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THE REFRACTION OF LIGHT IN STATIONARY AND MOVING REFRACTIVE MEDIA

C. K. Thornhill
39 Crofton Road
Orpington, Kent, BR6 8AE
United Kingdom

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Abstract

A new theory of the refraction of light is presented, using the mathematical fact that the equations of acoustics and optics are identical and that light may therefore be treated as waves in a fluid ether. Light waves are penetrated by the more slowly moving constituents of a refractive medium and so the rays behind them are perturbed and made wavy as they are diffracted around material particles. The arc-length along a wavy ray is thus increased by a factor $\mu(> 1)$. This simple observation permits the quasi-propagation of waves, in a refractive medium moving in any direction, to be handled mathematically. It is shown to conform to *true* wave-propagation in a fluid medium, with *true* wave-speed $\bar{c}/\mu$, but moving in the same direction with a velocity reduced by a factor $(1 - 1/\mu^2)$. Using this result the relation between the angles of incidence and refraction of an oblique wave-front, when the refractive medium is moving, is derived, so giving a relation more general than Snell's 'law'.

> *Light is nothing but an agitation or disturbance caused in the particles of the ether. The parallel between light and sound is, in this respect, so well established that we can boldly maintain that, if the air became as subtle and at the same time as elastic as the ether, the velocity of sound would also become as rapid as that of light.*
>
> Léonard Euler, 1760
>
> (*Lettres* à *une Princesse d'Allemagne*)

Introduction

The classical theory of the refraction of light, based on Snell's observed relation $\sin i = \mu \sin r$, and its subsequent interpretation as implying a wave speed $\overline{c}/\mu$ in the refractive medium, was discredited as long ago as 1851 when Fizeau first measured the speed at which light waves travel in flowing water. For, whereas wave theory predicts that plane wave fronts should travel at speed $(\overline{c}/\mu \pm q)$ in such a refractive medium moving with speed $\pm q$ normal to the wave fronts, Fizeau observed their speed in such circumstances to be $[\overline{c}/\mu \pm q(1 - 1/\mu^2)]$. Subsequently the experiment has been repeated when the light waves impinge on a moving surface, using solids such as glass and quartz. The experiment has also been extended to the case when the refractive medium moves transversely and the deflection of normal rays entering the refractive medium is observed to be $\tan^{-1}[q(\mu - 1/\mu)/\overline{c}]$ rather than $\tan^{-1}(\mu q/\overline{c})$ as predicted by wave theory.

Fresnal principally, and others, attempted to explain this apparent discrepancy between theory and observations in a moving refractive medium by postulating that the ether density inside a refractive medium is μ^2 times the density outside and that the part of the ether which constitutes this excess density is carried or dragged along by the refractive medium when it moves. Thus the centre of gravity of the ether moves at a speed $(1 - 1/\mu^2)$ times the speed of the refractive medium. Later Stokes obtained the same result by assuming that the ether was compressed as it entered a refractive medium and decompressed as it emerged, giving rise to a reduction in speed $1/\mu^2$ times the speed of the refractive medium. Since then the literature does not record any subsequent attempts to construct an alternative physical theory of refraction, based on the concept of an ether, that accords with observations in a moving refractive medium.

There is, however, no reason to question the mathematical validity of wave theory and, as the observations show, the reason for the discrepancy must lie with the fact that, in a moving refractive medium, the apparent wave speed (as distinct from the speed at which wave fronts travel) varies with direction. Thus the speed $\overline{c}/\mu$ in a moving refractive medium cannot be a *true* wave speed and so wave theory cannot be applied to the quasi-propagation of light through it; nor is there any reason why it should.

In the comparatively vast spaces between the constituents of a refractive medium there is no reason whatsoever why the wave speed of light should be any different from its wave speed in the absence of matter; nor, in these comparatively vast spaces, does it seem credible that these constituents could produce such a significant reduction in the speed of the ether. It is the purpose here to construct a new physical theory of refraction, based on the ether concept, which maintains the true wave speed $\overline{c}$ of the ether inside a refractive medium, neglects any ether drag by the constituents of a refractive medium, and accords with observations of the speeds of wave fronts in both stationary and moving refractive media.

Since the time of Fizeau's first measurements in flowing water various refinements have been introduced into take account that, although the wave speed of light is the same for all wave-lengths *outside* a refractive medium, the refractive index and therefore the apparent wave speed *inside* varies with wave-length. To take account of this dispersion, the speed of a wave front in a medium moving longitudinally must be adjusted to $\{\overline{c}/\mu \pm q[1 - 1/\mu^2 - (\lambda/\mu)d\mu/d\lambda]\}$ for Fizeau's water experiment or $\{\overline{c}/\mu \pm q[1 - 1/\mu^2 - (\lambda/\mu^2)d\mu/d\lambda]\}$ for solid refractive media (Jones 1972). Player (1975) suggested that both phase and group velocity were involved so that, in the case of transverse motion, the angle of deflection of a normal ray should be $\tan^{-1}[q(\mu_g - 1/\mu_\phi)/\overline{c}]$ rather than $\tan^{-1}[q(\mu_\phi - 1/\mu_\phi)/\overline{c}]$ or $\tan^{-1}[q(\mu_g - 1/\mu_g)/\overline{c}]$, where μ_ϕ and μ_g are, respectively, the phase and group refractive indices. Rogers (1975) derived the same expression by applying Abbe image theory and Jones (1975) verified this result experimentally. Here, these niceties will not be considered and attention will be confined to the basic physical theory for monochromatic light, and to solid refractive media.

It has been shown that the equations of acoustics and optics are mathematically identical so that light may be treated as waves in an ethereal fluid in the same way as sound waves in air. There can be no question, then, of light waves being propagated through the actual constituents of a refractive medium; rather, they are propagated through the ether between such constituents. As already stated, there can be no question of the wave speed of light being changed merely by the presence of an odd material particle here or there. Equally there can be no possibility of the rays being straight lines or smooth curves in the presence of matter. Material particles, even though in rapid motion themselves, are practically at rest as compared with the speed at which light waves are propagated. Thus, what are thought of as straight or smooth rays must in fact be wiggly or wavy as the light waves are diffracted around material particles; and what are thought of as smooth or plane wave fronts must be continually pierced by material particles and reconstituted behind them in much the same way as a self-sealing fuel tank responds to a continual penetration by pellets.

Very roughly, the three-dimensional wiggles in the light rays will have an amplitude of the order of the 'size' of material particles and a mean wave-length of the order of their separation distance or mean free path. It seems likely that, if the diffraction process produced too large an increase in the arc-length along a wavy ray, it would lose its integrity and the medium would be opaque and not translucent. This accords with the fact that refractive indices only exceed two in a few rare cases.

The treatment given here is essentially two-dimensional in so far as all the plane wave fronts considered are perpendicular to the (x, y)-plane and all the rays are effectively parallel to the (x, y)-plane; but the perturbations to the rays inside a refractive medium caused by diffraction are, of course, three-dimensional.

With this physical theory, it is possible to deal more generally with the propagation of light waves in a refractive medium moving in any direction. It is thus established that the apparent propagation of light waves in a moving refractive medium accords with the *true* propagation of waves in a fluid medium with a *true* wave speed $\overline{c}/\mu$ but moving with a velocity only $(1 - 1/\mu^2)$ times the velocity of the refractive medium.

Using this result it is then possible to derive the angle of deflection of normal rays entering a refractive medium moving in any direction and, in particular, transversely. It is also possible to deal with oblique incidence of waves entering a moving refractive medium and thus to derive a relation between the angles of incidence and refraction more general then Snell's 'law'.

1 Wave Fronts and Rays

The theory of characteristics of linear partial differential equations of the first order shows (Thornhill, 1993) that the equations for the wave fronts and rays in a general electromagnetic field are mathematically identical with those for wave fronts and rays in a general fluid in general motion. In particular, Maxwell's equations correspond to waves in a uniform fluid at rest. Thus the equations of optics and acoustics are identical and light waves may be treated mathematically as waves in an ethereal fluid in the same way as sound waves in air.

The condition for a hypersurface $\zeta(x_i, t) =$ constant, $(i = 1, 2, 3)$, to be a wave-hypersurface or an envelope of wave-hypersurfaces is (loc. cit),

$$\left(\frac{D\zeta}{Dt}\right)^2 = c^2 \left[\left(\frac{\partial\zeta}{\partial x_1}\right)^2 + \left(\frac{\partial\zeta}{\partial x_2}\right)^2 + \left(\frac{\partial\zeta}{\partial x_3}\right)^2\right] \tag{1.1}$$

or

$$\left(\frac{dx_1}{dt} - u_1\right)^2 + \left(\frac{dx_2}{dt} - u_2\right)^2 + \left(\frac{dx_3}{dt} - u_3\right)^2 = c^2 \tag{1.2}$$

where c is the local wavespeed, $\{u_i\}$ the local velocity of the fluid and $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$. The equation (1.2) expresses quite simply the physical fact that waves travel in all directions with the local wave speed c relative to the local fluid velocity $\{u_i\}$.

In two space dimensions (x, y), for a uniform fluid with constant velocity $(q, 0)$, the equation (1.2) reduces to

$$\left(\frac{dx}{dt} - q\right)^2 + \left(\frac{dy}{dt}\right)^2 = \bar{c}^2 \tag{1.3}$$

where $\bar{c}$ is the *constant* wavespeed.

The mathematical solution of the differential equation (1.3) is illustrated in figures 1a, b. Figure 1a shows the solution in the 3-space $(x, y, \bar{c}t)$. Through any line source such as OA in the plane $\bar{c}t = 0$ there are two wave surfaces, and all such wave surfaces have an envelope, a skew circular cone, which is the wave-surface from a point source at the origin O. The lines along which the wave-surfaces from line-sources through O touch the cone are its straight-line generators, such as OB, OC and these are the rays. The equation of the skew circular cone is

$$(x - qt)^2 + y^2 = (\bar{c}t)^2 \tag{1.4}$$

and the equations of the rays are

$$x = \left(\frac{q}{\bar{c}} + \cos\theta\right)\bar{c}t; \quad y = \sin\theta\,(\bar{c}t) \tag{1.5}$$

Figure 1b shows the solution in the 2-space (x, y) at time t. The rays are now the straight lines such as O'B, O'C. The circle BDCE is the wave-front at time t from a point source O', at $t = 0$, and the wave fronts from a line source such as O'A, at $t = 0$, are the tangents to the circle at B,C which are parallel to O'A.

Using the triangle O'FC, for instance, as a triangle of velocities it is seen that the normal velocity of the wave front CG is $(\bar{c} + q\cos\theta)$ and the velocity of the wave front along the ray O'C is $[q\cos\alpha + \bar{c}\cos(\theta - \alpha)]$.

2 Refraction in a Stationary Medium

In figure 2a a plane wave-front, parallel to the surface of a stationary refractive medium, is followed by normal rays and, as it enters the refractive medium, it remains parallel to the surface and is followed now by rays which are wavy, on account of diffraction around the constituents of the refractive medium, but which are still, in the large, normal to it. Along a wavy ray

$$\int_A^B dx = l; \quad \int_A^B dy = 0; \quad \int_A^B dz = 0\,. \tag{2.1}$$

Along any element ds of arc of the wavy ray AB, the wave-speed remains as $\bar{c}$, but the path length is increased by a factor $\mu > 1$. Thus

$$\int_A^B ds = \mu l; \quad \frac{ds}{dt} = \bar{c}\,. \tag{2.2}$$

The time of transit of the wave-front from A to B is then

$$\int_A^B dt = \int_A^B \frac{ds}{\bar{c}} = \frac{\mu l}{\bar{c}}\,, \tag{2.3}$$

and so the wave-front travels from A to B as if the wave-speed in the refractive medium were $\bar{c}/\mu$.

In figure 2b a plane wave-front followed by normal rays enters the refractive medium obliquely at an angle of incidence i and gives rise to a plane wave-front in the refractive medium inclined at an angle of refraction r to the surface of the refractive medium, followed by wavy rays which are, in the large, normal to it. Again the transit time of the refracted ray from A to B is $\mu AB/\bar{c}$, and this must be the same as the transit time at speed $\bar{c}$ from C to D along the ray CD outside the refractive medium. Thus

$$\frac{CD}{\bar{c}} = AD \sin\frac{i}{\bar{c}} = \frac{\mu AB}{\bar{c}} = \mu AD \sin\frac{r}{\bar{c}}$$

and so $\sin i = \mu \sin r$ in accordance with Snell's 'law'.

3 Refraction in a Moving Medium

In figure 3a plane wave fronts, parallel to the surface of a refractive medium moving with velocity $(q\cos\alpha, q\sin\alpha)$, are followed by normal rays. After entering the moving refractive medium they remain parallel to the surface but are now followed by wavy rays inclined at an angle ψ of order $(q/\bar{c})$. This frame of reference is designated frame 1.

Figure 3b shows the transformation to frame 2 in which the ether both inside and outside the refractive medium has velocity $(-q\cos\alpha, -q\sin\alpha)$ and the refractive medium is at rest. In this frame the rays in the ether outside the refractive medium are inclined at an angle $\zeta = \tan^{-1}(q\sin\alpha/\bar{c})$ to the first order in $q/\bar{c}$, and the wavy rays, such as AB, inside the refractive medium are inclined at an angle ω, of order $q/\bar{c}$.

Let W be the velocity of a wave-front along the wavy ray AB at any point where the element of arc-length ds has direction cosines $(\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$. Then, in the triangle of velocities shown in figure 3c, P is the point $(q\cos\alpha, q\sin\alpha, 0)$ and so R is the point

$$(q\cos\alpha + W\cos\theta\cos\phi, q\sin\alpha + W\sin\theta\cos\phi, W\sin\phi)\,,$$

relative to O. It follows that

$$\bar{c}^2 = (q\cos\alpha + W\cos\theta\cos\phi)^2 + (q\sin\alpha + W\sin\theta\cos\phi)^2 + W^2\sin^2\phi$$

whence

$$W = \bar{c} - q\cos\phi\cos(\theta - \alpha) \tag{3.1}$$

to the first order in $q/\bar{c}$.

The integral relations along the wavy ray AB give

$$\int_A^B dx = l; \quad \int_A^B dy = -\,l\tan\omega; \quad \int_A^B dz = 0; \quad \int_A^B ds = \frac{\mu l}{\cos\omega} \tag{3.2}$$

where $dx = \cos\theta\cos\phi\; ds, dy = \sin\theta\cos\phi\; ds, dz = \sin\phi\; ds$.

Also

$$\int_A^B dt = \int_A^B \frac{ds}{W} = \int_A^B \frac{ds}{\bar{c}} + \frac{q}{\bar{c}^2}\int_A^B \cos\alpha\; dx + \frac{q}{\bar{c}^2}\int_A^B \sin\alpha\; dy\,.$$

Thus

$$\begin{aligned}\int_A^B dt &= \frac{\mu l}{\bar{c}}\sec\omega + \frac{ql}{\bar{c}^2}\cos(\alpha+\omega)\sec\omega \\ &= \frac{\mu l}{\bar{c}} + \frac{ql}{\bar{c}^2}\cos\alpha \end{aligned} \tag{3.3}$$

to the first order in $q/\bar{c}$.

The normal velocity of a wave-front or its velocity along the wavy ray AB is then $[\bar{c}/\mu - (q/\mu^2)\cos\alpha]$ to the first order. It follows that, in frame 1, the normal velocity of a wave-front or the velocity along a wavy ray inclined at the angle ψ (of order $q/\bar{c}$) is $[\bar{c}/\mu + q(1 - 1/\mu^2)\cos\alpha]$ to the first order in $q/\bar{c}$.

For the longitudinal motion $\alpha = 0, \pi$, and the velocity along a wavy ray in frame 1 corresponding to longitudinal velocities $(\pm q, o)$ is

$$\frac{\overline{c}}{\mu} \pm q \left(1 - \frac{1}{\mu^2}\right) .$$

In this case the angles ψ, ω are zero. For transverse flow $\alpha = \pm\pi/2$ and the velocity along a wavy ray inclined at the angle ψ is $\overline{c}/\mu$ to first order. Without the unjustifiable use of wave theory there is no apparent way of deriving the angles ψ, ω in figures 3a, b, in the case of transverse flow or, indeed, for any value of α except $0, \pi$; but it is possible to show that $\psi + \omega = (q/\overline{c})\mu \sin\alpha$ to the first order in $q/\overline{c}$.

4 Equivalent True Wave Propagation

The observed normal velocity of a wave front in a refractive medium moving longitudinally with a velocity $(\pm q, 0)$ may be written as $[(\overline{c}/\mu \mp q/\mu^2) \pm q]$ or as $[(\overline{c}/\mu \pm q(1 - 1/\mu^2)]$. The former expression shows that the quasi-wave-propagation in a refractive medium does not conform to the acoustic/optic equations; these require the form $(a \pm q)$ for a uniform medium with a constant wave speed a moving with velocity $(\pm q, 0)$. The latter expression, however, does satisfy the acoustic/optic equations for a fluid medium with a constant wave speed $\overline{c}/\mu$ moving with velocity $[\pm q(1 - 1/\mu^2), 0]$.

It has now been shown more generally in section 3 above that, on the new approach presented here, the quasi-wave-propagation, in a refractive medium moving with speed q *in any direction* conforms to the acoustic/optic equations for a fluid medium, with true wave-speed $\overline{c}/\mu$, moving in the same direction with a reduced speed $q(1 - 1/\mu^2)$.

Figure 4a corresponds to figure 3a with the surface of the refractive medium still moving with velocity $(q\cos\alpha, q\sin\alpha)$ but with the refractive medium between these surfaces, moving with velocity $(q\cos\alpha, q\sin\alpha)$, replaced by a fluid medium with *true* wave-speed $\overline{c}/\mu$ moving with velocity $[q(1-1/\mu^2)\cos\alpha, q(1-1/\mu^2)\sin\alpha]$. The acoustic/optic equations now determine that the normal velocity to the wave-fronts in the fluid medium is $[\overline{c}/\mu + q(1-1/\mu^2)\cos\alpha]$ to the first order exactly as determined in section 3 above for the corresponding quasi-wave-propagation (c.f. figure 3a) in a medium with refractive index μ moving with velocity $(q\cos\alpha, q\sin\alpha)$. In figure 4a, however, the triangle of velocities which determines the velocity of the wave-fronts along the rays enables the angle ψ to be determined as $\tan^{-1}[q(\mu - 1/\mu)\sin\alpha/\overline{c}]$ to the first order in $q/\overline{c}$.

Figure 4b shows the flow corresponding to figure 3b. Here the normal velocity of the wave-fronts in the fluid medium is $[\overline{c}/\mu - q/\mu^2 \cos\alpha]$ exactly as determined for the quasi-wave-propagation in a refractive medium in frame 2 (c.f. figure 3b). The triangle of velocities now gives $\omega = \tan^{-1}(q\sin\alpha/\mu\overline{c})$, to the first order in $q/\overline{c}$, so that, again, $\psi + \omega = (q/\overline{c})\mu\sin\alpha$.

When $\alpha = \pm\pi/2$, for transverse flow of a refractive medium, $\psi = \pm\tan^{-1}[q(\mu - 1/\mu)/\overline{c}]$ in agreement with the observations of Jones (1972, 1975).

5 Oblique Incidence

The equivalent true wave propagation described above may be used to derive the relation between the angle of incidence i and the angle of refraction r of a plane wave-front entering obliquely a refractive medium moving with velocity $(q\cos\alpha, q\sin\alpha)$. Figure 5a shows such a wave-front in two positions separated by a time-interval t_1. In the ether at rest, outside the refractive medium, the rays are normal to the wave-fronts. In the fluid medium, which replaces the refractive medium the normal velocity of the wave-fronts is $[\overline{c}/\mu + q(1 - 1/\mu^2)\cos(r - \alpha)]$, to the first order.

Figure 5b shows the transformation to the frame of reference in which the surface of the refractive medium is stationary. In the ether outside the refractive medium, moving with velocity $(-q\cos\alpha, -q\sin\alpha)$ the normal velocity of the wave-fronts is $[\bar{c} - q\cos(i-\alpha)]$. In the fluid medium which replaces the refractive medium the normal velocity of the wave-fronts is $[\bar{c}/\mu - (q/\mu^2)\cos(r-\alpha)]$.

The time interval t_1, is then given by

$$t_1 = \frac{AD\sin i}{[\bar{c} - q\cos(i-\alpha)]} = \frac{AD\sin r}{\left[\frac{\bar{c}}{\mu} - \left(\frac{q}{\mu^2}\right)\cos(r-\alpha)\right]}$$

whence, to the first order in $q/\bar{c}$,

$$\sin i\left[1 + \frac{q}{\bar{c}}\cos(i-\alpha)\right] = \sin r\left[\mu + \frac{q}{\bar{c}}\cos(r-\alpha)\right] . \tag{5.1}$$

The same result must, of course, be derivable from figure 5a. Here, during the time-interval t_1, the boundary of the refractive medium moves a distance $qt_1\cos\alpha$ in the x-direction and $qt_1\sin\alpha$ in the y-direction. Thus, from the ether outside the refractive medium

$$t_1 = \frac{1}{\bar{c}}\left[DF\sin i + qt_1, cos\,(i-\alpha)\right] \tag{5.2}$$

and, from the flow inside the fluid medium

$$t_1 = \frac{\left[DF\sin r + qt_1, \cos(r-\alpha)\right]}{\left[\frac{\bar{c}}{\mu} + q\left(1 - \frac{1}{\mu^2}\right)\cos(r-\alpha)\right]} . \tag{5.3}$$

To the first order, equation (5.2) reduces to

$$t_1\left[\bar{c} - q\cos(i-\alpha)\right] = DF\sin i$$

and equation (5.3) reduces to

$$t_1\left[\frac{\bar{c}}{\mu} - \frac{q}{\mu^2}\cos(r-\alpha)\right] = DF\sin r$$

thus yielding again the relation (5.1).

For a refractive medium moving longitudinally with velocity $(\pm q, 0), \alpha = 0, \pi$ and so

$$\sin i\left[1 \pm \frac{q}{\bar{c}}\cos i\right] = \sin r\left[\mu \pm \frac{q}{\bar{c}}\cos r\right] . \tag{5.4}$$

For a refractive medium moving transversely with velocity $(0, \pm q), \alpha = \pm\pi/2$ and so

$$\sin i\left[1 \pm \frac{q}{\bar{c}}\sin i\right[= \sin r\left[\mu \pm \frac{q}{\bar{c}}\sin r\right] . \tag{5.5}$$

For a stationary refractive medium $q = 0$ and $\sin i = \mu\sin r$ in accordance with Snell's 'law'.

6 Čerenkov Radiation

If two stationary refractive media, with refractive indices μ_1, μ_2 have a common boundary, a particle moving with speed q may pass from one to the other. If q is such that $\bar{c} > \bar{c}/\mu_1 > q > \bar{c}/\mu_2$ it follows that, in the medium with quasi-wave- speed $\bar{c}/\mu_1$, the movement of the particle will be analogous to subsonic/subluminal motion; and in the second medium, with quasi-wave speed $\bar{c}/\mu_2$, it will be analogous to supersonic/superluminal motion. This quasi-superluminal motion in the second medium is the phenomenon known as Čerenkov radiation. Since, however, the true wave-speed in both media is $\bar{c}$, it follows that there is nothing whatsoever superluminal about Čerenkov radiation.

References

Jones, R. V. 'Fresnel aether drag' in a transversely moving medium. Proc. R. Soc. Lond. A328, 337-352, 1972.

'Aether drag' in a transversely moving medium. Ibid A345, 351-364, 1975.

Player, M. A. Dispersion and the transverse aether drag. Ibid A345, 343-344, 1975.

Rogers, G. L. The presence of a dispersion term in the 'transverse Fresnel aether drag' experiment. Ibid A345, 345-349, 1975.

Thornhill, C. K. Real and apparent invariants in the transformation of the equations governing wave motion in the general flow of a general fluid. Ibid A442, 495-504, 1993.

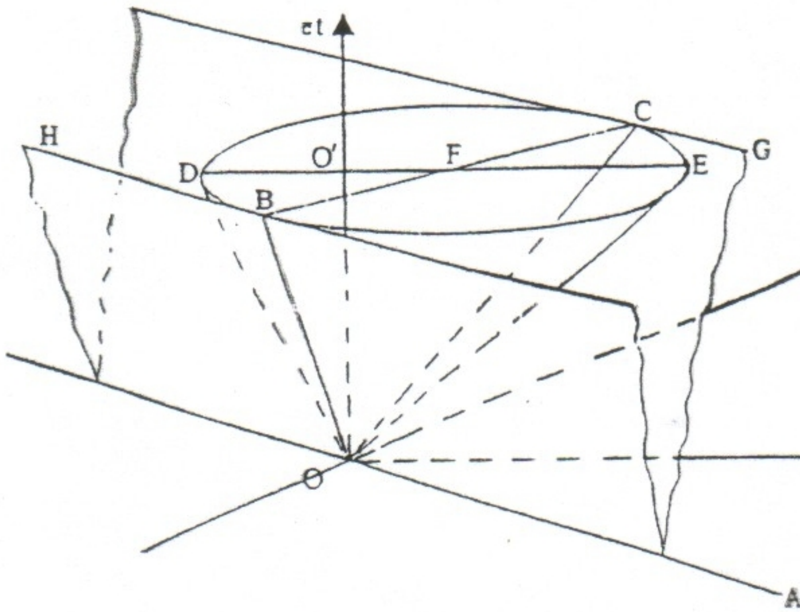


Figure 1a

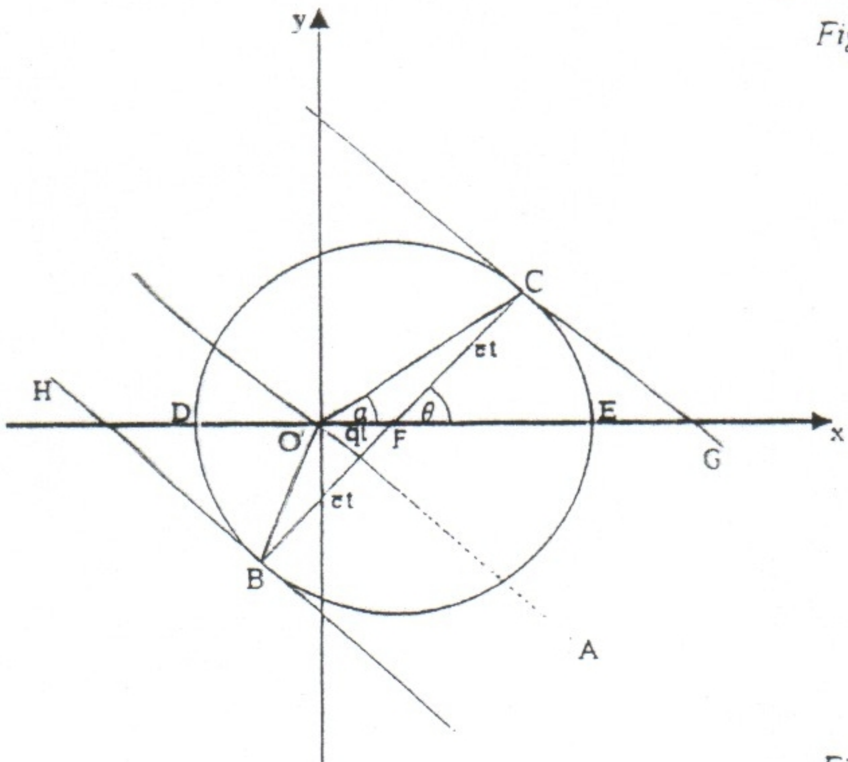


Figure 1b

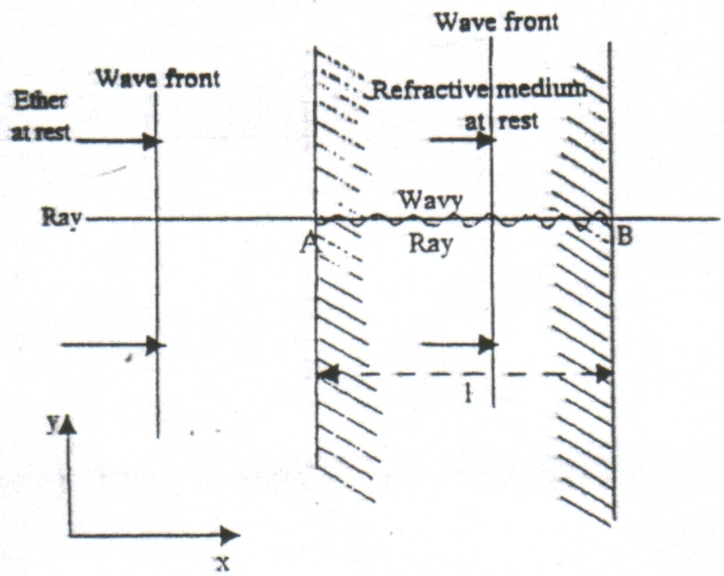
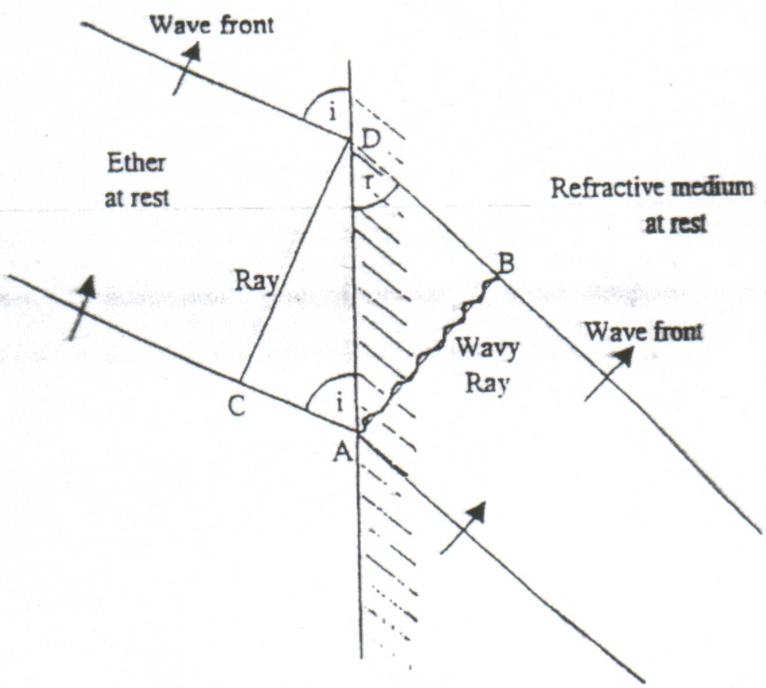


Figure 2a



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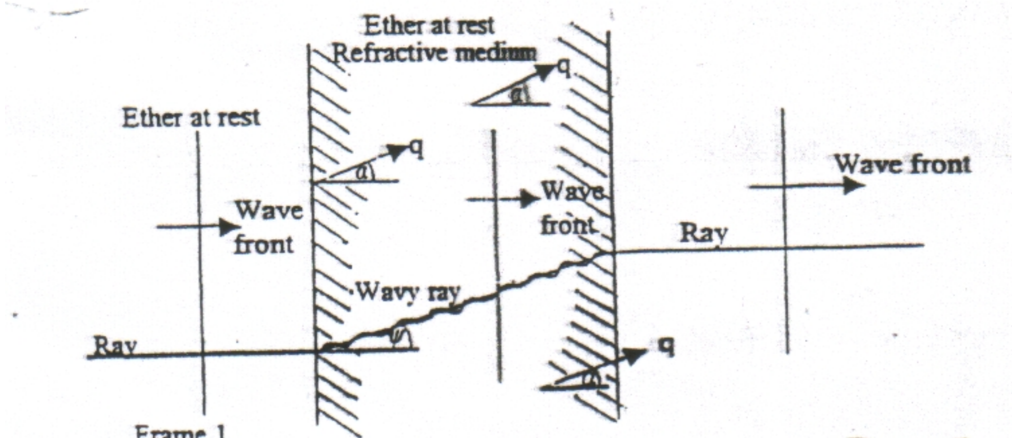
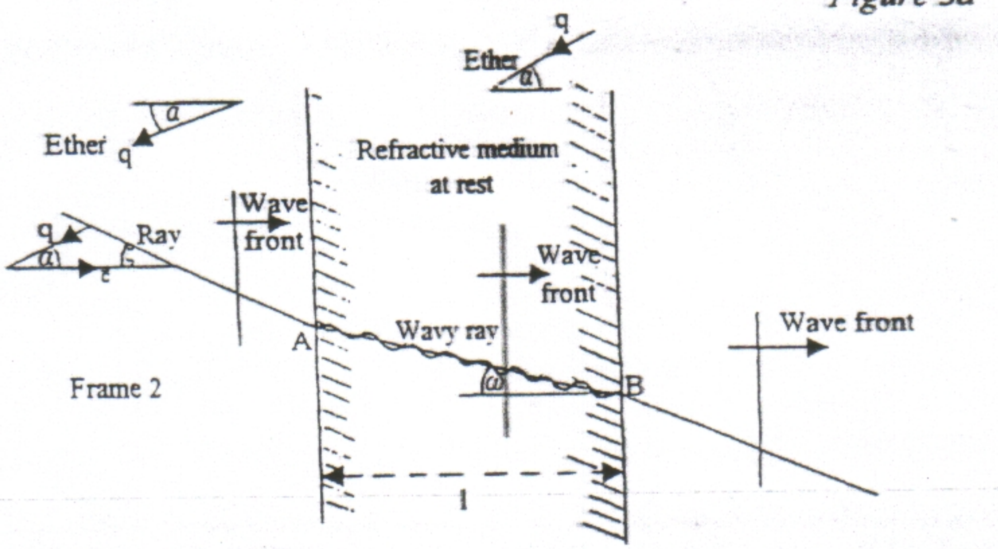


Figure 3a



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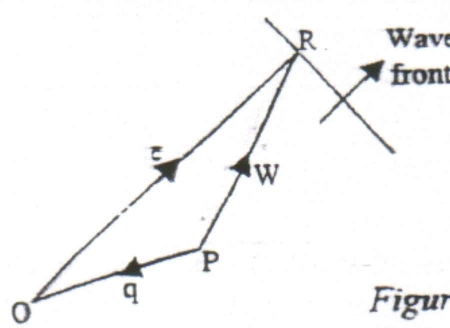

Figure 3c

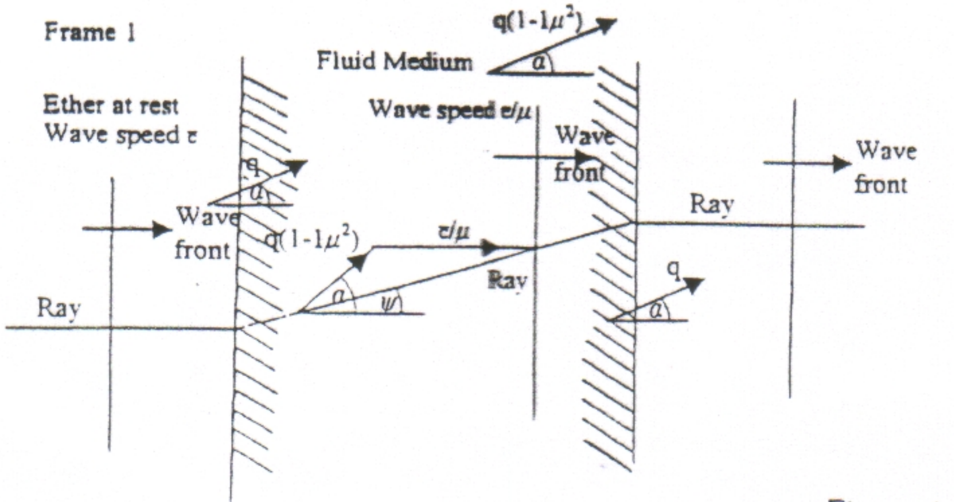

Figure 4a

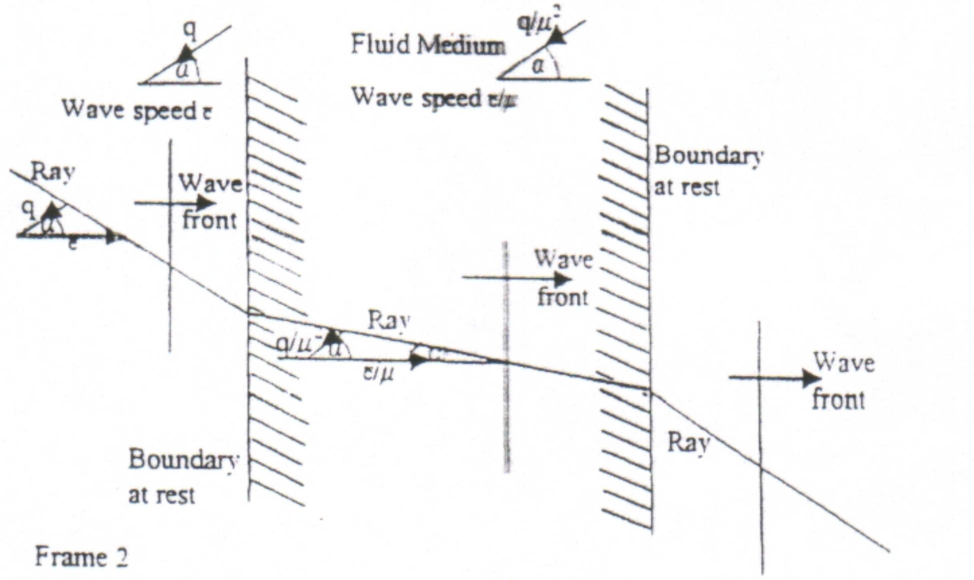

Figure 4b

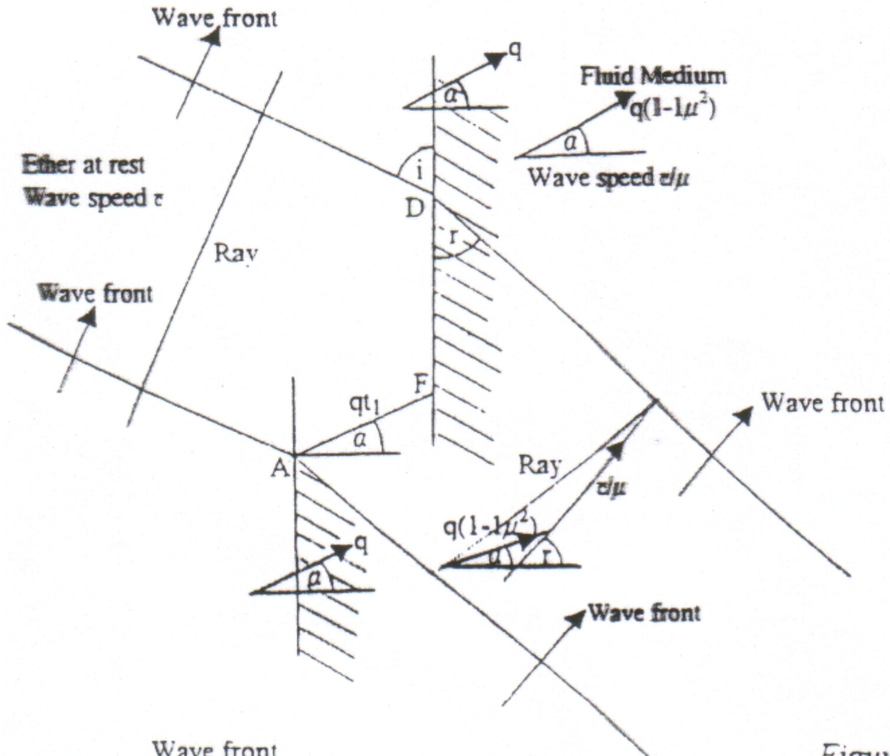

Figure 5a

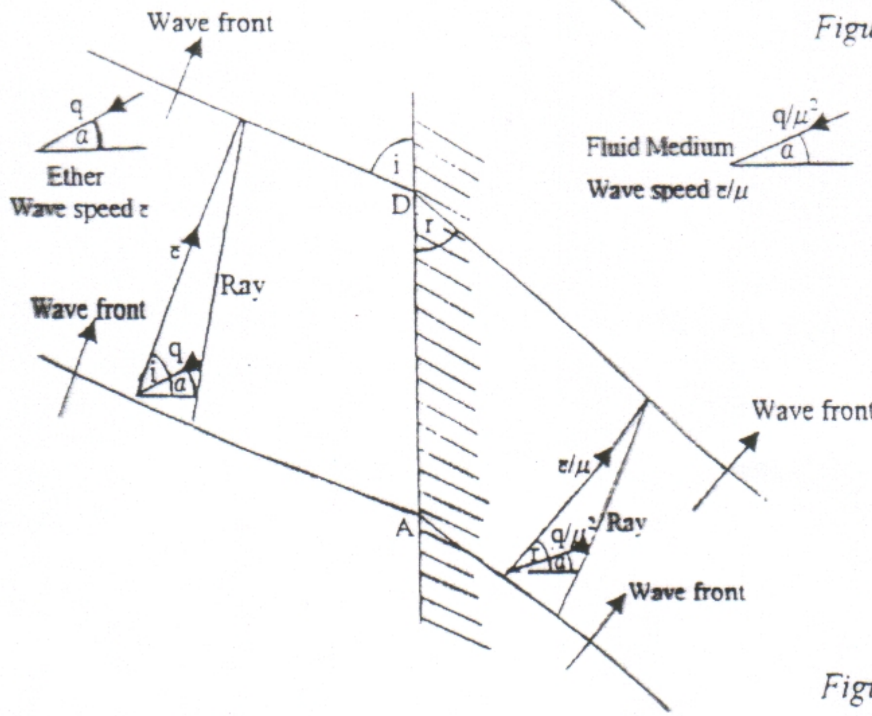

Figure 5b