, quantization methods for variable and equation are suitable for all the continuity problems.

5 Fractal quantization

The quantization realized by fractal method, could be reached through taking certain variables in fractal formula for the integers.

The fractal formula reads

$$N = \frac{C}{r^D} \qquad (16)$$

Now, in the fractal formula, we carry on the quantization processing to value of N, namely take the value of N for the arrange sequence number, and N =1, 2, 3….

For example, in the solar system for the orbital motion average velocities of the nine planets (unit: km/s), taking the characteristic dimension r for some planet orbital motion average velocity, taking the value of N for the serial number according to the size of the orbital motion average velocity, firstly considering the case of Mercury r=47.89, then we have N=1 (Mercury's orbital motion average velocity is the greatest), thereupon we have the coordinates point (47.89, 1), according to analogizes other 8 planets coordinates points are as follows: (35.03, 2), (29.79, 3), (24.13, 4), (13.06, 5), (9.64, 6), (6.81, 7), (5.43, 8), (4.74, 9). The above 9 coordinates points may be plotted on the double logarithmic coordinates, then we may obtain 8 straight-line segments. In order to adapt the application of Smarandache geometric and neutrosophic methods, here we do not find the fitting curve of these 8 straight-line segments with least squares method, but for the 8 straight lines, using the coordinates points data to determine accurately their fractal parameters (constant C and fractal dimension D). For example, according to Mercury's coordinates (47.89, 1) and Venus's coordinates (35.03, 2), may obtain the fractal parameters for the first straight-line segment: C =5302.684, D =2.216639. The fractal formula for the first straight-line segment reads: $N = \frac{5302.684}{r^{2.216639}}$. This formula may be used as the extrapolation formula to forecast the orbital motion average velocity of next planet (Earth) by substituting N =3 into this formula and solving the value of r. Similarly, all the forecasting results for other planets may be reached.

By using the 1st straight-line segment, the forecasting result of the orbital motion average velocity V of next planet (Earth) reads: V=29.17, the error is 2.07%.

By using the 2nd straight-line segment, the forecasting result of next planet (Mars) reads: V=26.55, the error is 10.0%.

By using the 3rd straight-line segment, the forecasting result of next planet (Jupiter) reads: V=20.49, the error is 59.9%.

By using the 4th straight-line segment, the forecasting result of next planet (Saturn) reads: V=7.91, the error is 18.0%.

By using the 5th straight-line segment, the forecasting result of next planet (Uranus) reads: V=7.46, the error is 9.51%.

By using the 6th straight-line segment, the forecasting result of next planet (Neptune) reads: V=5.04, the error is 7.19%.

By using the 7th straight-line segment, the forecasting result of next planet (Pluto) reads: V=4.45, the error is 6.18%.

By using the 8th straight-line segment, the forecasting result of next planet (tenth planet) reads: V=4.20, the error is indeterminacy, because the tenth planet is not discovered.

In addition, in the reference [9], the concept of variable dimension fractal was introduced. In which the fractal dimension D is a variable instead of a constant, for example

$$D = a_0 + a_1 r + a_2 r^2 + ... + a_n r^n \qquad (17)$$

In some cases, for the sake of convenience, the fractal formula can be written as the following form

$$\ln N = \ln C - D \ln r \qquad (18)$$

Substituting Eq.(17) into Eq.(18), the form easy to handle the fractal quantization can be obtained

$$\ln N = \ln C - (a_0 + a_1 r + a_2 r^2 + ... + a_n r^n) \ln r \qquad (19)$$

Namely

$$F = \ln C - (a_0 + a_1 r + a_2 r^2 + ... + a_n r^n) \ln r - \ln N = 0 \qquad (20)$$

ь Application of unified variational principle of quantization and fractal quantization

Now we discuss the unified processing for the problems of special relativity and quantum mechanics. Considering the unified processing for the problems of the wavelength of Balmer series in atomic spectrum of hydrogen and the ultimate speed experiment.

With the method of quantum mechanics, the wavelength of Balmer series in atomic spectrum of hydrogen may be obtained theoretically

$$\lambda(n) = 9.112 \frac{4n^2}{n^2 - 4} \qquad n = 3,4,5 \qquad (21)$$

From table 1 we can see that comparing with the experimental data $\lambda_0(3), \lambda_0(4), \lambda_0(5)$, the results of Eq.(21) have some small errors.

In order to obtain the better results, we choose the points with $n = 3,4,5$ as the representative points, using the fractal quantization method and supposing

$$\lambda_1(n) = \frac{C_1}{n^{D_1}} \qquad n = 3,4,5 \qquad (22)$$

The concrete form will be decided with variational principle of quantization.

Table1. Wavelength of Balmer series in atomic spectrum of hydrogen

Value of n	3	4	5
Experimental value of λ_0	6562	4861	4340
Quantum mechanics method	6561	4860	4339
Variational principle of quantization□non-coupled solution□	6562	4861	4340
Variational principle of quantization□coupled solution□	6562	4861	4340

In special relativity, as discussing the ultimate speed, it gives

$$v^2(E_k) = c^2[1-(1+\frac{E_k}{m_0c^2})^{-2}] \qquad \square 23\square$$

From table 2 we can see that comparing with the Bertozzi experimental value of v_0^2, the results of Eq.(23) also have some small errors.

In order to obtain the better results, using the fractal quantization method and supposing

$$v_1^2(E_k) = \frac{C_2}{E_k^{D_2}} \qquad E_k = 1.1,1.8,4.7 \qquad \square 24\square$$

Table 2. Bertozzi ultimate speed experiment

Value of energy E_k	1.1	1.8	4.7
Experimental value of v_0^2	7.5	8.2	8.5
Special relativity	8.09	8.55	8.91
Variational principle of quantization□non-coupled solution□	7.5	8.2	8.5
Variational principle of quantization□coupled solution□	7.5	8.2	8.5

In variational principle □15'□, adopting Least Square Method, and the weighted number being 1, then we have

$$\Pi = \Pi_1 + \Pi_2 = \min_0 \qquad \square 25\square$$

where□ $\Pi_1 = \sum_{n=3,4,5}[\ln\lambda_1(n) - \ln\lambda_0(n)]^2$

$$\Pi_2 = \sum_{E_k=1.1,1.8,4.7}[\ln v_1^2(E_k) - \ln v_0^2(E_k)]^2$$

Because the coupled equation of quantum mechanics and special relativity has not been established, there is not such term in Eq.(25).

Now we discuss the solution of Eq.(25).

First is the non-coupled solution, namely the quantum mechanics solution and the special relativity solution will have not any relations.

In Eqs.(22) and (24), supposing

$$D_1 = a_0 + a_1 n$$

$$D_2 = b_0 + b_1 E_k$$

Then the solution of Eq.(25) is decided with the following equations

$$\frac{\partial \Pi}{\partial C_i} = \frac{\partial \Pi}{\partial a_i} = \frac{\partial \Pi}{\partial b_i} = 0 \qquad (26)$$

Solving these equations, the non-coupled expressions of fractal quantization for the wavelength and ultimate speed are as follows

$$\lambda_1(n) = \frac{47542.69}{n^{2.275455 - 0.1576265n}} \qquad n = 3,4,5 \qquad (27)$$

$$v_1^2(E_k) = \frac{7.353035}{E_k^{-0.2424636 + 0.03165919E_k}} \qquad E_k = 1.1,1.8,4.7 \qquad (28)$$

From Eqs.(27) and (28), we can see that $C_1 = 6466.7C_2$ and $a_0 = -9.385b_0$.

The results of fractal formulas Eqs.(27) and (28) are shown in Table 1 and table 2, these results and the experimental results are completely same.

Secondly discussing the coupled solution, we take all the constant terms in the quantum mechanics solution to be equal to all these in the special relativity solution.

Namely in Eqs.(22) and (24) we should have

$$C_1 = C_2$$

$$a_0 = b_0$$

$$D_1 = a_0 + a_1 n + a_2 n^2$$

$$D_2 = b_0 + b_1 E_k + b_2 E_k^2$$

Solving Eqs.(26), the coupled expressions of fractal quantization for the wavelength and ultimate speed are as follows

$$\lambda_1(n) = \frac{4.365727}{n^{-16.41046 + 4.489881n - 0.4130926n^2}} \qquad n = 3,4,5 \qquad (29)$$

$$v_1^2(E_k) = \frac{4.365727}{E_k^{-16.41046 + 11.69979E_k - 1.765917E_k^2}} \qquad E_k = 1.1,1.8,4.7 \qquad (30)$$

The results of fractal formulas Eqs.(29) and (30) are shown in Table 1 and table 2, these results and the experimental results are completely same.

From the above results we may see that, for the questions of two completely different domains of quantum mechanics and special relativity, the extremely similar solutions can be obtained with the method presented in this paper. Moreover, although the coupled equation of quantum mechanics and special relativity has not been established, the

coupled solution for the quantum mechanics and the theory of relativity really can be obtained.

7 Further discussion

The unified variational principle of quantization in DSMS and the fractal quantization method, similarly can be used for the unified processing of the questions in different domains. Thus will pave the way for the unified processing of the theory of relativity and the quantum mechanics, and the unified processing of the four foundational interactions.

For example, we may discuss the simplest situation, namely the equations for describing the four foundational interactions read: $F_i = 0 \quad (i = 1 \sim 4)$, structuring their action domains as a DSMS, supposing that all the coupled equations and supplementary contact and boundary conditions read: $C_j = 0$, then the variational principle for the unified processing of the four foundational interactions may be written as follows

$$\Pi = \sum_{i=1}^{4} W_i \int_{V_i} F_i^2 dV_i + \sum_{1}^{j} W_j \int_{V_j} C_j^2 dV_j = \min_0$$

The further topics are how to derive the coupled equations in DSMS, as well as supplement suitable contact and boundary conditions and so on. When these equations have not been established, the coupled solution for different domains may be obtained firstly, in order to discover the common ground of them, and create the conditions for finally deriving the coupled equations and so on.

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