Nonlinear classical fields

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Abstract

We regard a classical field as medium. Then additional parameter is appearing. It is the local fourvelocity vector of field. If the one itself regard as potential of same field then all field's self energies became finite. As examples electromagnetic, mechanical, pionic, and somewhat gluonic fields are regarding.

Contents

1 Foundation

The internal contradictions of classical field theory well known. Mainly these are infinite self energies of Coulomb and Yukawa fields. First attempt to construct the electron as a field object with goal to delete these contradictions was done by G. Mie [1] near 1912 year. His work give strong impulse for development field's theory and then it was created many works in this direction. But always the models contradict at least one of the general physical principle. For example in Born - Infield model analyticity principle was not fulfilling. In work of a French physicist [2] about 1972 the current's vector of field was build but without C-symmetry. All these are because it is impossible to construct the fourvector of electromagnetic current from parameters of the electromagnetic field. A proof of this impossibility is in many textbooks from quantum electrodynamics.

Hence one of the ways for construction of model for electromagnetic field without internal contradictions is to take into account interaction of electromagnetic field with other fields. This task in area of quantum field theory was done where electromagnetic and week interactions was joined in one electroweek interaction.

What must we do in classical field theory of electromagnetic field or in area of low energy physics where infinities are remaining? In this article extension of classical field theory is done for reaching finite self energies of fields.

Main idea is following.

In general case any physical field have non-zero density of mass. Therefore in classical physics for any field the additional parameter $U(x)$ exist which is local fourvelocity vector of field. Both velocity and fourvelocity vectors are usual parameters in physics but typically they are regarded as property of particle. We will regard the velocity fourvector as local quantity $U = U(x)$ so $U^2 \equiv 1$ only if a field is continual variety of pointed non-interacted between themselves particles. In words, a field will be regarding as special medium with two parameters, these are the potential and the local fourvelocity vector of field.

However, the fourvelocity vector as parameter exists for any physical object and in any case it is essential quantity. At construction of any theoretical model for physical object - this in today physics mean the lagrangian building - this parameter need take into account. If that is done then fourvelocity vector may be regarding or as external parameter similar to forces in Newtonian mechanics, or as internal parameter. In last case one way is visible for differential equations system closing. It is regarding of quantity $U(x)$ itself as potential of some field. Then kinetic term $(DU)^2$ in lagrangian removes all problems with set upping of full and close equation system. Below we will regard the fourvelocity vector of the field as potential of some field which we call w-field.

Let us attempt to comprehend the physical meaning of w-field. The tensions (forces) of this field are following (Clifford's algebra [3] always is using)

$$DU = D \cdot U + D \wedge U$$

$$D \cdot U = \partial_0 U_0 + \nabla \cdot \vec{U}$$

$$D \wedge U = -\partial_0 \vec{U} - \nabla U_0 + i_c \nabla \times \vec{U}$$

In case $U^2 = 1$ the quantity $-\partial_0 \vec{U}$ is usual mechanical acceleration with opposite sign. Hence w-field forces contain inertial forces. In typical case the square of fourvelocity vector is not equal to unit. As example for scalar photons (these are electrostatic fields) $U_2 > 0$, for longitudinal photons (these are magnetic fields) $U_2 < 0$. We may feel almost certainly that by their physical meaning the w-field is bearer of the inertial (typically virtual) forces and expect that these forces create barrier which do not permit any field to concentrate in pointing particle with infinite self energy as it is in standard theory of strong interaction.

Field's theory internal contradictions are sufficiently for introducing new essence. If we do not wish to do this then w-field may be regarding as representation, of course partially, of week forces in area of classical physics. Or in simplest approach the fourvelocity vector regard as external parameter.

If take into account existence of w-field then it is not hard to build the lagrangian of fields. But before of lagrangian construction it is need to regard what is the coherence condition and some properties of wave function.

2 Coherence condition

Formally, this is trivial thing and this is more simply understood in examples.

Regard scalar field with potential $s(x)$ in one dimension space. Lagrangian for one take as following

$$L = \frac{s'^2}{2} + ss'^2$$

Variation of this lagrangian is

$\delta S = s'\delta s' + s'^2 \delta s + 2ss'\delta s'$

This is sum of few terms hence exist more then one solution of variation task. Regard some of these solutions.

In case

$s'' = 0$

$s = a + bx$

the coherence condition is following

$\int (s'^2 \delta s + 2ss'\delta s')dx = 0$

Variations in class of linear functions are

$\delta s = \delta a + x\delta b$
where $\delta a, \delta b$ are free numbers. Then
$\int [b^2(\delta a + x\delta b) + 2b(a + bx)\delta b]\, dx = 0$
Hence $b = 0$ and by their physical meaning this is vacuum state of this field.
Other solution is
$s'' = s'^2$
$s = a - ln(b + x)$
Coherence condition for this state

$$\int [a - ln(b + x)] \frac{dx}{(b + x)^3} = 0$$

connect between themselves integration's constants.
Next state is
$s''(1 + 2s) + 2s'^2 = 0$
$s + s^2 = a + bx$
with coherence condition

$$b^2 \int (\delta a + x\delta b) \frac{dx}{(1 + 2s)^3} = 0$$

In this state $b = 0$ and this is another vacuum state because potential's constant in the states may be differ.

The standard equation's system is
$s'' + 2(ss')' = s'^2$
without any restrictions on integration constants.

We see that a field may exit in few states. Additionally, we may put some parameter equal to fixed constant then variation of this constant is zero and coherence condition for such parameter is automatically fulfill (for example, electric charge is not variable constant). The coherence condition is strong tool for theoretical physics but it is unknown for most physicists.

3 Wave function. Boundary conditions

For bound states of particle in spherical symmetrical field the radial part of wave function take as following

$$F \sim R^B \prod_0^N (R - R_i) exp(DR)$$

where B, D, R_i are constants and zeros may be replicating.

This form of wave function is generalization of Sturm-Liouville oscillation theorem and analyticity principle forced such type of wave function for Coulomb-like forces. For oscillatory potential the wave function on infinity have other behavior

$\sim exp(\frac{1}{2}DR^2)$ and main source of information about a system contain the polynomial part of wave function. For some types of the differential equations such solution was known before quantum mechanics was appearing.

At first look this is trivial thing but direct solution of differential equation is fruitful.

Let us determinate additional boundary conditions for wave function in case when the potential of field have singularity in area of small distances. The exact mathematical solution of any equation is not exact physical solution of one. Having we deal with linear or nonlinear Coulomb field or with any other field always area of small distances is unknown land. In this area all known and unknown forces are working. For example, usual Coulomb potential have in center singularity which is non-physical and the one do not exist in nature so it is true regard the one from infinity but not as singularity in center. Physical sense for microscopic system has quantities that are seeing on infinity. In other words, it is necessary to formulate boundary conditions on infinity.

For these doing divide the Schroedinger equation on wave function-this mean that logarithmic derivative of wave function is using - and multiply the one expression by R^X where X is whole number. Then run distance to infinity. In this way we get $X + 1$ algebraic equations which restrict the unidentified parameters of wave function.

Which number must be the degree of this multiplier? Answer is it is must be equal to number of unknown parameters. For example, count this degree for upper wave function. Here are N unidentified wave functions zeros, constants D, B, energy E. In this case the degree of multiplier is $X = N + 2$.

But we may include in this list the scale parameters, potentials of vacuum and interaction's constants. If differential equation determinate all unknown parameters it is well. If for some parameters solutions are not exist the ones are regarding as external quantities. Below at least at first approach parameters in additional list will be regarding as external constants.

Constructing these boundary conditions we avoid uncertainties connected with knowledge physical quantities in non physical area of small distances. And this look as trivial but only look. From these boundary condition follow that a states with more wave function zeros contain more information about internal structure of the object. In other words the precise measurements of energy levels may replace high energy scattering experiments.

4 Nonlinear electromagnetic field

4.1 Lagrangian, current, Maxwell's equations

The lagrangian [4] of pair interacting between themselves fields take as following

$$L = L_1 + L_2 + L_3$$

We regard electromagnetic field with its shadow w-field in area which contain no any particle. As usually, this area is whole space-time.

The fourpotential of electromagnetic field $A(x)$ with tension

$F = D \wedge A = \vec{E} + i_c\vec{H}$

is restricted by Lorentz gage condition $D \cdot A = 0$

Lagrangian of free electromagnetic field is

$$L_1 = \frac{F^2}{8\pi c}$$

Here in general case need to denominate Dirac conjugation at one multiplier but for simplicity this sign is omitted.

Lagrangian for free w-field let us take in similar form

$$L_2 \sim (DU)(DU)$$

The square form for lagrangian of free electromagnetic field needs for compatibility with experimental Coulomb low. But for free w-field such form of lagrangian is assumption and it is strong assumption.

At first look about interaction's lagrangian nothing is known. It is true in general case but for electromagnetic field it is known enough. The fourvector

$$\frac{\delta L_3}{\delta A} = J$$

is density of current for electromagnetic field. For simplicity below we call it current. This is source of electromagnetic field.

From Bohr correspondence principle this fourvector is linear function from velocity fourvector and at all transformations the current must have exactly such properties as fourvelocity vector. General form of such quantity is following

$$J = c_1U + c_2(FU - UF) + c_3FUF$$

where c_n are scalar functions which do not depend from velocity fourvector. From gradient's symmetry of the electromagnetic field the ones depend from electromagnetic field tension F only.

The phase of any physical quantity is relative. This condition gives two restrictions. Firstly $c_2 \equiv 0$. Secondly scalar variable is F^2F^2 (one multiplier is conjugating).

Because here no particles with electric charge the condition $c_1(0) = 0$ must be valid. For usual transverse waves the current must be equal zero that at once confirms this restriction.

Then in first non-trivial approach the current of electromagnetic field is following

$$J = \frac{c}{4\pi g}FUF$$

where g is interaction constant which determinate scale of the electromagnetic potential. This field's jet need to compare with usual eU jet of a particle if the question about C-symmetry arise.

For w-field's current $\delta L_3/\delta U$ all above argument are true. Only restriction for first coefficient may be not valid and interaction's lagrangian may contain term $L_3 \sim U^2$. For simplicity here we will not regard self-interaction of w-field.

The fourvelocity vector is dimensionless quantity so it is convenient to take the potential of electromagnetic field in dimensionless form. It is

$$A(g,x) = gA(y = \frac{x}{a})$$

where g, a are scales of potential and length.

Then simplest action for electromagnetic field with its shadow w-field is following

$8\pi cS = e^2 \int \left[(DF)^2 + k^2(DU)^2 + 2A \cdot (FUF) + qU \cdot (FUF)\right] d^4y$

where all quantities after integral sign, including integration variables, are dimensionless.

This action does not contradict any general physical principle. The scale invariance of this lagrangian is general property of any model for fields without external particles. By construction the validity area of this lagrangian is $O(F^4)$ and, because the scale parameter is external quantity, for description of macroscopic as well for microscopic systems the model may be applied.

Owing to constriction, the current's fourvector is not variable by electromagnetic potential and Maxwell's nonlinear equations for dimensionless quantities are following

$DF = FUF$

or for usual dimensionless three-vector quantities

$$-\nabla \cdot \vec{E} = u_0(E^2 + H^2) + 2\vec{u} \cdot (\vec{H} \times \vec{E})$$

$$\nabla \cdot \vec{H} = 0$$

$$-\partial_0 \vec{E} + \nabla \times \vec{H} = \vec{u}(E^2 + H^2) + 2u_0(\vec{H} \times \vec{E}) - 2\vec{E}(\vec{u} \cdot \vec{E}) - 2\vec{H}(\vec{u} \cdot \vec{H})$$

$$\partial_0 \vec{H} + \nabla \times \vec{E} = 0$$

There always exist solutions with potentials equal to constant anywhere. By physical meaning these are vacuum states of a field.

The electromagnetic bevector, Maxwell's equations, and Lorentz gage condition do not changing at transformation

$A \rightarrow A + constant$

Therefore for pure electromagnetic system the value of scalar part of the electromagnetic potential at infinity is free number.

Now it is need to close the system of equations.

4.2 Closing equations of field

The simplest state of electromagnetic field is that when the w-field is in vacuum state. In this case equation for fourvelocity vector is

$D^2U = 0$

Because this is isolate equation the coherence condition take in form

$\int(\delta U \cdot (FAF + qFUF))d^4y = 0$

$\int \delta_A J \cdot (2A + qU)d^4y = 0$

Next states we call the coherence states. In this case equation for velocity fourvector is

$k^2 D^2 U = FAF$

with restrictions

$\int \left[(2A + qU) \cdot \delta_A J + qU \cdot \delta_U J\right] d^4y = 0$

All other states involve in equation system the self interaction of w-field and for simplicity we regardless ones here.

In this way a system of differential equations is closing.

Now regard nonlinear Coulomb field.

4.3 Nonlinear Coulomb field.

This is spherically symmetrical electrostatic field. Dimensionless potential and space variable of one denominate as following

$$s(x), \;\; x = \frac{a}{R}$$

The Maxwell's equation in this case is following

$s'' = us'^2$

where denomination of scalar velocity is clear.

This electrostatic field has non-zero electric charge and non-zero density of electric charge what is new phenomena not only for theory of classical fields.

In case system with full positive electric charge equal to unit the solution for tension is following

$$s' = exp \int_{s_0}^{s} u(s)ds = [1 - \int_0^x u(x)dx]^{-1}$$

In physical area - this is area of big distances - electrostatic potential is following

$$s = s_0 + x + s_2\frac{x^2}{2} + s_3\frac{x^3}{6} + ..$$

or for physical potential of system with electric charge e

$$A_0 = \frac{es_0}{a} + \frac{e}{R} + \frac{eas_2}{2R^2} + ..$$

In this expression the coefficients s_i depend from interaction constant of free w-field and from vacuum potential of electrostatic field. Then implicitly here the vacuum potential of electrostatic field is observable. In this point the nonlinear model differ from linear model where vacuum potential is free number. Of course, for different states these are different constants.

When w-field is in vacuum state the equation for scalar velocity is equation of free field

$u'' = 0$

$u = u_0 + bx$

Any physical field must have finite self energy. Hence constant $b = 0$ and electrostatic potential in this state is

$s = s_0 - \frac{1}{u_0} ln(1 - u_0 x)$

$u_0 < 0$

Here coherence conditions take in form

$\int (s + qu_0) s'^2 \delta u_0 dx = 0$

$\int (2s + qu_0) s' \delta s' dx = 0$

then

$u_0(s_0 + qu_0) = 1$

$u_0(2s_0 + qu_0) = 3$

In this case for system with positive electric charge the scalar velocity $u_0 = -1$. Then $s_0 = -2$ and $q = -1$. This is very exotic state and below will be regarding only coherence states of electromagnetic field.

In coherence states the equation and solutions for scalar velocity are following

$$k^2 u'' = s s'^2$$

$$u = -\frac{s}{k} + 2 \sum_n \frac{1}{s - s_n}$$

where Ermit's numbers s_n are determining by equations

$$s_n = 2k \sum_{i \neq n} \frac{1}{s_n - s_i}$$

The electrostatic tensions in these states are following

$$s' \sim \prod_0^N (s - s_n)^2 exp(-\frac{s^2}{2k}) \quad s'(0) = 1$$

so we have the variety of states similar to Glauber states.

For these coherence states always must be $s_0 > s_N$ then self energy of free w-field is finite.

From physical reason, any motion on infinity is free motion. Correspondingly, the scalar velocity on infinity u_0 have one of three possible values $u_0 = \{-1, 0, +1\}$. For states with positive electric charge $\frac{du}{ds} < 0$ and in case $u_0 = +1$ the ones have complicated electromagnetic structure because the density of electric charge change sign in internal area of field.

As another example, regard the nonlinear electrostatic field in light nucleus. In nucleus the scalar velocity u depend both from parameters of electromagnetic and strong forces. In light nucleus nucleon's motion mainly is ruled by strong interaction. The nucleons in nucleus are near mass shell so distribution of scalar velocity for nucleons is almost constant. Then scalar velocity of electromagnetic field in light nucleus also is almost constant. In other worlds, the scalar velocity in equation for determination potential of electrostatic field is external parameter which is zero in external area and it is constant in internal area of nucleus. However, the u_0 is not necessary unit because a nucleon move in cloud of virtual bosons but not in empty space. If b is radius of nucleus then electrostatic tension is

$$E = \frac{eZ}{R} \qquad u_0 = 0 \qquad R > b$$

$$E = \frac{d}{R(R+b)} \qquad u_0 = const \qquad R < b$$

where constant $d = -\frac{bg}{u_0}$ by their physical meaning is polarization of internal nucleus medium.

Correspondingly, distribution of electric charge in light nucleus at small distances essentially differs from one in heavy nucleus.

Remark that for nonlinear Coulomb field the Earnshow theorem is not valid because the laplacian of electrostatic potential is not zero.

Additionally, regard qualitatively the electron levels in nonlinear Coulomb field and nonlinear electromagnetic waves.

4.4 Electron levels

The spin effects are essential here so Dirac equation must be using. Electromagnetic field itself is fourvector field but it is joining with spin framework. For example, proton's electrostatic field in general case must be sum of two terms with spin equal 1/2 and 3/2. To avoid this mainly technical barrier regard here the electron connecting with alpha particle. The one has zero spin and variety of data exist for it. It is He II levels and Dirac equation for this system is following

$(iD - eA)\Psi = m\Psi$

$c = 1, \ \hbar = 1$

Let us go to usual three-spinors, separate angle dependence of waves function and convert two equations for two radial wave functions into one equation. Then Schroedinger-like equation is appearing

$$F'' + \frac{2}{R}F' + \frac{F'V'}{W-V} = \left[\frac{l(l+1)}{R^2} - \frac{fV'}{R(W-V)} + 2EV - V^2 + m^2\right] F$$

where E, V, m, j, l are electron's energy, potential energy, mass, moment, orbital moment and $f = -l$ if $j = l + \frac{1}{2}$, $f = l + 1$ if $j = l - \frac{1}{2}$

In this form analytical properties of wave function are more realize.

According to boundary conditions it is possible to make following replacement

$$V \to V_0 - \frac{Z}{R} - \frac{Zau_0}{2R^2} - \frac{Za^2 s_3}{6R^3} + ..$$

but these are not singularities in zero.

In Dirac equation electron's energy and term V_0 which by their physical sense is energy interaction of electron with vacuum always are joined so below last term omit when it itself is not necessary. Additionally, electric charge number and fine structure constant are joined then both ones are denominating as Z.

First four equations for unknown parameters calculation are following

$$D^2 = m^2 - E^2$$

$$D(N + B + 1) = -EZ$$

$$B^2 + B + D(\frac{Z}{W} + 2\sum R_i) + 2BN + N^2 + N = l(l+1) - Z^2 - ZEas_2$$

$$D\left[\frac{Z}{W}(as_2 - \frac{Z}{W}) + 2\sum R_i^2\right] + 2(B+N)\sum R_i + (B+N+f)\frac{Z}{W} =$$

$$= -Z^2 as_2 - \frac{1}{3} ZEs_3 a^2$$

Firstly, roughly regard He I levels neglecting spin's effects. Here assumed scale of length is $a = \frac{d}{Z} + ..$

Then binding energies of states with maximal orbital moment (N=0) are following

$$\varepsilon = \frac{m\alpha^2}{2\left[\frac{1}{2} + \sqrt{(l+\frac{1}{2})^2 - du_0}\right]^2}$$

In He I the electric charge density on infinity is negative hence $u_0 = +1$ and electron's bound energy is bigger compare with linear model. From ionization energy data we roughly have $u_0 d = 0.2$ so with increasing of moment the spectrum quickly became hydrogen-like. The picture is true.

Regard the electron levels in HeII (alpha particle). Here on infinity the electric charge density is positive hence $u_0 = -1$. Because electric charge is not concentrating in point all levels have shift up compare with levels in usual Coulomb field.

The strong interaction between nucleons in nucleus ties up particles - and with ones the electric charge - what may compensate nonlinear effects. Such is qualitative picture.

For calculation it is need knowledge of the scale parameter order. It is $a = .. + \frac{a_1}{Z} + a_0 + dZ + ..$
Certainly for HeII in this sum the left coefficient is zero and last is not zero. With middle coefficient is uncertainly but from equations the one may put equal to zero. Remark, when asymptotic procedure of solution searching is using then we have no single-valued result and for separation of physical solutions it is need to use the correspondence principle. Then B-coefficient takes as following

$$B = j - \frac{1}{2} - \frac{Z^2}{2j+1} + bZ^2 + ..$$

corresponding, bound energy is following

$$\varepsilon = \frac{Z^2}{2n^2} - \frac{3Z^4}{8n^4} + \frac{Z^4}{n^3(2j+1)} - \frac{bZ^4}{n^3}$$

$$n = N + j + \frac{1}{2}$$

In states with maximal moment (N=0) nonlinear adding to bound energy is $b(2j+1) = -du_0$

Comparing with data and linear approach show that both sign and moments dependence of adding are true.

But regard 2S, 2P states with moment $j = \frac{1}{2}$. In linear model these states are degenerate as any levels on fixed shell with equal moment and opposite parity. In both these states the wave function have one zero. However, the ones have different order - nonrelativistic

$$R_S = \frac{2}{Z} + ..$$

for S-state and relativistic

$$R_P = -\frac{3Z}{4} + .$$

for P-state. Correspondingly, $b_S = -du_0 + ..$ in S and $3b_P = -du_0 + ..$ in P states.

So bound energy of S-state is less than one in P-state. This is Lamb shift phenomena. The Lamb shift value is equal $-\frac{1}{12}u_0 dZ^4 mc^2$. From quantum electrodynamics calculations and data the value of Lamb shift is $0.41\alpha Z^4 mc^2$. It mean the value of parameter $-du_0$ is not sufficient for fitting electron's ionization energy in He II. Then it is need take into account the interacting energy of electron with vacuum and the vacuum potential of electrostatic field for this system is directly observable quantity. Because at estimation here do not used s_3 and the next coefficients in decompose of electrostatic potential this result is grounded mainly on analyticity principle.

In usual model we have as external parameters only electric charge then electromagnetic system has zero size and infinite self energy. In nonlinear model charge and scale of length are external parameters, all other quantities, including self energy of a field, must be calculated. There is similarity with quantum electrodynamics where electric charge and mass of particle are external parameters for electromagnetic system.

4.5 Nonlinear electromagnetic waves

For classification states of electromagnetic field the signs of invariants

$$E^2 - H^2,\ \vec{E}\cdot\vec{H}$$

are using. In nonlinear model somewhat another classification field's states is more convenient. Indeed, the sign of electromagnetic jet square is

$$J^2 \sim [(E^2 - H^2)^2 + (\vec{E}\cdot\vec{H})^2]u^2$$

Correspondingly, the states of field may be distinguishable via $u^2 = u_0^2 - \vec{u}^2$ sign.

The states with $u^2 = 0$ contain the usual electromagnetic waves. In case $u^2 > 0$ the field have not zero electric charge. If $u^2 < 0$ then it is magnetic state of field.

Typically, the electromagnetic waves are states without electric charge and with periodical phase. Thus nonlinear electromagnetic waves are magnetic states of field. Of course, if the waves exist.

For description of nonlinear waves it is convenient to choose the coordinate system where the scalar part of electromagnetic potential is zero. From Maxwell's nonlinear equations follow that fourvectors A, u are collinear then both $A_0 = 0, u_0 = 0$ in the appropriate coordinate system.

It is convenient take the four potentials of field for flat electromagnetic waves as following

$$A = A(x)\gamma_y exp\{i(\omega t - kz)\}$$

$$u = -u(x)\gamma_y exp\{i(\omega t - kz)\}$$

With such choice of potentials the four tension of electromagnetic field is given by expression

$$F = (-i\omega A e_y + i i_c k A e_x + i_c A' e_z)\, exp\{(i(\omega t - kz)\}$$

what is more complicated form compare with usual description of the vector field without using Clifford's algebra. However, the equations that needing contains the amplitudes of potentials so this change has no matter. If we wish divide the tension into electric and magnetic parts then suitable phase need to take. Usage of both Clifford's algebra and the phase multiplier in potentials create same theoretical troubles because this convert the vector field in mixture of vector and pseudovector fields. For simplicity we go round of that by usual manner - rewriting the definition of jets as $FuF \to Fu^+F,\ FAF \to FA^+F$.

Simplest waves are those where w-field is in free state. Nevertheless, we consider the coherence states of field.

For plainness we regard only slow waves (put $\omega = 0$) and denominate the dimensionless amplitudes of potentials as following

$A(kx) = g\sqrt{k_{int}}p(s)$
$gv(s) = \sqrt{k_{int}}u(kx)$
$s = kx$

where k_{int} is integration constant of free w-field.

Then from nonlinear equations for coherence states of field the equations for calculation of dimensionless amplitudes read

$$p'' - p = v(p'^2 - p^2)$$

$$v'' - v = p(p'^2 - p^2)$$

with symmetry $p(-x) = p(x)$ and boundary $p'(\infty) = 0,\ p(\infty) = constant$ conditions for paramagnetic waves. The last condition looks surprisingly because then the density of field energy on infinity contains constant terms. However by suitable choosing of interaction constant q in the last term of lagrangian for electromagnetic field the ones may be annulled. These are boundary condition for paramagnetic waves. In these waves the local currents are parallel therefore on ground of Ampere low the paramagnetic waves are stable.

Few simple exact solutions of these equations exist. First is trivial $p \equiv v \equiv 0$ and it correspond to pure vacuum state of field. Second is $p \equiv v \equiv 1$, because the phase of field is not zero these are usual waves with fixed constant amplitude. The solution $p \sim exp(\pm x),\ v \sim exp(\pm x)$ represent the free states of field, in this case the equations $p'^2 = p^2, v'' = v$ are equations of free field and the interaction between electromagnetic and w-field is absent.

Hence at least the free coherence nonlinear electromagnetic waves exist in the model. In these waves the field is concentrating near surface $x = 0$ and they have no the internal structure along x-axis.

Apparently, the variety of more complex waves is here. For its detecting consider some example. Let us take the simplest connection $v \equiv p$ between electromagnetic and w-field. By his physical meaning, the velocity parameter v is polarization of vacuum with nonlinear dependence from value of electromagnetic field. The solution $p \equiv v$ correspond to linear connection between polarization of vacuum and potential of electromagnetic field. In this case

$$p' = \pm\sqrt{p^2 + C\,exp(p^2)}$$

and here are the periodical solutions for potential if the first integration constant C is small negative number - then under square root expression is positive in area $p_- < p < p_+$. With conditions $p'(0) > 0$ the amplitude of potential arise at moving along s-axis and reach the value $p = p_+$ in same point $s = s_1$. After that point we may or put $p \equiv p_+$ or change the sign of derivative. In last case the amplitude grow down to value $p = p_-$ in point $s = s_2$. These circles may be repeated not once but on big distances need to put $p \equiv p_+$ or $p \equiv p_-$. The situation is similar to usual trigonometrical states where $p' = \sqrt{1 - p^2}$ therefore or $p(x) = sin(x)$ or $p(x) \equiv \pm 1$. The energy of these states is sum of bits that give additional chance for stability of these waves.

For axial symmetrical waves the electromagnetic jet is or flowing along the axis of symmetry or revolving about of one. In first case the electromagnetic potential for slow waves take in form

$$A = A(\rho)\gamma_z exp(ik_1\varphi)$$

and similarly for potential of w-field. Remark that here $k_1 = n$ and the wave's vector is discrete quantity in fixed point. The variable ρ convert in variable $x = ln(\rho)$ then equations for determination field's amplitudes are exactly the same as for system with flat symmetry. Then, for example, the field in the free slow paramagnetic axial symmetrical wave is condensed near surface $\rho = 1$ what is similar to skin effect for usual neutral electric current in conductors.

In this way, the states of nonlinear electromagnetic field in form of nonlinear waves exist on paper. These states have richer structure than usual electromagnetic waves. For theirs existence the external mechanical wells are not demanding.

Nonlinear waves interact with external electromagnetic field. Indeed, in this model all fields are interacting, however, not with themselves but with w-field. The simplest lagrangian for interaction of nonlinear wave with external electromagnetic field having the potential A_{ex} is $\sim A_{ex} \cdot J_w$ where of wave jet J_w is linear function from velocity fourvector. Due to nonzero phase of wave potentials, the simplest jet is not FuF but it is $F \cdot u$. Correspondingly, the Maxwell's equation remain without changing but the equation for fourvelocity vector gets adding and became

$$D^2 u = Fu^+F + g_e A_{ex} \cdot F$$

where g_e is constant.

All effects that are observable at spreading of usual electromagnetic waves via mechanical medium may be observable at spreading of nonlinear waves via external electromagnetic field.

5 Fluid as mechanical field

5.1 Introduction

For shortness here the fluid is continual isotropic homogeneous medium.

The description of continual mechanical states is grounded on Newtonian lows. At this approach from impulse conservation low and phenomenological properties of a system the Navier-Stokes equations are constructed. Such method is extension of particle dynamics in area of field objects. Seldom the fluid regard as field and the lagrangian formalism is using as framework of fluid dynamics [5]. The lagrangian formalism is general method for description of any field. But when this method is used in mechanics of continual system then main property of any field lose of sight. This property is spreading of the internal interaction in any field from point to point with finite velocity.

The internal mechanical interactions in fluid are transmitting with velocity of sound. This property may be taking into account if Lorentz (but not Galileo) transformations with parameter c=velocity of sound in fluid are using at coordinate system changing. In this article such road is choosing for description of fluid.

The Clifford's algebra and standard lagrangian formalism take as tool for delineation dynamics of fluid. The short review Clifford's algebra properties are in the end of article.

For going in this way the usual three-vector of mechanical shift $\vec{\xi}(t, \vec{x})$ will be regarded as space part of the shift fourvector $\xi(x)$

In detail this look as following

$\xi = \xi_0\gamma_0 + \xi_n\gamma_n$

$x = x_0\gamma_0 + x_n\gamma_n$

$x\gamma_0 = x_0 + \vec{x}$

$\xi\gamma_0 = \xi_0 + \vec{\xi}$

When absolute time is taking - this is usual representation for mechanical systems because the fundamental interactions are transmitting with velocity of light - then time's part of the shift fourvector became known

$\xi_0 = ct$

The space variety also became absolute in this case. Due to existence of fundamental interactions the relative velocities in fluid may exceed velocity of sound. Of course, they are always less of light velocity.

5.2 Fluid parameters

It is well approach regard any mechanical system as variety of particles. The interaction between particles is so small that ones are on mass shell and each particle move on trajectory $x(s)$. The tangent fourvector $u(s)$ to trajectory of the particle with mass M and fourimpulse P is

$$P = Mu$$

$$u = \frac{dx}{ds}$$

$$u\gamma_0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(1+\frac{\vec{v}}{c})$$

For continual state it is need to put $M = \rho d^3x$ where ρ is density of mass and d^3x is small volume. Then two independent parameters exist. These are fourscalar $m = \rho\sqrt{1-\frac{v^2}{c^2}}$ and fourvector of the mass current J_m

$J_m\gamma_0 = \rho(1+\frac{\vec{v}}{c})$

In mechanical interactions the mass is conserving quantity so

$\partial_t\rho + \nabla\cdot(\rho\vec{v}) = 0$

Other set of parameters are the relative shifts (these are deformations) which in local limit are following

$D\xi = s + f$

$s = D\cdot\xi = \partial_0\xi_0 + \nabla\cdot\vec{\xi}$

$f = D\wedge\xi = -\partial_0\vec{\xi} - \nabla\xi_0 + i_c\nabla\times\vec{\xi}$

When absolute time is taking then deformations are following

$$s = 1 + \nabla\cdot\vec{\xi}$$

$$f = -\frac{\vec{v}}{c} + i_c\vec{h}$$

$$\vec{h} = \nabla\times\vec{\xi}$$

where $\vec{v}$ is local velocity of fluid.

If non relativistic formalism is using then

$\nabla\vec{\xi} = \nabla\cdot\vec{\xi} + i\vec{h}$

Hence appearance of pseudovector $\vec{h} = \nabla\times\vec{\xi}$ as one part of deformations is inevitable in field's model of fluid. As standard in the usual model of fluid the variable $\nabla\times\vec{v}$ is taking as one of parameters.

The deformations are field's parameters. They are pure mathematical but not physical objects. In other words, these quantities do not exist in nature. It is because $m\xi$ but not ξ itself may exist as real quantities. We get out these variables only with goal to make construction of the fluid lagrangian more evident.

The speed of the fundamental interactions spreading is velocity of light. Hence the photography of stream may be done and picture of the current lines will be appearing. On current's line the interval is space-like then tangent three-vector on a line is unite pseudovector $i\frac{\vec{v}}{v}$. For simplicity below the existence of such additional parameter is ignored. Remark that in work of a soviet physics near 1972 year a unit vector was introduced as the external parameter for description of solid state.

5.3 Lagrangian of fluid

For any field its lagrangian may be regarded as sum of two terms

$L = L_0 + L_{int}$

Here L_0 is lagrangian of the free field. This mean that external and self interaction forces are switching off and only interaction between different parts of field (if ones exist) is working. In case when forces between different parts of field disappear at big distances the lagrangian of the free fundamental field is square form of field tensions

$L_0 = kF^2$

$k = \text{constant}$

For fluid the condition decreasing of forces when distances increase is valid. From the Hook low the fluid tensions are linear functions from deformations. But in fluid the parameter k is not constant. The dimension of this parameter is density of energy. Because self interaction is switching out only the quantity $m = c^2\rho\sqrt{1-\frac{v^2}{c^2}}$ have need dimension. And from Bohr correspondence principle - this is one of the general physical principle - it is need to take $k \sim c^2\rho$. Hence for fluid the lagrangian of the free field (in Hook's low area) is following

$L_0 = m(s^2 - f^2)$

where parameter m is not variable by shift's fourvector. Corresponding, field's equation for ideal fluid is

$D(mF) = 0, \;\; F = s + f$

The self-interaction lagrangian in any case may be taking as following

$L_{sint} = \xi \cdot J$

where fourvector J is not variable quantity. By physical meaning this is fourvector of the self interaction forces in fluid. For phenomenological construction of this fourvector we have variety of odd derivatives from deformations and even derivatives from the mass fourvector. But in the field theory any lagrangian of self-interaction do not contain the times derivative with degree more than unite - if this restriction is violated then Newtonian low is not valid. So DF and J_m are accessible quantities for building of fourvector J. Because using derivatives from deformations create only renormalization of mass we regard only quantity $J \sim J_m$. From physical reason the internal self interaction disappear in case $f \to 0$ then $J \sim fJ_m$.

In this way the equation of the fluid dynamics in four dimension form is following

$D(mF) = af \cdot J_m + bf \wedge J_m$

In general case the parameters a, b are scalar functions from deformations but below the ones are taking as constants. Owing to kinematical restrictions not always this is possible for b-coefficient

There are two essential differences with standard approach.

Upper space derivatives are absent here because in field theory both time and space derivatives are symmetrical. The standard model contain upper space derivative.

Because parameter k is not constant the pseudo-fourvector part of field equation in general case is not zero as it is must be for fundamental fields. Here the symmetry of lagrangian at time reversing (in relativistic field's model the time and space inverse are indivisible) and the energy dissipation are not in contradiction.

5.4 Equations of stream

For passing to three-dimension equations of motion it is need to take the absolute time, then multiply basic equation on matrix e_0, and put together terms with equal O3 properties. It is convenient denominate
$$p_c = -c^2 m(1 + \nabla \cdot \vec{\xi}) = (p - c^2\rho)\sqrt{1 - \frac{v^2}{c^2}}$$
where the note of the hydrostatic pressure p is clear. In result the equations of stream are appearing as following (c=1)
$$\partial_t p_c + \nabla \cdot (m\vec{v}) = a\rho v^2$$
$$\partial_t(m\vec{v}) + \nabla p_c + \nabla \times (m\vec{h}) = -a\rho(\vec{v} + \vec{h} \times \vec{v})$$
$$\nabla \cdot (m\vec{h}) = -b\rho\vec{v} \cdot \vec{h}$$
$$\partial_t(m\vec{h}) - \nabla \times (m\vec{v}) = b\rho\vec{h}$$
$$m = \rho\sqrt{1 - v^2}$$
From Le Chatelier principle the constant a is positive and the constant b is negative numbers. Usual boundary conditions are valid with one supplement. Because any mass move with finite speed any singular solutions for velocity must be rejected. For example if the axial symmetrical stream have singularity $v = q\ ln(\sqrt{x^2 + y^2})$ with q=constant then self energy of this state is finite but because at small distances the velocity growth to infinity we must put q=0. Other essential difference with usual model is that in general case a stream is not continual. When velocity of stream reach value of the sound velocity then phase of quantity m is changing
$$\rho\sqrt{1 - v^2} \rightarrow i\rho\sqrt{v^2 - 1}$$
and we are going in area which is space-like for mechanical but time-like for fundamental interactions. Then for example the three-vector's equation in space-like area is
$$\partial_t(m\vec{h}) - \nabla \times (m\vec{v}) = -a\rho(\vec{v} + \vec{h} \times \vec{v})$$
Both intuitively and formally, this is abeyance area because here is solution $f = 0$ which is trivial for usual zone but unexpected for space-like area.

The mass conservation low is valid anywhere if relative velocities in stream are less essentially of the light velocity. Remark once again that density of mass is external parameter.

Because here always sound's velocity is constant this is somewhat unusual thermodynamic situation but it is real in many cases. The not thermodynamic but field's approach was using, however the scalar equation after integration give the thermodynamic connection.

At once, the $m\xi$ but not ξ itself exist in natere so initially the $m\xi$ need take as potential of mechanical field. Here we use simplest approach and it is acceptable if the kinematic connection between $h,\ v$ discard, it is replaced by 1^+ equation for observables. Other alternative in this case is to put $h \equiv 0$ or $b \equiv 0$ but this mean that transverse to velocity forces do not exist in fluid. Of course, this is first simplest step in construction of field model for fluid.

Regard few simple streams for example.

5.5 Examples of stream

The simplest system is ideal fluid. In this case all internal interactions may be neglecting and equations for extension pressure and impulse are isolating wave equations

$(\partial_t^2 - \Delta)p_c = 0$

$(\partial_t^2 - \Delta)m\vec{v} = 0$

Next approach is semi-ideal fluid. This means following. The three vector's part of internal forces have clear physical sense. Such quantity $-a\rho\vec{v}$ is force of internal frontal self-friction. The quantity $a\rho\vec{v} \times \vec{h}$ is force of internal lateral self-friction. These forces exclude stationary stream. However, if external forces are acting by such way that the coefficient of frontal friction effectively is zero then stationary motion is possible. In this case in usual zone the equations of the stationary stream are following

$\nabla\rho\vec{v} = \nabla m\vec{v} = 0$

$b\nabla p_c = \nabla \times [(\sqrt{1-v^2})\nabla \times (m\vec{v})]$

and even at small velocities the ones do not coincide fully with Navier-Stocks equations. For axial symmetrical motion in tube along z-axis let us take

$p_c = zp_1(r)$

$\vec{v} = v(r)\vec{e}_z$

$r^2 = x^2 + y^2$

Then in time-like area

$$p = \rho + \frac{zp_2}{\sqrt{1-v^2}}$$

$$v(1 - \frac{2}{3}v^2) = -\frac{1}{4}bp_2r^2 + t_1 + t_2 lnr$$

where p_2, t_k are integration constants. If velocity of stream is less of the sound velocity then it is need to put the last integration constant equal zero. This is nonlinear Poiseuille stream. If velocity of stream reach sound's velocity on surface $r = r_0$ then in area $r > r_0$ is more complicated Poiseuille stream because here the last integration constant may be not zero. This deviation from the Poiseuille low may be checking and it has immediate interest for technical applications.

What is happen in abeyance area is more interesting to regard in case of atmosphere whirls. For axial symmetrical motion in atmosphere at first step neglect by weight and Coriolis forces. Then

$\vec{v} = v(r)\vec{e}_\varphi, \quad p_c = p_c(r)$

$\nabla \times [\sqrt{1-v^2}\nabla \times (m\vec{v})] = 0$

In usual area for simplicity let us neglect change of density. Then in this zone equation for velocity is

$$v'(1-2v^2)+\frac{v}{r}(1-v^2)=g=\frac{1}{a}=constant$$

The velocity must be finite then in case $g\neq 0$ the whirl has finite size. It is fully concentrated in area $r\sim a$.

If velocity $v\rightarrow 1$ at surface $r=r_0$ then near dividing surface in times area $v=\pm 1-q(r-r_0)$

When this is internal area of stream the velocity increase at moving to small distances and solution in the space-like area is

$\rho v\sqrt{v^2-1}=\frac{q_1}{r}=0$

where integration constant q_1 must be zero because velocity must be finite and here it is no necessary regard density as constant. There are three solutions: or $v=1$, or $\rho=0$, or $v=0$. First solution is not stable, second solution is not stable from physical reason. But similar not stable objects are observable in whirls as tower of dust above hard surface or as waters loom above sea surface. The solution $v=0$ is stable and in this case in internal area of whirl the medium is immovable. Such behavior of the atmosphere whirls is observable in strong cyclones. But internal area may be zone of usual stream because velocity may be not continual. Then whirl in whirl structure may be observable and some of ones may have zero velocity. If external area of stream is space-like (anticyclone) then picture is less exotic. It is hard to comprehend such phenomena in usual model of fluid.

In Newtonian mechanics any interaction spread with infinite velocity and it is well approach if velocity of particles is more less than light's velocity. But in fluid dynamics certainly it is need take into account field's conception of point to point spreading of the mechanical interaction with velocity of sound. This may be make up or using field's theory tools as it is done in this article or using methods of relative mechanics or by other way but this need to do.

6 Potentials of pionic field

6.1 Extended Yukawa potential

Here the classical potential of pionic field is constructed using usual lagrangian formalism.

In nucleus the nucleons are near mass shell. Then in low energy area it is possible regard the nucleon as moving in potential well which is generated by cloud of virtual particles. The pions give main contribution to nucleon's interaction with cloud of virtual mesons. Correspondingly, the pionic field is main part of strong interaction in nucleus. Simplest potential of pionic field is Yukawa potential

$$p\sim\frac{1}{R}exp(-mR)$$

where m is mass of free pion.

Both from experiment and from theory this potential is not valid at small distances. From data the nucleus forces do not grow in center of field. In theory the self energy of any field must be finite. Then at phenomenological description of nucleus the potentials of Woods-Saxon or Bonn-Paris type are using.

It is need extend the Yukawa potential in area of small distances. This may to do just as it was making in previous section with potential of electromagnetic field.

The first step is regarding the cloud of virtual particles as continual state. This is possible because in general case any physical field have non zero density of mass. Hence in general case exist additional parameter $u(x)$ which is local fourvelocity vector of virtual medium. The pion in cloud is far off mass shall and for moving a pion in free space it is need to spend the energy no less of mc^2. Therefore for pionic cloud $u^2 < 0$. Of course, this condition is valid in many other cases. And here we will regard both the potential and local fourvelocity vector as essential parameters of pionic field.

The next step is regarding of fourvelocity parameter as potential of w-field. The virtual inertial forces will prevent tightening of pionic cloud and nucleon in point with infinite self energy.

For quantitative description of virtual pionic cloud in classical physics it is need to build the lagrangian of field. The pion is pseudoscalar isovector particle then pionic field have these properties. Simplest lagrangian of pionic field with its shadow w-field take as following (always using Clifford's algebra)

$$L \sim \frac{1}{2}(Dp)^2 + \frac{k^2}{2}(Du)^2 + \frac{g_1}{2}p^2 + g_2 p(u \cdot D)p$$

$$D = \gamma_0 \partial_0 - \gamma_n \partial_n$$

Here first two terms are lagrangians of free pionic and w-fields. For usual interactions the square form of ones is almost axiom.

Third term is simplest self-interaction lagrangian for pionic field. It is widely known and using. However, as it was seeing with Coulomb field, the potential of pionic field may be not zero on infinity.

Last term is interaction lagrangian for pionic and w-field. This form of one is possible because the quantity $D \cdot (up^2)$ effectively is zero. Remark that we neglect by possible term in lagrangian $\sim u^2$ because the self interaction of w-field at least in area of small energies look too exotic.

The variation of this lagrangian is following

$\delta L \sim (Dp)(D\delta p) + k^2(Du)(D\delta u) + g_1 p\delta p +$
$+ g_2 \delta p(u \cdot D)p + g_2 p(\delta u \cdot D)p + g_2 p(u \cdot D)\delta p$

Using standard way of variation formalism we get field's equations as following

$$D^2 p = g_1 p - g_2 p D \cdot u$$

$$k^2 D^2 u = \frac{g_2}{2} D p^2$$

For static spherical symmetrical field

$$p = p(R), \ \ ue_0 = u(r)\vec{e}_R$$

and equation for pionic potential after velocity parameter excluding is

$$p'' + \frac{2}{R}p' = -g_1 p + cp^3$$

With boundary condition $p(0) = 0$ the equation in area of small distances have no analytical solutions and we reject this equation.

Due to coherence condition here exist other solutions of variation task. The solution similar to Glauber state is following (for velocity the equation is without changing)

$$D^2 p = g_1 p + g_2(u \cdot D)p$$

with restriction on interaction constants as coherence condition

$$\int p(u \cdot D)\delta p d^4x = 0$$

Here all function after interaction sign are solutions of upper system of differential equations. So

$$\delta p = \frac{\partial p}{\partial c_n}\delta c_n$$

where c_n are integration constants. If the ones are fixed numbers ('charges') then coherence condition is automatically fulfill. For simplicity below the last case of coherence condition is using.

The equations for static spherical symmetrical field are following

$$p'' + \frac{2}{R}p' = -g_1 p - g_2 u p'$$

$$u' + \frac{2}{R}u = cp^2$$

In lower equation we take into account that self energy of any field must be finite and for simplicity the pionic potential is taking vanishing on infinity. After excluding of velocity parameter the equation for pionic potential is following

$$p'' + \frac{2}{R}p' = -g_1 p + \frac{p'}{R^2}(s_1 + s_2 \int_0^R p^2 R^2 dR)$$

Now linear equation has physical solution at small distances. On big distances it is need to take into account also nonlinear term. From physical reason at approximating searching of pionic potential it is natural to use as first input the potential of free field. The one exist with potential

$$p_f = k_1 + \frac{k_2}{R}$$

Once again the self energy of any field must be finite then it is need to put last integration constant equal zero.

In this way approximating equation for potential of pionic field is following

$$p'' + \frac{2}{R}p' = -g_1 p + p'(\frac{c_1}{R^2} + c_2 R)$$

with solution

$$p \sim exp(-\frac{a}{R} + \frac{g_1}{6}R^2)$$

At first look the constant g_1 determine potential's behavior on infinity. At small distances the constant a, which is scale of free w-field, set up the activity of potential. This is potential well but it is imaginary well because the pionic field is pseudoscalar field. It is simply visible when Clifford's algebra is using. In this case the coordinate vectors are Pauli matrixes. In space algebra the matrix

$$\vec{e}_1\vec{e}_2\vec{e}_3 = i1$$

change sign at parity transformation. Hence imaginary unit of the complex number algebra at that time is pseudoscalar of space algebra.

Remark that in spacetime Clifford's algebra two pseudoscalars exist (or two different imaginary unity if we wish). Then in general case the pionic potential is mixture of usual $\sim i$ and chiral $\sim i_c$ pseudoscalars.

This extended Yukawa potential may be useful in area of low energy nuclear physics.

6.2 Shell model for light nuclei

Regard simplest shell model (the pointlike nucleon is moving in classical field) using extended Yukawa potential. All essential features of one are generating by free w-field. So put the constant of self-interacting pionic field $g_1 = 0$. This not mean that pion's mass is zero because in this model any particle is not local field's object which has non zero size and mass of particle is integral from tensions and potentials of field. Switching off the pure self interaction of pionic field contradict with common approach but it has hard physical ground because pure field's self interaction and third Newtonian low (the impulse conservation low) are not in harmony. Additionally, existence of bound states in spherical symmetrical potential which decrease on infinity faster than R^{-2} is doubtful when asymptotical boundary conditions are applying (another arguments of one are in [6]).

In this case the exact vacuum solution exist

$$p \equiv constant, \;\; R^2 u \equiv constant$$

and in low energy area the not constant part of pionic potential may be regarding as small perturbation of vacuum potential. On whole R-axis it always will be true. For simplicity we restrict a potential by condition $p(0) = 0$ and will regard the one as pure chiral pseudoscalar. Then we get potential of pionic field as following

$$p = i_c \, \vec{\tau} g \; exp(-\frac{a}{R})$$

More than hundred years is recognizing that particles interact with fields but not with themselves. Therefore, this pionic field regard as external field whose parameters depend from number of nucleons in nucleus. At big distances behavior of this potential is similar to Coulomb field. At small distances this potential quickly disappears. This mean that pionic forces act only on nucleons which are near surface $R = a$. In other words these are surfacing forces. The parameters of pionic field depend both from neutron and proton numbers in nucleus. For simplicity below we regard only nuclei which contain equal number of protons and neutrons. So proton's number $2Z = A$. Then

$$a \sim (A-1)^{\frac{2}{3}}$$

with may be additional slow dependence from mass number. We take into account the simplest kinematical multiplier and put

$$Ga = k\frac{(A-1)^{\frac{5}{3}}}{A} \qquad k = constant$$

In this way we take into account essential properties of pionic field generated by free w-field.

For simplicity neglect by neutral pions - this is more general restriction than equality of proton and neutron numbers. In addition, we neglect by electromagnetic interaction of nucleons. Then proton's and neutron's wave functions differ only by constant phase.

The Dirac equation initially is grounding on Clifford's algebra and including interaction of the fermion with having any algebraic structure field have no problems. The equation for wave function of nucleon in pionic field is following

$$(iD - i_c V)\Psi = M\Psi$$

$$V = Gexp(-\frac{a}{R})$$

$$2Z = A, \hbar = 1, c = 1$$

For stationary states with energy E the equation for upper part of wave function is

$$(\Delta + V'\vec{e}_R)\Psi_{up} = (M^2 - E^2 + V^2)\Psi_{up}$$

Obviously, this wave function is not simple factorisable term, it is sum of two terms with different algebraic structure, and differential equations for radial parts of wave function has forth order. The term $V'\vec{e}_R$ by their physical meaning represent spin - orbital interaction and from data the one may be discarded for light nuclei. In this case for radial part $F(R)$ of nucleon wave function we have usual Schroedinger equation

$$F'' + \frac{2}{R}F' = [\frac{l(l+1)}{R^2} + M^2 - E^2 + V^2]F$$

For free motion of nucleon his energy $E^2_{free} = M^2 + G^2$ so pionic field eat up the part of free nucleon mass. At that time for free nucleon's motion $E_{free} = m_N$ where m_N is mass of free nucleon. Correspondingly, the bound energy of nucleon in this field determinate as $\varepsilon = m_N - E,\ \varepsilon > 0$. Then boundary conditions connect the bound energy with other parameters

$$\varepsilon(A,N,l) = \frac{G^2}{2m_N}\frac{G^2a^2}{(N+1+B)^2}$$

$$G^2a^2 = k^2\frac{(A-1)^2}{A^2}(A-1)^{\frac{4}{3}}$$

where N is number of wave function zeros. The coefficient $B(A,N,l,Ga)$ must be calculated so we have two unknown parameters. These are depth $\frac{G^2}{m_N}$ and constant k in slope Ga of potential well.

At searching B-coefficient it is need to use asymptotical boundary conditions. This method step by step take into account the terms $(\frac{2a}{R})^n$ in decomposition of potential on big distances. But in this way we have no visible expression for bound energy at all quantum numbers. Then for simplicity let us replace $V^2 \to G^2(1 - \frac{2a}{R} + \frac{2a^2}{R^2})$ what take into account surfacing nature of pionic field and regard these singularities as real. Remark that commonly and anywhere at calculations with Coulomb-like field his singularities are regarding as real quantities without warning.

With these approximations the B-coefficient is following

$$B = \frac{1}{2}\left[-1 + \sqrt{(2l+1)^2 + 8G^2a^2}\ \right]$$

At first approach the minimization of unknown parameter numbers is more useful than exact fitting of binding energy. From usual shell model we know that for heavy nucleus the lowest bound energy of nucleon is near 40MeV. So we put

$$\frac{G^2}{2m_N} = 80MeV$$

Using the deuteron as caliber - but not exactly - we find

$$k^2 = 0.055$$

Evidently, interaction's energy of nucleon with pionic field is overestimated. Additionally, the electromagnetic forces are not accounting. So we may expect that calculating levels of energy will exceed observable numbers. From this reason we take the value 1MeV as separation energy for deuteron.

In this way the expression for energy levels of nucleon in this pionic field is following

$$\varepsilon(A,N,l) = \frac{(A-1)^{\frac{10}{3}}}{A^2}\left[2N + 1 + \sqrt{(2l+1)^2 + \frac{0.44(A-1)^{\frac{10}{3}}}{A^2}}\right]^{-2} \cdot 17.6MeV$$

As in any model with spherical symmetrical potential without spin-orbital interaction the nucleon state depend from number of wave function zeros N and orbital moment l. The states may be numerated as (N, l). From correspondence principle the restriction $l \le N$ need to put into play for stationary state of nucleus. It is quit noticeable that changing of zeros number create bigger shift of levels than changing of orbital moment. This mean the quantity N but not N+l+constant is main quantum number of levels. Then we denominate the basic state in shell as $(N, 0)$ with subshells as $(N, 0 < l \le N) + (N-1, 0 < l \le N)$. These states are enough for placement of light nuclei.

Additionally, let us pair off the nuclei:

$$^2H,^4He;\ ^6Li,^8Be;\ ^{10}B,^{12}C;\ ^{14}N,^{16}O;\mathrm{etc}$$

In any pair the first nucleus have unpairing nucleons. From data at passing from last nucleus in pair to first nucleus in next pair the binding energy per one nucleon downfall. In any shell model this mean the new shell or subshell is opening at going from pair to pair. Or may be changing subshell's filling is happing. The last case seems doubtful and we reject it. Also propose that additional nucleons are jointed to shell with unpairing nucleons at passing from first to second nucleus in pair.

The state (0, 0) contain $^2H,\ ^4He$ pair with binding energy per one nucleon for deuteron (2,0,0)=1MeV and for alpha particle (4,0,0)=7.2MeV.

Next shell (1,0) contain subshells (0,1) and (1,1). The $^6Li,\ ^8Be$ pair have lower position with (1,0)+(0,1) subshells.

For ^6Li the energy levels are (6,0,1)=5.38MeV and (6,1,0)=4.35MeV. The unpairing nucleons have lower bound energy so in ^6Li are two nucleon with l=0 and four nucleons with l=1. The binding energy per one nucleon is 5.0(5.3)MeV/A. In bracket are observable numbers taking from [7]. But it may be all nucleons have l=1. Placing the nucleons on subshells is problem because no one possibility exist.

The ^8Be levels are(8,0,1)=8.25MeV and (8,1,0)=6.3MeV with four nucleons on each subshell. The binding energy is 7.27(7.06)MeV/A.

Next shell (1,0)+(1,1) contain $^{10}B,^{12}C$ pair.

The levels of ^{10}B are (10,1,0)=8.0MeV and (10,1,1)=5.5MeV. There are four nucleons with l=0 and six with l=1. The binding energy is 6.5(6.47) MeV/A.

The ^{12}C levels are (12,1,0)=9.5MeV with four nucleons and (12,1,1)=6.9MeV with eight particles. The binding energy is 7.75(7.68) MeV/A.

Numerical results are well enough for such rough approximations so let us make the nuclei levels calculation more exactly. The deuteron and alpha particle take as caliber. In alpha particle the electrostatic interaction between protons take as usually

$$\varphi = \frac{e}{R}$$

Then we find

$$\frac{G}{2m_N} = 73.67MeV,\ k^2 = 0.06487$$

For light nuclei it is need take into account polarization effects for electrostatic interaction - see for example the end of subsection 4.3. Because polarization effects partly compensate the pure Coulomb forces below the electrostatic interaction is neglecting fully. Even so the polarization effects partly are taking into account because the adapting of alpha particle levels was done exactly.

For states with N=0 the previous expression for levels calculation is valid. For N=1 the B-coefficient is set up by connection

$$3(B+2)\left[B^2+3B+2-l(l+1)+2G^2a^2\right]\left[B^2+B-l(l+1)-2G^2a^2\right] = 8G^4a^4$$

If solutions of this equation are not unique then correspondence principle need to use for separation of physical solutions. For next N-states the set up equations are more complicated - see subsection 4.4.

In this way the structure of light nuclei is following (all energies are in MeV and in bracket are observable value [7]).

In ^6Li nucleus on shell (6,1,1)=5.618 are four and on shell (6,1,0)=4.419 are two nucleons. The binding energy of this nucleus is 31.51(31.99).

In ^8Be nucleus are states (8,0,1)=8.566 and (8,1,0)=6.244 with four nucleons in each state. The binding energy of this nucleus is 59.24(56.49).

In ^{10}B nucleus are four nucleons on shell (10,1,0)=7.775 and six nucleons on shell (10,1,1)=5.635 with binding energy of nucleus 64.9(64.70).

In ^{12}C nucleus are four nucleons on shell (12,1,0)=9.063 and eight nucleons on shell (12,1,1)=6.928 with binding energy of nucleus 91.67(92.16).

For next nuclei the new basic shell (2,0) need to open with nearby subshells (2,1), (2,2), (1,1), (1,2).

In ^{14}N nucleus the states energies are: (14, 2, 0)=6.025, (14, 2, 1)=5.069, (14, 1, 1)=8.083, and (14, 1 2)=5.835. Because the states (2, 0) and (1, 2) have almost equal energies it will be true the unpairing nucleons set up to (2, 1) subshell. When two nucleons are on (2, 1), four on (2, 0) and eight on (1, 1) subshells then binding energy of this nucleus is 98.9 (98.73).

In ^{16}O the levels energies are:(16, 2, 0)=6.747, (16, 2, 1)=5.804,(16, 1 1)=9.129 and (16, 1, 2)=6.789. And here the states (2, 0) and (1, 2) have almost equal energies. Because all nucleons are pairing we put four nucleons on (1, 2), four on (2, 0) and eight on (1, 1) subshells. The binding energy of this nucleus is 127.74 (127.6).

For pair $^{18}F,\ ^{20}Ne$ it is need to replace the (1, 1) subshell with biggest energy to (1, 2) subshell.

Then in ^{18}F nucleus the levels are:(18, 2, 0)=7.402 with four , (18, 1, 2)=7.682 with twelve and (18, 2, 1)=6.48 with two nucleons. The binding energy of this nucleus is 134.752 (137.369).

In ^{20}Ne nucleus the levels energies are:(20, 2, 0)=8.001 with four, (20, 1, 2)=8.524 with twelve, (20, 2, 1)=7.104 with four nucleons. The binding energy of this nucleus 162.708 (160.644).

Of course, in this model many states exist and placing of nucleons on subshells is matter of opinion and we will do not regard next nuclei.

Additionally, let us regard lower excitation levels in ^{6}Li. From physical reason the one nucleon transitions without orbital moment change are long-living compare with transitions when orbital moment change. Similarly, when two nucleons both go on the same upper state we expect bigger time of life compare with situation when the nucleons go to different subshells.

Lower empty subshells in 6Li are (6,1,1)=2.693 and (6,2,0)=2.257. Then here are following long-living transitions

$(1,0) \to (2,0) = 2.162$
$2(0,1) \to 2(1,0) = 2.498$
$(0,1) \to (1,1) = 2.975$
$2(1,0) \to 2(1,1) = 3.452$
$2(1,0) \to 2(2,0) = 4.324$
$2(0,1) \to 2(1,0) = 4.996$

Moreover, because for these nuclei is (p,n) symmetry we may expect that one particle passing is suppressed compare with p+n transition. Observable [8] are states with excitation energy 2.186, 3.562, 4.312, 5.366.

Excitation energies are not in well agreement with data but these numbers are accepted. Here are somewhat disagreements with customary model. The lowered state is state with l=1 but not with l=0 and four but not two nucleons have l=1. Moreover, additional excited long living and variety of short living states are here. These are possible to check in precise experiment.

Numerical results are satisfactory and may be useful for nuclear physics.

At that time these results give additional support for rejecting of pure field's self-interaction for any field. In whole, that without relations with other field the pure self interaction of field is source of unphysical value of field energies and that interaction with other field cut down the ones is quite noticeable.

7 Free gluonic field

By their physical meaning the gluonic field is primary field that binds quarks in particle. Similarly to electromagnetic field this is fourvector field with gradient's symmetry and fourpotential $G(x) = G_0\gamma_0 + G_n\gamma_n$ which is restricted by Lorentz gage condition.

Physical differences with electromagnetic field are following. Electromagnetic field may exist in three forms: as charged or as magnetic field or as transverse waves. Gluonic field always exist as charged field. On infinity the charged electromagnet field disappears and on small distances both linear and nonlinear Coulomb potentials have singularity. Gluonic forces do not vanished on infinity and their potential has no singularities in center of field. As for any physical quantity these properties of gluonic field and field's existence itself are grounded on data and their theoretical explanations.

Here we regard the gluonic field as classical object in static spherical symmetrical state. So potential of one is $Ge_0 = g(R)$

Simplest state of any field is free field. In classical physics, the lagrangian of free field always is square form from field's tensions. If we take such lagrangian

$$L_0 \sim (\nabla g)^2$$

then we get Coulomb-like potential

$$g(R) = g_1 + \frac{g_2}{R}$$

which has infinite self energy on small distances. Due the asymptotical freedom of strong interaction just the free gluonic field act in area of small distances so Coulomb-like potential is not potential of free gluonic field. We must accept that lagrangian of free gluonic field have more complicated form. If the one depend only from field's tension then another, not Coulomb-like, solution of variation task exists. It is

$$(\nabla g(R))^2 = constant$$

and it have no matter how complicated is lagrangian of free field. In this case the potential of free gluonic field is following

$$g(R) = g_1 + g_2 R$$

where constant g_2 determinate scale of strong forces. It is fundamental quantity similarly to electric charge in electromagnetic interaction. Remark that this is not faultless because it is general result and the unknown field may exist.

As application example, regard the bound states of particle in this field. Gluonic field tie up only quarks but for simplicity regard a scalar particle in this field. For particle with mass m, energy E, and orbital moment l the Klein-Gordon equation for radial part of wave function is following

$$F'' + \frac{2}{R} = \left[\frac{l(l+1)}{R^2} + m^2 - (E - g_0 - bR)^2 \right] F$$

Effective potential energy goes to minus infinity on big distances so this field is unrestricted source of kinetic energy. If we do not believe in existence of one then virtual inertial forces need take into account.

Yet we regard pure free gluonic field. Particle's wave function takes as follow

$$F(R) \sim R^l \prod_0^N (R - R_n) exp(AR + \frac{1}{2} BR^2)$$

Here restriction $ImE < 0$ is needing because the full wave function contain multiplier $exp(-iEt)$.

The solutions exist if condition

$$m^2 = -ib(2N + 2l + 3)$$

is valid. So in these field only resonances may to be and square of their mass have linear dependence from own spin.

Such connection between mass and spin of resonances from experiment know few ten years. Firstly the Redge-pole than the string model of strong interaction was set up taking this connection as base [9]. However, here simplest tools of semi-classical physics enable to sight on high-energy physics phenomena.

8 Clifford's algebra

It is adding for reader that is not familiar with this algebra.

Any algebra is richer variety than vector space. In algebra the sum and multiplying of elements with different algebraic structure are defining.

Is it possible extension of the vector space variety to algebra? W. K. Clifford give the answer in 1876 year. For this doing it is enough regard a coordinate vectors as matrixes.

In physics, the space algebra L3 and the spacetime algebra L4 are essential. Let us regard their properties briefly. Remark, from relativity principle it have no matter which coordinate system is using. But it became as standard to divide a vectors on components. Which troubles this dividing create easy is seeing on example switching interaction of the electron with external magnetic field in quantum mechanics. We avoid such way. Then a flat coordinate system is using in general case. Only when numerical calculations are doing the suitable coordinates are taking.

In space algebra the coordinate orts $\vec{e}_n$ are two dimension matrixes of Pauli with properties

$\vec{e}_n = \sigma_n$

$e_n^2 = 1$ n=1, 2, 3

$\vec{e}_1\vec{e}_2\vec{e}_3 = i1$

The last matrix change sign at parity transformations so imaginary unite of the complex numbers algebra at that time is pseudoscalar of space algebra. Then general element in the space algebra is sum of scalar, pseudoscalar, vector, and pseudovector.

The gradient operator in L3 algebra is following

$\nabla = \vec{e}_n \partial_n$

Few examples of calculations in space algebra.

$\vec{a}\vec{b} = a_n b_k \vec{e}_n \vec{e}_v = \vec{a} \cdot \vec{b} + i\vec{a} \times \vec{b}$

$\nabla(\vec{a}\vec{b}) = (\nabla\vec{a})\vec{b} - \vec{a}(\nabla\vec{b}) + 2(\vec{a} \cdot \nabla)\vec{b}$

$\nabla R^N \vec{e}_z = NR^{(N-1)}\vec{e}_R\vec{e}_z = NR^{(N-1)}(cos\theta - isin\theta\vec{e}_\varphi)$

In the algebra of spacetime the coordinate vectors u_n are four dimension matrixes of Dirac with properties

$u_0^2 = 1$ $u_k^2 = -1$ k=1, 2, 3

$u_0u_1u_2u_3 = i_c$

The last matrix change sign at parity transformations and the standard is to denominate the one as $i\gamma_5$ matrix. We use almost denominate of G. Casanova because here are two pseudoscalar which coincide at passing to space algebra. Remark that existence of two pseudoscalar i, i_c in the spacetime algebra is missing as implicit standard. De facto space and spacetime algebras are complex varieties. General element in spacetime algebra is sum of scalars, pseudo scalars, fourvectors, pseudo fourvectors and bevectors. If A, B are two fourvectors then bevector F is external multiply of ones

$F = A \wedge B = \frac{1}{2}(AB - BA)$

The matrix $e_n = u_nu_0$ just is four dimension representation (anti-diagonal) of Pauli matrix. Then any bevector have other form

$F = \vec{V} + i_c\vec{H}$

where $\vec{V}$, $\vec{H}$ are the space vectors in four dimensional representation. This property makes easy the crossing between the space and the spacetime algebras.

The gradients operator in L4 algebra is following

$D = u_0\partial_0 - u_n\partial_n$

$\partial_0 = \frac{1}{c}\partial_t$

With common convention about the phases of physical quantities the operator of fourimpulse is

$$\hat{p} = i\hbar D$$

Few examples of calculations in this algebra

$DA = Du_0u_0A = (\partial_0 - \nabla)(A_0 - \vec{A}) = D \cdot A + D \wedge A$

$D \cdot A = \partial_0 A_0 + \nabla \cdot \vec{A}$

$D \wedge A = -\partial_0\vec{A} - \nabla A_0 + i_c\nabla \times \vec{A}$

For more details see any textbook on Clifford's algebra but it is need to read the ones critically.

9 As summary

This article contains few news. The coherence condition and direct solution of Dirac equation are technical tools. The w-field conception is physical assumption and it is working. Remark that any field has this w-field as shadow. We may regard this model as the description of virtual states in classical physics, especially if the local fourimpulse vector take as potential of w-field. However, in quantum field theory the local fourimpulse is variable of integration but not field's potential.

This extension of classical field theory after it is make became almost trivial in case of electromagnetic field. For nucleus forces in simplest case the situation is even more simple than for electromagnetic field. However for mechanical continual state the model is not extension but another way and by this or other manner this need to do because the interaction travel with finite velocity.

Also in general case the density of energy in any physical field is not zero, so one more shadow, however scalar, field may exist. Therefore, no one feedback may be in any physical field. J guess these will have interest for physicist and will be useful.

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