A DERIVATION OF THE FINE STRUCTURE CONSTANT FROM FIRST PRINCIPLES

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Abstract

It is shown that the value of $\alpha = (1/137.036...)$ can be derived from first physical principles by modeling the renormalized charge differences in QED, between three prescribed scales, with the definite integrals of the ground state probability density of the Hydrogen atom. This requires choosing the three fundamental scales to be the Bohr radius, the Compton wavelength and the Rydberg scale, whose ratios among themselves are related to α. The derivation is also compatible with the observed Astropysical variations of the fine structure constant which results from the cosmological expansion of the Universe. Concluding remarks pertaining Diophantine equations, number theory and physical constants are made.

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1. Introduction

For many years there have been many attempts to find a theoretical basis for the values of the coupling constants of the four forces in Nature and the masses of the fundamental particles. To our knowledge the first person who presented a formula for the electromagnetic coupling constant was the mathematician Wyler [1] . Smith later [2] has derived the values of all the couplings and masses of fundamental particles with remarkable precision using hyper-diamond lattices based on discrete Clifford algebras. In particular he reproduced Wylers heuristic formula for the fine structure constant :

$$\alpha = \frac{8\pi V(D_5)^{1/4}}{V(S^4)V(Shilov_5)} = \frac{9}{8\pi^4}(\frac{\pi^5}{2^4 5!})^{1/4} = \frac{1}{137.03608245} \quad (1)$$

which agrees with the observed value of the fine structure constant in five-parts in ten-million

We have used in (1) the following numerical values for the volumes of the respective spaces : The volume of the 5-dim bounded homogeneous complex domain D_5 was calculated by Hua to be $\pi^5/2^4 5!$ [2] . The volume of the four-sphere S_4 is $8\pi^2/3$ and the volume of the Shilov boundary (of the five-dim complex domain) $Shilov_5$ whose topology is $RP^1 \times S^4$ is $8\pi^3/3$.

Most recently Smilga [3] has obtained Wyler and Smith's results based on the conformal group $SO(3,2)$. Beck [4], as well, has been able to derive the values of *all* the Standard Model parameters, including neutrino masses, based on a deterministic chaotic model using Kaneko coupled-map lattices in two-dimensions, the so-called Chaotic Strings , which is the stringy version of the Higgs fields. Goldfain [6] recently has proposed a derivation based on an anomalous fractal difusion problem. For further references on the numerical calculations of α over the years we refer to [9] .

In this letter we will show how to obtain the value of α from a straightforward QM model of the Hydrogen atom and argue why our derivation is also compatible with the alleged Astrophysical variations of the fine structure constant, with the cosmological evolution of the universe [7] .

2. Hydrogen Atom and the Fine Structure Constant

The reader might be inclined to say that all this is just a fruitless task because charges get renormalized in QFT. On the contrary, we will show precisely why the scaling behaviour of the charge, at the Rydberg, Bohr, and the Comptonscales , in powers of α, is what determines the value of α.

In QED, to first loop order, it is well known that the behavour of the renormalized electric-charge with distance $r < r_B$ goes as follows :

$$\frac{1}{e^2(r)} - \frac{1}{e^2(r_B)} = -\frac{1}{6\pi^2} ln(r_B/r) \Rightarrow$$

$$e^2(r) = e^2(r_B)[1 - (e^2(r_B)/6\pi^2)ln(r_B/r)]^{-1}. \tag{2}$$

where $e^2(r_B)$ is the fiducial the value of $\alpha = e^2/\hbar c$ at the Bohr radius, in units of $\hbar = c = 1$, that we wish to determine. The above equations only yields the charge *differences* in terms of the scaling ratio of distances, but does not determine the value of $e^2(r_B)$.

One explanation for the behaviour of the renormalization of charge with scale, at one loop, is based on an analogy with the vacuum polarization effects induced by the negative bare charge. A sea of virtual electron-positron pairs, electric dipoles , will surround the bare charge and screen it with distance, since the positively charged pole points towards the center, and the negatively charged pole points radially outwards. We will propose in this letter an alternative and *different* interpretation of the scale dependence of the renormalized charge. We shall *model* such scale variations of the renormalized electric charge as those due to an effective electronic probability cloud surrounding the charge, similar to the probability density cloud of an electron revolving around the Hydrogen nucleus and subjected to the standard Coulomb potential. We wish to emphasize that we are *not* studying the vacuum polarization and self energy effects of QED on a bound system of electrons and protons. This would require complicated techniques of QED, like the introduction of the Uehling potential and the solution of the Dirac equation, etc....

Since one-loop effects in QFT can be understood as a semiclassical approximation, beyond the tree level results (classical fields) , it is not unreasonable to model the one-loop effects within a pure QM framework. Simply put, by having a varying charge $e(r)$, one may define the charge density as just the derivative $\rho(r) = de(r)/dr$. We cannot, of course, equate the charge density distribution $\rho(r)$ derived from eq-(2), with the probability density, let us say, of the ground state of the Hydrogen atom, since their functional forms are clearly very different from each other.

Instead, we will equate the *difference* between the values of the renormalized charge, at two different points r_1, r_2, with the definite *integral* of the electron-cloud probability density, associated with the ground state of the Hydrogen atom, between those two points r_1, r_2; i.e. the expressions for $\rho(r)$ and $4\pi r^2 \psi^2(r)$ differ clearly, however the definite *integrals* between two *prescribed* points can be made equal. For this reason, we shall focus on *two* definite integrals , one of which involves the two fundamental scales that delineate the regime of the QM world with the one of relativistic QM (QFT), which are the Bohr radius r_B and the Compton wavelength of the electron r_C respectively . Since one does not know a priori the value of the electric charge e one does not know the values of the Bohr radius, per se. Nevertheless, it is solely in the *ratios* of scales that we shall focus our attention on. We know that there are 4 scales : the Rydberg length, the Bohr radius, the Compton wave length and the classical electron radius, whose respective ratios scale relatively to each other with powers of α itself ! . This is all one needs to know to *derive* the explicit functional form of a *trascendental* equation whose solution determines the sought-after values of $\alpha = 1/137.036....$

Hence, upon equating the charge difference, between the two prescribed points, with the probability integral between the r_C and the r_B limits, up to an overall numerical factor λ, gives for the integral I_1 :

$$e(r_e) - e(r_B) = I_1 = e\lambda \int_{r_e}^{r_B} 4\pi r^2 \psi^2(r) dr. \tag{3}$$

where we have introduced a factor of charge e in the r.h.s to match dimensions , and convert an ordinary probability to a charge density. One has to rescale all the quantities in units of the Bohr radius in order to work with dimensionless variables in the definite integral. Doing so for the normalized Hydrogen atom ground state:

$$\psi^2(x) = \frac{1}{\pi} exp[-2x]. \tag{4}$$

with $x = r/r_B$, gives for the charge difference :

$$e(r_C) - e(r_B) = e(r_B)[1 - (e^2(r_B)/6\pi^2)ln(r_B/r_e)]^{-1/2} - e(r_B) =$$

$$e\lambda \int_{r_C/r_B}^{r_B/r_B} 4\pi(r/r_B)^2 \frac{1}{\pi} exp\ [-2r/r_B] d(r/r_B). \tag{5}$$

setting $e = e(r_B)$ allows to cancel out the e factors on both sides of the equation (5) giving a relationship between $\alpha|ln\ \alpha|$, the integral I_1, and the numerical constant λ , after recalling that the ratio of the upper and lower limits is related to α itself :

$$\frac{r_C}{r_B} = (\alpha)^1. \tag{6}$$

. The ratio of the Rydberg scale R with the Bohr radius is also related to α and is given by $(R/r_B) = (1/\alpha) > 1$.

After straightforward algebra in (5) one arrives at the first relation :

$$\alpha|ln\ \alpha| = 6\pi^2[1 - (1 + \lambda I(n = 1))^{-2}]. \tag{6}$$

In general, evaluating the integral $I(n)$ between $r_n = r_B(\alpha)^n$ and r_B gives the most general relation of the form :

$$\alpha|ln\ \alpha| = F(n, \lambda, \alpha)$$

where

$$F = F(n, \lambda, \alpha) \equiv \frac{6\pi^2}{n}[1 - (1 + \lambda I(n))^{-2}]. \tag{7}$$

i.e. the solutions to the trascendental equations of the type :

$$\alpha|ln\ \alpha| = F(\alpha). \tag{8}$$

for a family of functions $F(\alpha)$, will furnish all the plausible values of α. Whereas, it is the *physical* constraints imposed by the scaling relations of the 3 scales : the Bohr radius, the Compton wavelength and the Rydberg scale, that will fix the value of $\alpha = 1/137$ uniquely.

The family of functions $F(n, \lambda, \alpha)$ have important properties . F can be viewed as an infinity family of functions of n parametrized by the values of λ and α. We should stress that n is in general a *real* variable, not necessary an integer. When $n = 0$, $I(n = 0) = 0$, since the upper and lower limits coincide. Thus, for finite positive λ, one has from eq-(7) that $F(n = 0) = 0/0 =$ undetermined, so a straightforward use of L' Hopital's rule yields for the magnitude of F at the origin $n = 0$:

$$|F(n = 0; \lambda, \alpha)| = 12\pi^2\lambda|ln\ \alpha|exp[-2]. \tag{9}$$

Thus we arrive at the first important result of this work, upon equating : $|F(n = 0)|$ with $\alpha|ln\ \alpha|$ one immediately *deduces* that :

$$\lambda_0 = \lambda(n = 0; \alpha) = \frac{\alpha exp[2]}{12\pi^2}. \tag{10}$$

We should emphasize that we are *not* choosing this value of $\lambda(n = 0)$ ad-hoc ! , on the contrary, it is fully *determined* by using L' Hopital's rule in the definition of the generalized function F, for the limiting case $n = 0$, and which in turn, is furnished by the ground state wave function of the Hydrogen atom.

Another interpretation of the family of functions $F(n, \lambda, \alpha)$ may be understood as follows. Given the function $f(\alpha) \equiv \alpha|ln\ \alpha|$, there exist an uncountably-infinite number of functions $g = g(\alpha)$ such that their intersections with $f(\alpha) = g(\alpha)$ occurs at $\alpha_o = 1/137$,

$$f = \alpha_o|ln\ \alpha_o| = g(\alpha_o) = 0.0359048. \tag{11}$$

Among this uncountably-infinite number of functions $g(\alpha)$ we are going to prove that there exists at least *wo* functions :

$$F_2 = F(n = -1; \lambda_2; \alpha).$$

$$F_1 = F(n = 1; \lambda_1; \alpha). \tag{12}$$

whose intersection with $f(\alpha) = F_1 = F_2$ occurs at $\alpha_o = 1/137$. The latter three functions correspond precisely to the lower limits related to the Bohr (n = 0) , Compton (n = 1) and Rydberg scale (n = - 1), respectively.

Now we must turn to find a solution to the system of equations (11,12); i.e. we must find the triple intersection point of the 3 functions $f(\alpha); F_0; F_1$. The unknowns are $\alpha; \lambda_1, \lambda_2,$. From the equations : $f = F_1$ and $f = F_2$ one infers that $F_1 = F_2$, which is really one independent equation for the 3 unknowns, $(\alpha; \lambda_1; \lambda_2)$. Since one is only interested in finding the *ratios* of the lambdas $z = \lambda_2/\lambda_1$, this reduces the problem further from 3 to only 2 unknowns $z; \alpha$ and one single equation only. However, the single independent trascendental equation is still under-determined.

How do we solve this problem ? Imposing more equations does not help because this would be tantamount of introducing more unknowns $\lambda_3; \lambda_4$... as we go along. Imposing a normalization of the integral $\rho(r)$ equal to unity , like it was proposed in [6] , will not do the job either because there are infinities in the integral due to the charge singularities of eq-(2) at the Landau pole , $r_o > 0$, when $1 = (e(r_B)^2/6\pi^2) ln\ (r_B/r_o)$, signaling a breakdown of the theory.

Our *only* postulate to solve the single under-determined trascendental equation, is that the values of the lambda ratio $z = (\lambda_2/\lambda_1)$ are determined by a more fundamental theory at the Planck scale (unified field theory, quantum gravity, strings..), and once the ratio z is fixed by the fundamental theory, then one can proceed to go ahead and solve for the observed value, today, of α_o.

We will show why the lambda ratios, as observed today, are very close to the values :

$$\lambda_2 : \lambda_0 : \lambda_1 = 1 : 1 : 2 \tag{13}$$

The reasons why we believe that these ratios of lambdas 1 : 1 : 2 don't involve any " fudge " factors whatsoever, but instead may have deep cosmological implications, is because in recent years there has been mounting evidence for the validity of the old ideas of Dirac and Eddington concerning the Astrophysical *variations* of the fundamental constants in Nature with the cosmological evolution of the universe. Different values of the lambda ratios (as the universe evolves) will furnish $different$ trascendental equations, and consequently, $different$ values of α .

A plausible explanation of the cosmological variations of the fine structure constant, based on the maximal acceleration relativity principle in phase spaces, associated with the existence of a minimal Planck scale [5] , and the accelerated expansion of the Universe, was given in [8] . The variations of α in [8] were due to the maximal-acceleration relativistic corrections to the spacetime metric which involved an effective $conformal$ factor, resulting from the existence of a maximal acceleration : $a = c^2/L_P$, where L_P is the minimal distance = Planck length.

It was shown why this effective conformal factor in the accelerated expansion of the universe = scale size of universe = cosmological clock = redshift factor, is what determines the cosmological evolution of α and, consequently, the ratio $z = (\lambda_2/\lambda_1)$, as seen today z_o, is different than its value in the distant past. Therefore it is no surprise that the scaling lambda factors appearing in front of the integrals, involving the ground state wavefunction of the Hydrogen atom, capture the effects of the $conformal$ scaling factor of the spacetime metric. We believe that it is for this reason that the value of α, as observed today, happens to be given by 1/137.036..., measured at the Bohr radius scale .

If the fundamental constants of Nature evolve with the cosmological clock, not unlike the Renormalization group [8], all boils down once again to the values of the cosmological constant (vacuum energy). The ratio of the lambda, parameters (today) $z = z_o$ should be fixed by the values of the cosmological constant. According to Nottale [5], the cosmological constant is is *not* a constant, but also $flows$ with the universe's expansion = cosmological clock of the universe. For this reason, the lambda ratio z also $flows$ with the cosmological clock. (we shouldn't confuse the notaion of z with the cosmological redshift).

Having argued in favour of support of the variation of the fundamental constants with the cosmological evolution of the Universe, Mach principle, one begins by evaluating the integral $I(n = 1)$ in (5), in dimensionless form :

$$I = \int 4\pi x^2 \frac{exp[-2x]}{\pi} dx = -(2x^2 + 2x + 1)exp[-2x]. \quad (14)$$

between the Bohr radius $x = 1$ and the the Compton scale $x = \alpha^1$ which gives $I(n = 1; \alpha) \sim 0.3224 - 4\alpha^2$, when $\alpha << 1$.

This integral leads to the *trascendental* equation

$$\alpha|ln\ \alpha| = 6\pi^2[1 - (1 + \lambda_1 I(n = 1))^{-2}] \sim$$

$$12\pi^2\lambda_1 I(n = 1). \quad (15)$$

to a first oder approximation, after having performed a Taylor expansion of the binomial term in the r.h.s (15), $(1 + z)^{-2} = 1 - 2z + ...$. Evaluating the integral $I(n = 1)$ for small values of $\alpha << 1$ yields, finally :

$$\alpha|ln\ \alpha| \sim 12\pi^2\lambda_1 I(n = 1; \alpha) \sim 12\pi^2\lambda_1(0.3224 - 4\alpha^2) \quad (16)$$

Turning now to the calculations with the Rydberg scale (n = - 1), after performing the binomial expansion and upon evaluating the integral, it gives :

$$\alpha|ln\ \alpha| = F(n = -1; \lambda_2; \alpha) = (\frac{6\pi^2}{(-1)})\ [1 - (1 + \lambda_2 I(n = -1))^{-2}] \sim$$

$$12\pi^2\lambda_2(-I(n = -1)). \quad (17)$$

The minus sign is compatible with the fact that we are evaluating the integral between the Rydberg scale and the Bohr radius , $R/r_B = \alpha^{-1} > 1$, which accounts for an extra minus sign, whence $-I(n = -1) > 0$. After evaluating the magnitude of this integral it yields, for small values of α, $\alpha^{-1} = large$, the value of $5exp[-2]$, giving the other relation :

$$\alpha|ln\ \alpha| \sim 12\pi^2\lambda_2\ |I(n = -1; \alpha)| \sim$$

$$(12\pi^2\lambda_2)\ (5exp[-2]). \quad (18)$$

¿From eqs-(16, 18) we obtain finally the trascendental equations that governed the values of α :

$$(12\pi^2\lambda_2)\ 5exp[-2] = \alpha\ |ln\ \alpha| = (12\pi^2\lambda_1)\ (0.3224 - 4\alpha^2). \quad 19)$$

We recall that $\lambda_0 = (\alpha exp[2]/12\pi^2)$ is a well known expression *derived* explicitly above (10) by using L' Hopital's rule. Upon dividing and multiplying simultaneously the above equation (19) by λ_0, and after substituting its *known* expression in terms of α into (19) we get finally :

$$5\ (\frac{\lambda_2}{\lambda_0})\ \alpha = \alpha|ln\ \alpha| =$$

$$(\alpha exp[2])\ (\frac{\lambda_1}{\lambda_0})\ (0.3224 - 4\alpha^2). \quad (20a)$$

¿From eq-(20 a) one obtains the single equation in terms of the ratio $z = (\lambda_2/\lambda_1)$:

$$5\ (\frac{\lambda_2}{\lambda_1})\ \alpha = 5z\alpha = (\alpha\ exp[2])\ (0.3224 - 4\alpha^2). \quad (20b)$$

In order to find the *triple* intersection point of the 3 functions of α appearing in eq-(20a) , which is equivalent to finding out the solution of the single eq-(20b), one must know the values (today) of the ratios of the lambdas. The choice given by eq-(13) :

$$\lambda_2 : \lambda_0 : \lambda_1 = 1 : 1 : 2. \quad (21)$$

yields in eq-(20 a) :

$$5\alpha = \alpha|ln\ \alpha| = 2\ (\alpha\ exp[2])\ (0.3224 - 4\alpha^2). \qquad (22)$$

which implies then that the z appearing in eq- (20b) must be $z = 1/2$

To check that the last expression (22) is indeed consistent with the *nontrivial* value of $\alpha_o = 1/137$, since $\alpha = 0$ is a trivial solution that is disregarded, it is a simple matter to see that the middle term gives $\alpha_o|ln\ \alpha_o| = 0.03590...$; the last term gives 0.03474 and the first term gives 0.0365. Hence one gets for $\alpha_o = 1/137$ in (22), the sequence of decreasing numbers :

$$0.0365\ >\ 0.0359\ >\ 0.0347. \qquad (23)$$

An more precise match in (23) would require that the ratios of the lambdas should have been close to :

$$\lambda_2 : \lambda_0 : \lambda_1 = 0.984 : 1 : 2.066 \Rightarrow z = 0.476 \sim 0.5 \qquad (24)$$

which are *not* so different from the ratios given already in (13, 21). Of course, we could tune these ratios to any number of digits if desired. The point we will raise at the end is why these ratios are so close to pure *integers* ! Plugging a value of $1/137.036... < 1/137$ into (22) gives even better results since it *narrows* down the intervals between the upper and lower bounds of $\alpha|ln\ \alpha|$ given by eq-(23).

What is important is that the functional $form$ of all the functions involved in the trascendental equations (20a, 20b) remains *unchanged*, since its functional form is fixed by evaluating the integrals associated with the ground state of the Hydrogen atom. It is only the numerical scaling factors given by the lambdas which change. An even more precise evaluation of $\alpha(today)$ requires, of course, to solve the full-fledged trascendental equations (15, 17), maintaining the unknown value of α in the evaluation of all the integrals, and not to perform a Taylor (binomial) expansion, nor to insert the known values of 1/137 inside the integrals.

Therefore, we have shown that this numerical calculation corroborates very satisfactory that upon selecting the limits of the integrals to coincide precisely with the Compton wavelength; the Bohr radius, and the Rydberg scale, and by choosing the ratios (today) given aproximately by eqs-(24), it yields a correct value for the fine structure constant (as observed today) and measured at the Bohr radius scale (as observed today) since its value may also change as a result of the variation of the fundamental constants in Nature :

$$r_B = \frac{\hbar}{mc}\frac{\hbar c}{e^2} = \frac{\hbar^2}{me^2}. \qquad (25)$$

To conclude, we have shown that indeed the value of $\alpha = (1/137.036...)$ can be consistently *derived* from first physical principles by modeling the renormalized charge differences between 3 *prescribed* scales with the definite integrals of the ground state probability density of the Hydrogen atom. This requires choosing the 3 fundamental scales to be the Bohr radius, the Compton wavelength and the Rydberg scale, whose ratios among themselves are related to α, and by setting the ratios (today) of the lambda parameters to be those given aproximately by $1 : 1 : 2$, and more closely by $0.984 : 1 : 2.066$. This procedure allows one to find the value for $\alpha_o = (1/137.036...)$ as observed today (at the Bohr scale).

We also may notice that it was not neccesary to go through the complicated procedure of [6] based on an anomalous fractal diffusion equation using Uehling's potential; introducing and fixing explicitly certain numerical parameters, making assumptions about an Ising model to fix an exponent, etc.... However, this is not to say that the procedure in [6] does not have some validity, and that there couldn't be an underlying fractional dynamical (fractal spacetime) description of the electron that allows to determine the value of α [5] . For this reason, we believe that these ideas shouldn't be disregarded.

We have shown that a simple model of the Hydrogen atom, after invoking the scaling relations of the Compton, Bohr, Rydberg scales, based on α itself, was all that was needed to determine α (today), once the ratios of the lambdas were set aproximately to $1 : 1 : 2$, To our knowledge, the approach of Smith [2] is the most elegant because it is based on just the mere $definition$ of charge, as a probability to emit a photon, and invokes a minimum number of steps. However, it does not raise the issue of the variations of the fundamental constants with the evolution of te Universe, because at the time there was no Astrophysical data to support it.

3. Concluding Remarks

To conclude we must point out that we believe there is the deep connection between number theory, Diophantine equations and physical constants.

• We have seen that the geometrical expression for α given by eq-(1) is explicitly related to the number π (as Feynman postulated long ago). We can interpret this π dependence resulting from the spherical symmetry of the ground state of the Hyrogen atom. As well as a dependence on the logarithm function as a result of scaling.

• We mentioned above why the ratios of the lambdas was very close to 1 : 1 : 2 , which are pure integers, and why the integrals involved $n = -1$ (Rydberg scale) ; $n = 0$ (Bohr radius) and $n = 1$ (Compton scale) were also related to integers. From the number theory perspective we just have in eqs-(11, 12) a generalized system of Diophantine-like equations, when we choose the domain of integrations to fall between the limits $r_n = \alpha^n$, $n = integer$, (postive, negative) and the Bohr radius .

The generalized system of Diophantine-like equations (trascendental) given in (11, 12) are :

$$\alpha|ln\ \alpha| = F(n; m; \alpha). \quad where\ m = \frac{\lambda_n}{\lambda_0}. \tag{26}$$

where λ_0 is the well known parameter given by eq-(10) after using L'Hopital's rule.

The solutions, roots, of these equations (26) can be written symbolically as : $\alpha = \alpha(n; m)$ that depend solely on the *two* parameters n, m (say integers) and where the form $\alpha(n, m)$ is some very complicated expression, not analytical in general. The reason we mention all this is because Gilson [9] has found an empirical solution for α that depends precisely on *two* parameters (integers) : N_1, N_2 :

$$\alpha = \alpha(N_1, N_2) = N_2\ cos\ (\pi/N_1)\ \frac{tan\ (\pi/N_1 N_2)}{\pi}. \tag{27}$$

The scaling variable N_2 in (27) plays the similar role as our scaling variable λ_n, in units of the λ_0 ; i.e, to our m variable, the ratios of lambdas. Whereas, the variable N_2 plays the same role as our exponent n, appearing in powers of α, that determines the limits of integration.

In fact, Gilson [9] found out that for the values of the *primes* : $N_1 = 137$ (in essence the inverse of the fine structure constant) , and $N_2 = 29$, the above formula gives :

$$\alpha = \alpha(137; 29) = 29\ cos(\pi/137)\ \frac{tan(\pi/137 \times 29)}{\pi} = 0.00729735253186..... \tag{28}$$

which matches extremely well the latest CODATA recommended experimental value for this quantity with the uncertainty of ± 27 range centered on the last two digits : 0.007297352533(27). Numerical coincidence or design ?

• Mersenne primes have been known long ago, in the Pierre Noyes program of Bit-Strings physics, to reproduce the counting number 137 from the following iterated sequence of Mersenne primes :

$$2^2 - 1 = 3. \quad 2^3 - 1 = 7. \quad 2^7 - 1 = 127. \quad 3 + 7 + 127 = 137. \tag{29}$$

Dyson interpreted 137 as the maximum number of virtual electron-positron pairs which signal a breakdown of the QED power series perturbation expansion. The next iterated Mersenne prime is huge $2^{127} - 1 = prime$. It is unknown if this iterated sequence produces further Mersenne primes. This iterated hierarchy in binary powers is consitent with Smith's derivation [2] of the fundamental constants based on hyper-diamond lattices and (discrete) Clifford algebras. A Clifford algebra Cl (N) has 2^N basis elements.

We believe that this is *not* senseless numerology but there is no doubt that number theory must be at the very core of the values of the physical constants. For references on the website on alpha see [9], and for the growing importance of the interplay between number theory and physics see [10]

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References

1-A. Wyler , C. R. Acad. Sc. Paris **269** (1969) 743-745.

2-F.D. Smith Jr , Int. J. Theor. Phys **24** (1985) 155-174 . Int. J. Theor. Phys **25** (1985) 355-403 . From Sets to Quarks hep-ph/9708379.

3-W. Smilga : Higher order terms in the contraction of $S0(3,2)$ hep-th/0304137.

4-C.Beck , Spatio-Temporal Vacuum fluctuations of Quantized Fields World Scientific series in Advances in Nonlinear Dynamics, vol. 21 (2002).

5-L. Nottale : " Fractal Spacetime and Microphysics, towards the Theory of Scale Relativity " World Scientific, Singapore, (1992).

6-E. Goldfain : " Derivation of the fine structure constant using fractional dynamics " Chaos, Solitons and Fractals, (2003) in press.

7- D. F. Mota and J.D. Barrow : " Local and Global variations of the Fine strycture Constant " astro-ph/ 0309273.

8- C. Castro : " On the variable fine structure constant, strings and maximal acceleration phase space Relativity " Int. Jour. Mod. Phys. **A** (2003) in press.

9- The Fine Structure Constant Collection ; http://www.geocities.com/Area51/Nebula/3735/fine.html J.G. Gilson : " The fine structure constant, a 20 th centrury mystery " http: //www.btinternet.com/ ugah174/[redacted]

10. M.Watkins : http://www.maths.ex.ac.uk/ mwatkins/zeta