About Correspondence Between Infinite Primes, Space-time Surfaces, and Configuration Space Spinor Fields

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Abstract

The idea that configuration space CH of 3-surfaces, "the world of classical worlds", could be realized in terms of number theoretic anatomies of single space-time point using the real units formed from infinite rationals, is very attractive.

The correspondence of CH points with infinite primes and thus with infinite number of real units determined by them realizing Platonia at single space-time point, can be understood if one assume that the points of CH correspond to infinite rationals via their mapping to hyper-octonion real-analytic rational functions conjectured to define foliations of HO to hyper-quaternionic 4-surfaces inducing corresponding foliations of H.

The correspondence of CH spinors with the real units identified as infinite rationals with varying number theoretical anatomies is not so obvious. It is good to approach the problem by making questions.

a) How the points of CH and CH spinors at given point of CH correspond to various real units? Configuration space Hamiltonians and their super-counterparts characterize modes of configuration space spinor fields rather than only spinors. Does this mean that only ground states of super-conformal representations, which are expected to correspond elementary particles, correspond to configuration space spinors and are coded by infinite primes?

b) How do CH spinor fields (as opposed to CH spinors) correspond to infinite rationals? Configuration space spinor fields are generated by elements of super-conformal algebra from ground states. Should one code the matrix elements of the operators between ground states and creating zero energy states in terms of time-like entanglement between ground states represented by real units and assigned to the preferred

points of H characterizing the tips of future and past light-cones and having also interpretation as arguments of n-point functions?

The argument is in a nutshell following.

a) CH itself and CH spinors are by super-symmetry characterized by ground states of super-conformal representations and can be mapped to infinite rationals defining real units U_k multiplying the eight preferred H coordinates h^k whereas configuration space spinor fields correspond to discrete analogs of Schrödinger amplitudes in the space whose points have U_k as coordinates. The 8-units correspond to ground states for an 8-fold tensor power of a fundamental super-conformal representation or to a product of representations of this kind.

b) General states are coded by quantum entangled states defined as entangled states of positive and negative energy ground states with entanglement coefficients defined by the product of operators creating positive and negative energy states represented by the units. Normal ordering prescription makes the mapping unique.

c) The condition that various symmetries have number theoretical correlates leads to rather detailed view about the map of ground states to real units. As a matter fact one ends up with a detailed view about number theoretical realization of fundamental symmetries of standard model.

d) It seems that quantal generalization of the fundamental associativity and commutativity conditions might be needed in the sense that quantum states are superpositions over all possible associations associated with a given hyper-octonionic prime. Only infinite integers identifiable as many particle states would reduced to infinite rational integers mappable to rational functions of hyper-octonionic coordinate with rational coefficients. Infinite primes could be genuinely hyper-quaternionic. This would imply automatically color confinement but would allow colored partons.

Contents

1 Introduction

The notion of configuration space CH, the "world of classical worlds", identified as the space of 3-surfaces in $H = M^4 \times CP_2$ defines the arena of quantum dynamics in quantum TGD [1, 2, 3]. Configuration space spinor fields in turn represent the quantum states of the Universe and correspond naturally to many particle states of a super-conformal quantum field theory in a generalized sense.

The notion of infinite prime is a central notion in physics as a generalized number theory vision [E3]. The construction recipe for infinite primes has interpretation as a repeated second quantization of a super-symmetric arithmetic quantum field theory. At the lowest level ordinary primes label the states labelled by infinite primes. The infinite integers at given level representing many-particle states become elementary particles of the next level corresponding to infinite primes at this level. This gives rise to an infinite hierarchy of infinite primes, integers and rationals. Hence a highly attractive hypothesis is that the infinite rationals are able to represent quantum physical states of the universe or at least some subset of these states, most naturally the ground states of super-conformal representations.

The real units defined as ratios of infinite rationals have arbitrarily complex number theoretical anatomy. This inspires the notion of algebraic holography. Algebraic Brahman=Atman identity states that the number theoretical anatomy of space-time point capable of representing both CH and the configuration space spinors (but not necessarily configuration space spinor fields). Therefore single space-time point would become Platonia able to represent everything which is mathematically representable and configuration space would be realized at the level of imbedding space.

The correspondence of CH points with infinite primes and thus with real units can be understood if one assumes that the points of CH correspond to infinite rationals via their mapping to hyper-octonion real-analytic rational functions conjectured to define foliations of HO to hyper-quaternionic 4-surfaces inducing corresponding foliations of H [C1, E3]. The correspondence of CH spinors with the real units identified as infinite rationals with varying number theoretical anatomies is not so obvious. It is good to approach the problem by making questions.

a) How the points of CH and CH spinors at given point of CH correspond to various real units? Configuration space Hamiltonians and their super-counterparts characterize modes of configuration space spinor fields rather than only spinors. Does this mean that only ground states of super-conformal representations [C1], which are expected to correspond elementary particles, correspond to configuration space spinors and are coded by infinite primes?

b) How do CH spinor fields (as opposed to CH spinors) correspond to infinite rationals? Configuration space spinor fields are generated by elements of super-conformal algebra from ground states. Should one code the matrix elements of the operators between ground states and creating zero energy states in terms of time-like entanglement between ground states represented by real units and assigned to the preferred points of H characterizing the tips of future and past light-cones and having also interpretation as arguments of n-point functions?

The argument to be represented is in a nutshell following (see also [C1, E3]).

a) CH itself and CH spinors are by super-symmetry characterized by ground states of super-conformal representations and can be mapped to infinite rationals defining real units U_k multiplying the eight preferred H coordinates h^k whereas configuration space spinor fields correspond to discrete analogs of Schrödinger amplitudes in the space whose points have U_k as coordinates. The 8-units correspond to ground states for an 8-fold tensor power of a fundamental super-conformal representation or to a product of representations of this kind.

b) General states are coded by quantum entangled states defined as entangled states of positive and negative energy ground states with entanglement coefficients defined by the product of operators creating positive and negative energy states represented by the units. Normal ordering prescription makes the mapping unique.

c) The condition that various symmetries have number theoretical correlates leads to rather detailed view about the map of ground states to real units.

d) It seems that quantal generalization of the fundamental associativity and commutativity conditions might be needed.

Before continuing it is perhaps good to represent the most obvious objection against the idea. The correspondence between CH and CH spinors with infinite rationals and their discreteness means that also CH (world of classical worlds) and space of CH spinors should be discrete. First this looks non-sensible but is indeed what one obtains if space-time surfaces correspond to light-like 3-surfaces expressible in terms of algebraic equations involving rational functions with rational coefficients.

2 How CH and CH spinor fields correspond to infinite rationals?

The basic question is how CH and CH spinor fields on quantum fluctuating degrees of freedom (degrees of freedom for which configuration space metric is non-vanishing) correspond to infinite rationals.

2.1 Associativity and commutativity or only their quantum variants?

Associativity and commutativity conditions are absolutely essential notions in quantum TGD and also in the mapping of infinite primes to the space-time sheets. Associativity, guaranteed by hyper-octonion real-analyticity and implying rational infinite primes [E3], seems to be necessary in order to obtain well-defined representations but might be too strong a condition.

Associativity implies hyper-quaternionicity and commutativity requirement in turn leads to rational infinite primes. Since one can decompose rational primes to hyper-quaternionic and even hyper-octonionic primes, one might hope that this could allow to represent states which consist of colored constituents. This representations has however the flavor of a formal trick and the considerations related to concrete representations of infinite primes suggest that the rationality of infinite primes might be a too restrictive condition.

A more radical possibility is that physical states are only quantum associative. This means that they are obtained as quantum superpositions in the space of real units over all possible associations performed for a given product of hyper-octonion primes (for instance, $|A(BC)\rangle + |(AB)C\rangle$). These states would be associative in quantum sense but would not reduce to hyper-quaternionic primes. Also the notion of quantum commutativity makes sense. The fact that mesons are quantum superpositions of quark-antiquark pairs which each corresponds to different pair of hyper-quaternionic primes and are thus not representable classically, suggests that one can require only quantum associativity and quantum commutativity.

How this idea relates to the representation of space-time surfaces in terms of rational functions of hyper-octonionic variable obtained as an image of rational infinite prime? If one replaces the coefficients of the polynomial which complex or more complex rational, hyper-octonion real analyticity is lost and one must consider some manner to map associative quantum state defined as superposition of various associations to single hyper-quaternionic prime.

a) The first approach is based on the assumption that only infinite integers reduce to infinite rational integers in the sense that the corresponding rational function has rational coefficients. This would allow partons as colored partons represented as non-associative constituents of infinite integers and there would be no problems with space-time correlates. It is however not clear whether this kind infinite integers are possible.

b) In the case of non-commutative group one can speak about commutator group and define Abelian group as coset group of these. Could it be that one can speak about associator algebra and define associative algebra by identifying additive associators $A(BC)-(AB)C$ with zero or multiplicative associators $(A(BC))((AB)C)^{-1}$ with unit. Hyper-octonionic primes would be mapped to something represented by matrices. A good guess for the representation is in terms of 8-D analog of Pauli spin matrices.

2.2 Basic assumptions

The following assumptions serve as constraints when one tries to guess the map of quantum states to infinite primes.

a) Free many-particle states correspond to infinite integers and bound states to infinite primes mappable to irreducible polynomials. The numerator/denominator of the infinite rational should correspond to positive/negative energy states of which zero energy states consist of. At higher levels the mapping should be induced from that for the lowest level. Bosonic (fermionic) elementary particles in ground states should correspond to bosonic (fermionic primes). Phase conjugation as a generalization of that for laser beams) would correspond to the replacement of infinite integer with its inverse.

b) Concerning charge conjugation one can imagine several options but the detailed study of the realization of color symmetry leaves only one option. For this option the two singlets $1 \pm ie_7$ and triplet and antitriplet correspond to leptons and quarks with spin and electro-weak spin represented by the moduli space associated with the hyper-octonionic structures. One must leave open the interpretation of the change of the sign of the small part of the infinite prime, which looks excellent candidate for some discrete symmetry (parity perhaps?).

b) Discrete super-canonical and Super Kac-Moody algebras with bosonic and fermionic generators label the states [C1]. One should map the ground states of these representations to infinite primes and thus to real units in a natural manner. The requirement that standard model symmetries reduce to number theory serves as a powerful constraint and will be analyzed in detail later.

c) The excited states of various super-conformal representations can be mapped to quantum superpositions of many particle states formed from infinite primes. The operators creating the positive and negative energy parts are unique combinations of the operators of algebra if normal ordering prescription is applied. The matrix elements of these operators between ground states can be calculated. The entangled state formed from ground states with entanglement coefficients represented by these matrix elements gives the representation of the general state. Note that the real units would be associated with different points of H identifiable as arguments of n-point function in S-matrix elements.

2.3 How to map ground states of super-conformal representations to infinite primes?

Under the assumptions just stated the problem reduces to that of guessing the detailed form of the map of the ground states of super-conformal representations to primes at the first level of the hierarchy. The mapping of infinite primes to rational functions could provide a clue about how to achieve a natural one-to-one correspondence.

a) The decomposition of the irreducible polynomials in the algebraic extension of rationals gives interpretation in terms of many-particle states labelled by primes in the extension. This brings in Galois groups and their representations. This seems to be something new to present day physics. Note that color group plays the role of Galois group for octonions regarded as extension of reals.

b) Partonic two-surfaces should correspond to infinite primes but in such a manner that an infinite number of infinite primes are mapped to the same partonic 2-surface since given 3-surface should be able to to carry an arbitrary state of super-canonical and super Kac-Moody representation. This is the case since each light-like 3-surface traversing a given partonic 2-surface corresponds to an infinite prime in turn assumed to code for a foliation of hyper-quaternionic or co-hyper-quaternionic surfaces via corresponding rational function of hyper-octonionic variable. Light-like 3-surfaces and corresponding 4-D space-time sheets would thus code for the ground states of super-conformal representations. Quantum classical correspondence would apply to ground states but not to the excited states of super-conformal representations.

c) One should also understand how light-like partonic 3-surfaces are mapped to the number theoretic anatomies of a point of imbedding space. The natural choice for this point would be the preferred point of H defining the tip of the light-cone and the origin of complex coordinates of CP_2 transforming linearly under $U(2) \subset SU(3)$. This choice should be coded as a zero/pole of infinite rational with unit real norm coding for the zero energy states. Zeros would correspond to the positive energy state and poles to the negative energy state.

2.4 The treatment of zero modes

There are also zero modes which are absolutely crucial for quantum measurement theory. They entangle with quantum fluctuating degrees of freedom in quantum measurement situation and thus map quantum numbers to positions of pointers. The interior degrees of freedom of space-time interior must correspond to zero modes and they represent space-time correlates for quantum states realized at light-like partonic 3-surfaces.

As long as states associated with zero modes are represented by operators (such as CH Hamiltonians), the same description applies to them as to the representation of excited states of super-conformal representations. The absence of metric in zero modes means that there is no integration measure. The problems are avoided if one assumes that wave functions in zero modes have a discrete locus as suggested already earlier.

According to the argument represented in [C1], the quantum fluctuating configuration space degrees of freedom are by definition super-symmetrizable since configuration space gamma matrices correspond to the super counterparts of Hamiltonians in the case of super-canonical algebra. Super-symmetrizability condition means that the Poisson brackets of bosonic Hamiltonians reduce to 1-dimensional integrals over "stringy" curves of partonic 2-surface [C1]. This happens for the sub-algebra of super-canonical algebra having vanishing S^2 spin and color charges.

This would mean that zero modes include also the charged Hamiltonians of the super-canonical algebra. This brings in mind induced representations for which one has coset space structure with entire super-canonical group divided by the group generated by neutral super-canonical algebra. The necessary discretization zero modes of freedom suggests a reduction of the representations of isometry groups of H and CH to those for discrete subgroups of isometry groups which indeed appear naturally in Jones inclusions.

One must take this suggestion with some grain of salt. The coset construction for Kac-Moody representations allows to consider the possibility of extending the representations to charged Hamiltonians in such a manner that "stringy" commutators are preserved. The generation of Virasoro and Kac-Moody central extension parameters might be seen as the price paid for the stringy commutation relations.

2.5 Configuration space spinor fields as discrete Schrödinger amplitudes in the space of number theoretic anatomies?

It would seem that the analog of a complex Schrödinger amplitude in the space of number-theoretic anatomies of a given imbedding space point represented by single point of H and represented as 8-tuples of real units could naturally represent the dependence of CH spinors understood as ground states of super-conformal representations obtained as an 8-fold tensor power of a fundamental representation or product of representations perhaps differing somehow. The open question is why eight of them are needed. The excited states of super-conformal representations would be represented as time entangled states with entanglement between real units associated with the preferred points characterizing the tips future and past directed light-cones.

This picture conforms with the simple idea that infinite primes label the points in the fibers of the spinor field bundle having CH_h, h a preferred point of H characterizing the preferred origin of hyper-octonion structure, as a base space and that physical states correspond to discrete analogs of Schrödinger amplitude in this kind of bundles and product bundles formed from them. These 8-tuples define a number theoretical analog of $U(1)^8$ group in terms of which all number theoretical symmetries are represented.

3 Can one understand fundamental symmetries number theoretically?

One should understand symmetries number theoretically.

a) The basic idea is that color $SU(3) \subset G_2$ acts as automorphisms of hyper-octonion structure with a preferred imaginary unit and preferred point with respect to which hyper-octonionic power series are developed [E3]. $SO(7,1)$ would act as symmetries in the moduli space of hyper-octonion structures. Associativity implies symmetry breaking so that only hyper-quaternionic structures are considered and $SO(3,1) \times SO(4)$ acts as symmetries of the moduli space for these structures.

b) Color group is the analog of Galois group for the extension of reals to octonions and has a natural action on the decompositions of rational infinite primes to hyper-octonionic infinite primes. Color confinement is implied by hyper-quaternionicity of primes implied by associativity necessary to assign space-time surfaces to the infinite rationals. If one assumes only quantum associativity, one should have a generalization of the condition guaranteing color confinement. A possible more general condition is that infinite integers give rise to rational polynomials whereas infinite primes can be non-associative and non-commutative if they appear as constituents of N-particle state. This would predict that free quarks are not possible.

c) Electro-weak symmetries and Lorentz group act in the moduli space of hyper-octonionic structures and their actions deform space-time in H picture. CP_2 parameterizes the moduli space of hyper-quaternionic structures induced from a given hyper-octonionic structure with preferred imaginary unit.

d) Four-momenta correspond to translational degrees of freedom associated with the preferred points of M^4 coded by the infinite rational (tip of the light-cone). Color quantum numbers in cm degrees of freedom can be assigned to the CP_2 projection of the preferred point of H. As a matter fact, the definition of hyper-octonionic structure involves the choice of origin of HO giving rise to the preferred point of H

3.1 Automorphisms and the symmetries of moduli space of hyper structures as basic symmetries

Consider now in more detail various symmetries.

a) G_2 acts as automorphisms on octonionic imaginary units and $SU(3)$ respects the choice of preferred imaginary unit [E3]. Associativity requires a reduction to hyper-quaternionic primes and implies color confinement.

For hyper-quaternionic primes the automorphisms restrict to $SO(3)$ which has right/left action of fermionic hyper-quaternionic primes and adjoint action on bosonic hyper-quaternionc primes. The choice of hyper-quaternionic structure is global as opposed to the local choice of hyper-quaternionic tangent space of space-time surface assigning to a point of $HQ \subset HO$ a point of CP_2. $U(2) \subset SU(3)$ leaves invariant given hyper-quaternionic structure which are thus parameterized by CP_2. Color partial waves can be interpreted as partial waves in this moduli space.

b) The choice of global hyper-octonionic coordinate is dictated only modulo a transformation of $SO(1,7)$ acting as isometries of hyper-octonionic norm and as transformations in moduli space of hyper-octonion structures $SO(7)$ acting leaves invariant the choice of real unit. $SO(1,3) \times SO(4)$ acts in the moduli space of global hyper-quaternionic structures identified as sub-structures of hyper-octonionic structure. The choice of global HO structures involves also choice of origin implying preferred point of H. The M^4 projection of this point corresponds to the tip of light-cone. Since the integers representing physical states must be hyper-quaternionic by associativity conditions, the symmetry breaking ("number theoretic compactification") to $SO(1,3) \times SO(4)$ occurs very naturally. This group acts as spinor rotations in H picture and as isometries in HO picture.

c) $SO(1,7)$ allows 3 different 8-dimensional representations (8_v, 8_s, and $\overline{8}_s$). All these representations must decompose under $SU(3)$ as $1+1+3+\overline{3}$ as little exercise with $SO(8)$ triality demonstrates. Under $SO(6) \cong SU(4)$ the decompositions are $1+1+6$ and $4+\overline{4}$ for 8_v and 8_s and its conjugate. Both hyper-octonion spinors and gamma matrices are identified as hyper-octonion units rather than as matrices. It would be natural to assign to bosonic HO primes 8_v and to fermionic HO primes 8_s and $\overline{8}_s$. One can distinguish between $8_v, 8_s$ and $\overline{8}_s$ for hyper-octonionic units only if one considers the full $SO(1,3) \times SO(4)$ action in the moduli space of hyper-octonionic structures.

3.2 Physical interpretation of the decomposition of rational primes to various hyper-primes

Consider now the physical interpretation for the decomposition of rational primes to hyper-complex, hyper-quaternionic, and hyper-octonionic primes. Here one must keep doors open by allowing also the notion of quantum commutativity and quantum associativity so that infinite hyper-octonionic primes would not in general have these properties whereas their images to gamma matrices would define primes of an associative algebra so that a unique space-time representation in terms of hyper-octonionic polynomial would result. Abelianization would produce a generalization of hyper-complex algebra with 7 commuting imaginary units satisfying $e_i^2 = 1$. I have considered earlier also the possibility that hyper-analytic functions of this kind of variable could define space-time surfaces. At this stage one cannot distinguish between this and hyper-octonion real-analytic option.

a) The net quantum numbers of physical states must vanish in zero energy ontology. This is implied by the reduction of infinite rationals to infinite rationals associated with rationals but one must consider also more general options. The vanishing of net quantum numbers could be achieved in many manners. In the most general case the quantum numbers of positive and negative energy state represented by integers in the numerator and denominator of the infinite rational would compensate. If one requires only associativity for infinite primes (or integers) then positive (negative) energy state can correspond to hyper-quaternionic integer and one ends up with H picture and breaking of HO symmetries to those of H.

b) Commutativity condition implies a restriction to hyper-complex numbers. The only restriction would be due to fermion number conservation. Bosonic rational primes could be decomposed to fermionic and antifermionic hyper-quaternionic/octonionic primes such that the net fermion number vanishes. Fermionic primes could correspond to neutrinos and antineutrinos.

c) Giving up commutativity condition but requiring that the primes are associative gives hyper-quaternionic primes and color confinement. One obtains two states which possess non-vanishing and opposite color hypercharges equal to $\pm 2/3$. Thus only the interpretation as lepton, antilepton, quark and antiquark with no color isospin is possible. Spin, weak spin, and color would not be manifest since it would correspond to degree of freedom in the moduli space of hyper-quaternionic structures.

d) Hyper-quaternionic primes can be decomposed to hyper-octonionic primes. In the fermionic sector the three quark states consisting of hyper-octonion units would give color singlets as linear combination of hyper-octonion real unit and the preferred imaginary unit. A state analogous to baryon would result. Is this representation just a formal trick or does it have a real physical content must be left open. In TGD framework, color quantum numbers correspond to color partial waves in CP_2 labelling the moduli space of hyper-quaternionic structures associated with a given hyper-octonionic structure. One might hope that the decomposition provides a formal representation of information about these partial waves.

d) Giving up also associativity for single hyper-octonionic prime and requiring only quantum associativity and requiring that only infinite integers reduces to rational infinite integers leads to the most general framework allowing to describe entangled many particle states formed from elementary particles with quantum numbers of quark and lepton and basic gauge bosons. Gauge bosons and would correspond to locally entangled fermion antifermion pairs (as predicted by TGD) represented as locally entangled real units.

3.3 Electro-weak and color symmetries

The crucial test for this picture is whether color and electro-weak symmetries can be understood number theoretically.

Electro-weak group acts as transformations in the hyper-quaternionic moduli space inducing left or right actions of fermions which cannot interpreted as $U(2) \subset SU(3)$ automorphisms realized via adjoint action. For bosons one adjoint action results. Therefore color singlet states can possess non-vanishing electro-weak quantum numbers as also spin. For bosonic hyper-quaternionic primes one obtains singlet and triplet and for fermionic primes two doublets. The interpretation in terms of electro-weak gauge bosons and electro-weak doublets seems natural. Spin degrees of freedom are not manifestly visible but correspond to the moduli space resulting by $SL(2,C)$ action on hyper-quaternionic units.

Some more detailed comments about color symmetries are in order.

a) Color group SU(3) corresponds to subgroup of G_2 which acts as a Galois group for the extension of reals to octonions. $SU(3)$ leaves invariant real unit and a preferred octonionic imaginary unit. As noticed 8_v, 8_s and $\overline{8}_s$ decompose in a similar manner under $SU(3)$ and only the action of $SL(2,C) \times SO(4)$ modifying hyper-octonionic structure can distinguish between them.

b) Color group would act as a symmetry group on the composites of hyper-octonionic primes and color confinement in spinorial degrees of freedom would follow automatically from (complex) rationality (and even hyper-quaternionicity) of infinite integers necessitated by associativity. This does not however imply color singlet property in color rotational degrees of freedom in imbedding space. The value of color hypercharge (em charge) assignable to the spinors is the only signature of whether lepton or quark is in question.

3.4 Relationship to HO-H duality and two paradoxes

$HO-H$ duality states that the descriptions based on the use of HO and H as imbedding space are equivalent. This can be the case only if one assumes that the breaking of $SO(1,7)$ symmetry to $SO(1,3) \times SO(4)$ symmetry implied by mere associativity is present in both cases as indeed assumed in previous considerations. This forces to reconsider what one really means with HO and H pictures.

What looks to be the basic difference is that the notion of spinor is different in the two cases besides different identification of the imbedding space.

a) Hyper-octonionic spinors identifiable as hyper-octonionic units are used in HO picture. HO spinors reduce to HQ spinors by associativity and $SO(1,3) \times SO(4)$ symmetries act in the moduli space of hyper quaternion structures equivalent with the space of complex 8-spinors. Spin and electroweak spin quantum numbers are thus only implicitly present. For hyper-quaternionic structures induced from a fixed hyper-octonionic structure CP_2 is the moduli space. Color can be represented if one allows decomposition of hyper-quaternionic primes to products of hyper-octonionic primes.

b) In H picture one uses H spinors with CP_2 identified as the space of hyper-quaternionic tangent planes at a given point of HO. Spin and electroweak spin are explicitly present but not color.

c) One might perhaps say that in HO picture the roles of spinor rotations and isometries are changed. Color group takes the role of $SO(3,1) \times SO(4)$ and acts as automorphisms of hyper-octonion structure. In H picture one uses 8-D complex spinors on which $SO(3,1) \times SO(4)$ acts naturally and color groups acts as isometries. Instead of color group $SO(3) \times SO(4)$ would characterize HO Hamiltonians.

Both the spin puzzle of proton [4] implied by the observation that quarks do not seem to contribute to the spin of proton and the statistics paradox implied by the non-visibility of color can be understood in this framework.

a) For HO picture color confinement implies the vanishing of the net spin if attention is restricted to single hyper-octonion structure neglecting thus the zero modes defined by hyper-octonionic moduli parametrized by $SL(2,C)$. Also electroweak quantum numbers vanish under analogous conditions. If the experimental findings correspond to what one observes by using HO picture with a fixed space-time surface, then the spin puzzle of proton can be understood as a neglect of the moduli degrees of freedom characterized by $SO(3,1)$.

b) At low energy limit of hadron physics color is not visible and H picture is natural. This would mean that there is no manifest color and one ends up with spin-statistics paradox if one does not take into account the moduli characterizing hyper-quaternionic structure associated with given hyper-octonionic structure.

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